Problem Set 3 - ECON 880

Spring 2022 - University of Kansas

Gunawan, Minh Cao

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Problem 1

In this exercise, we are interested in solving $Ax = b$, where

$$A = \begin{pmatrix} 54 & 14 & -11 & 2 \\ 14 & 50 & -4 & 29 \\ -11 & -4 & 55 & 22 \\ 2 & 29 & 22 & 95 \end{pmatrix}, \quad b = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

using Gauss-Jacobi and Gauss-Seidel method. Both methods yield the same result

$$x = \begin{pmatrix} 0.0189 \\ 0.0168 \\ 0.0234 \\ -0.0004 \end{pmatrix}.$$

Gauss-Jacobi method required 0.0207 seconds with 45 iterations until convergence. The residual is given by

$$10^{-11} \times \begin{pmatrix} 0.0361 \\ -0.1211 \\ -0.0922 \\ 0.2351 \end{pmatrix}$$

Gauss-Seidel method required 0.0193 seconds with 23 iterations until convergence. The residual is given by

$$10^{-12} \times \begin{pmatrix} 0.1013 \\ -0.1849 \\ -0.1201 \\ -0.0002 \end{pmatrix}$$

Problem 2

In this exercise, we are interested in solving $Bq = r$ using extrapolation, where

$$B = \begin{pmatrix} 1 & 0.5 & 0.3 \\ 0.6 & 1 & 0.1 \\ 0.2 & 0.4 & 1 \end{pmatrix}, \quad r = \begin{pmatrix} 5 \\ 7 \\ 4 \end{pmatrix}.$$

Following Ken Judd's definition[†], we first define $G = I - B$, and run the following iteration

$$q^{k+1} = \omega G q^k + \omega r + (1-\omega) q^k,$$

where we pick $\omega = 1.05$, tolerance level 10^{-13}, and initial value $q_0 = (0,0,0)'$. The extrapolation converged after $k = 97$ iterations, with the residual

$$Bq - r = 10^{-12} \times \begin{pmatrix} 0.1670 \\ -0.2371 \\ 0.1279 \end{pmatrix}.$$

The solution to the linear equation system is

$$q = \begin{pmatrix} 1.6716 \\ 5.8651 \\ 1.3196 \end{pmatrix}$$

Problem 3

We want to solve the following functions

1. $\sin(2\pi x) - 2x = 0$
2. $\sin(2\pi x) - x = 0$
3. $\sin(2\pi x) - 0.5x = 0$

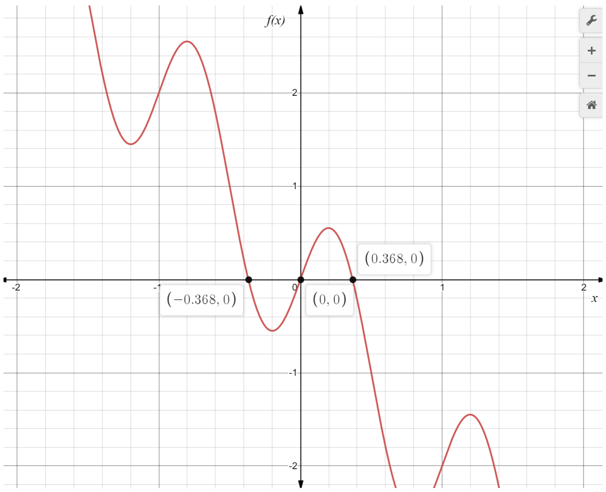
using 1) Bisection, 2) Newton method, 3) Secant method, and 4) fixed-point iteration. We want to evaluate for what value of initial guess $x_0 \in [-2, 2]$ these methods converge. We proceed by first plotting all the three functions on Figure 1. From these graphs, we see that within the interval $[-2, 2]$, function 1 and 2 have both two roots, while function 3 has seven roots.

