Problem Set #6: information theory

1. INFORMATION IN SPIKE TRAINS. Consider a Poisson neuron firing at a constant rate throughout the duration of a 1-second-long experiment (over which many stimuli are presented).

   A. Single-bin information capacity. Suppose that the neuron had a firing rate of r = 40 Hz throughout, and analyze its spike train at 1 ms resolution. Calculate how much information COULD be conveyed by the response in a single bin (i.e., the response entropy), and convert this into a rate (i.e., in bits per second).

   B. Optimal firing rates. Given the analysis at 1 ms resolution, show how the information capacity (entropy rate) depends on the firing rate up to 1000 Hz. Is there an optimal firing rate? Why (or why not)?

   C. Time resolution of the analysis. Calculate the entropy rate for the 40 Hz neuron using ½ millisecond resolution. Does it change from part (A)? Why? Does this concern you? Why or why not?

   D. Information versus entropy rate. So far you have been calculating the entropy rate (i.e., information capacity) of the neuron. Now calculate the mutual information rate.

2. CHANGE FIRING RATES TO GET BETTER PEAK/SLOPE TUNING CURVE PEAK OR FLANK: WHICH IS MORE INFORMATIVE? Consider the a tuning curve for motion direction of an MT neuron, considered in Problem Set #3:

$$r(\theta) = r_0 + r_{max}\, e^{-(\theta-\theta_0)^2/(2\sigma^2)}$$

   As in the last problem set, the neuron is described by the following parameters: r_0 = 10 Hz, r_{max} = 40 Hz, sigma = 30 degrees, and theta_0 = 180 degrees (leftward motion). Please resurrect the joint probability distribution **`jpd`** that you calculated in the last problem set, and/or use the one from the solution set. Remember that the range of stimulus and response should be: **`rs=0:1:80; thetas=2:2:360;`**

   A. Assuming that the prior p(s) is uniform (and given the binning of theta shown above), what is the entropy of the prior H[S]?

   B. Now calculate the specific information of each response (equation given above in problem #2), and plot as a function of the **`rs`**. Remember, to calculate the specific information for a response, you first need to extract p(s|r), as you did in the last problem set, and compare its entropy to that of the prior. Use your function **`myentropy`** from the last problem set where ever possible.

   C. Calculate the mutual information between the stimulus ensemble and the neuronal response.

   D. Now calculate and plot the stimulus-specific information (SSI) for all stimuli. The equation for SSI is given in the last problem set, and also requires extracting p(r|s). Note that you can directly use the specific information calculated in Part B.

   E. Where is the stimulus-specific information maximum? Why?

   F. Now consider a neuron whose stimulus-driven firing rate (r_{max}) is double that if the neuron considered above. In other words, hold every other parameter constant, and simply use a new r_{max} = 80 Hz and a larger range for the response ensemble: for **`rs=0:1:150;`**. Recalculate the specific information and plot.

   G. Now recalculate the stimulus-specific information (SSI) and plot. If you like, compare with Fig. 2A of Butts and Goldman (2006)

   H. Explain why the peak SSI has changed. Make sure to include in your answer why this change came about by simply increasing the firing rate of the neuron.