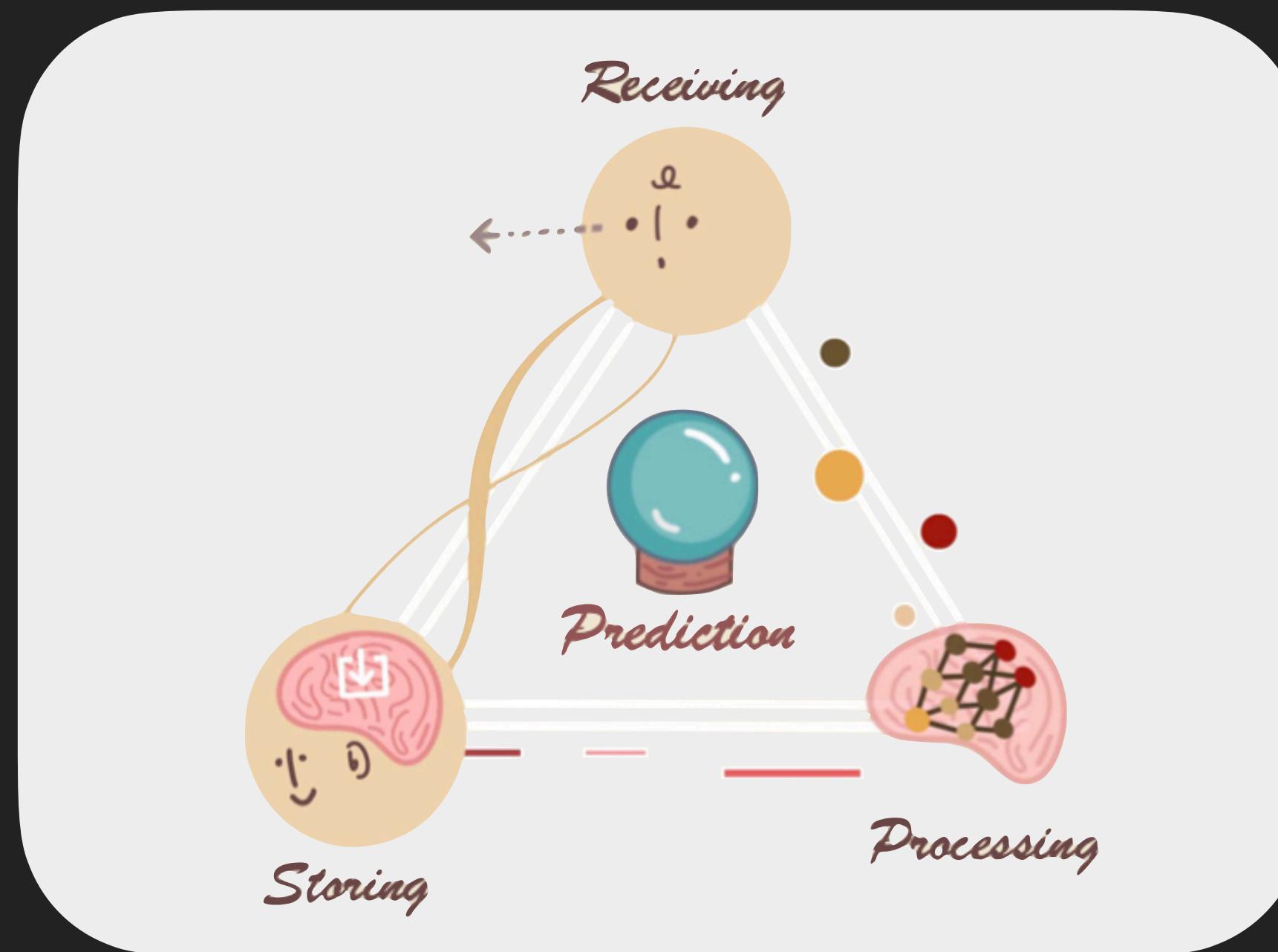


Ph 220: Quantum Learning Theory

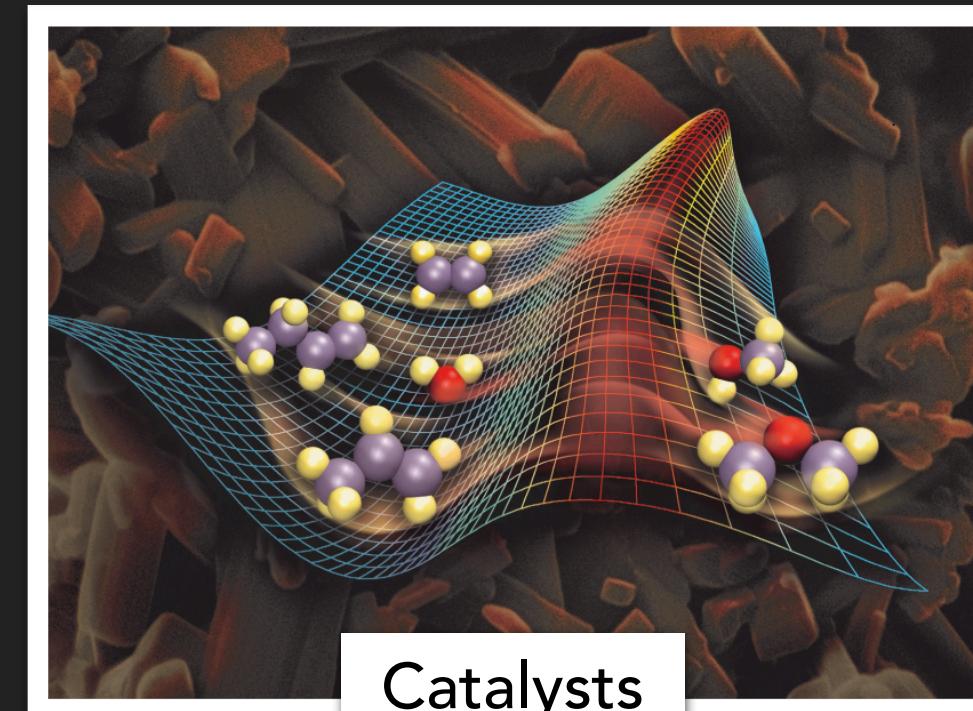
Hardness of Learning

Motivation

- Are some basic aspects of our universe fundamentally **hard to learn?**



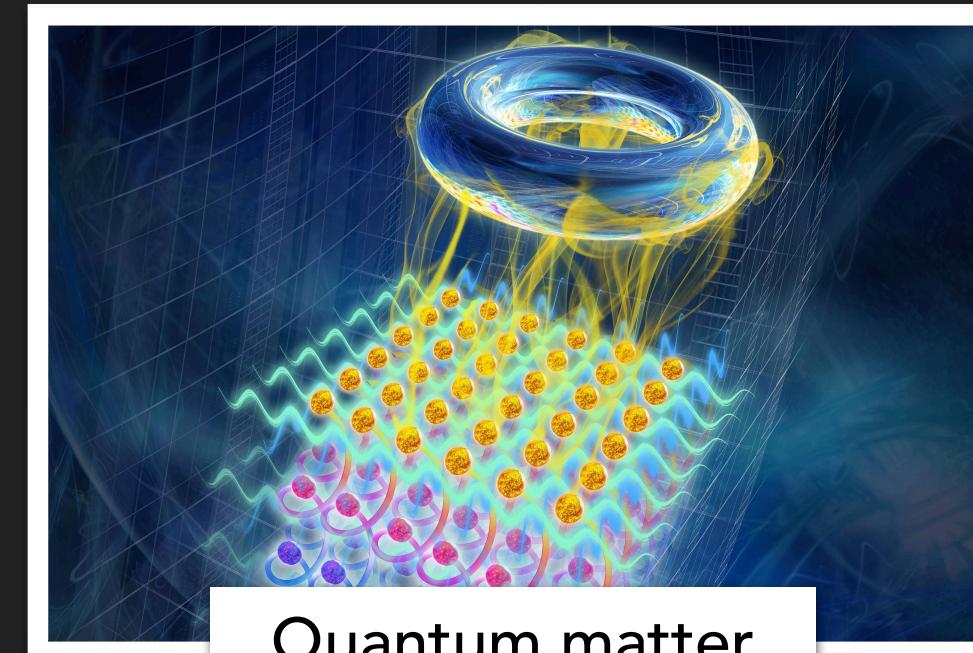
A cartoon depiction of learning



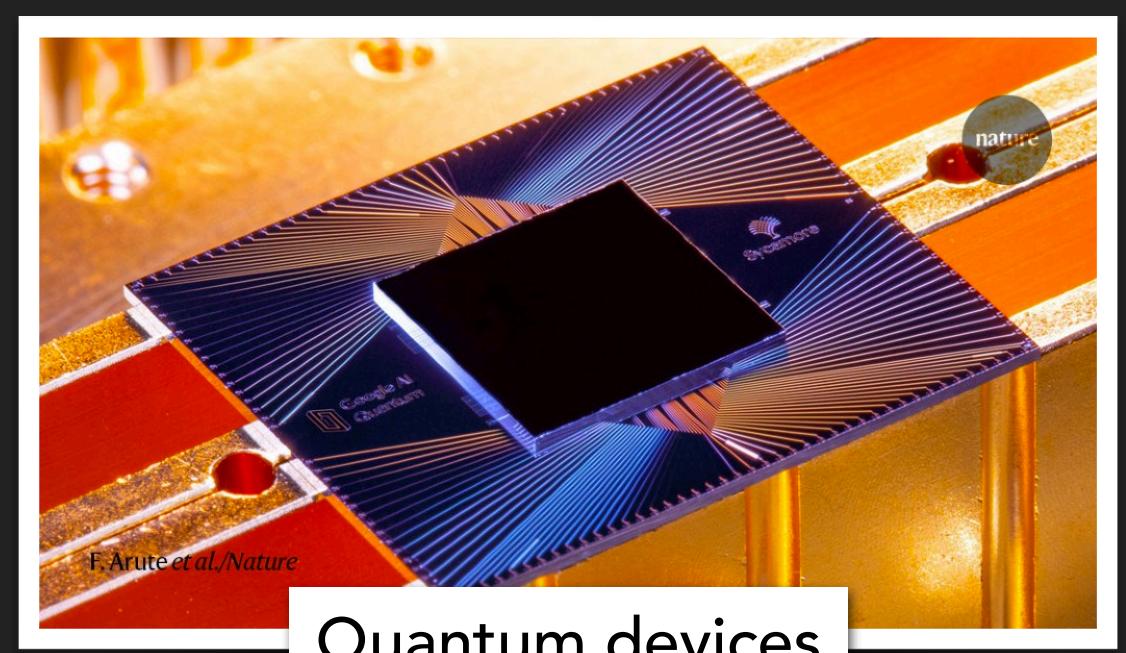
Catalysts



Pharmaceutics



Quantum matter

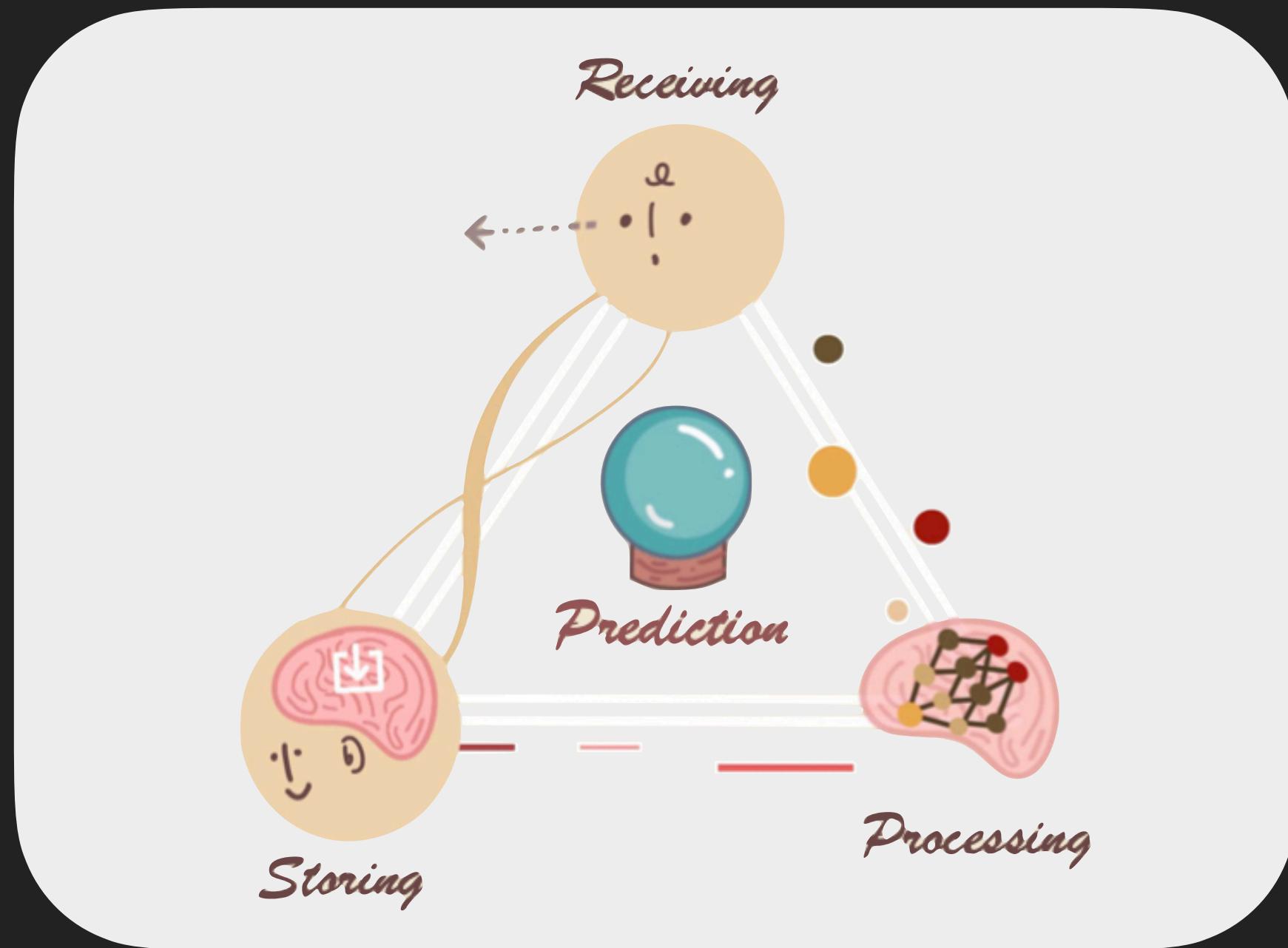


Quantum devices

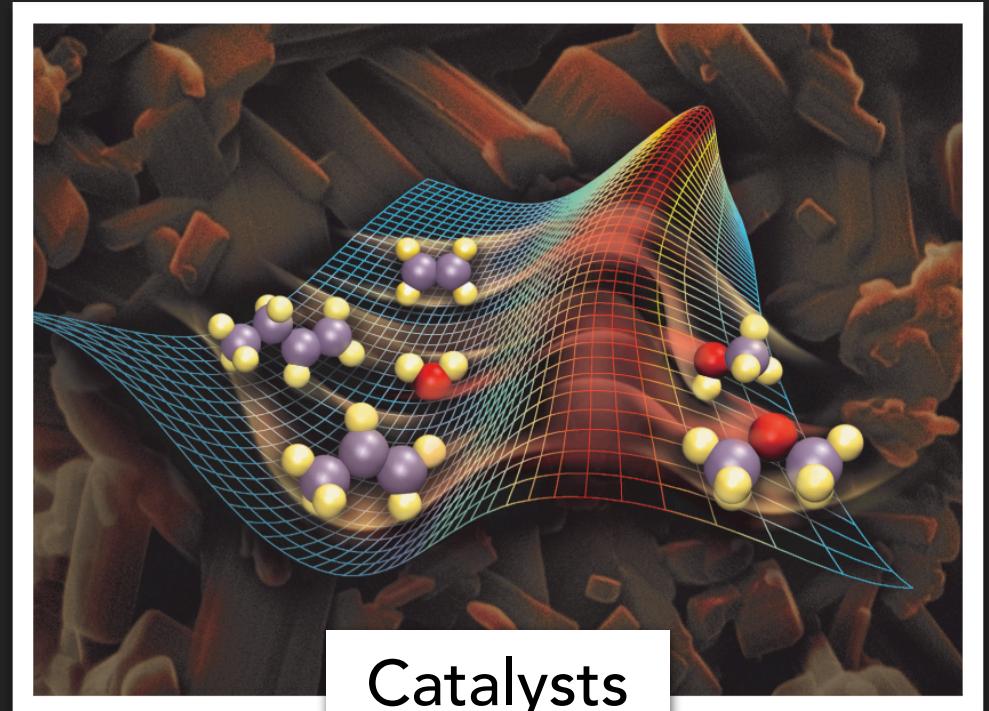
Motivation

- Are some basic aspects of our universe fundamentally

hard to measure
hard to see
hard to learn?
hard to feel



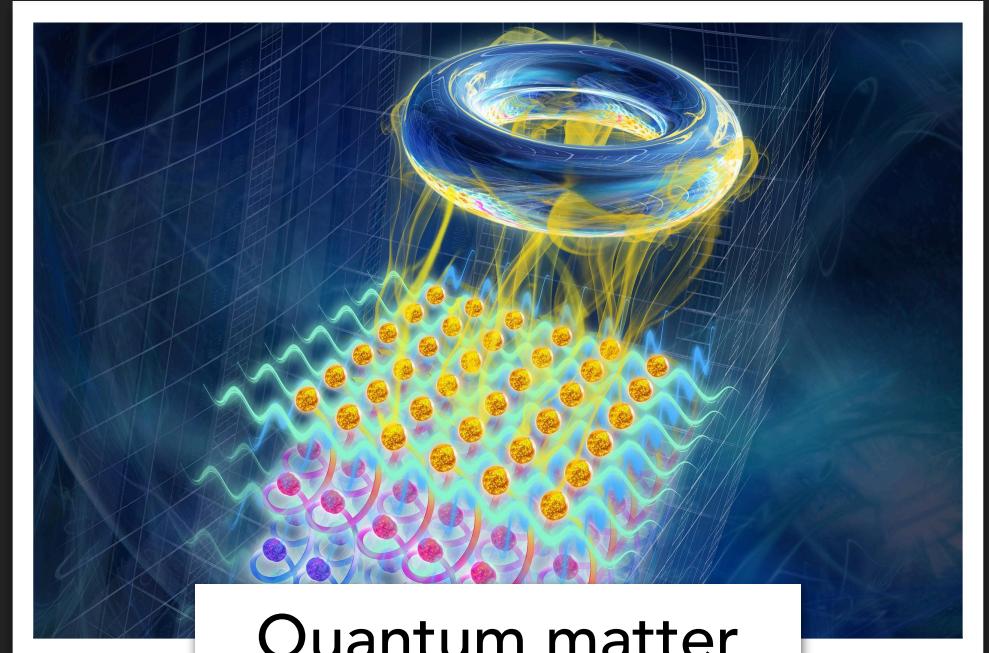
A cartoon depiction of learning



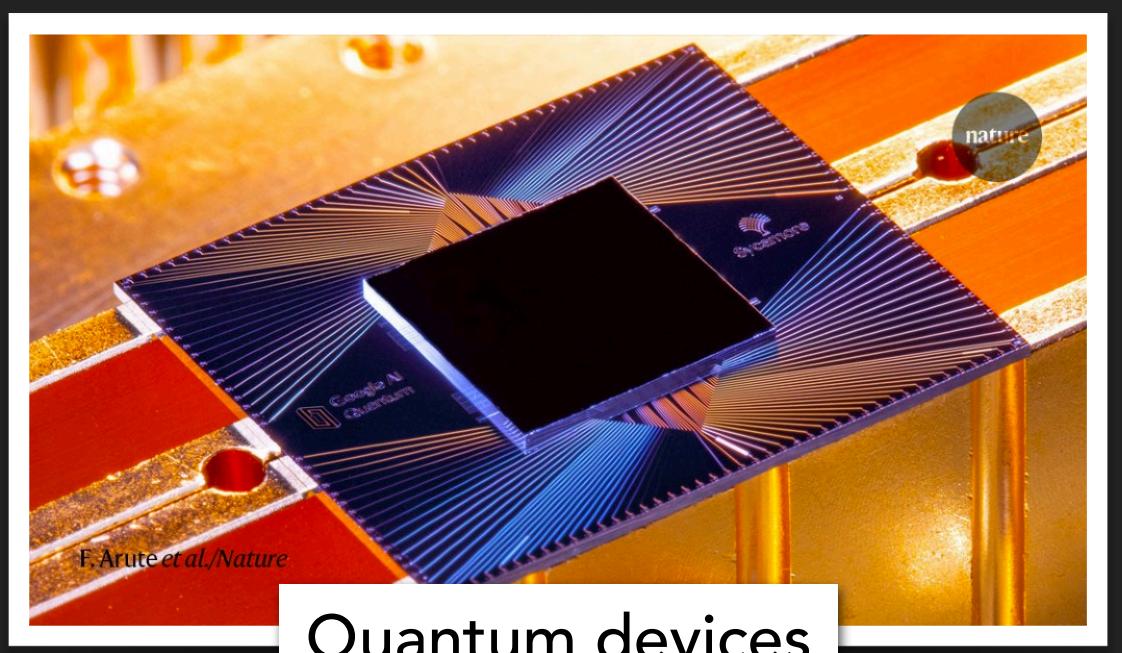
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Pharmaceutics



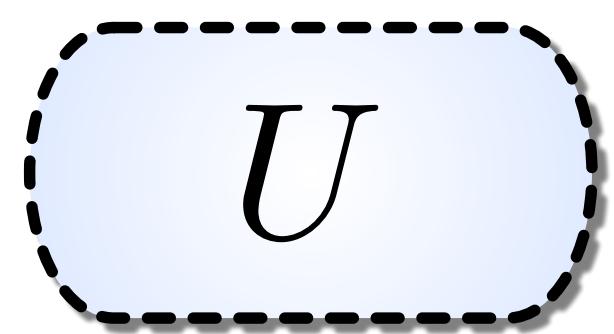
Quantum matter



Quantum devices

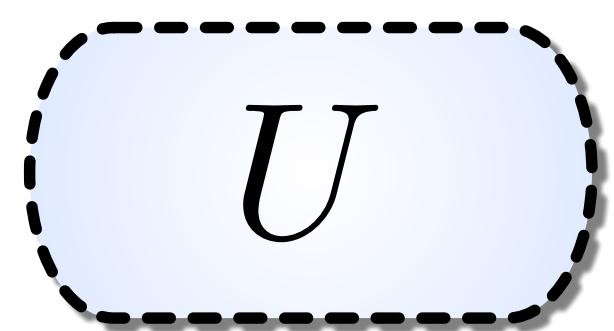
Can we feel time?

There is an unknown unitary

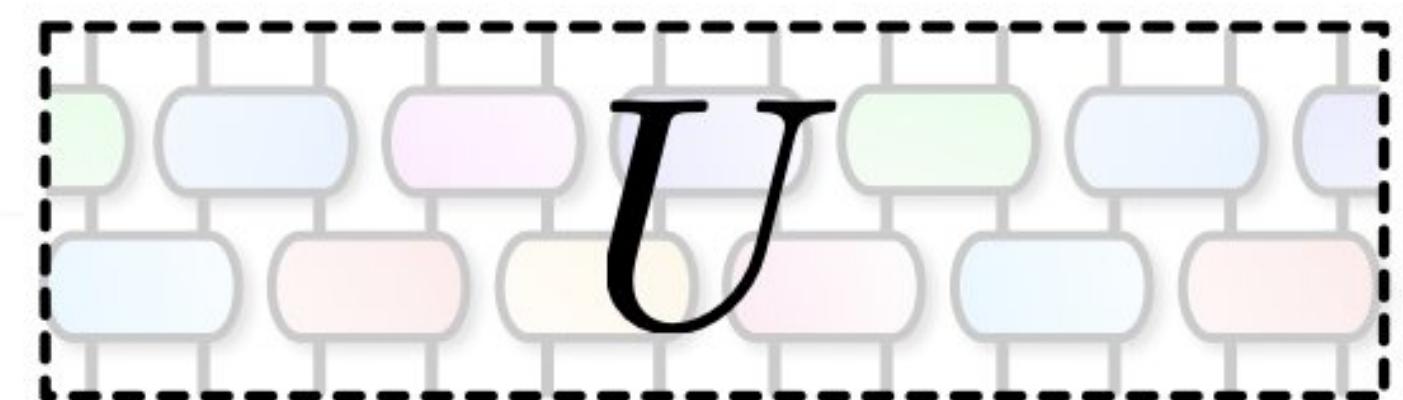


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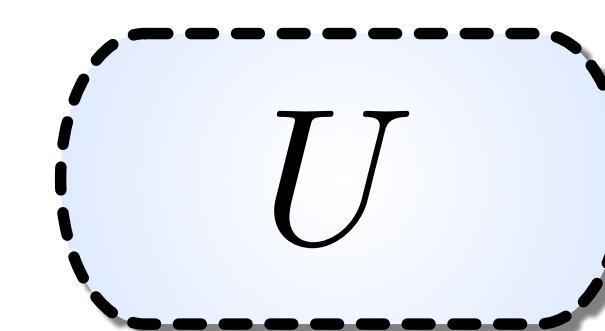
It is either



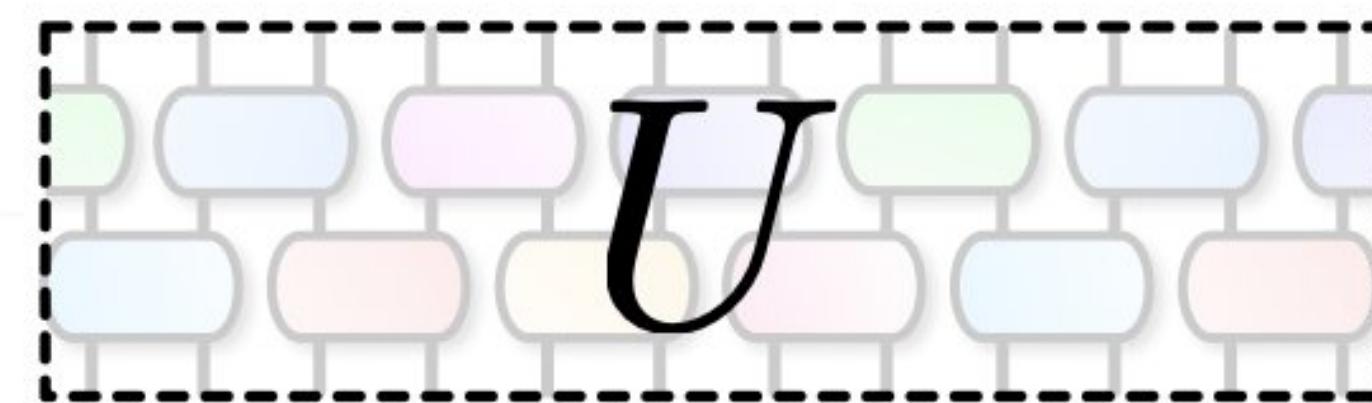
1D short-time dynamics

Can we feel time?

There is an unknown unitary

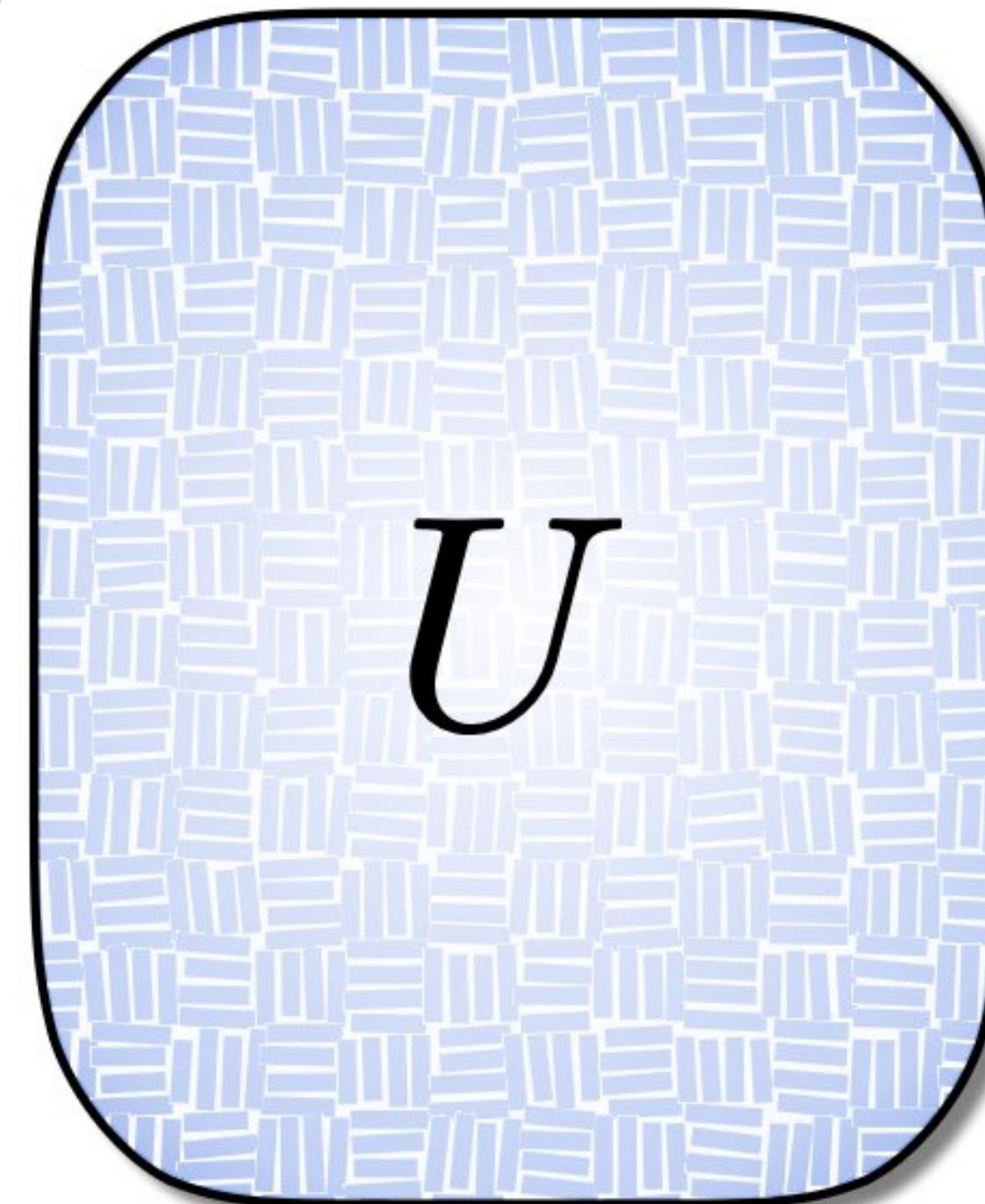


It is either



1D short-time dynamics

or

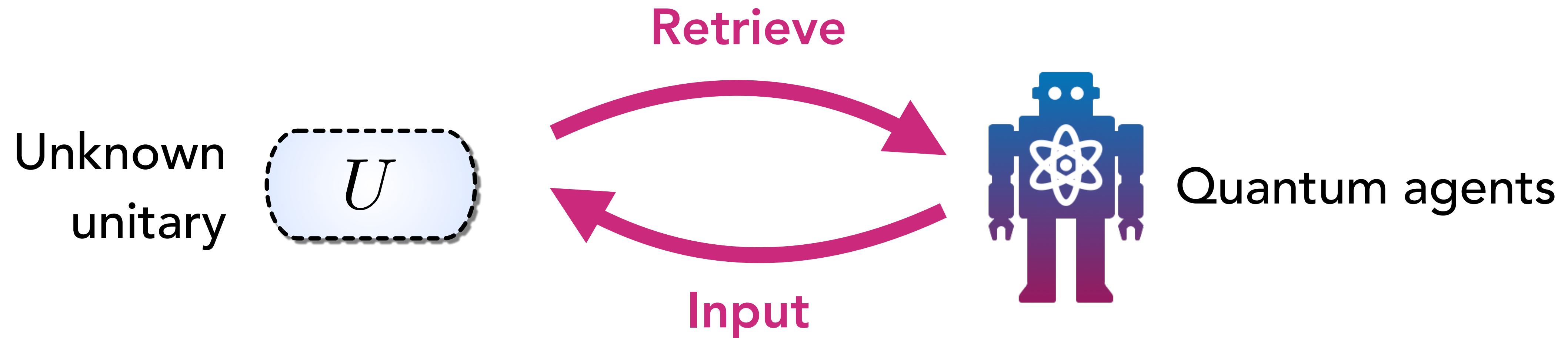


Generic, Haar-random

1D exponential-time dynamics

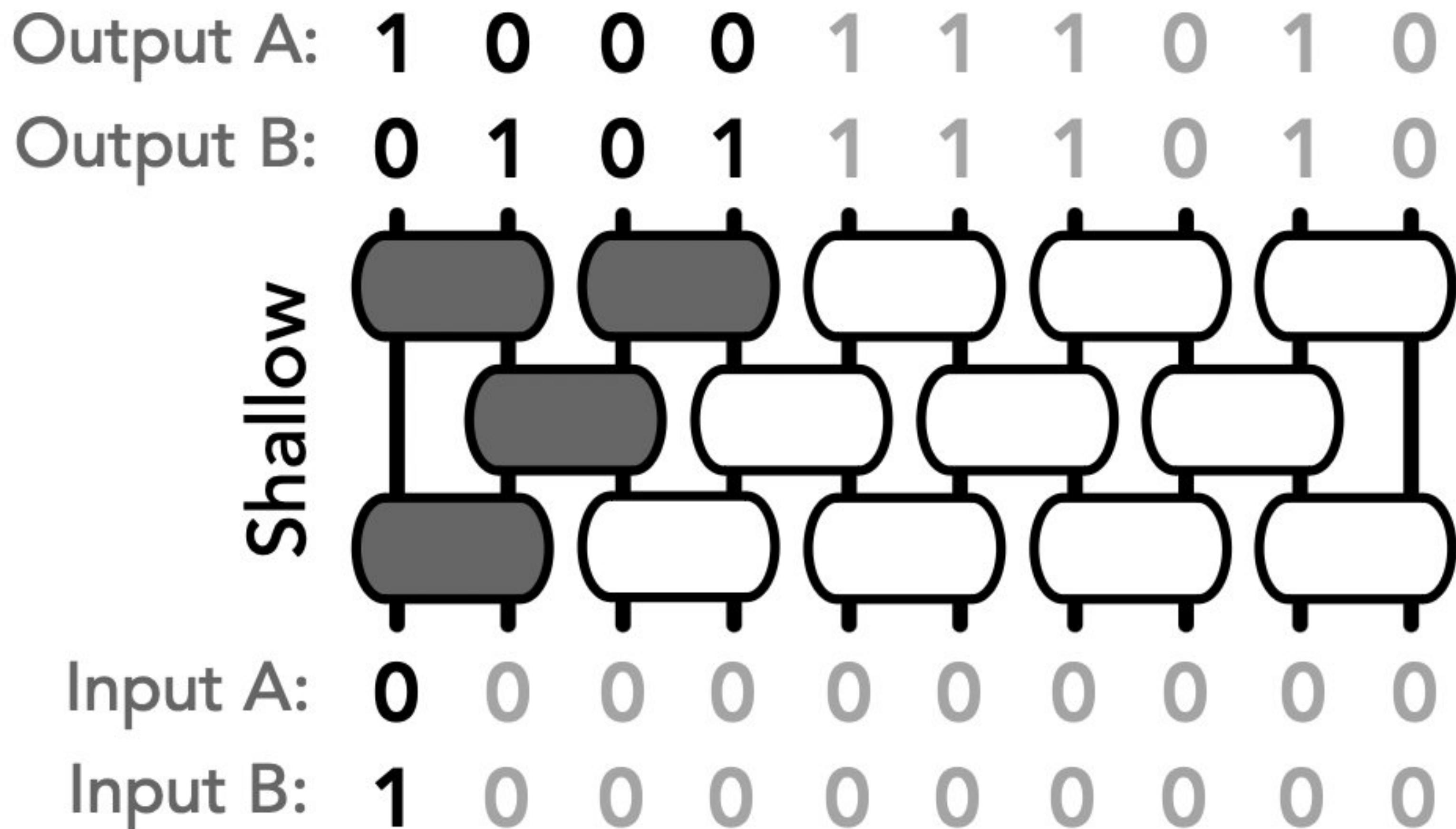
Classical dynamics

Can quantum agents distinguish **short-** vs **exponential-time**
in 1D **classical** dynamics?



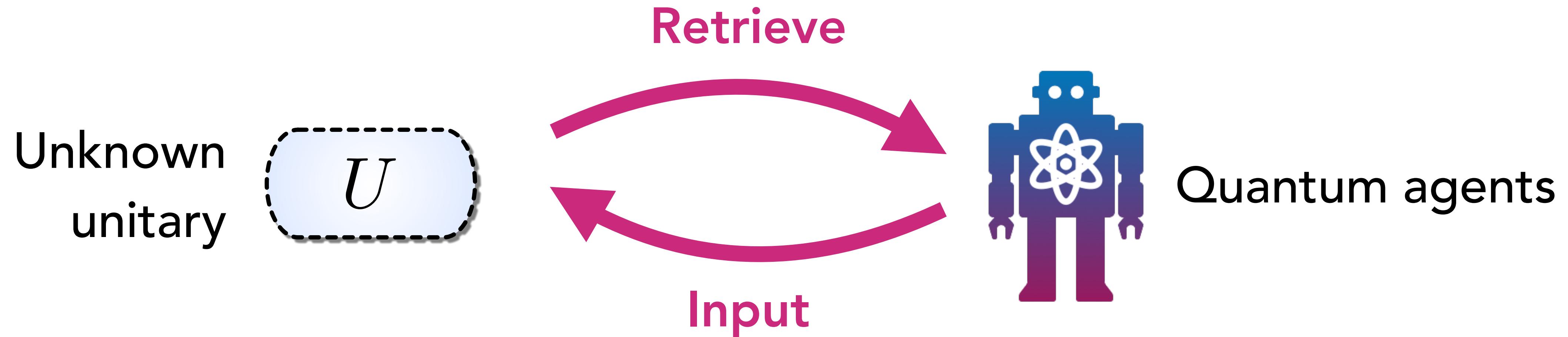
Classical dynamics

This is **easy!**
(by testing causality)



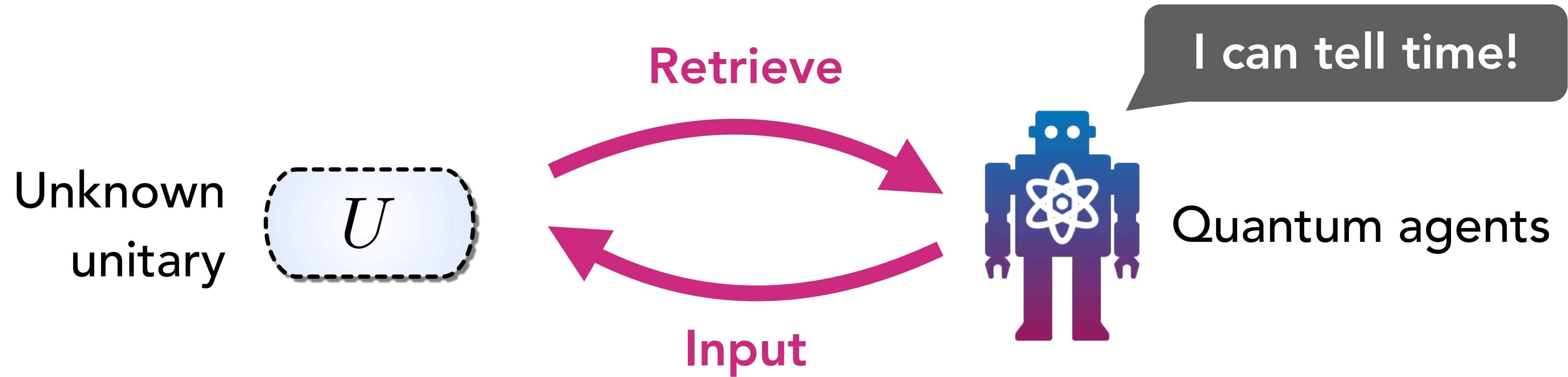
Classical dynamics

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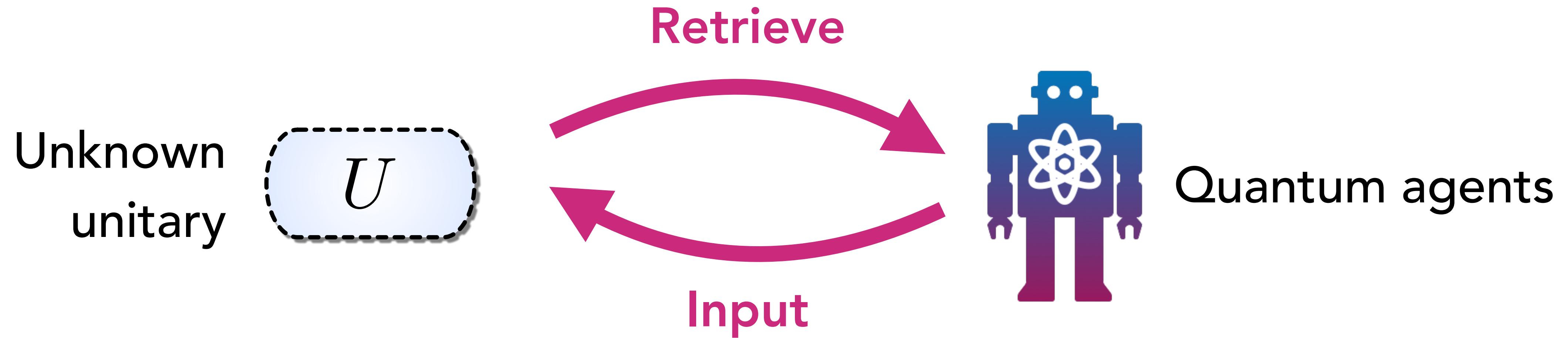
Classical dynamics

Can quantum agents distinguish **short-** vs **exponential-time** in 1D **classical** dynamics? **Yes!**



Quantum dynamics

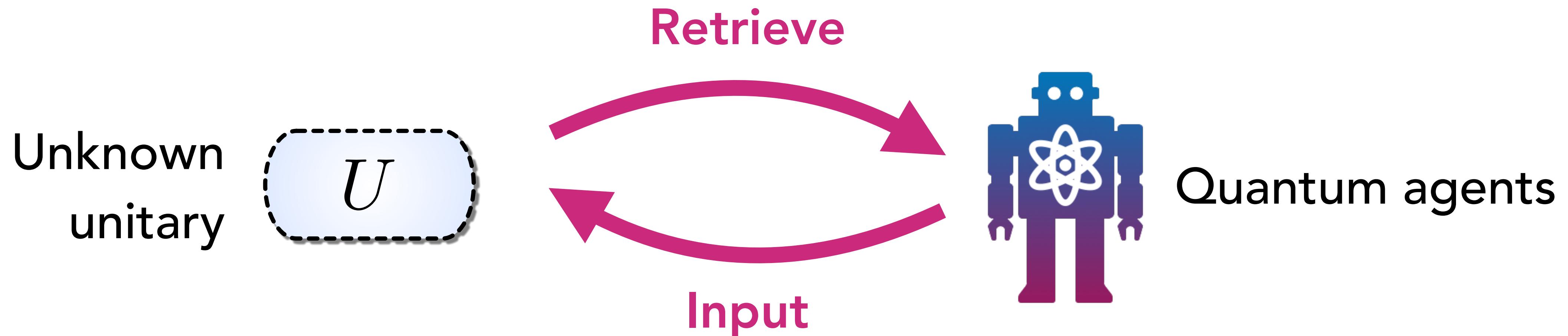
Can quantum agents distinguish **short-** vs **exponential-time**
in 1D **quantum** dynamics?



Quantum dynamics

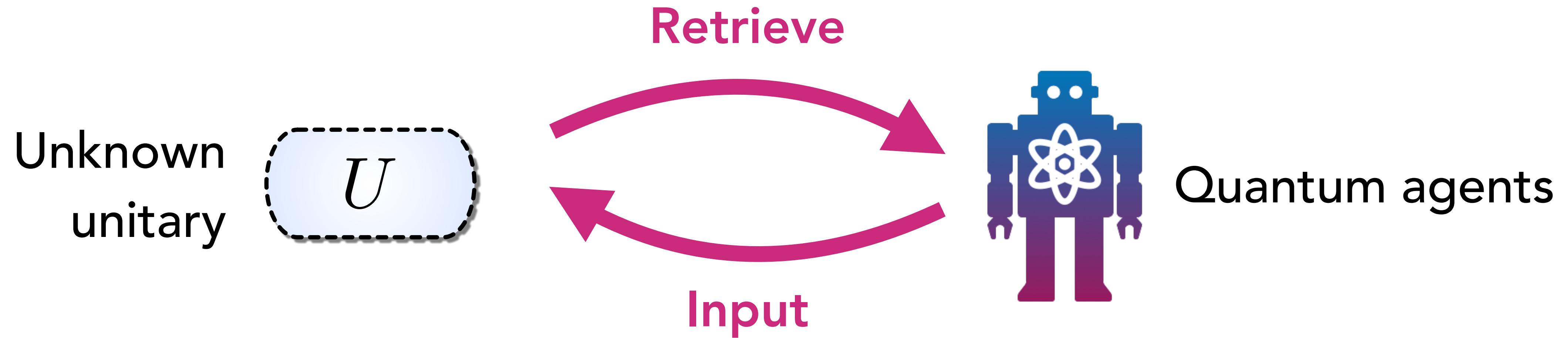
Theorem 1 (under standard crypto conjecture)

No polynomial-time quantum agents can tell apart
poly(log n)- vs **exp(n)-time** 1D quantum dynamics.



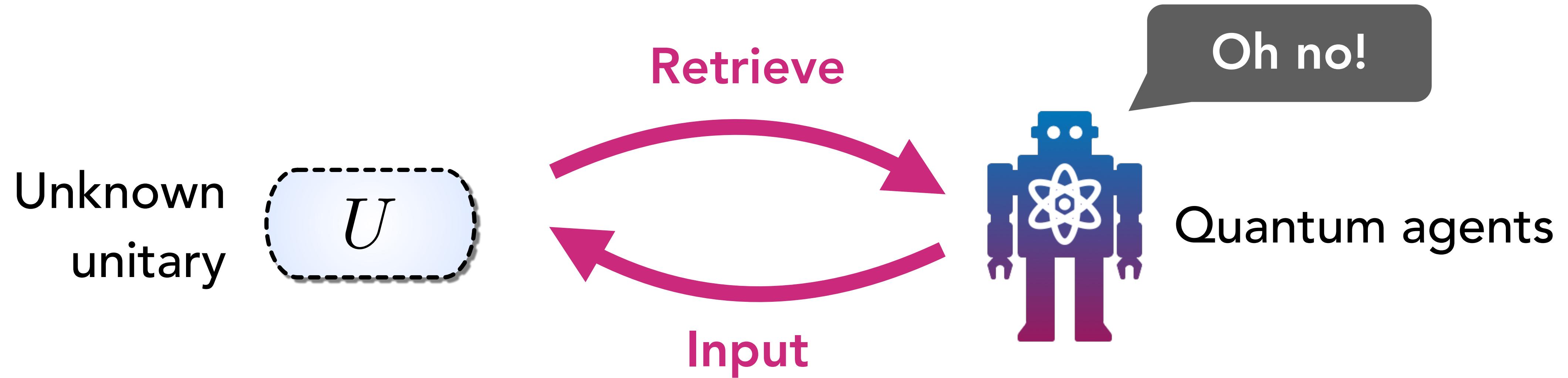
Quantum dynamics

Can quantum agents distinguish **short-** vs **exponential-time**
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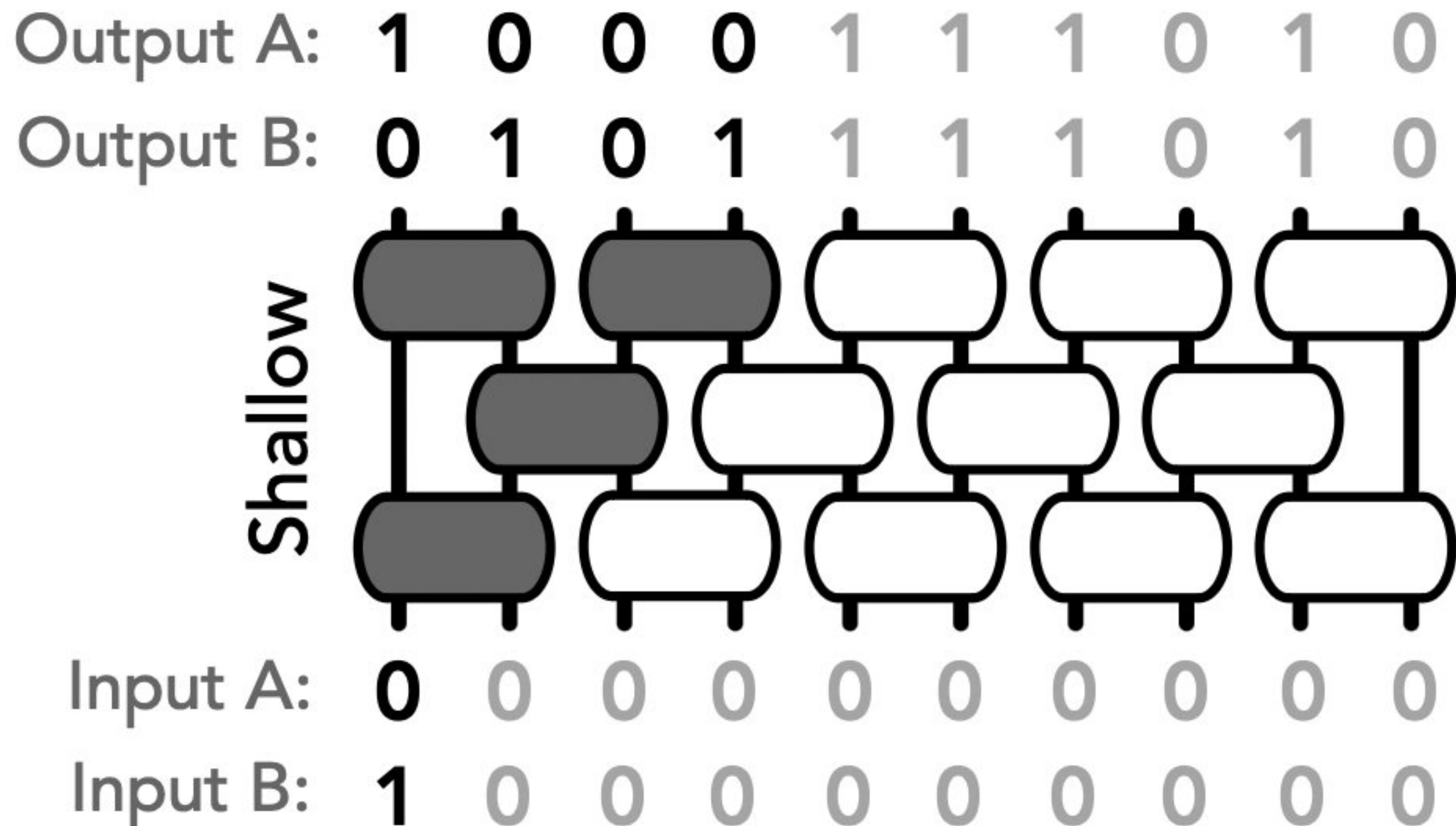
Quantum dynamics

Can quantum agents distinguish **short-** vs **exponential-time** in 1D **quantum** dynamics? **No!**



Classical dynamics

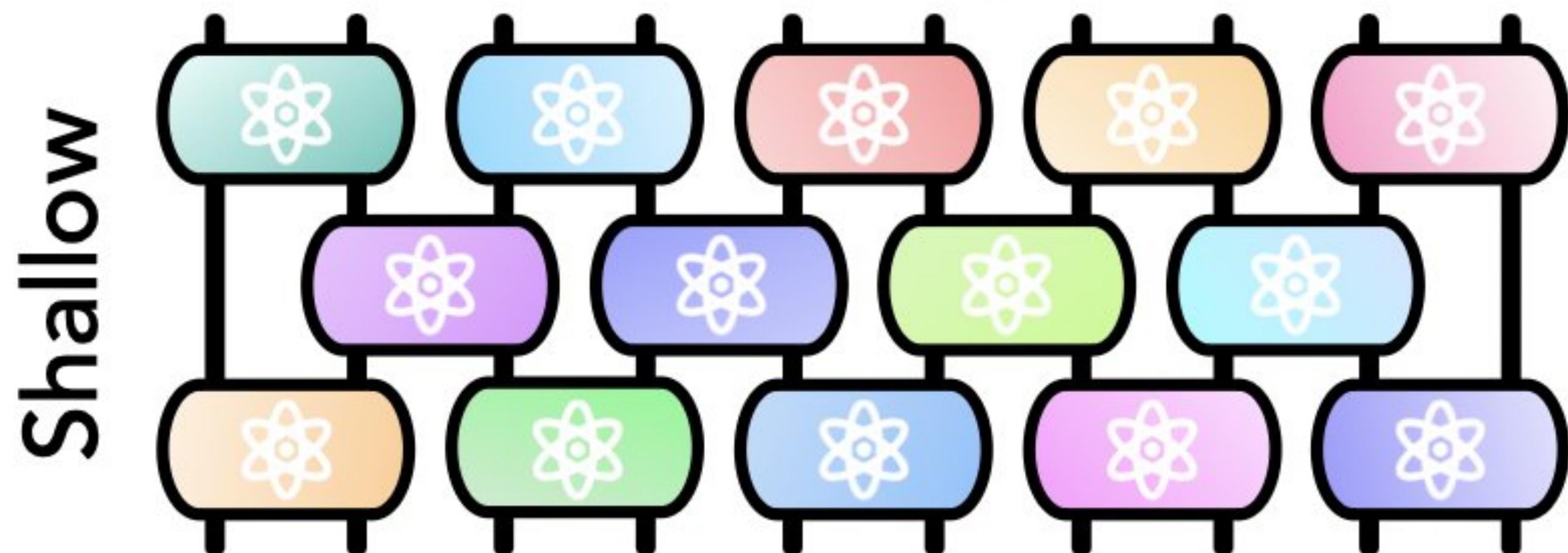
One needs at least
linear time to
behave random.



Quantum dynamics

One needs **only**
poly-log time to
behave random.

Output A: **Both output states A and B**
Output B: **look completely random**



Input A: 0 0 0 0 0 0 0 0 0 0

Input B: 1 0 0 0 0 0 0 0 0 0

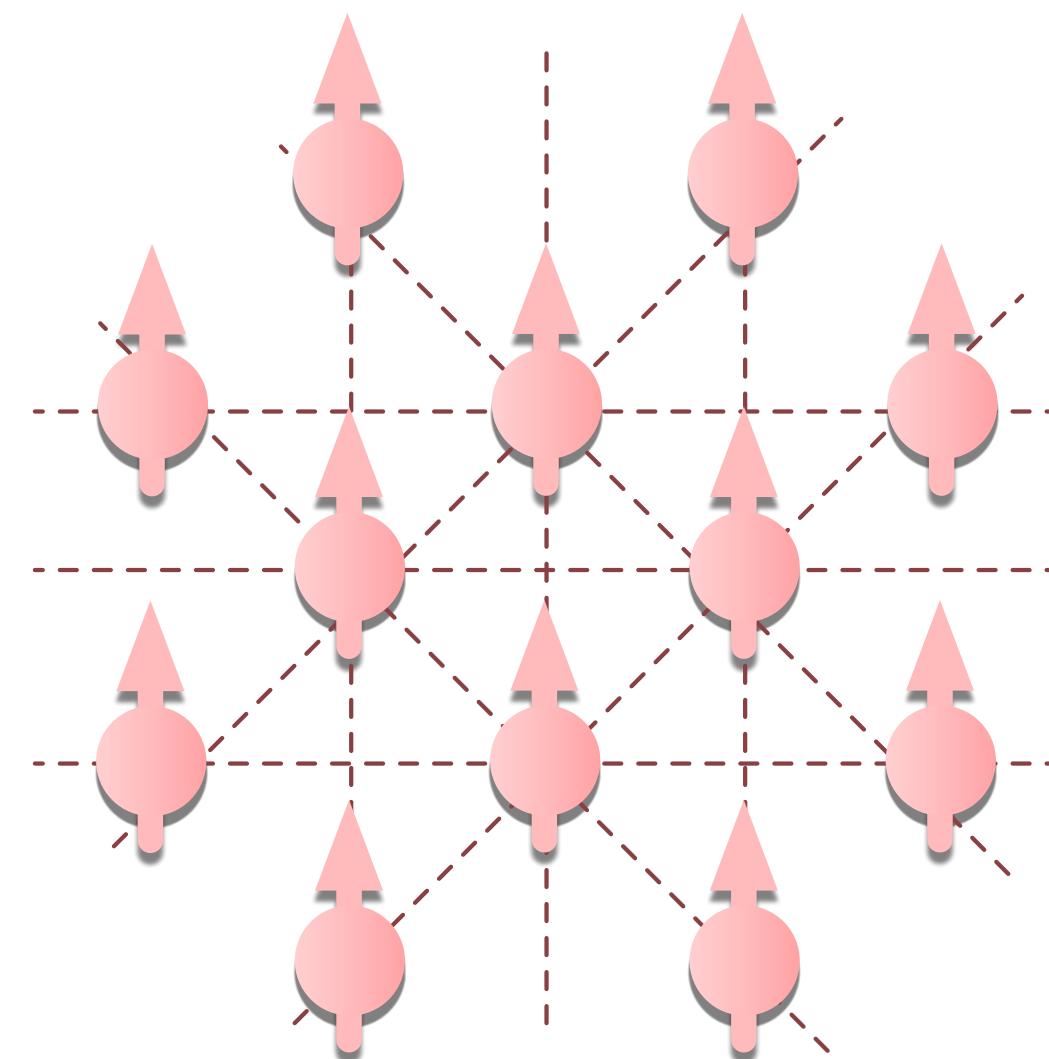
Is causality observable?

- Consider unitaries generated by quantum dynamics consisting of:

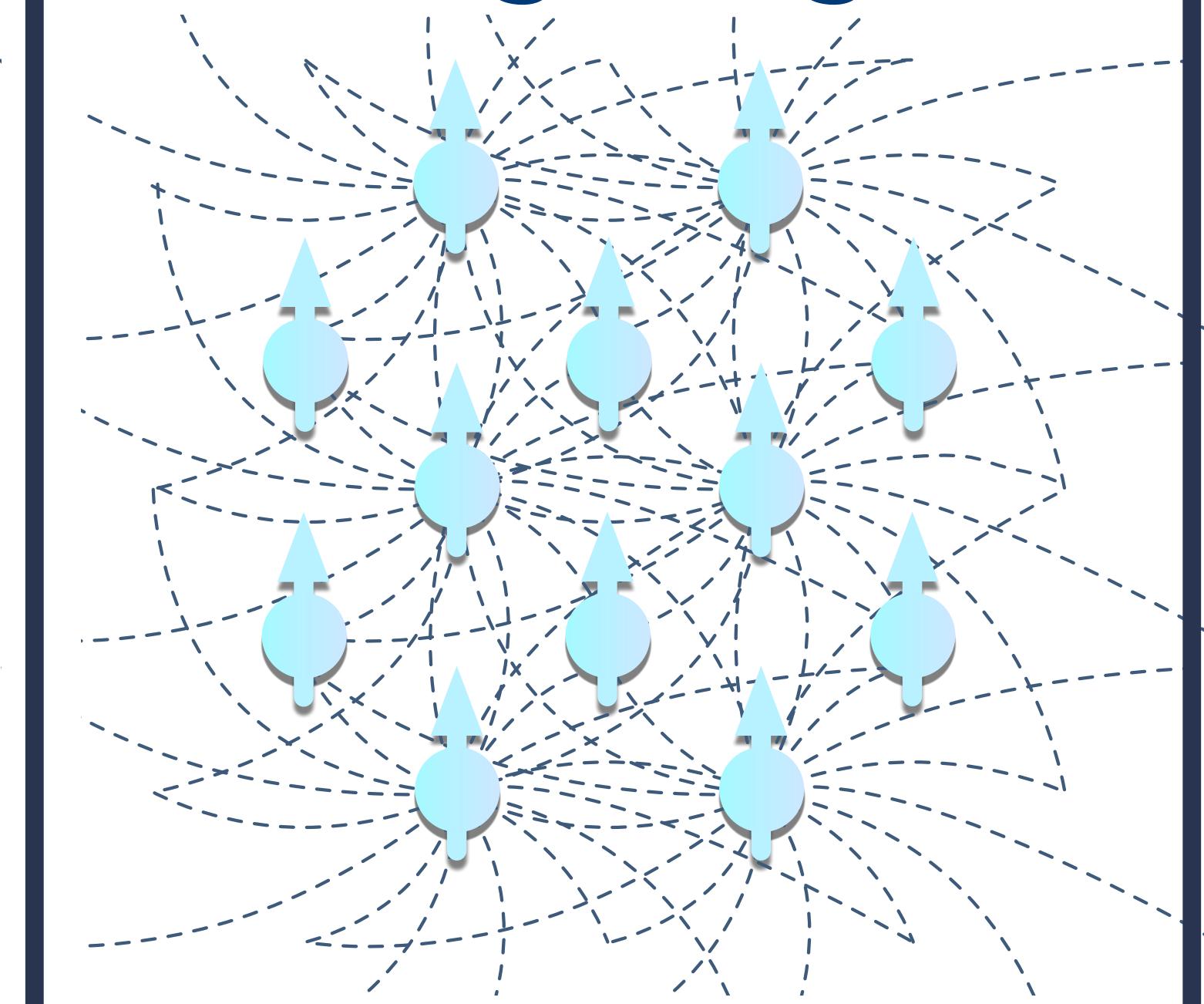
Is causality observable?

- Consider unitaries generated by quantum dynamics consisting of:
short-range interactions vs **long-range** interactions.

Short-range



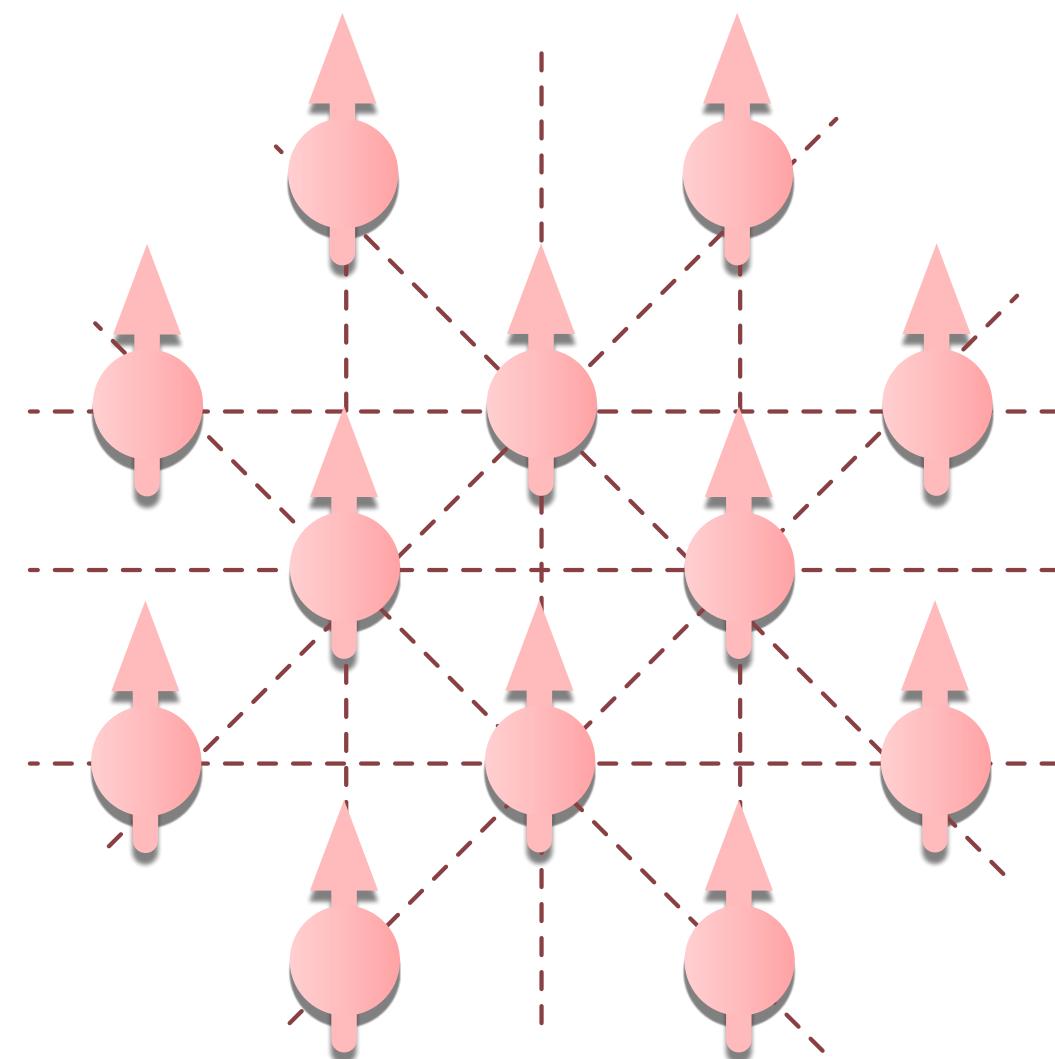
Long-range



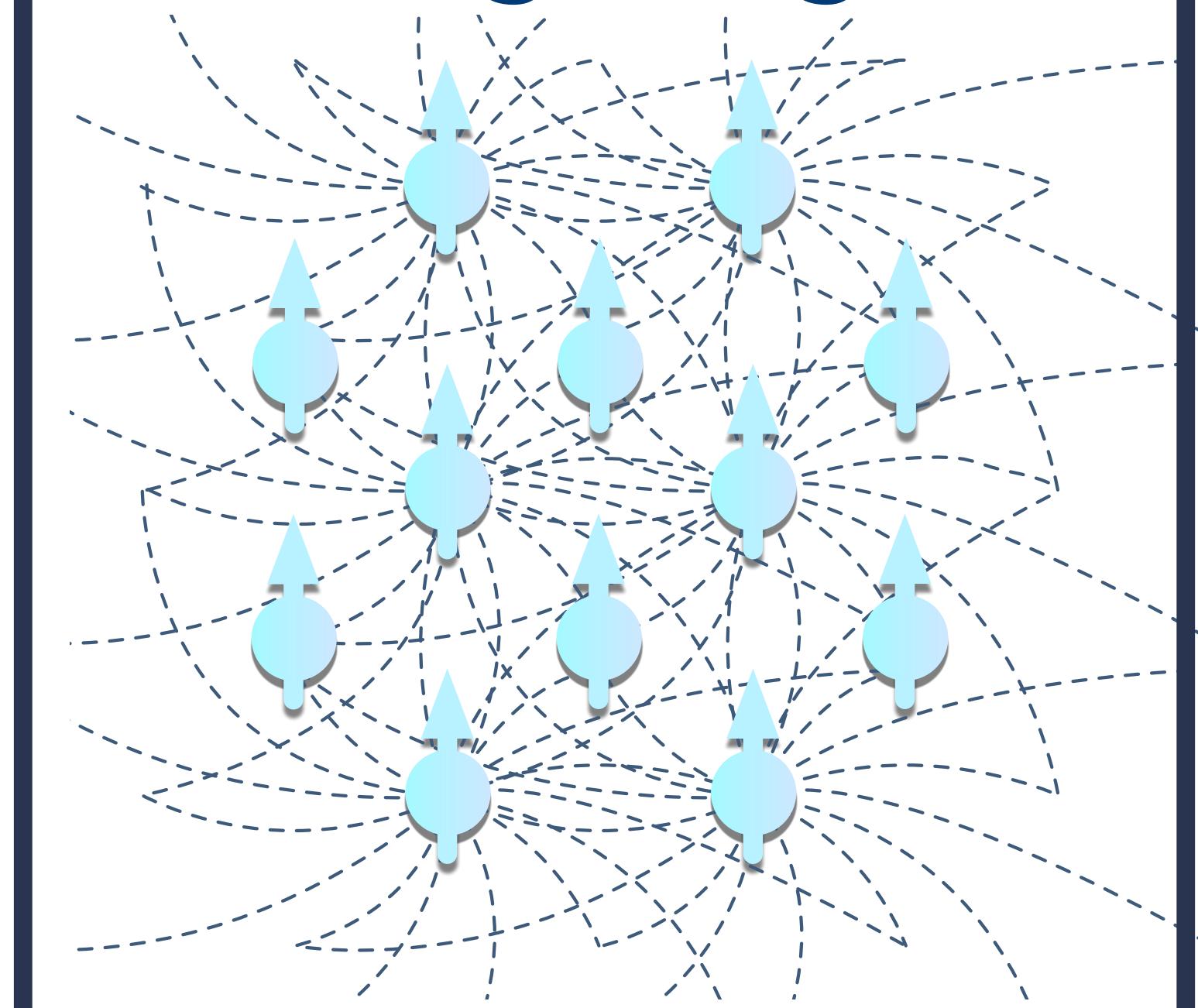
Is causality observable?

- In a short time, the two have **vastly different** causal structure.

Short-range

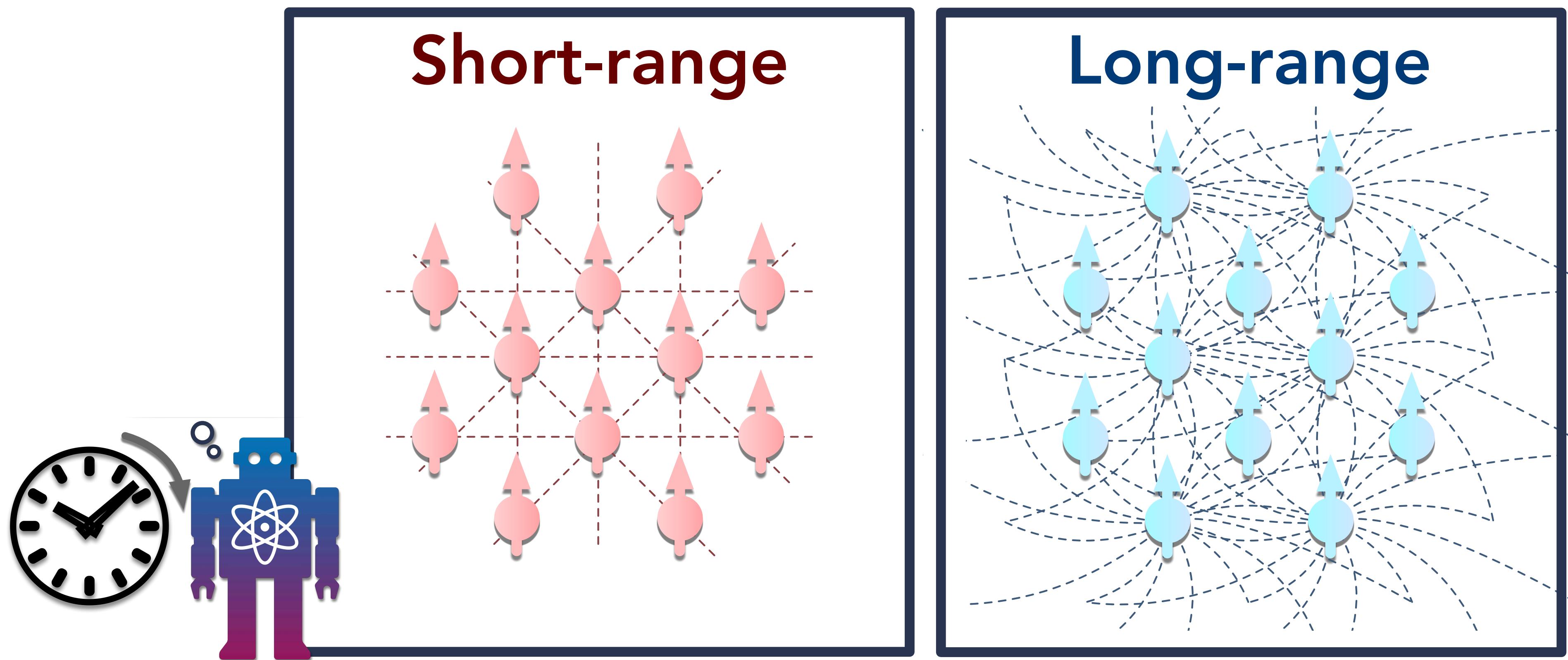


Long-range



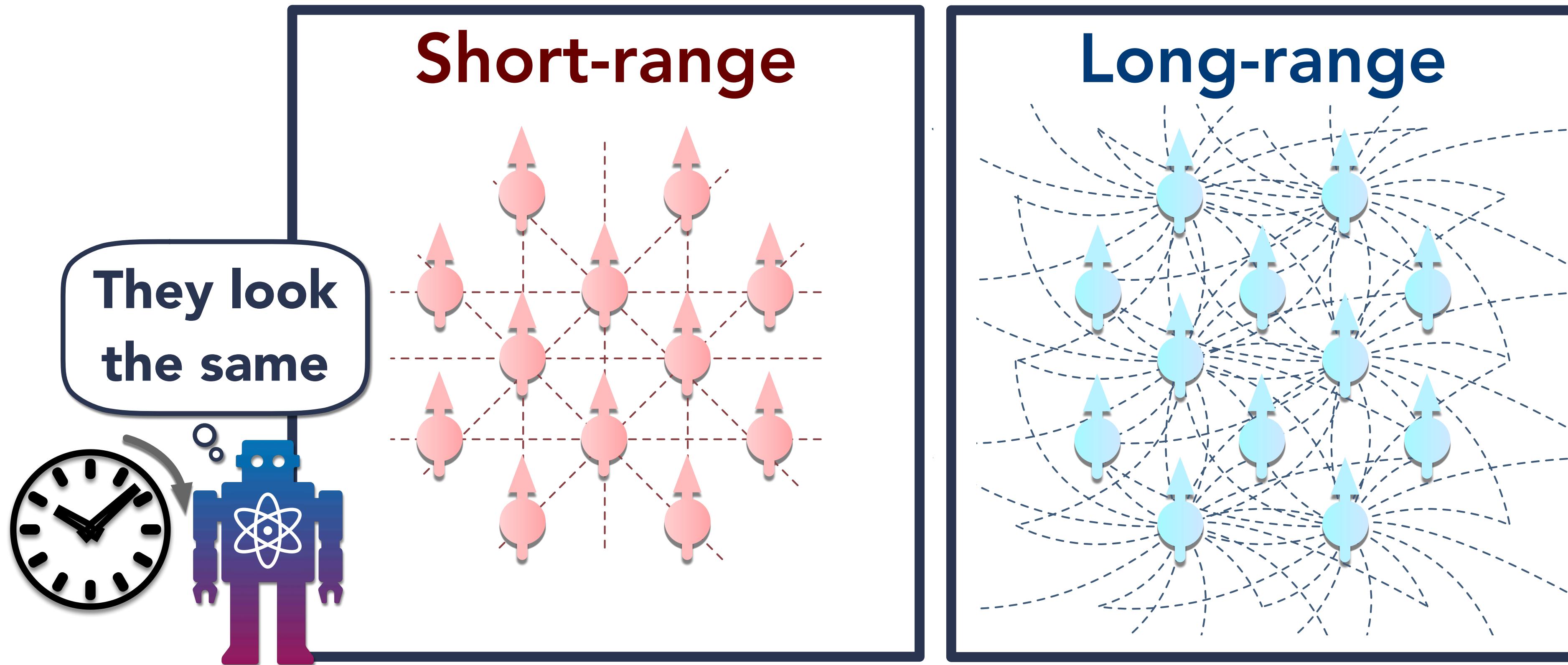
Is causality observable?

- In a short time, the two have **vastly different** causal structure.
- Can quantum agents tell the distinct causal structure apart?



Is causality observable?

- In a short time, the two have **vastly different** causal structure.
- Can quantum agents tell the distinct causal structure apart? No.



Is causality observable?

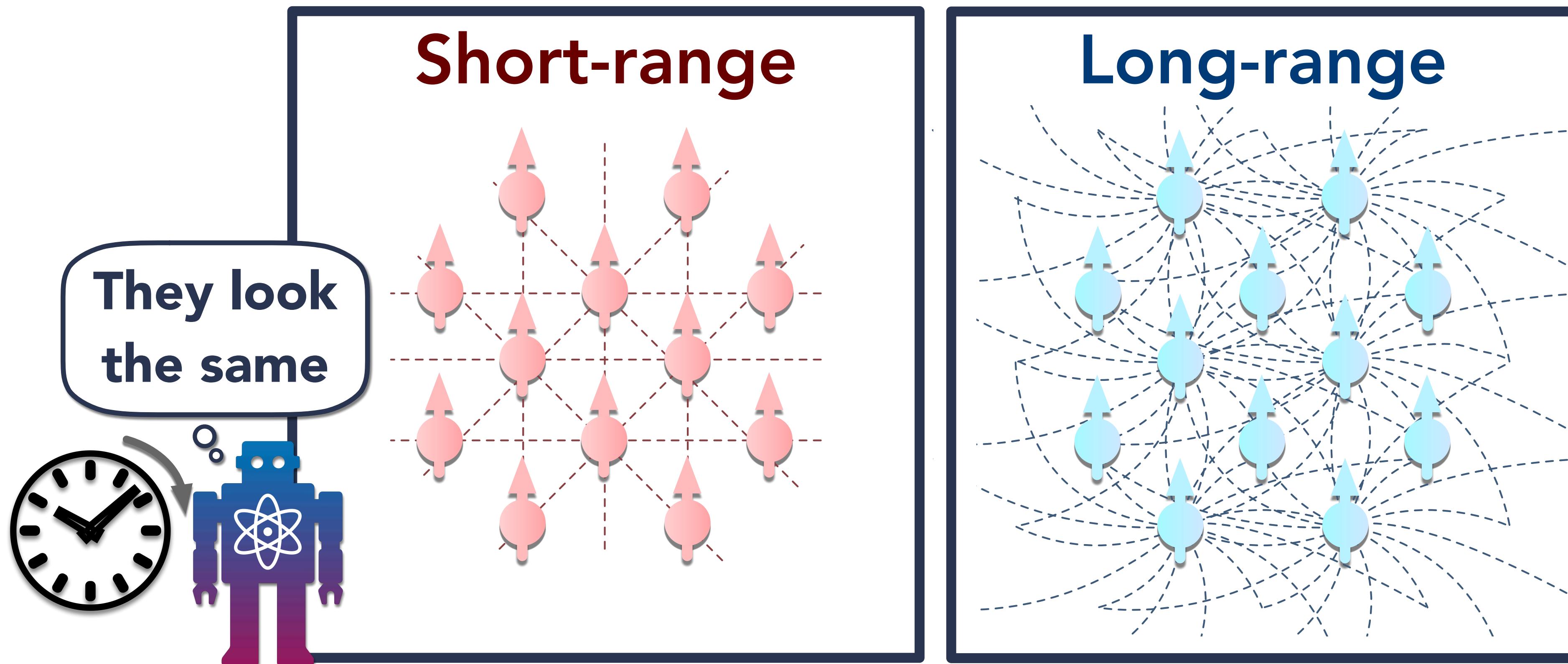
- In a short time, the two have **vastly different** causal structure.
- Can quantum agents tell the distinct causal structure apart? No.

Theorem 2 (under standard crypto conjecture)

No polynomial-time quantum agents can tell apart
short- vs **long-range** short-time dynamics.

