Imaginary Numbers

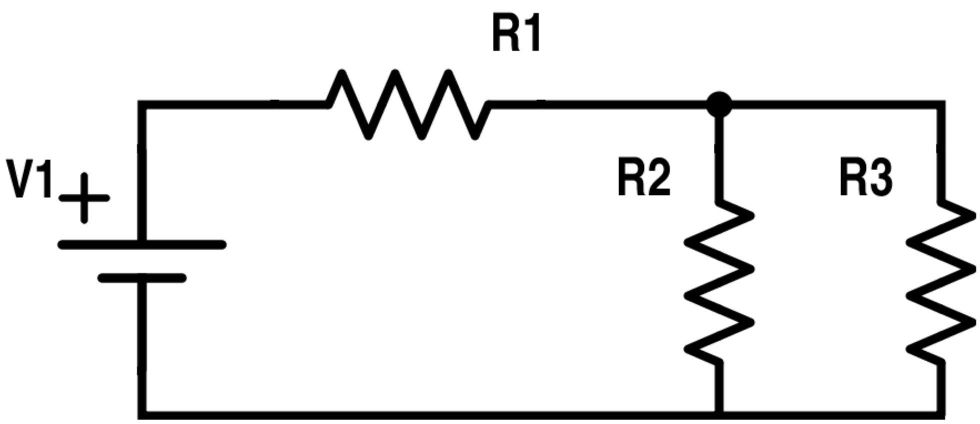
An imaginary number $a + jb$, has magnitude $M = \sqrt{a^2 + b^2}$, and phase $\phi = \tan^{-1}(b/a)$

$$a + jb = Me^{-j\phi} = M\angle\phi$$

Where $j = i = \sqrt{-1}$

KVL/KCL

KVL $\rightarrow$ The sum of all voltages in a loop must be zero.
KCL $\rightarrow$ The sum of currents in and out of a node must be zero.



KVL example, using "positive voltage" as $+ \rightarrow -$:

$$-V_1 + V_{R1} + V_{R2} = 0$$
$$-V_1 + V_{R1} + V_{R3} = 0$$
$$-V_{R2} + V_{R3} = 0$$

KCL example, using the node where the resistors meet:

$$i_{R1} = i_{R2} + i_{R3}$$

Impedance

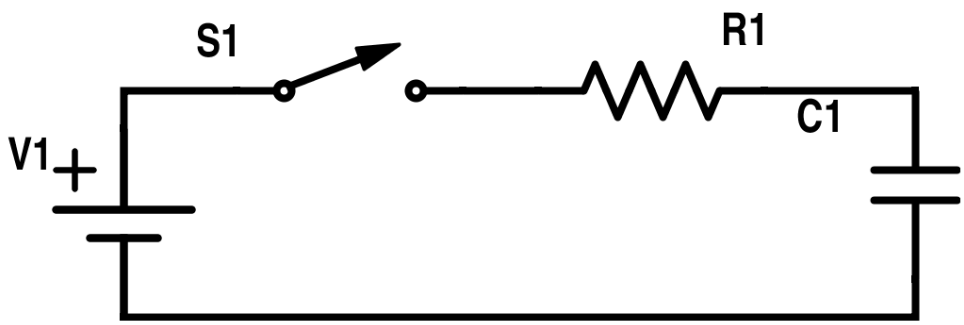
Resistors have real resistance $R = V/I$, measured in Ohms (Ω)
Resistors in series add cumulatively: $R_{ser} = R_1 + R_2$
Resistors in parallel add inversely: $1/R_{par} = 1/R_1 + 1/R_2$, or $R_{par} = \dfrac{R_1 R_2}{R_1 + R_2}$

Capacitors have imaginary reactance $I = C\dfrac{\partial V}{\partial t}$, or $Z_c = 1/j\omega C$, measure in Farads (F)
Capacitors in series add inversely: $1/C_{ser} = 1/C_1 + 1/C_2$, or $C_{ser} = \dfrac{C_1 C_2}{C_1 + C_2}$
Capacitors in parallel add cumulatively: $C_{par} = C_1 + C_2$

Inductors have imaginary reactance $V = L\dfrac{\partial i}{\partial t}$, measured in Henries (H)
Inductors in series add cumulatively: $L_{ser} = L_1 + L_2$
Inductors in parallel add inversely: $1/L_{par} = 1/L_1 + 1/L_2$, or $L_{par} = \dfrac{L_1 L_2}{L_1 + L_2}$

Time Domain RLC

Important constants $100\% * e^{-1} = 36\%$ and $100\% * (1 - e^{-1}) = 63\%$



RC Circuits have a time constant $\tau = RC$, and are exponential functions
Discharging capacitors decay at a rate of $V(t) = V_0 e^{-t/\tau}$
Charging capacitors charge at a rate of $V(t) = V_0(1 - e^{-t/\tau})$

RL Circuits have a time constant $\tau = R/L$, and are exponential functions

LC Circuits have a time constant $\tau = 1/\sqrt{LC}$, and are sinusoidal functions

RLC Circuits have time constant $\tau = 1/\sqrt{LC}$, and are dampened sinusoidal functions

Power

Passing current through a resistor consumes power. Capacitors and inductors store energy, but do not dissipate power. The instantaneous power burned by a resistor is:

$$P_{inst} = V \cdot I = \frac{V^2}{R} = \frac{I^2}{R}$$

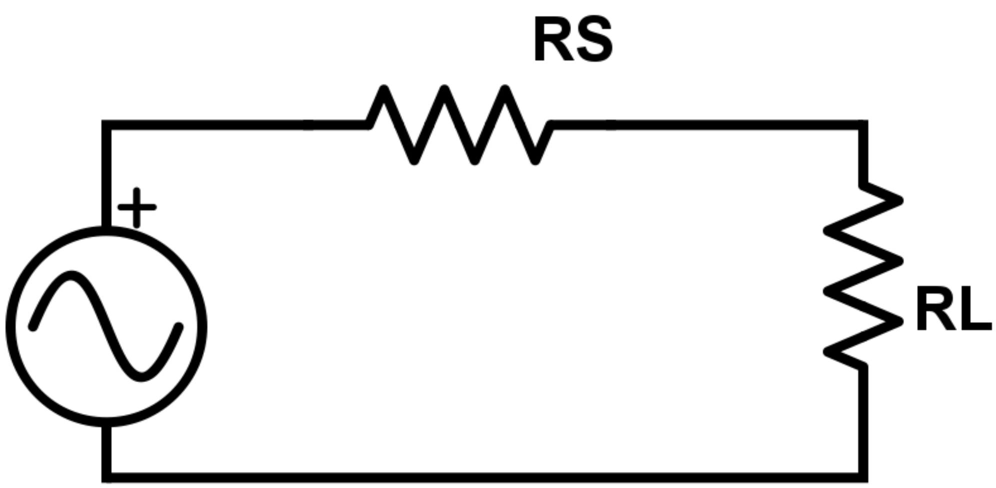
We can also define average power of some periodic waveform:

$$P = \frac{1}{T}\int_{t=0}^{T} v(t)i(t)dt = \frac{1}{2}\text{Re}\{VI^*\}$$

It's easier to see this with a sine wave and RMS values ($V_{pk} = \sqrt{2}V_{rms}$):

$$P = \frac{{V_{rms}}^2}{R} = \left(\frac{V_{pk}}{\sqrt{2}}\right)^2 \frac{1}{R} = \frac{{V_{pk}}^2}{2R}$$

Another important metric is available power, or how much power can a source with some resistance deliver to a load.



We can prove that the maximum power is delivered to the load when $R_S = R_L$. When the resistance is equal, each resistor sees half the voltage, so the maximum power that the load resistor can use is:

$$P_a = \left(\frac{V_{rms}}{2}\right)^2 \frac{1}{R_L} = \frac{V_{rms}}{4R_L} = \frac{V_{pk}}{8R_L}$$

Fourier/Transfer Function

We use Fourier transforms to go from time to frequency domain. Some time domain signal $f(t)$ can represented in the frequency domain as $F(w)$ through:

$$F(w) = \int_{-\infty}^{\infty} f(t)e^{jwt}dw$$

These are pretty ugly math expressions in general, so only a few are worth memorizing:

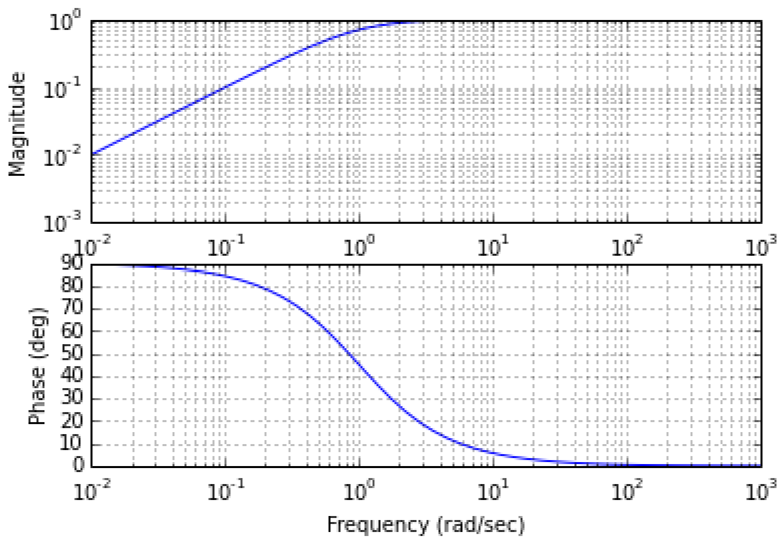
$$\cos(w_0 t) \leftrightarrow \pi[\delta(w - w_0) + \delta(w + w_0)]$$
$$\frac{df(t)}{dt} \leftrightarrow jwF(w)$$

