Observer Design: Cart-Pendulum System

Lab 06 - EGH445 Modern Control - 2020

Electrical Engineering & Robotics (EER)
Queensland University of Technology

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Objectives

The objective of this Lab is to design and simulate observers for the cart-pendulum system using Matlab-Simulink.

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1 Inverted pendulum on a cart

We use the cart-pendulum system as a benchmark to design state observers. Figure 1 shows the idealised model of the system that consists of a pendulum of mass m and length ℓ attached to a cart of mass M_c. The pendulum moves under the action of the gravity (g) and the cart moves on the horizontal direction and is actuated by the control force F. The state-space model can be written as follows

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix} x_3 \\ x_4 \\ \frac{x_4^2 \ell m \sin(x_2) - gm\sin(x_2)\cos(x_2) + F}{M_c + m - m\cos^2(x_2)} \\ \frac{-\ell m \sin(x_2)\cos(x_2) x_4^2 + g(M_c+m)\sin(x_2) - \cos(x_2)F}{\ell[M_c + m - m\cos^2(x_2)]} \end{bmatrix}, \tag{1}$$

where the states are

- x_1: the position of the cart,
- x_2: the angle of the pendulum,
- x_3: the velocity of the cart,
- x_4: the angular velocity of the pendulum.

We consider the values of the model parameters given in Table 1.

Table 1: Model parameters.

Parameter	value
m	0.15kg
M_c	0.4kg
ℓ	0.2m
g	9.81m/s^2

1.1 Problem formulation.

In this lab, we consider the observer design problem of the cart-pendulum system using the linearised model about the equilibriums

$$\bar{x}_a = \begin{bmatrix}\bar{x}_{1a}\\ \bar{x}_{2a}\\ \bar{x}_{3a}\\ \bar{x}_{4a}\end{bmatrix} = \begin{bmatrix}0\\0\\0\\0\end{bmatrix}; \qquad \bar{x}_b = \begin{bmatrix}\bar{x}_{1b}\\ \bar{x}_{2b}\\ \bar{x}_{3b}\\ \bar{x}_{4b}\end{bmatrix} = \begin{bmatrix}0\\ \pi\\0\\0\end{bmatrix}. \tag{2}$$

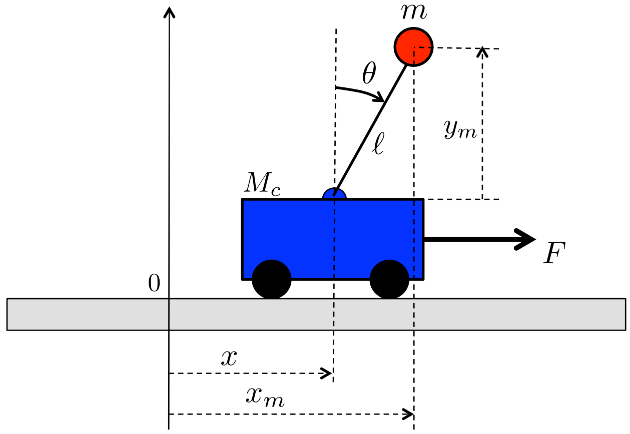

Figure 1: Cart-pendulum system.

The objective is to build observers in the general form

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y}),$$

where $\hat{x}$ is the state estimation, $\hat{y} = C\hat{x}$ is the output estimation, u and y are the input and output of the system. The matrix L is the observer gain that has to be selected to ensure convergency of the observer error to zero.

1.2 Observer for the cart-pendulum system with the pendulum close to the upright position ($\bar{x}_a$).

The task in this section is to design an observer using the linearised model. To do that, consider the linearised model of the cart-pendulum about $\bar{x}_a$:

$$\dot{\tilde{x}}_a = A_a\tilde{x}_a + B_aF, \tag{3}$$
$$y = C_a\tilde{x}_a + D_aF, \tag{4}$$

where

$$A_a = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{m\,g}{M_c} & 0 & 0 \\ 0 & \frac{g(M_c+m)}{\ell M_c} & 0 & 0 \end{bmatrix}; \quad B_a = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{M_c} \\ -\frac{1}{\ell M_c} \end{bmatrix}; \quad C_a = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}; \quad D_a = 0_{2\times 1}. \tag{5}$$

Notice that we assume that the position of the cart and the angle of the pendulum are the only measurements available. Write a script that performs the following tasks (build on scripts created in previous labs):

a) Define the parameters of the model.

