

Homework 03, Isaac Hancock, ST371

Import Section

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In [1]: import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
import pandas as pd
import stemgraphic as stg
import statistics
```

Problem 74

Table	A	B	AB	O
P	0.4	0.11	0.04	0.45

Independent if $P(A \cap B) = P(A) * P(B)$ Since these are independent, we can find the probability that two randomly phenotypes are O by multiplying the probabilities. $0.45 * 0.45 = 0.2025$

The probability that two random phenotypes match is the sum of the probability of drawing two of the same for each phenotype.

- A : $0.4 * 0.4 = 0.16$
- B: $0.11 * 0.11 = 0.0121$
- AB : $0.04 * 0.04 = 0.0016$
- O: $0.45 * 0.45 = 0.2025$

Total is $0.16 + 0.0121 + 0.0016 + 0.2025 = 0.3762$

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Problem 80

$P(1) = P(2) = 0.9$ and $P(3) = P(4) = 0.8$ find $P(\text{system works})$

The probability for the system working is $P(3 \cap 4 \cup S_{12})$ where $S_{12} = P(1 \cup 2)$ By independence,

$$P(3 \cap 4) = P(3)P(4) = 0.64$$

$$P(1 \cup 2) = P(1) + P(2) - P(1 \cap 2) = P(1) + P(2) - P(1)P(2) = 0.9 + 0.9 - (0.9 * 0.9) = 0.99$$

$$P(3 \cap 4 \cup S_{12}) = P(3 \cap 4) + S_{12} - P(3 \cap 4 \cap S_{12}) = 0.64 + 0.99 - (0.64 * 0.99) = 0.9964$$

Therefore $P(\text{system works}) = 0.9964$

Problem 8

Discrete Random Variables $T(S) = 1, T(F) = 0$ For the set, the five smallest possible values are as follows.

- $Y = 3: S = \{SSS\}$
- $Y = 4: \{FSSS\}$
- $Y = 5: \{FFSSS\}, \{SFSSS\}$
- $Y = 6: \{FFFSSS\}, \{SFFSSS\}, \{FSFSSS\}, \{SSFSSS\}$
- $Y = 7:$
 $\{FFFFSSS\}, \{SFFFSSS\}, \{FSFFSSS\}, \{SSFFSSS\}, \{FFSFSSS\},$
 $\{SFSFSSS\}, \{FSSFSSS\}$

Thus for Y the 5 smallest values are 3,4,5,6,7

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Problem 13

Part A

$$P(X \leq 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) = 0.1 + 0.15 + 0.2 + 0.25 = 0.7$$

Part B

$$P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2) = 0.1 + 0.15 + 0.2 = 0.45$$

Part C

$$P(X \geq 3) = 1 - P(X < 3) = 1 - 0.45 = 0.55$$

Part D

$$P(2 \leq X \leq 5) = P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) = 0.2 + 0.25 + 0.2 + 0.06 = 0.71$$

Part E

$$P(2 \leq X \leq 4) = 1 - P(X = 2) + P(X = 3) + P(X = 4) = 0.2 + 0.25 + 0.2 = 0.65$$

Part F

This can also be expressed as $P(X \leq 2) = P(X < 3) = 0.45$

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Problem 24

Part A

The pmf of $F(x)$ can be derived by subtracting overlapping terms to find the appropriate probability values.

$$\begin{aligned} P(X = n) &= F(n) - F(n - 1) \\ P(X = 0) &= 0 \\ P(X = 1) &= F(1) - F(0) = 0.3 - 0 = 0.3 \\ P(X = 2) &= F(2) - F(1) = 0.3 - 0.3 = 0 \\ P(X = 3) &= F(3) - F(2) = 0.4 - 0.3 = 0.1 \\ P(X = 4) &= F(4) - F(3) = 0.45 - 0.4 = 0.05 \\ P(X = 5) &= F(5) - F(4) = 0.45 - 0.45 = 0 \\ P(X = 6) &= F(6) - F(5) = 0.6 - 0.45 = 0.15 \\ P(X = 7) &= F(7) - F(6) = 0.6 - 0.6 = 0 \\ P(X = 8) &= F(8) - F(7) = 0.6 - 0.6 = 0 \\ P(X = 9) &= F(9) - F(8) = 0.6 - 0.6 = 0 \\ P(X = 10) &= F(10) - F(9) = 0.6 - 0.6 = 0 \\ P(X = 11) &= F(11) - F(10) = 0.6 - 0.6 = 0 \\ P(X = 12) &= F(12) - F(11) = 1 - 0.6 = 0.4 \end{aligned}$$

x	0	1	2	3	4	5	6	7	8	9	10	11	12
p(x)	0	0.3	0	0.1	0.05	0	0.15	0	0	0	0	0	0.4

Part B

$$P(3 \leq X \leq 6) = F(6) - F(2) = 0.6 - 0.3 = 0.3$$

$$P(4 \leq X) = P(3 < X) = 1 - F(3) = 1 - 0.4 = 0.6$$

In []: