Problem Set 1 - ECON 880

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Problem 1

Problem 1a

We can evaluate the polynomial as follow:

$$
\begin{aligned}
f(x,y) &= 83521y^8 + 578x^2y^4 - 2x^4 + 2x^6 - x^8 \\
&= y^4(83521y^4 + 578x^2) + x^4(-2 + 2x^2 - x^4)
\end{aligned}
$$

Problem 1b

Test

Problem 1c

test 2

Problem 2

In this exercise, we write an algorithm to determine the relative speeds of addition, multiplication, division, exponentiation, and the logarithmic function of our computer. The computer uses Intel(R) Core(TM) i5-1035G4 CPU @ 1.10GHz, 1.50 GHz with 64-bit operating system and 8.00 GB installed RAM. We proceed by generating two matrices A and B of size $10^4 \times 10^4$ using the rand() function in Matlab. Then, we use them in element-wise operations of addition $(A + B)$, multiplication $(A. * B)$, division $(A./B)$, exponentiation $(A.^B)$, as well as the logarithmic function $(\log(A))$. The statements `tic` and `toc` are used around them to measure the computation time. This procedure is iterated 100 times, and the average computation time for each operation is computed. The results are as follows:

- Average computation time for variable initiation is 1.54030 seconds
- Average computation time for addition is 0.12073 seconds
- Average computation time for multiplication is 0.12083 seconds
- Average computation time for division is 0.11823 seconds
- Average computation time for exponentiation is 3.70586 seconds
- Average computation time for log function is 0.91528 seconds

Problem 3

In order to find our machine ε, we follow Ken Judd's definition[†] by writing a while loop to subtract (resp. add) progressively smaller numbers ϵ from (resp. to) 1 until the condition $1 + \epsilon > 1 > 1 - \epsilon$ is no longer satisfied. Repeat the exercise using 0.001 and 1000 instead of 1. We verify our results by comparing them with the epsilons delivered by the built-in Matlab function `eps(x)`, and the exponents with `log2(eps(x))`. The results are shown on Table 1.

x	$\varepsilon(x)$ - decimal	$\varepsilon(x)$ - exponential
0.001	2.16840434497101e-19	2^{-62}
1	2.22044604925031e-16	2^{-52}
1000	1.13686837721616e-13	2^{-43}

Table 1: Machine ε for diverse values of x

Comment: As x increases, the machine $\varepsilon(x)$ also does increase. This is to be expected, since $\varepsilon(x) \approx x\varepsilon(1)$. Thus, $\varepsilon(1000) > \varepsilon(1) > \varepsilon(0.001)$.

Problem 4

We wrote a loop to evaluate the convergence of the following sequences:

(4a) $x_k = \sum_{k=1}^{n} \frac{1}{2^n}$, where $\lim_{k\to\infty} x_k = 1$

(4b) $y_k = \sum_{k=1}^{n} \frac{1}{n}$, where $\lim_{k\to\infty} y_k = \infty$

We use absolute and relative convergence criteria, and tolerance distance $\delta \in \{10^{-2}, 10^{-4}, 10^{-6}\}$ as stopping rule. We limit the maximum number of iterations to 100,000. The number of iterations before convergence, as well the final guess are reported on Table 2 and 3 for x_k and y_k, respectively.

δ	10^{-2}	10^{-4}	10^{-6}
no. of iteration - absolute criteria	7	14	20
final guess - absolute criteria	0.992187500000000	0.999938964843750	0.999999046325684
no. of iteration - relative criteria	7	14	20
final guess - relative criteria	0.992187500000000	0.999938964843750	0.999999046325684

Table 2: Convergence for x_k

δ	10^{-2}	10^{-4}	10^{-6}
no. of iteration - absolute criteria	100	10,000	100,000
final guess - absolute criteria	5.18737751763962	9.78760603604435	12.0901461298633
no. of iteration - relative criteria	100	10,000	100,000
final guess - relative criteria	5.18737751763962	9.78760603604435	12.0901461298633

Table 3: Convergence for y_k

[†]Kenneth L. Judd, 1998. "Numerical Methods in Economics," MIT Press Books, The MIT Press, p.30

Comment: Table 2 shows that both absolute and relative convergence criteria lead to the same number of iterations for the sequence x_k. As tolerance distance δ lowers, the final guess for x_k approaches its true limit 1. Table 3 also shows that both absolute and relative criteria lead to the same number of iterations for the sequence y_k. Since y_k is a divergent sequence, the final guess will be higher (with no upper bound), the more we iterate. By lowering tolerance distance δ, the algorithm needs to iterate longer in order to satisfy the stopping rule, which in turn results in higher final guess. Since there is no upper bound for y_k, we can always make the tolerance distance δ even lower, and obtain even higher final guess for y_k.