Problem Set 1 - ECON 880

Spring 2022 - University of Kansas

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January 30, 2022

Problem 1

Problem 1a

We can evaluate the polynomial as follow:

$$\begin{aligned} f(x,y) &= 83521y^8 + 578x^2y^4 - 2x^4 + 2x^6 - x^8 \\ &= y^4(83521y^4 + 578x^2) + x^4(-2 + 2x^2 - x^4) \end{aligned}$$

Problem 1b

Test

Problem 1c

test 2

Problem 2

Problem 3

In order to find our machine ε, we write a while loop to subtract progressively smaller numbers from 1 until the result is not distinguishable from 1. Repeat the exercise using 0.001 and 1000 instead of 1. We verify our results by comparing them with the epsilons delivered by the built-in Matlab function `eps(x)`, and the exponents with `log2(eps(x))`. The results are shown on Table 1.

x	$\varepsilon(x)$ - decimal	$\varepsilon(x)$ - exponential
0.001	2.16840434497101e-19	2^{-62}
1	2.22044604925031e-16	2^{-52}
1000	1.13686837721616e-13	2^{-43}

Table 1: Machine ε for diverse values of x

Comment: As x increases, the machine $\varepsilon(x)$ decreases.

Problem 4

We wrote a loop to evaluate the convergence of the following sequences:

(4a) $x_k = \sum_{k=1}^{n} \frac{1}{2^n}$, where $\lim_{k\to\infty} x_k = 1$

(4b) $y_k = \sum_{k=1}^{n} \frac{1}{n}$, where $\lim_{k\to\infty} y_k = \infty$

We use absolute and relative convergence criteria, and tolerance distance $\delta \in \{10^{-2}, 10^{-4}, 10^{-6}\}$ as stopping rule. We limit the maximum number of iterations to 100,000. The number of iterations before convergence, as well the final guess are reported on Table 2 and 3 for x_k and y_k, respectively.

δ	10^{-2}	10^{-4}	10^{-6}
no. of iteration - absolute criteria	7	14	20
final guess - absolute criteria	0.992187500000000	0.999938964843750	0.999999046325684
no. of iteration - relative criteria	7	14	20
final guess - relative criteria	0.992187500000000	0.999938964843750	0.999999046325684

Table 2: Convergence for x_k

δ	10^{-2}	10^{-4}	10^{-6}
no. of iteration - absolute criteria	100	10,000	100,000
final guess - absolute criteria	5.18737751763962	9.78760603604435	12.0901461298633
no. of iteration - relative criteria	100	10,000	100,000
final guess - relative criteria	5.18737751763962	9.78760603604435	12.0901461298633

Table 3: Convergence for y_k

Comment: Table 2 shows that both absolute and relative convergence criteria lead to the same number of iterations for the sequence x_k. As tolerance distance δ lowers, the final guess for x_k approaches its true limit 1. Table 3 also shows that both absolute and relative criteria lead to the same number of iterations for the sequence y_k. Since y_k is a divergent sequence, the final guess will be higher (with no upper bound), the more we iterate. By lowering tolerance distance δ, the algorithm needs to iterate longer in order to satisfy the stopping rule, which in turn results in higher final guess. Since there is no upper bound for y_k, we can always make the tolerance distance δ even lower, and obtain even higher final guess for y_k.