COVID-19 Return to Campus

Slide for use in Semester 2 classes

- Go to ***Return to campus resources and posters*** at [COVID-19: Information for staff]
- Contact your HSE Partners Amanda Burns or Matt Mackay for local area support and advice

School of Electrical Engineering and Robotics

EGB348 Electronics

Op Amps and Filters

Jasmine Banks (2020)

Recommended Readings:

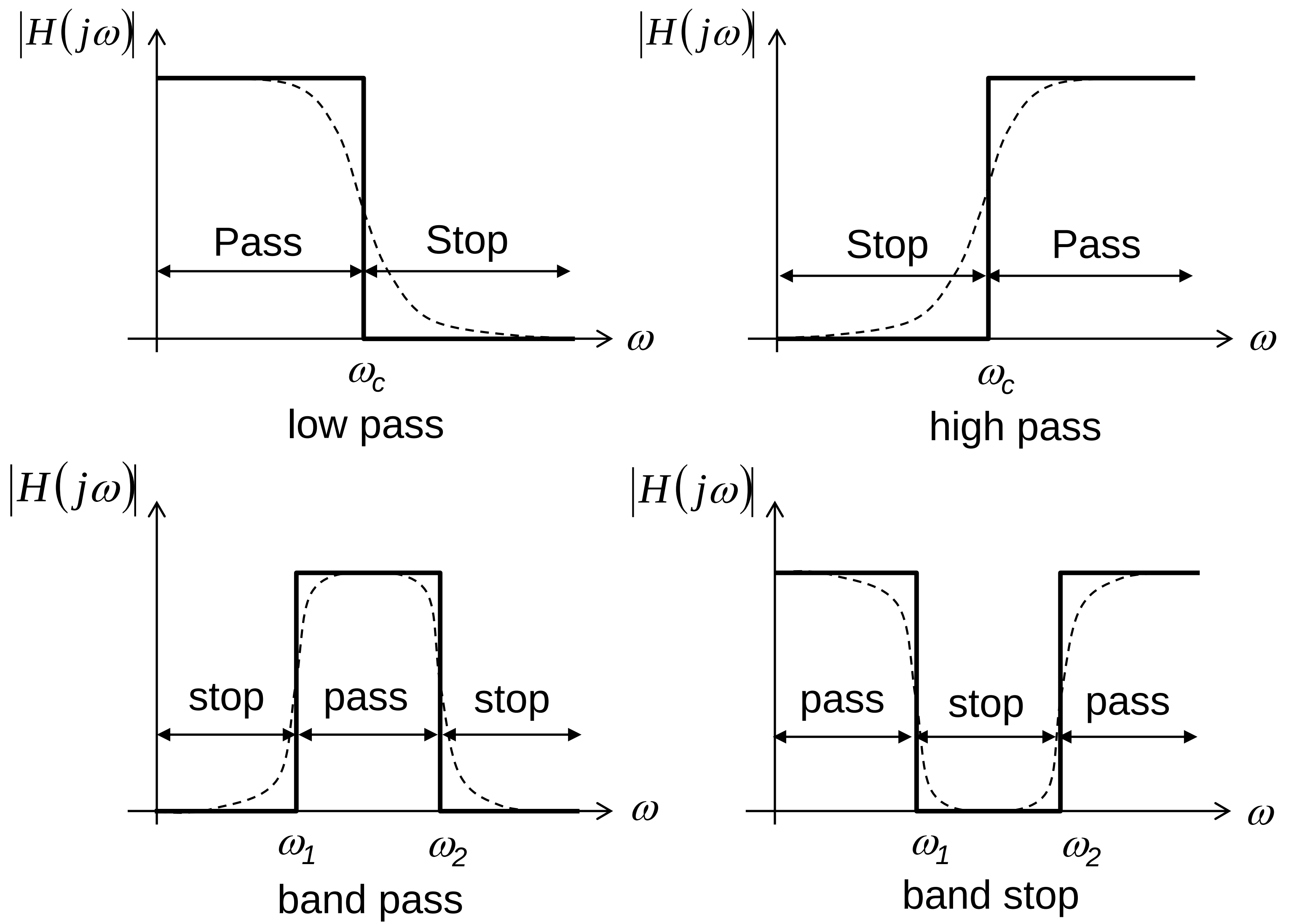
Hambley: Chapter 6,14, Horowitz and Hill: Chapter 6

Types of Filters

- A filter is a circuit designed to pass signals with desired frequencies and attenuate others.

- Types of filters – low pass, high pass, band pass, band stop.

Types of Filters

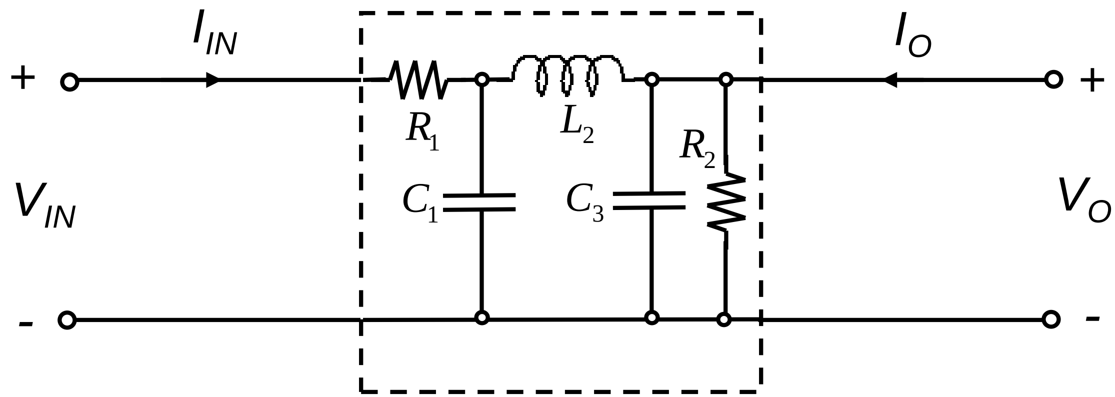


Analogue Filters

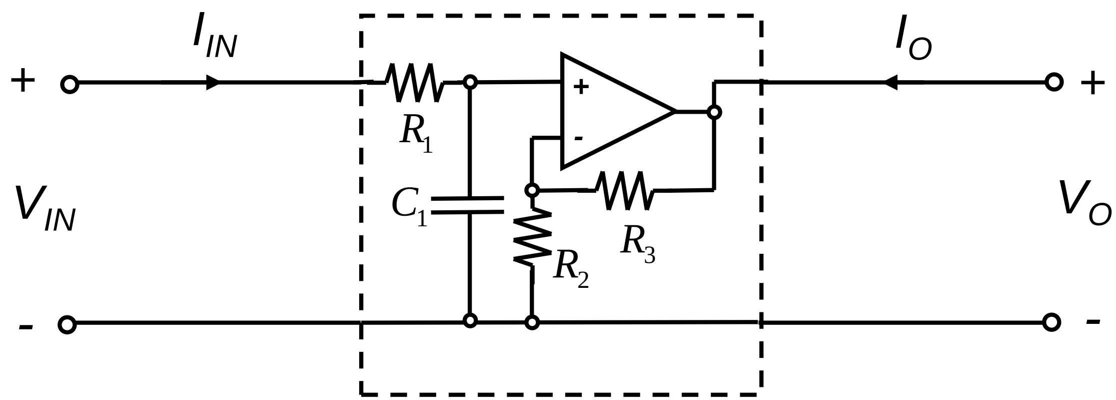
- Passive
  - Use only passive components (Resistors, Capacitors, Inductors)
  - Cannot amplify a signal
- Active
  - Use active components such as transistors or op amps
  - Need a power supply

Analogue Filters

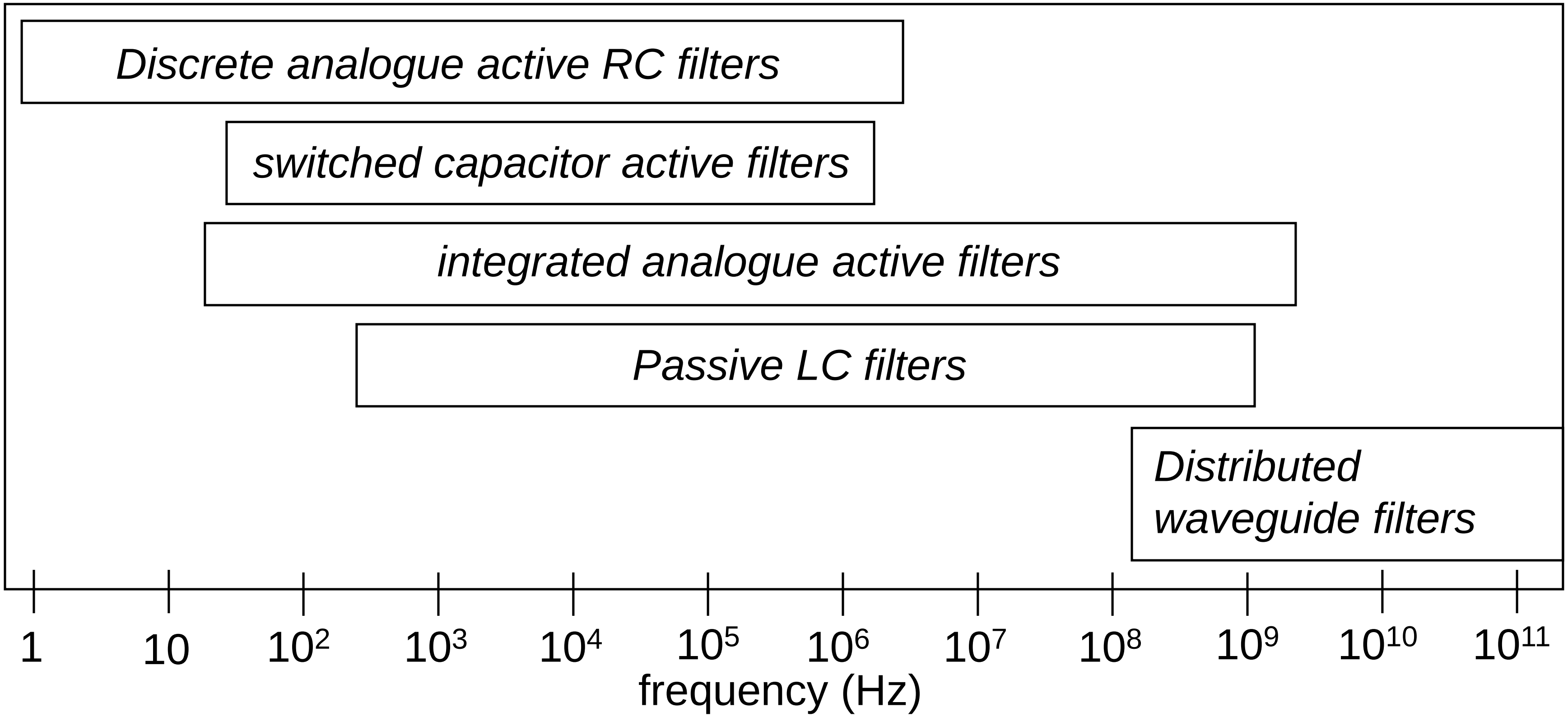
Passive filter example:



Active filter example:



Analogue Filters



Transfer Functions

- The ***Transfer Function*** of the filter is normally given by:

$$H(s) = \frac{v_O(s)}{v_{IN}(s)}$$

- The roots of the numerator are called *zeros*, while the roots of the denominator are called *poles*.

Transfer Functions

- For example:

$$H(s) = \frac{10}{(s+1)(s+2)}$$

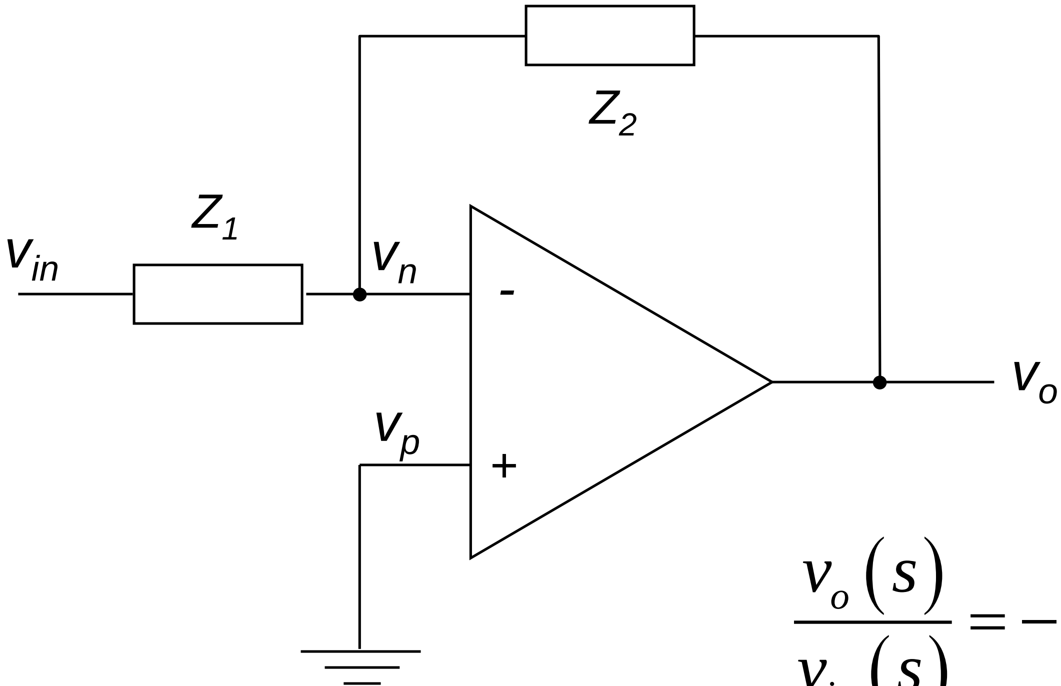
$$H(s) = \frac{1}{s^2 + \sqrt{2}s + 1}$$

$$H(s) = \frac{2(s+3)}{(s+1)(s+2)}$$

$$H(s) = \frac{1}{\left(s + \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right)\left(s + \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right)}$$

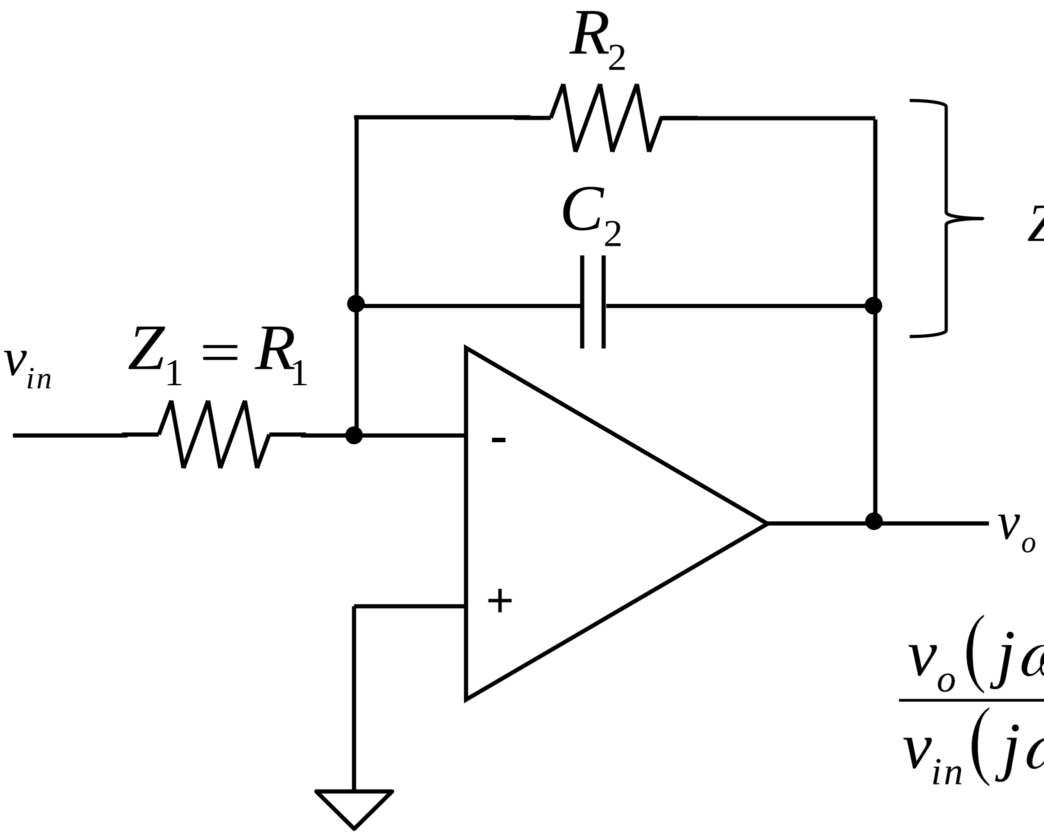
Simple Active Filters

- Inverting Circuit:



$$\frac{v_o(s)}{v_{in}(s)} = -\frac{Z_2}{Z_1}$$

Active Filters – Low Pass Filter 1



$$Z_2 = \frac{\frac{R_2}{j\omega C}}{\frac{1}{j\omega C} + R_2} = \frac{R_2}{1 + j\omega C R_2}$$

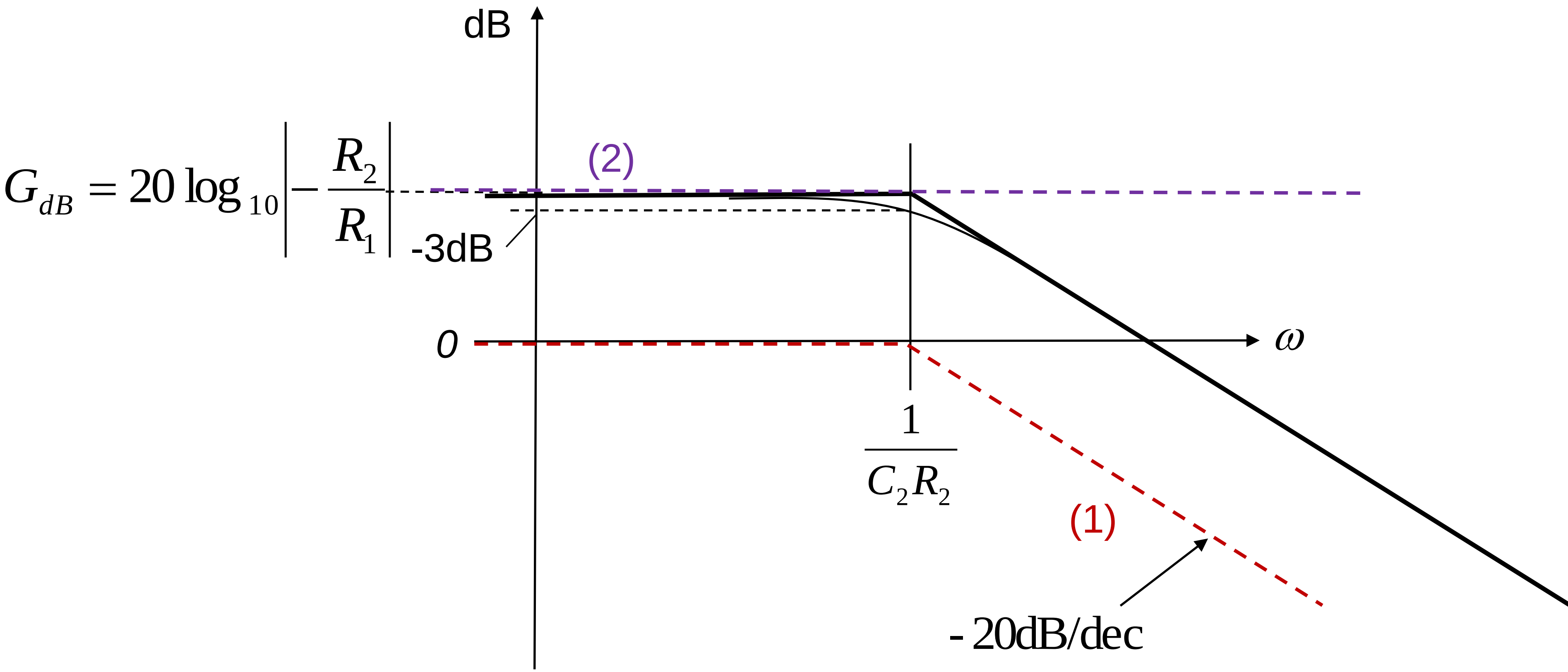
$$\frac{v_o(j\omega)}{v_{in}(j\omega)} = -\frac{Z_2}{Z_1} = -\frac{R_2}{R_1}\left(\frac{1}{1 + j\omega C_2 R_2}\right)$$

(2) (1)

$$\frac{v_o(s)}{v_{in}(s)} = -\frac{R_2}{R_1}\left(\frac{1}{1 + sC_2R_2}\right)$$

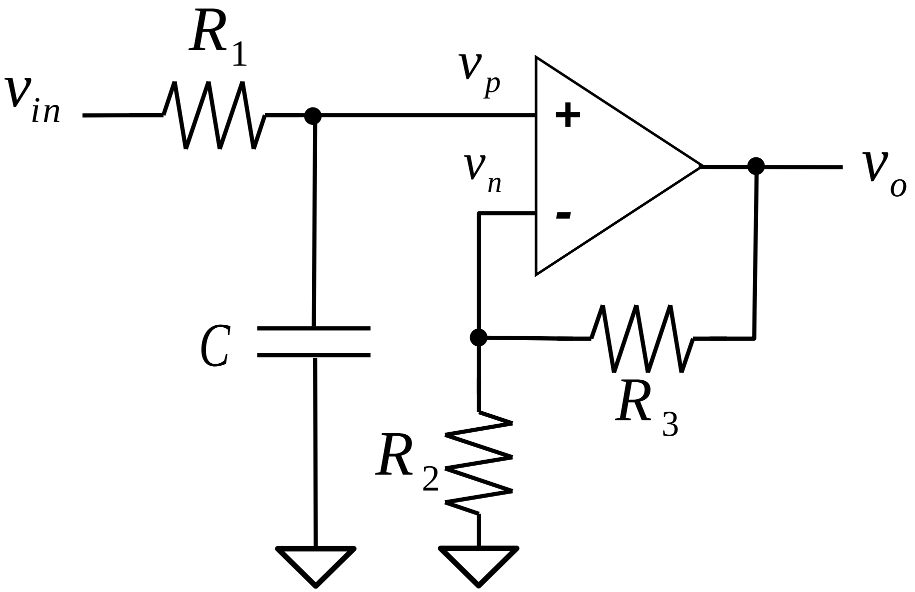
Active Filters – Low Pass Filter 1

$$G_{dB} = 20\log_{10}\left|H(\omega)\right|$$



Active Filters – Low Pass Filter 2

- First order Voltage-Controlled Voltage-Source (VCVS)



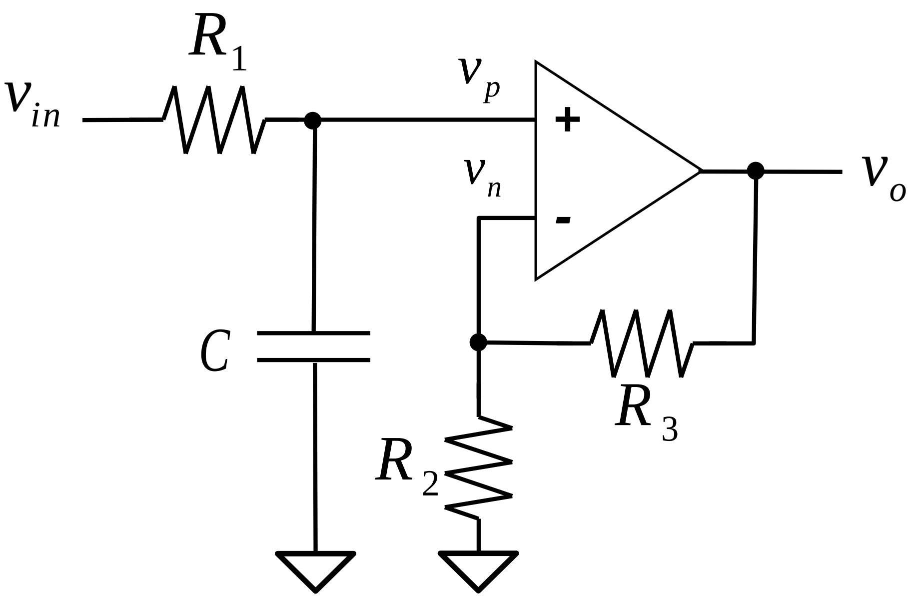
$$v_p = \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R_1} v_{in} = \frac{1}{1 + j\omega C R_1} v_{in}$$

$$v_n = \frac{R_2}{R_2 + R_3} v_o$$

$$\frac{R_2}{R_2 + R_3} v_o = \frac{1}{1 + j\omega C R_1} v_{in}$$

Active Filters – Low Pass Filter 2

- First order Voltage-Controlled Voltage-Source (VCVS)



$$\frac{v_o}{v_{in}} = \left(\frac{R_2 + R_3}{R_2}\right)\left(\frac{1}{1 + j\omega CR_1}\right)$$

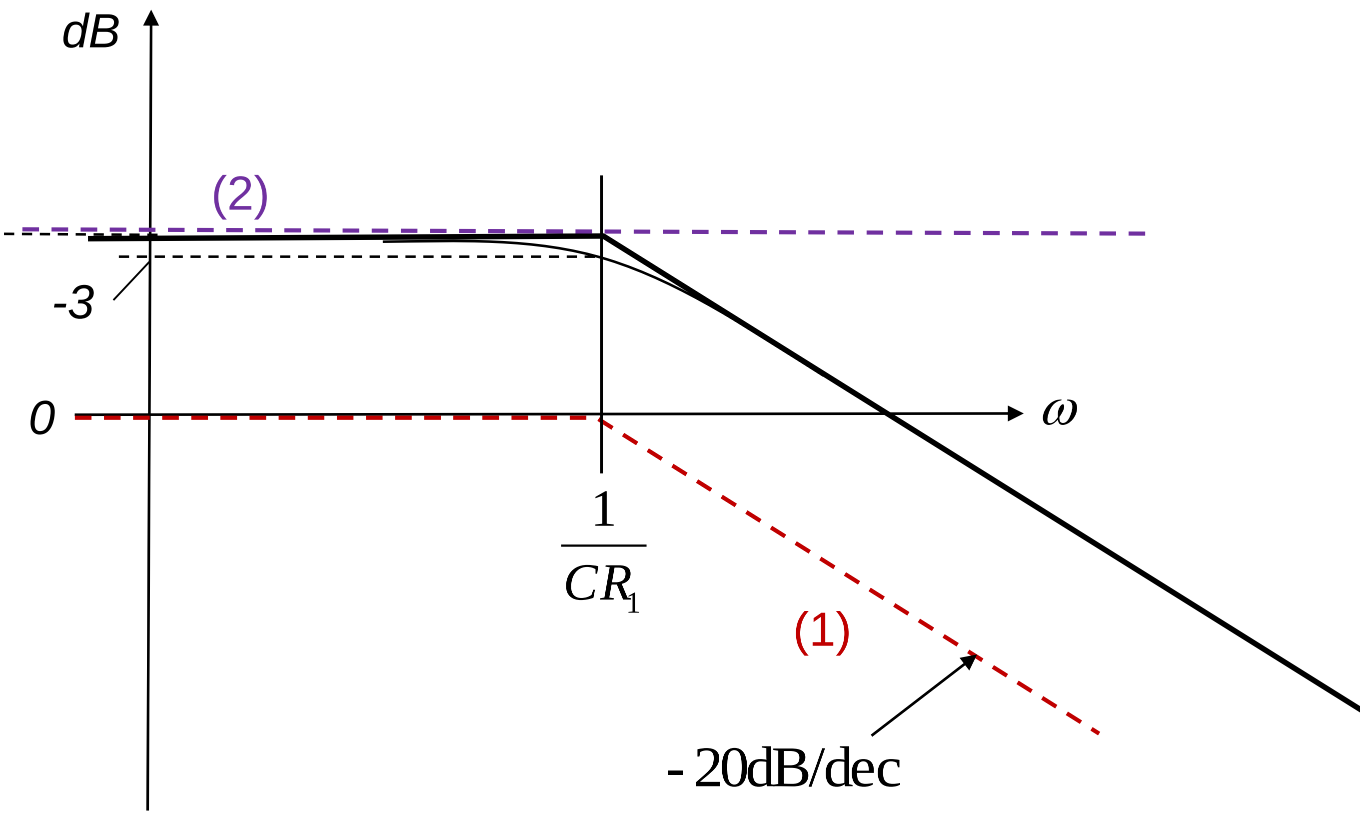
$$\frac{v_o}{v_{in}} = \underbrace{\left(1 + \frac{R_3}{R_2}\right)}_{(2)}\underbrace{\left(\frac{1}{1 + j\omega CR_1}\right)}_{(1)}$$

$$\frac{v_o(s)}{v_{in}(s)} = \left(1 + \frac{R_3}{R_2}\right)\left(\frac{1}{1 + sCR_1}\right)$$

Active Filters – Low Pass Filter 2

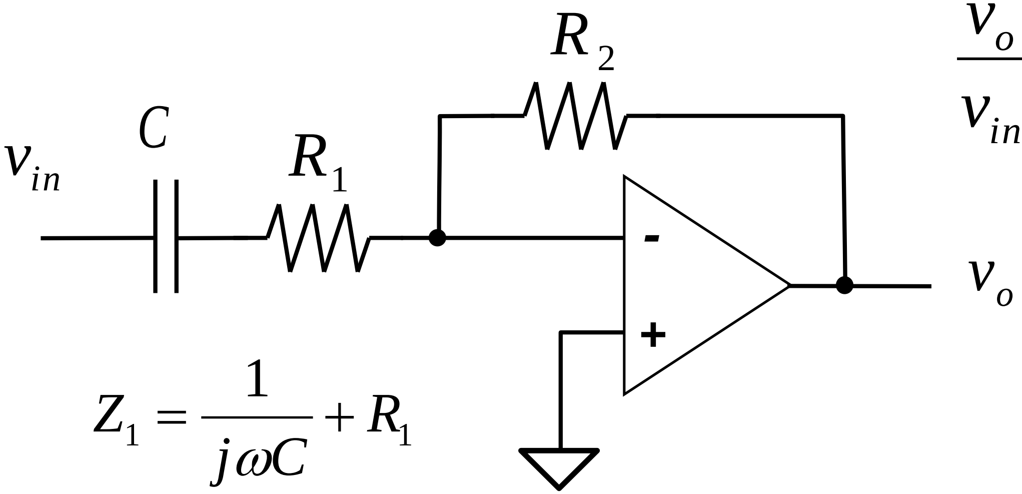
$$G_{dB} = 20\log_{10}\left|H(\omega)\right|$$

$$G_{dB} = 20\log_{10}\left(1+\frac{R_3}{R_2}\right)$$



Active Filters – High Pass Filter 1

$$Z_2 = R_2$$



$$Z_1 = \frac{1}{j\omega C} + R_1$$

$$Z_1 = \frac{1 + j\omega C R_1}{j\omega C}$$

(1)

$$\frac{v_o(\omega)}{v_{in}(\omega)} = -\frac{Z_2}{Z_1} = -\frac{j\omega C R_2}{1 + j\omega C R_1}$$

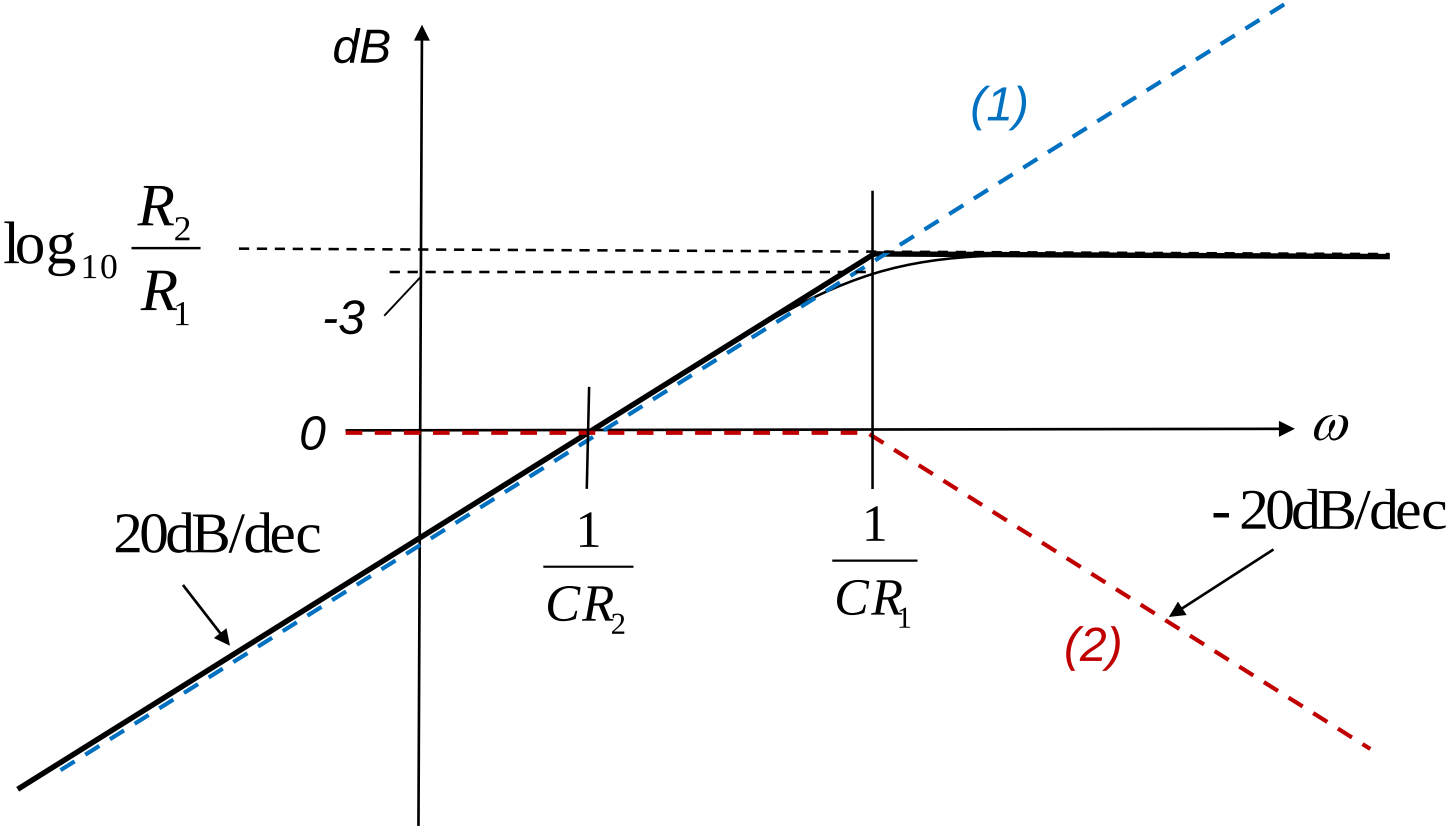
(2)

$$\frac{v_o(s)}{v_{in}(s)} = -\frac{Z_2}{Z_1} = -\frac{sCR_2}{1 + sCR_1}$$

Active Filters – High Pass Filter 1

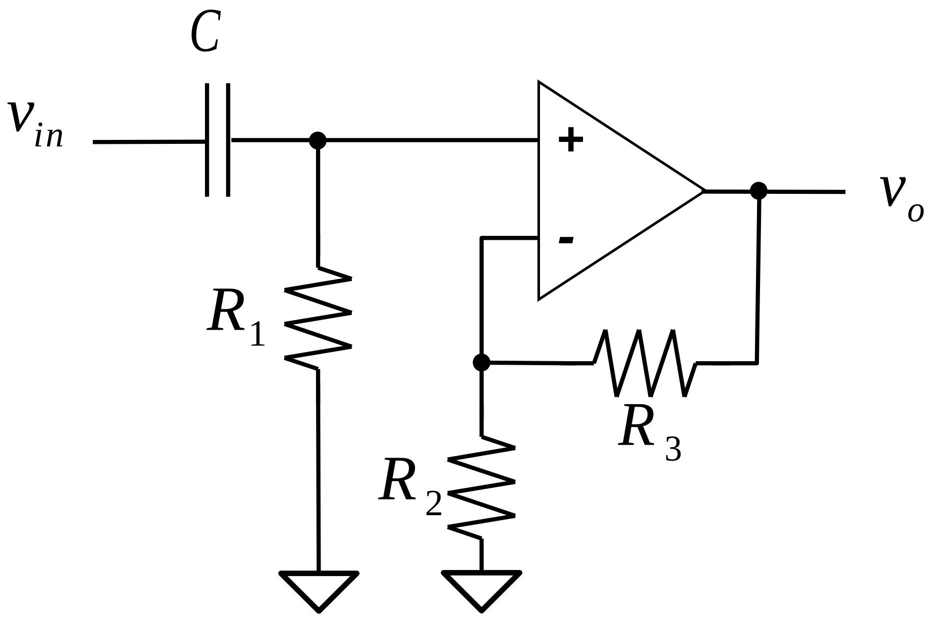
$$G_{dB} = 20\log_{10}\left|H(\omega)\right|$$

$$G_{dB} = 20\log_{10}\frac{R_2}{R_1}$$



Active Filters – High Pass Filter 2

- First order Voltage-Controlled Voltage-Source (VCVS)



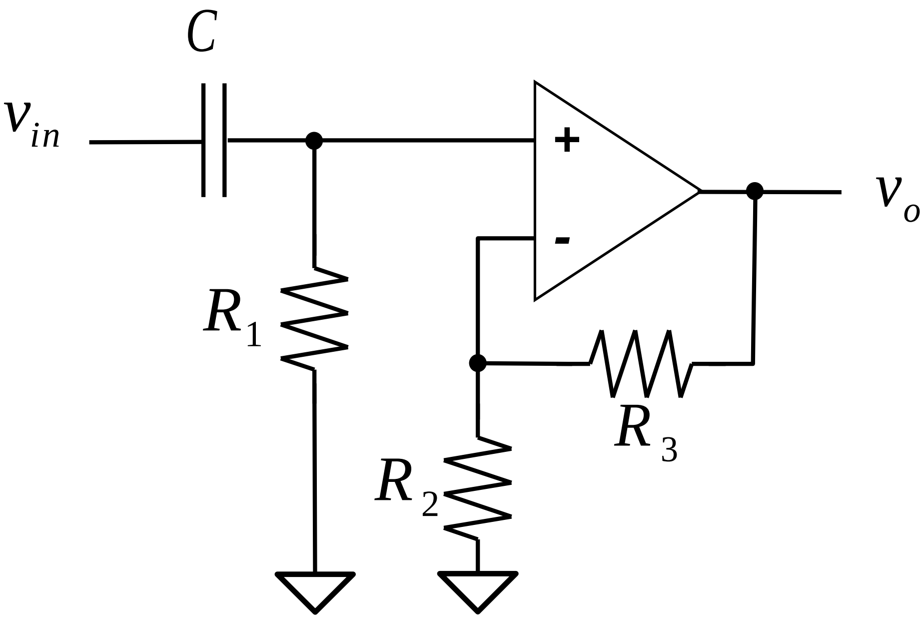
$$v_p = \frac{R_1}{\frac{1}{j\omega C} + R_1} v_{in} = \frac{j\omega C R_1}{1 + j\omega C R_1} v_{in}$$

$$v_n = \frac{R_2}{R_2 + R_3} v_o$$

$$\frac{R_2}{R_2 + R_3} v_o = \frac{j\omega C R_1}{1 + j\omega C R_1} v_{in}$$

Active Filters – High Pass Filter 2

- First order Voltage-Controlled Voltage-Source (VCVS)



$$\frac{v_o}{v_{in}} = \left(\frac{R_2 + R_3}{R_2}\right)\left(\frac{j\omega CR_1}{1 + j\omega CR_1}\right)$$

