Problem Set #5: Bayes vs. Information Theory

Please turn in your homework (m-file and published PDF) on the ELMs site.

1. BAYESIAN MULTI-SENSORY INTEGRATION. Consider the Kording/Wolpert experiment discussed in class, where the subject attempts to reach a target using prior experience with the task and brief, uncertain visual feedback. For this problem, use a resolution of 0.01 cm and the range of -1 cm to +4 cm, i.e., `x=-1:0.01:4;`

   A. PRIOR. Assume the prior experience allows the subject to expect the distribution of targets given by the prior, defined by a Gaussian with a mean of 1 cm and standard deviation of 0.5 cm. Plot the prior distribution as the first of three subplots (use the function **`subplot`** to make a vertical stack).

   B. VISUAL UNCERTAINTY. Assume there are four conditions of visual feedback (like the paper), corresponding to standard deviations of 0.1 cm, 0.2 cm, 0.5 cm, and no visual feedback. These are again given by Gaussians, centered on the position chosen in a given trial. On the same plot (subplot 2), plot the distributions given by the three visual feedback conditions centered on the position of 2 cm.

   C. POSTERIOR. For the trial described in (B), calculate the probability of true location given the visual feedback for all four conditions. Plot the resulting probability distributions for all four conditions in the third subplot. Be sure to label with a **`legend`**. What is the optimal choice in each case?

   D. ERROR CALCULATION. Calculate the mean squared error from the visual feedback for the probability distributions in C. For a given probability distribution p(x), the mean squared error is given by:

   $$\mathrm{MSE} = \sum_x p(x)\,(x - x_{true})^2$$

   For this position (x_{true} = 2 cm), how does the resulting average error (the square root of the MSE) compare to that of a simpler strategy that just uses the visual feedback alone?

   E. ADVANTAGE OF BAYESIAN ESTIMATION. Recalculate B-D for the position of 1 cm (same position as prior), and compare the average error at this position to that of the simple model that uses visual feedback alone. How do things change, and why?

   F. [OPTIONAL] Calculate and plot the mean squared error across all positions from -0 to 2 cm for the optimal strategy (i.e. B-D), and that responses will be distributed like the posterior. Note that this will not look exactly like Fig. 1e in the Kording and Wolpert paper, but will have some of the same underlying features. Compare this to the error level expected by the simple model that uses visual feedback alone. Does the Bayesian strategy ever do worse? Why or why not?

   G. [OPTIONAL] Compare this average mean squared error across all positions to that of the simple model that uses visual feedback alone. Which method is better? Remember to use the prior to properly weight the error at each location.

[see next page for #2-3]

2. ENTROPY FUNCTIONS. I want you to practice building m-files, and create a few useful ones for this problem set. For a brief tutorial on making functions in MATLAB, look at the "Demos" under the Help menu of MATLAB, and watch the third: "Writing a MATLAB program".

   A. Create a simple function called **mylog2.m**, which will return the log-base-2 of any number. However, add an if-statement to that it will return a '0' if given a zero. This is useful for calculating entropies of probability distributions, which often have zeros in some bins. The result is something you should be able to run in the command line:

```
>> mylog2( 0.5 )

ans =

    -1
```

   You can have this function in your directory when you publish, but please also include your function as a "comment" in this section '%% 2A' so I can see your function code. You can also attach the function as part of your submission (with m and PDF files)

   B. Create a function called **myentropy.m**, which can take an array that represents a probability distribution and calculate the entropy. It should also make sure that the probability distribution is properly normalized. It can use **mylog2.m**. Running it should look like

```
>> p = [0.25 0.25 0.25 0.25];
>> myentropy(p)

ans =

    2
```

3. SIMPLE INFORMATION MEASURES OF STIMULUS AND RESPONSE. Consider the simple neuron example from Problem Set #3 that is presented is 5 stimuli. As a reminder, it responds to the first stimulus (spike = 1, no spike = 0), but only with 50 percent probability. It does not respond to any of the other stimuli, and each stimulus is equally probable. Please re-enter the variable **jpd** from last problem set, and make sure it is correct (see problem set #3 solutions). [Note that I will not have your last problem set solutions loaded when running this problem set, so be sure that all necessary variables are included in your solutions.]

   A. Calculate the specific information of the two possible responses (0,1)? Remember that specific information of a response r is given by $i_{sp}(r)$ = H[S]-H[S|r]. Use your function **myentropy** where ever possible.

   B. What is the mutual information between stimulus and response of this neuron? Calculate this two ways: (1) using the specific informations calculated in part A, and (2) using the symmetric formula for mutual information based on the joint probability distribution and the marginal and prior. [Make sure they match!]

   C. The stimulus-specific information (SSI) is given by:

$$\mathrm{SSI}(s) = \sum_r p(r|s)\, i_{sp}(r)$$

   Calculate the SSI for all 5 stimuli. Verity that the weighted sum of SSI is the same as the mutual information.