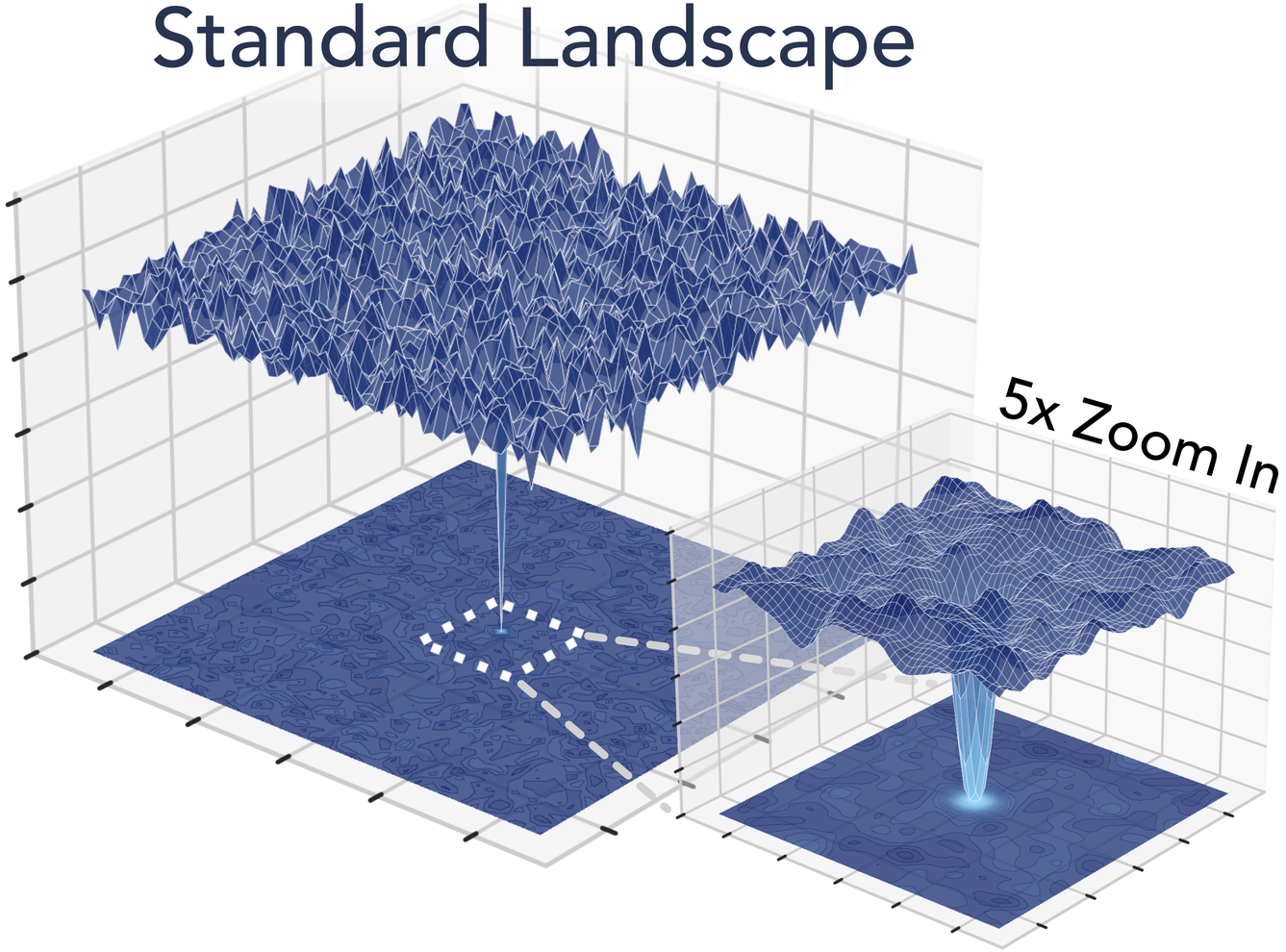
Ph 220: Lecture 11

Learning QNNs

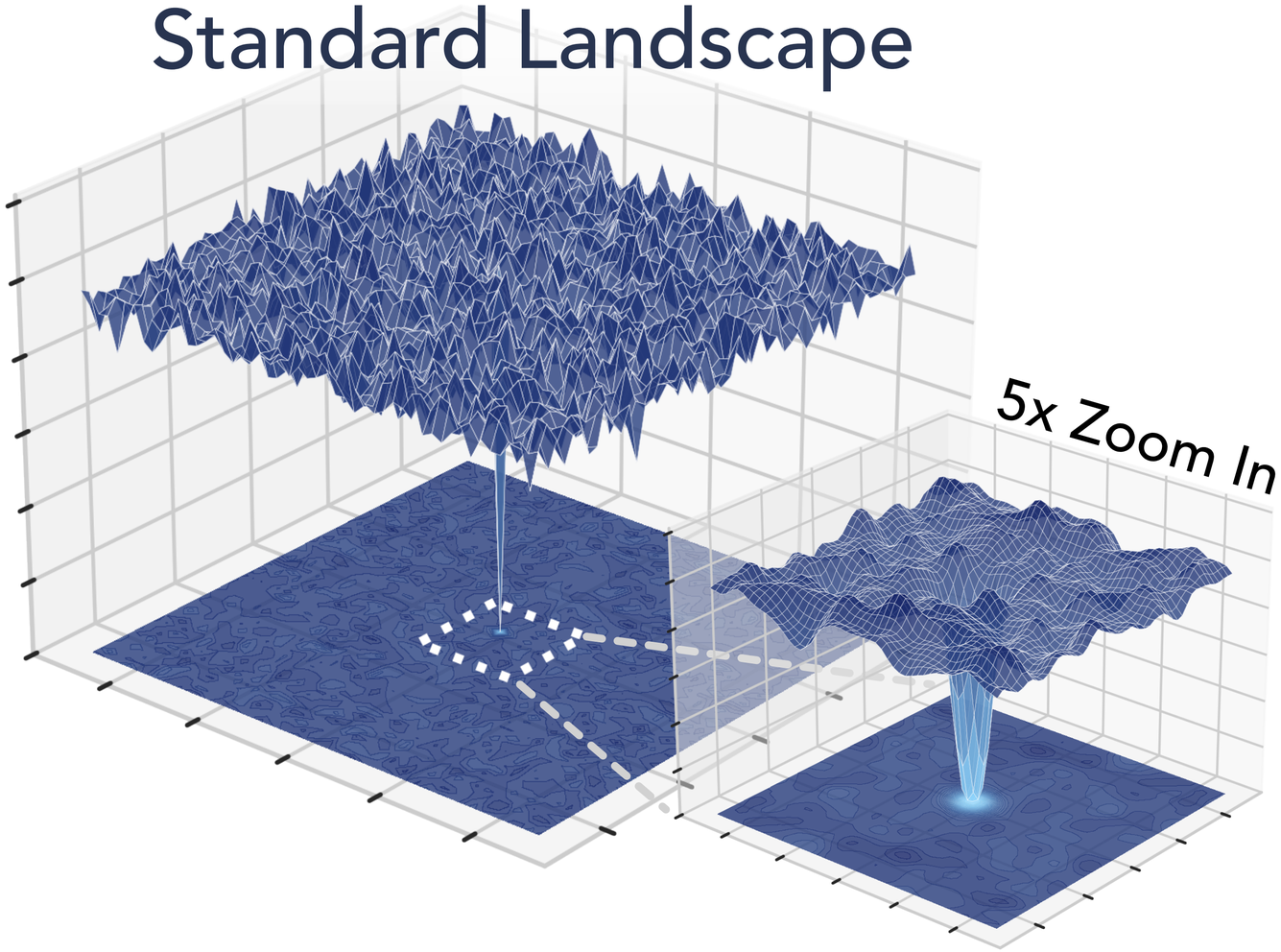
Bad Loss Landscape

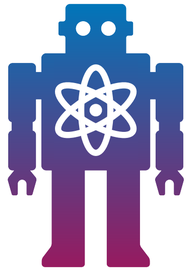
Shallow QNNs have **extremely bad** landscape.



Bad Loss Landscape

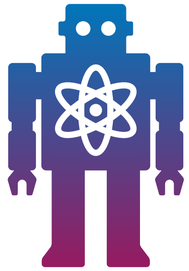
Gradient descent
and other methods
get stuck and fail.





Shallow QNNs

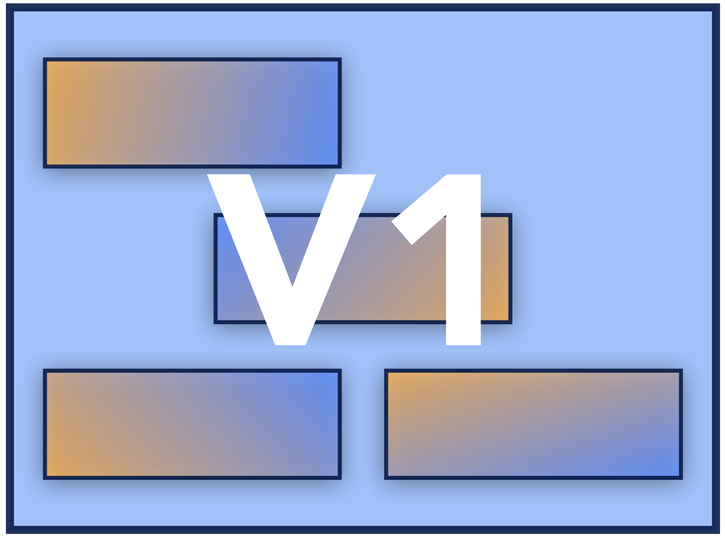
Despite significant interest in Shallow QNNs, **no efficient learning algorithm** was previously known.



