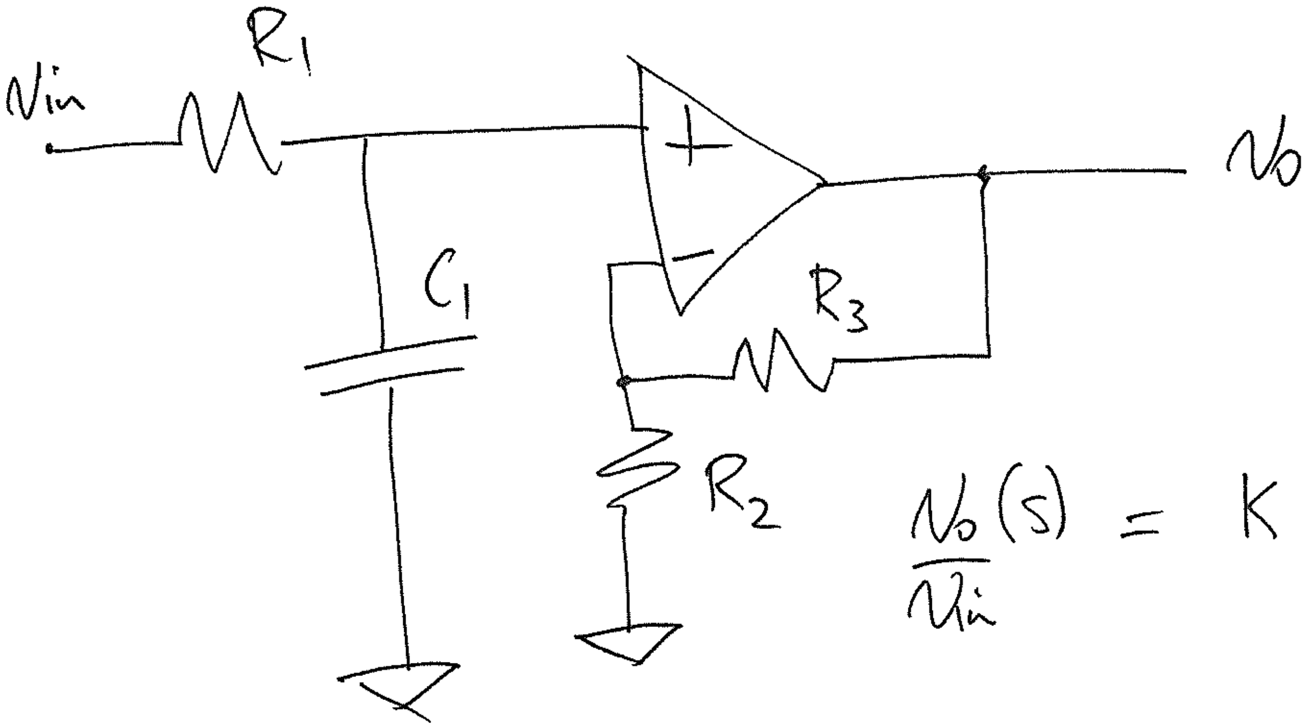
Tutorial 5

Q1

(a)



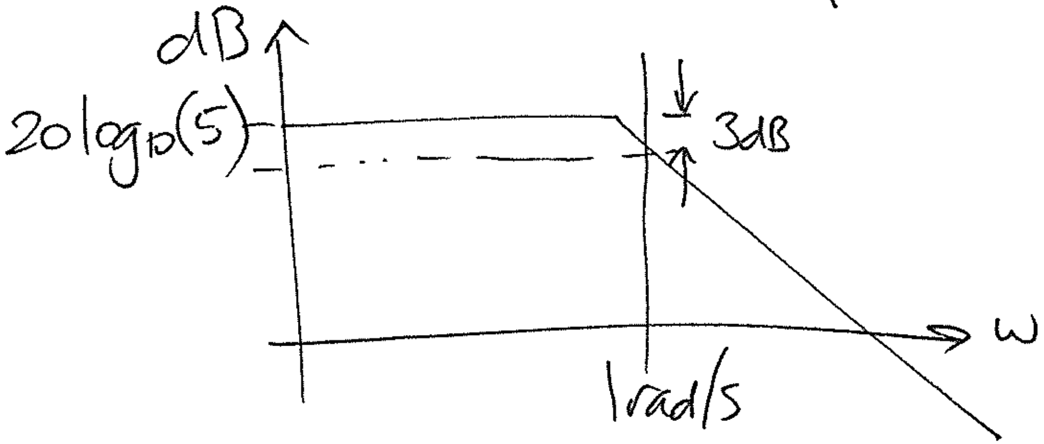
$$\frac{V_o}{V_{in}}(s) = K \frac{\frac{1}{R_1C_1}}{s+\frac{1}{R_1C_1}}$$