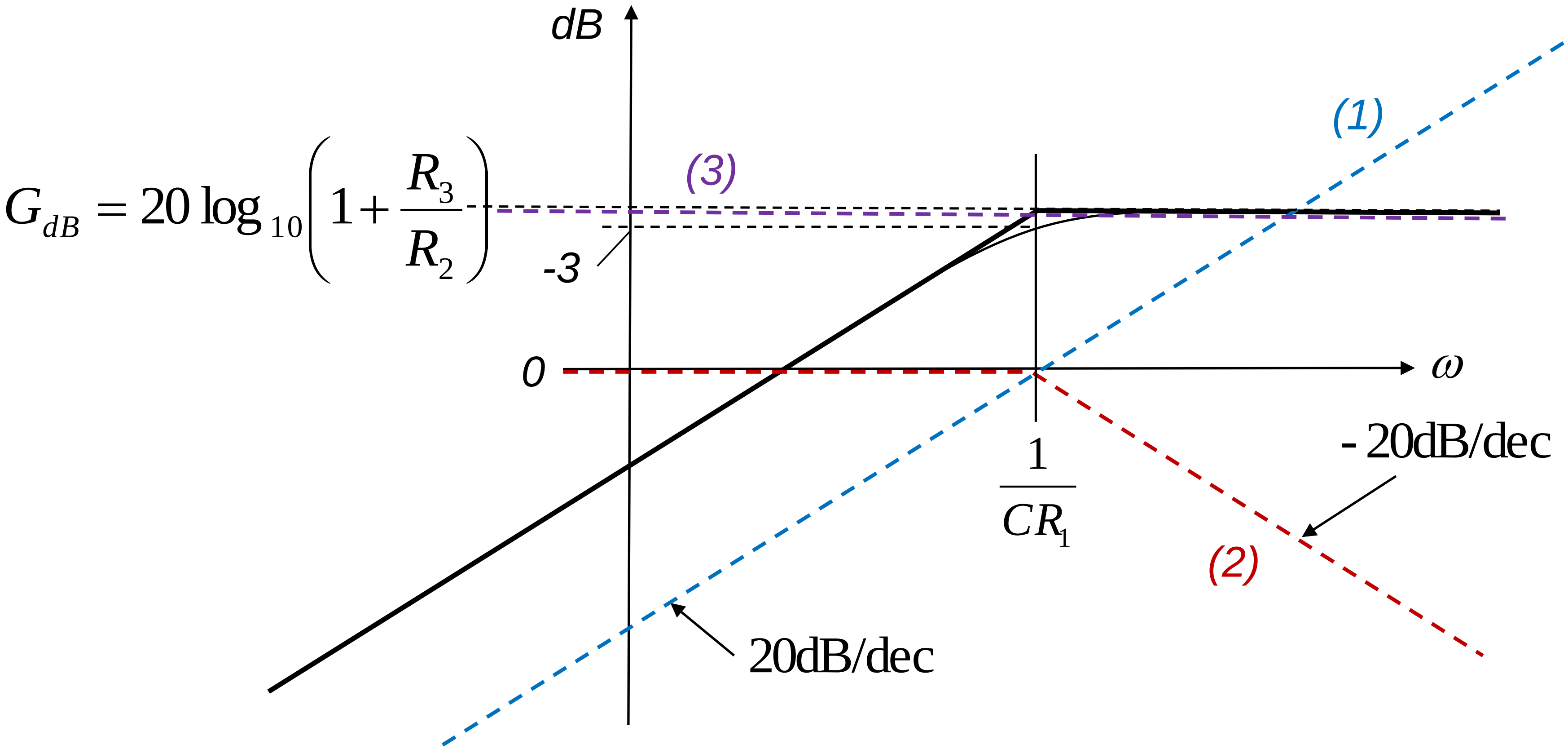
$$\frac{v_o}{v_{in}} = \underbrace{\left(1 + \frac{R_3}{R_2}\right)}_{(3)}\left(\frac{j\omega CR_1}{1 + j\omega CR_1}\right)$$

(1)

(2)

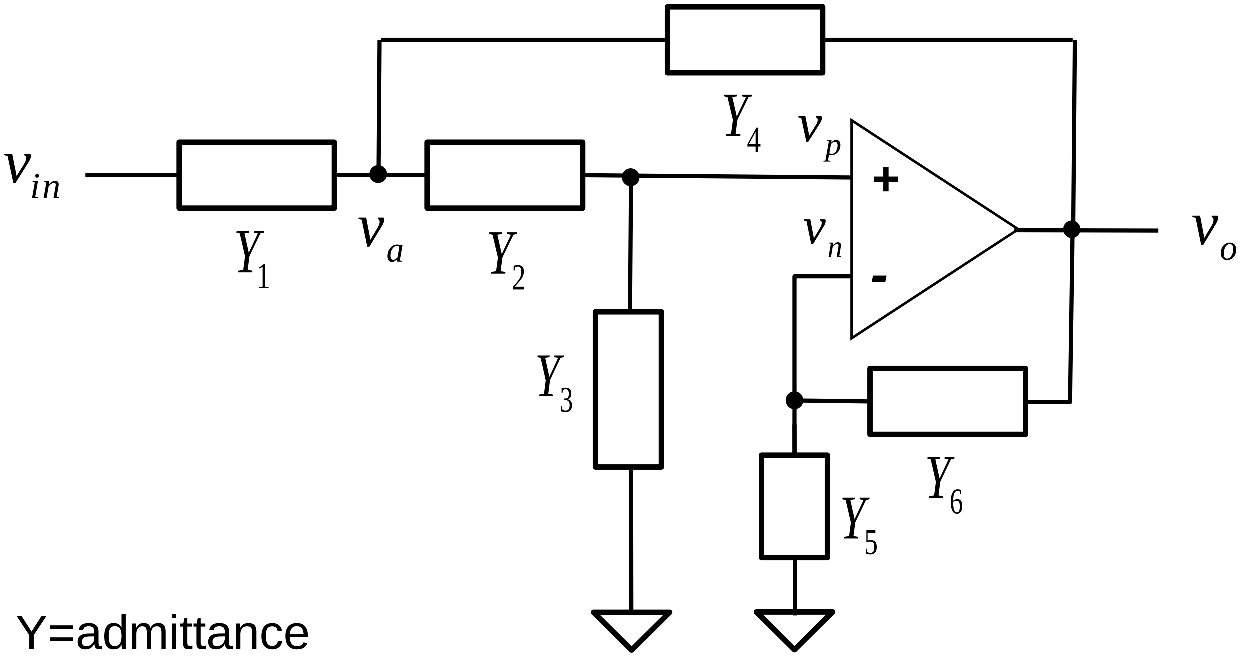
Active Filters – High Pass Filter 2

$$G_{dB} = 20\log_{10}\left|H(\omega)\right|$$



Sallen Key Circuit

- Second order Voltage-Controlled Voltage-Source (VCVS)



Y=admittance

Sallen Key Circuit

Node v_a :
$$(v_{in} - v_a)Y_1 + (v_o - v_a)Y_4 = (v_a - v_p)Y_2$$

$$v_a = \frac{1}{Y_1 + Y_2 + Y_4}\left(v_{in}Y_1 + v_pY_2 + v_oY_4\right) \qquad \ldots(1)$$

Node v_p :
$$(v_a - v_p)Y_2 = v_pY_3 \quad \Rightarrow \quad v_p = \frac{Y_2}{Y_2 + Y_3}v_a \qquad \ldots(2)$$

Node v_n :
$$(v_o - v_n)Y_6 = v_nY_5 \quad \Rightarrow \quad v_n = \frac{Y_6}{Y_5 + Y_6}v_o \qquad \ldots(3)$$

Sallen Key Circuit

Subst (1) into (2):

$$v_p = \left(\frac{Y_2}{Y_2+Y_3}\right)\left(\frac{1}{Y_1+Y_2+Y_4}\right)\left(v_{in}Y_1 + v_pY_2 + v_oY_4\right)$$

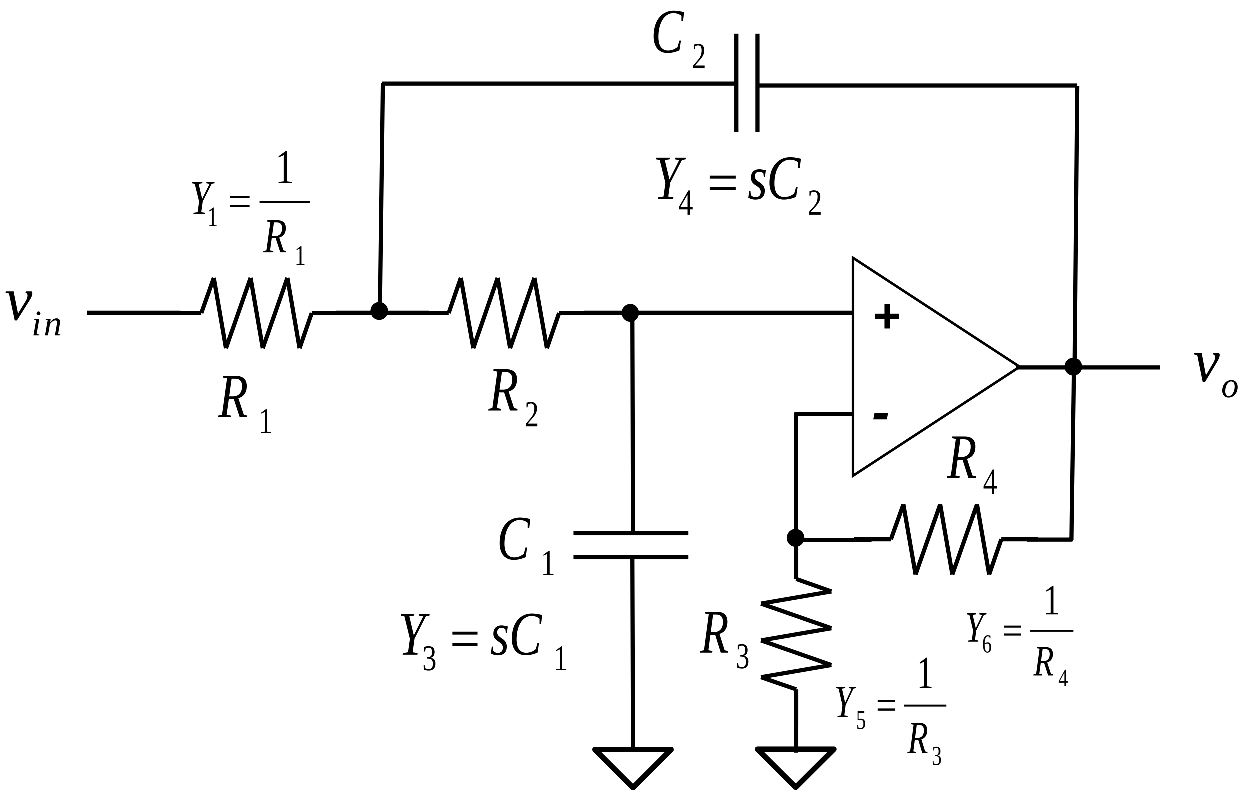
$$v_p = \frac{v_{in}Y_1Y_2 + v_oY_2Y_4}{(Y_2+Y_3)(Y_1+Y_2+Y_4) - Y_2^2} \qquad \ldots(4)$$

Since $v_n = v_p$ we can equate (3) and (4):

$$\frac{Y_6}{Y_5+Y_6}v_o = \frac{v_{in}Y_1Y_2 + v_oY_2Y_4}{(Y_2+Y_3)(Y_1+Y_2+Y_4) - Y_2^2}$$

$$\frac{v_o}{v_{in}} = \frac{Y_1Y_2(Y_5+Y_6)/Y_6}{(Y_2+Y_3)(Y_1+Y_2+Y_4) - Y_2^2 - Y_2Y_4(Y_5+Y_6)/Y_6} \qquad \ldots(5)$$

Sallen Key Low Pass

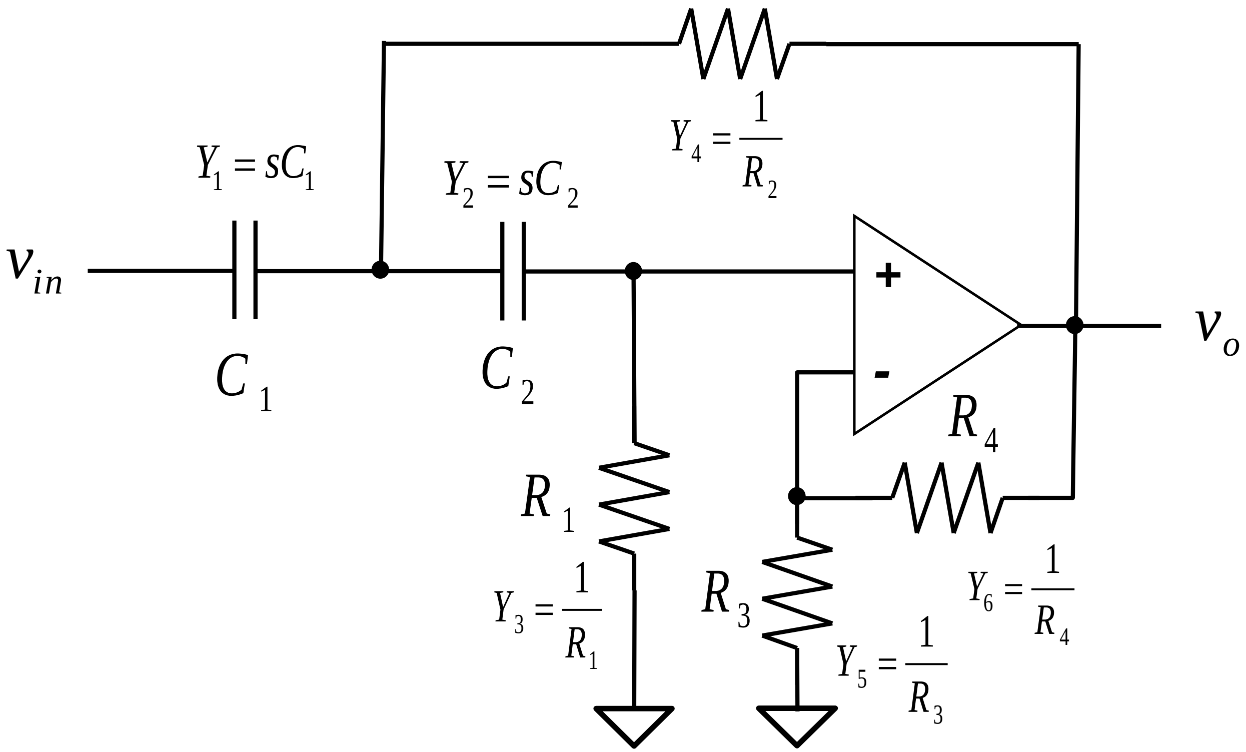


Sallen Key Low Pass

$$\frac{v_o}{v_{in}} = \frac{\frac{1}{R_1R_2}\left(1+\frac{R_4}{R_3}\right)}{\left(\frac{1}{R_2}+sC_1\right)\left(\frac{1}{R_1}+\frac{1}{R_2}+sC_2\right)-\left(\frac{1}{R_2}\right)^2-\frac{1}{R_2}sC_2\left(1+\frac{R_4}{R_3}\right)}$$

$$\frac{v_o}{v_{in}} = \frac{\frac{K}{R_1R_2C_1C_2}}{s^2+s\left(\frac{1}{R_1C_2}+\frac{1}{R_2C_2}+\frac{1}{R_2C_1}-\frac{K}{R_2C_1}\right)+\frac{1}{R_1R_2C_1C_2}}; \qquad \left(K = 1+\frac{R_4}{R_3}\right)$$

Sallen Key High Pass



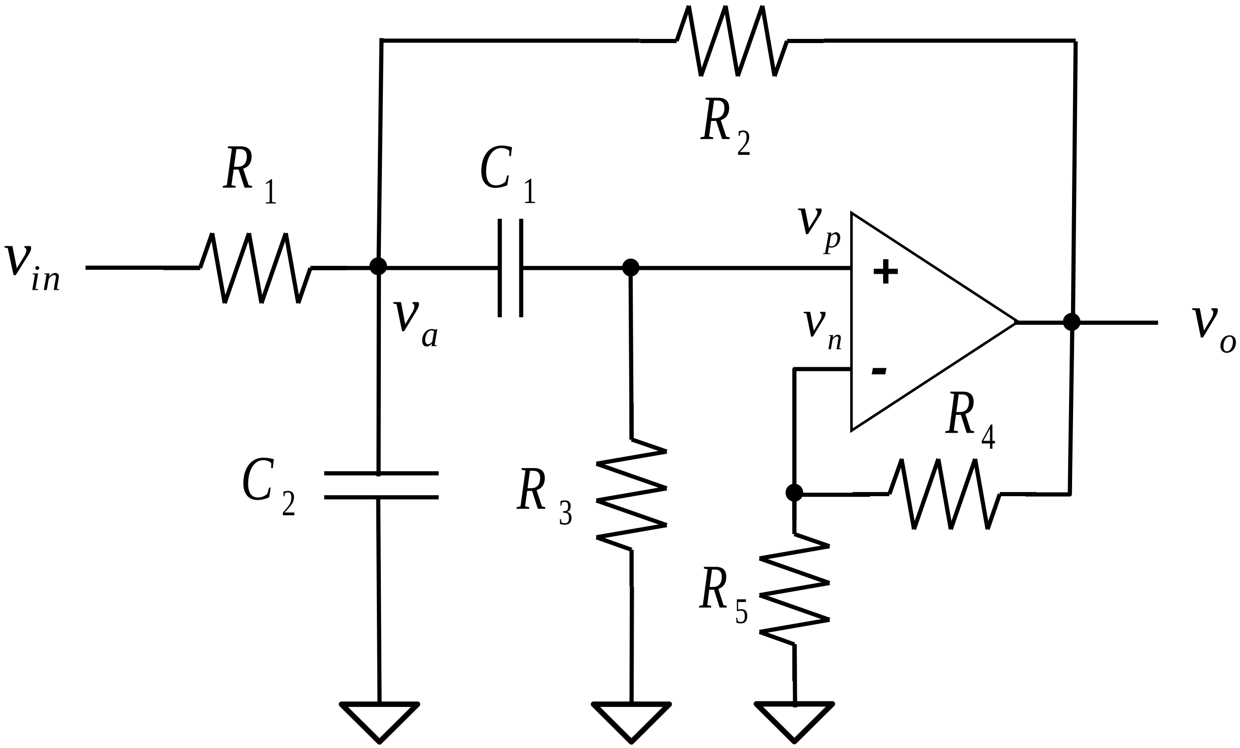
Sallen Key High Pass

$$\frac{v_o}{v_{in}} = \frac{sC_1 sC_2 K}{\left(sC_2 + \frac{1}{R_1}\right)\left(sC_1 + sC_2 + \frac{1}{R_2}\right) - s^2 C_2^2 - \frac{sC_2}{R_2} K}$$

$$\frac{v_o}{v_{in}} = \frac{Ks^2}{s^2 + s\left(\frac{1}{R_1 C_1} + \frac{1}{R_1 C_2} + \frac{1-K}{R_2 C_1}\right) + \frac{1}{R_1 R_2 C_1 C_2}}$$

$$\left(K = 1 + \frac{R_4}{R_3}\right)$$

Sallen Key Band Pass



Sallen Key Band Pass

Node v_a :

$$\frac{(v_{in}-v_a)}{R_1}+\frac{(v_o-v_a)}{R_2}Y_4 = v_a sC_2 + (v_a - v_p)sC_1$$

$$\frac{v_{in}}{R_1}+\frac{v_o}{R_2}+v_p sC_1 = v_a\left(\frac{1}{R_1}+\frac{1}{R_2}+sC_1+sC_2\right) \qquad \ldots(1)$$

Node v_p :

$$(v_a - v_p)sC_1 = \frac{v_p}{R_3} \quad \Rightarrow \quad v_a sC_1 = v_p\left(\frac{1}{R_3}+sC_1\right) \qquad \ldots(2)$$

Node v_n :

$$v_n = v_o / K \qquad \ldots(3)$$

$$K = 1+\frac{R_4}{R_5}$$

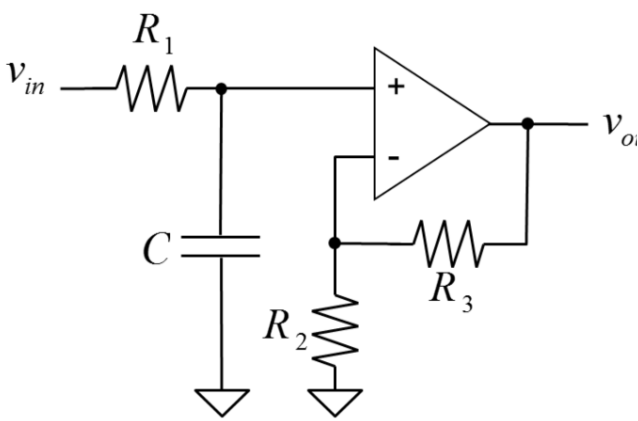
Sallen Key Band Pass

- Setting $v_n = v_p$, substituting (2) and (3) into (1) and rearranging yields:

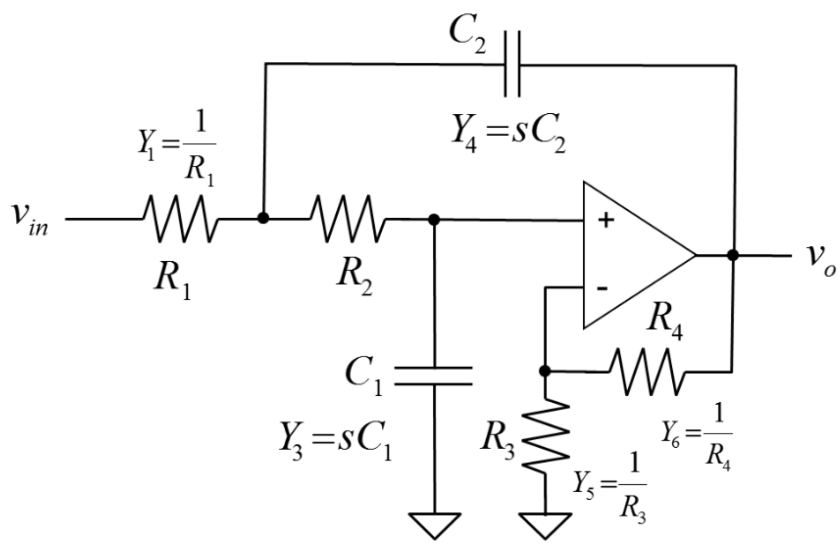
$$\frac{v_o}{v_{in}} = \frac{sK/R_1C_2}{s^2 + s\left(\frac{1}{C_2R_3} + \frac{1}{C_1R_3} + \frac{1}{C_2R_1} + \frac{(1-K)}{C_2R_2}\right) + \frac{R_1 + R_2}{R_1R_2R_3C_1C_2}}$$

Higher Order Filters

- VCVS (Sallen Key) circuits are a good choice to implement first and second order stages.



$$H(s) = \frac{K_1}{(s+a)}$$



$$H(s) = \frac{K_2}{s^2 + bs + c}$$

Higher Order Filters

- VCVS filters have good isolation properties (high input impedance, low output impedance), which means they may be cascaded to form higher order filters.