Question

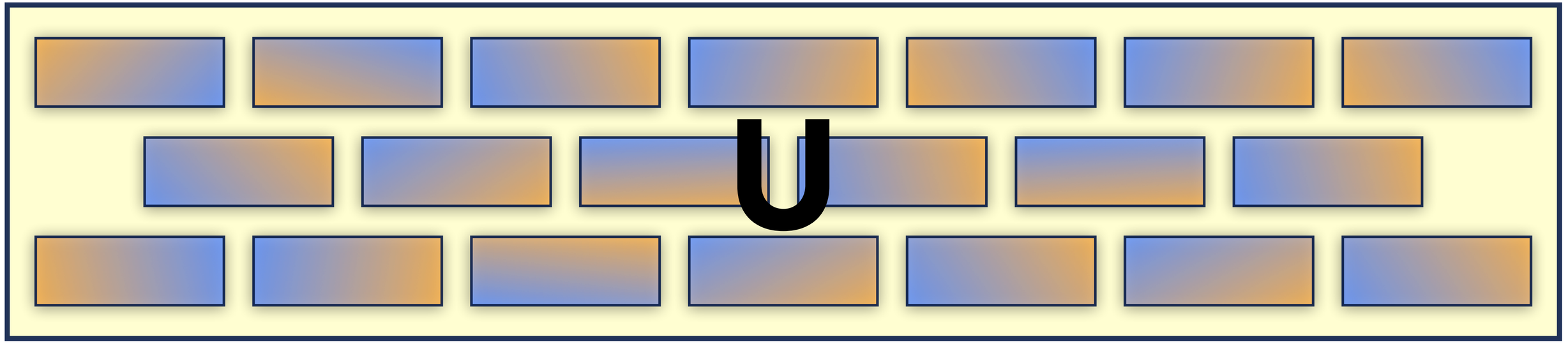
Are shallow QNNs fundamentally **hard to learn**?
(as a price of their beyond-classical power)

Parameterization by Local Inversion



Given an n-qubit unitary U.

V1 is a local inversion of U for qubit 1.



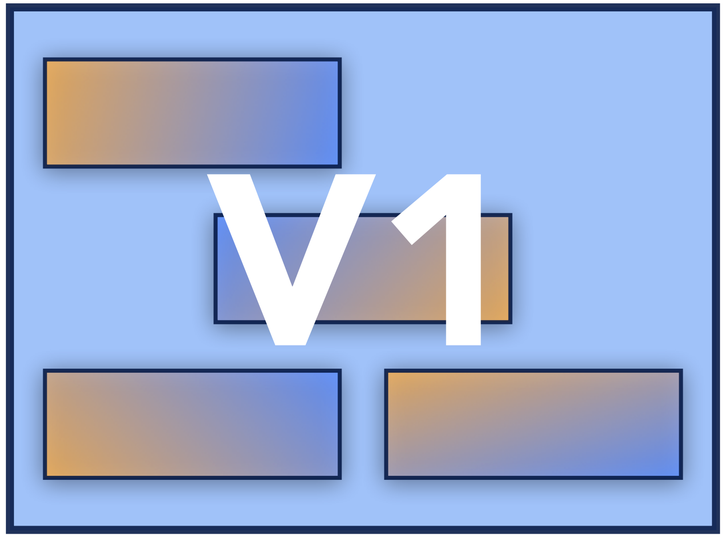
Parameterization by Local Inversion

Given an n-qubit unitary U.

Identity on qubit 1

V1 is a **local inversion** of U for qubit 1.

Parameterization by Local Inversion



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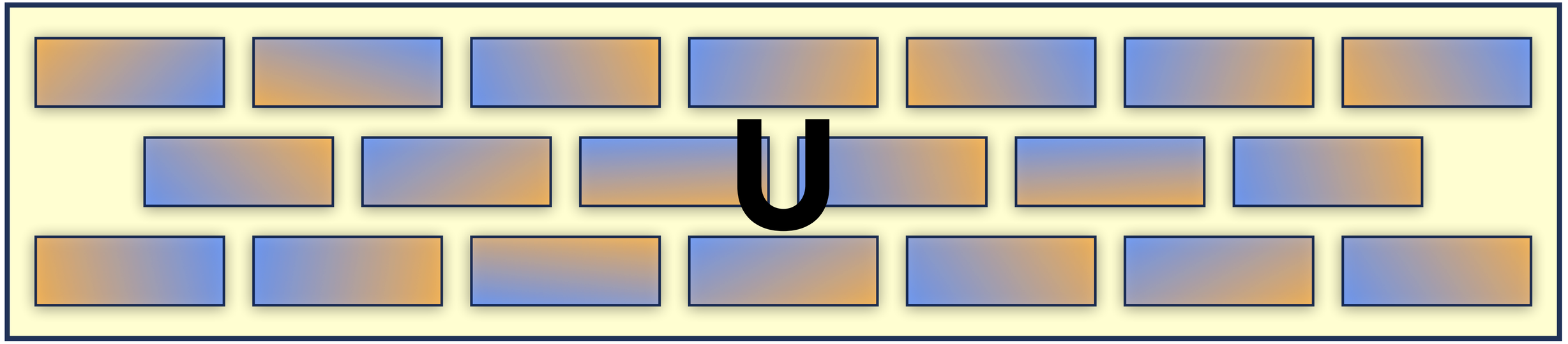
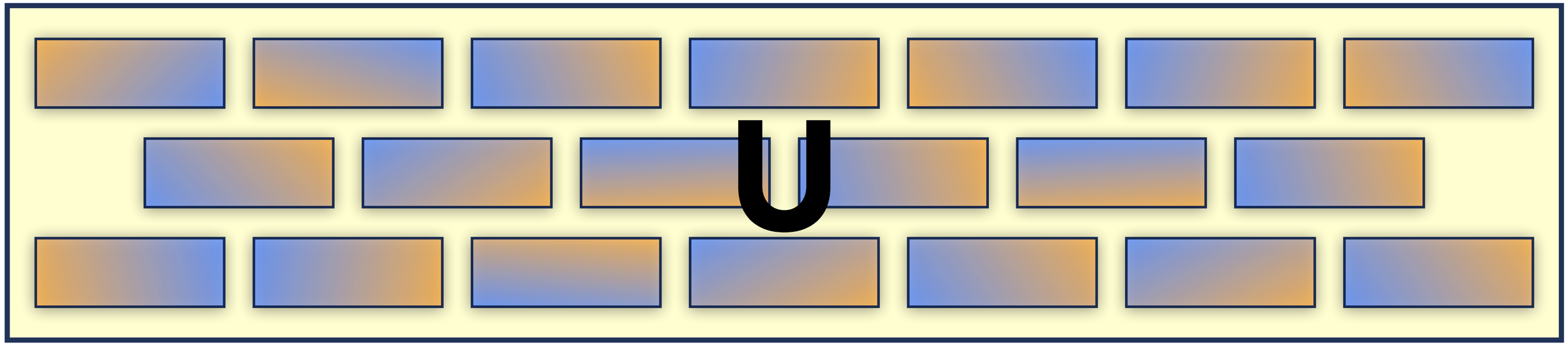


Parameterization by Local Inversion

Given an n-qubit unitary U.

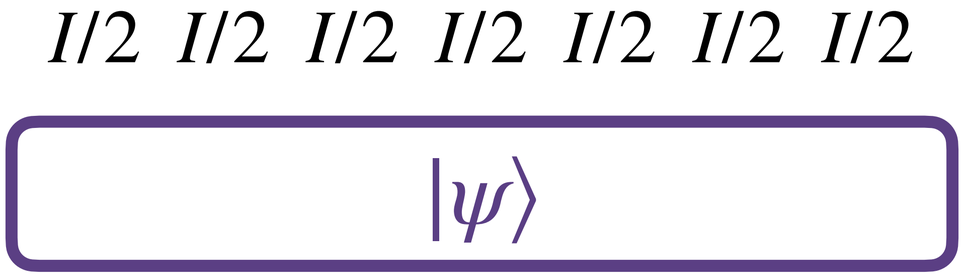
local inversions **V1**, …, **Vn** uniquely determine U.



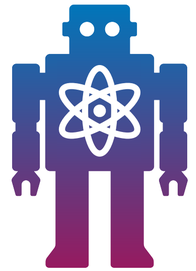
Sewing local inversions

- Given local inversions $V_1, \ldots, V_n$ of U on each qubit.
- We can sew them together to form the n-qubit unitary U.

Initialize



$2n$ qubits



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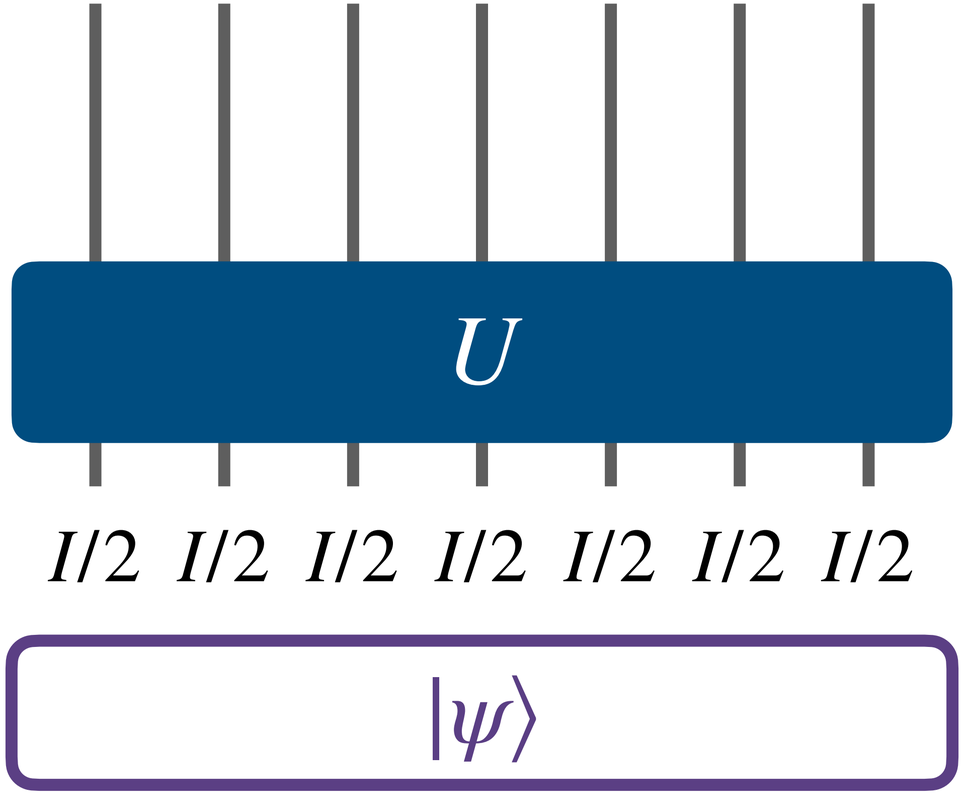
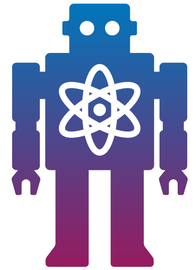
Imagine



$2n$ qubits



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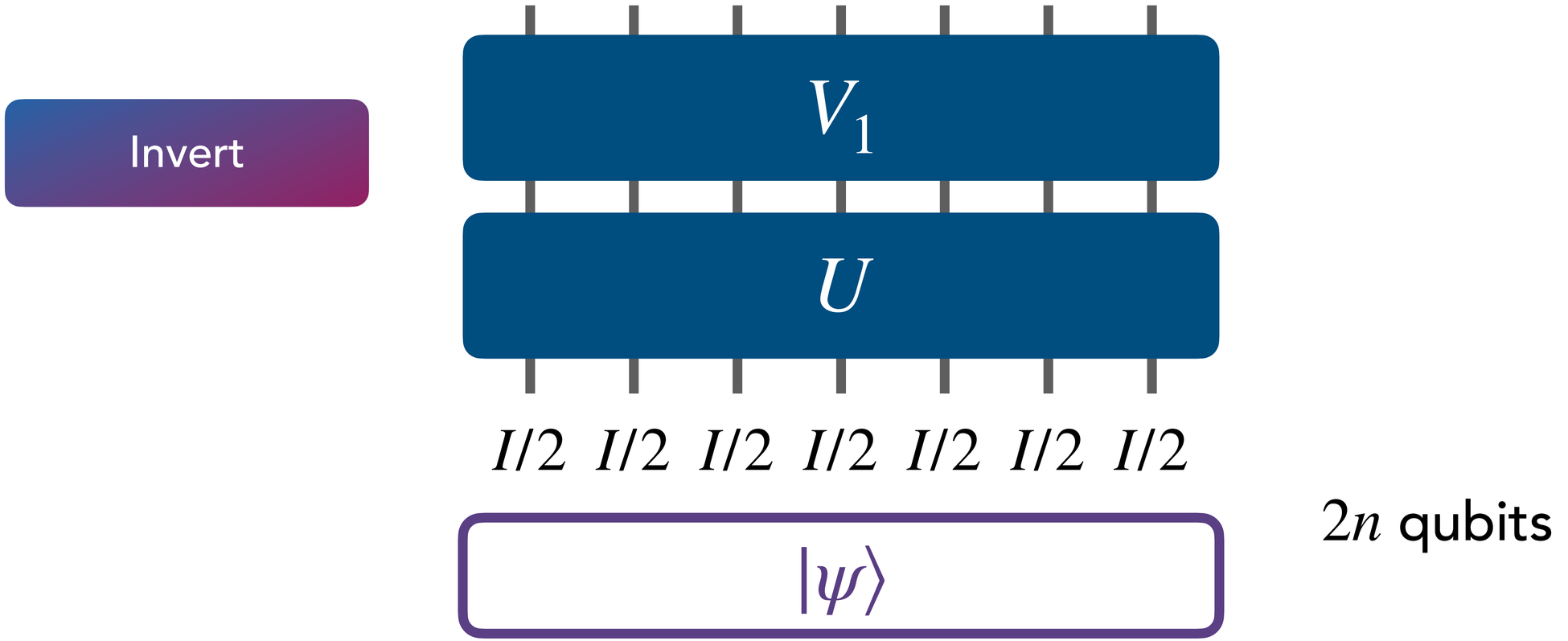

Sewing local inversions

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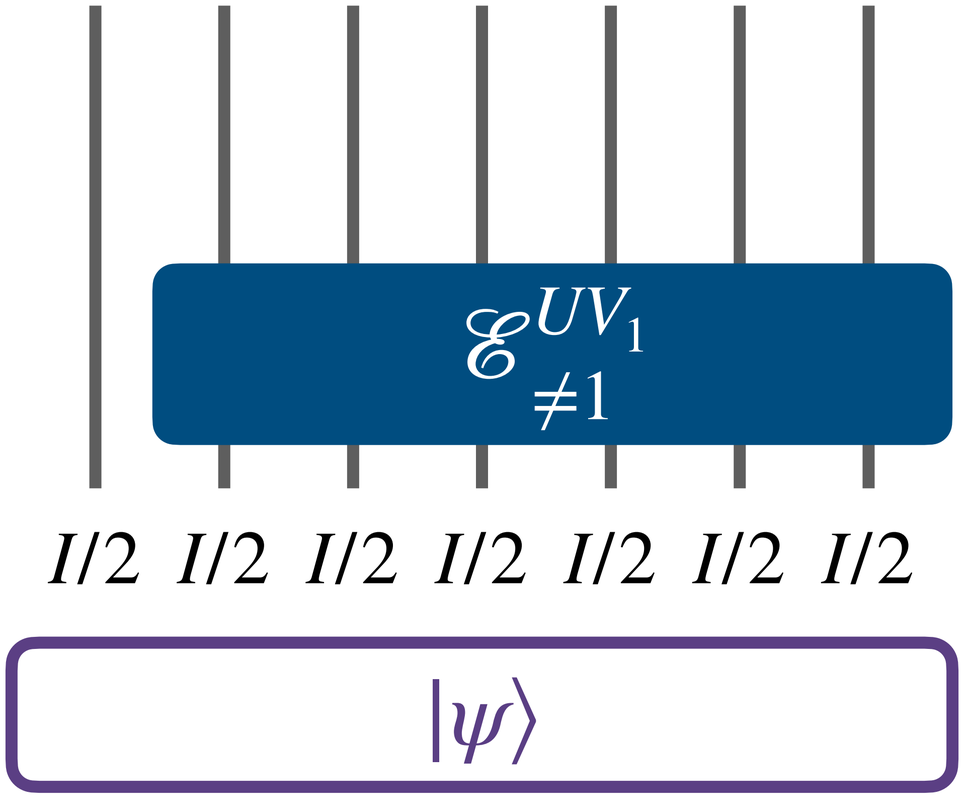
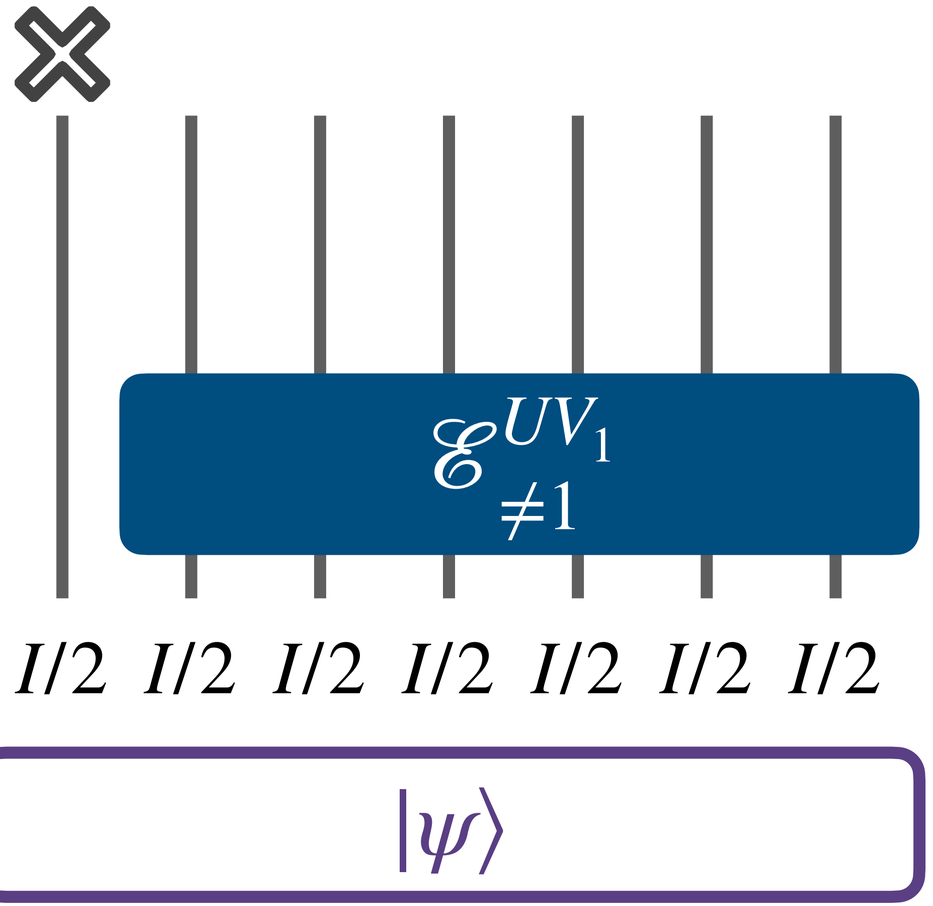
Uncover



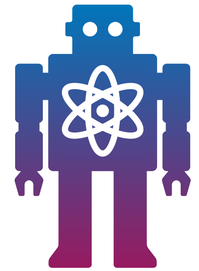
$2n$ qubits

Sewing local inversions

Dispose



$2n$ qubits



Sewing local inversions

- Given local inversions $V_1, \ldots, V_n$ of U on each qubit.
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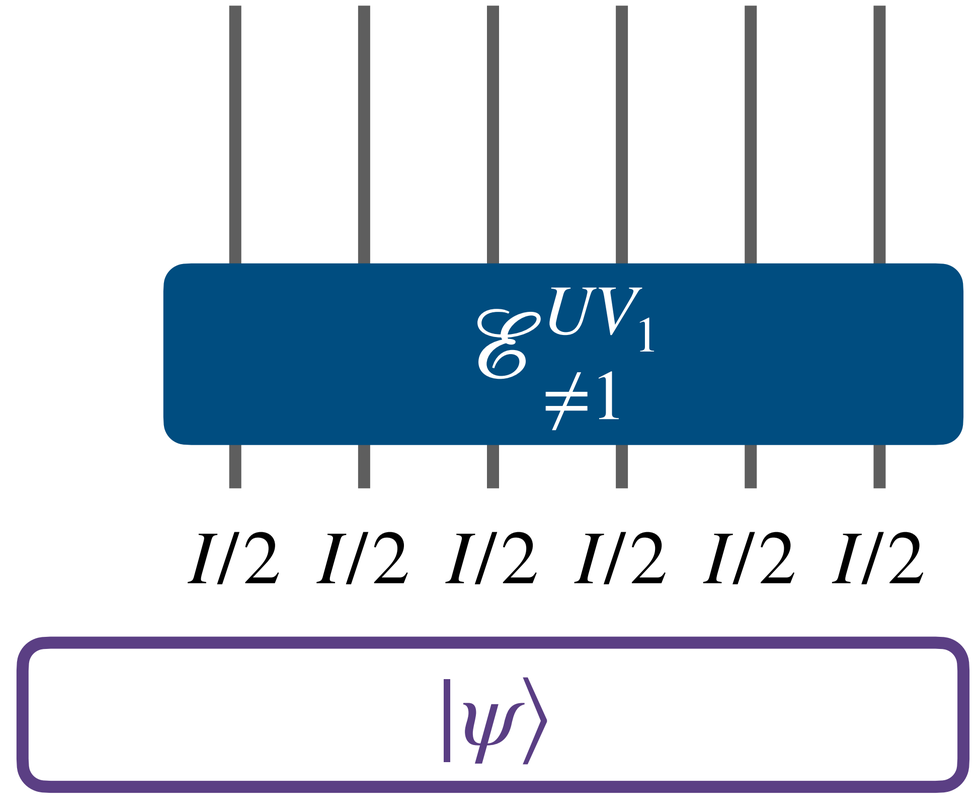
Dispose



$2n - 1$ qubits

Sewing local inversions

- Given local inversions $V_1, \ldots, V_n$ of U on each qubit.
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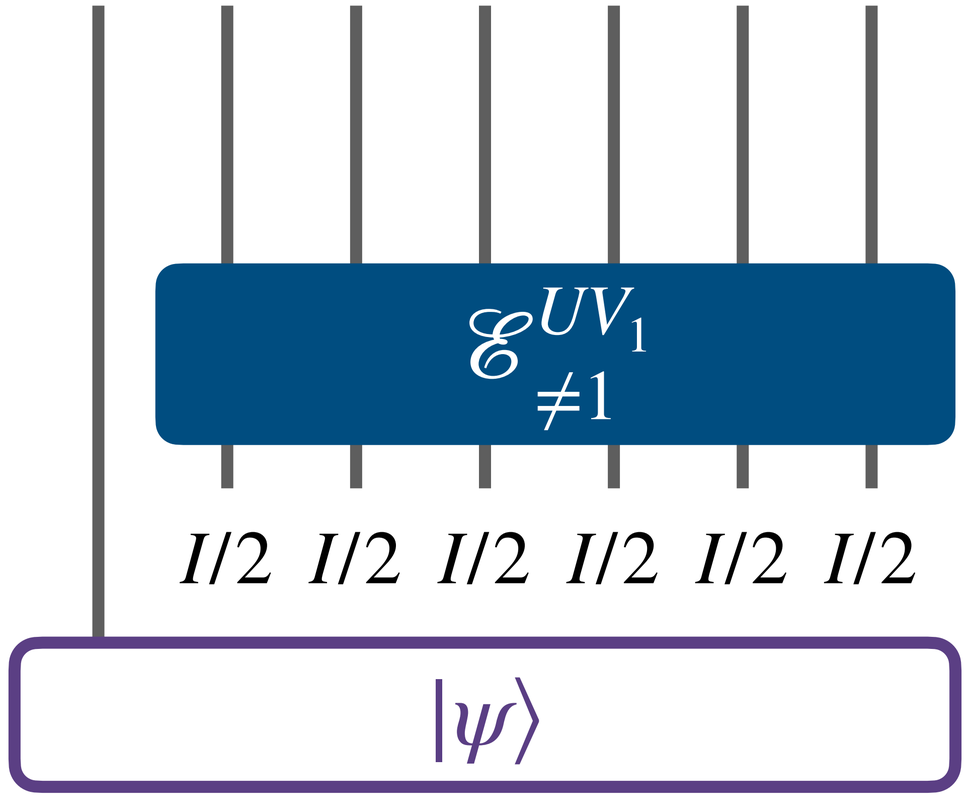
Rewire

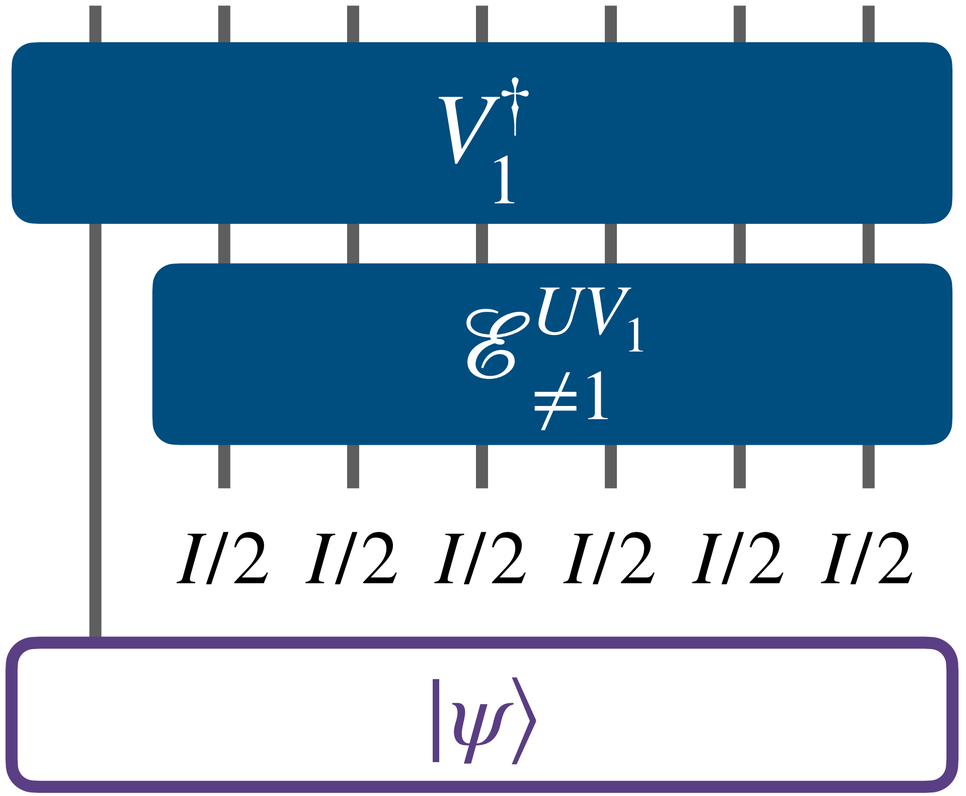


$2n - 1$ qubits

Sewing local inversions

- Given local inversions $V_1, \ldots, V_n$ of U on each qubit.
- We can sew them together to form the n-qubit unitary U.

Un-invert

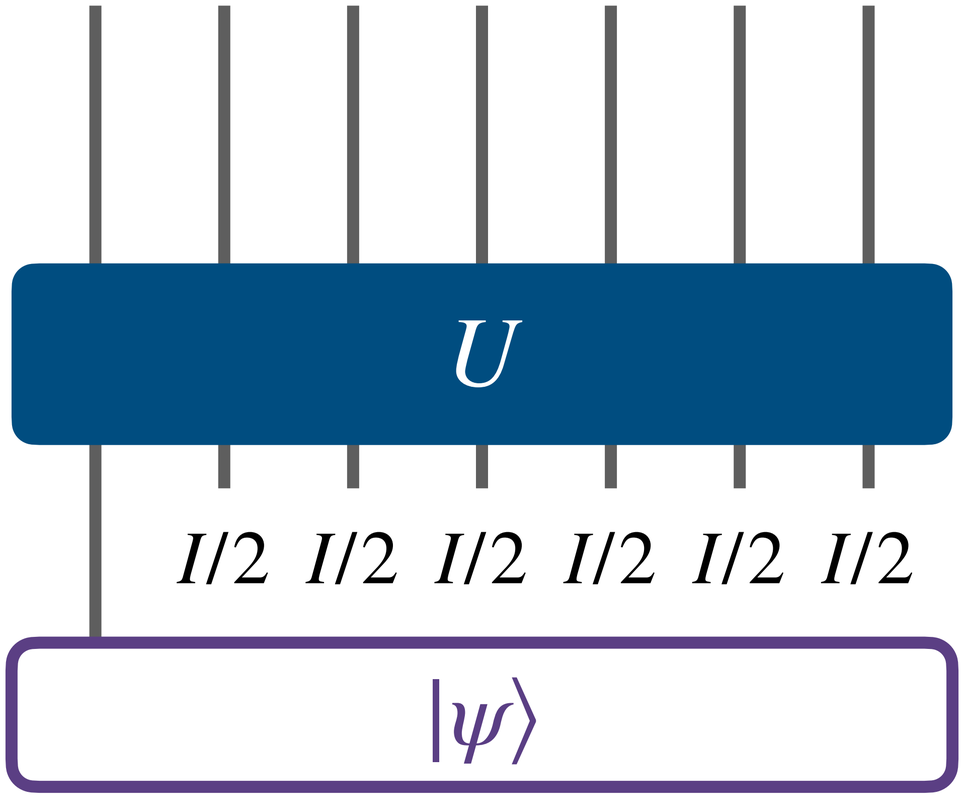


$2n - 1$ qubits

Sewing local inversions

- Given local inversions $V_1, \ldots, V_n$ of U on each qubit.
- We can sew them together to form the n-qubit unitary U.

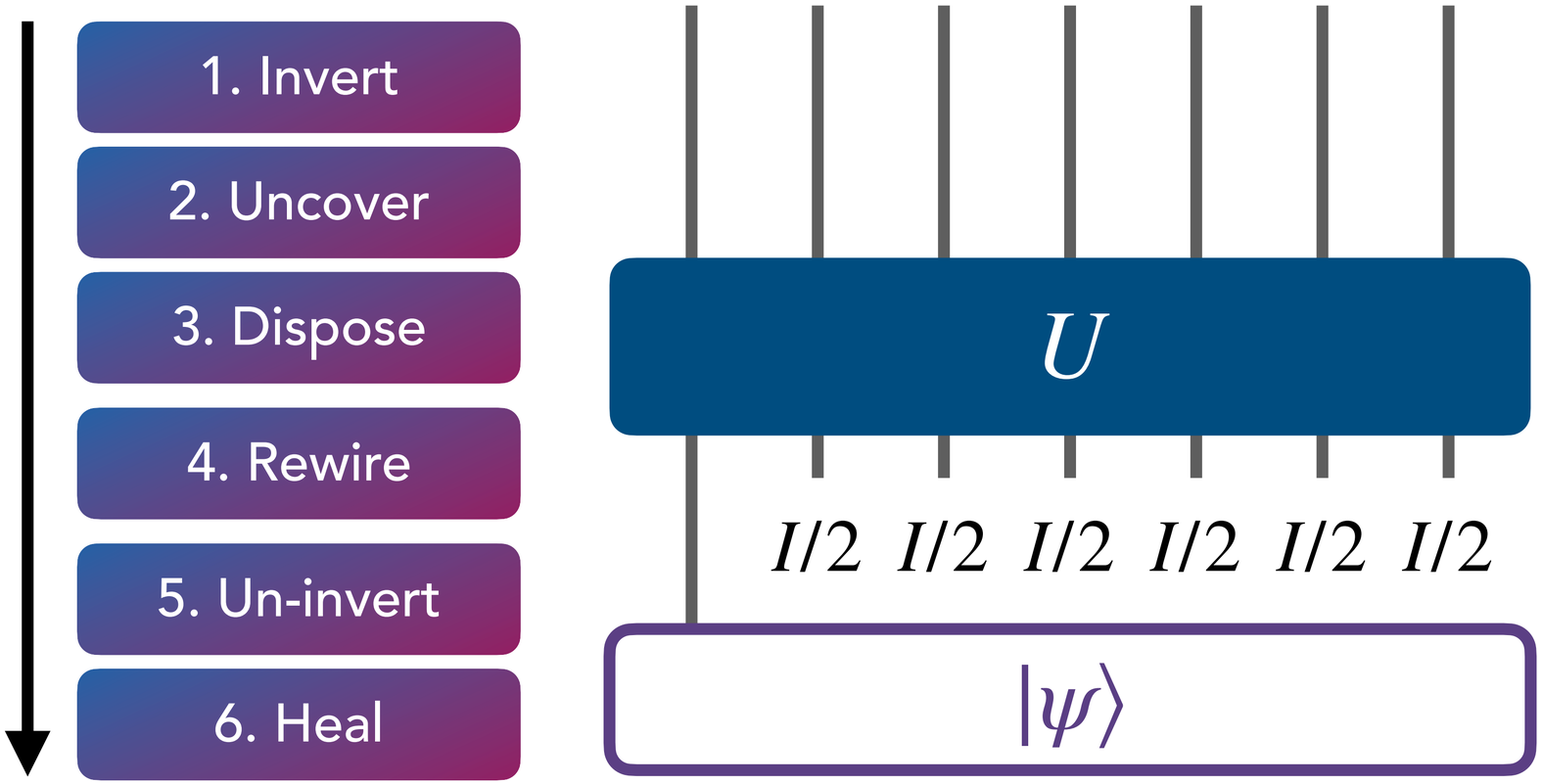
Heal



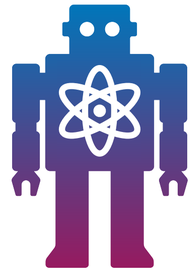
$2n - 1$ qubits

Sewing local inversions

- Given local inversions $V_1, \ldots, V_n$ of U on each qubit.
- We can sew them together to form the n-qubit unitary U.

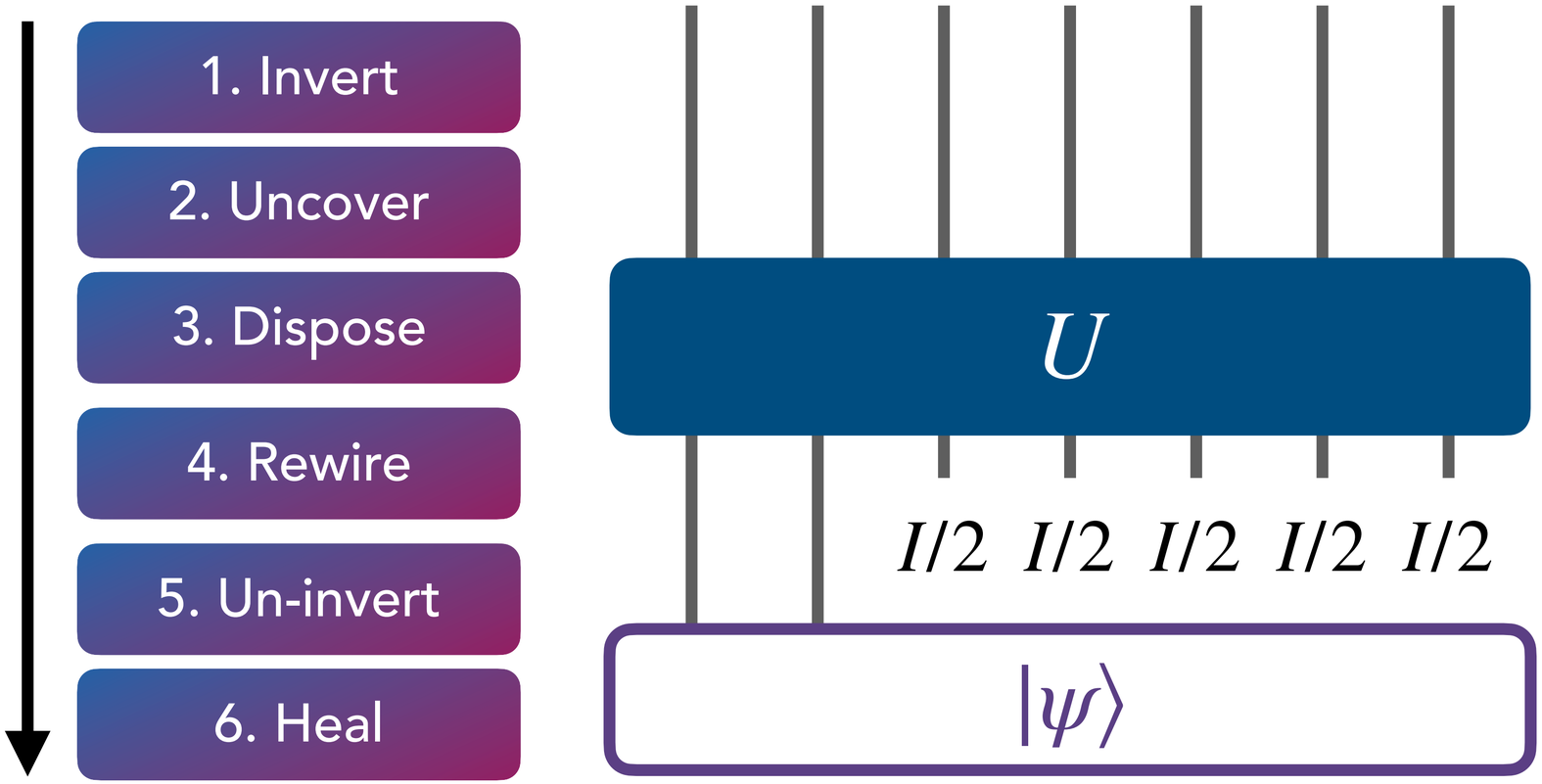


$2n - 1$ qubits

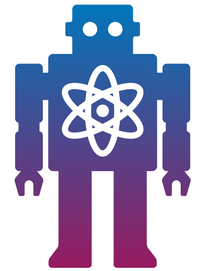


Sewing local inversions

- Given local inversions $V_1, \ldots, V_n$ of U on each qubit.
- We can sew them together to form the n-qubit unitary U.