$$K = 1 + \frac{R_3}{R_2}$$

$$\frac{V_o}{V_{in}}(j\omega) = \left(1+\frac{R_3}{R_2}\right)\left(\frac{1}{1+j\omega R_1C_1}\right)$$

(b)

$$H(s) = \frac{5}{s+1} \quad \text{(cutoff 3dB down at 1rad/s)}$$



$$H(s) = \frac{5}{s+1} \quad \cdots (1)$$

$$\frac{V_o}{V_{in}}(s) = \frac{K \frac{1}{R_1 C_1}}{s + \frac{1}{R_1 C_1}} \quad \text{where } K = 1 + \frac{R_3}{R_2} \quad \cdots (2)$$

Compare coefficients:

$$1 = \frac{1}{R_1 C_1} \quad \text{— (3)}$$

$$K = 5 = 1 + \frac{R_3}{R_2} \quad \cdots (4)$$

Let $C_1 = 1F$:

From (3) : $R_1 = \frac{1}{C_1} = 1\,\Omega$

To reduce the effect of input bias current:

$$R_1 = R_2 \,//\, R_3 \qquad K = 5 = \frac{R_2 + R_3}{R_2}$$

$$R_1 = \frac{R_2 R_3}{R_2 + R_3}$$

$$R_1 = \frac{R_3}{K}$$

$$R_3 = K R_1 = 5 \times 1 = 5\,\Omega$$

From (4) : $K = \frac{R_2 + R_3}{R_2}$

$$K R_2 = R_2 + R_3$$

$$(K-1) R_2 = R_3$$

$$R_2 = \frac{R_3}{K-1} = \frac{5}{4}\,\Omega$$

$C_1 = 1F$

$R_1 = 1\,\Omega$

$R_2 = \frac{5}{4}\,\Omega$

$R_3 = 5\,\Omega$

(c) $C_1' = 10nF$

$f_p = 1kHz$

$$K_f = \frac{\text{new freq cutoff}}{\text{old freq cutoff}} = \frac{2\pi(1k)\text{ rad/s}}{1\text{ rad/s}} = 2\pi(1k)$$

$$C_1' = \frac{C_1}{K_f K_m}$$

$$K_m = \frac{C_1}{C_1' K_f} = \frac{1F}{10nF(2\pi)(1k)} = 15.9 \times 10^3$$

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Scale Components:

$$C_1' = \frac{C_1}{K_f K_m} = \frac{1F}{2\pi(1k)(15.9 \times 10^3)} = 10nF$$

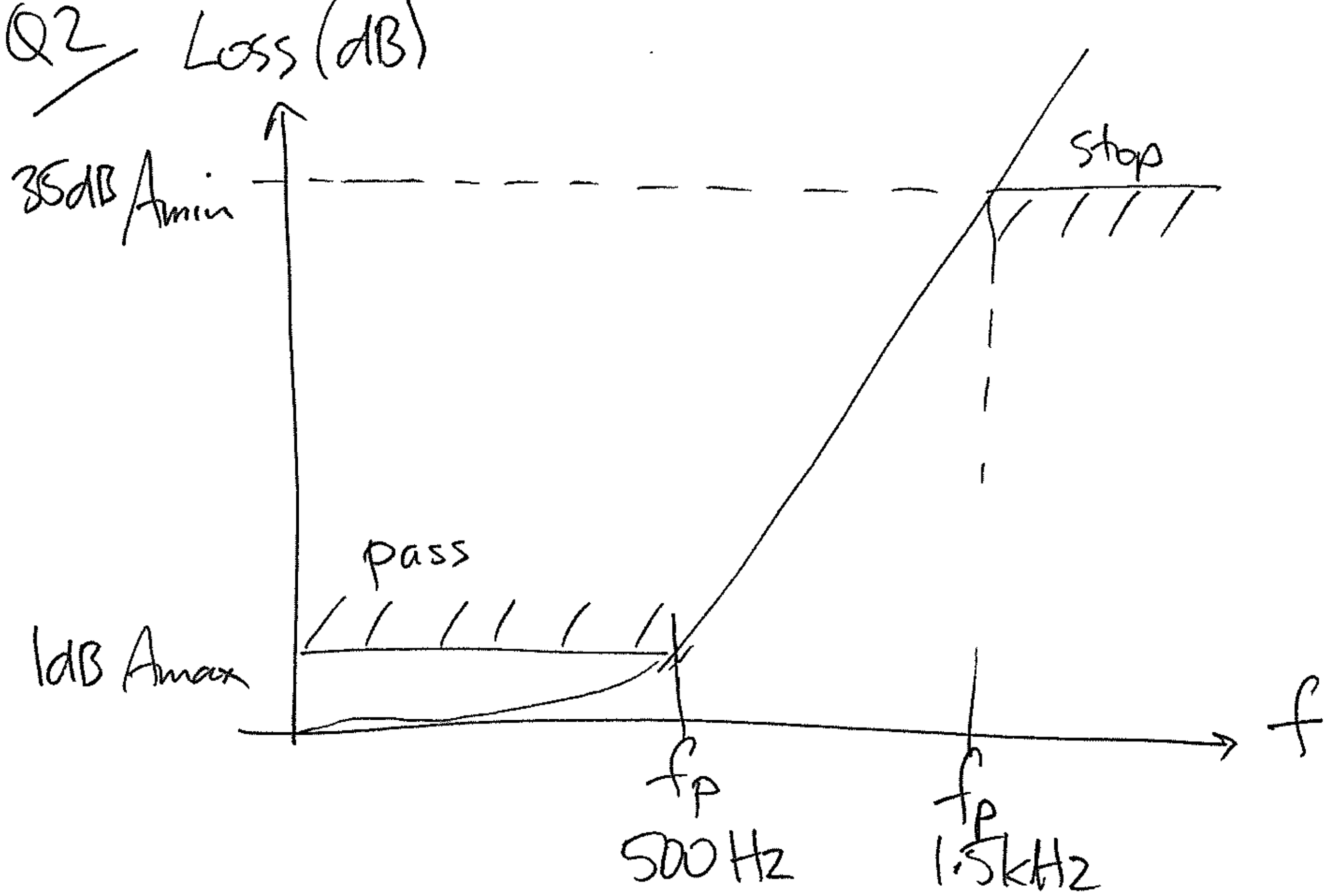
(given anyway)

$$R_1' = K_m R_1 = K_m(1) = 15.9k\Omega$$

$$R_2' = K_m R_2 = K_m\left(\frac{5}{4}\right) = 19.9k\Omega$$

$$R_3' = K_m R_3 = K_m(5) = 79.6k\Omega$$

Q2



(a) Butterworth

$$\varepsilon = \sqrt{10^{0.1 A_{max}} - 1} = \sqrt{10^{0.1 \times 1} - 1} = 0.5088$$

$$n = \frac{\log_{10}\left(\frac{10^{0.1 A_{min}} - 1}{\varepsilon^2}\right)}{2 \log_{10}\left(\frac{\omega_s}{\omega_p}\right)}$$