Filter Design – Frequency and Magnitude Scaling

- To simplify calculations in filter design, it is sometimes convenient to work with element values of 1Ω, 1F and 1H.

- The values are then transformed to realistic values by *scaling*.

Magnitude Scaling

- *Magnitude scaling* increases all impedances by a factor, K_m, but the frequency response is unchanged.

- Resistors and inductors are multiplied by K_m and capacitors are multiplied by $1/K_m$.

$$R' = K_m R$$

$$L' = K_m L$$

$$C' = C / K_m$$

Frequency Scaling

- *Frequency scaling* shifts the frequency response, while leaving the impedance unchanged.

- Resistors are unaffected by frequency scaling.
- Both inductors and capacitors are divided by scaling factor K_f.

$$R' = R$$

$$L' = L/K_f$$

$$C' = C/K_f$$

Magnitude and Frequency Scaling

- Magnitude and frequency scaling can be combined:

$$R' = K_m R$$

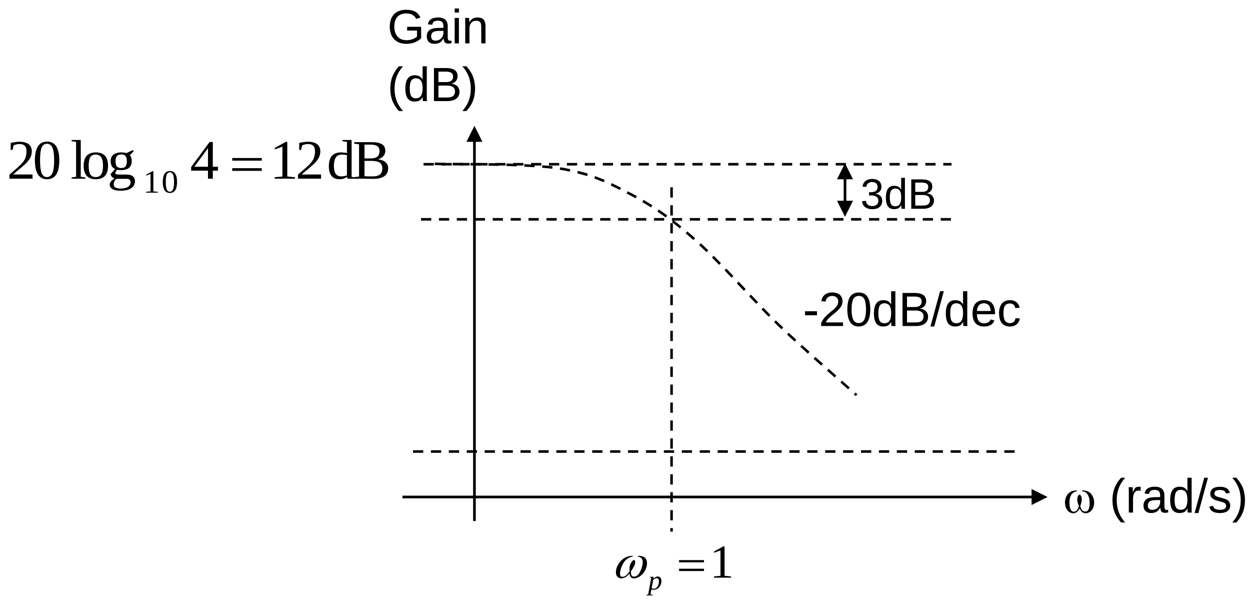
$$L' = \frac{K_m}{K_f} L$$

$$C' = \frac{1}{K_m K_f} C$$

First Order Low Pass

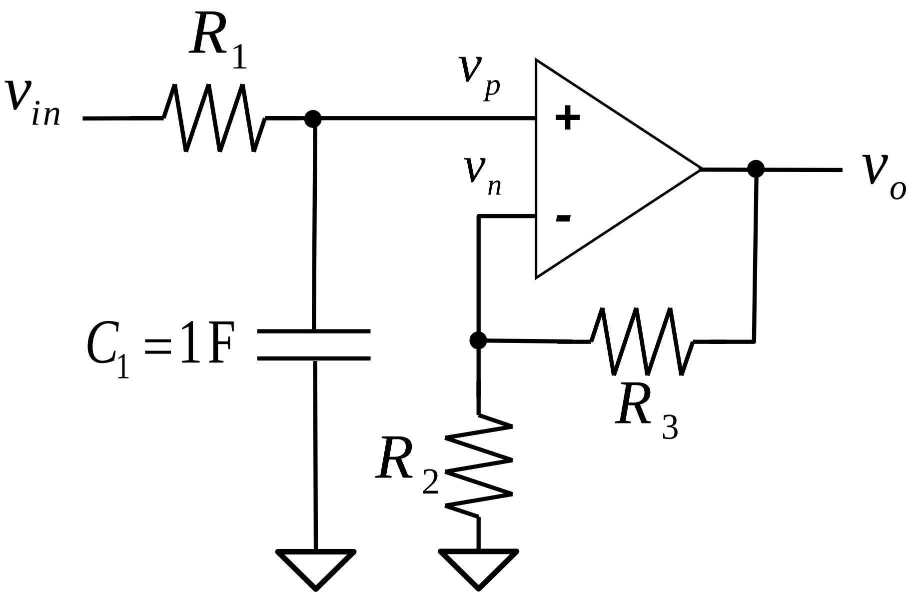
Example 2(a)

- Use the first order VCVS to design a prototype first order low pass filter with low frequency gain of 4, and with a 3dB cutoff frequency of ω_p=1rad/s. Let C_1 = 1F.



First Order Low Pass

Example 2(a)



$$\frac{v_o(s)}{v_{in}(s)} = \left(1 + \frac{R_3}{R_2}\right)\left(\frac{1}{1 + sC_1R_1}\right)$$

$$\frac{v_o(s)}{v_{in}(s)} = K\left(\frac{1/C_1R_1}{s + 1/C_1R_1}\right) \qquad \ldots(1)$$

where $K = 1 + \dfrac{R_3}{R_2}$

First Order Low Pass

Example 2(a)

- First order low pass filter, gain = 4, cutoff frequency = 1 rad/s:

$$\frac{V_o(s)}{V_{in}(s)} = \frac{4}{s+1} \qquad \ldots(2)$$

- Compare (1) and (2):

First Order Low Pass

Example 2(a)

- Choose C_1 = 1F
- Therefore since $\dfrac{1}{C_1 R_1} = 1$, $R_1 = \dfrac{1}{C_1} = 1\Omega$.
- Gain, K:

First Order Low Pass

Example 2(a)

- To reduce offset current effect, resistance seen by each input should be equal at DC:

$$R_3 // R_2 = R_1$$

$$\frac{R_3 R_2}{R_3 + R_2} = R_1$$

$$\frac{R_3}{K} = R_1$$

- Values of R_2 and R_3:

First Order Low Pass

Example 2(a)

- Component values:

$$C_1 = 1\text{F}$$

$$R_1 = 1\Omega$$

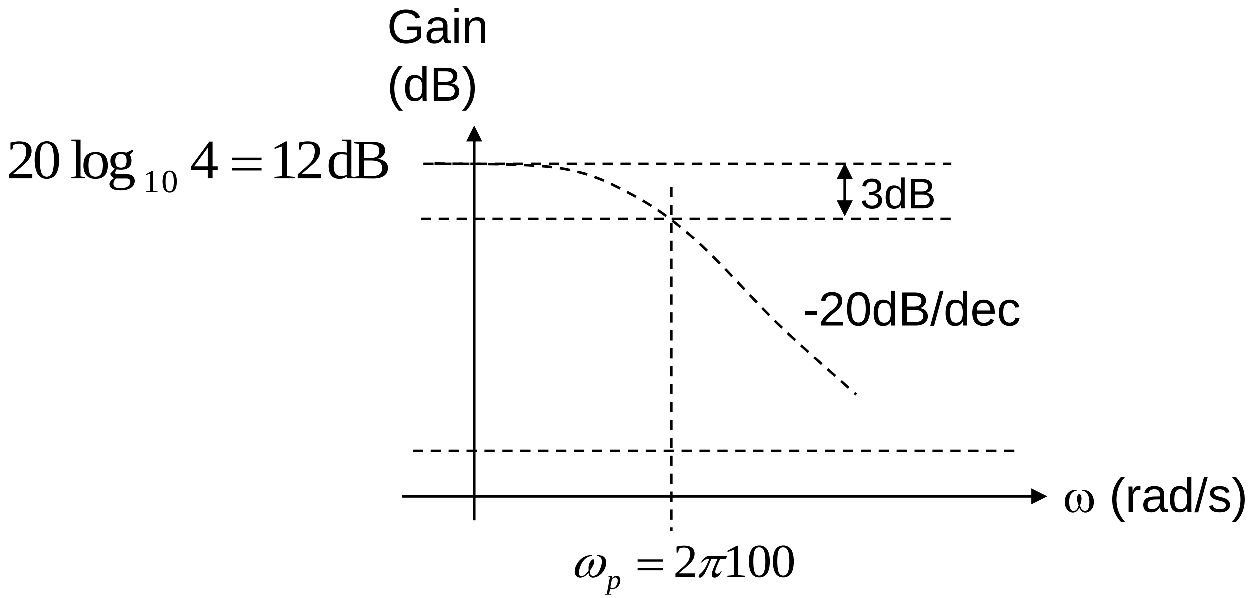
$$R_2 = 4/3\,\Omega$$

$$R_3 = 4\Omega$$

First Order Low Pass

Example 2(b)

- Choose a realistic capacitor value, and scale the design so that the 3dB cutoff frequency is 100Hz.



First Order Low Pass

Example 2(b)

- Rough estimate of C_1':

$$C_1' = \frac{10}{f_p}\mu\text{F}$$

- Frequency scaling:

$$K_f = \frac{\text{new cutoff freq.}}{\text{old cutoff freq.}} =$$

First Order Low Pass

Example 2(b)

- Work out magnitude scale factor.
- Since:

$$C_1' = \frac{C_1}{K_m K_f}$$

- Rearranging:

$$K_m = \frac{C_1}{K_f C_1'} =$$

- Now that we have K_f and K_m, all components can be scaled to their new values.

First Order Low Pass

Example 2(b)

- New component values:

$$R_1' = K_m R_1 = 15.9\text{k} \times 1 = 15.9\text{k}\Omega$$

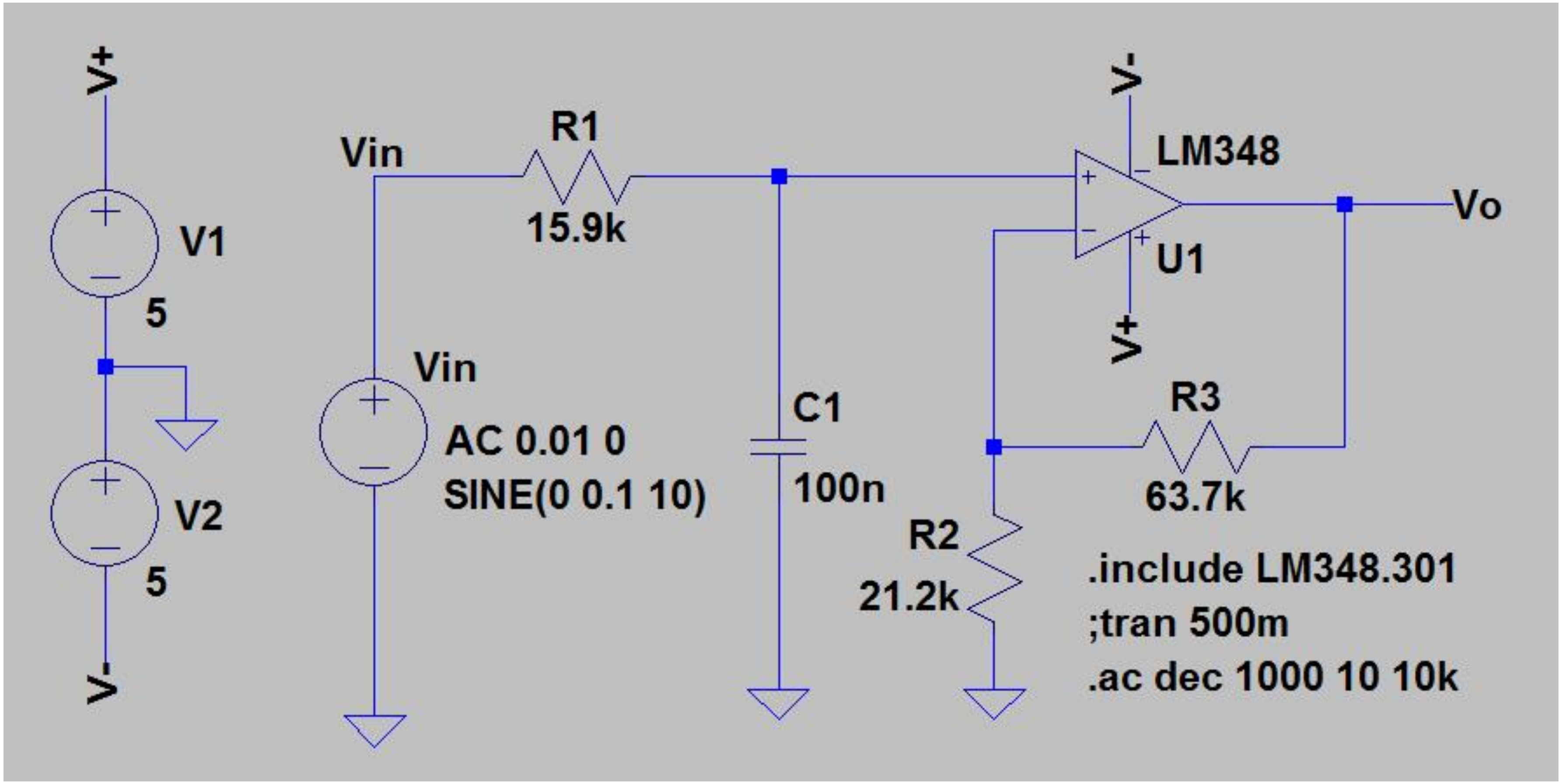
$$R_2' = K_m R_2 = 15.9\text{k} \times 4/3 = 21.2\text{k}\Omega$$

$$R_3' = K_m R_3 = 15.9\text{k} \times 4 = 63.7\text{k}\Omega$$

$$\text{C}_1' = \frac{C_1}{K_m K_f} = 100\text{nF} \qquad \text{(already determined anyway)}$$

First Order Low Pass

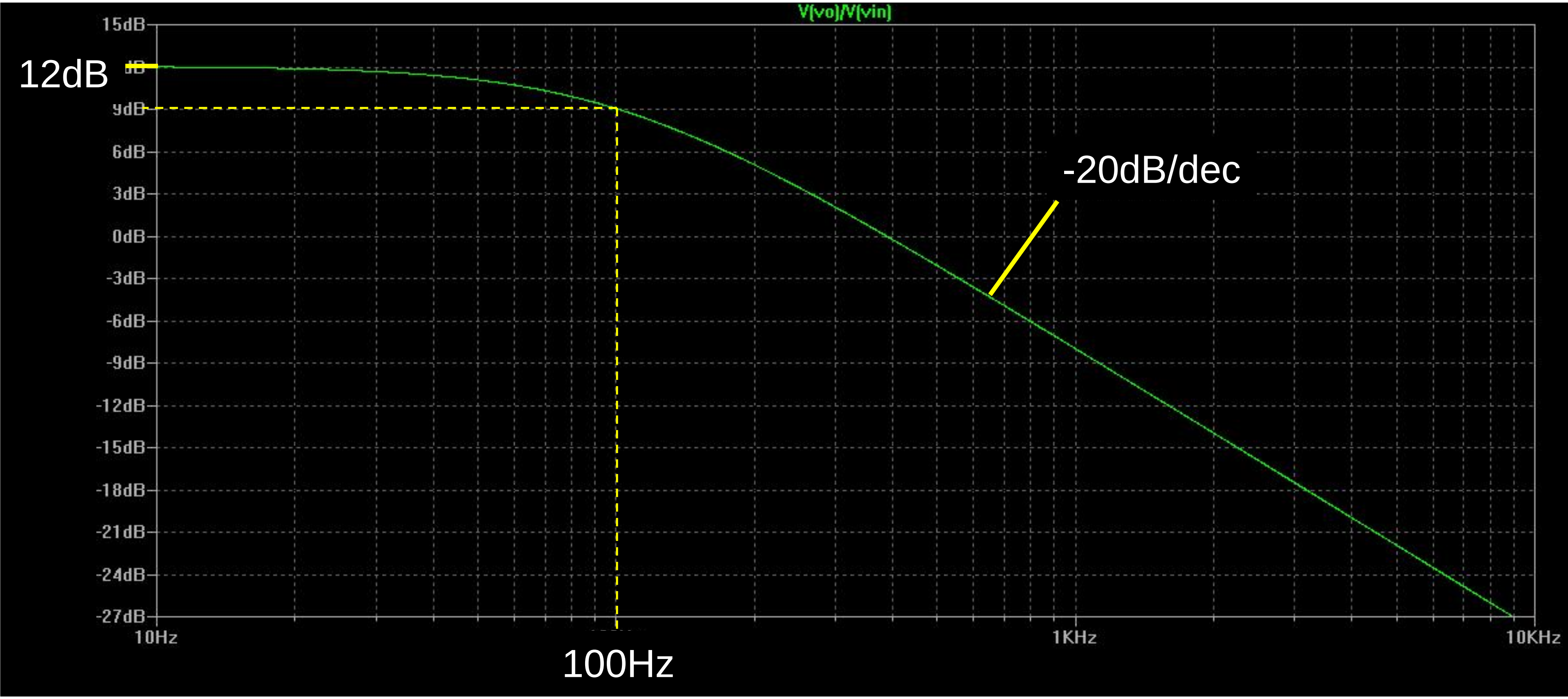
Example 2(b)



First Order Low Pass

Example 2(b)

- Frequency response in LTSpice:



First Order Low Pass

Example 2(b)

- Transfer function:

$$\frac{v_o(s)}{v_{in}(s)} = K\frac{1/C_1R_1}{s+1/C_1R_1} = \left(\frac{R_2+R_3}{R_2}\right)\frac{1/C_1R_1}{s+1/C_1R_1}$$

$$\frac{v_o(s)}{v_{in}(s)} = \frac{2518.7}{s+628.9}$$

- To display Bode plot in Matlab:

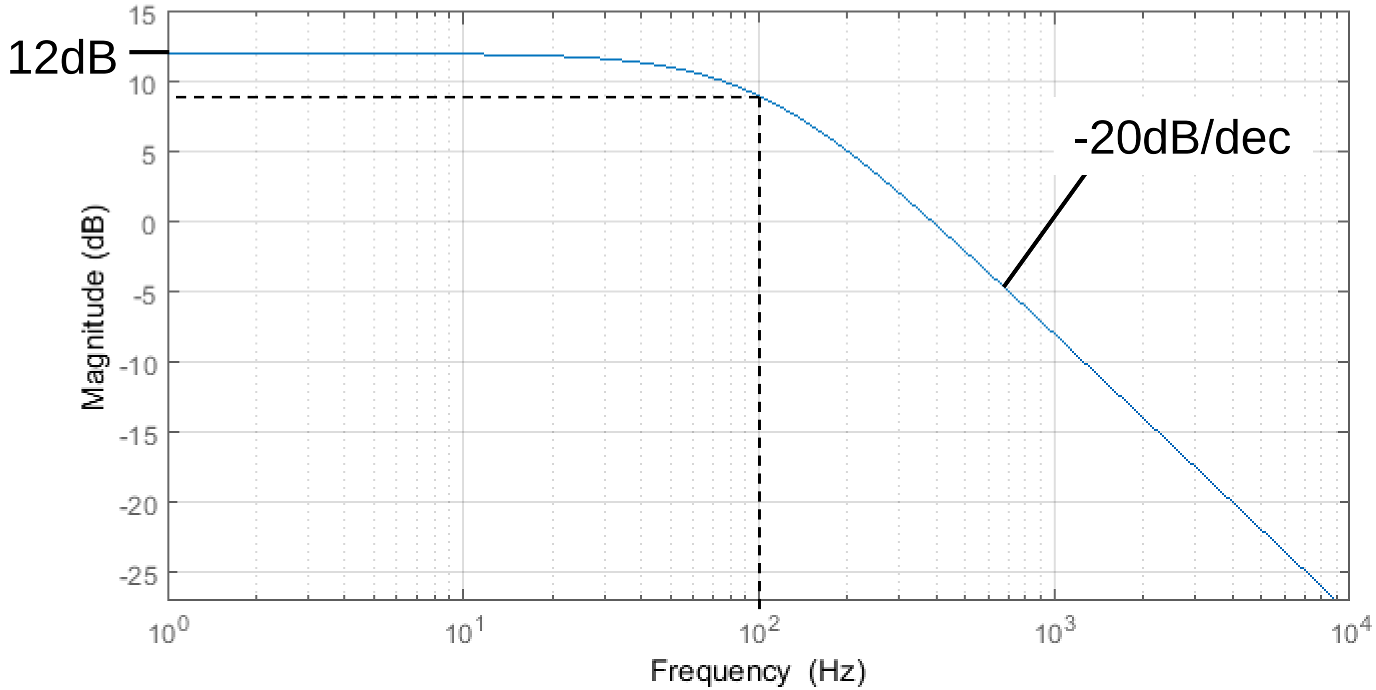
```
K1 = (R2A+R3A)/R2A;
t1 = tf ([0 K1/(C1A*R1A)], [1 1/(C1A*R1A)]);

figure(1);
h = bodeplot(t1);
setoptions(h,'FreqUnits','Hz');
grid on;
```

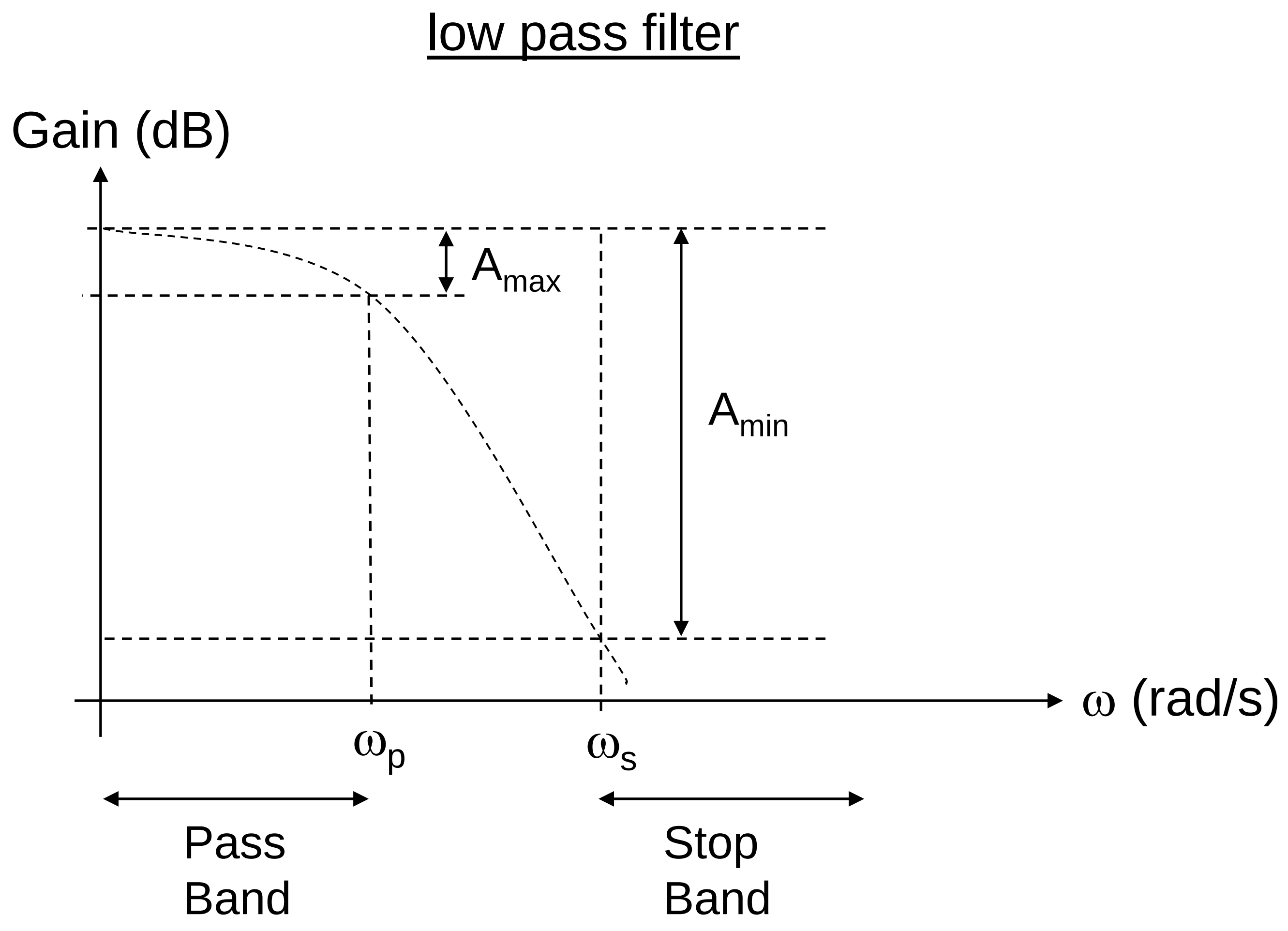
First Order Low Pass

Example 2(b)

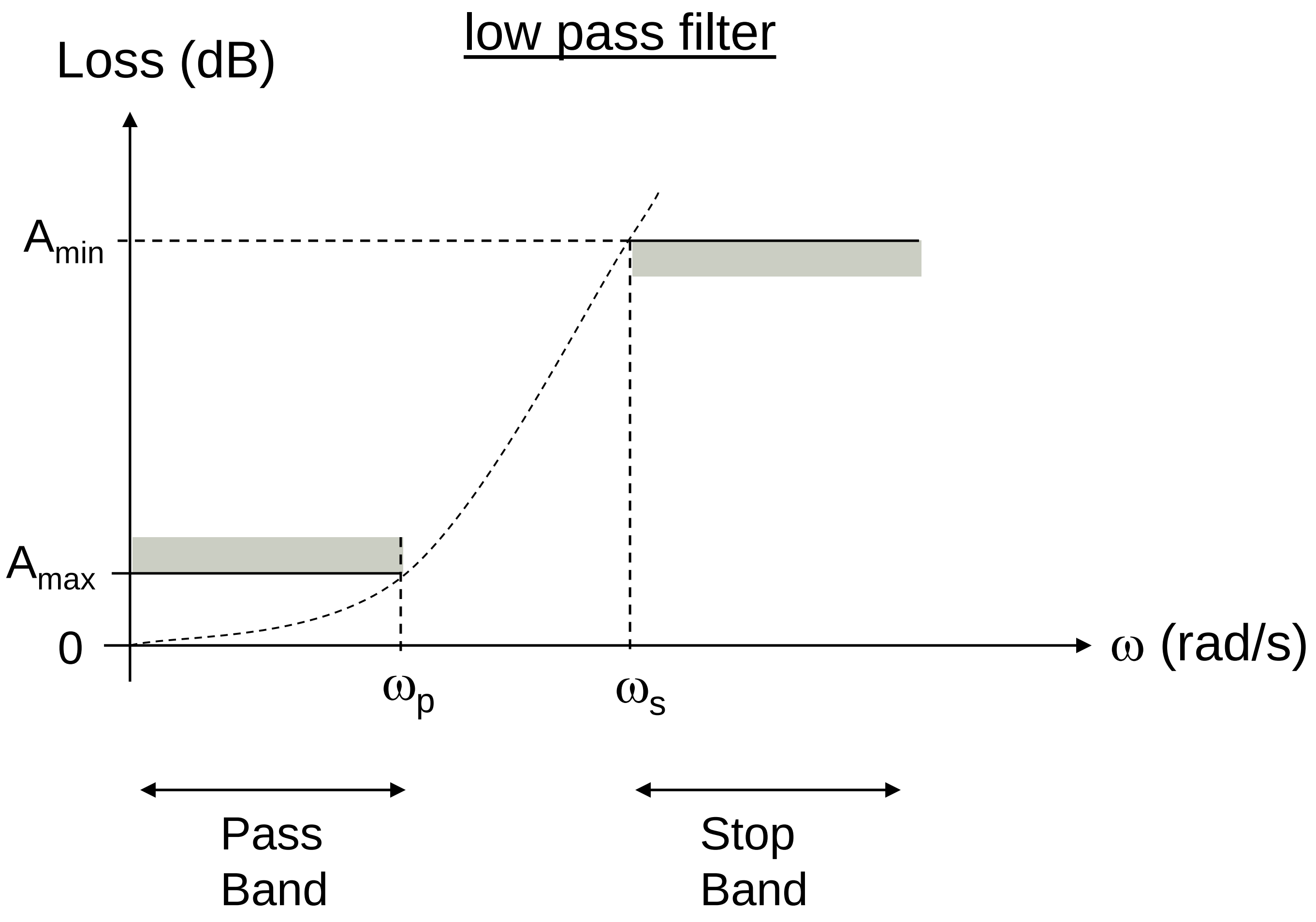
- magnitude plot in Matlab:



Filter Gain Characteristics



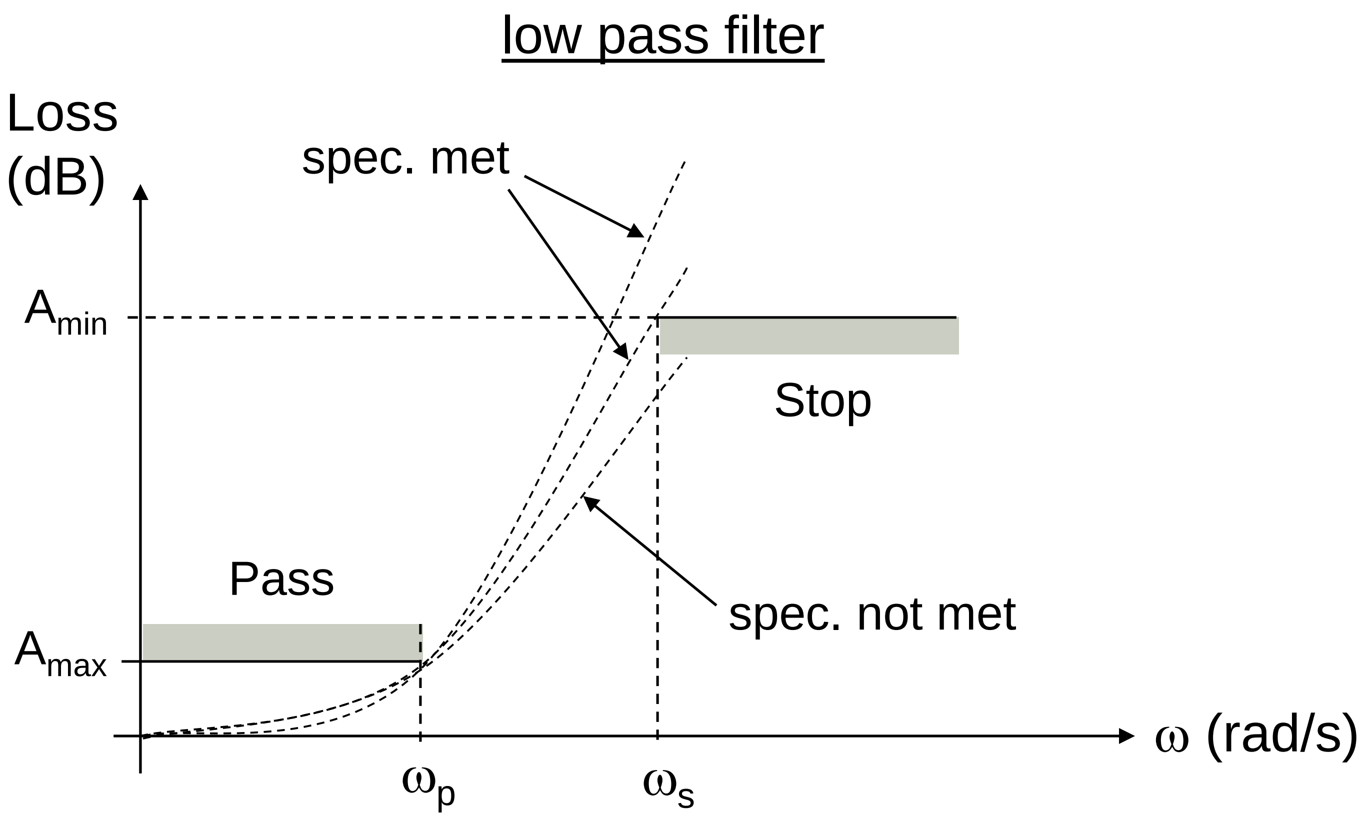
Filter Attenuation Characteristics



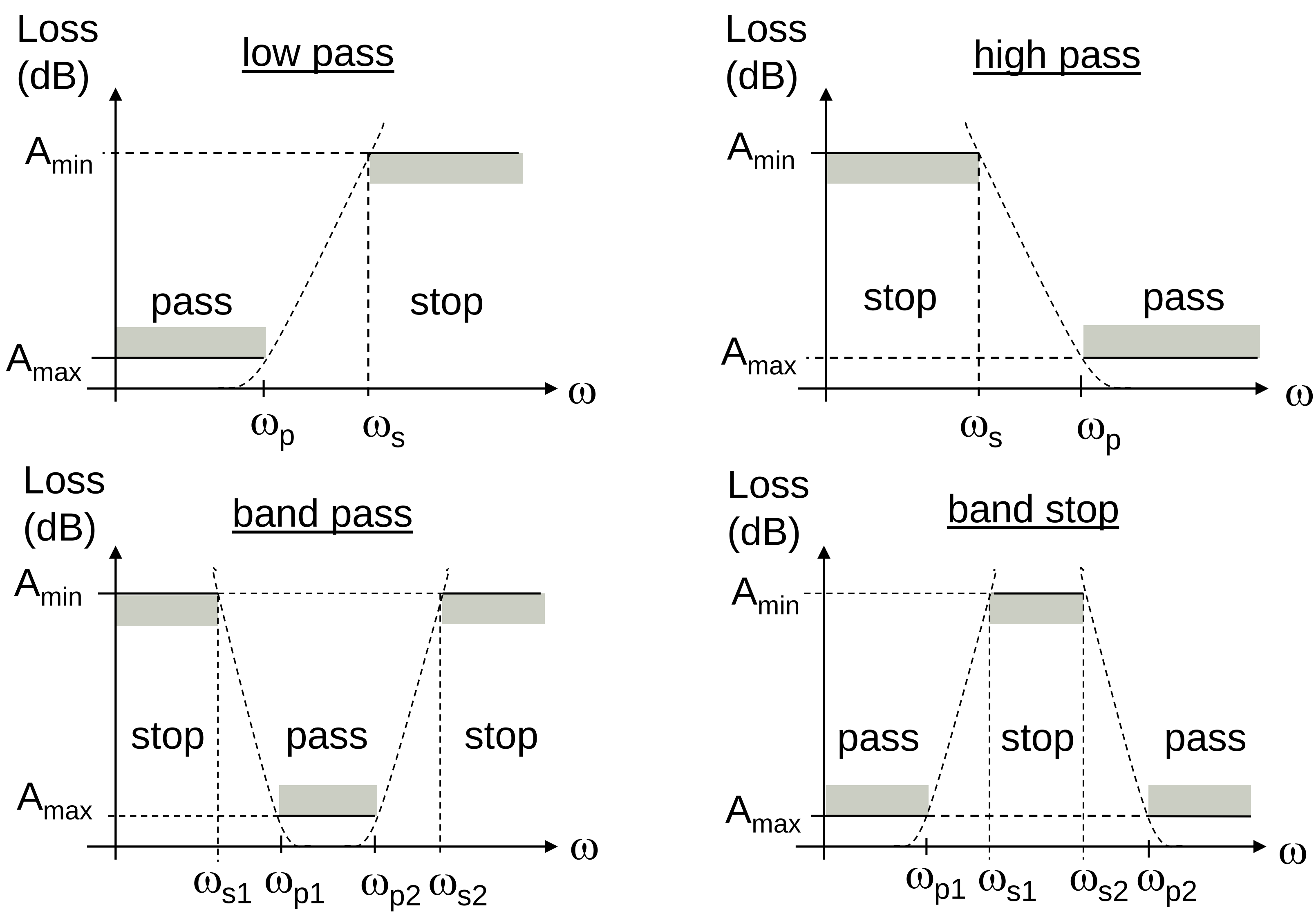
Filter Attenuation Characteristics

- Filter attenuation characteristic must stay outside the shaded region.
- A_{max} maximum attenuation that is allowed in the passband.
- A_{min} minimum attenuation that is required in the stopband.

Filter Attenuation Characteristics



Filter Attenuation Characteristics



Butterworth Filters

- Butterworth filter:

$$\left|\mathrm{T}(\mathrm{j}\Omega)\right|^2 = \frac{1}{1+\Omega^{2n}} \qquad \ldots(1)$$

$$\left|\mathrm{T}(\mathrm{j}\Omega)\right| = \frac{1}{\sqrt{1+\Omega^{2n}}}$$

- Maximally flat, because the first $2n-1$ derivatives of the denominator are zero at $\Omega = 0$.

Butterworth Filters

- “Adjustable” Butterworth function: $$\Omega = \varepsilon^{\frac{1}{n}} \frac{\omega}{\omega_p}$$

$$\left|\mathrm{T}(\mathrm{j}\omega)\right|^2 = \frac{1}{1+\varepsilon^2\left(\omega/\omega_p\right)^{2n}}$$

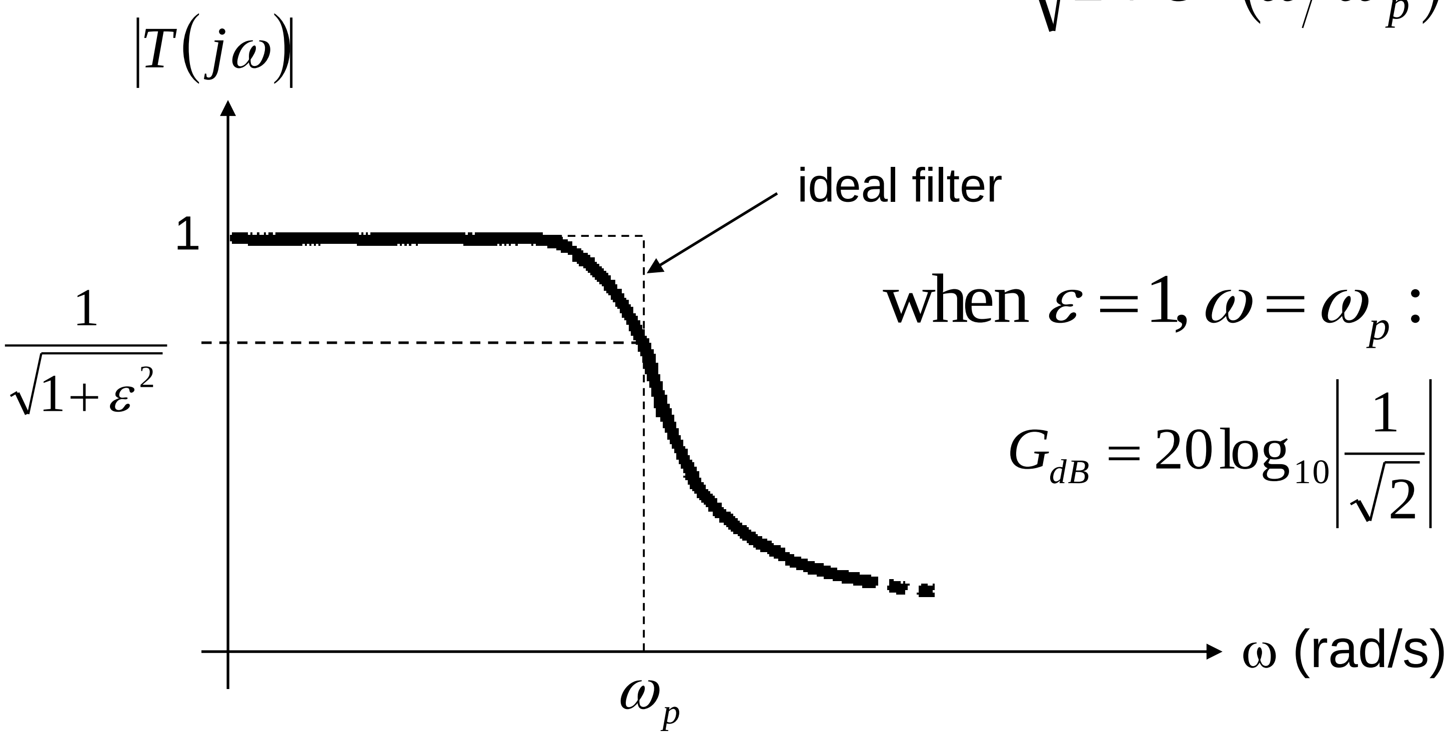
$$\left|\mathrm{T}(\mathrm{j}\omega)\right| = \frac{1}{\sqrt{1+\varepsilon^2\left(\omega/\omega_p\right)^{2n}}}$$