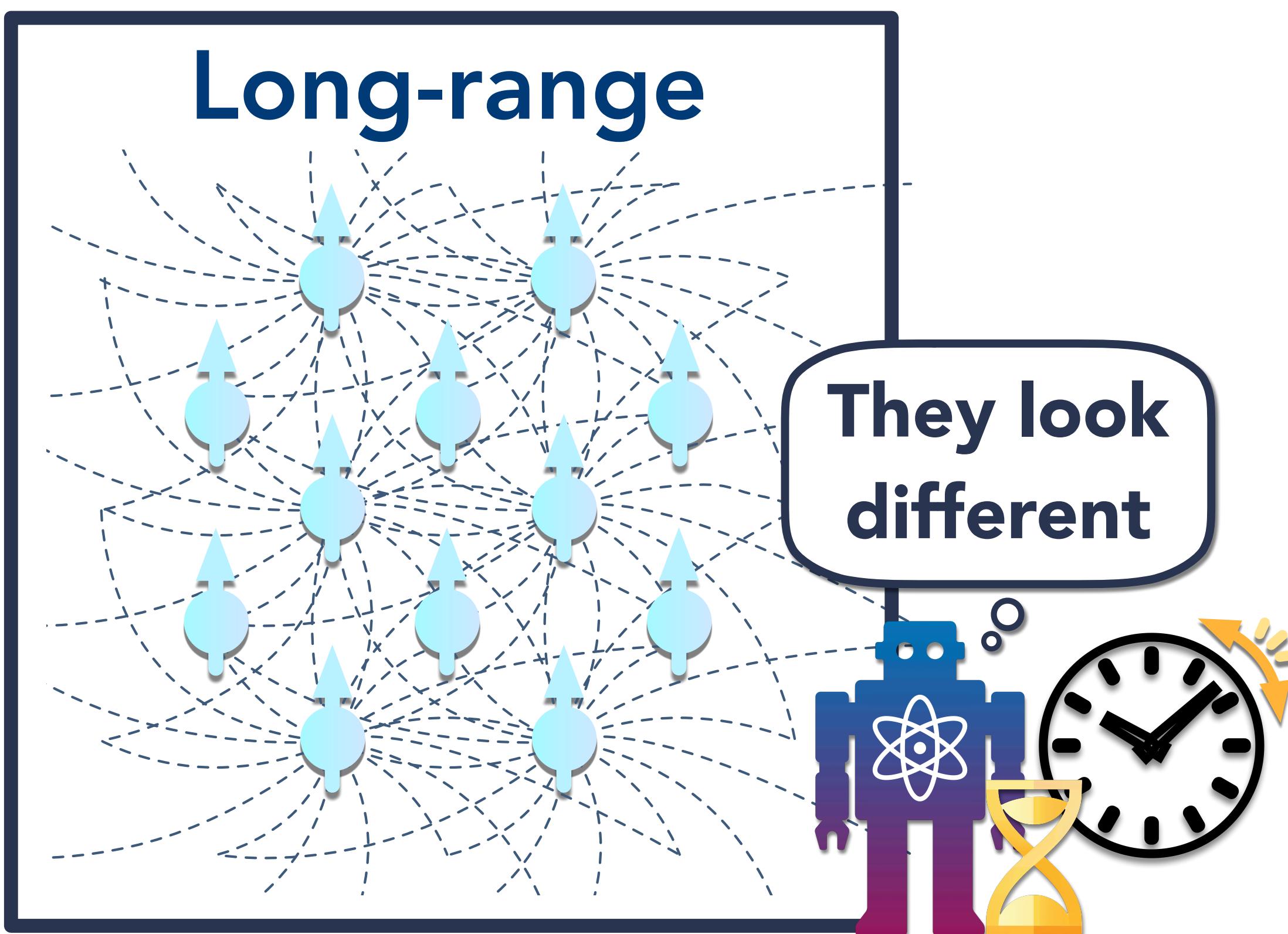
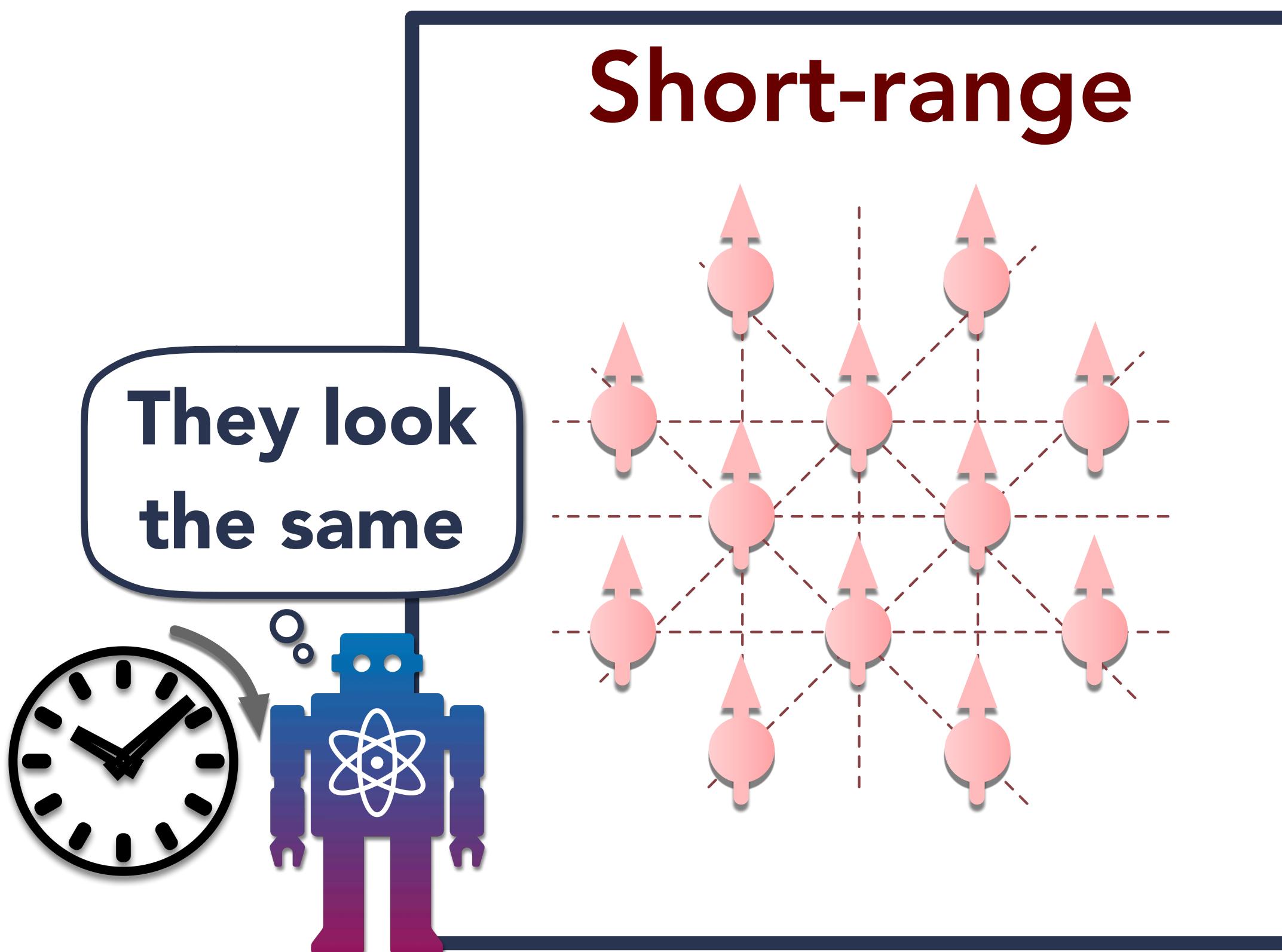
Is causality observable?

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Is causality observable?

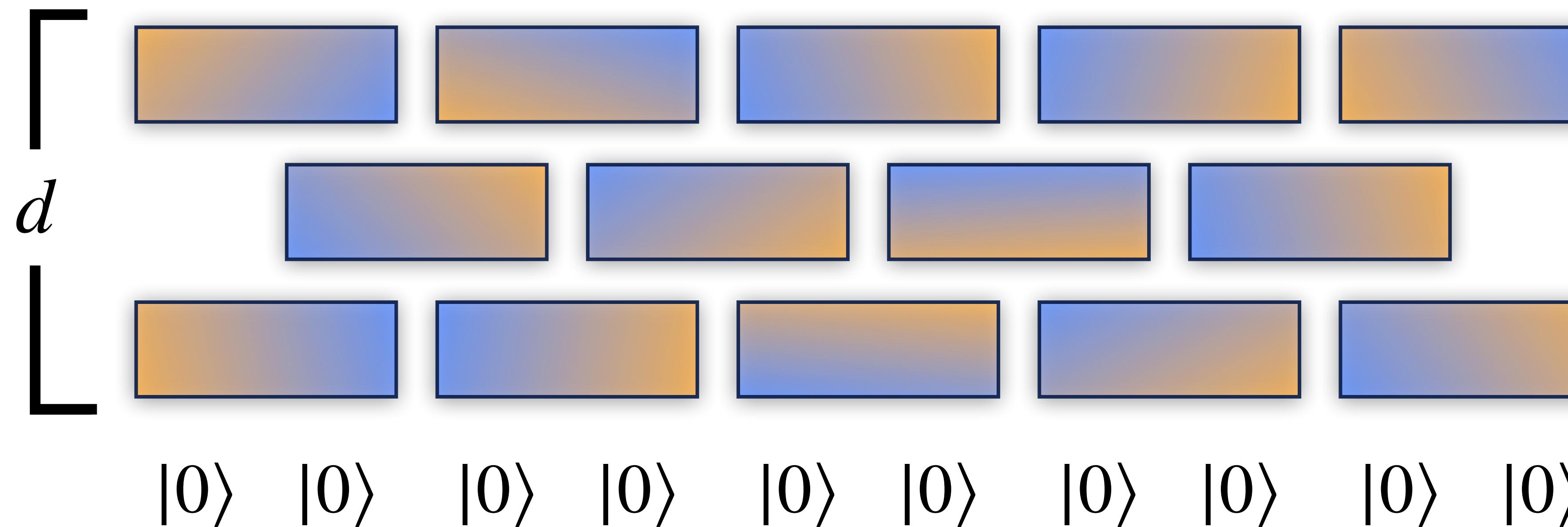
- If we consider a **more powerful observer** that can reverse time, then such an observer can distinguish the two easily.



Classifying topological order

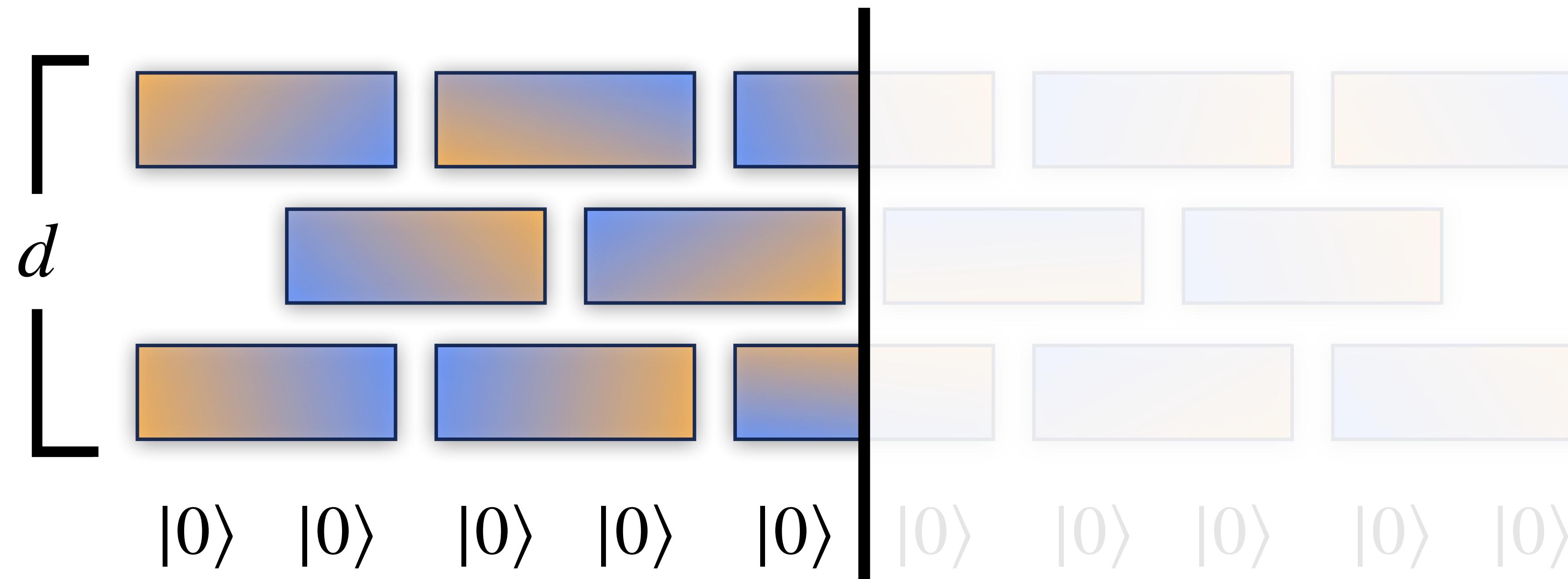
Classifying topological order

- **Entanglement:** Geometrically-local shallow quantum circuits can only generate **short-range entanglement**.



Classifying topological order

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Entanglement entropy = $\mathcal{O}(d)$

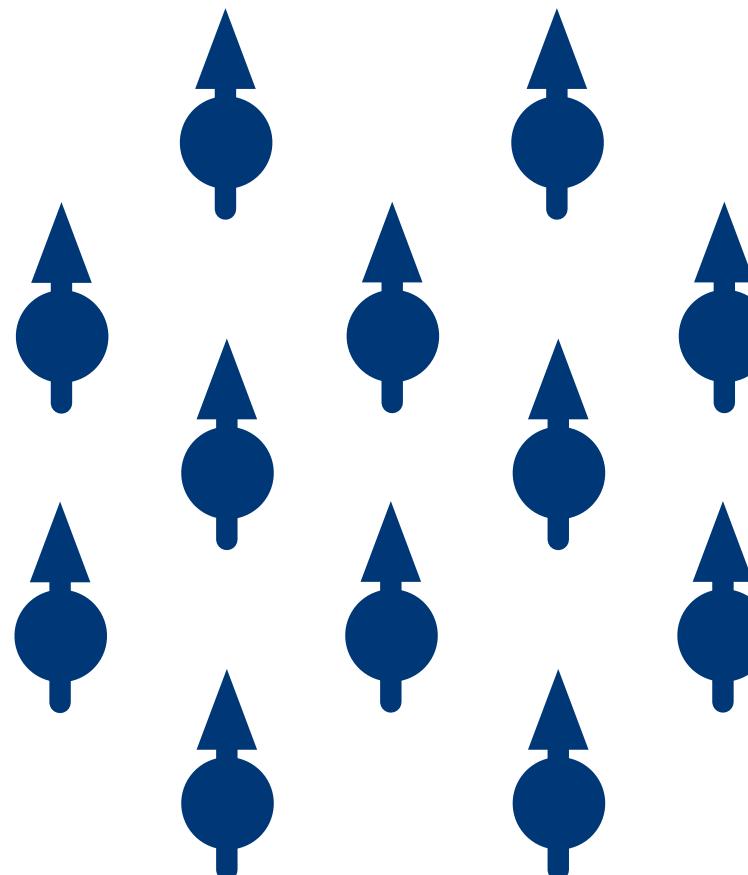
Classifying topological order

- **Entanglement:** Geometrically-local shallow quantum circuits
cannot alter long-range entanglement.

Classifying topological order

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Product state

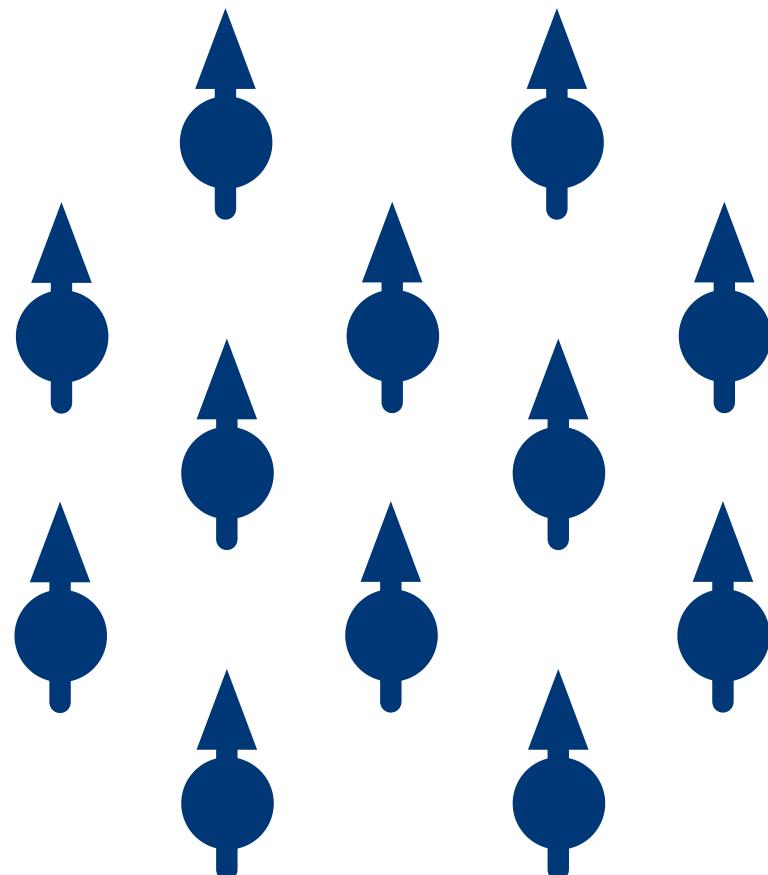


Classifying topological order

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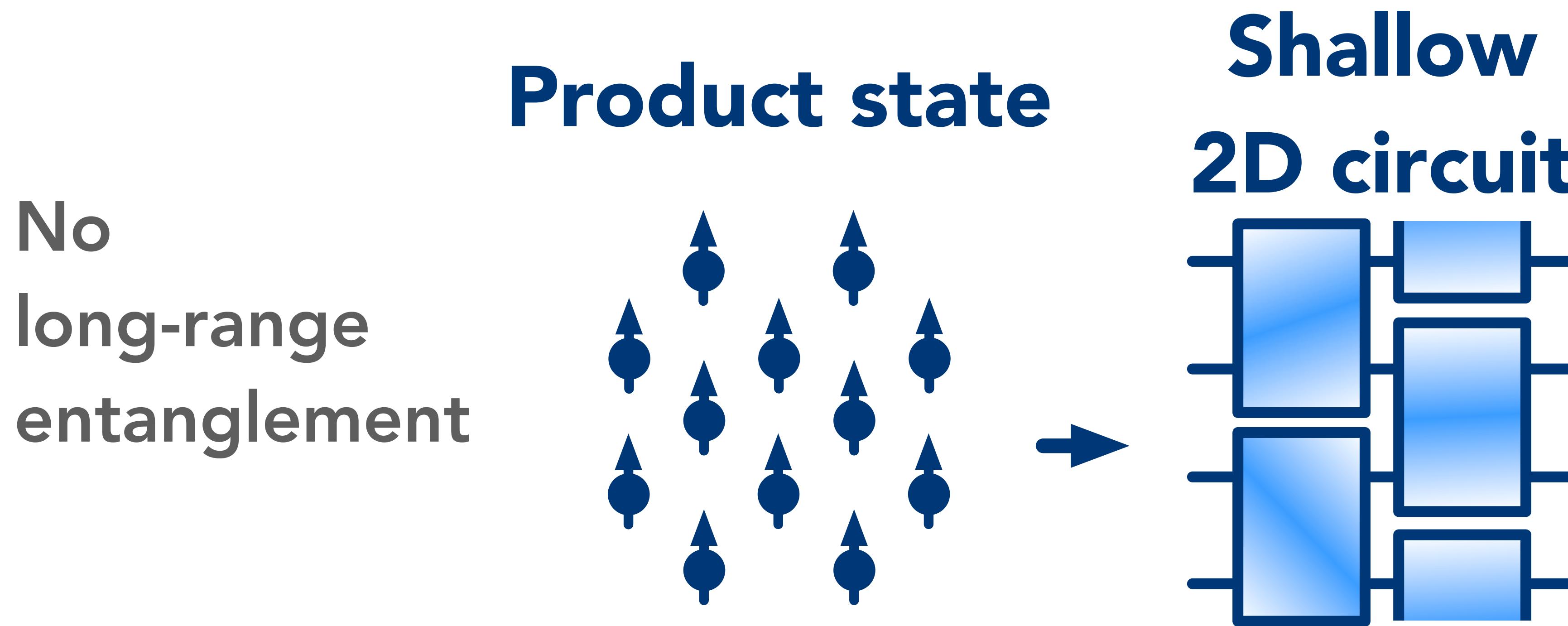
Product state

No
long-range
entanglement



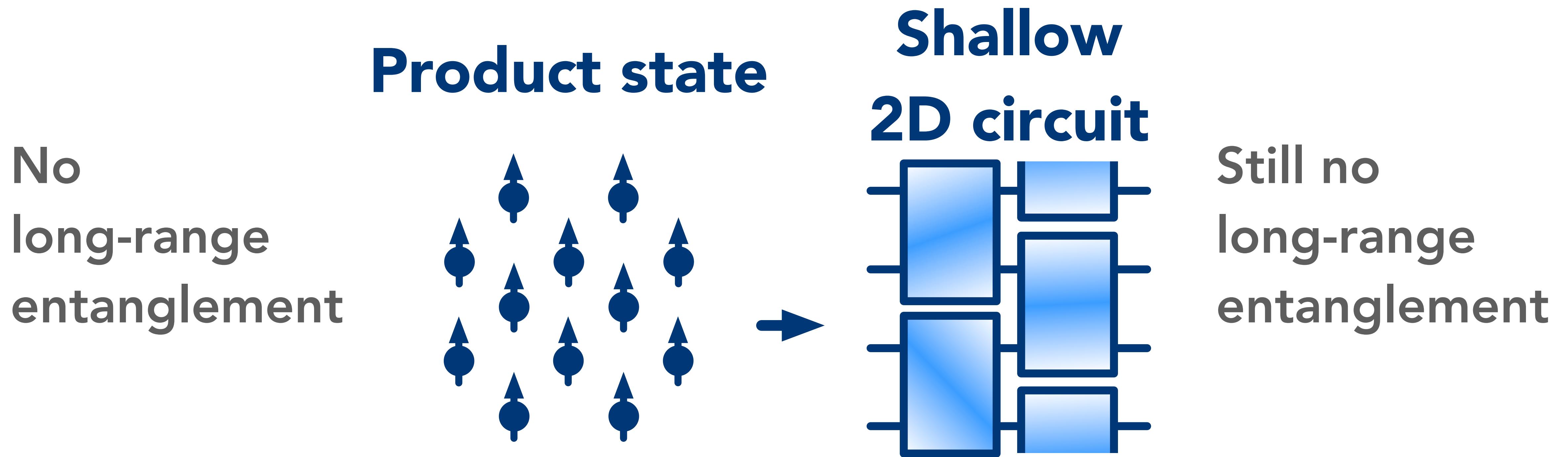
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Classifying topological order

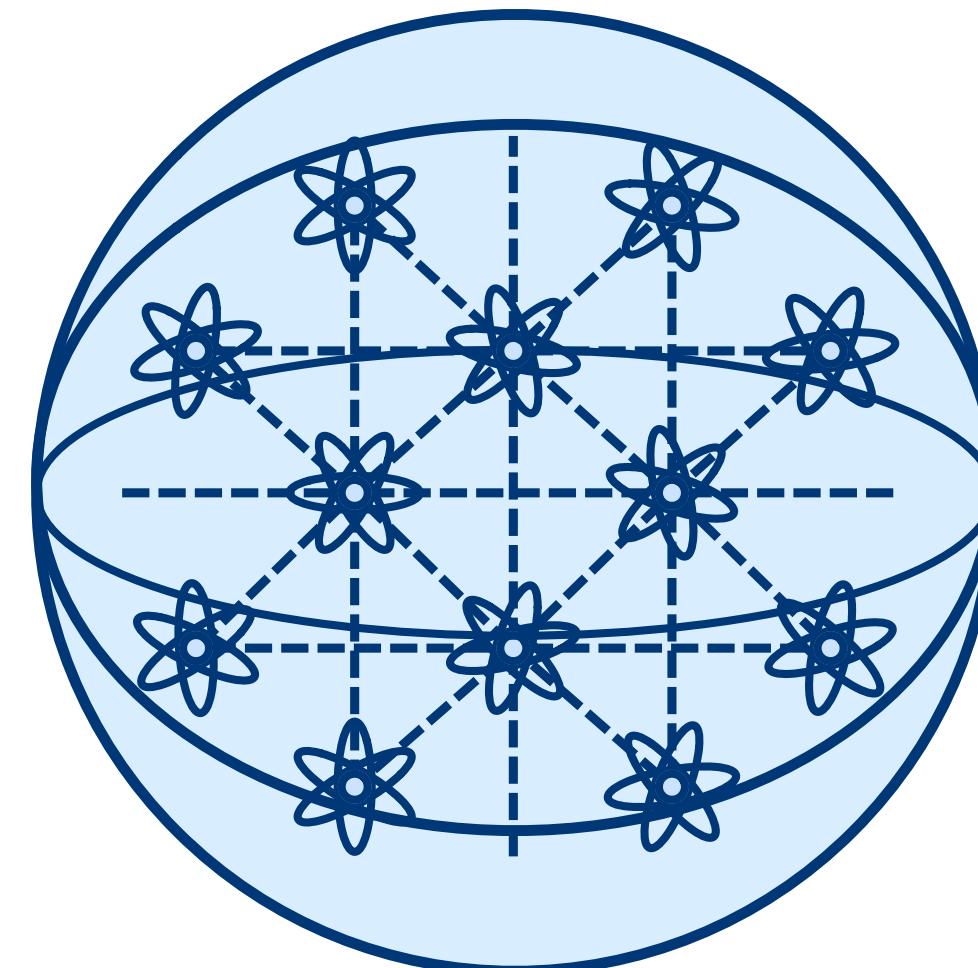
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Classifying topological order

- **Entanglement:** Geometrically-local shallow quantum circuits
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**Toric code
ground state**

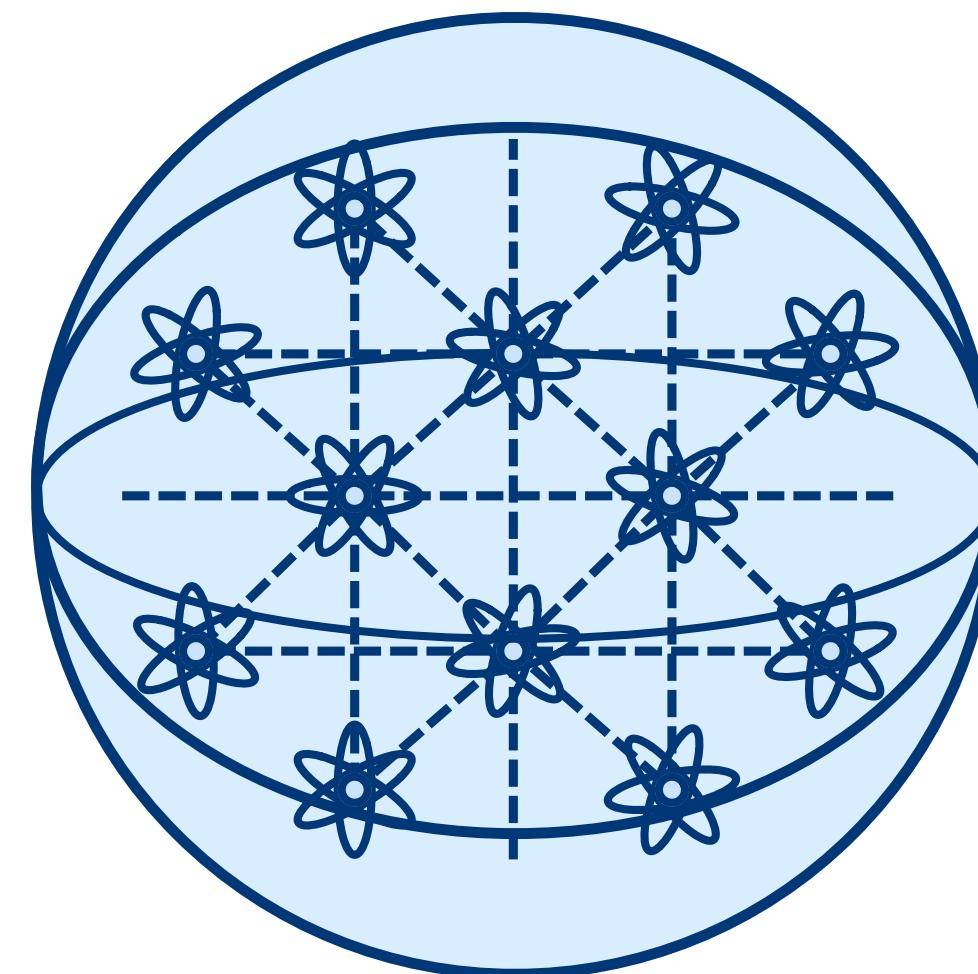


Classifying topological order

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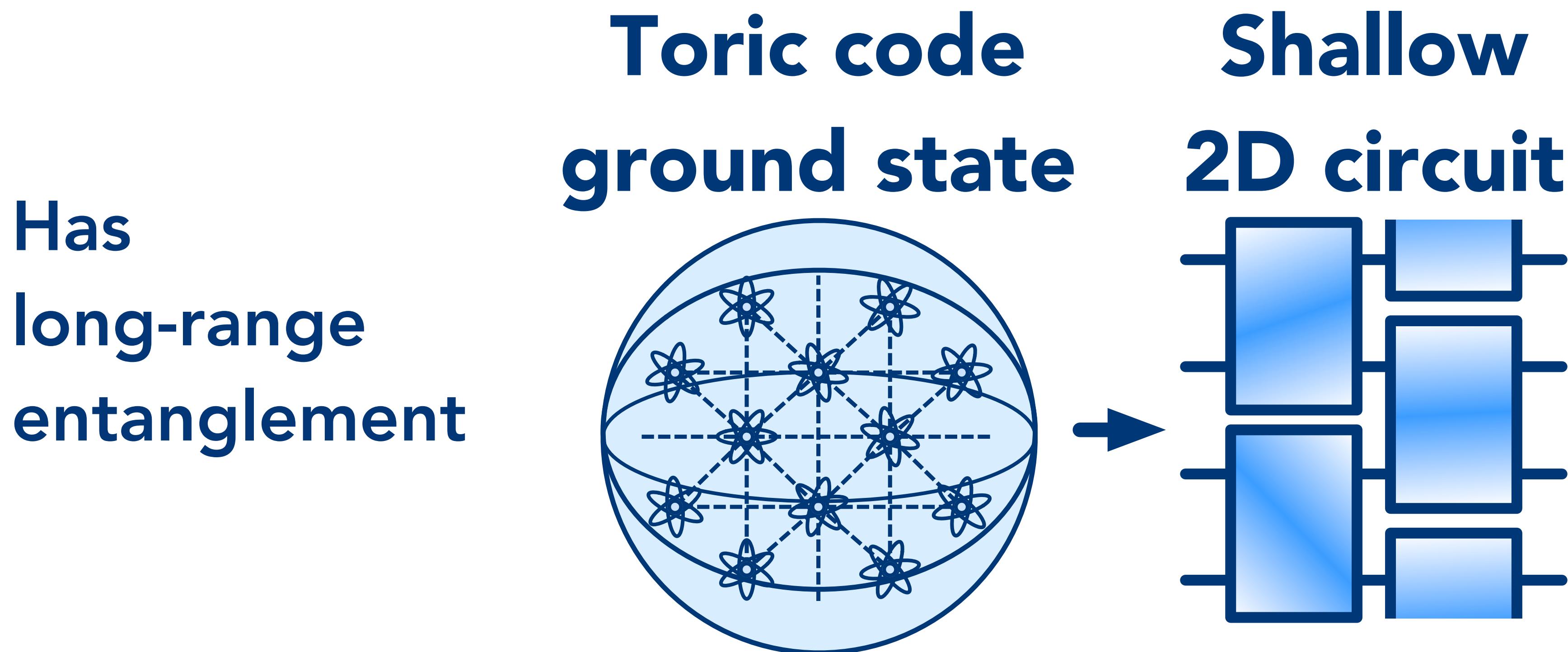
Has
long-range
entanglement