$$x(t) * h(t) \leftrightarrow X(w)H(w)$$

This helps us represent circuits in another way: transfer functions. These are defined by their magnitude and phase. We noramlly draw them on a log-log (log-dB) scale. The two classical ones are high pass and low pass filters.

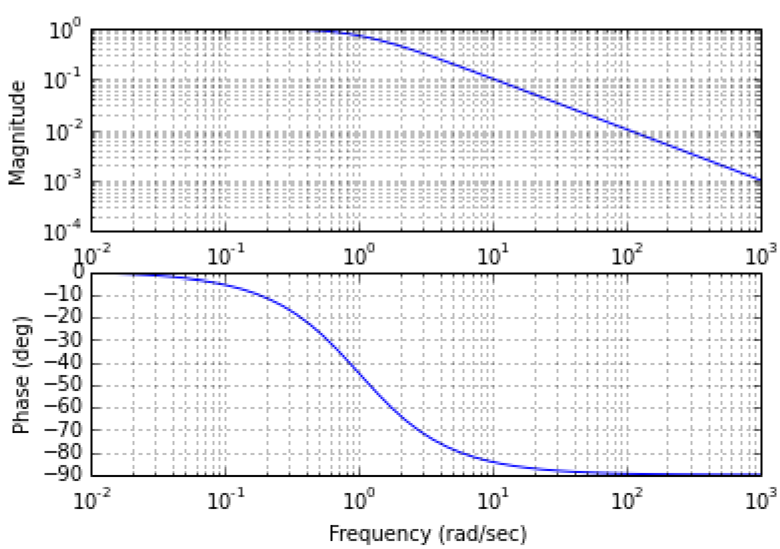
High pass filters have a magnitude of 1 as $w \to \infty$, and take the form:

$$H(w) = \frac{jw}{1 + jw}$$



Low pass filters have a magnitude of 1 as $w \to 0$, and take the form:

$$H(w) = \frac{1}{1 + jw}$$



Frequency Domain RLC

Sometimes it's simpler to represent RLC circuits in frequency domain rather than time domain. We can now define each RLC element as an impedance Z:

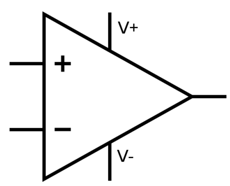
- $Z_R = R$
- $Z_C = \dfrac{1}{jwC}$
- $Z_L = jwL$

This modifies Ohm's Law to be $V = IZ$. We treat impedances like resistors, summing them cumulatively in series, and inversely in parallel.

Op Amps

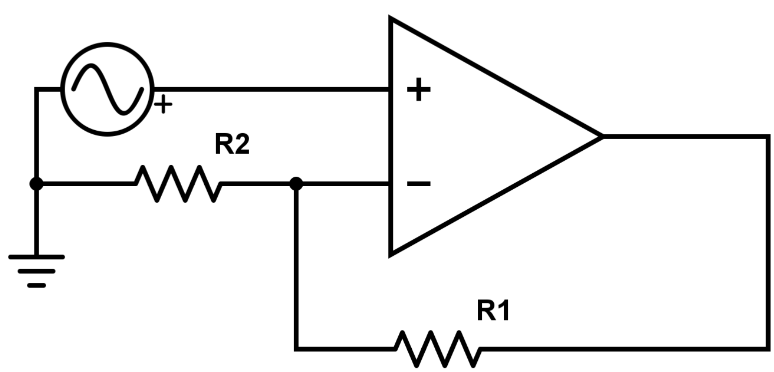
Op amps are a common active component in EE. They have five terminals:

1. Positive Input
2. Negative Input
3. Output
4. Positive Supply
5. Negative Supply



While they are complex devices, we can abstract them with three rules:

- No current flows into the positive/negative inputs
- The voltage at the positive/negative inputs is the same
- The output voltage can't go above or below the supply



As an example of how to solve this circuit:

$$V_+ = V_{in} = V_-$$

$$i_{R2} = \frac{Vin}{R2} = i_{R1}$$

$$V_{out} - V_- = i_{R1} R_1$$

$$V_{out} = V_{in} + V_{in} \frac{R1}{R2}$$

$$V_{out} = V_{in} \left(1 + \frac{R1}{R2} \right)$$

AM Radio/Mixing