Homework 02, Isaac Hancock, ST371

Import Section

In [9]:

```
Powersave-Tweaks import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
import pandas as pd
import stemgraphic as stg
import statistics
```

Problem 17

Part A

There is most likely another statistical software package that is not being accounted for here. We can look at a nice punnet square to help with the rest of the problems

Table	A	A'	Total
B	0	0.5	0.5
B'	0.3	0.2	0.5
Total	0.3	0.7	1

Part B

$P(A') = 1 - P(A) = 1 - 0.3 = 0.7$

Part C

Because P(A) and P(B) are mutually exclusive, we know that
$P(A \cup B) = P(A) + P(B) = 0.3 + 0.5 = 0.8$

Part D

By Demorgans we can do the following. $P(A \cup B)' = P(A' \cap B')$ Thus we are looking for $P(C')$ where $P(C) = P(A \cup B)$. As a result, $(P(C') = 1 - P(C) = 1 - 0.8 \rightarrow P(A' \cap B') = 0.2$

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Problem 26

Part A

$P(A_1) = 1 - P(A_1') = 1 - 0.12 = 0.88$

Part B

$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) \rightarrow P(A_1 \cap A_2) = P(A_1) + P(A_2) - P(A_1 \cup A_2)$
$= 0.12 + 0.07 - 0.13 \rightarrow P(A_1 \cap A_2) = 0.06$

Part C

Trying to solve $P(A_1 \cap A_2 \cap A_3')$
$P(A_1 \cap A_2 \cap A_3') + P(A_1 \cap A_2 \cap A_3) = P(A_1 \cap A_2) \rightarrow P(A_1 \cap A_2 \cap A_3') = P(A_1 \cap A_2)$
$- P(A_1 \cap A_2 \cap A_3) = 0.06 - 0.01 \rightarrow P(A_1 \cap A_2 \cap A_3') = 0.05$

Part D

This means we need to find the probably of at least 1 part functioning properly. This also can be expressed as $P(D) = 1 - P(A_1 \cap A_2 \cap A_3)$. $P(D) = 1 - 0.01 \rightarrow P(D) = 0.99$

Problem 56

Prove that For any events A and B with $P(B) > 0$ that $P(A|B) + P(A'|B) = 1$

We know that $P(A|B) = \frac{P(A \cap B)}{P(B)}$ and $P(A'|B) = \frac{P(A' \cap B)}{P(B)}$. Therefore
$P(A|B) + P(A'|B) = \frac{P(A \cap B)}{P(B)} + \frac{P(A' \cap B)}{P(B)} = \frac{P(A' \cap B) + P(A' \cap B)}{P(B)}$. We also know that
$P(A \cap B) + P(A' \cap B) = P(B)$, so it follows that $P(A|B) + P(A'|B) = \frac{P(B)}{P(B)}$ and that for $P(B) > 0$ to avoid divergence, $P(A|B) + P(A'|B) = 1$

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Problem 59

$P(A_1) = 0.40, P(A_2) = 0.35, P(A_3) = 0.25, P(B|A_1) = 0.3, P(B|A_2) = 0.6, P(B|A_3) = 0.5$

Part A

We can determine this by what we know about conditional probability.
$P(B|A_2) = \frac{P(A_2 \cap B)}{P(A_2)} \rightarrow P(A_2 \cap B) = P(A_2)P(B|A_2) = 0.35 * 0.6 \rightarrow (A_2 \cap B) = 0.21$

Part B

For this we must use our answer to Part A. We are trying to find $P(B)$ This can be accomplished by finding $P(A_1 \cap B)$ and $P(A_3 \cap B)$

Building off of the logic described in Part A $P(A_1 \cap B) = P(A_1)P(B|A_1) = 0.4 * 0.3 = 0.12$

$P(A_3 \cap B) = P(A_3)P(B|A_3) = 0.25 * 0.5 = 0.125$

We know that
$P(A_1 \cap B) + P(A_2 \cap B) + P(A_3 \cap B) = p(B) \rightarrow P(B) = 0.21 + 0.12 + 0.125 = 0.455$

Part C

We now must find $P(A_1|B)$,$P(A_2|B)$, and $P(A_3|B)$.

$P(A_1|B) = \frac{P(A_1 \cap B)}{P(B)} = \frac{0.12}{0.455} = 0.264$

$P(A_2|B) = \frac{P(A_2 \cap B)}{P(B)} = \frac{0.21}{0.455} = 0.462$

$P(A_3|B) = \frac{P(A_3 \cap B)}{P(B)} = \frac{0.125}{0.455} = 0.275$

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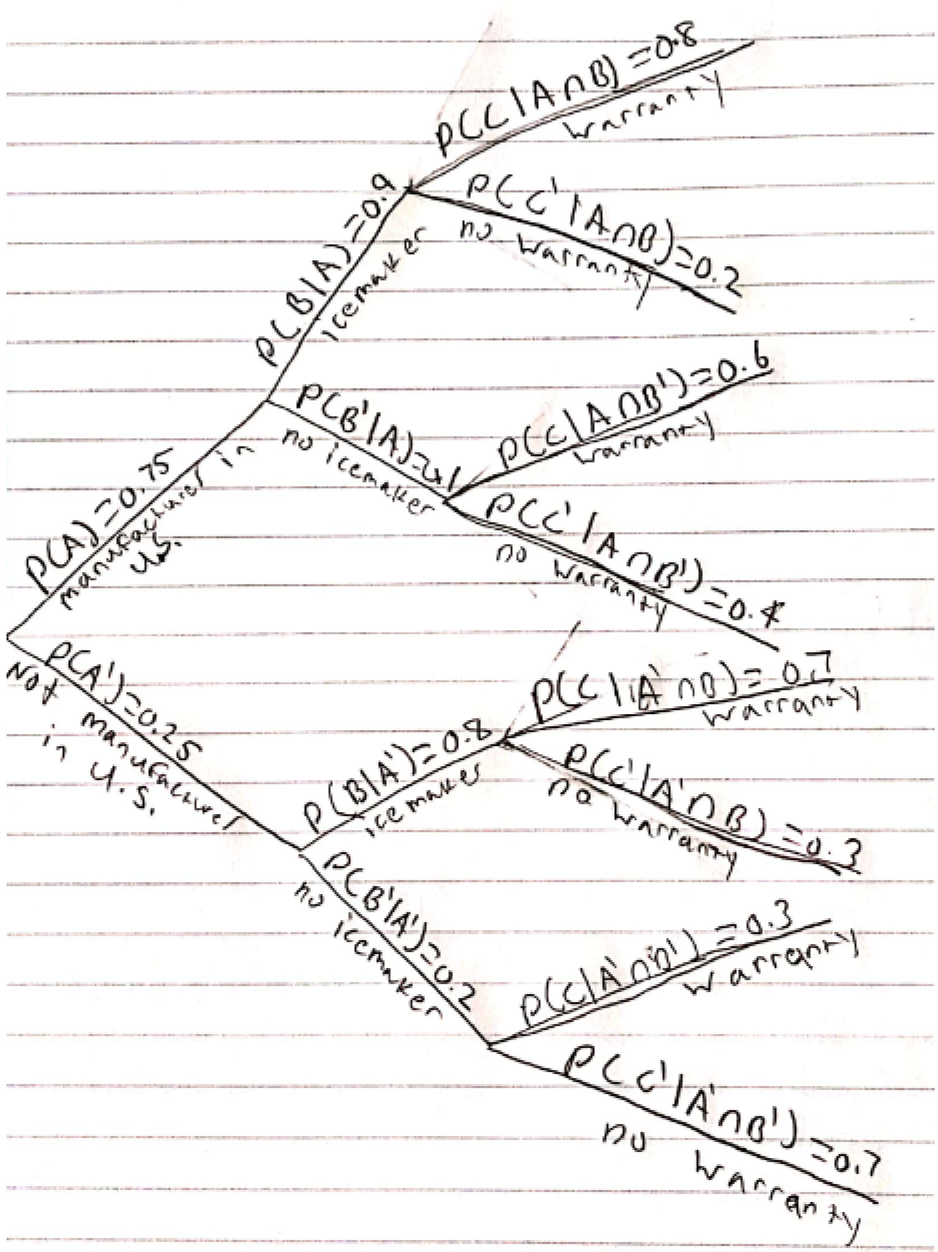
$P(A) = 0.75, P(B|A) = 0.9, P(B|A') = 0.8, P(C|A \cap B) = 0.8, P(C|A \cap B') = 0.6,$
$P(C|A' \cap B) = 0.7, P(C|A' \cap B') = 0.3$