Toric code
ground state



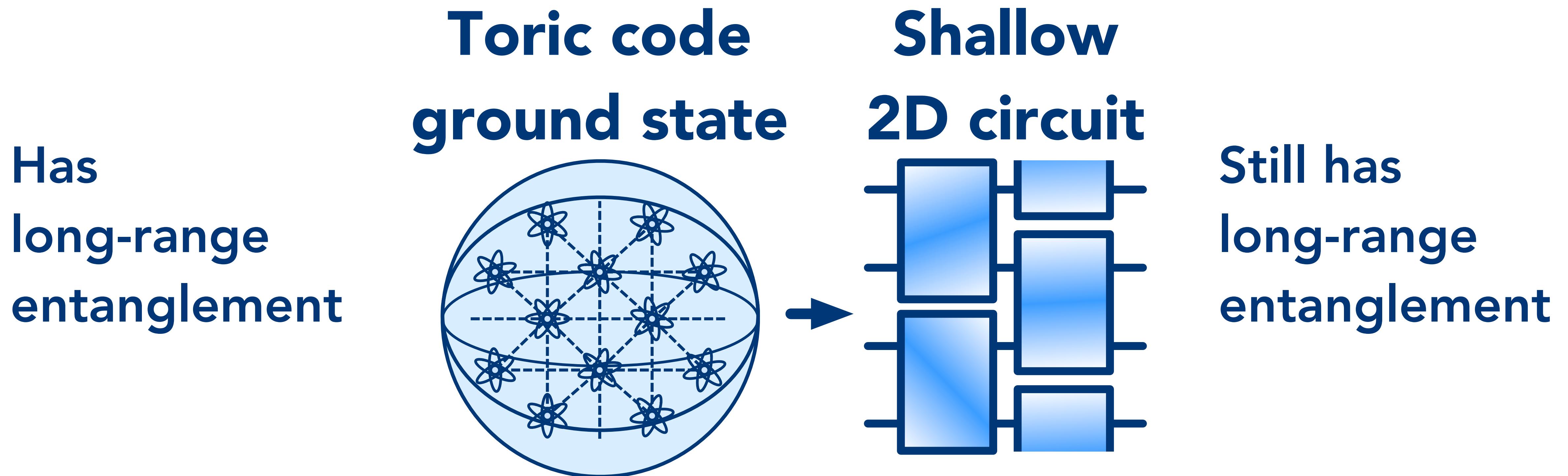
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Classifying topological order

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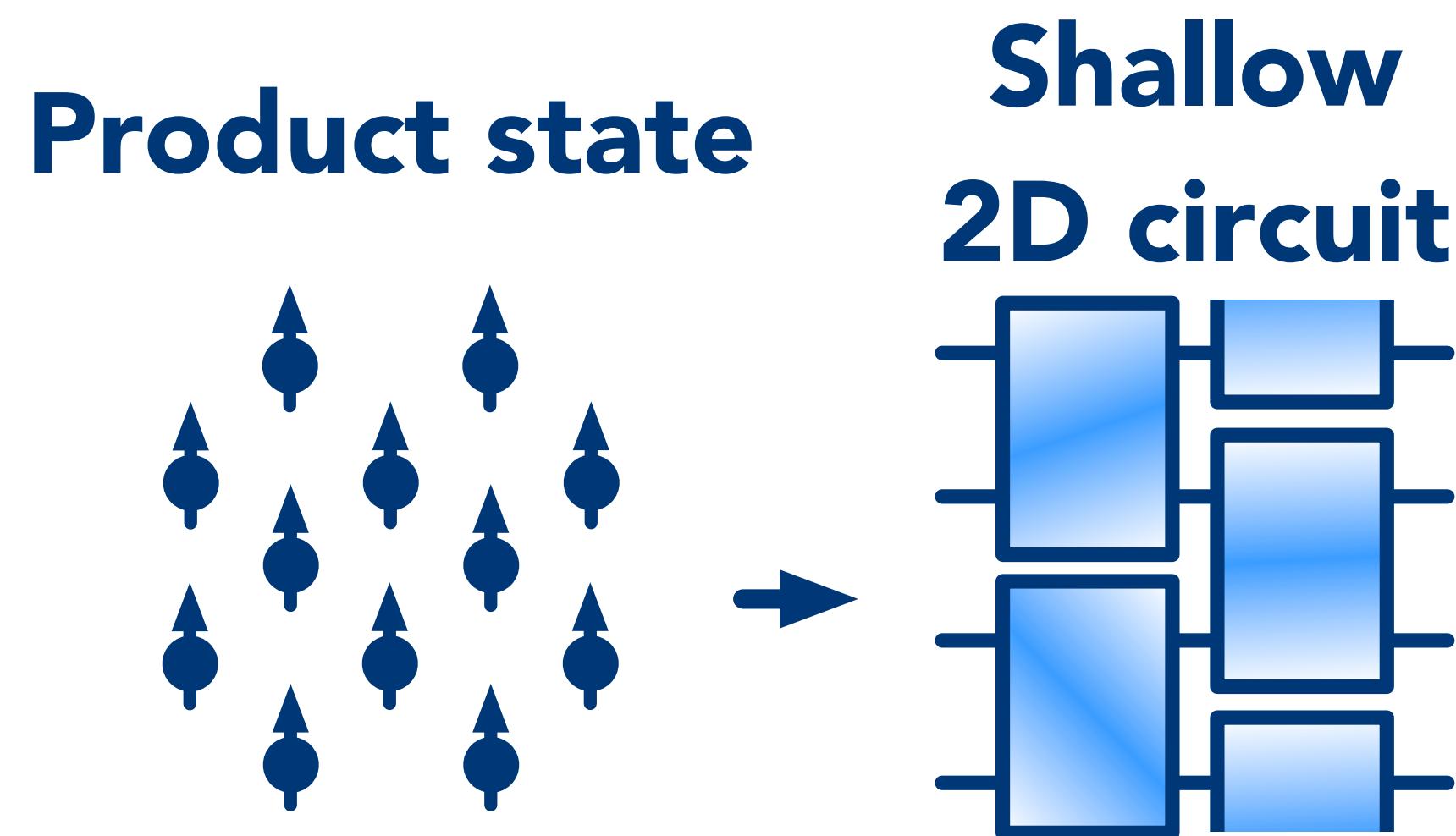


Classifying topological order

- **Topological order:** Two states have the same topological order if they have the same **long-range entanglement** structure.

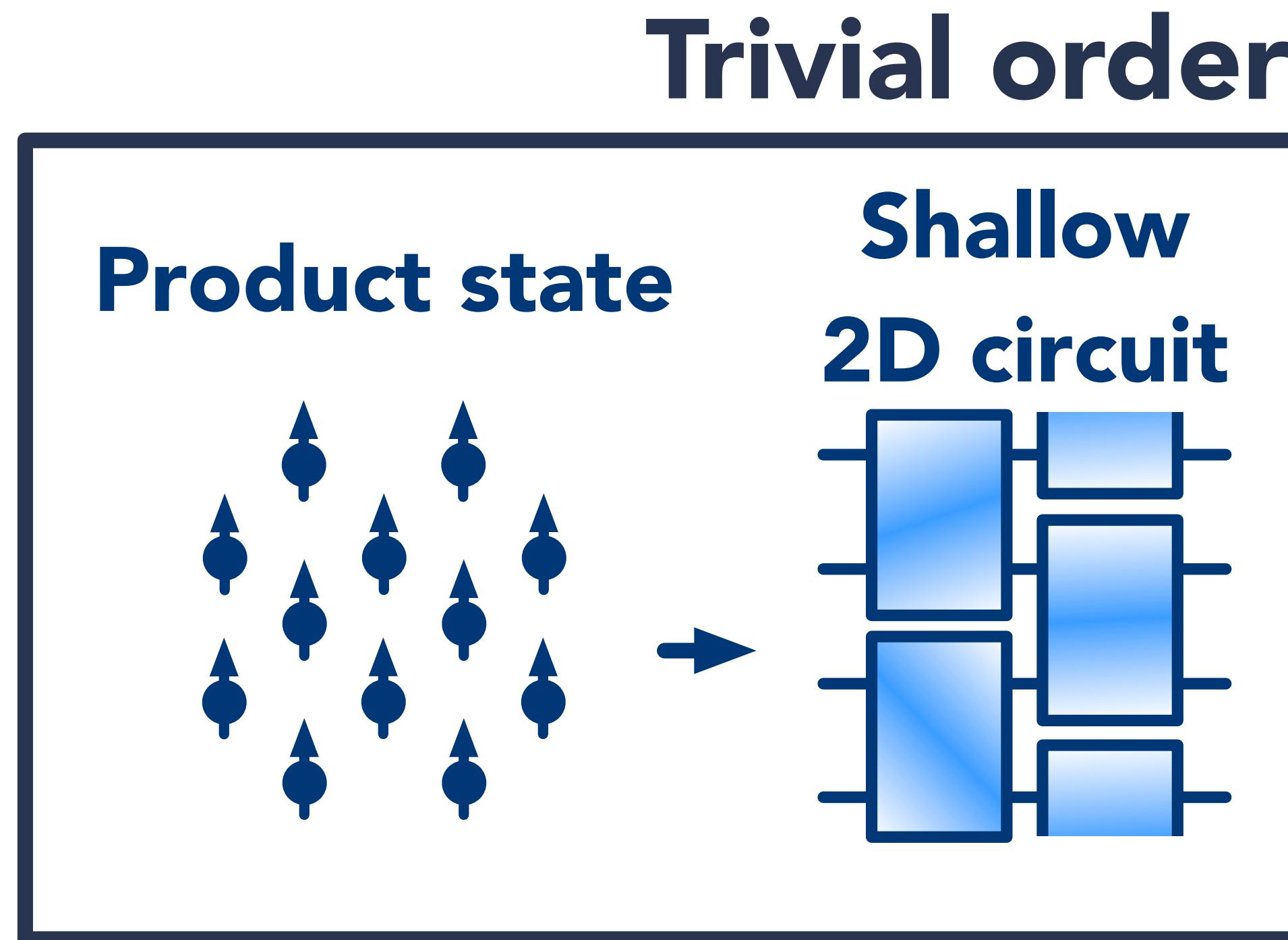
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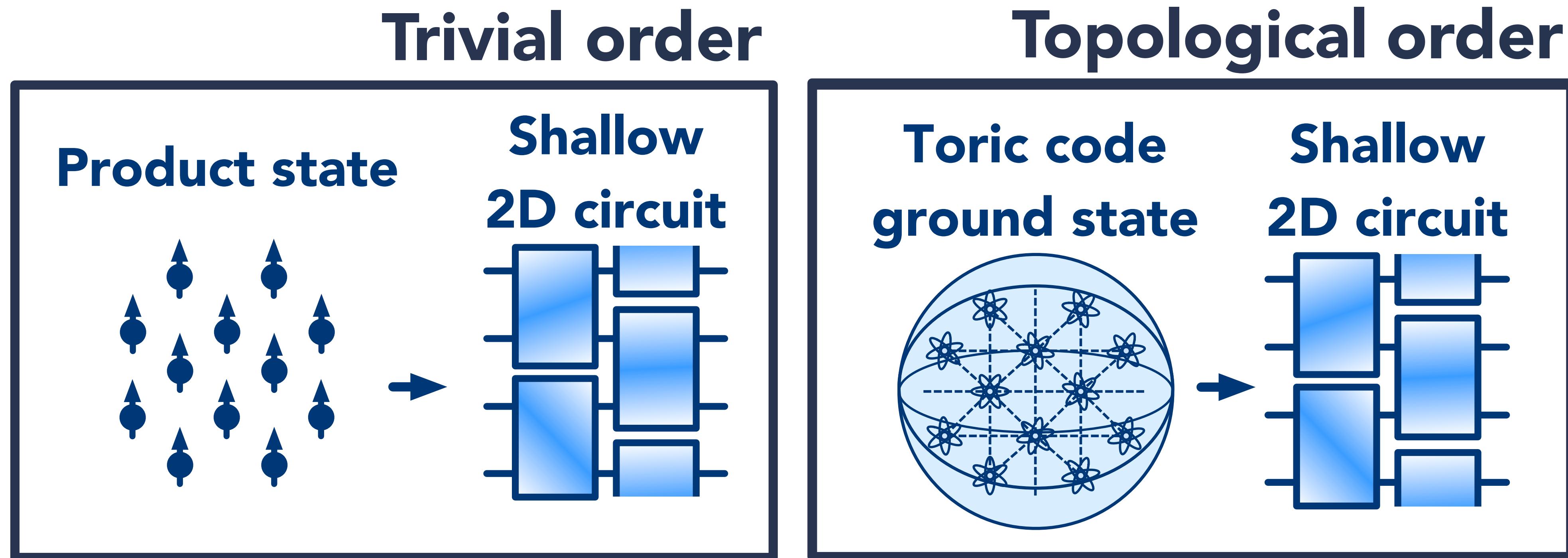
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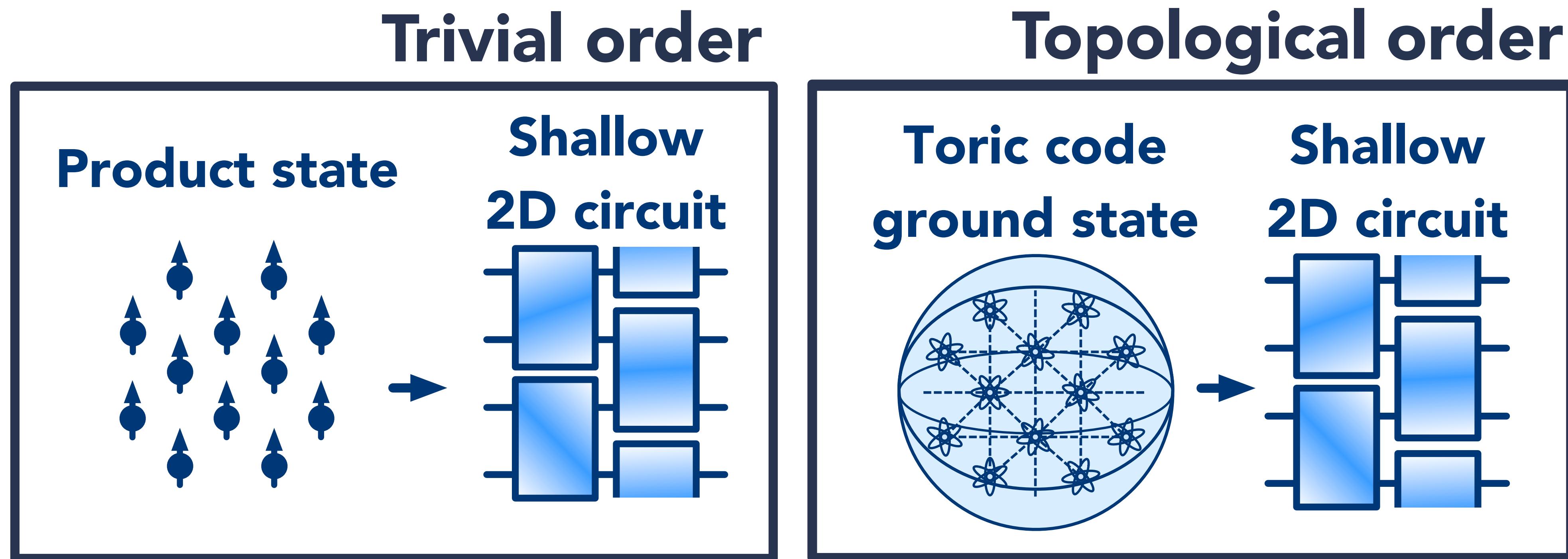
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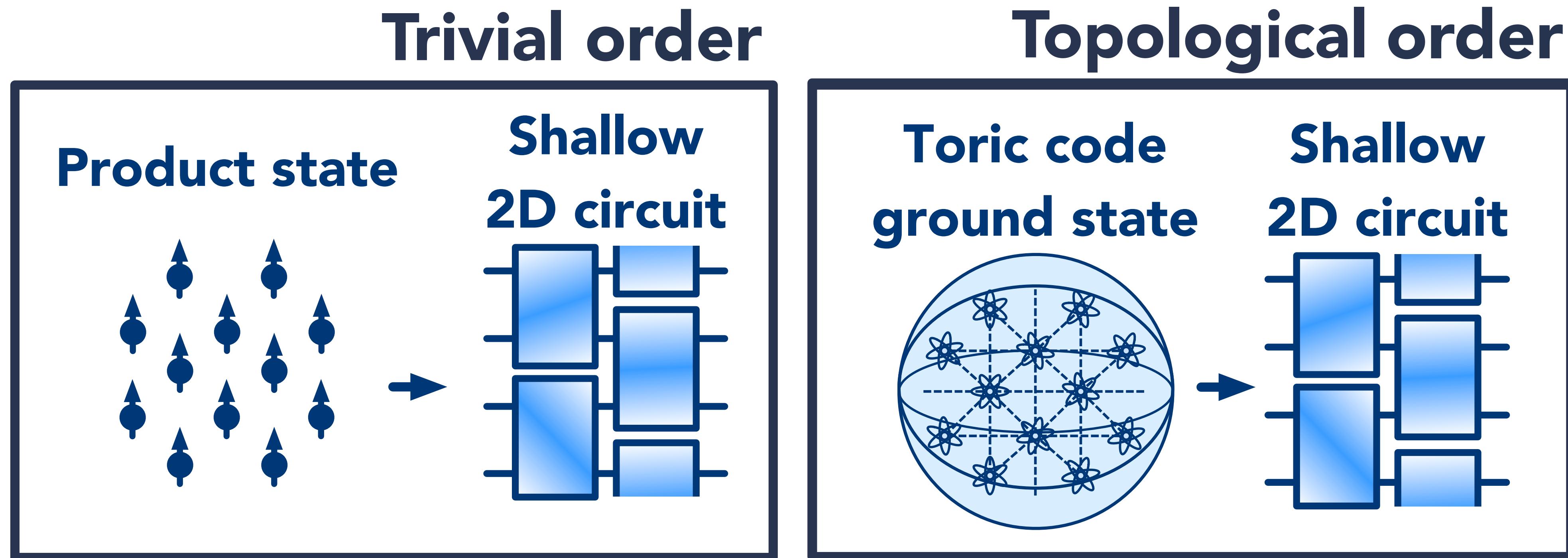
Classifying topological order

- An open question in ML \cap quantum: Given copies of a state $|\psi\rangle$, can we efficiently **classify** if $|\psi\rangle$ is topological or not?



Classifying topological order

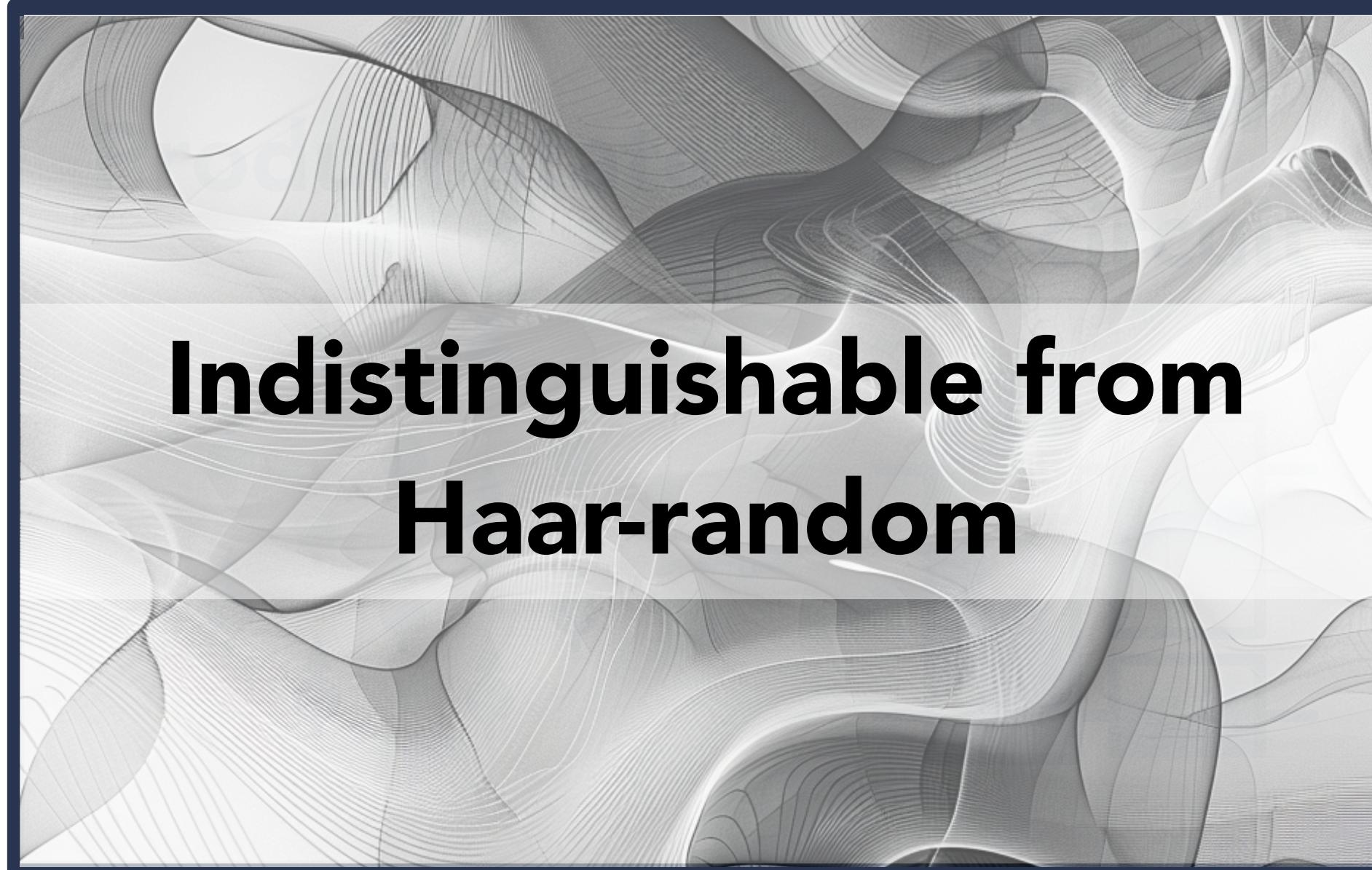
- No, shallow 2D circuits can hide topological order.



Classifying topological order

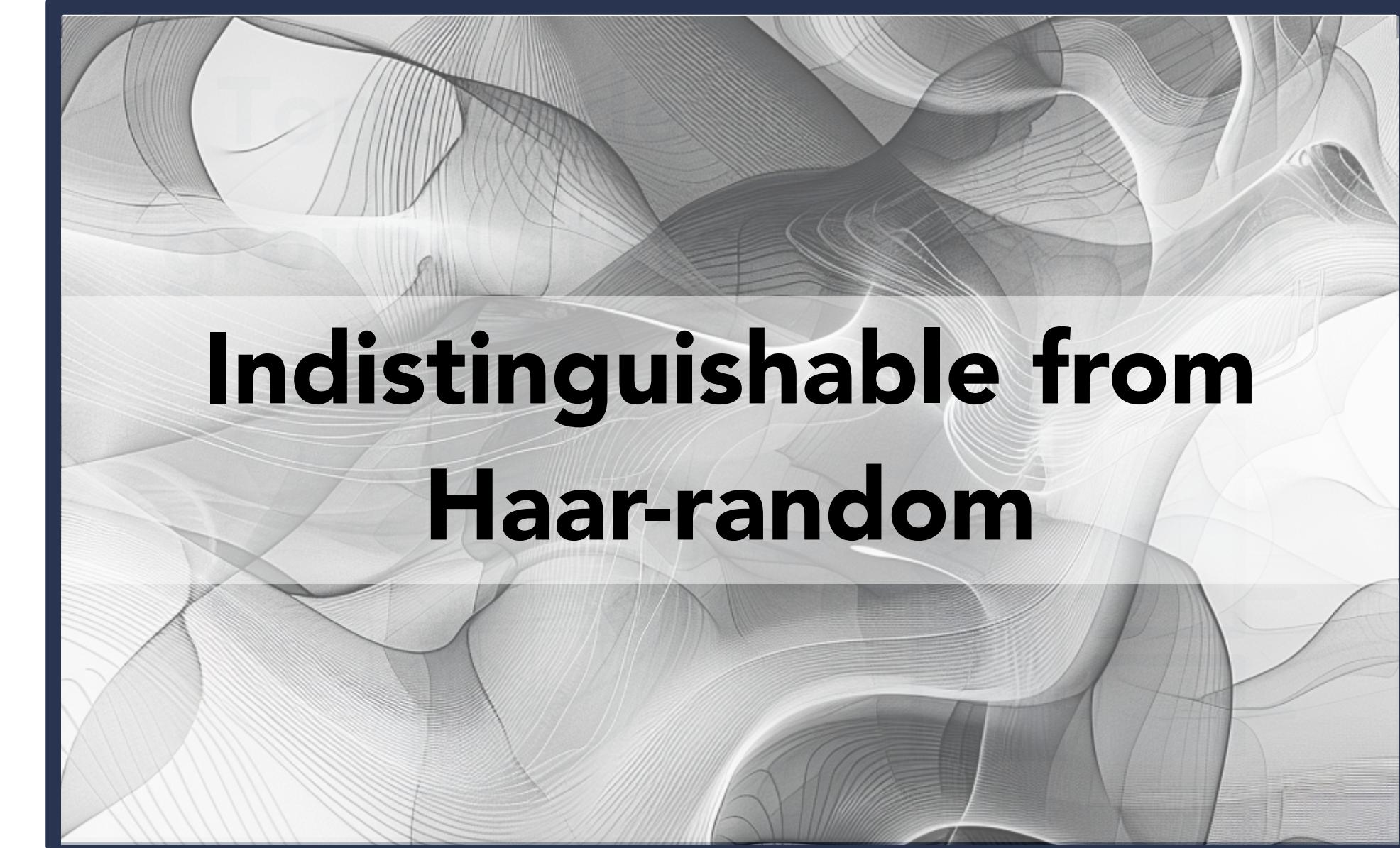
- **No**, shallow 2D circuits can **hide** topological order.
- Random topological-ordered / trivial states both look **Haar-random**.

