Problem Set #4: joint probability distributions

Please turn in your homework (m-file and published PDF) on the ELMs site. For this problem set, you will need the data-file **`PCAdata.mat`**, which is located in the ELMs site under **"`SharedCode`"**.

---

1. HIDDEN STRUCTURE IN DATA.

   A. Consider the 2-dimensional data generated in the Oct 10 lab (or use **`Xdata1`**). They will differ only due to a different random number seed. Plot the histogram of data points along the first and second dimension. Report the standard deviation in each dimension.

   B. Use PCA to find the principle axes. Plot the data in 2 dimensions, and the two principle axes on top of the data. Make sure these axes make sense.

   C. Plot histograms of the data projected onto each principle component. Calculate the standard deviation of the data along each dimension from the data, using these projections. Compare to the eigenvalues generated by the PCA function.

   D. [Optional] Perform PCA on **`Xdata2`**, which in 10-dimensional space. Report the variance along each axis, and compare to the eigenvalue spectrum. How do they relate to eachother?

   E. [Optional 2] Plot the data in the 2-D space spanned by the first two PC components. Why was it not possible to see this hidden structure without PCA?

2. A SIMPLE JOINT PROBABILITY DISTRIBUTION. Consider a simple neuron that is presented with 5 stimuli (all equally probable). It spikes to the first stimulus (spike = 1, no spike = 0), but only with 50 percent probability. It does not respond to any of the other stimuli.

   A. What does the joint probability distribution of this neuron look like? Enter it as a variable **`jpd`** in MATLAB. The size of **`jpd`** should be 2x5 [...and of course be normalized.] Display the JPD, and verify that the sum over the whole JPD is equal to 1.

   B. Plot the two conditional probability distributions p(s|r) for no response and response. Again, be sure that each is properly normalized.

   C. Without yet knowing formal measures of information theory, hypothesize which response (spike or no spike) is more informative. Why? Also hypothesize about which stimulus is best encoded by the neuron, and explain why.

3. JOINT PROBABILITY DISTRIBUTIONS AND TUNING CURVES. Consider a tuning curve for motion direction of an MT neuron. It is of similar form to that of the V1 neuron considered in the last problem set:

$$r(\theta) = r_0 + r_{max}\, e^{-(\theta-\theta_0)^2/(2\sigma^2)}$$

   The neuron has similar firing values as the V1 neuron considered in the last problem set, though with a higher spontaneous firing rate with r_0 = 10 Hz, r_{max} = 40 Hz, sigma = 30 degrees, and theta$_0$ = 180 degrees (leftward motion). Note that it is now sensitive to direction of motion rather than orientation, which can vary between 0 and 360 degrees. Like the V1 neuron, we will assume that its spike-count variability is Poisson.

   For the purposes of this problem, assume all experiments (counting windows) are 1 second long. An important aspect of all these problems is that your probability distributions are normalized (i.e., they sum to 1). Be sure of this at every stage!

   A. Calculate the conditional probability distribution of responses (numbers of spikes) p(r|s) for the following angles: 40 degrees, 140 degrees, and 180 degrees. To do this, assume that each distribution is given by a Gaussian, with its mean (and variance) given by the tuning curve above. Please use a response resolution of 1 spike, such that all plots of the response distribution are along the x-axis: `rs=0:1:80;` (note that we go up to 80 because it is sufficiently high to to encompass all possible responses. Plot the three distributions on the same plot in different colors. [Remember to normalize!]

B. Calculate the joint probability distribution for all possible combinations of responses and stimuli p(r,s). For the purposes of this problem, please represent the directions `thetas=2:2:360;` and responses given above in part A. Plot the JPD using the function `imagesc`, and please add `colorbar`. Assume that all directions are equally likely (meaning a uniform prior).

C. Calculate (and plot) the marginal distribution of responses p(r).

D. Calculate the conditional [decoding] probability distributions p(s|r) for seeing a response of 60 spikes, for 50 spikes, for 25 spikes, and for 10 spikes. Plot them all on the same plot, and use `legend` to label them. Which response do you think is the best at labeling stimuli? Which is the worst? Why?

4. SIMPLE DECODING. To perform this problem, you will need the JPD from Problem #3, so make sure that you have it right! We now want to do some "experiments". Imagine we want to know how well a neuron with this tuning curve can perform "fine discrimination" at different points along the tuning curve.

   A. Tuning curve flank. Calculate (and plot) the conditional probability distributions of responses p(r|s) for stimuli at +/- 2 degrees from 140 degrees (138 and 142 degrees).

   B. Best decision threshold. Using a threshold decision rule, calculate (and plot) the fraction correct as a function of threshold. Choose the best threshold to maximize your accuracy in guessing correctly based on the neural data, and report the fraction of correct responses, assuming 138 and 142 was presented an equal number of times. Finally, plot the threshold on the plot from A as a vertical line that maximizes the number of correct answers.

   C. Tuning curve peak. Perform the same steps (and answer the same questions) for the tuning curve peal: distinguishing between 178 and 182 degrees. What happened?

   D. Which stimuli are better encoded by this neuron that is "tuned" to 180 degrees? Can you reconcile this with why you might still say it is tuned to 180 degrees? [Hint: think of when you would hear spikes if you were presenting different stimuli to a neuron.]