Econ 780 HW Week 2

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Due:

Exercises for chapter 3

1. Exercise 1

2. Exercise 2.
   Consider augmented the matrix.

   $\overline{D} = \begin{bmatrix} 1 & 1 & 2 & 3 \end{bmatrix}.$

   There is no arbitrage. since there only 1 matrix, so the only choice to make the value of the profolio is zero in day zero is buy nothing, but it will turnout that the value of the porlio at day 1 will be zero. So there will be no arbitrage opportunity.

   let $Q(w_1), Q(w_2), Q(w_3)$be Martingale measure for the market.
   We have the formular.

$$1 = \beta(Q(w_1 + 2Q(w2) + 3Q(w_3))$$

   and

$$Q(w_1) + Q(w_2) + Q(w_3) = 1$$

   .
   For some β
   For simplicity, I will use Q_1, Q_2, Q_3 instead of $Q(w_1), Q(w_2), Q(w_3)$

$$1 = \beta(Q(w_1 + 2Q(w2) + 3Q(w_3)) \implies \frac{1}{\beta} = Q_1 + 2Q_2 + 3Q_3$$

   Let write $Q_1 = 1 - Q_2 - Q_3$ and substitiue to the equation above, we obtain:

$$\frac{1}{\beta} = 1 - Q_2 - Q_3 + 2Q_2 + 3Q_3 = 1 + Q_2 + 2Q_3 \implies Q_2 = \frac{1}{\beta} - 1 - 2Q_3$$

again, substitute $Q_2 = 1 - Q_1 - Q_3$ Obtain

$$\frac{1}{\beta} = Q_1 + 2(1 - Q_1 - Q_3) + 3Q_3 = 2 - Q_1 + Q_3 \implies Q_1 = 2 - \frac{1}{\beta} + Q_3$$

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Now we will find the bounds for Q_2 to be legitimate probability measure.

- $0 < Q_1 < 1 \implies 0 < 2 - \frac{1}{\beta} + Q_3 < 1 \implies \frac{1}{\beta} - 2 < Q_3 < \frac{1}{\beta} - 1$
- $0 < Q_2 < 1 \implies 0 < \frac{1}{\beta} - 1 - 2Q_3 < 1 \implies \frac{\frac{1}{\beta} - 2}{2} < Q_3 < \frac{\frac{1}{\beta} - 1}{2}$
  Combine these two inequalities.
  $$\frac{1 - 2\beta}{2\beta} < Q_3 < \frac{1 - \beta}{2\beta}$$
  Hence the family of martingale measure can be writte as:
  $$\{2 - \frac{1}{\beta} + Q_3, \frac{1}{\beta} - 1 - 2Q_3, Q_3 \mid \frac{1 - 2\beta}{2\beta} < Q_3 < \frac{1 - \beta}{2\beta}\}$$
- Consider $\overline{D} = \begin{bmatrix} 1 & 1.2 & 1.2 & 1.2 \\ 2 & 2.4 & 2.5 & 2.4 \end{bmatrix}$.
  Let transfer the market to nomarlized market Z:
  $D^Z = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & \frac{2.5}{1.2} & 2 \end{bmatrix}$..

  Here asset 1 is risk-free asset with the interest rate i 0.
  let $\alpha_1, \alpha_2, \alpha_3$strictly positive and $\alpha_1 + \alpha_2 + \alpha_3 = 1$ the equation:
  $2 = 2\alpha_1 + \ frac2512\alpha_2 + 2(1 - \alpha_1 - \alpha_2) \implies \alpha_2 = 0$ a contradiction.
  hence the market is not arbitrage free.

  In fact, in the original market. Let consider the portfolio $h = (-2, 1)$, hence ${V_0}^h = -2 + 2 = 0$ and $P({V_1}^h \geq 0) = 1, P({V_1}^h > 0) > 0$ Q.E.D
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