b) Compute the matrices of the linearised model.

c) Determine if the linearised model is observable by computing the observability matrix and checking its rank. Hint: use the command `obsv`.

d) Compute the observer gain `La` such that the eigenvalues of the observer error dynamics are $\lambda_1 = -63$, $\lambda_2 = -64$, $\lambda_3 = -65$ and $\lambda_4 = -66$. Hint: use the command `place`.

e) Simulate the linearised and nonlinear closed-loop systems:

   i) Use the Simulink model of the linearised system in closed loop with the state-feedback controller created in Lab 05 (`CP_SFC_Lin_a_yourstudentnumber.slx`) and add the observer. Save your model as `CP_Obs_Lin_a_yourstudentnumber.slx`. An example of the Simulink model is shown in Figure 2. Notice that the block diagram of the linearised model is an alternative to the block `State-Space` used in Lab 05).

   ii) Use a Simulink model of the nonlinear system in closed loop with the state-feedback controller created in Lab 05 (`CP_SFC_NLin_yourstudentnumber.slx`) and add the observer. Select `K_SF=Ka`, `L_NL=La`, `A_NL=Aa`, `B_NL=Ba` and `C_NL=Ca`. Save your model as `CP_Obs_NLin_yourstudentnumber.slx`. An example of the Simulink model is shown in Figure 3.

iii) Export the states of the system, the estimation of the states and the simulation time from Simulink to Matlab.

iv) Simulate both the linearised and the nonlinear closed-loop models with the cart stating at 0.2m and the pendulum at 20deg. Set the initial conditions of the velocities to zero. That is $x(0) = \begin{bmatrix} 0.2 & 20\pi/180 & 0 & 0 \end{bmatrix}^\top$. Suggestion for the simulation: use the fixed-step solver `ode4` and select 0.02 as step time. Use the state of the system x to close the loop.

f) Plot the results of the simulations in a figure that shows the time histories of the position of the cart, the angle of the pendulum, the velocity of the cart, the angular velocity of the pendulum and the estimation of the states for both the linearised and nonlinear control systems. An example of the simulation results is shown in Figure 4.

g) Save the script as `CP_Obs_Lin_a_MainFile_yourstudentnumber.m`.

h) Run the script using different initial conditions for the simulation and analyse the results. The initial conditions should be defined in the script.

i) (Optional) Use the function `Cart_Pendulum_Animation.m` to create an animation of your control system.

Important: The plots and animation should be produced automatically when the script is executed without further user intervention.

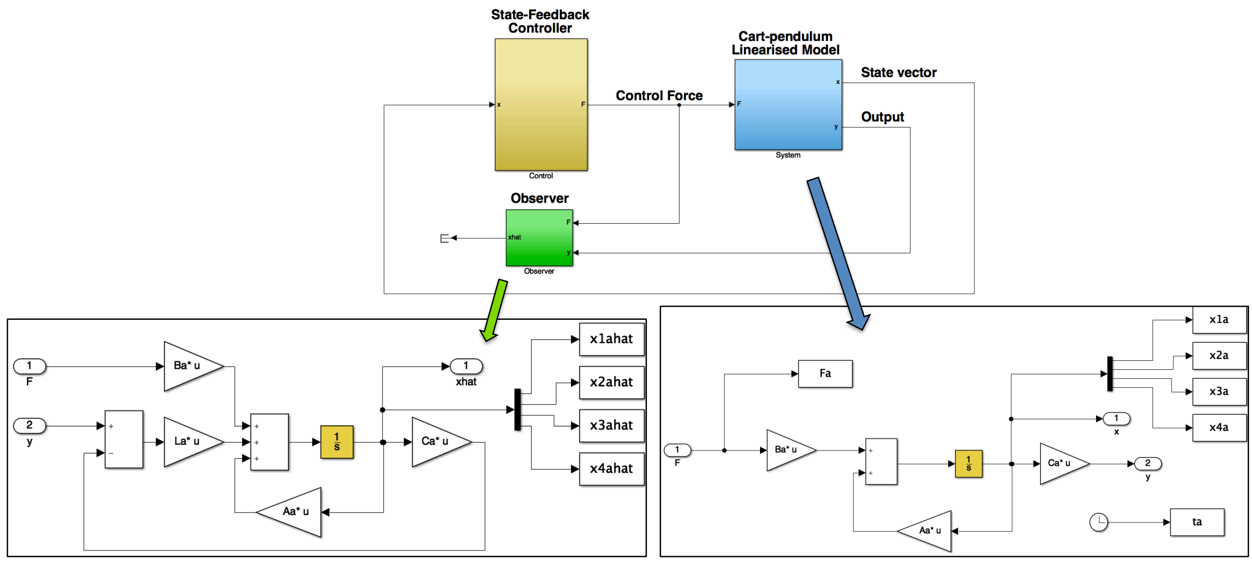

Figure 2: Linearised control system.

1.3 Observer for the cart-pendulum system with the pendulum pointing downwards ($\bar{x}_b$).

The task in this section is to design an observer using the linearised model. To do that, consider the linearised model of the cart-pendulum about $\bar{x}_b$:

$$\dot{\tilde{x}}_b = A_b \tilde{x}_b + B_b F, \tag{6}$$

$$y = C_b \tilde{x}_b + D_b F, \tag{7}$$

where

$$A_b = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{m\,g}{M_c} & 0 & 0 \\ 0 & -\frac{g(M_c+m)}{\ell M_c} & 0 & 0 \end{bmatrix}; \quad B_b = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{M_c} \\ \frac{1}{\ell M_c} \end{bmatrix}; \quad C_b = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}; \quad D_b = 0_{2\times 1}. \tag{8}$$

Write a script that performs the following tasks (build on scripts created in previous labs):

a) Define the parameters of the model.

b) Compute the matrices of the linearised model.

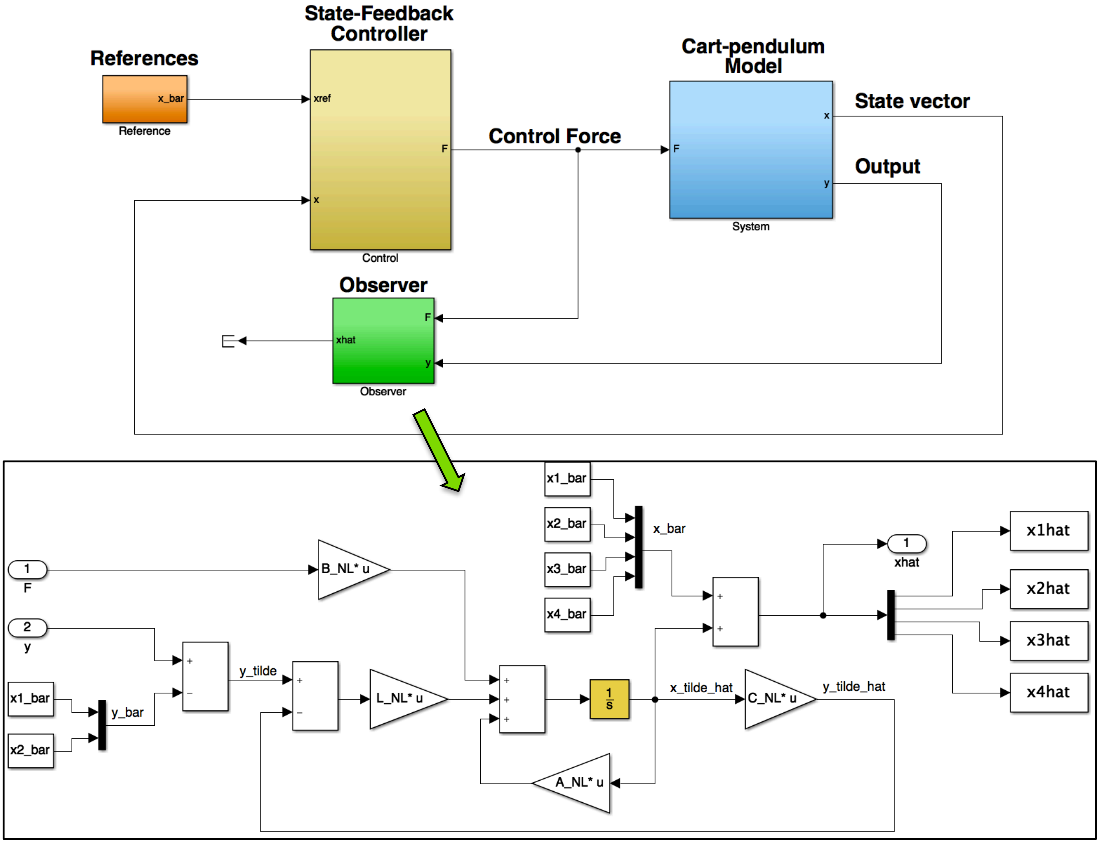

Figure 3: Nonlinear control system.

c) Determine if the linearised model is observable by computing the observability matrix and checking its rank.

d) Compute the observer gain `La` such that the eigenvalues of the observer error dynamics are $\lambda_1 = -63$, $\lambda_2 = -64$, $\lambda_3 = -65$ and $\lambda_4 = -66$.

e) Simulate the linearised and nonlinear closed-loop systems:

   i) Use the Simulink model of the linearised system in closed loop with the state-feedback controller created in Lab 05 (`CP_SFC_Lin_b_yourstudentnumber.slx`) and add the observer. Save your model as `CP_Obs_Lin_b_yourstudentnumber.slx`.

   ii) Reuse the Simulink model of the nonlinear system in closed loop with the state-feedback controller you created in Section 1.2 (`CP_Obs_NLin_yourstudentnumber.slx`). Select `K_SF=Kb`, `L_NL=Lb`, `A_NL=Ab`, `B_NL=Bb` and `C_NL=Cb`.

   iii) Export the states of the system, the state estimation and the simulation time from Simulink to Matlab.

   iv) Simulate both the linearised and the nonlinear closed-loop models with the cart stating at 0.2m and the pendulum at 200deg. Set the initial conditions of the velocities to zero. That is $x(0) = \begin{bmatrix} 0.2 & 200\pi/180 & 0 & 0 \end{bmatrix}^\top$ (adapt the initial conditions for the linearised model). Suggestion for the simulation: use the fixed-step solver `ode4` and select 0.02 as step time.

f) Plot the results of the simulations in a figure that shows the time histories of the position of the cart, the angle of the pendulum, the velocity of the cart, the angular velocity of the pendulum and the estimation of the states for both the linearised and nonlinear control systems. An example of the simulation results is shown in Figure 5.

g) Save the script as `CP_Obs_Lin_b_MainFile_yourstudentnumber.m`.

h) Run the script using different initial conditions for the simulation and analyse the results. The initial conditions should be defined in the script.

i) (Optional) Use the function `Cart_Pendulum_Animation.m` to create an animation of your control system.