Problem 63

Part A

Below I have drawn a 3 segment tree for all possible branches.



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Part B

Compute $P(A \cap B \cap C)$

$P(C|A \cap B) = \frac{P(A \cap B \cap C)}{P(A \cap B)} \rightarrow P(B|A) = \frac{P(A \cap B)}{P(A)} \rightarrow P(A \cap B) = P(B|A) * P(A) = 0.9 * 0.75 = 0.675 \rightarrow P(A \cap B \cap C) = P(C|A \cap B) * P(A \cap B) = 0.8 * 0.675 = 0.54$

Part C

Compute $P('A \cap B \cap C)$ and then add that to what was found in Part C.

$P(C|A' \cap B) = \frac{P(A' \cap B \cap C)}{P(A' \cap B)} \rightarrow P(B|A') = \frac{P(A' \cap B)}{P(A')} \rightarrow P(A' \cap B) = P(B|A') * P(A') = 0.8 * 0.25 = 0.2 \rightarrow P(A' \cap B \cap C) = P(C|A' \cap B) * P(A' \cap B) = 0.7 * 0.2 = 0.14$

$P(B \cap C) = P('A \cap B \cap C) + P(A \cap B \cap C) = 0.14 + 0.54 = 0.68$

Part D

To find $P(C)$ we must find $P(B' \cap C)$ which means determing $P(A \cap B' \cap C)$ and $P(A' \cap B' \cap C)$.

First we find $P(A \cap B' \cap C)$
$P(C|A \cap B') = \frac{P(A \cap B' \cap C)}{P(A \cap B')} \rightarrow P(B'|A) = \frac{P(A \cap B')}{P(A)} \rightarrow P(A \cap B') = P(B'|A) * P(A) = 0.1 * 0.75 = 0.075 \rightarrow P(A \cap B' \cap C) = P(C|A \cap B') * P(A \cap B') = 0.6 * 0.075 = 0.045$

Second we find $P(A' \cap B' \cap C)$
$P(C|A' \cap B') = \frac{P(A' \cap B' \cap C)}{P(A' \cap B')} \rightarrow P(B'|A') = \frac{P(A' \cap B')}{P(A')} \rightarrow P(A' \cap B') = P(B'|A') * P(A') = 0.2 * 0.25 = 0.05 \rightarrow P(A' \cap B' \cap C) = P(C|A' \cap B') * P(A' \cap B') = 0.3 * 0.05 = 0.015$

Finally we add these to get $P(B' \cap C)$
$P(B' \cap C) = P(A \cap B' \cap C) + P(A' \cap B' \cap C) = 0.045 + 0.015 = 0.06$

$P(C) = P(B' \cap C) + P(B \cap C) = 0.06 + 0.68 = 0.74$

Part E

$P(A|B \cap C) = \frac{P(A \cap B \cap C)}{P(B \cap C)} = \frac{0.54}{0.68} = 0.794$

In [ ]: