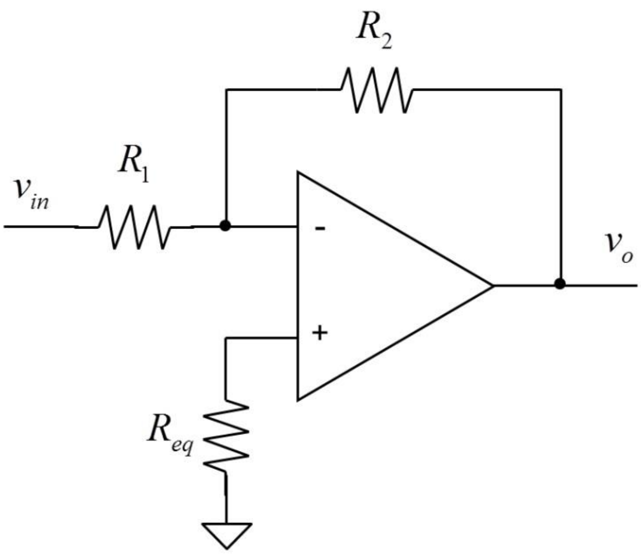
EGB348 Electronics
Tutorial 4 – Operational Amplifiers

Q1. Inverting Amplifier

Design an inverting amplifier with a voltage gain of -10. Sketch the circuit and indicate resistor values.

Circuit for inverting amp:



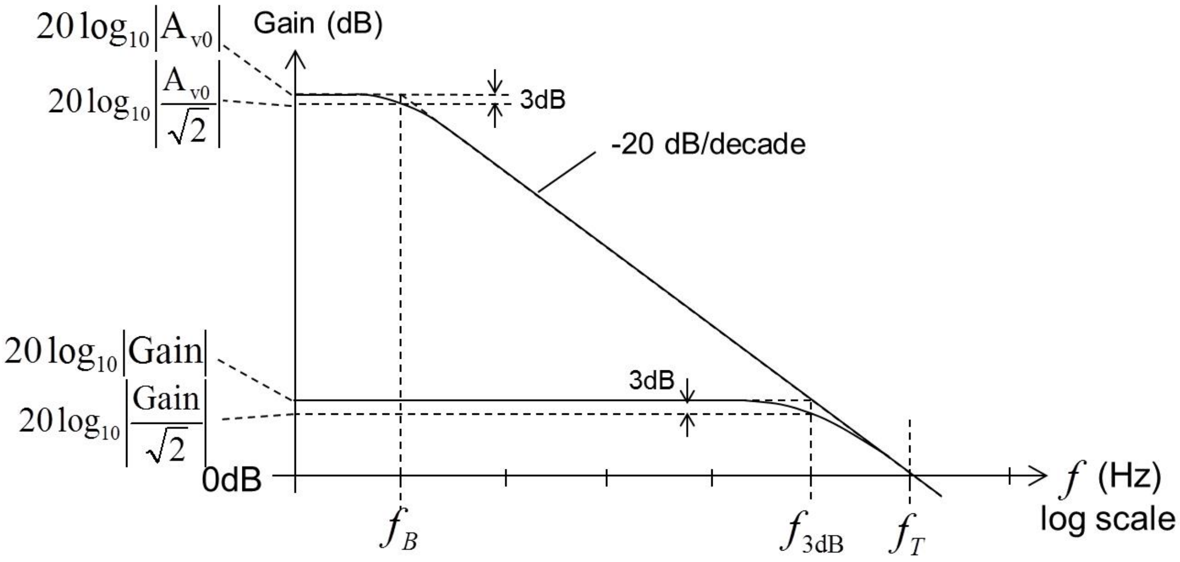
$$\text{Gain} = -10$$

$$\frac{v_o}{v_{in}} = -\frac{R_2}{R_1} = -10$$

$R_1 = 1\text{k}\Omega$ and $R_2 = 10\text{k}\Omega$ will give the desired gain.

(a) If the gain bandwidth product f_T= 1MHz, estimate the closed loop bandwidth of the amplifier.

The figure before shows the magnitude frequency response of the op amp open loop and closed loop gain. The response intersects the 0dB line at $f_T = 1\text{MHz}$.



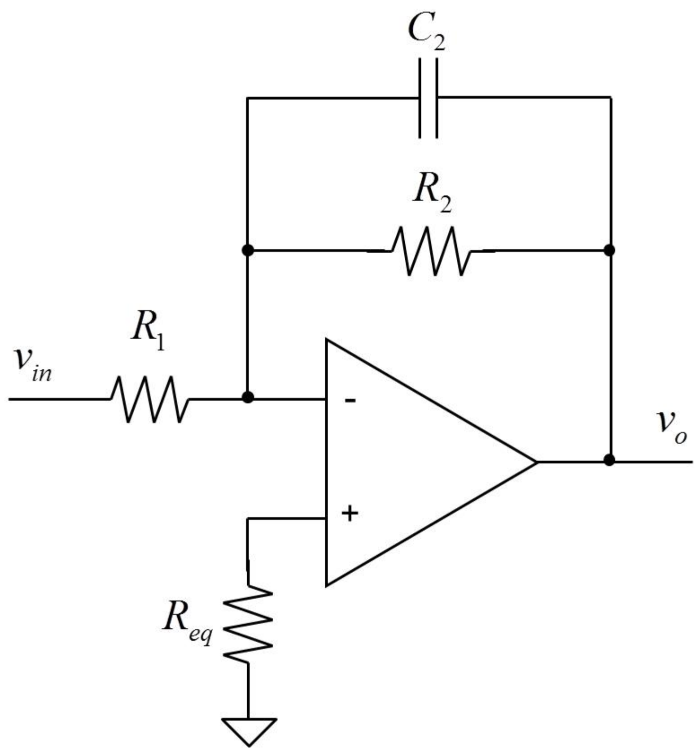
To work out the 3dB bandwidth of the closed loop amplifier:

$$f_T = |\text{Gain}| \cdot f_{3dB}$$

$$f_{3dB} = \frac{f_T}{|\text{Gain}|} = \frac{1\text{MHz}}{10} = 100\text{kHz}$$

(b) Show how the bandwidth could be reduced to 10kHz while maintaining a DC gain magnitude of 10.