3(a) Bisection

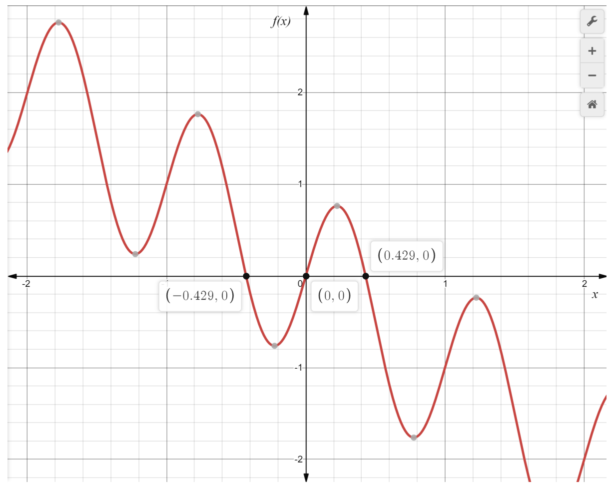
In order for Bisection to work, we need to pick two values x_{low} and x_{high} so that $f(x_{low}) \cdot f(x_{high}) < 0$. The range of admissible values for x for the three functions above is summarized in Table 1

3(b) Newton Method

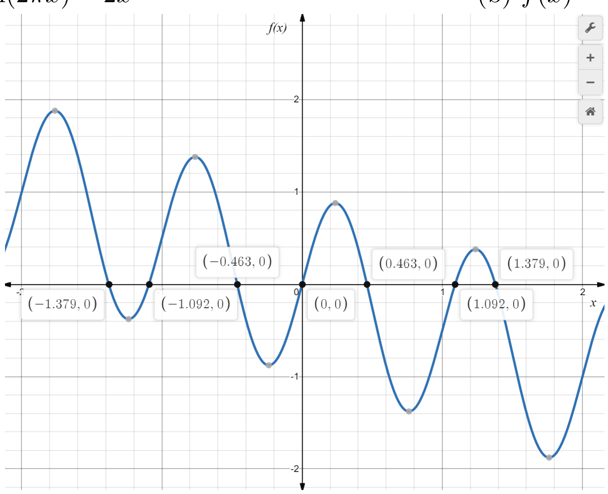
[†] Kenneth L. Judd, 1998. "Numerical Methods in Economics," MIT Press Books, The MIT Press, p.78-79



(a) $f(x) = \sin(2\pi x) - 2x$



(b) $f(x) = \sin(2\pi x) - x$



(c) $f(x) = \sin(2\pi x) - 0.5x$

Figure 1: Function plots for Problem 3

Function	Root	Admissible Range	
		x_{low}	x_{high}
$\sin(2\pi x) - 2x = 0$	$x_1 = -0.368$	$[-2, -0.368)$	$(-0.368, 0)$
	$x_2 = 0$	$(-0.368, 0)$	$(0, 0.368)$
	$x_3 = 0.368$	$(0, 0.368)$	$(0.368, 2]$
$\sin(2\pi x) - x = 0$	$x_1 = -0.429$	$[-2, -0.429)$	$(-0.429, 0)$
	$x_2 = 0$	$(-0.429, 0)$	$(0, 0.429)$
	$x_3 = 0.429$	$(0, 0.429)$	$(0.429, 2]$
$\sin(2\pi x) - 0.5x = 0$	$x_1 = -1.379$	$[-2, -1.379)$	$(-1.379, -1.092)$
	$x_2 = -1.092$	$(-1.379, -1.092)$	$(-1.092, -0.463)$
	$x_3 = -0.463$	$(-1.092, -0.463)$	$(-0.463, 0)$
	$x_4 = 0$	$(-0.463, 0)$	$(0, 0.463)$
	$x_5 = 0.463$	$(0, 0.463)$	$(0.463, 1.092)$
	$x_6 = 1.092$	$(0.463, 1.092)$	$(1.092, 1.379)$
	$x_7 = 1.379$	$(1.092, 1.379)$	$(1.379, 2]$

Table 1: Polynomial evaluation costs