Trivial order



Indistinguishable from
Haar-random

Topological order



Indistinguishable from
Haar-random

Theorem 3 (under standard crypto conjecture)

No poly-time quantum agents can learn
any properties **invariant** under shallow circuits.

Trivial order

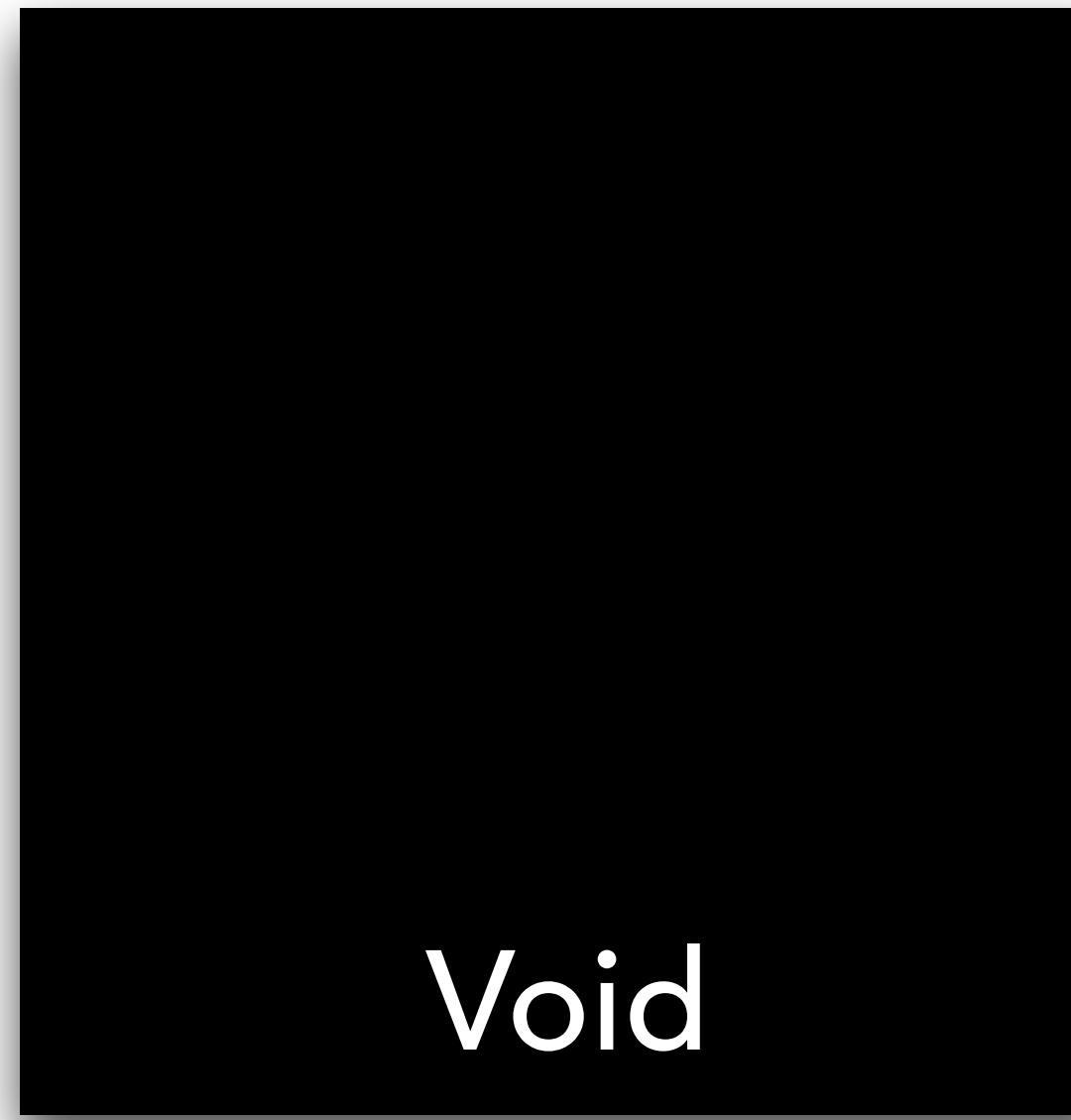
Indistinguishable from
Haar-random

Topological order

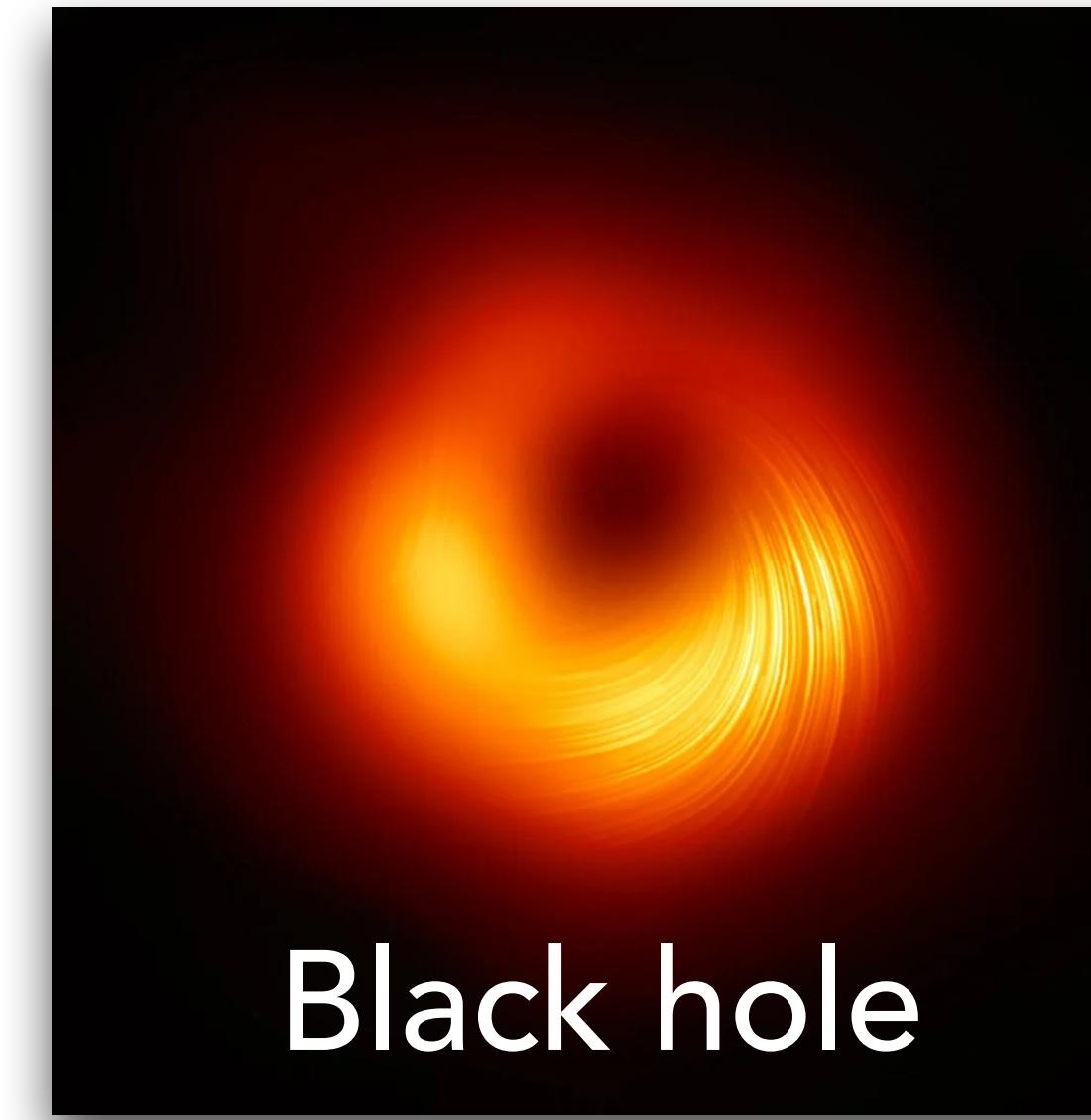
Indistinguishable from
Haar-random

Spacetime Geometry

- Consider two vastly different spacetime geometries.



Void



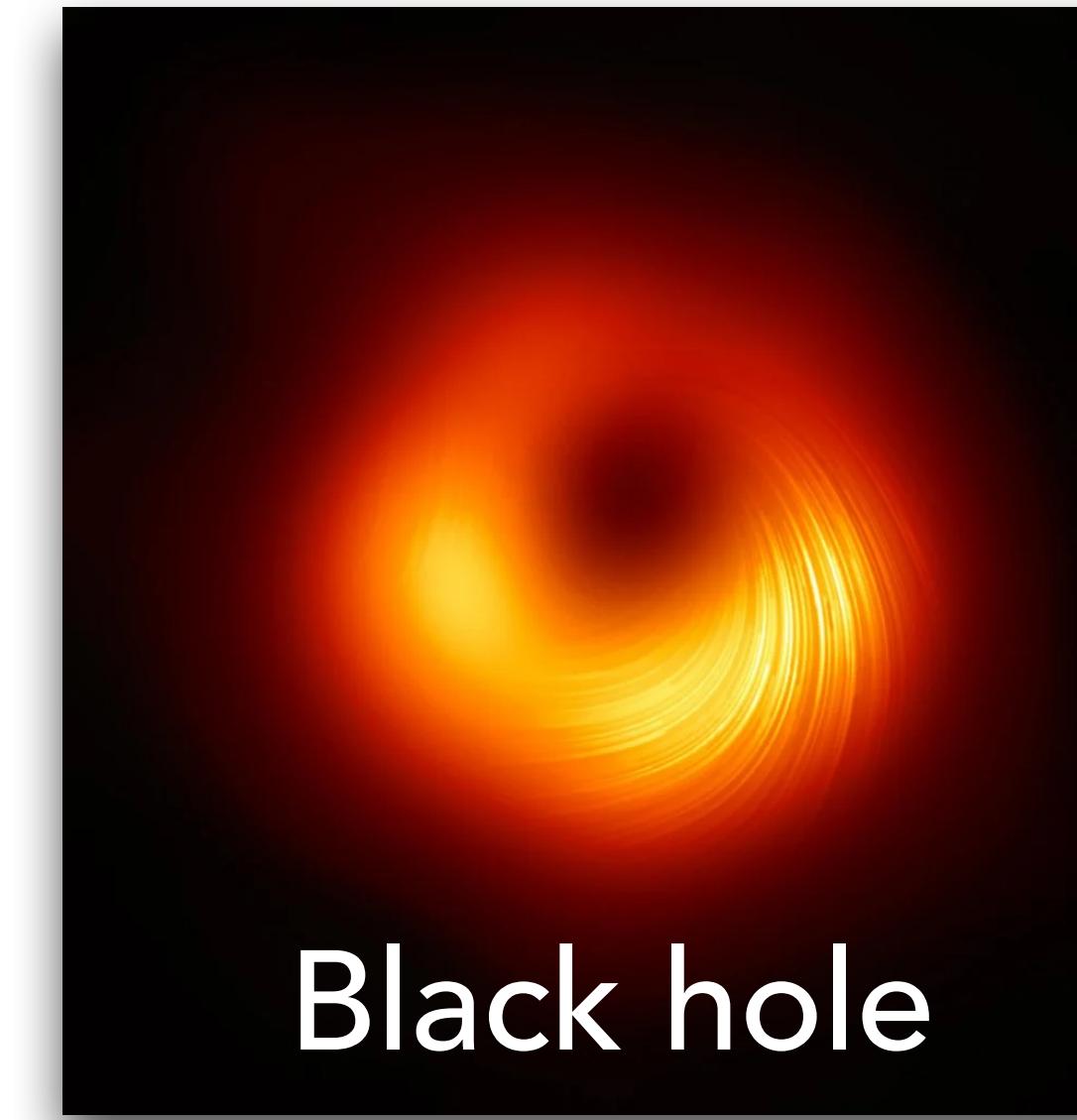
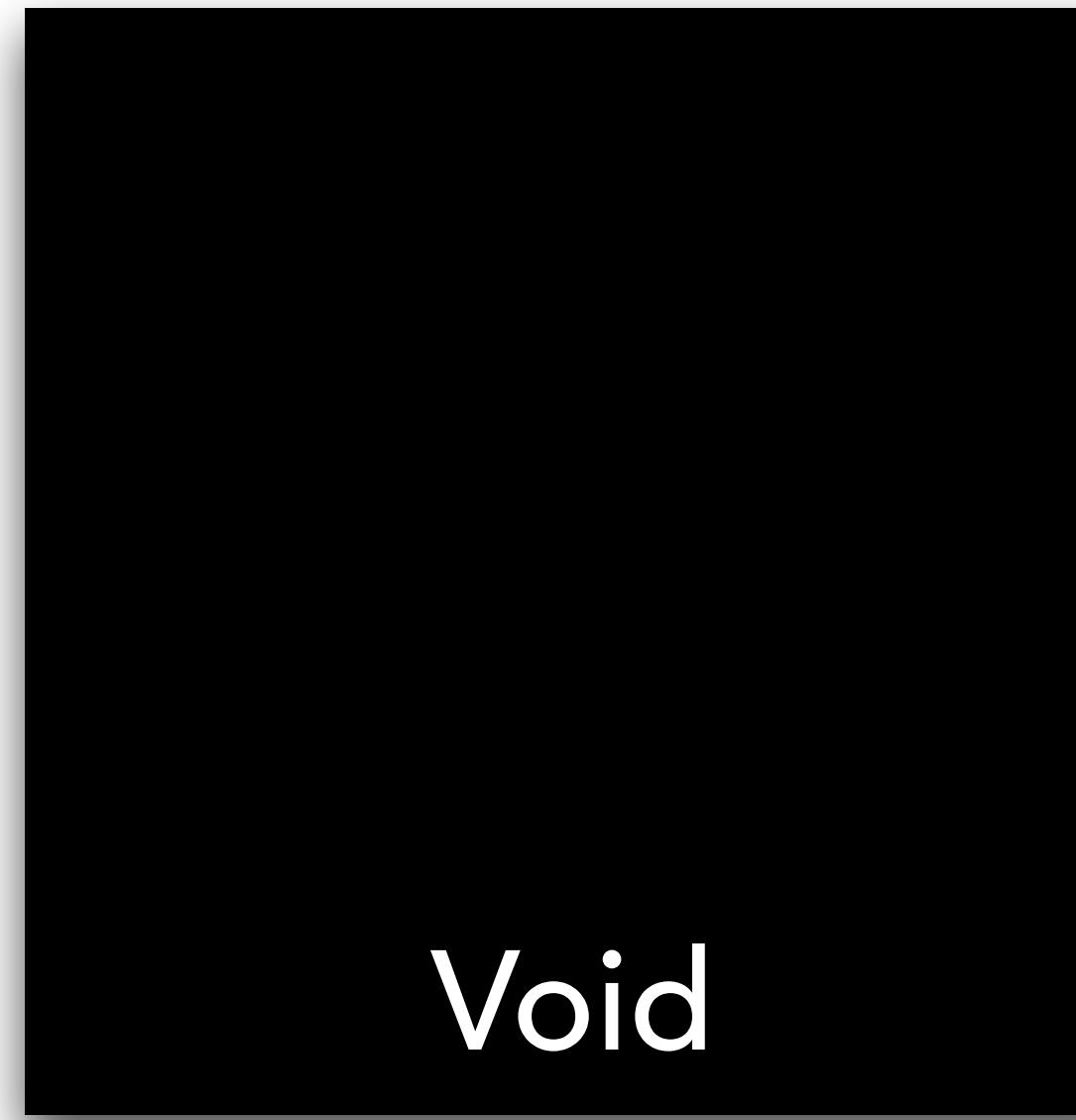
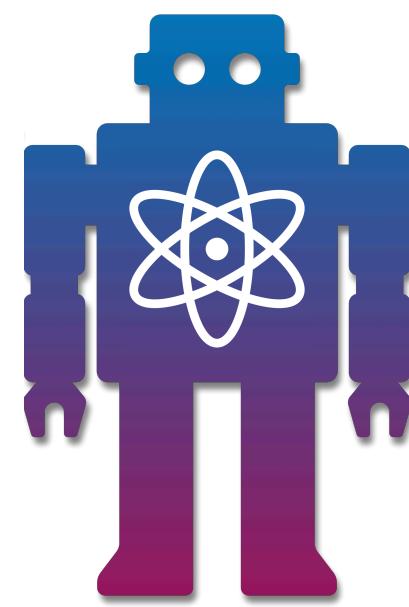
Black hole

Akers, Bouland, Chen, Kohler, Metger, Vazirani.

Holographic pseudoentanglement and the complexity of the AdS/CFT dictionary.

Spacetime Geometry

- Consider two vastly different spacetime geometries.
- Can an **outside observer** distinguish between the two?



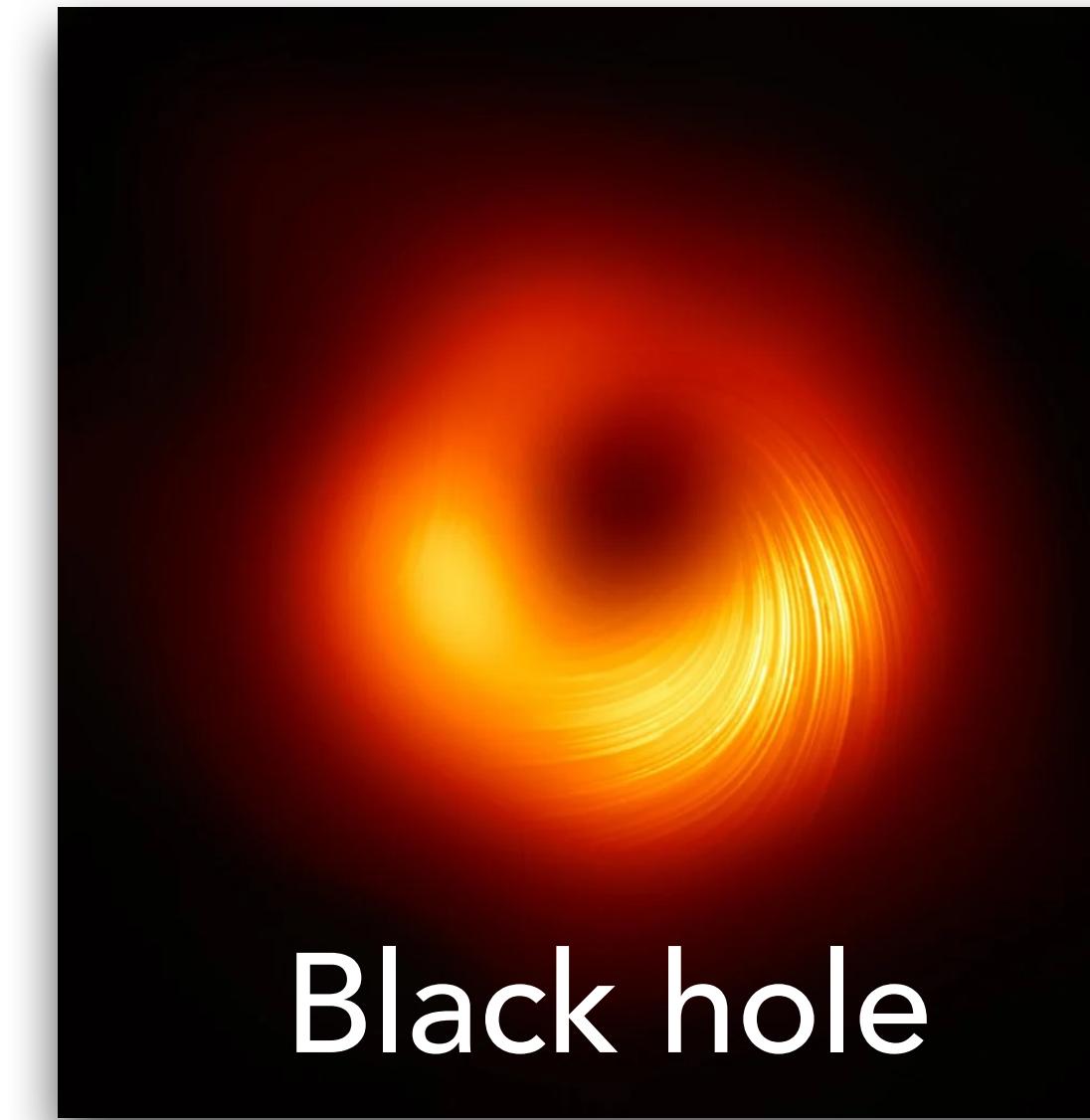
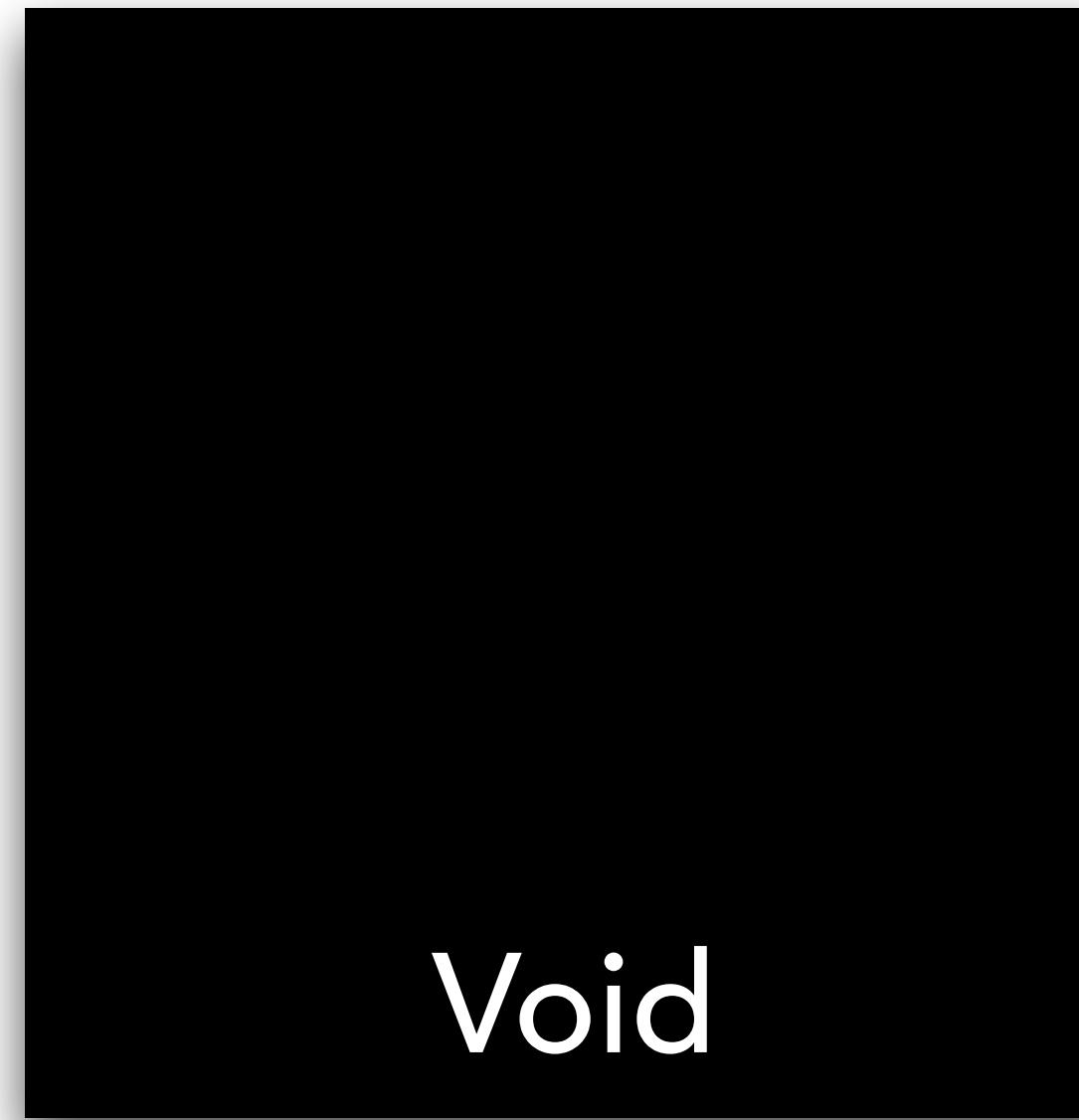
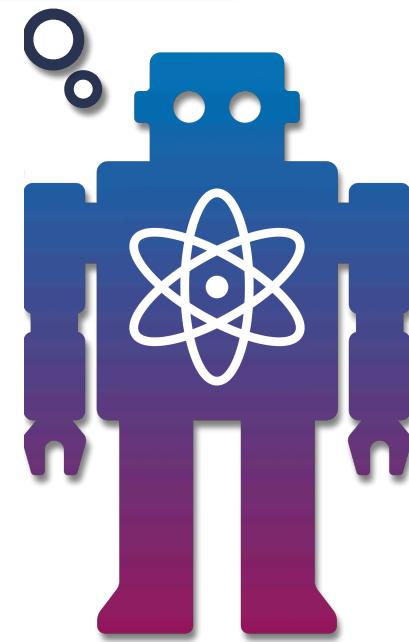
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Spacetime Geometry

- Consider two vastly different spacetime geometries.
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They look
the same



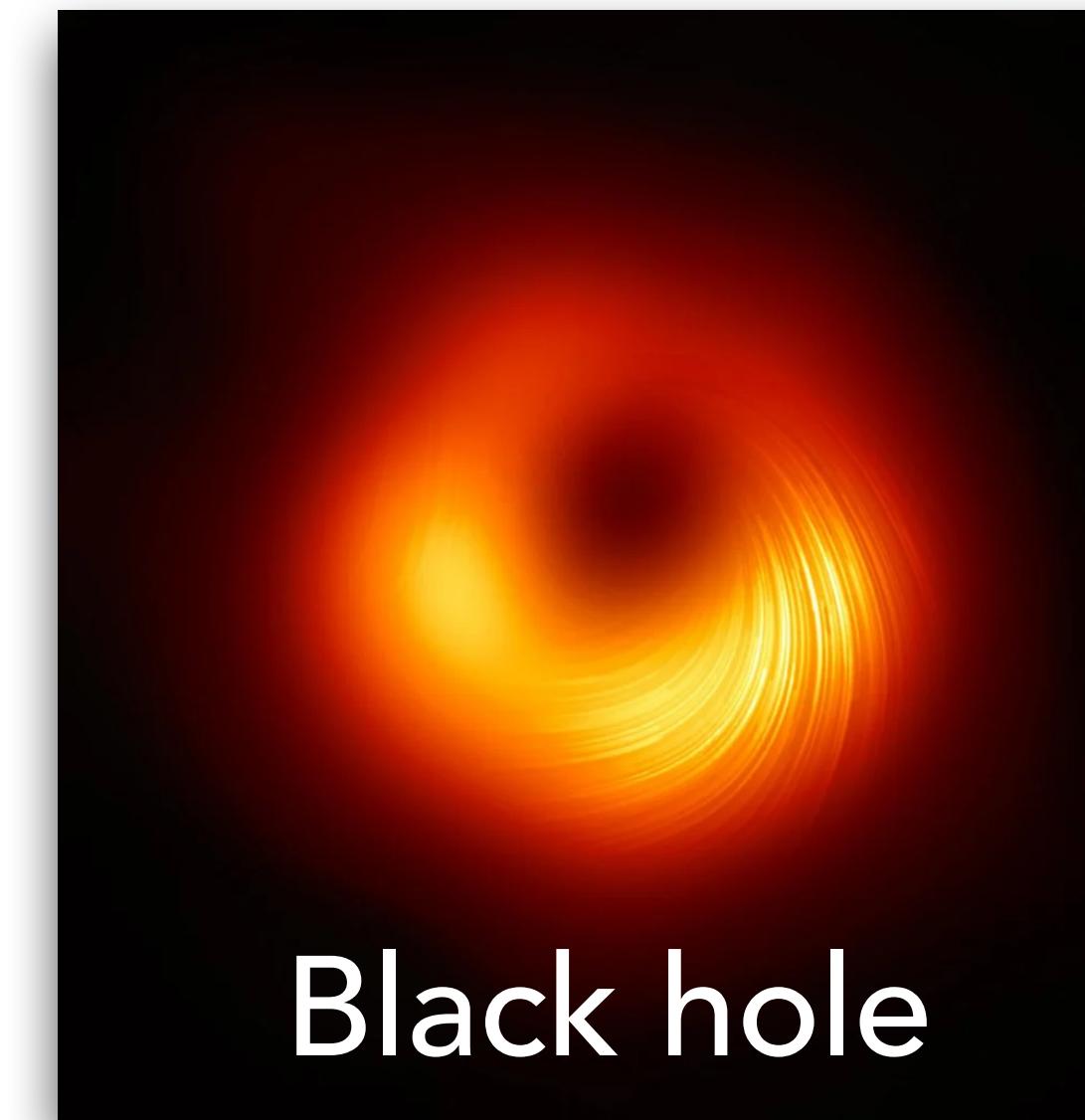
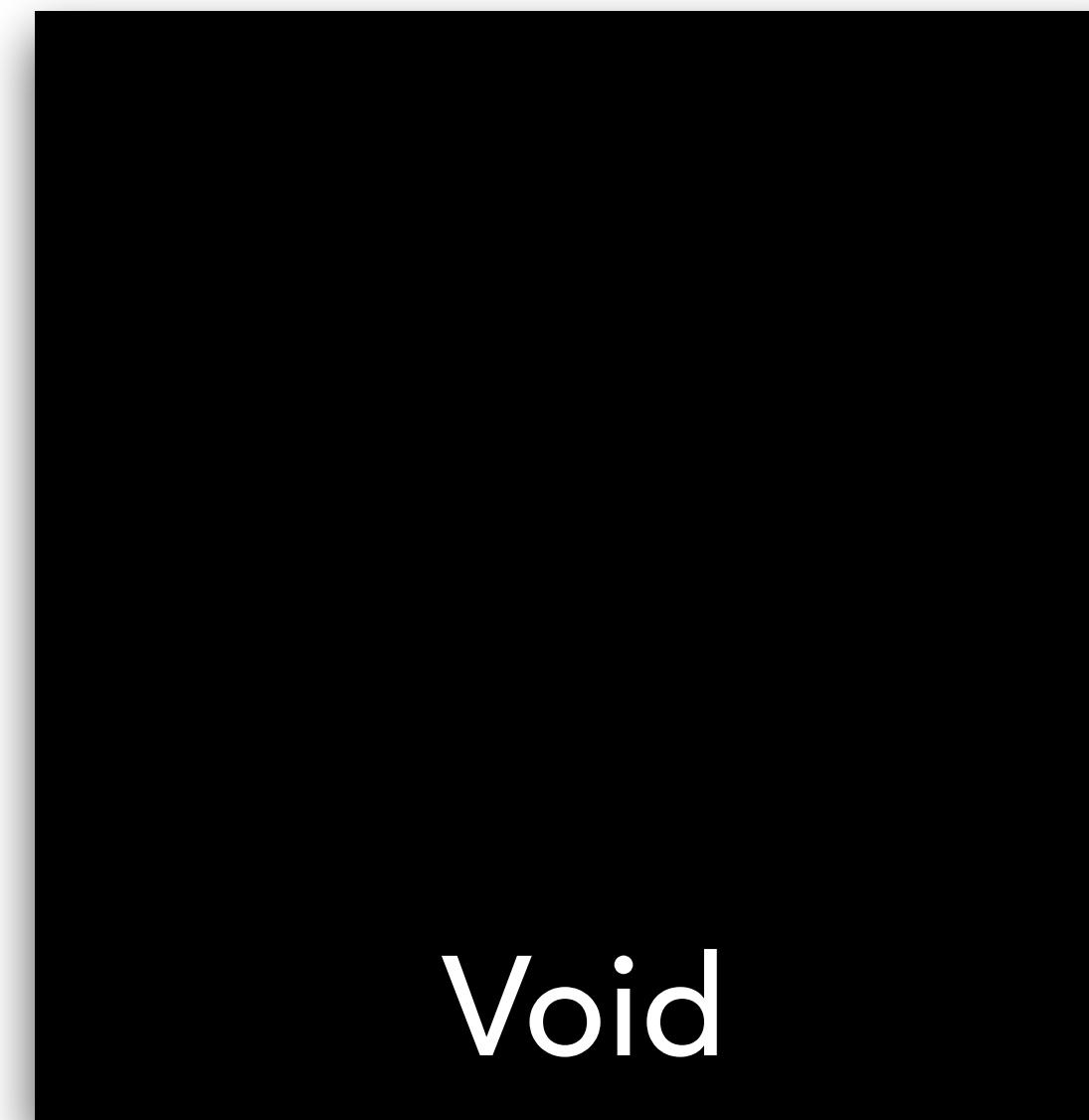
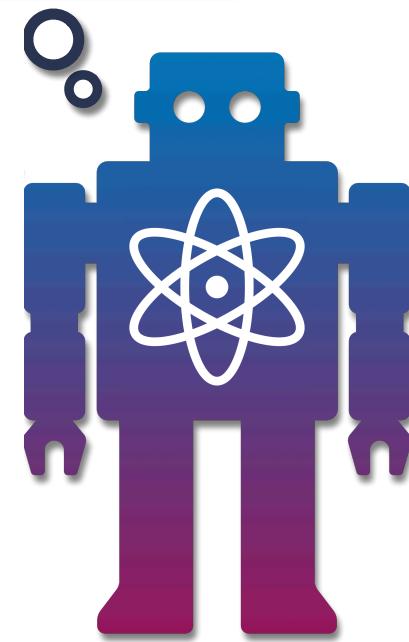
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Spacetime Geometry

- Consider two vastly different spacetime geometries.
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They look
the same



Event horizon
without blackhole?

Quantum gravity
breaks hard crypto?

Akers, Bouland, Chen, Kohler, Metger, Vazirani.

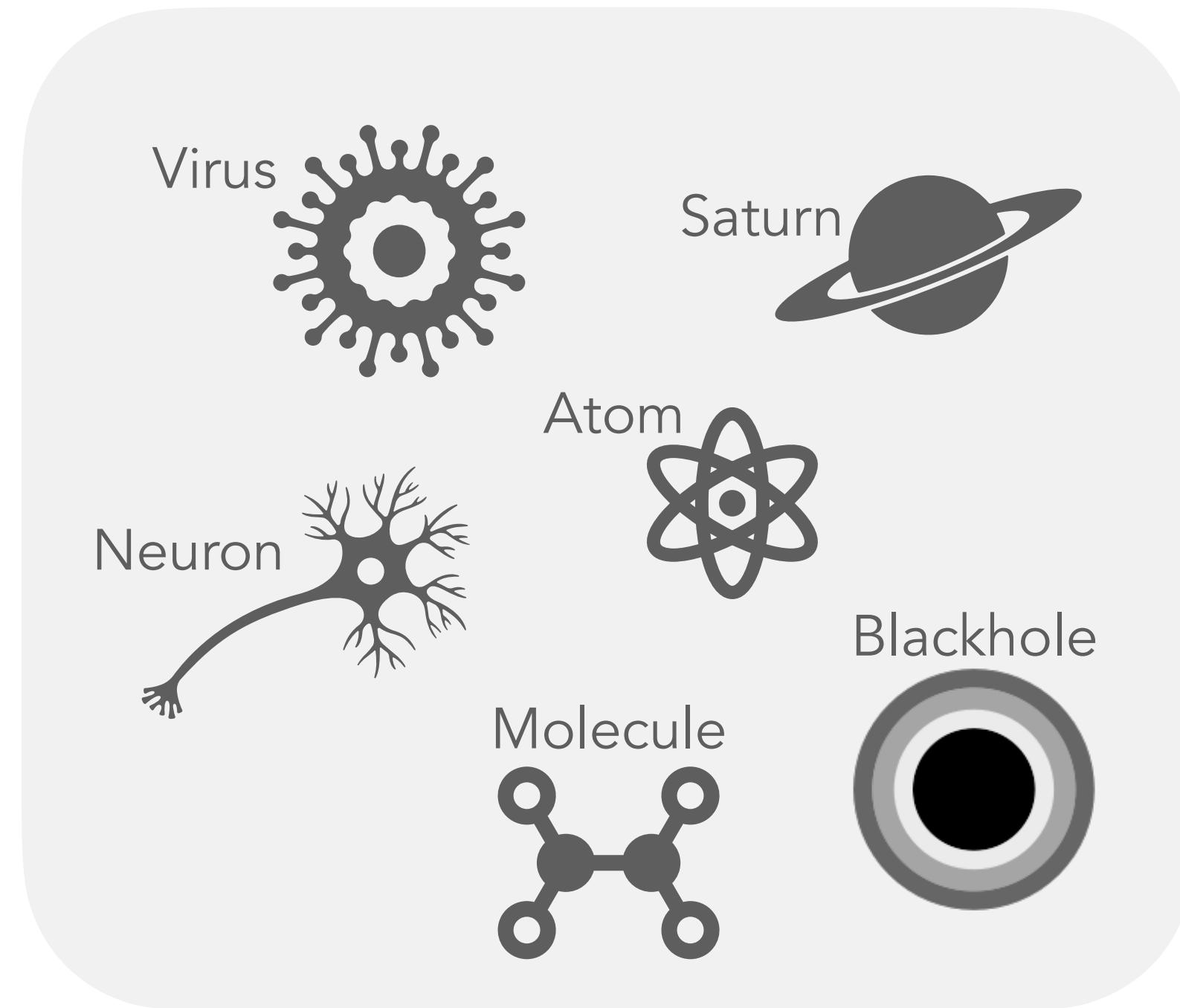
Holographic pseudoentanglement and the complexity of the AdS/CFT dictionary.

