System Modelling: Cart-Pendulum System
Lab 04 - EGH445 Modern Control

Electrical Engineering & Robotics (EER)
Queensland University of Technology

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Objectives

The objective of this Lab is to build a simulator for the cart-pendulum system using Matlab-Simulink.

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1 Inverted pendulum on a cart

The cart-pendulum system is a benchmark that has been widely used to study control system designs. Figure 1 shows the idealised model of the system that consists of a pendulum of mass m and length ℓ attached to a cart of mass M_c. The cart moves on the horizontal direction and is actuated by the input force F. The equations of motion of the system are as follows

$$
\begin{aligned}
(M_c + m)\,\ddot{q}_1 + m\,\ell\,\cos(q_2)\,\ddot{q}_2 - m\,\ell\,\sin(q_2)\,\dot{q}_2^2 &= F, && (1)\\
m\,\ell\,\cos(q_2)\,\ddot{q}_1 + m\,\ell^2\,\ddot{q}_2 - m\,g\,\ell\,\sin(q_2) &= 0, && (2)
\end{aligned}
$$

where q_1 and q_2 are the position of the cart and the angle of the pendulum respectively, and g is the gravitational constant. We consider the values of the model parameters given in Table 1.

Table 1: Model parameters.

Parameter	value
m	0.15kg
M_c	0.4kg
ℓ	0.2m
g	9.81m/s^2

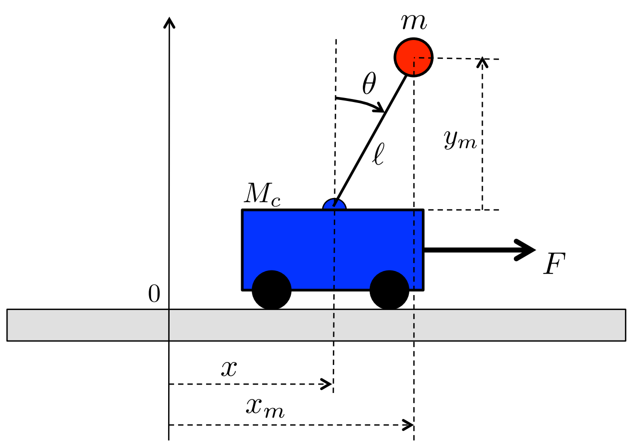

Figure 1: Cart-pendulum system.

1.1 Nonlinear Simulink model.

The task is to build a simulator of the cart-pendulum system. To do that, use the following procedure:

a) Verify that the dynamics (1)-(2) can be written in state-space form as follows

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_3 \\ x_4 \\ \frac{x_4^2 \ell m \sin(x_2) - gm \sin(x_2)\cos(x_2) + F}{M_c + m - m\cos^2(x_2)} \\ \frac{-\ell m \sin(x_2)\cos(x_2) x_4^2 + g(M_c+m)\sin(x_2) - \cos(x_2) F}{\ell[M_c + m - m\cos^2(x_2)]} \end{bmatrix}, \tag{3}$$

where $x_1 = q_1$, $x_2 = q_2$, $x_3 = \dot{q}_1$ and $x_4 = \dot{q}_2$ are the states.

b) Build a Simulink model for the cart-pendulum system that returns the position and velocity of the cart, the angle and angular velocity of the pendulum, input force and the simulation time to the workspace: `x1`, `x2`, `x3`, `x4`, `F` and `tout` respectively. Save the model as `Cart_Pendulum_studentnumber.slx`, where `studentnumber` is your student number. Hint: you may use the blocks `Fcn`, `Mux` and `Demux` to write the state equations.

c) Write a script that performs the following actions:

   i) Assign the parameter values of the cart-pendulum Simulink model.

   ii) Simulate the model.

   iii) Plot the input and the states.

   Hint: You have used a similar script in Lab 2 (Week 3).

d) Save the script as `MainFile_studentnumber.m`.

e) Simulate the model using different initial conditions and analyse its behaviour.

1.2 Linearised Simulink model.

Consider the following equilibrium points of the system (1)-(2):

$$\bar{x}_a = \begin{bmatrix} \bar{x}_{1a} \\ \bar{x}_{2a} \\ \bar{x}_{3a} \\ \bar{x}_{4a} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad ; \qquad \bar{x}_b = \begin{bmatrix} \bar{x}_{1b} \\ \bar{x}_{2b} \\ \bar{x}_{3b} \\ \bar{x}_{4b} \end{bmatrix} = \begin{bmatrix} 0 \\ \pi \\ 0 \\ 0 \end{bmatrix}, \tag{4}$$

where $\bar{x}_a$ and $\bar{x}_b$ correspond to the pendulum in the upright and down positions respectively, with the cart at the origin of the coordinate frame.

a) Linearise the model (1)-(2) about the equilibrium point $\bar{x}_a$. Hint: See Week 3's tutorial.

b) Build a Simulink model for the linearised model that returns the states, the input force and the simulation time to the workspace: `x1a_tilde`, `x2a_tilde`, `x3a_tilde`, `x4a_tilde`, `Fa_tilde`, `touta_tilde`. Use the block `State-Space`. Save the model as `Cart_Pendulum_Linearised_a_studentnumber.slx`.

c) Write a script that performs the following actions:

   i) Assign the parameter values of the linearised Simulink model

   ii) Simulate the model.

   iii) Plot the input and the states.

d) Save the script as `MainFile_Linearised_a_studentnumber.m`.

e) Simulate the model using different initial conditions and analyse its behaviour.

f) Consider now the equilibrium point $\bar{x}_b$ and repeat the tasks in the items *a*) to *e*) of this subsection.

g) Compare the trajectories of the nonlinear and linearised models of the cart-pendulum.

1.3 Comparison of the nonlinear and linearised models.

In this section, we analyse the time histories of the states of the nonlinear model and the linearised approximation. To compare the time histories of the states use the following procedure:

a) Consider a set of initial conditions about the equilibrium point $\bar{x}_a$.

b) Simulate the nonlinear model and linearised model using the same initial conditions.

c) Plot the time histories of the states of the nonlinear and linearised models in the same figures, e.g. plot `x1` and `x1a_tilde` together.

d) Consider a set of initial conditions about the equilibrium point $\bar{x}_b$.

e) Simulate the nonlinear model and linearised model using the same initial conditions.

f) Plot the time histories of the states of the nonlinear and linearised models in the same figures, e.g. plot `x1` and `x1b_tilde` together.

1.4 Animation.

We provide the function `Cart_Pendulum_Animation(time,x1,x2,x1_bar,x2_bar)` that creates an animation of the cart-pendulum system. The function requires the vectors time, x_1 and x_2 from the simulation, and the constants $\bar{x}_{1i}$ and $\bar{x}_{2i}$, with $i = \{a, b\}$.

The animation will pop-up in the screen when the function is executed. The function will also create the file `Cart_Pendulum_Animation.avi`. You can disable the creation of the video file by commenting the last section of the function.