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$$n = \frac{\log_{10}\left(\frac{10^{0.1\times 35} - 1}{\varepsilon^2}\right)}{2\log_{10}\left(\frac{1.5k}{500}\right)}$$

$n = 4.28$

$n = \text{ceil}(4.28) = 5$

require $n=5$

(b) Chebyshev

$$n = \frac{\cosh^{-1}\sqrt{\frac{10^{0.1 A_{max}} - 1}{\varepsilon^2}}}{\cosh^{-1}\left(\frac{\omega_s}{\omega_p}\right)}$$

$$n = \frac{\cosh^{-1}\sqrt{\frac{10^{0.1\times 35} - 1}{\varepsilon^2}}}{\cosh^{-1}\left(\frac{1.5k}{500}\right)}$$

$n = 3.06$

$n = \text{ceil}(n)$

$n = 4$

Q3/(a)

5th order Butterworth, 3dB attenuation at 1rad/s

$$H(s) = \frac{1}{(S+1)(S^2+0.618S+1)(S^2+1.618S+1)}$$

Incorporating gain:

$$H(S) = \frac{2}{(S+1)} \times \frac{3}{(S^2+0.618S+1)} \times \frac{1}{(S^2+1.618S+1)}$$

To adjust Amax from 3dB to 1dB, use the substitution:

$$S = \frac{\varepsilon^{1/n}}{\Omega_p} \Delta = \frac{(0.5088)^{1/5}}{1} \Delta$$

$$S = 0.8736\Delta = B\Delta$$

$$H(s) = \frac{2}{(Bs+1)} \times \frac{3}{(B^2s^2+0.618Bs+1)} \times \frac{1}{(B^2s^2+1.618Bs+1)}$$

$$= \frac{2/B}{s+\frac{1}{B}} \times \frac{3/B^2}{\left(s^2+\frac{0.618}{B}s+\frac{1}{B^2}\right)} \times \frac{\frac{1}{B^2}}{\left(s^2+\frac{1.618}{B}s+\frac{1}{B^2}\right)}$$

$$H(s) = \underset{A}{\frac{2\times 1.145}{s+1.145}} \times \underset{B}{\frac{3\times 1.310}{s^2+0.7074s+1.310}} \times \underset{C}{\frac{1.310}{s^2+1.852s+1.310}}$$

(b)

Stage A:

$$H(s) = \frac{K_A \frac{1}{R_{1A}C_{1A}}}{s+\frac{1}{R_{1A}C_{1A}}}$$

$$K_A = 1 + \frac{R_{3A}}{R_{2A}}$$

Comparing coefficients:

$$1.145 = \frac{1}{R_{1A}C_{1A}} \quad \ldots (1)$$

$$K_A = 2 = 1 + \frac{R_{3A}}{R_{2A}} \quad \ldots (2)$$

Let $C_{1A} = 1F$:

$$R_{1A} = \frac{1}{1.145 \times C_{1A}} = 0.8736\,\Omega$$

To reduce the effect of input bias current:

$$R_{1A} = R_{2A} \,//\, R_{3A}$$

$$R_{1A} = \frac{R_{2A}R_{3A}}{R_{2A}+R_{3A}}$$

$$K_A = 1 + \frac{R_{3A}}{R_{2A}}$$

$$K_A = \frac{R_{2A}+R_{3A}}{R_{2A}}$$

$$R_{1A} = \frac{R_{3A}}{K}$$

$$R_{3A} = K R_{1A} = 2 \times 0.8736 = 1.7472\,\Omega$$

$$K_A R_{2A} = R_{2A} + R_{3A}$$

$$(K_A - 1) R_{2A} = R_{3A}$$

$$R_{2A} = \frac{R_{3A}}{K_A - 1} = \frac{1.7472}{2-1} = 1.7472\,\Omega$$

Stage A: $C_{1A} = 1F$

$R_{1A} = 0.8736\,\Omega$

$R_{2A} = 1.7472\,\Omega$

$R_{3A} = 1.7472\,\Omega$