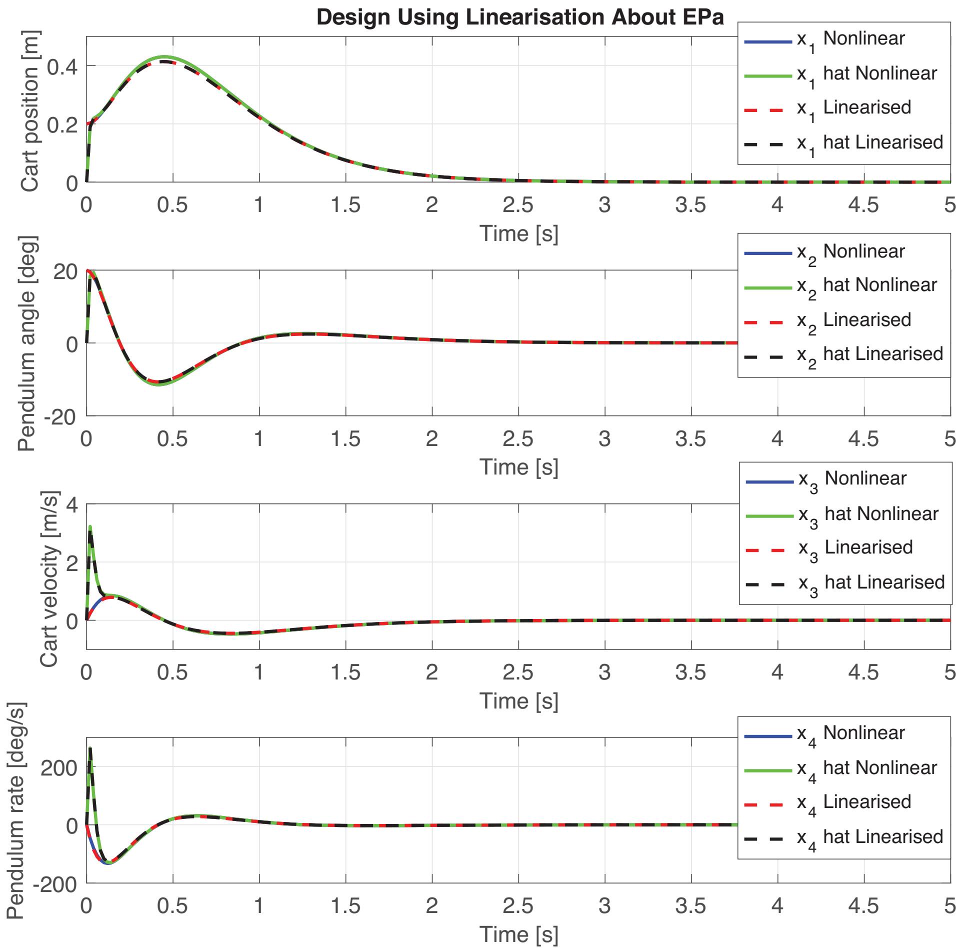

Figure 4: Time histories of the system states and the state estimations.

Important: The plots and animation should be produced automatically when the script is executed without further user intervention.

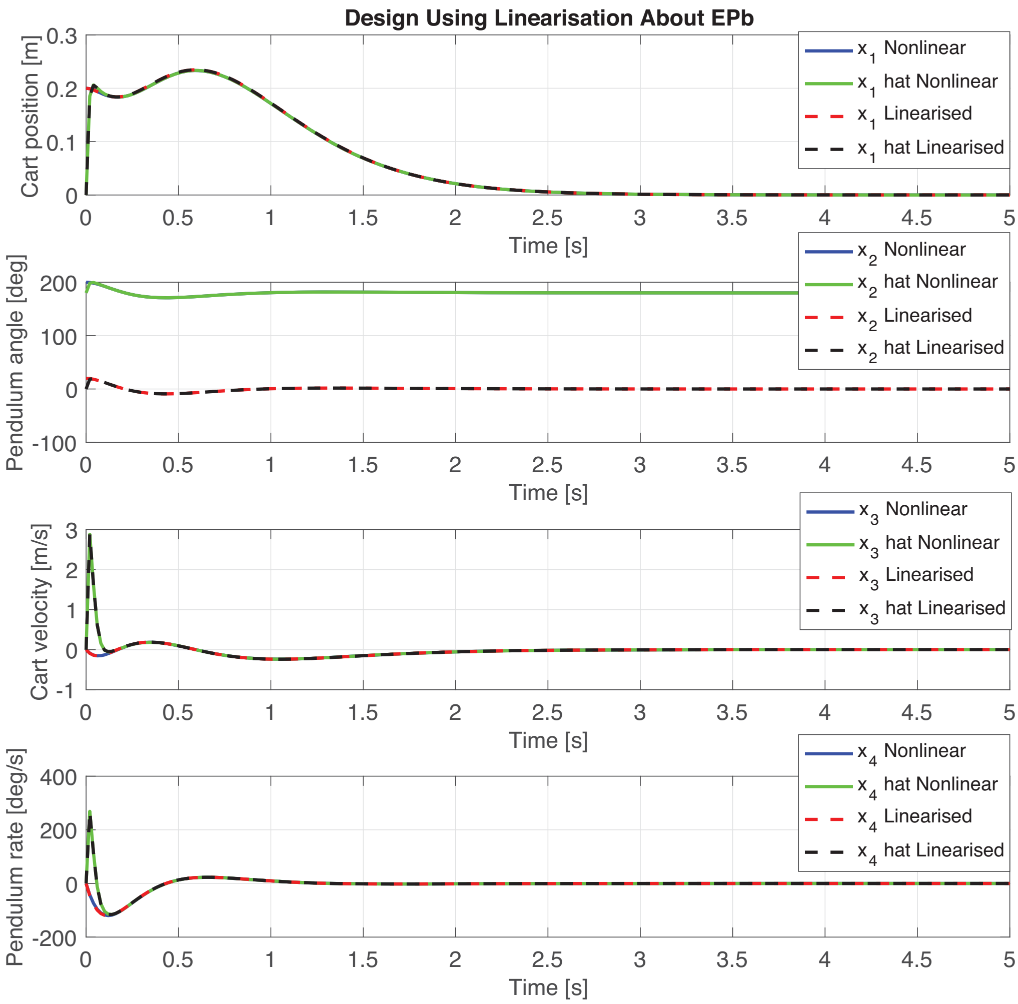

Figure 5: Time histories of the system states and the state estimations.