We could place a capacitor in parallel with R_2 as shown below. This introduces a pole that rolls off the gain at a lower frequency.



The transfer function of this circuit is given by:

$$\frac{v_o}{v_{in}} = -\frac{Z_2}{Z_1}$$

where

$$Z_1 = R_1$$

and

$$Z_2 = \frac{R_2 \frac{1}{sC_2}}{R_2 + \frac{1}{sC_2}} = \frac{R_2}{1 + sC_2R_2}$$

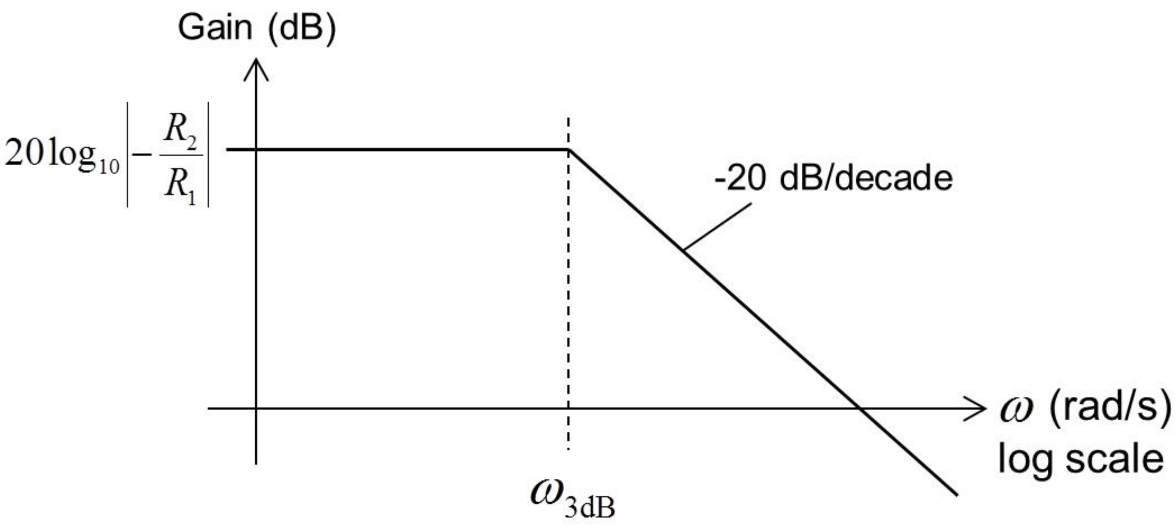
Therefore

$$\frac{v_o(s)}{v_{in}(s)} = -\frac{Z_2}{Z_1} = -\frac{R_2}{R_1}\left(\frac{1}{1 + sC_2R_2}\right)$$

or in terms of $j\omega$:

$$\frac{v_o(j\omega)}{v_{in}(j\omega)} = -\frac{Z_2}{Z_1} = -\frac{R_2}{R_1}\left(\frac{1}{1 + j\omega C_2R_2}\right)$$

A Bode plot of the magnitude frequency response is as shown below:



The DC gain magnitude is given by R_2/R_1 and is unchanged.

The 3dB bandwidth is given by:

$$\omega_{3dB} = \frac{1}{C_2 R_2}$$

Therefore:

$$f_{3dB} = \frac{1}{2\pi C_2 R_2}$$

$$10\text{kHz} = \frac{1}{2\pi C_2 (10\text{k})}$$

$$C_2 = \frac{1}{2\pi (10\text{k})(10\text{k})} = 1.6\text{nF}$$

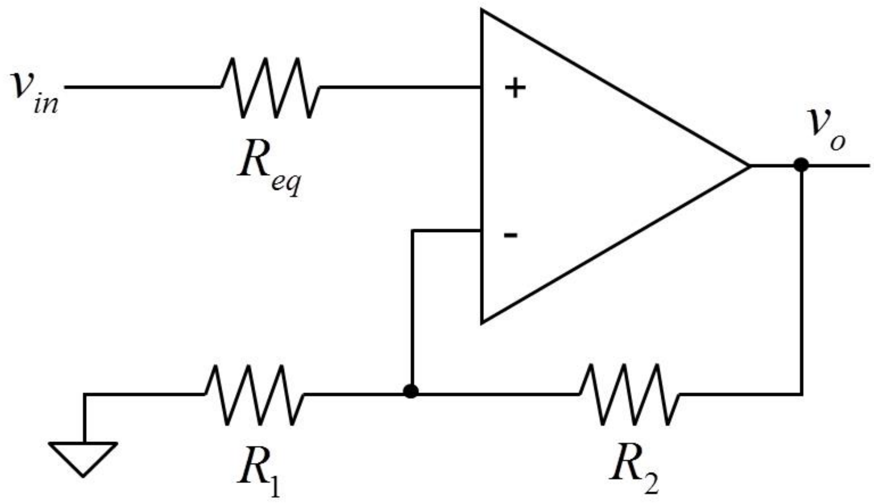
(c) What value of resistor, when placed at the non-inverting input, would reduce the effect of bias currents?

$$R_{eq} = R_1 // R_2 = 1\text{k}//10\text{k} = 909\Omega$$

Q2. Non-Inverting Amplifier

Design a non-inverting amplifier with a voltage gain of 10. Sketch the circuit and indicate resistor values.

Circuit for non-inverting amp:



$$\text{Gain} = 10$$

$$\frac{v_o}{v_{in}} = 1 + \frac{R_2}{R_1} = 10$$

$R_1 = 10\text{k}\Omega$ and $R_2 = 90\text{k}\Omega$ will give the desired gain.

What value of resistor, when placed at the non-inverting input, would reduce the effect of bias currents?

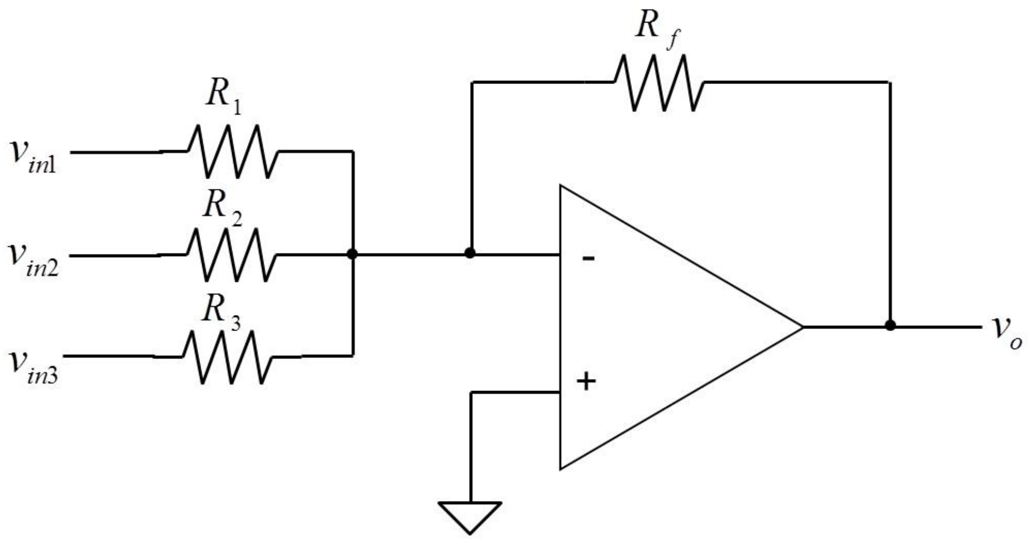
$$R_{eq} = R_1 // R_2 = 10\text{k}//90\text{k} \approx 9\text{k}\Omega$$

Q3. Summing Amp

Design a circuit using a single op amp to produce the output:

$$v_o = -(4v_{in3} + 2v_{in2} + v_{in1})$$

Circuit for the summing amp is below:



Output voltage of summing amp circuit is given by:

$$v_o = -\left(\frac{R_f}{R_3}v_{in3} + \frac{R_f}{R_2}v_{in2} + \frac{R_f}{R_1}v_{in1}\right)$$

Let:

$$R_f = 10\text{k}\Omega$$

Then, values of R_1, R_2, R_3 to produce the required output are:

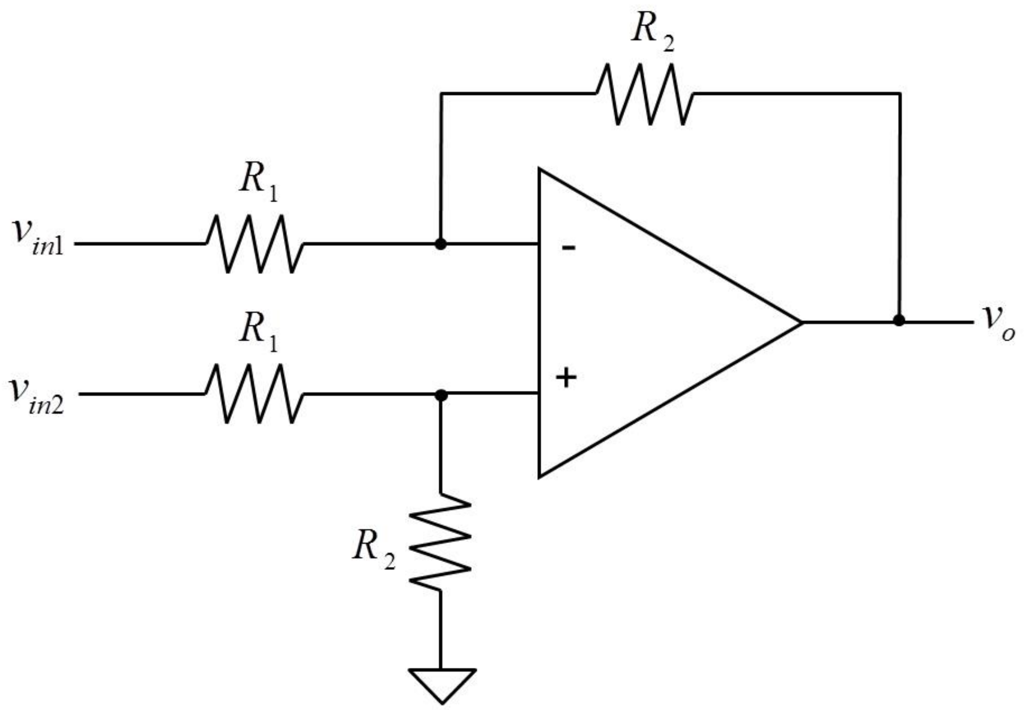
$$R_1 = 10\text{k}\Omega$$
$$R_2 = 5\text{k}\Omega$$
$$R_3 = 2.5\text{k}\Omega$$

Q4. Difference Amp

Design a circuit using a single op amp to produce the output:

$$v_o = 100(v_{in2} - v_{in1})$$

Circuit for the difference amp is below:



Output voltage of summing amp circuit is given by:

$$v_o = \frac{R_2}{R_1}(v_{in2} - v_{in1})$$

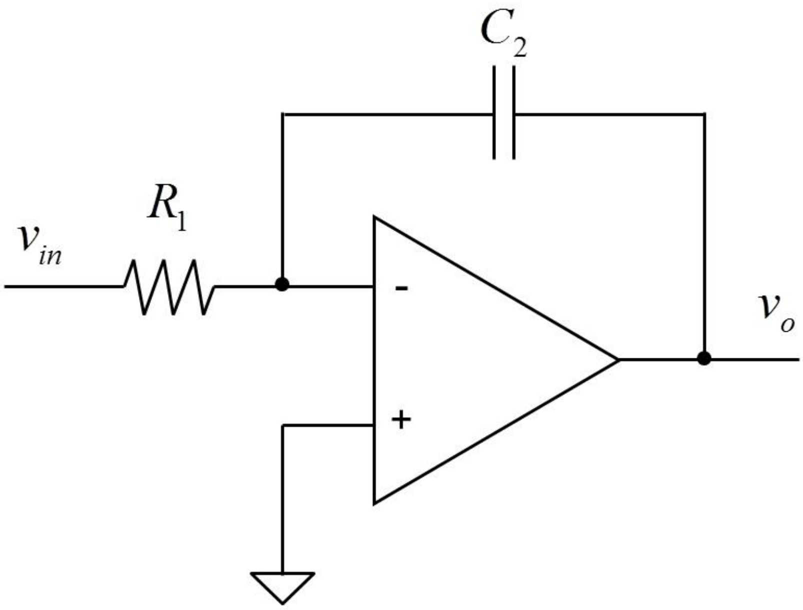
Let:

$$R_1 = 1\text{k}\Omega$$
$$R_2 = 100\text{k}\Omega$$

Q5. Integrator

The input to an integrator is a 500Hz square wave of ±5V. If R_1 = 10kΩ, and C_2 = 0.1μF, sketch the output waveform and find its magnitude. Sketch the magnitude frequency response. For this question, assume power supply voltages of ±15V.

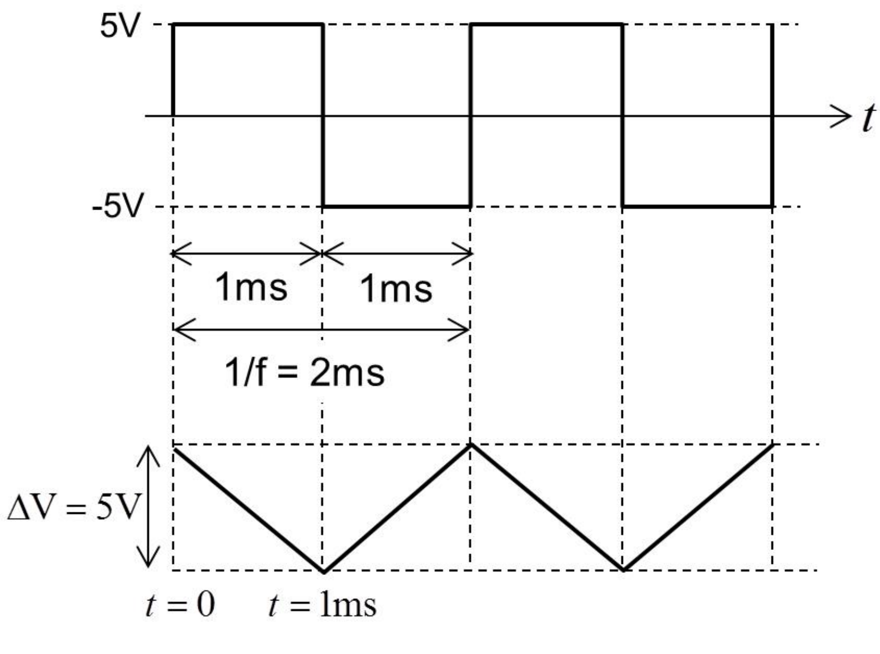
Circuit for the idea integrator is below:



Integrator output:

$$v_o = -\frac{1}{R_1 C_2}\int v_{in}dt$$

Input and output waveforms:



The result of integrating the square wave will be a triangular wave. We do not know the initial value of v_o. To work out ΔV for the triangular wave, we will assume a starting point of $t = 0$, $v_o = 0$.

$$v_o = -\frac{1}{R_1 C_2}\int_0^{1\mathrm{ms}} 5dt$$
$$v_o = -\frac{1}{0.1\mu \times 10\mathrm{k}}[5t]_0^{1\mathrm{ms}}$$
$$v_o = -\frac{1}{0.1\mu \times 10\mathrm{k}}(5 \times 1\mathrm{ms})$$
$$v_o = -5\mathrm{V}$$

Therefore, the output decreases by 5V during the 1ms while the square wave has its positive value. The 1ms where the square wave has its negative value will be a mirror image of the positive half, and the output will increase by 5V. The result is that the output is a triangular wave.

The transfer function of this circuit is given by:

$$\frac{v_o}{v_{in}} = -\frac{Z_2}{Z_1}$$

where

$$Z_1 = R_1$$

and

$$Z_2 = \frac{1}{sC_2}$$

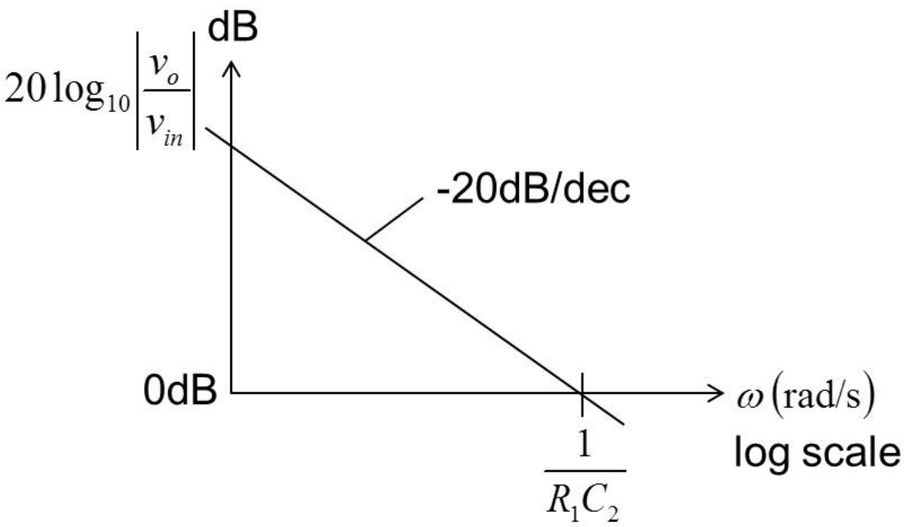
Therefore

$$\frac{v_o(s)}{v_{in}(s)} = -\frac{Z_2}{Z_1} = -\frac{1}{sR_1C_2}$$

or in terms of $j\omega$:

$$\frac{v_o(j\omega)}{v_{in}(j\omega)} = -\frac{Z_2}{Z_1} = -\frac{1}{j\omega R_1C_2}$$

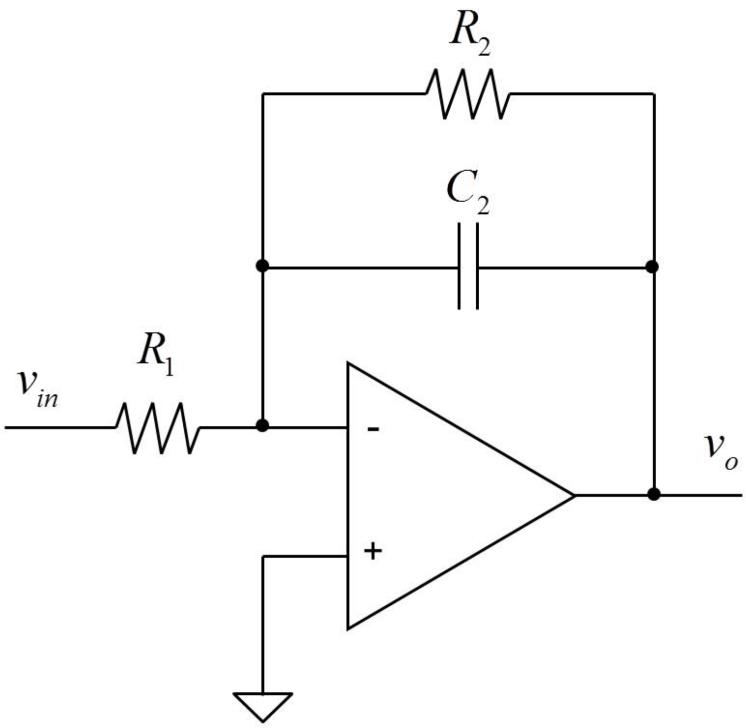
The ideal integrator magnitude frequency response is as follows:



The magnitude frequency response will intersect 0dB at $\omega = 1/(R_1C_2)$.

Show how the circuit could be modified to have a pole at 10 Hz, to limit the gain at low frequencies. Sketch the frequency response.

To limit the gain at low frequencies, include a resistor R_2:



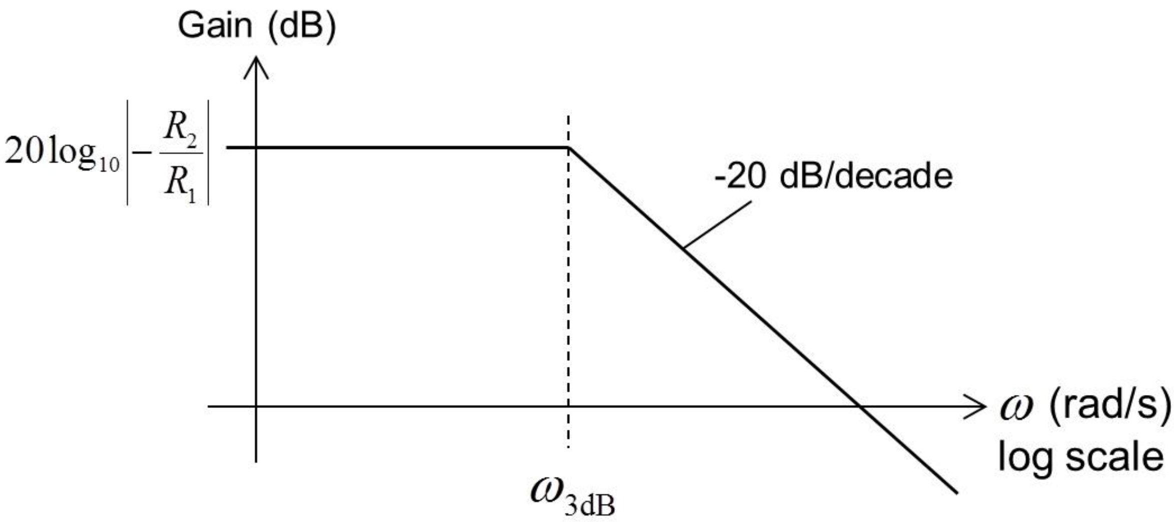
As shown in Q1(b), the transfer function of the above circuit is given by:

$$\frac{v_o(s)}{v_{in}(s)} = -\frac{R_2}{R_1}\left(\frac{1}{1 + sC_2R_2}\right)$$

or in terms of $j\omega$:

$$\frac{v_o(j\omega)}{v_{in}(j\omega)} = -\frac{R_2}{R_1}\left(\frac{1}{1 + j\omega C_2 R_2}\right)$$

The magnitude frequency response is as shown below:



We are given that frequency of the pole should be 10Hz. Therefore, the value of R_2 can be worked out as follows:

$$\omega_{3dB} = \frac{1}{C_2 R_2}$$

$$f_{3dB} = \frac{1}{2\pi C_2 R_2}$$

$$10\text{kHz} = \frac{1}{2\pi(0.1\mu\text{F})R_2}$$

$$R_2 = \frac{1}{2\pi(0.1\mu)(10\text{k})} = 160\text{k}\Omega$$

The DC gain is given by:

$$\text{DC Gain} = -\frac{R_2}{R_1} = -\frac{160\text{k}}{10\text{k}} = -16$$

DC gain in dB:

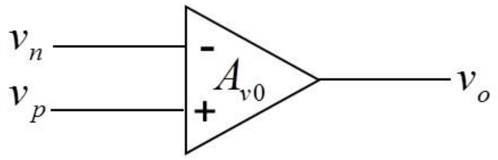
$$\text{DC Gain in dB} = 20\log_{10}\left|-\frac{R_2}{R_1}\right| = 20\log_{10}(16) = 24\text{dB}$$

EGB348 Electronics
Practice Problems – Op Amps

Q6. Open Loop Gain

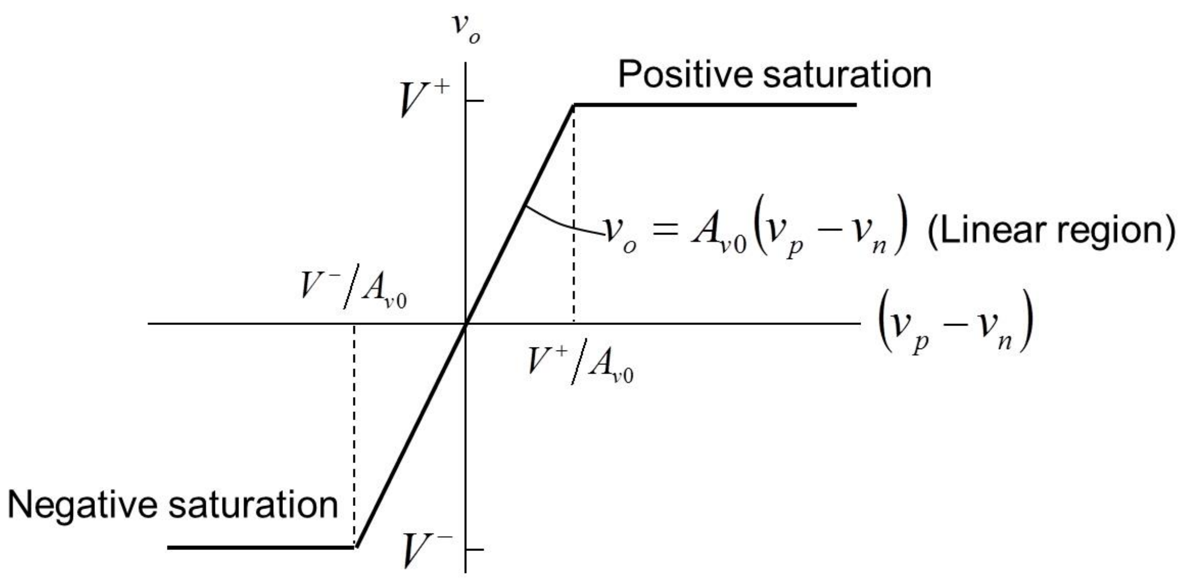
The open loop gain of an op amp is $A_{v0} = 10^5$ V/V. If the power supply voltage is ±15 V, and the op amp is connected in open loop mode, what difference between the inverting and non-inverting inputs (i.e. $|v_p - v_n|$) will result in the output saturating?

The op amp connected in "open loop" is shown below:



$$v_o = A_{v0}(v_p - v_n)$$

The output v_o plotted versus the difference between the inputs $(v_p - v_n)$, is shown below:



The difference between the inputs $(v_p - v_n)$, that will result in the output saturating can be found as follows:

$$v_o = A_{v0}(v_p - v_n)$$
$$15 = 10^5(v_p - v_n)$$
$$v_p - v_n = \frac{15}{10^5} = 0.15\text{mV}$$

Q7. Instrumentation Amp

An instrumentation amp consists of three op amps, six internal resistors R_1 and one external variable resistor R_2 as shown in Figure 1.

We wish like to design the circuit so that a maximum difference between the inputs $v_2 - v_1$ of 0.01V results in an output voltage v_o of -1.2V. If the internal resistor R_1 = 10kΩ, what should be the value of external variable resistor R_2?

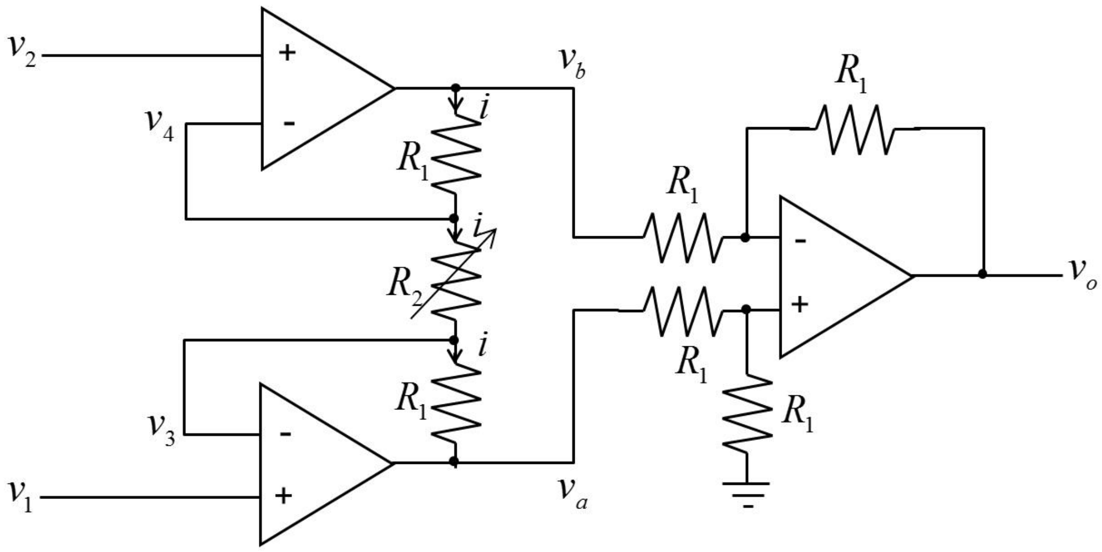


Figure 1: Instrumentation Amp.

For an instrumentation amp:

$$v_o = -(v_2 - v_1)\left(\frac{2R_1}{R_2} + 1\right)$$

To work out R_2:

$$-1.2 = -(0.01)\left(\frac{2(10\text{k})}{R_2} + 1\right)$$

$$\frac{1.2}{0.01} - 1 = \frac{2(10\text{k})}{R_2}$$

$$R_2 = \frac{2(10\text{k})}{\frac{1.2}{0.01} - 1}$$

$$R_2 = 168\Omega$$

Q8. Precision Half Wave Rectifier

A non-saturating precision half wave rectifier is connected as shown in Figure 2.

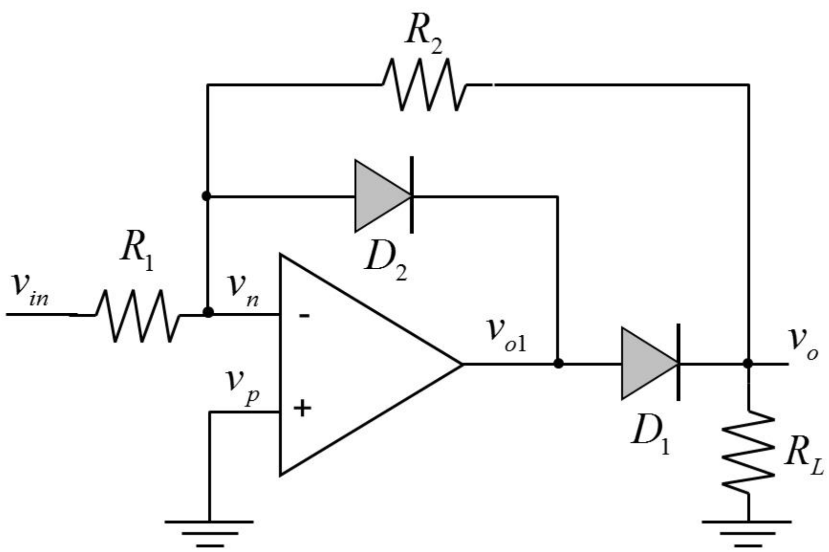


Figure 2: Precision Half Wave Rectifier.

If R_1 = 15kΩ, R_2 = 100kΩ, diode forward voltage drop = 0.6V, and the power supply = ±15V, then:

(a) What is the output when the input v_{in} = -0.5V?

Output will be rectified for negative input values.

$$v_o = -\frac{R_2}{R_1} v_{in}$$
$$v_o = -\frac{100}{15}(-0.5)$$
$$v_o = 3.33\text{V}$$

(b) What is the output when the input v_{in} = +0.5V?

Output will be zero for positive input values.

$$v_o = 0\text{V}$$

(c) Presuming the maximum and minimum output voltage is close to the power supply rails, what is the most negative input voltage for which the circuit will operate as a rectifier?

$$v_{o1(max)} = 15\text{V}$$

$$v_{o(max)} = 15 - 0.6 = 14.4\text{V}$$

When v_{in} is negative and the circuit is operating like a rectifier:

$$v_o = -\frac{R_2}{R_1} v_{in}$$

Rearranging:

$$v_{in} = -\frac{R_1}{R_2} v_o$$

Therefore, the most negative input voltage for which the circuit will operate as a rectifier is found from:

$$v_{in} = -\frac{R_1}{R_2} v_{o(max)}$$

$$v_{in} = -\frac{15}{100}(14.4)$$

$$v_{in} = -2.16\text{V}$$

Q9. Square Wave Generator

A circuit which generates a square wave (astable multivibrator) is shown in Figure 3.

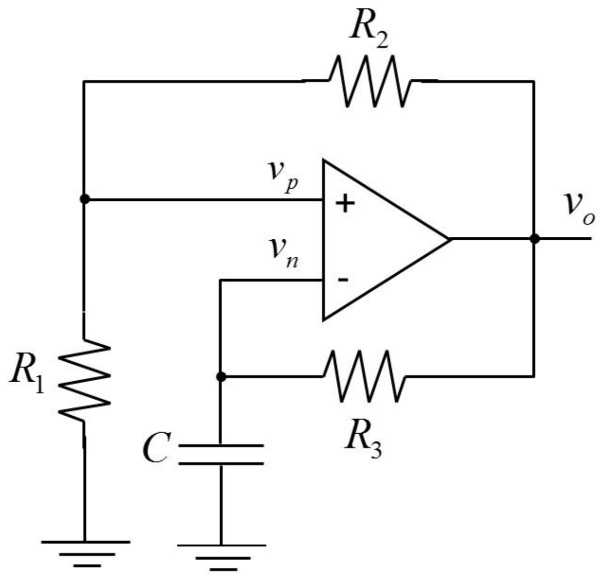
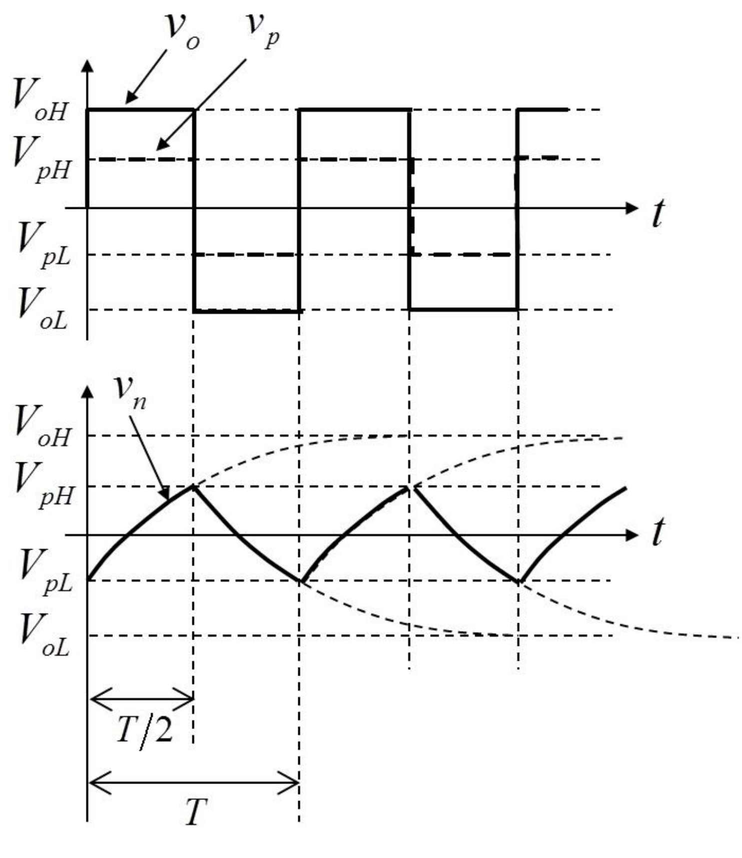


Figure 3: Square wave generator.

If R1 = 10kΩ, R₂ = 22kΩ, R₃ = 100kΩ, C = 0.1μF, high state of the output waveform (V_{oH}) = 15V and low state of the output waveform (V_{oL}) = -15V, what is the frequency of the output square wave?



The waveforms for v_o, v_n, and v_p are shown above.

If we assume $V_{OH} = 15\text{V}$, V_{pH} can be estimated as follows:

$$v_{pH} = v_{oH}\left(\frac{R_1}{R_1 + R_2}\right) = 15\left(\frac{10}{10 + 22}\right) = 4.7\text{V}$$

Similarly:

$$v_{pL} = -4.7\text{V}$$

Using the capacitor charge/discharge equation:

$$v_c(t) = V_{\text{final}} + (V_{\text{initial}} - V_{\text{final}})e^{-\frac{t}{\tau}}$$
$$v_n = V_{oH} + \left(V_{\text{pL}} - V_{oH}\right)e^{-\frac{t}{R_3C}}$$

When the capacitor voltage v_n has reached its maximum value:

$$V_{pH} = V_{oH} + \left(V_{\text{pL}} - V_{oH}\right)e^{-\frac{T/2}{R_3C}}$$

Rearranging to find T:

$$\frac{V_{pH} - V_{oH}}{V_{\text{pL}} - V_{oH}} = e^{-\frac{T/2}{R_3C}}$$
$$\ln\left(\frac{V_{pH} - V_{oH}}{V_{\text{pL}} - V_{oH}}\right) = -\frac{T/2}{R_3C}$$
$$\ln\left(\frac{V_{\text{pL}} - V_{oH}}{V_{pH} - V_{oH}}\right) = \frac{T/2}{R_3C}$$
$$\frac{T}{2} = R_3C\,\ln\left(\frac{V_{\text{pL}} - V_{oH}}{V_{pH} - V_{oH}}\right)$$
$$\frac{T}{2} = 100\text{k} \times 0.1\mu \times \ln\left(\frac{-4.7 - 15}{4.7 - 15}\right)$$
$$\frac{T}{2} = 6.47\text{ms}$$
$$T = 12.93\text{ms}$$

Frequency of output square wave:

$$f = \frac{1}{T} = 77.3\text{Hz}$$

Q10. Pulse Generator

A circuit which generates a fixed duration output pulse, when a trigger input v_T is detected (monostable multivibrator), is connected as shown Figure 4.

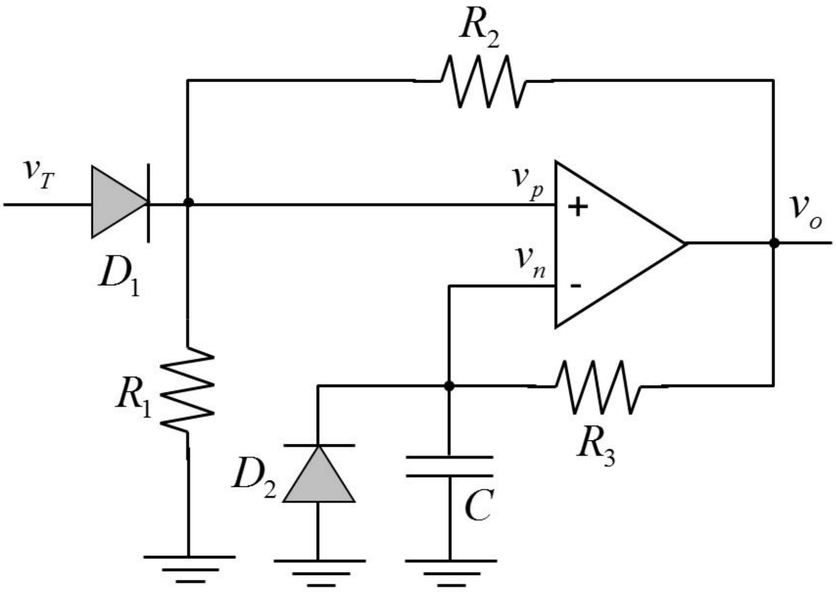
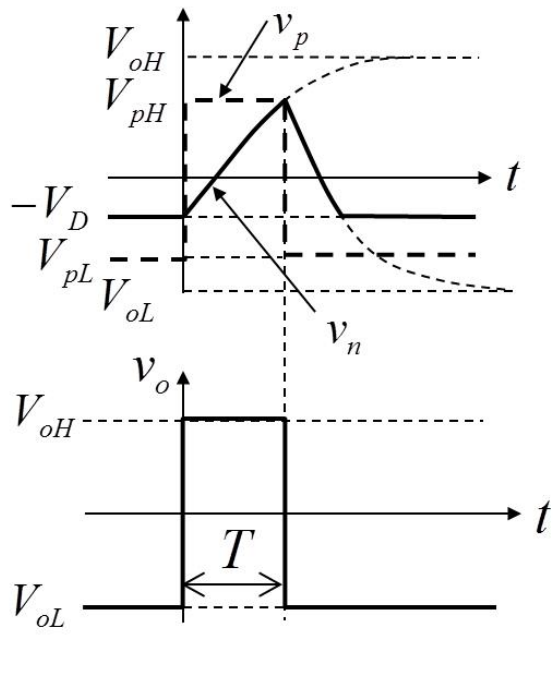


Figure 4: Pulse Generator.

If R_1 = 22kΩ, R_2 = 18kΩ, R_3 = 100kΩ, C = 0.1μF, high state of the output waveform (V_{oH}) = 15V, low state of the output waveform (V_{oL}) = -15V, and diode forward voltage drop = 0.7V, then what is the duration of the output pulse?



The waveforms for v_o, v_n, and v_p are shown above.

If we assume $V_{OH} = 15\text{V}$, V_{pH} can be estimated as follows:

$$v_{pH} = v_{oH}\left(\frac{R_1}{R_1 + R_2}\right) = 15\left(\frac{22}{18+22}\right) = 8.25\text{V}$$

Similarly:

$$v_{pL} = -8.25\text{V}$$

Using the capacitor charge/discharge equation:

$$v_c(t) = V_{\text{final}} + (V_{\text{initial}} - V_{\text{final}})e^{-\frac{t}{\tau}}$$

$$v_n = V_{oH} + (-V_{\text{D}} - V_{oH})e^{-\frac{t}{R_3C}}$$

When the capacitor voltage v_n has reached its maximum value:

$$V_{pH} = V_{oH} + (-V_{\text{D}} - V_{oH})e^{-\frac{T}{R_3C}}$$

Rearranging to find T:

$$\frac{V_{pH} - V_{oH}}{-V_{\text{D}} - V_{oH}} = e^{-\frac{T}{R_3C}}$$

$$\ln\left(\frac{V_{pH} - V_{oH}}{-V_{\text{D}} - V_{oH}}\right) = -\frac{T}{R_3C}$$

$$\ln\left(\frac{-V_{\text{D}} - V_{oH}}{V_{pH} - V_{oH}}\right) = \frac{T}{R_3C}$$

$$\text{T}{=}R_3C\ln\left(\frac{-V_{\text{D}} - V_{oH}}{V_{pH} - V_{oH}}\right)$$

$$\text{T}{=}100\text{k} \times 0.1\mu \times \ln\left(\frac{-0.7 - 15}{8.25 - 15}\right)$$

$$\text{T}{=}8.44\text{ms}$$