Ph 220: Quantum Learning Theory

Quantum Advantage

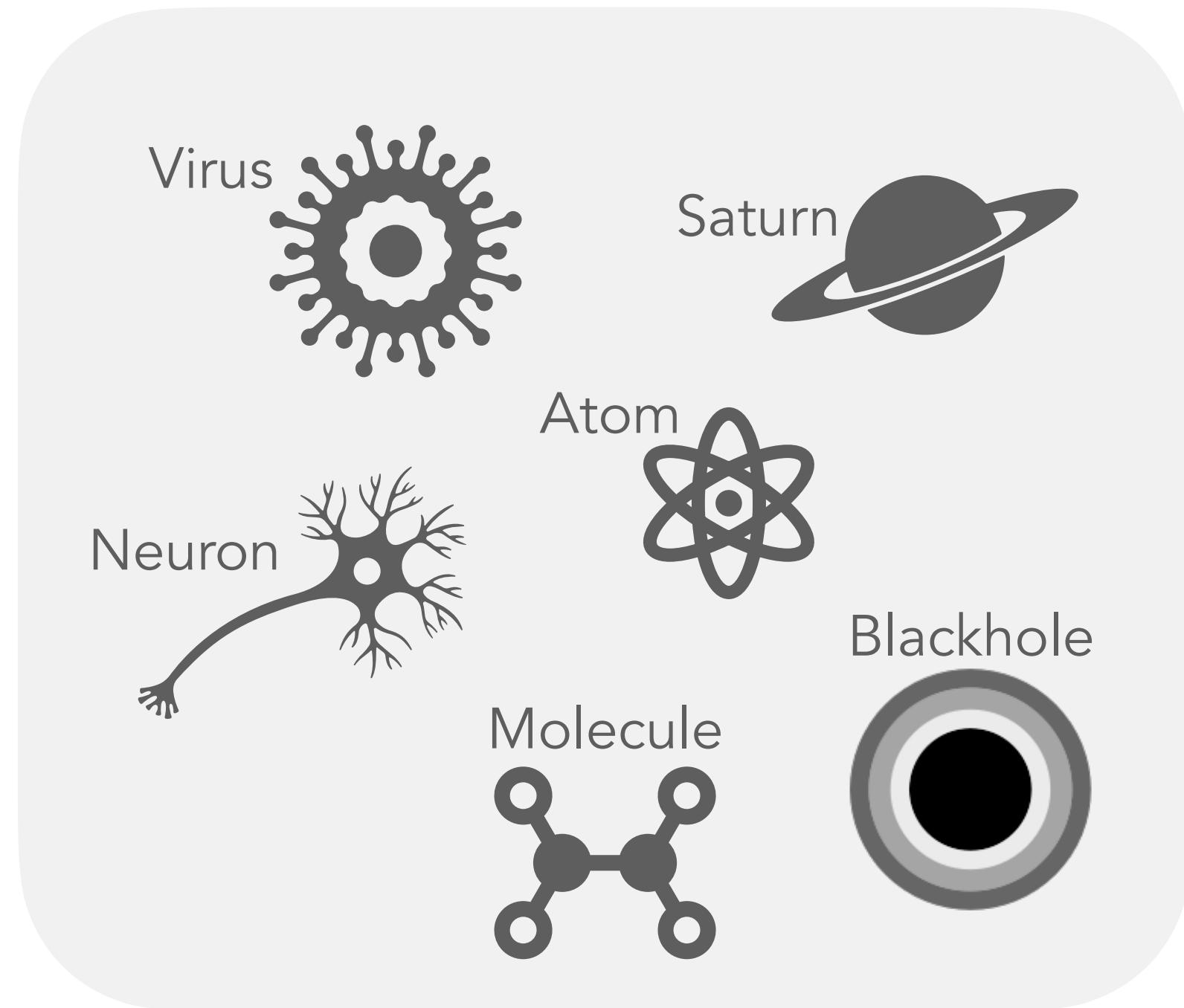
in Learning

Definition: Learning

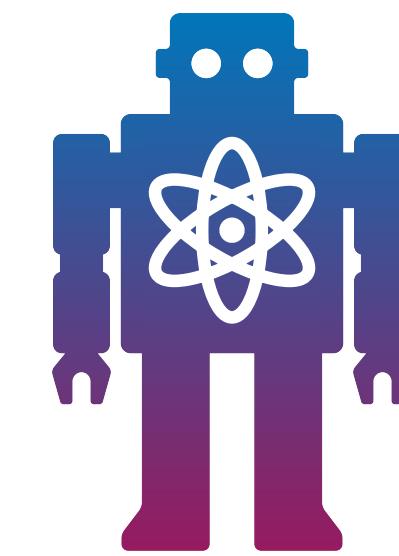


External world

Definition: Learning

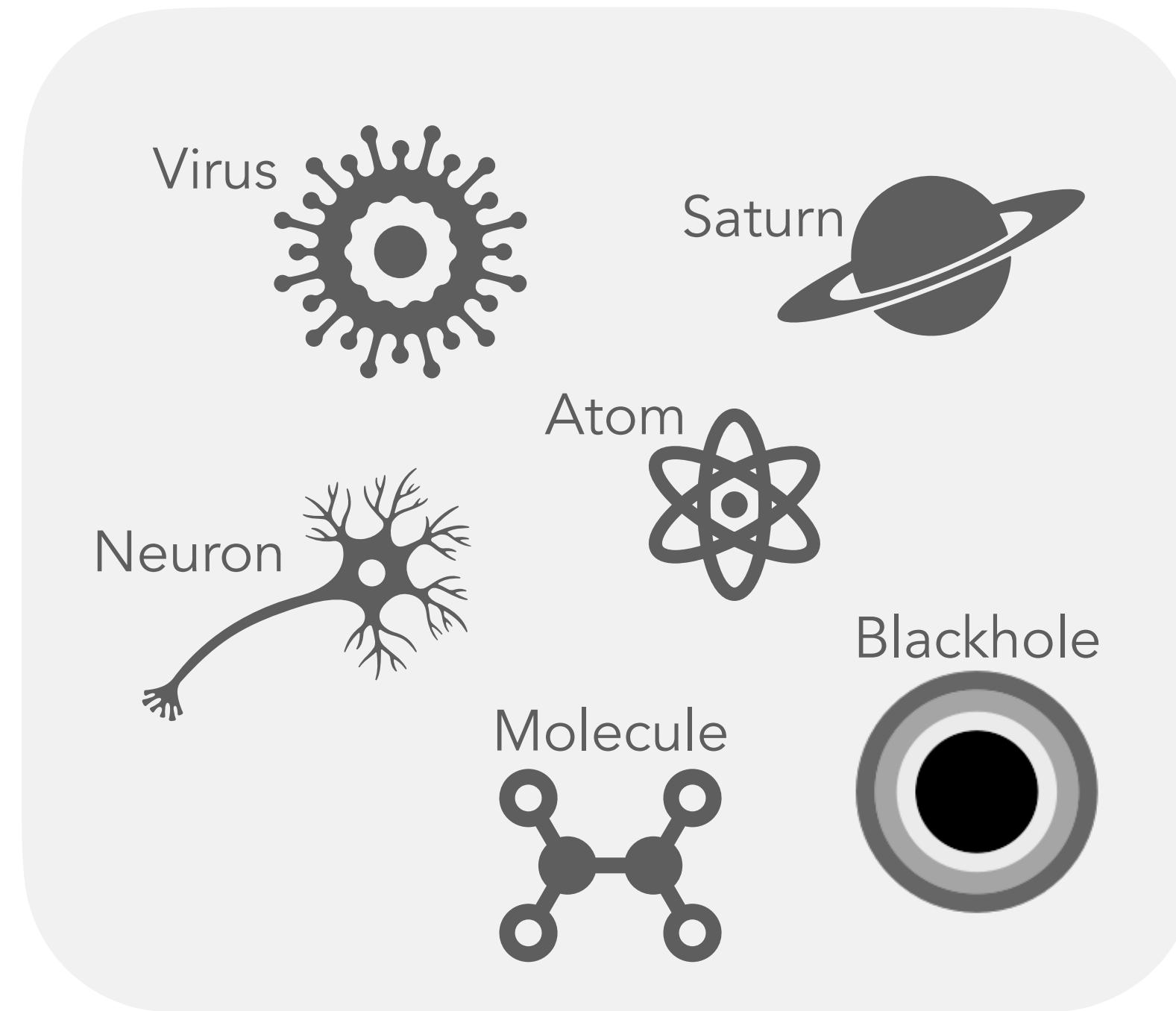


External world

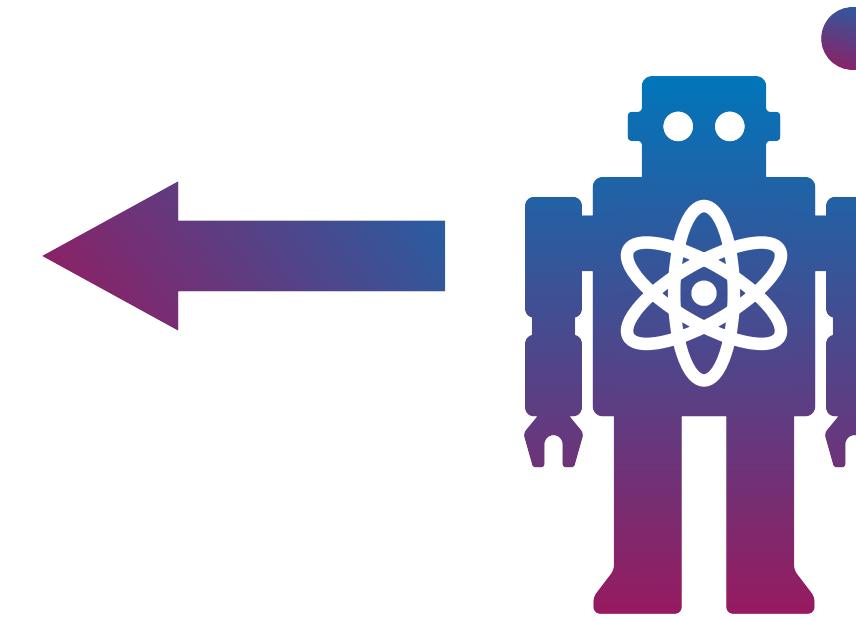


Advanced Intelligence

Definition: Learning

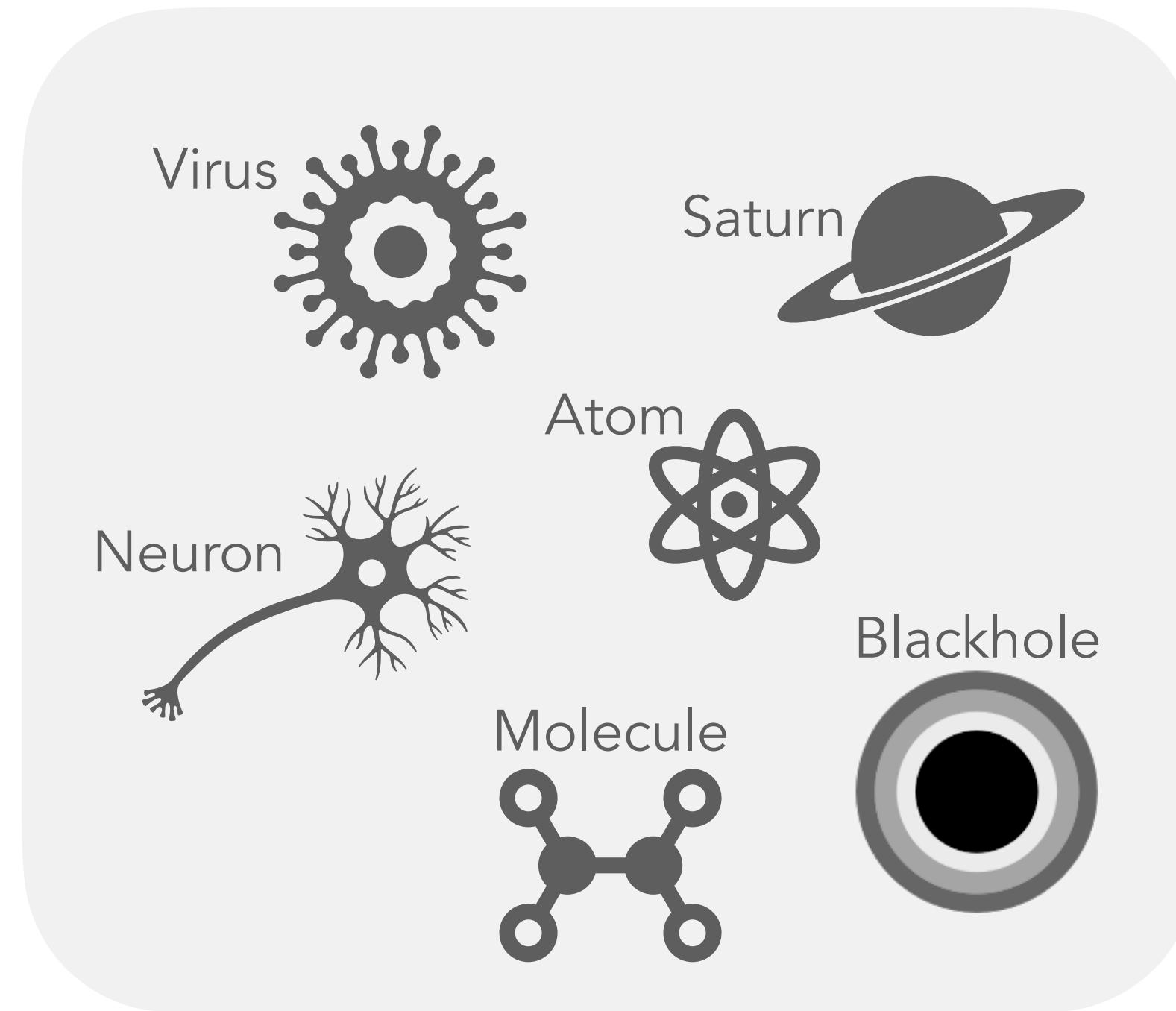


External world

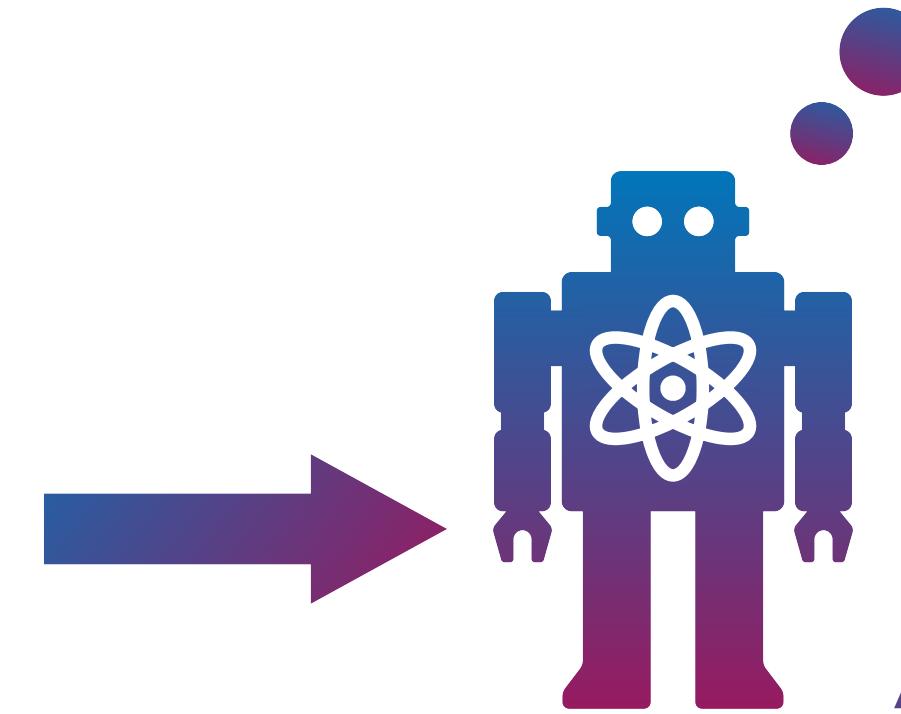


Advanced Intelligence

Definition: Learning

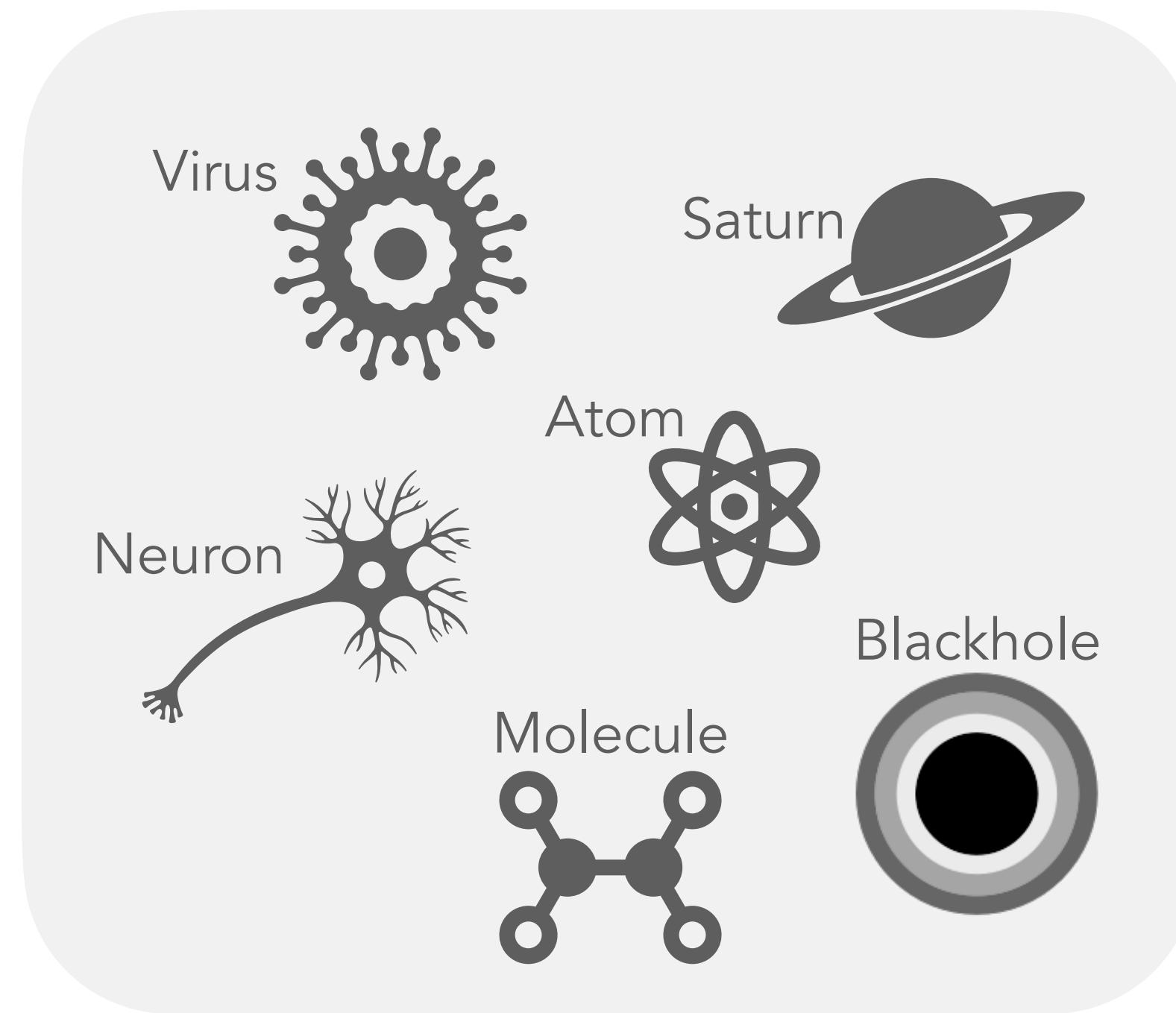


External world

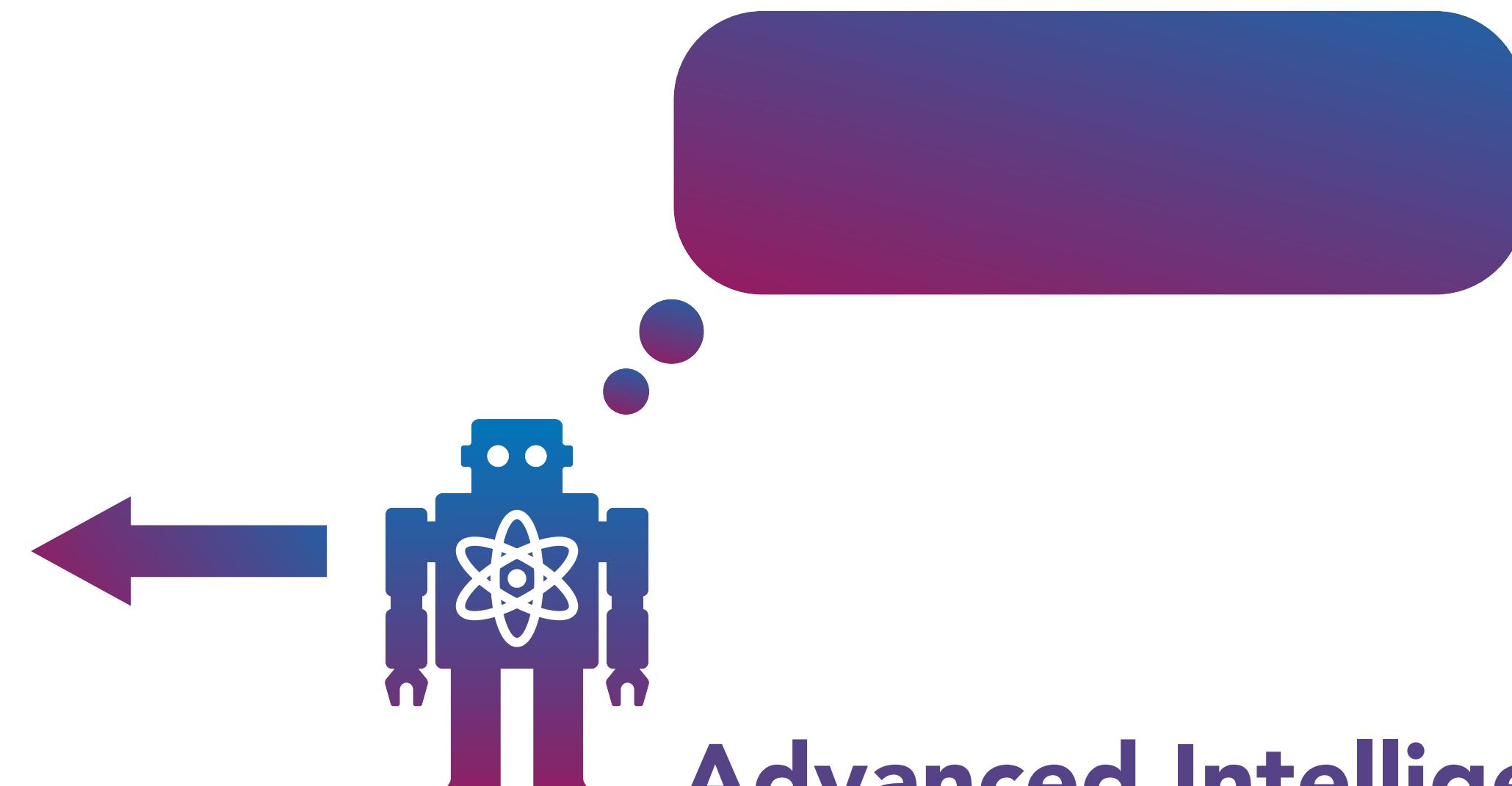


Advanced Intelligence

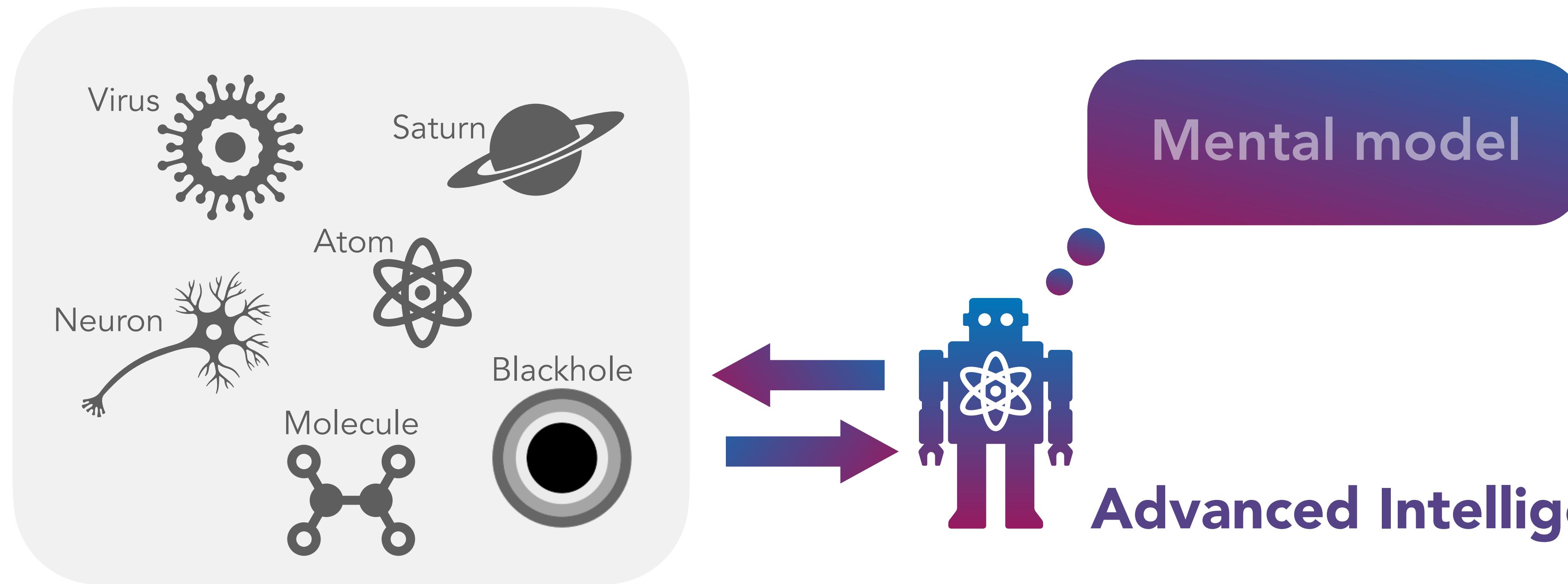
Definition: Learning



External world



Definition: Learning

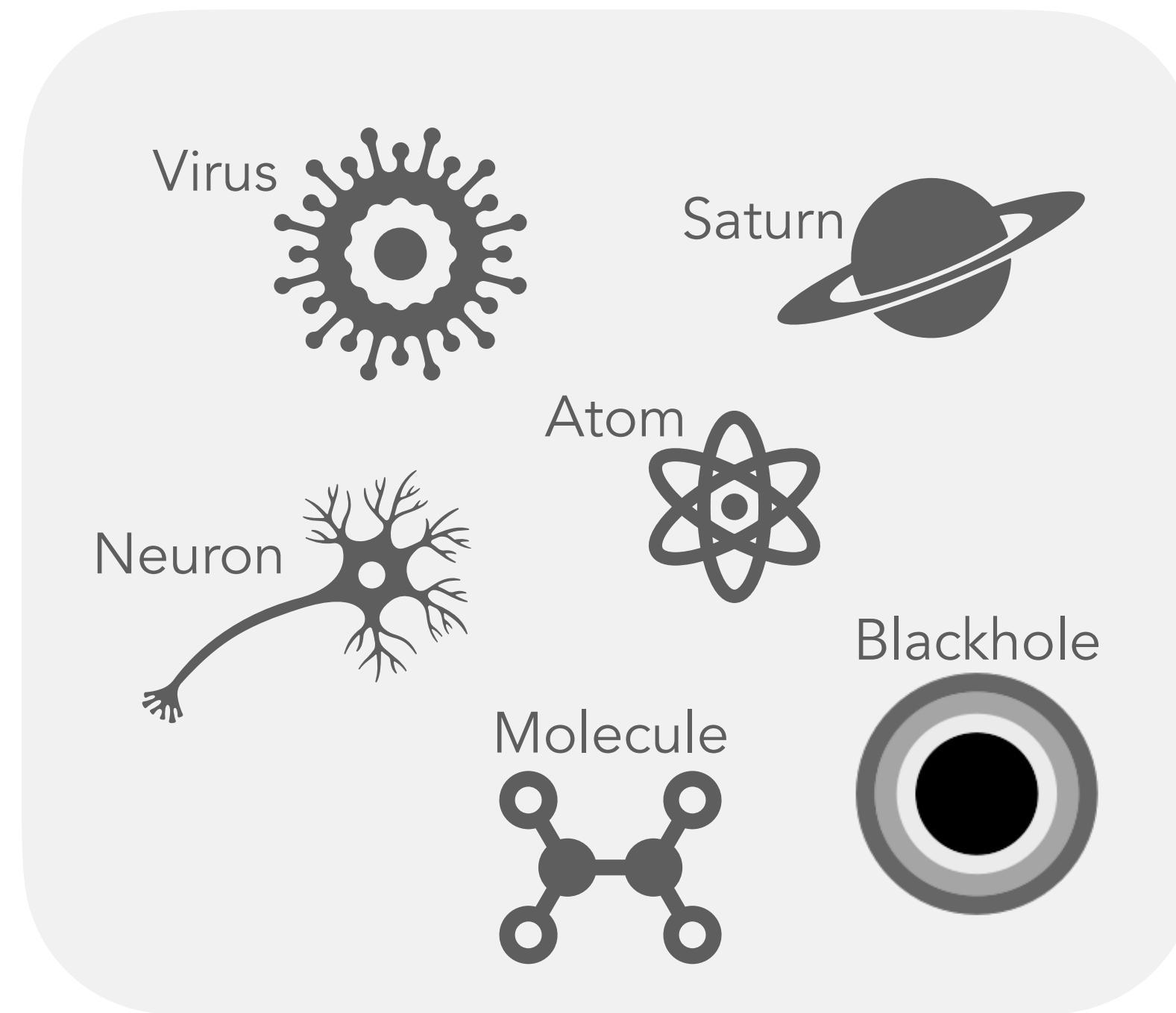


External world

Mental model

Advanced Intelligence

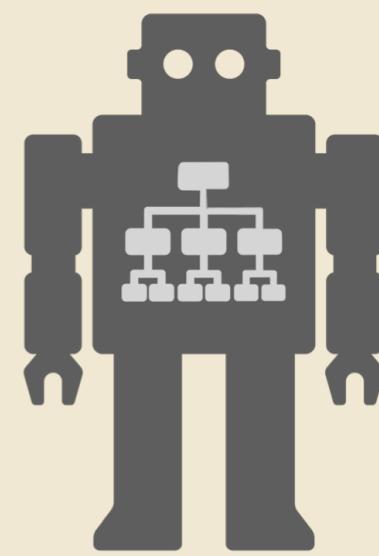
Definition: Learning



External world

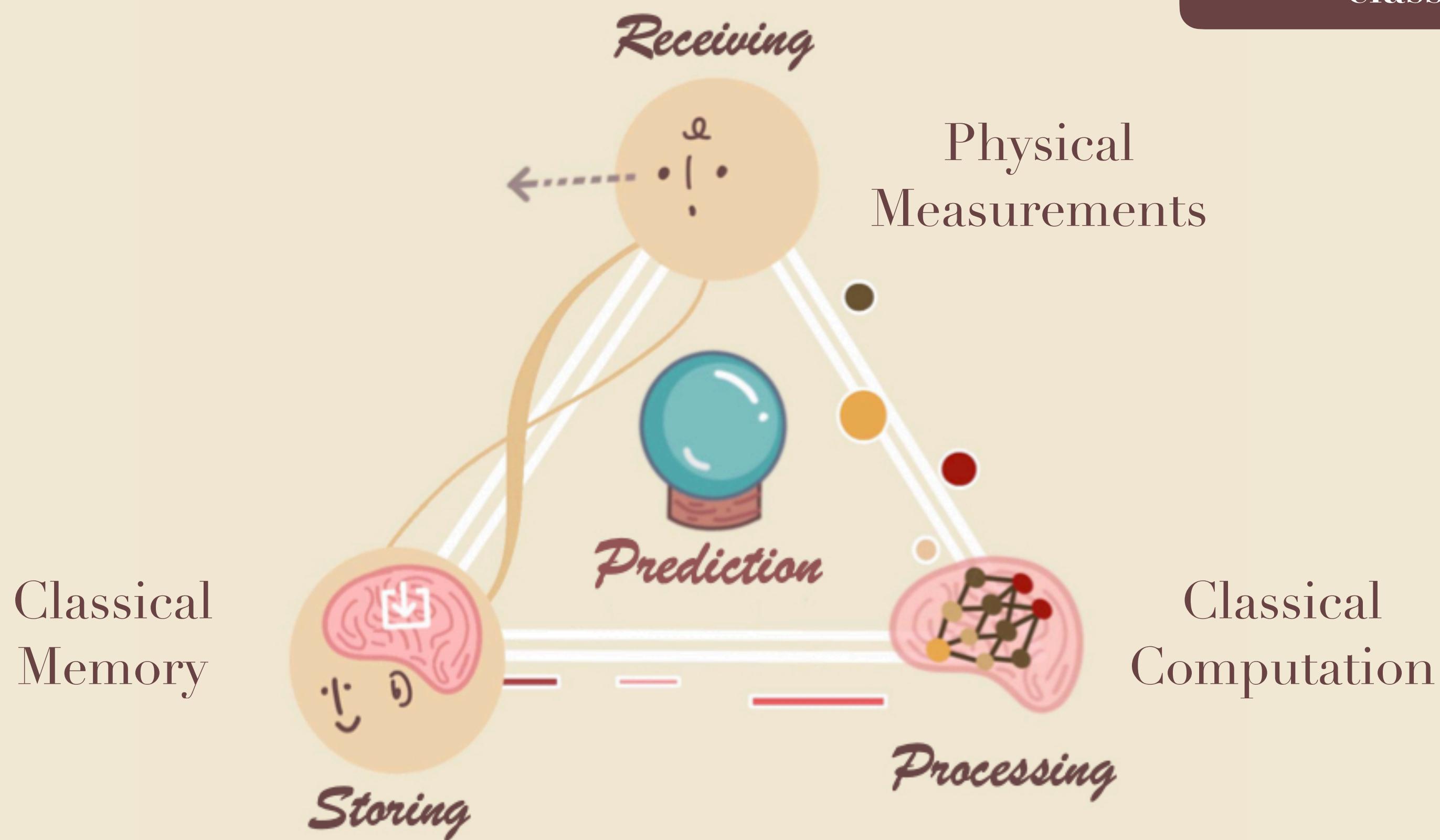


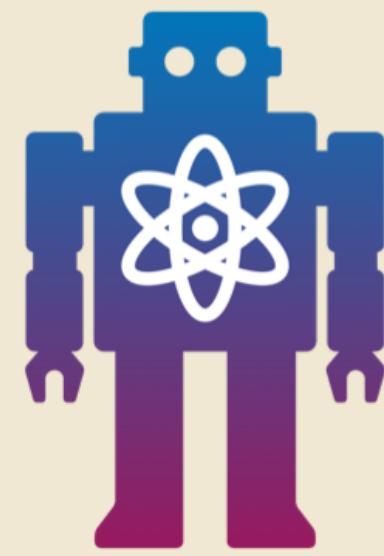
Advanced Intelligence



Classical AI

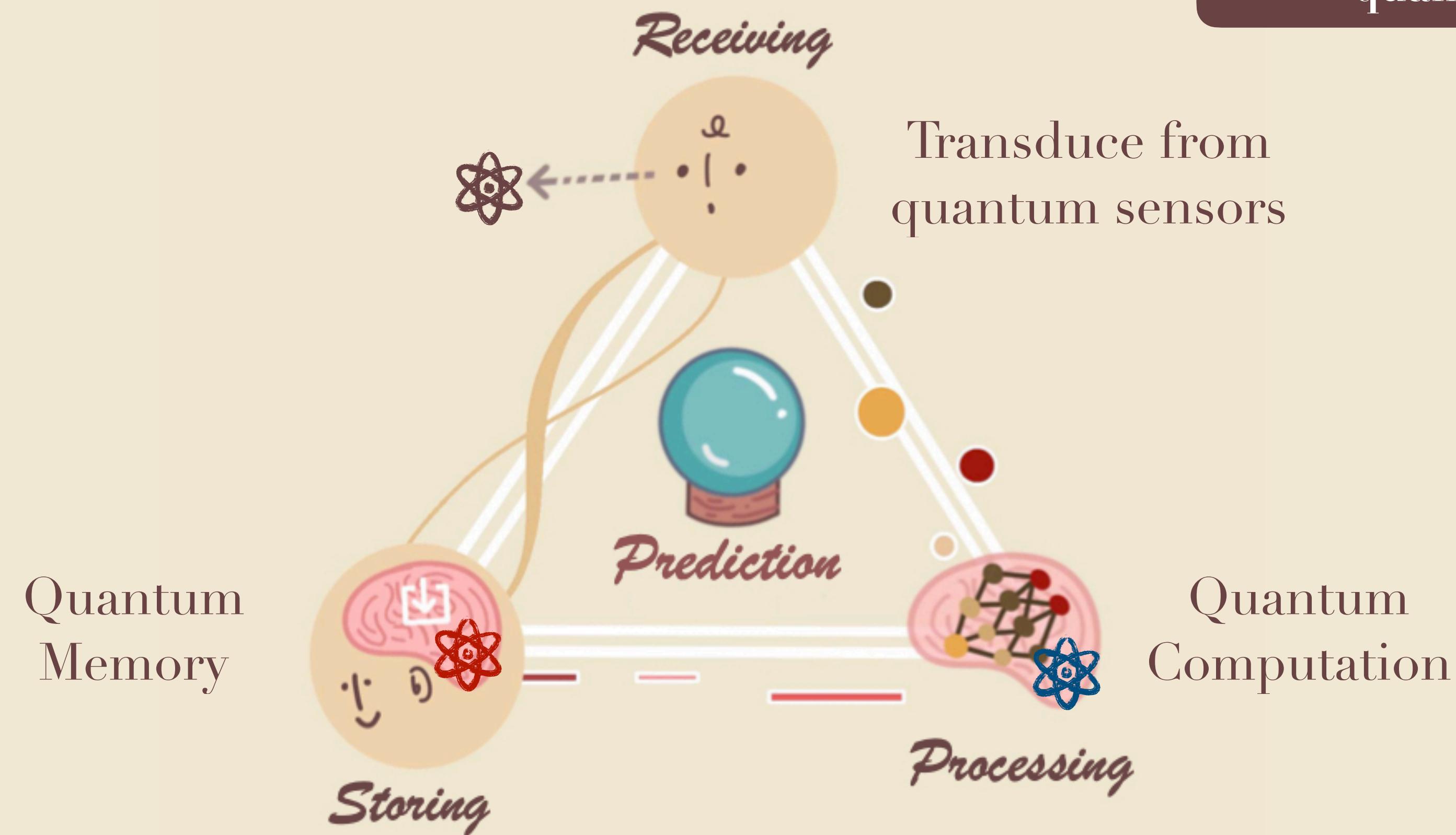
Receive, process, and store
classical information





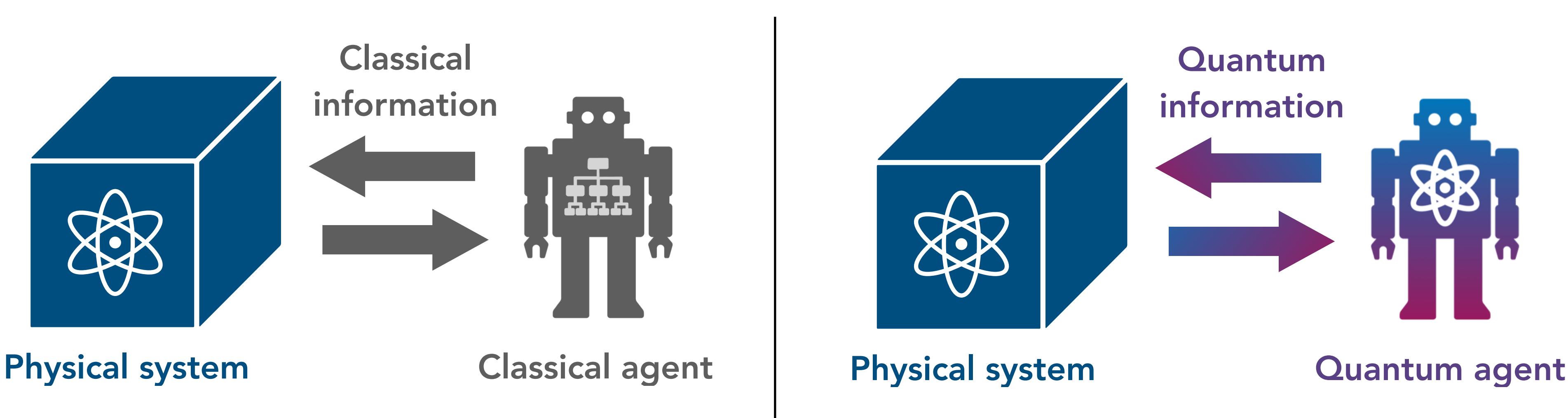
Quantum AI

Receive, process, and store
quantum information



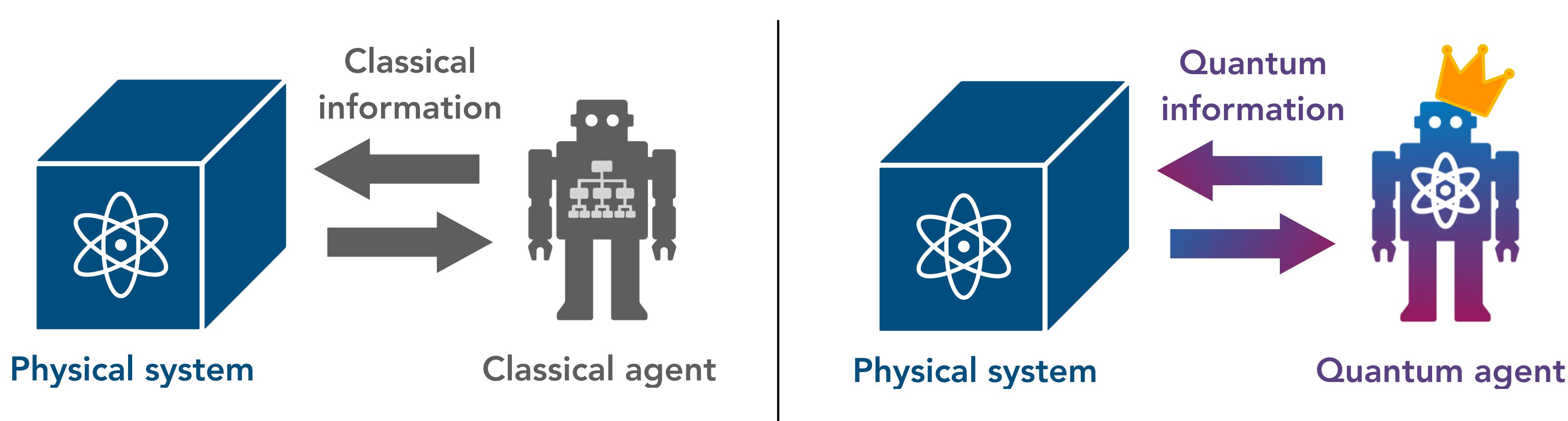
Central Question

- How can quantum AI agents yield **significant advantage** in learning about our physical world?



Quantum Advantage

- There are **exponentially large** quantum advantage in **predicting observables, quantum PCA, uncovering symmetry...**



Quantum Advantage

- The agent can see many samples of an **unknown n -qubit state ρ** .
- After learning, the agent needs to form a model for predicting $\text{Tr}(P\rho)$ for any observable P chosen from $\{I, X, Y, Z\}^{\otimes n}$.

Theorem (Predicting Observables)

Classical agents require $\Omega(2^n)$ **samples** to predict.

Quantum agents only need $\mathcal{O}(n)$ **samples** to predict.