Econ 780   HW Week 2

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Due:

Exercises for chapter 3

1. Exercise 1

2. Exercise 2.
   Consider augmented the matrix.

   $$\overline{D} = \begin{bmatrix} 1 & 1 & 2 & 3 \end{bmatrix}.$$

   There is no arbitrage. since there only 1 matrix, so the only choice to make the value of the profolio is zero in day zero is buy nothing, but it will turnout that the value of the porlio at day 1 will be zero. So there will be no arbitrage opportunity.

   let $Q(w_1), Q(w_2), Q(w_3)$ be Martingale measure for the market.
   We have the formular.

   $$1 = \beta(Q(w_1 + 2Q(w2) + 3Q(w_3))$$

   and

   $$Q(w_1) + Q(w_2) + Q(w_3) = 1$$

   .
   For some β
   For simplicity, I will use Q_1, Q_2, Q_3 instead of $Q(w_1), Q(w_2), Q(w_3)$

   $$1 = \beta(Q(w_1 + 2Q(w2) + 3Q(w_3)) \implies \frac{1}{\beta} = Q_1 + 2Q_2 + 3Q_3$$

   Let write $Q_1 = 1 - Q_2 - Q_3$ and substitiue to the equation above, we obtain:

   $$\frac{1}{\beta} = 1 - Q_2 - Q_3 + 2Q_2 + 3Q_3 = 1 + Q_2 + 2Q_3 \implies Q_2 = \frac{1}{\beta} - 1 - 2Q_3$$

again, substitute $Q_2 = 1 - Q_1 - Q_3$ Obtain

$$\frac{1}{\beta} = Q_1 + 2(1 - Q_1 - Q_3) + 3Q_3 = 2 - Q_1 + Q_3 \implies Q_1 = 2 - \frac{1}{\beta} + Q_3$$

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Now we will find the bounds for Q_2 to be legitimate probability measure.

- $0 < Q_1 < 1 \implies 0 < 2 - \frac{1}{\beta} + Q_3 < 1 \implies \frac{1}{\beta} - 2 < Q_3 < \frac{1}{\beta} - 1$

- $0 < Q_2 < 1 \implies 0 < \frac{1}{\beta} - 1 - 2Q_3 < 1 \implies \frac{\frac{1}{\beta} - 2}{2} < Q_3 < \frac{\frac{1}{\beta} - 1}{2}$
  Combine these two inequalities.

$$\frac{1 - 2\beta}{2\beta} < Q_3 < \frac{1 - \beta}{2\beta}$$

Hence the family of martingale measure can be writte as:

$$\{2 - \frac{1}{\beta} + Q_3, \frac{1}{\beta} - 1 - 2Q_3, Q_3 \mid \frac{1 - 2\beta}{2\beta} < Q_3 < \frac{1 - \beta}{2\beta}\}$$

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Solve for this system of equation, we can find the family of martingale measure.