$\varepsilon =$ adjustment factor for max. passband attenuation

$\omega_\mathrm{p} =$ cut off frequency at edge of passband

Butterworth Filters

- Gain: $$\left|T(j\omega)\right| = \frac{1}{\sqrt{1+\varepsilon^2\left(\omega/\omega_p\right)^{2n}}}$$



when $\varepsilon = 1, \omega = \omega_p$:

$$G_{dB} = 20\log_{10}\left|\frac{1}{\sqrt{2}}\right| \approx -3\text{dB}$$

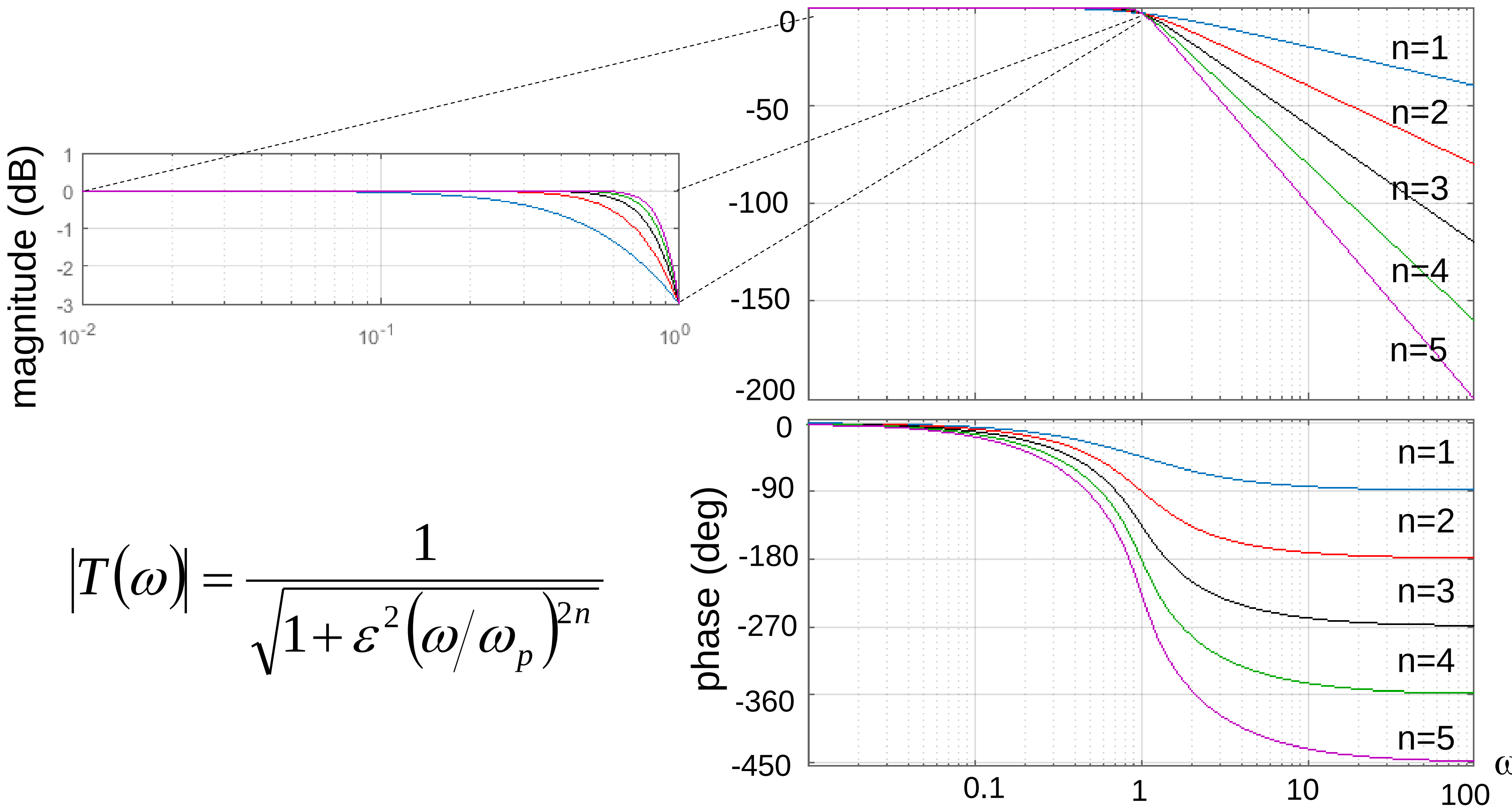
low pass filter

Butterworth Filters

- Butterworth functions:

n	Butterworth function
1	$(s+1)$
2	$(s^2+1.414s+1)$
3	$(s+1)(s^2+s+1)$
4	$(s^2+0.76537s+1)(s^2+1.8477s+1)$
5	$(s+1)(s^2+0.61803s+1)(s^2+1.61803s+1)$

Butterworth Filters

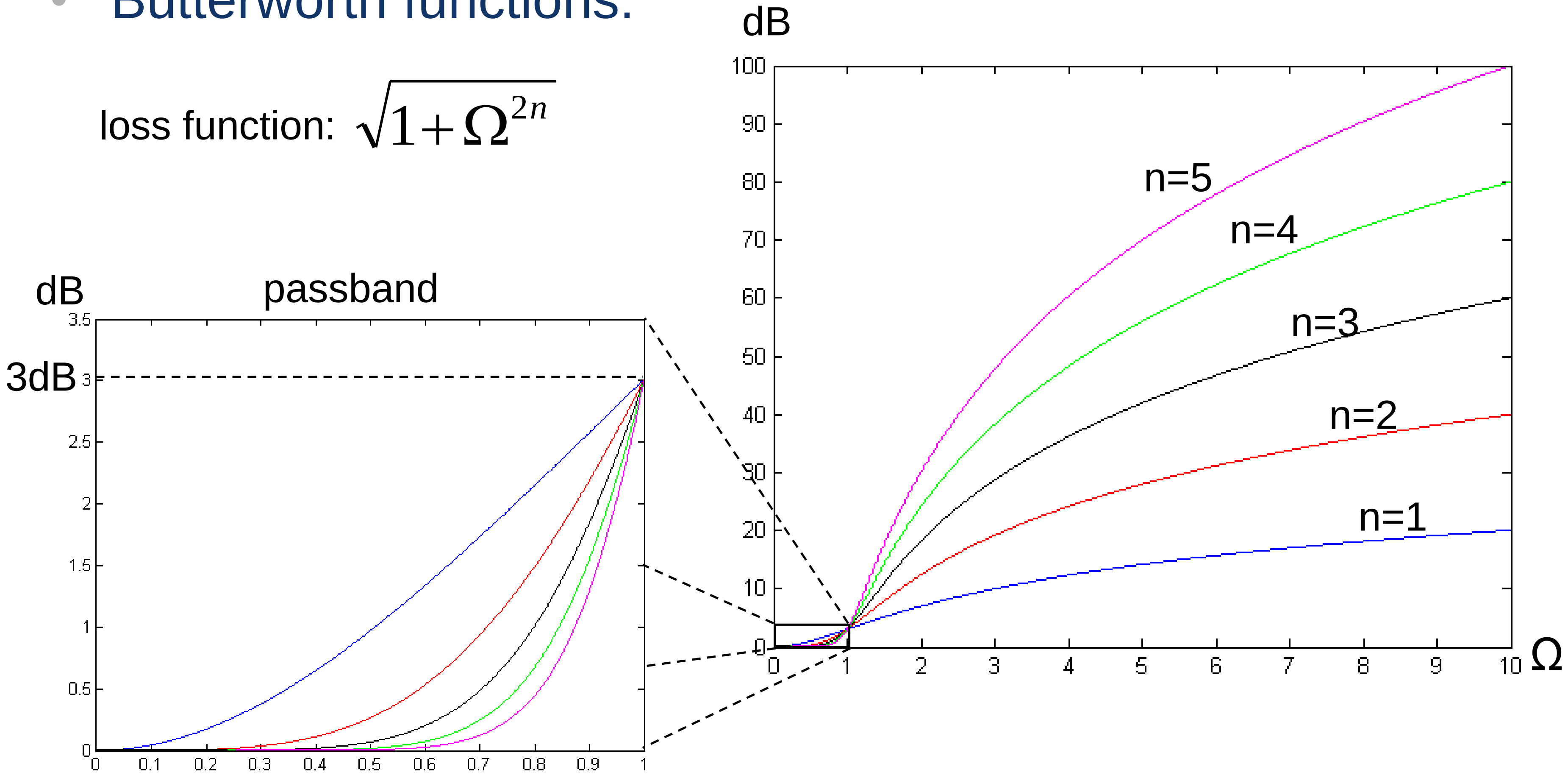


$$|T(\omega)| = \frac{1}{\sqrt{1+\varepsilon^2\left(\omega/\omega_p\right)^{2n}}}$$

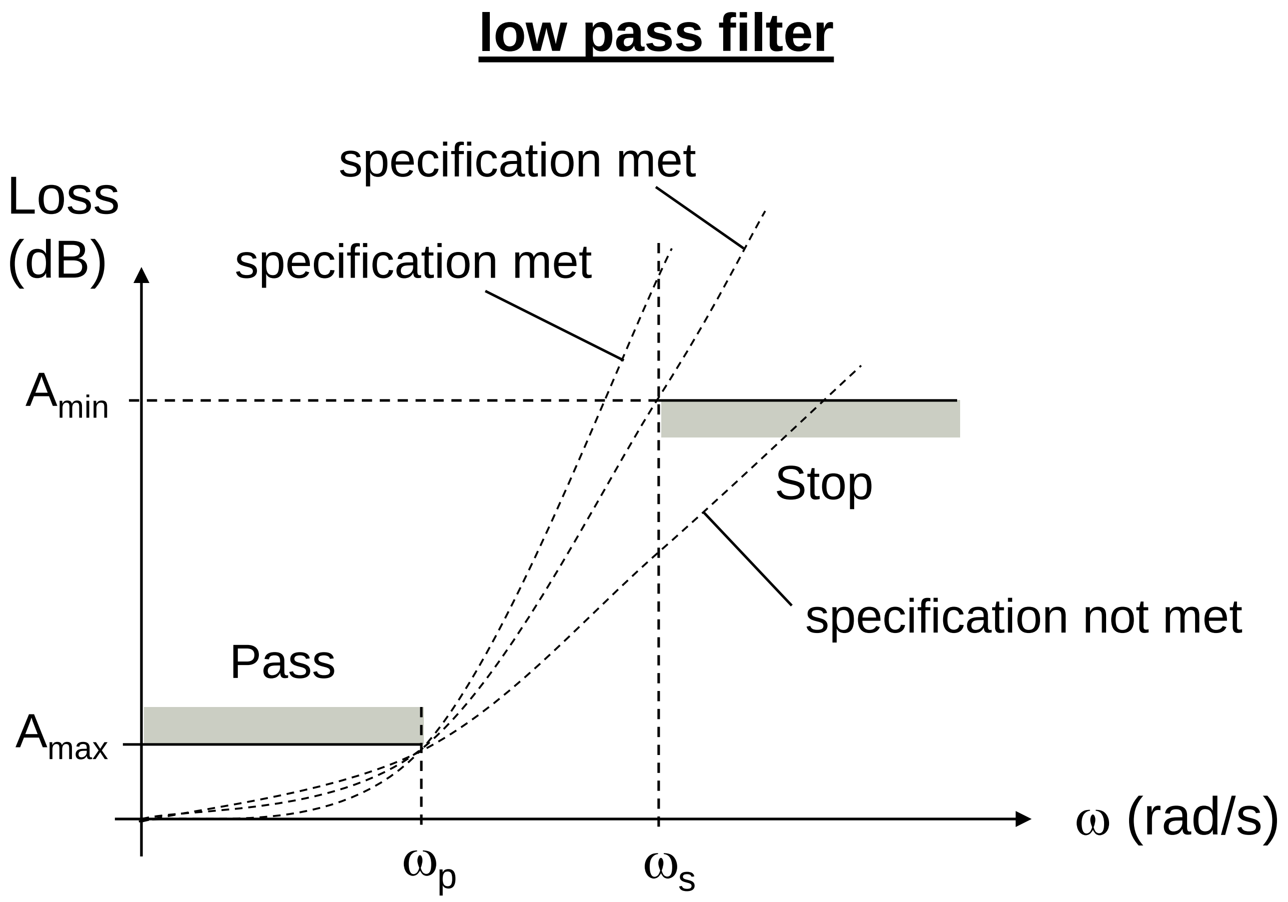
Butterworth Filters

- Butterworth functions:

loss function: $\sqrt{1+\Omega^{2n}}$



Butterworth Filters



Butterworth Filters

$$\varepsilon = \sqrt{10^{0.1A_{\max}} - 1}$$

$$n = \frac{\log_{10}\left(\dfrac{10^{0.1A_{\min}} - 1}{\varepsilon^2}\right)}{2\log_{10}\left(\dfrac{\omega_s}{\omega_p}\right)}$$

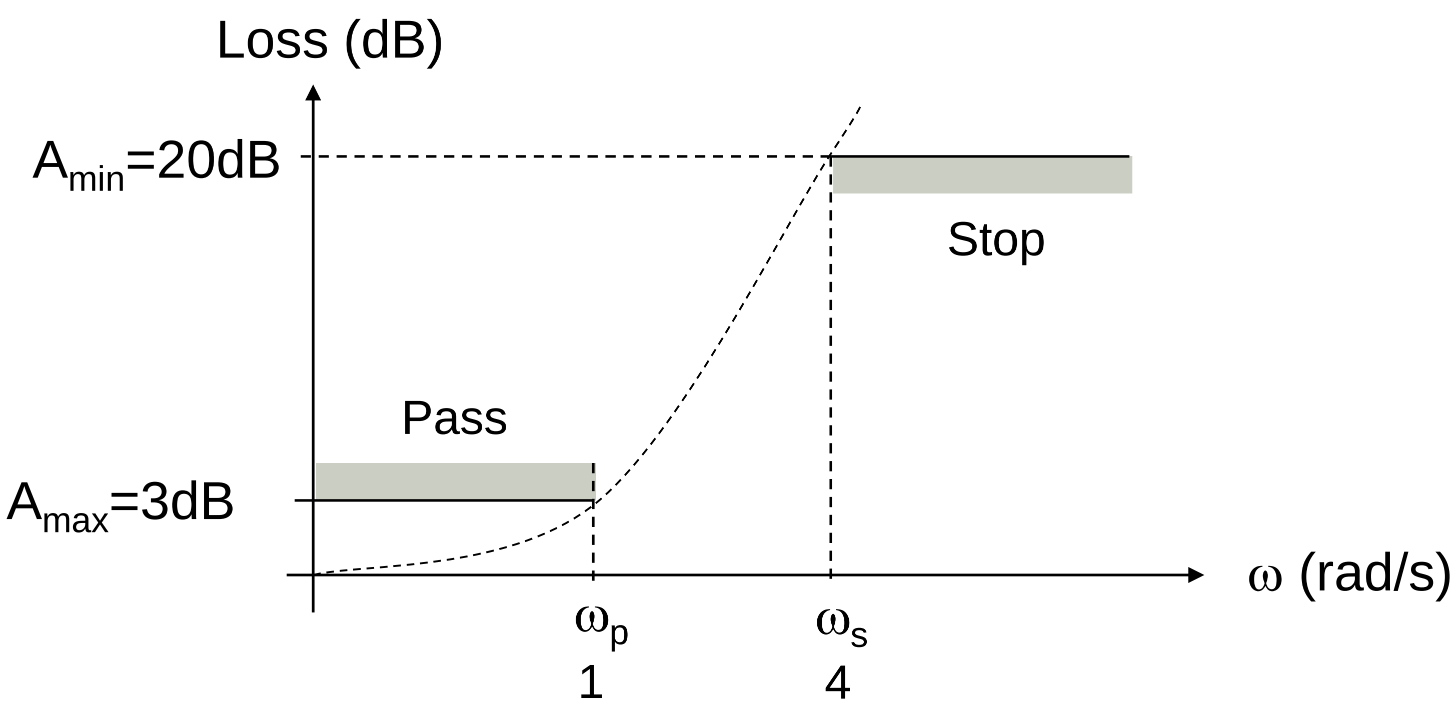
- Given A_{max}, A_{min}, ω_p and ω_s, the order of Butterworth filter required can be calculated.

Butterworth LP Filter

Example 3(a)

- Find the Butterworth approximation for a low pass filter whose requirements are characterised by:

  A_{max} = 3dB, A_{min} = 20dB, ω_p = 1, ω_s = 4rad/s



Butterworth LP Filter

Example 3(a)

- First find ε:

$$\varepsilon = \sqrt{10^{0.1A_{\max}} - 1} = \sqrt{10^{0.3} - 1} \approx 1$$

- Now find order of filter required:

$$n = \frac{\log\left(\dfrac{10^{0.1A_{\min}} - 1}{\varepsilon^2}\right)}{2\log\left(\dfrac{\omega_s}{\omega_p}\right)} = \frac{\log\left(\dfrac{10^{2.0} - 1}{\varepsilon^2}\right)}{2\log\left(\dfrac{400}{100}\right)} = 1.66$$

- Therefore we will choose n = 2.

Butterworth LP Filter

Example 3(a)

- Second order Butterworth function:

$$T(S)=\frac{1}{S^{2}+\sqrt{2}S+1}$$

- Normally would substitute $S=\left(\frac{\varepsilon^{1/n}}{\omega_p}\right)s=Bs$,

But in this case, $\varepsilon=1$ and $B=1.$

$$T(s)=\frac{1}{s^{2}+\sqrt{2}s+1}$$

Butterworth LP Filter

Example 3(a)

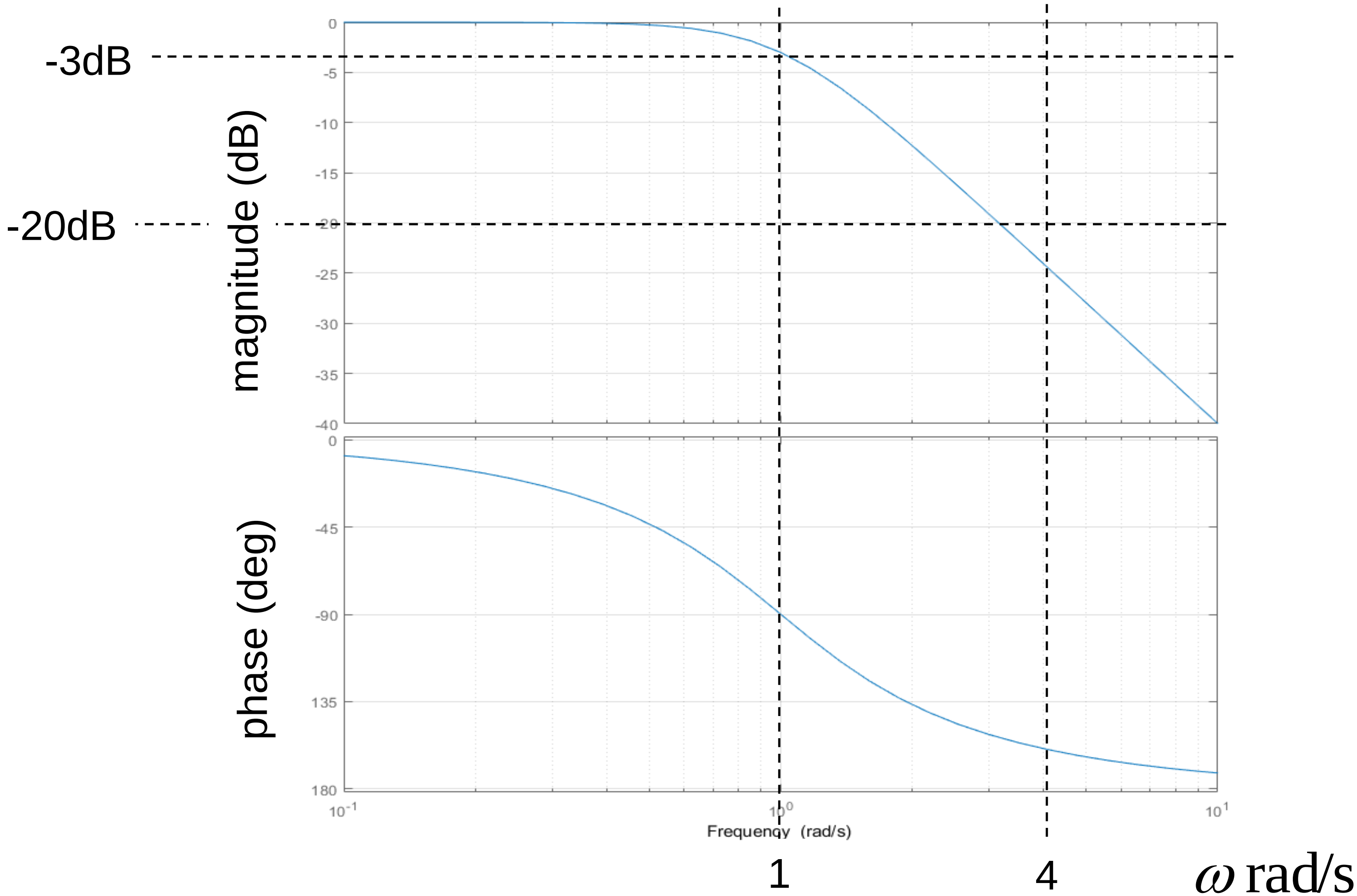
In Matlab:

```
Amax = 3;
Amin = 20;
wp = 1;
ws = 4;

epsilon = sqrt(10^(0.1*Amax) - 1);
n = log10((10^(Amin*0.1) - 1) / (10^(0.1*Amax) - 1)) /
(2*log10(ws/wp));
n = ceil(n);

t1 = tf([0 0 1],[1 sqrt(2) 1]);
figure(1);
bodeplot(t1);
grid on;
```

Butterworth LP Filter



Butterworth LP Filter

Example 3(b)

- Design a prototype Sallen-Key circuit for this filter, with a gain of 10 in the passband, $\omega_p = 1\,\mathrm{rad/s}$, and $C_1 = C_2 = 1\,\mathrm{F}$.

$$T(s) = \frac{10}{s^2 + \sqrt{2}s + 1}$$

- Circuit transfer function:

$$\frac{v_o(s)}{v_{in}(s)} = \frac{\dfrac{K}{R_1R_2C_1C_2}}{s^2 + s\left(\dfrac{1}{R_1C_2} + \dfrac{1}{R_2C_2} + \dfrac{1}{R_2C_1} - \dfrac{K}{R_2C_1}\right) + \dfrac{1}{R_1R_2C_1C_2}} \qquad K = 1 + \frac{R_4}{R_3}$$

Butterworth LP Filter

Example 3(b)

- Let $C_1 = C_2 = 1\,\mathrm{F}$

- Equating denominator coefficients we have:

$$K = 1 + \frac{R_4}{R_3} = 10 \qquad \ldots(1)$$

$$\frac{1}{R_1} + \frac{(2-K)}{R_2} = \sqrt{2} \qquad \ldots(2)$$

$$\frac{1}{R_1 R_2} = 1 \qquad \ldots(3)$$

Butterworth LP Filter

Example 3(b)

- Substituting (3) in (2):

$$R_2 + \frac{(2-K)}{R_2} = \sqrt{2}$$

$$R_2^2 - \sqrt{2}R_2 + (2-K) = 0$$

- Solution of a quadratic:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Butterworth LP Filter

Example 3(b)

- Therefore:

$$R_2 = \frac{\sqrt{2} \pm \sqrt{2-4(2-K)}}{2}$$

- Since K=10:

$$R_2 = \frac{\sqrt{2} \pm \sqrt{2-4(2-10)}}{2}$$

$$R_2 = 3.6\Omega \quad \text{or} \quad -2.2\Omega$$ (use positive answer)

Butterworth LP Filter

Example 3(b)

- Use eqn(3) to calculate R_1:

$$R_1 = \frac{1}{R_2} = 0.276\Omega$$

- Now calculate R_3 and R_4.
- To reduce offset current effect, resistance seen by each input should be equal at DC:

$$R_3 \,//\, R_4 = R_1 + R_2$$

$$\frac{R_3 R_4}{R_3 + R_4} = R_1 + R_2$$

Butterworth LP Filter

Example 3(b)

- However, we know that: $K = 1 + \frac{R_4}{R_3} = \frac{R_3 + R_4}{R_3}$

- Therefore:

$$\frac{R_4}{K} = R_1 + R_2$$

$$R_4 = K(R_1 + R_2) = 10(3.62 + 0.276) = 38.98\Omega$$

and from re-arranging $K = 1 + \frac{R_4}{R_3}$:

$$R_3 = \frac{R_4}{K-1} = \frac{38.98}{9} = 4.33\Omega$$

Butterworth LP Filter

Example 3(b)

- The component values now are:

$$C_1 = 1\text{F}$$
$$C_2 = 1\text{F}$$
$$R_1 = 0.276\Omega$$
$$R_2 = 3.6\Omega$$
$$R_3 = 4.33\Omega$$
$$R_4 = 38.98\Omega$$

- These values will now be scaled to new values:

$$C_1', C_2', R_1', R_2', R_3' \text{ and } R_4'$$

Butterworth LP Filter

Example 3(c)

- Choose a value for the capacitors, and scale the circuit so that the edge of the passband (f_p) is at 2kHz.

- Frequency scale factor:

$$K_f = \frac{\text{new cutoff freq.}}{\text{old cutoff freq.}} = \frac{2\pi 2\text{k}}{1} = 2\pi 2\text{k}$$

Butterworth LP Filter

<u>Example 3(c)</u>

- Choose C_1' and C_2', using rough rule:

$$\text{new value of } C = \frac{10}{f_c}\,\mu\text{F}$$

- Therefore:

$$C_1' = C_2' = \frac{10}{f_c}\,\mu\text{F} = \frac{10}{2\text{k}}\,\mu\text{F} = 0.005\,\mu\text{F} = 5\text{nF}$$

Butterworth LP Filter

<u>Example 3(c)</u>

- Work out magnitude scale factor.
- Since:

$$C_1' = \frac{C_1}{K_m K_f}$$

- The magnitude scale factor can be calculated:

$$K_m = \frac{C_1}{K_f C_1'} = \frac{1\text{F}}{(2\pi 2\text{k})5n\text{F}} = 15.9\text{k}$$

Butterworth LP Filter

Example 3(c)

- The new component values are as follows:

$$C_1' = \frac{C_1}{K_m K_f} = 5\text{nF}$$

$$C_2' = \frac{C_2}{K_m K_f} = 5\text{nF}$$

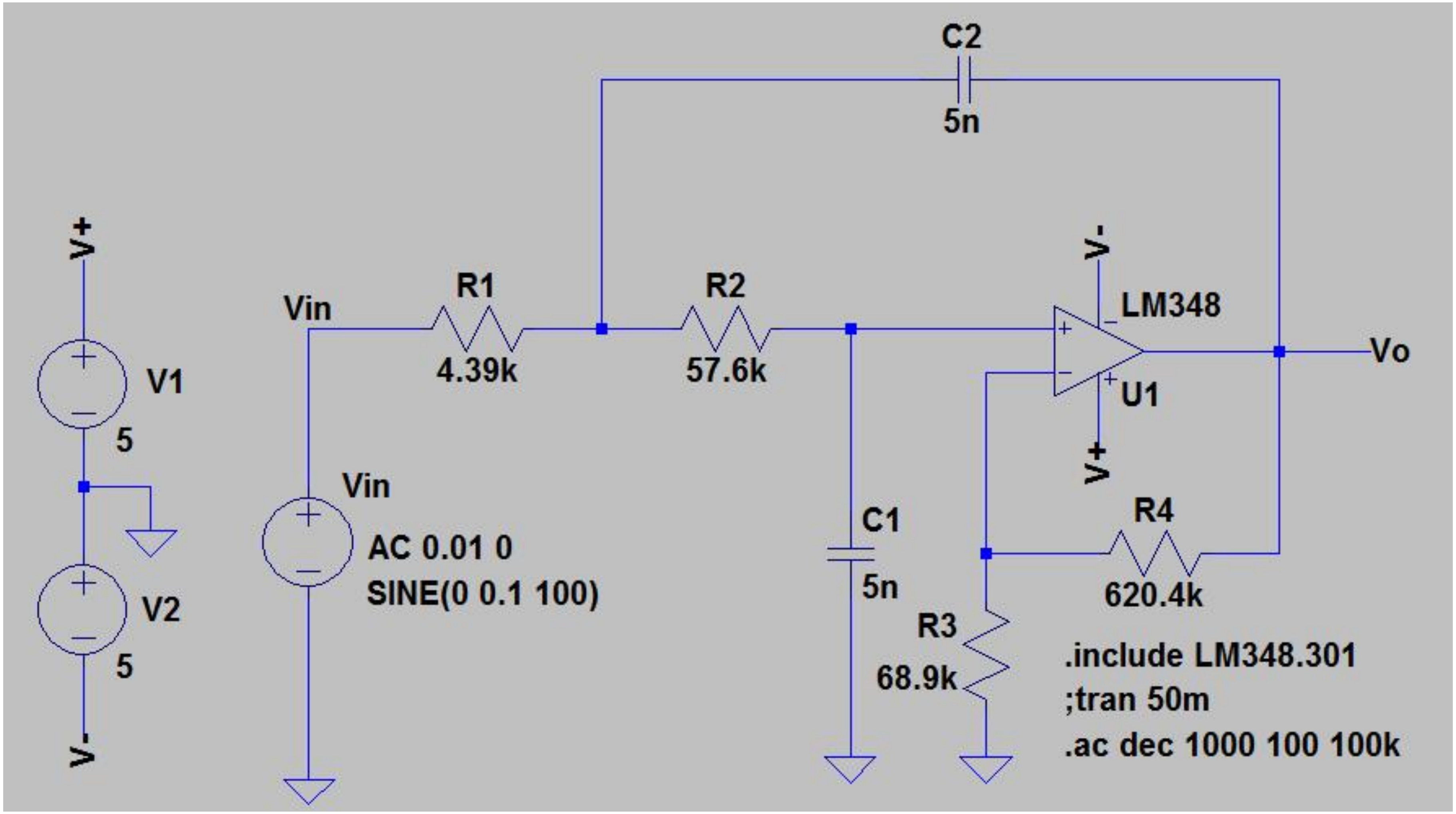
$$R_1' = K_m R_1 = 0.276 \times 15.9\text{k} = 4.39\text{k}\Omega$$

$$R_2' = K_m R_2 = 3.62 \times 15.9\text{k} = 57.6\text{k}\Omega$$

$$R_3' = K_m R_3 = 4.33 \times 15.9\text{k} = 68.9\text{k}\Omega$$

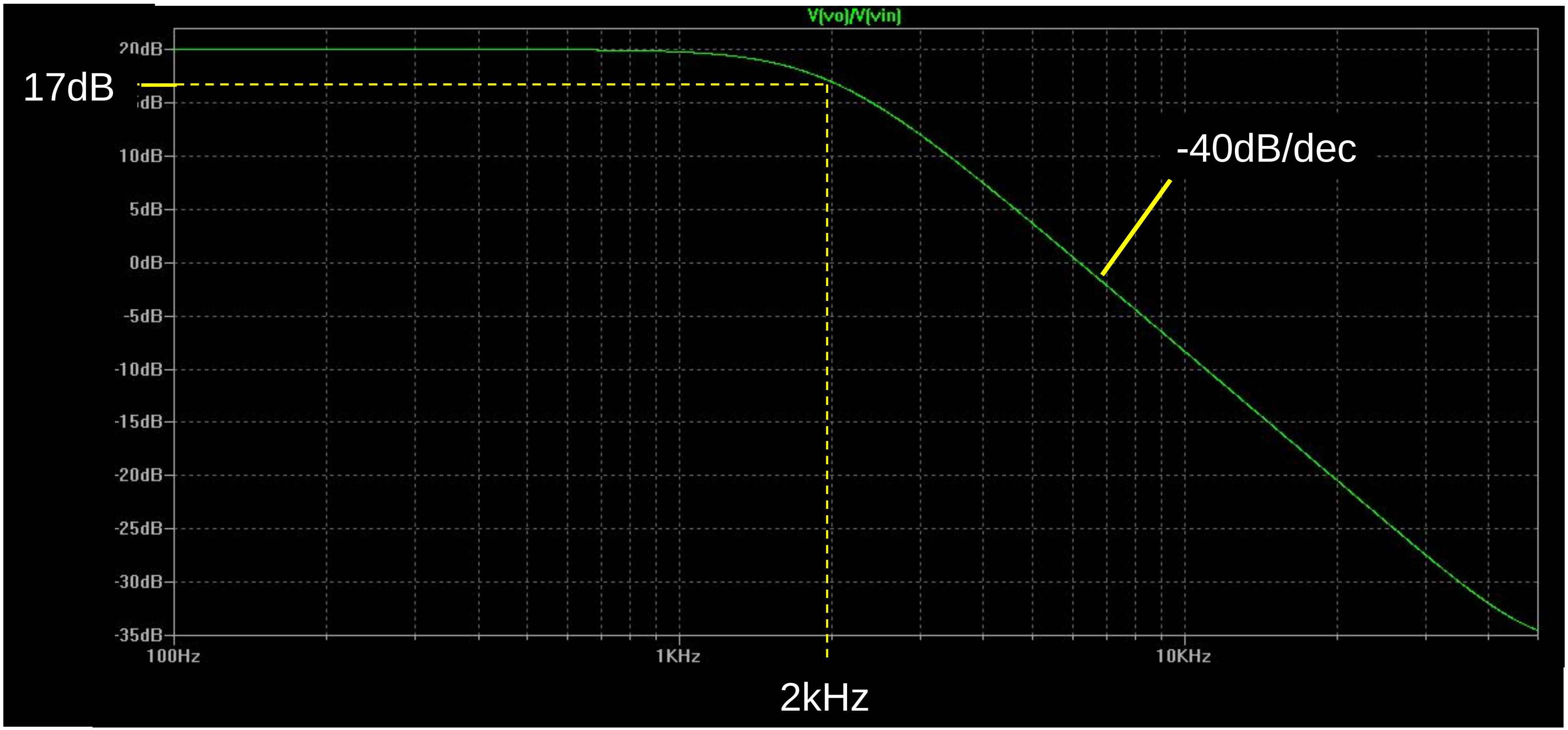
$$R_4' = K_m R_4 = 38.98 \times 15.9\text{k} = 620.4\text{k}\Omega$$

Butterworth LP Filter



Butterworth LP Filter

Frequency response in LTSpice:



Butterworth LP Filter

Example 3(c)

- Transfer function:

$$\frac{V_o}{V_{in}} = \frac{\dfrac{K}{R_1R_2C_1C_2}}{s^2 + s\left(\dfrac{1}{R_1C_2} + \dfrac{1}{R_2C_2} + \dfrac{1}{R_2C_1} - \dfrac{K}{R_2C_1}\right) + \dfrac{1}{R_1R_2C_1C_2}}$$

$$\frac{V_o}{V_{in}} = \frac{1.58\times10^9}{s^2 + 17.8\times10^3 s + 1.58\times10^8}$$

- To display Bode plot in Matlab:

```
t1 = tf([1.58e9],[1 17.8e3 1.58e8]);
bode (t1);
grid on;
```

Butterworth LP Filter

Example 3(c)

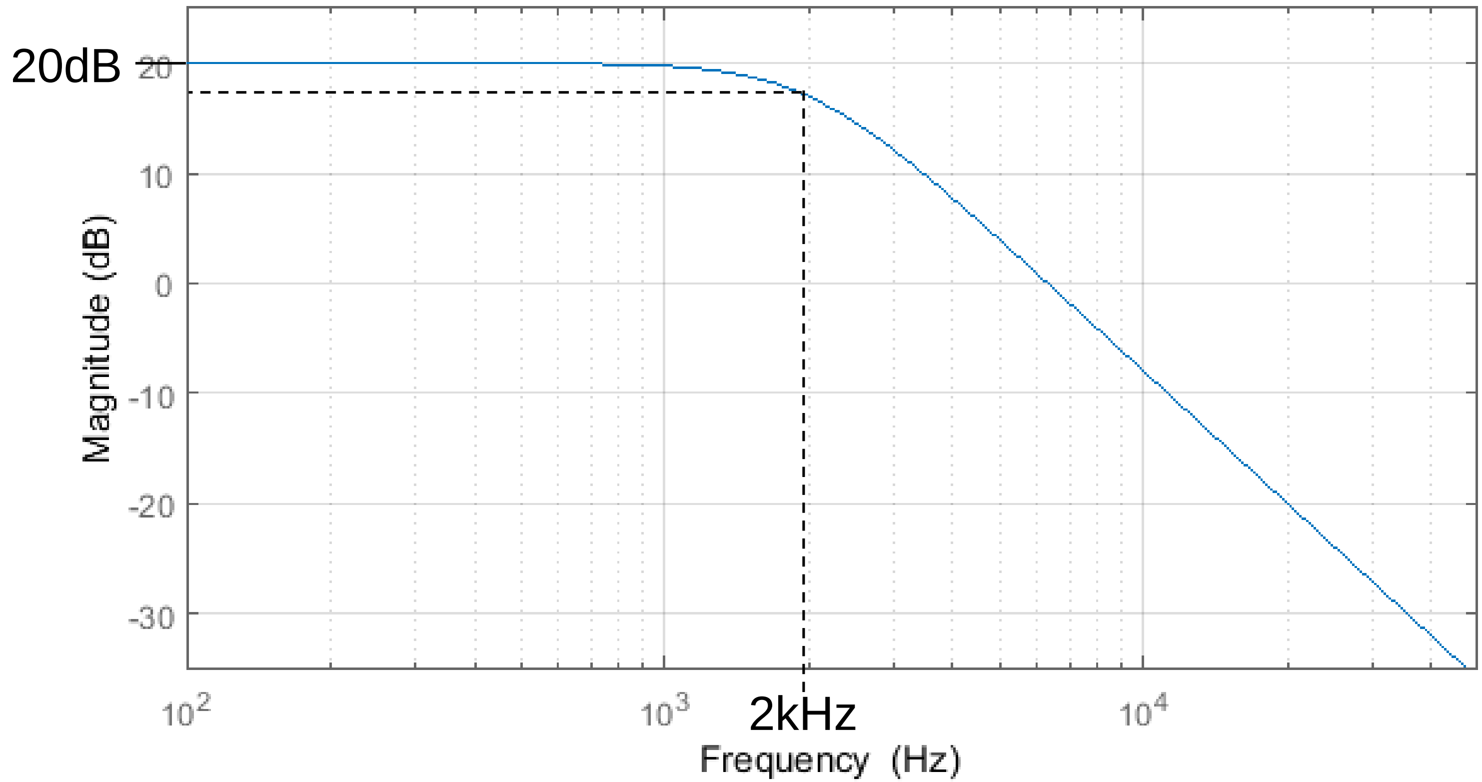
- To display Bode plot in Matlab:

```
K1 = 1 + R4A/R3A;
num = K1/(R1A*R2A*C1A*C2A);
den1 = 1/(R1A*C2A) + 1/(R2A*C2A) + (1-K1)/(R2A*C1A);
den2 = 1/(R1A*R2A*C1A*C2A);

t1 = tf([0 0 num],[1 den1 den2]);
figure(2);
h = bodeplot(t1);
setoptions(h,'FreqUnits','Hz');
grid on;
```

Butterworth LP Filter

Example 3(c)



Chebyshev Filters

- Butterworth approximation is maximally flat at DC.
- The approximation to a flat passband gets progressively poorer as ω approaches ω_p.

- Chebyshev uses an equiripple characteristic in the passband.

- Usually requires a lower order than Butterworth for same stopband attenuation.

Chebyshev Filters

- Chebyshev filter:

$$\left|\mathrm{T}(\mathrm{j}\Omega)\right| = \frac{1}{\sqrt{1+\varepsilon^2 C_n^2(\Omega)}}$$

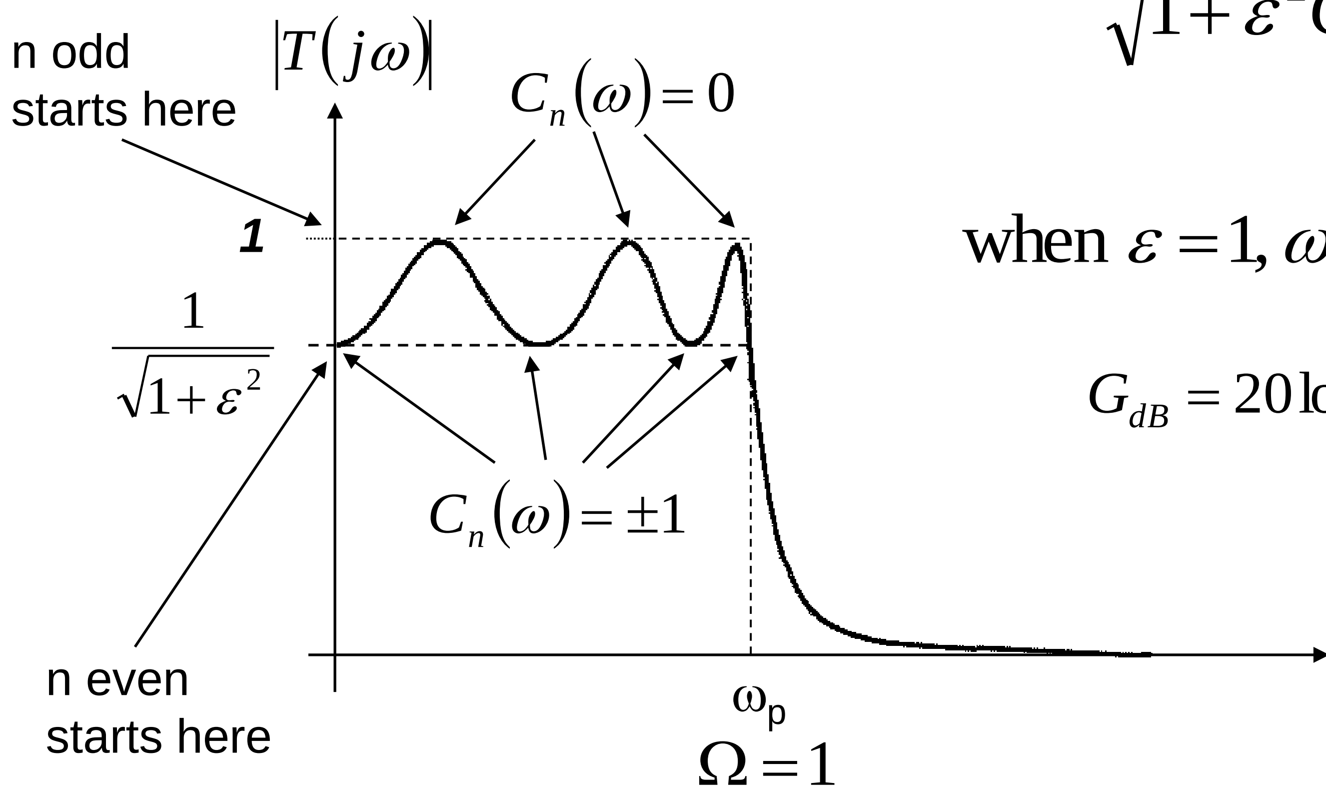
where $C_n(\Omega)$ is Chebyshev polynomial of the first kind of degree n.

- Ω is the standardised frequency:

$$\Omega = \frac{\omega}{\omega_p}$$

Chebyshev Filters

- Gain: $$\left|\mathrm{T}(\mathrm{j}\omega)\right| = \frac{1}{\sqrt{1+\varepsilon^2 C_n^2\left(\omega/\omega_p\right)}}$$



when $\varepsilon = 1, \omega = \omega_p$:

$$G_{dB} = 20\log_{10}\left|\frac{1}{\sqrt{2}}\right| \approx -3\mathrm{dB}$$

<u>low pass filter</u>

Chebyshev Filters

- Chebyshev functions:

$$\Omega = \omega / \omega_p$$ standardised frequency

$$C_n(\Omega) = \begin{cases} \cos\left(n\cos^{-1}(\Omega)\right) & \text{for } |\Omega| \le 1 \\ \cosh\left(n\cosh^{-1}(\Omega)\right) & \text{for } |\Omega| > 1 \end{cases}$$

n	Chebychev Polynomial
0	$C_0(\Omega) = 1$
1	$C_1(\Omega) = \Omega$
2	$C_2(\Omega) = 2\Omega^2 - 1$
3	$C_3(\Omega) = 4\Omega^3 - 3\Omega$
4	$C_4(\Omega) = 8\Omega^4 - 8\Omega^2 + 1$
5	$C_5(\Omega) = 16\Omega^5 - 20\Omega^3 + 5\Omega$
n	$C_{n+1}(\Omega) = 2\Omega C_n(\Omega) - C_{n+1}(\Omega)$

Chebyshev Filters

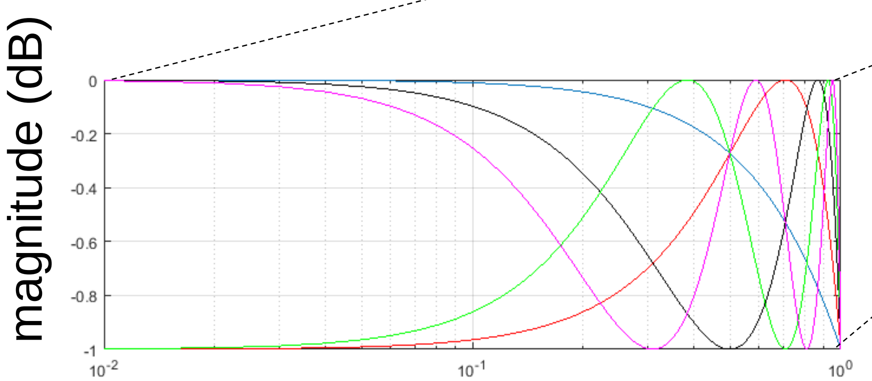
- Hyperbolic functions: sinh, cosh, tanh

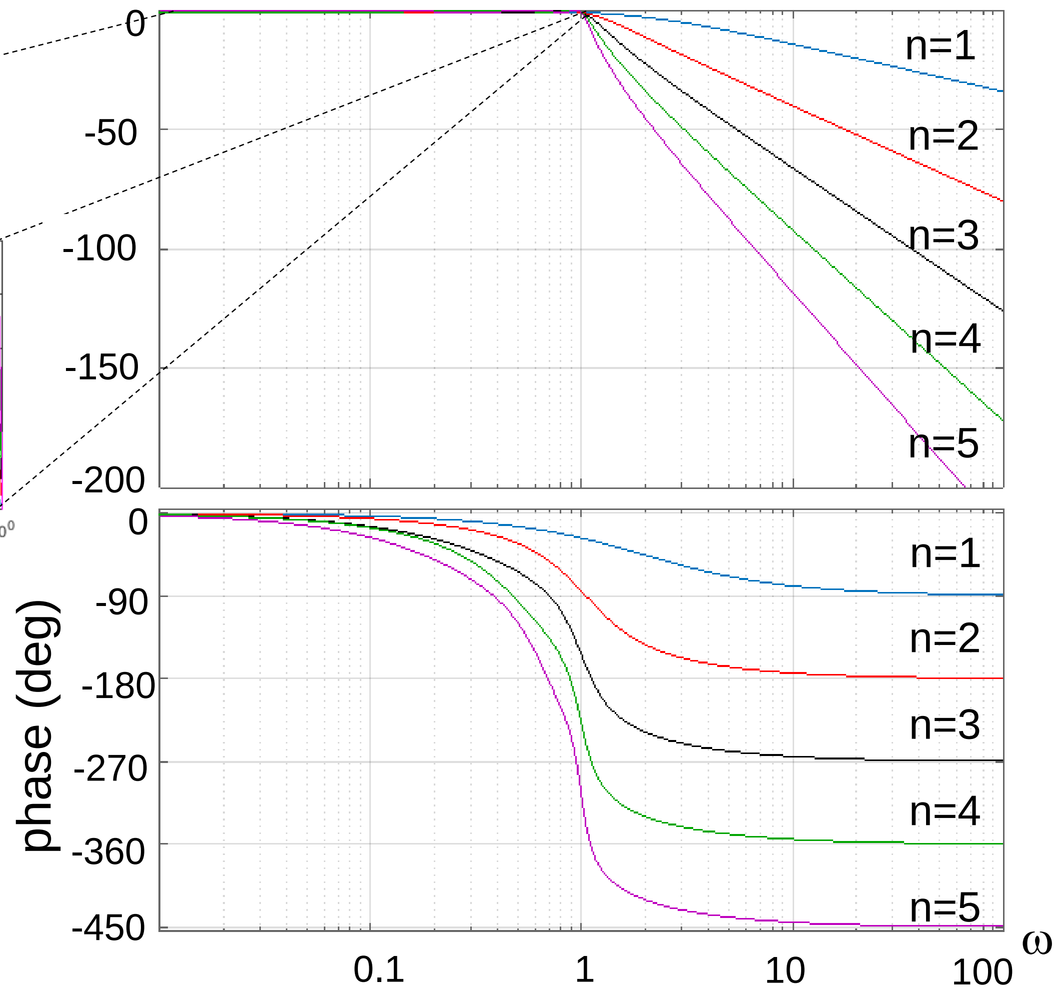
$$\sinh x = \frac{1}{2}\left(e^{x} - e^{-x}\right) \; ; \qquad \cosh x = \frac{1}{2}\left(e^{x} + e^{-x}\right) \; ; \qquad \tanh x = \frac{\sinh x}{\cosh x}$$

- Trigonometric functions:

$$\sin x = \frac{1}{2j}\left(e^{jx} - e^{-jx}\right) \; ; \qquad \cos x = \frac{1}{2j}\left(e^{jx} + e^{-jx}\right) \; ; \qquad \tan x = \frac{\sin x}{\cos x}$$

Chebyshev Filters





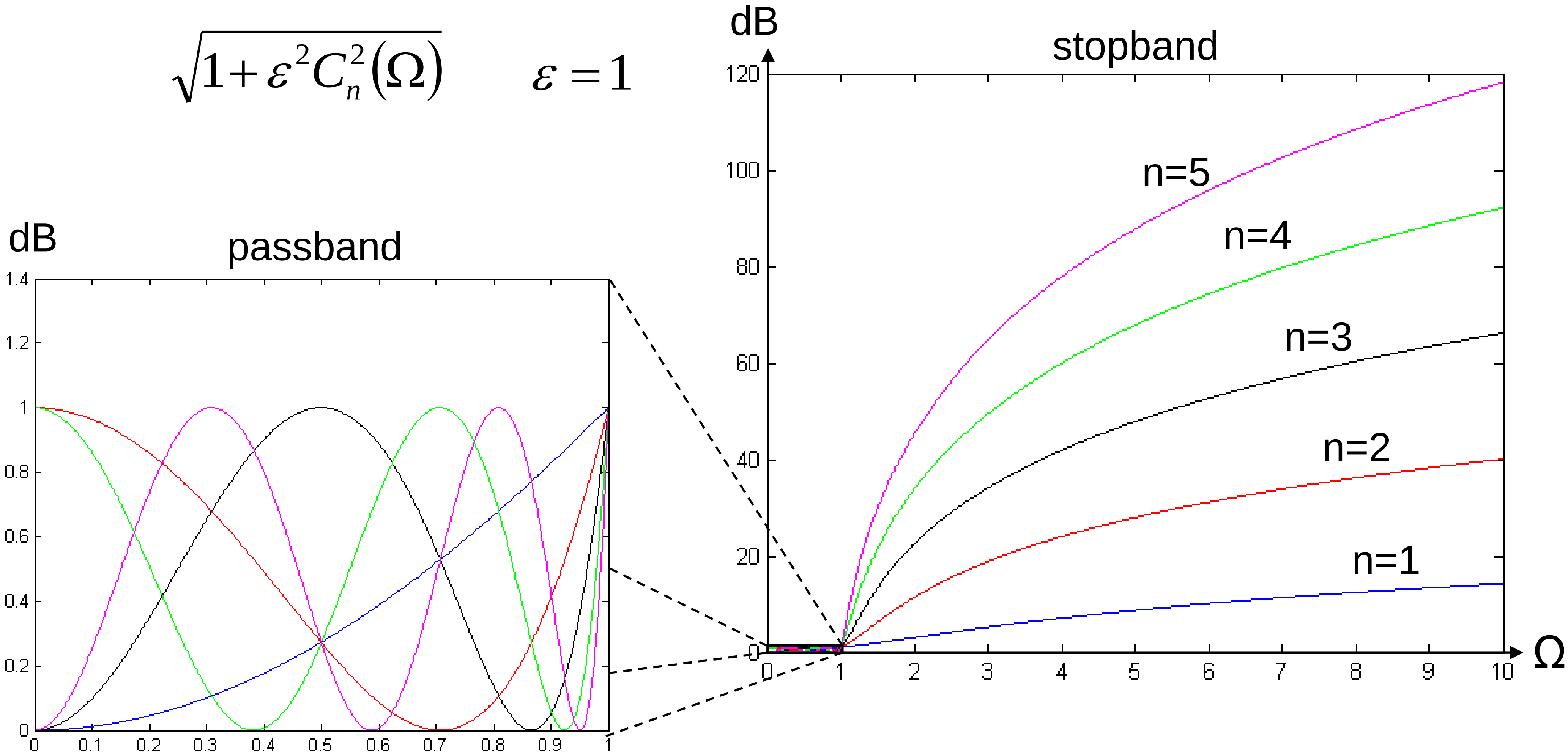
$$|H(\omega)| = \frac{1}{\sqrt{1+\varepsilon^2 C_n^2\left(\omega/\omega_p\right)}}$$

Sharper “knee”, generally lower order required than a Butterworth.

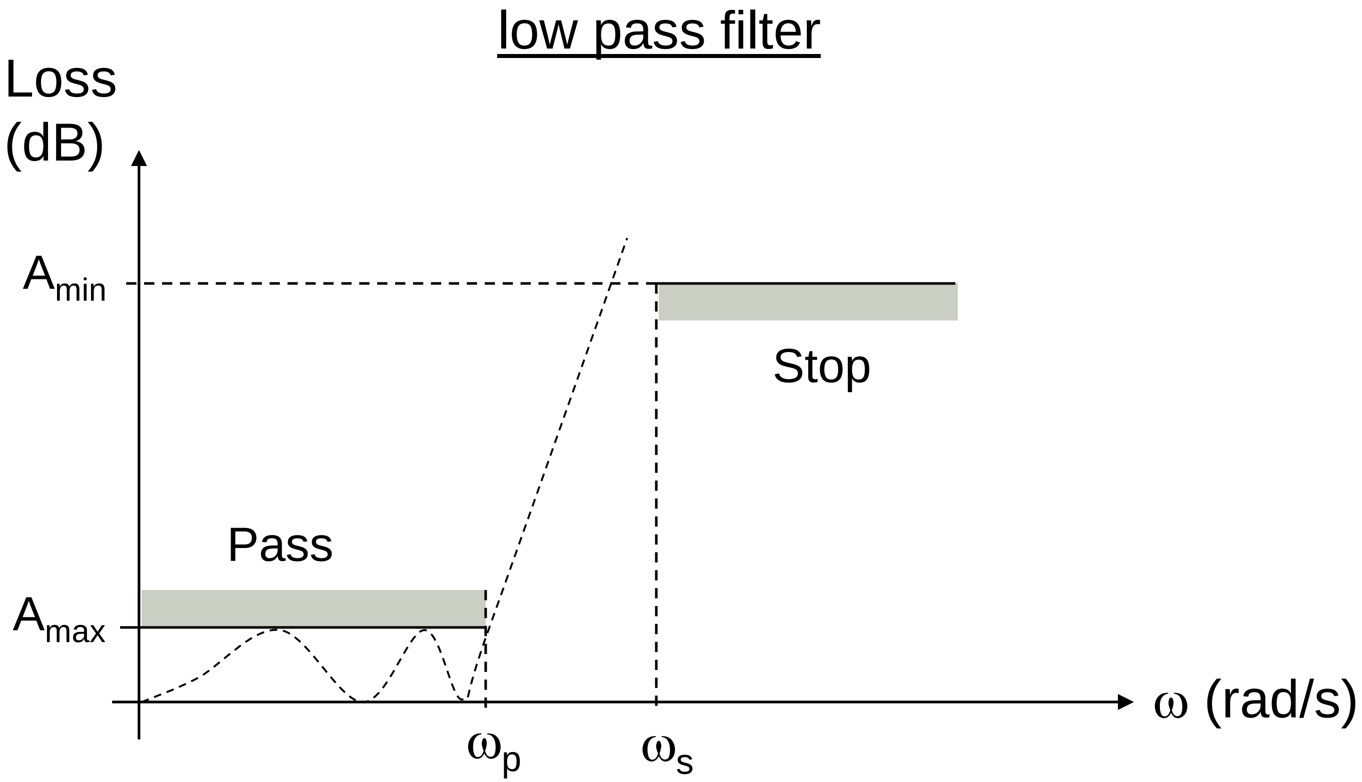
Chebyshev Filters

Plot of loss function:

$$\sqrt{1+\varepsilon^2 C_n^2(\Omega)} \qquad \varepsilon = 1$$



Chebyshev Filters



Chebyshev Filters

$$\varepsilon = \sqrt{10^{0.1A_{\max}} - 1}$$

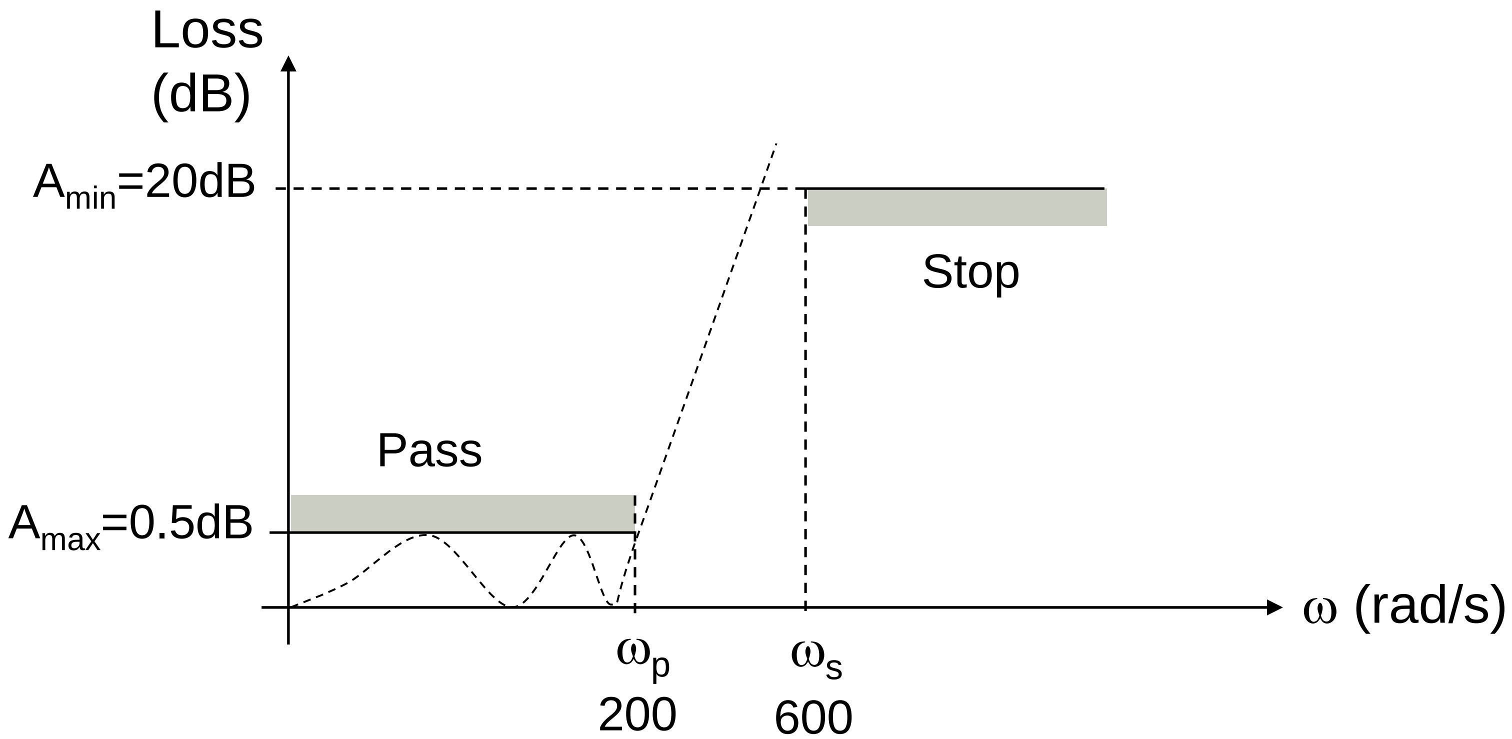
$$n = \frac{\cosh^{-1}\sqrt{\dfrac{10^{0.1A_{\min}} - 1}{\varepsilon^{2}}}}{\cosh^{-1}\left(\omega_s / \omega_p\right)}$$

Chebyshev LP Filter

Example 4

- Find the order of Chebyshev required for a low pass filter whose requirements are:

  Amax = 0.5dB, Amin = 20dB, ω_p = 200, ω_s = 600rad/s



Chebyshev LP Filter

Example 4

- First find ε:

$$\varepsilon = \sqrt{10^{0.1A_{\max}} - 1} = \sqrt{10^{0.05} - 1} = 0.35$$

- Now find order of filter required:

$$n = \frac{\cosh^{-1}\sqrt{\dfrac{10^{0.1A_{\min}} - 1}{\varepsilon^2}}}{\cosh^{-1}\left(\omega_s / \omega_p\right)} = \frac{\cosh^{-1}\sqrt{\dfrac{10^{2.0} - 1}{\varepsilon^2}}}{\cosh^{-1}\left(600/200\right)} = 2.3$$

- Therefore we will choose n = 3.

(Note that Butterworth would have required n=4)

Chebyshev LP Filter

Example 4

- From tables, for A_{max}=0.5dB, and n=3:

$$T(s)=\frac{0.71570}{\left(s^2+0.62646s+1.14245\right)\left(s+0.62646\right)}$$

- Substitute $\frac{s}{\omega_p}=\frac{s}{200}$:

$$T(s)=\frac{5725600}{\left(s^2+125.3s+45698\right)\left(s+125.3\right)}$$

Chebyshev LP Filter

Example 4

$$T(s)=\frac{5725600}{\left(s^2+125.3s+45698\right)\left(s+125.3\right)}$$