# Problem Set 6 - ECON 880

Spring 2022 - University of Kansas

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## Question 1

- The trapezoid is given by:

$$\int_a^b f(x)dx = \frac{h}{2}\left[f_0 + 2f_1 + \cdots + 2f_{n-1} + f_n\right] - \frac{h^2(b-a)}{12}f''(\xi)$$

- The Gauss-Chebyshev is given by:

$$\int_a^b f(y)dy \doteq \frac{\pi(b-a)}{2n}\sum_{i=1}^{n} f\left(\frac{(x_i+1)(b-a)}{2} + a\right)\left(1-x_i^2\right)^{1/2}$$

  where the $x_i$ are the Gauss-Chebyshev quadrature nodes over $[-1,1]$. Where the quadrature nodes are

$$x_i = \cos\left(\frac{2i-1}{2n}\pi\right), \quad i = 1,\ldots,n.$$

- The Gauss-Legendre is given by: A linear change of variables is necessary to apply Gauss-Legendre quadrature to general integrals. In general.

$$\int_a^b f(x)dx \doteq \frac{b-a}{2}\sum_{i=1}^{n}\omega_i f\left(\frac{(x_i+1)(b-a)}{2} + a\right),$$

  where the $w \quad i$ and xiq are the Gauss-Legendre quadrature weights and nodes over $[-1,1]$

- True value of the integral = 0.7500

| The errors for three methods | | | |
|---|---|---|---|
| | $n=3$ | $n=5$ | $n=11$ |
| Trapezoid | 0.081359 | 0.041771 | 0.01481 |
| Gauss-Chebyshev | 0.036964 | 0.012382 | 0.0024208 |
| Gauss-Legendre | 0.0051406 | 0.0015098 | 0.00020904 |

## Question 2

- True value of the integral = 34.920804

- It is clear that the pseudo-random error does not converge to zero, however it is bounded by 0.08. The ) uniformly spaced grid method converges to zero, however if the number of nodes are less than 3000, the pseudo-random method does better the uniformly spaced grid since the lower bound of the uniformly spaced grid method is 0.1.
- The pseudo-random is easier to implement than the uniformly spaced grid method

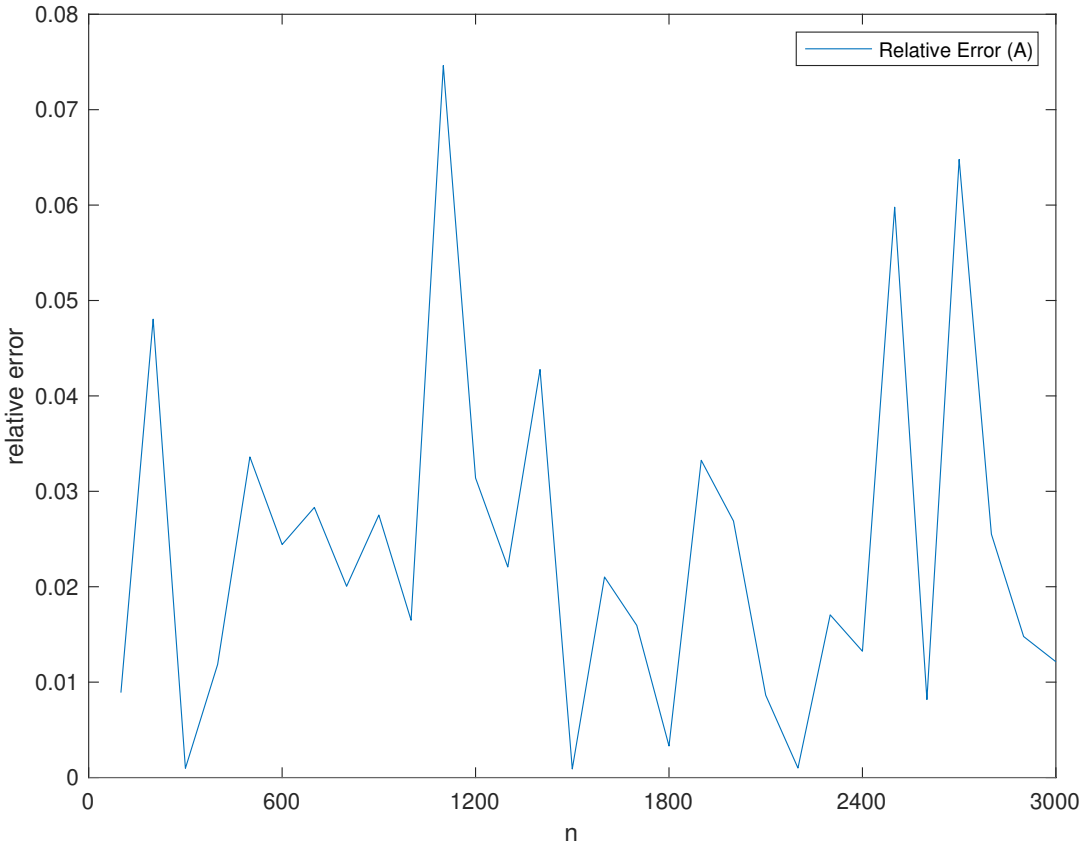


Figure 1: Pseudo-random error

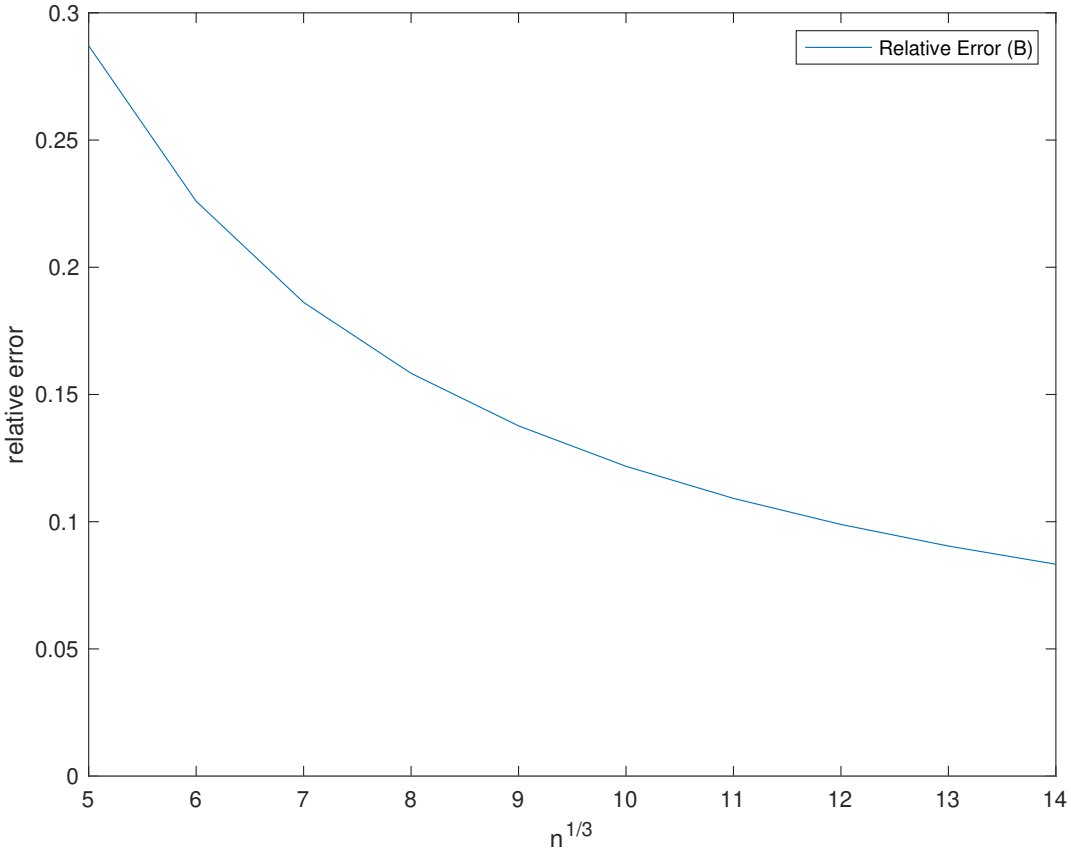


Figure 2: Uniformly spaced grid error