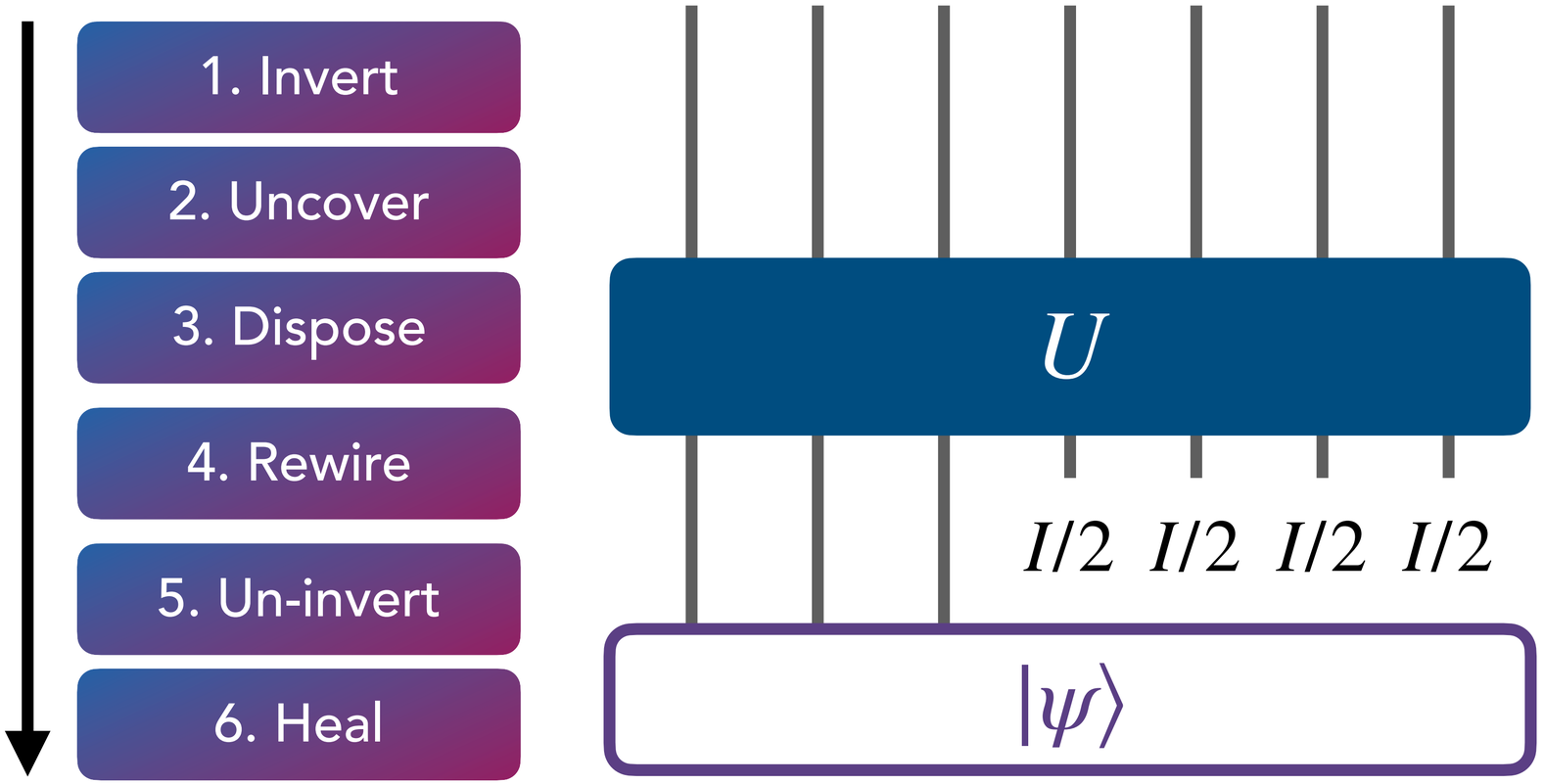


$2n - 2$ qubits

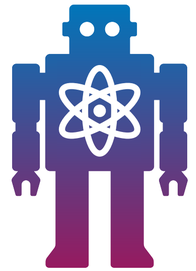


Sewing local inversions

- Given local inversions $V_1, \ldots, V_n$ of U on each qubit.
- We can sew them together to form the n-qubit unitary U.

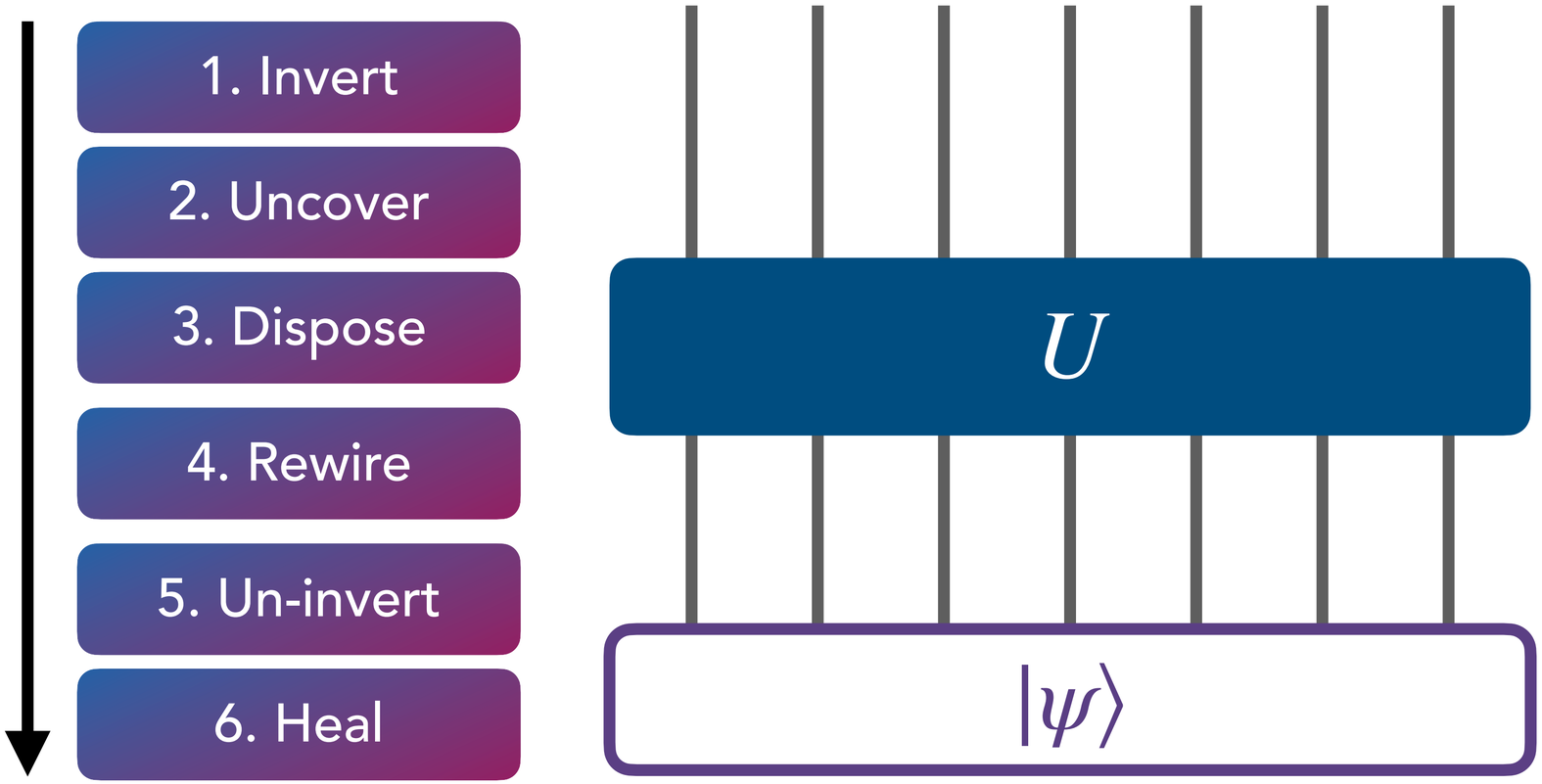


$2n - 3$ qubits



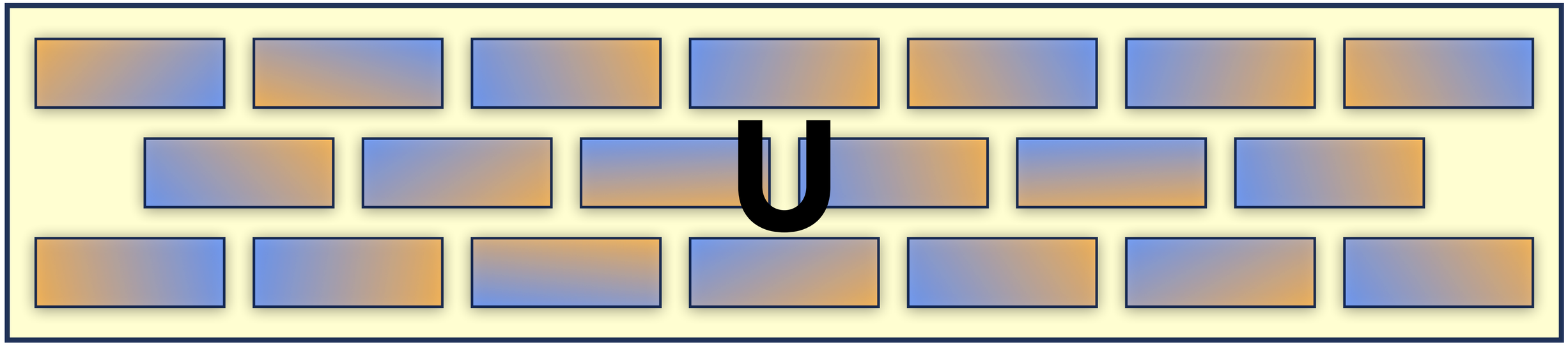
Sewing local inversions

- Given local inversions $V_1, \ldots, V_n$ of U on each qubit.
- We have sewn them together to form the n-qubit unitary U.



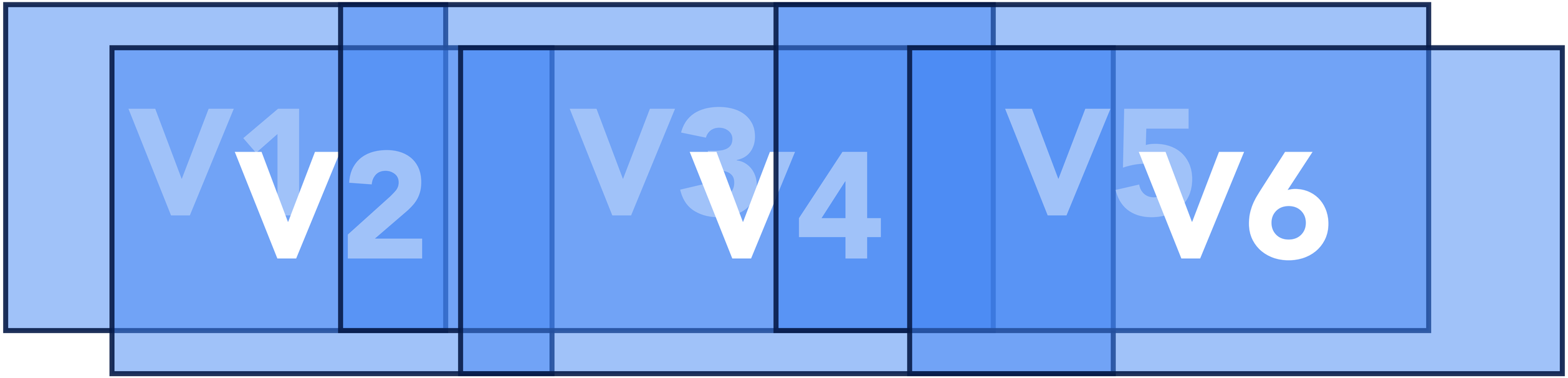
Key Algorithmic Idea

To train/learn an n-qubit QNN U,



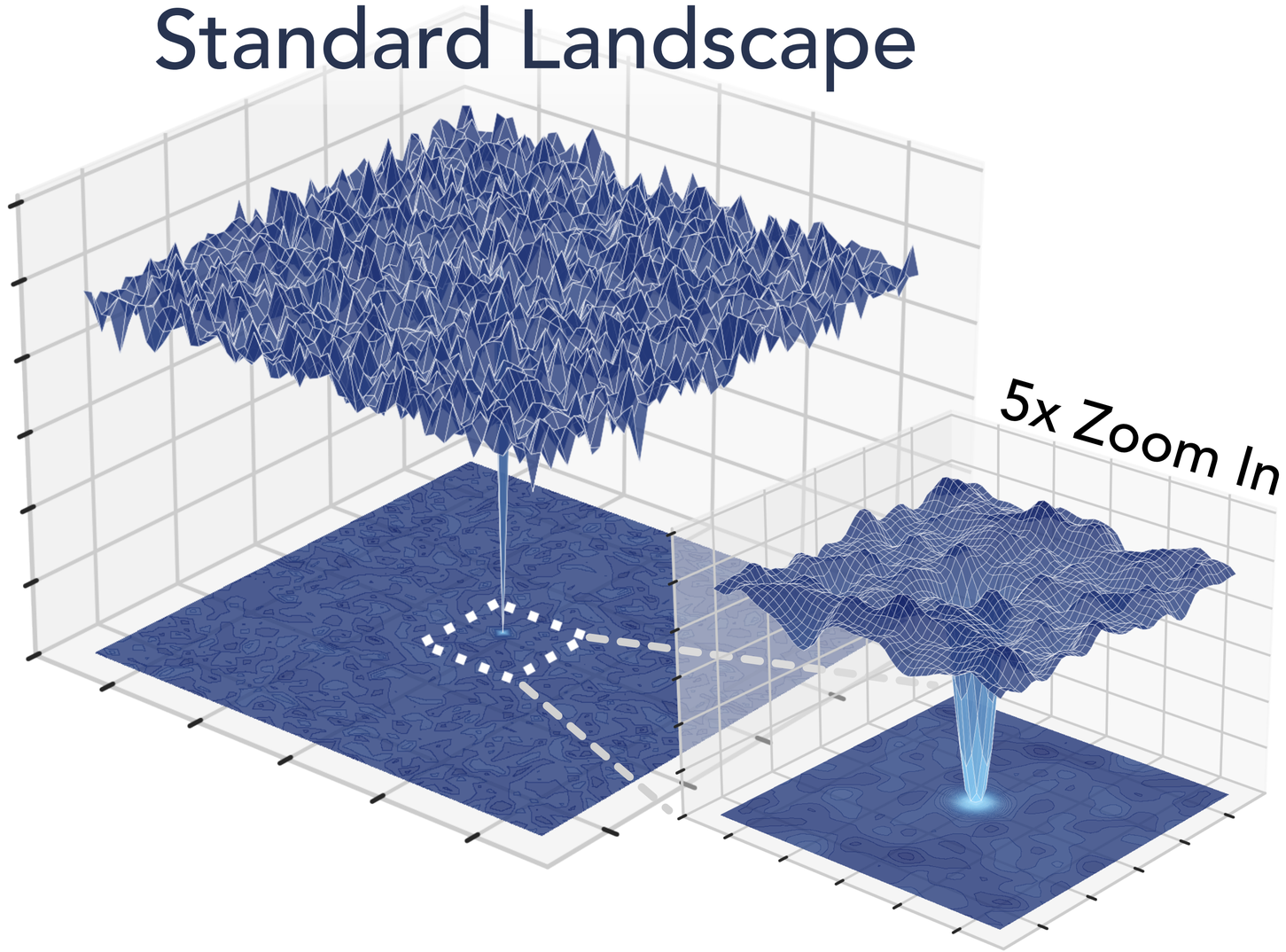
Key Algorithmic Idea

To train/learn an n-qubit QNN U,
train/learn the local inversions V1, …, Vn instead.



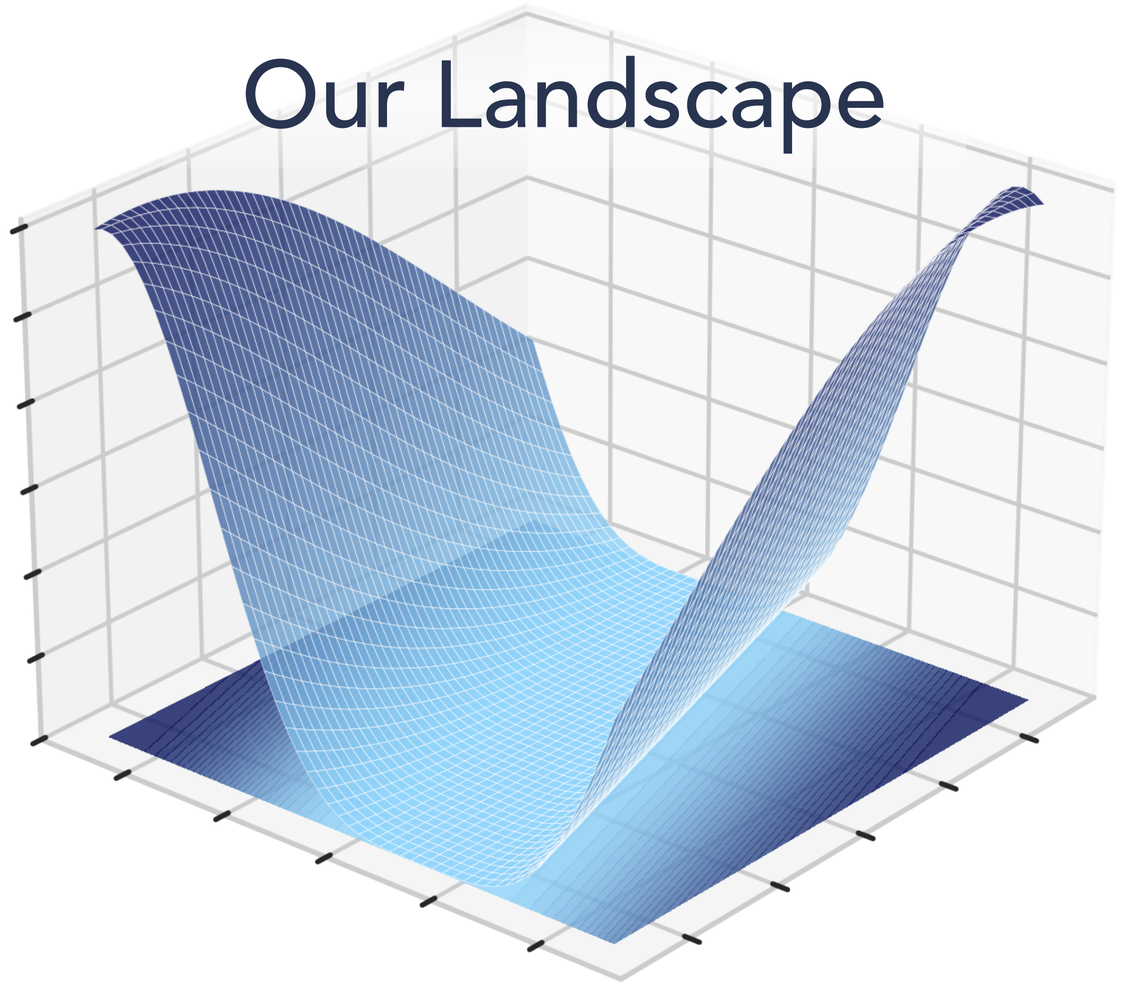
Optimization Landscape

When we learn a shallow QNN by optimizing parameterized quantum gates

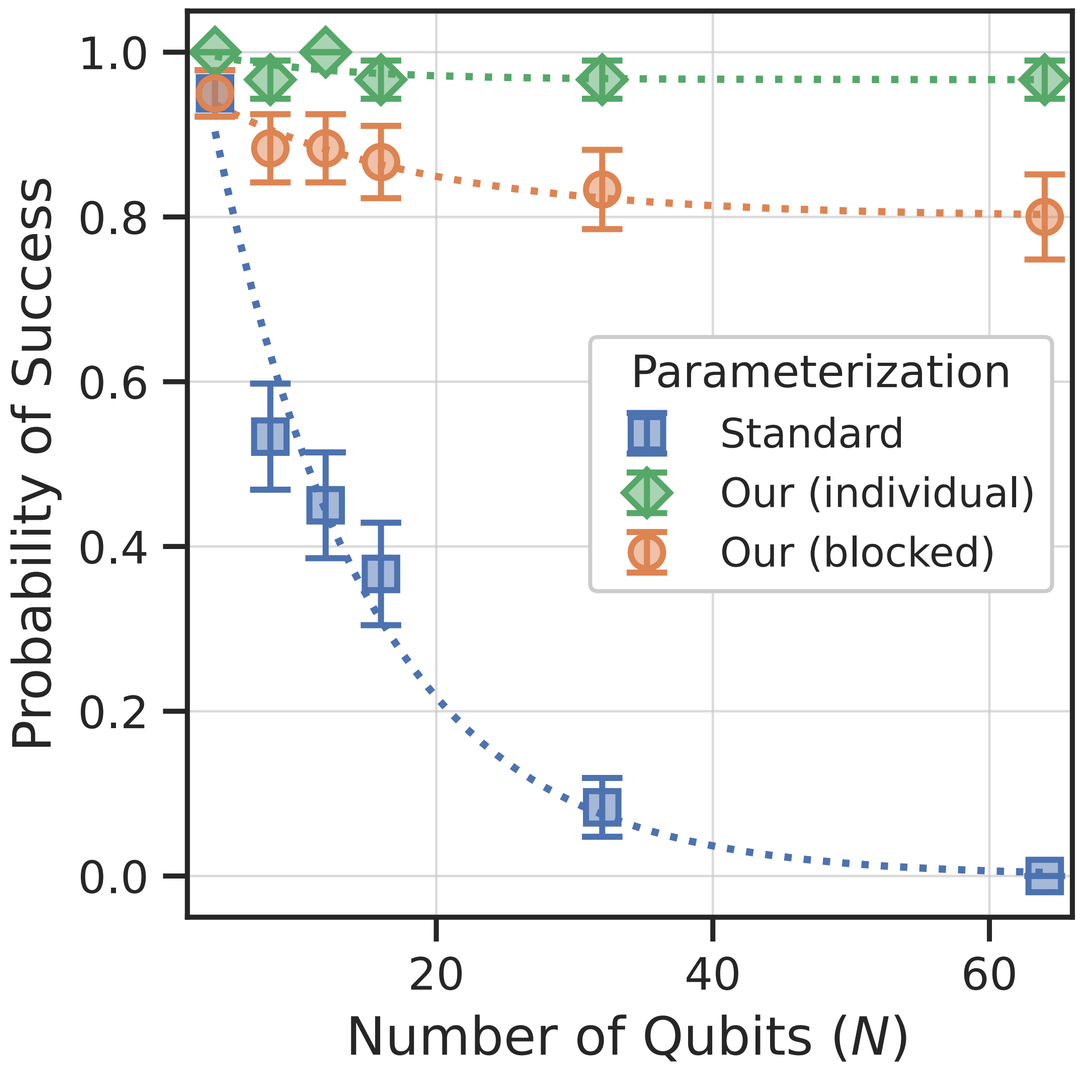


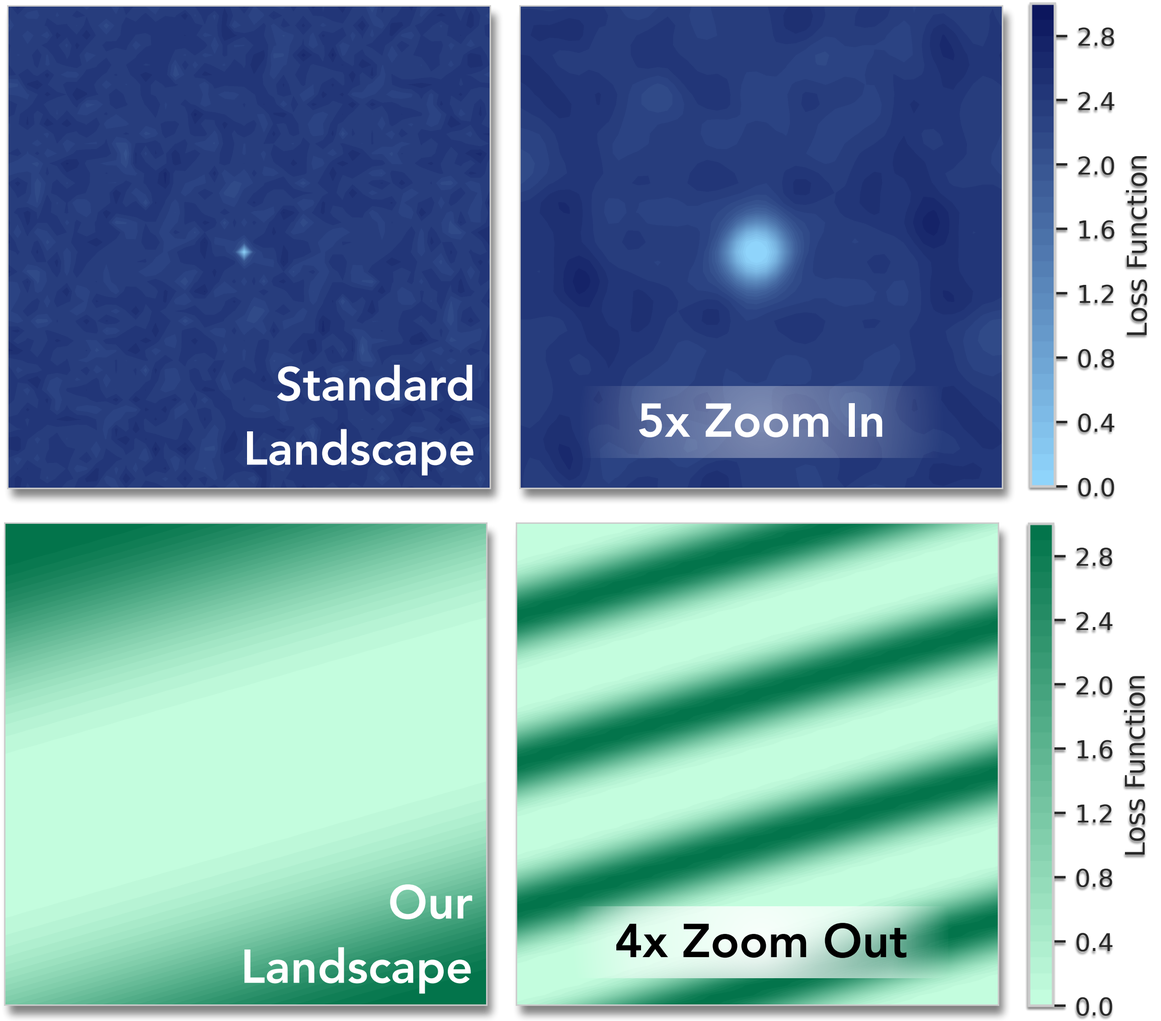
Optimization Landscape

When we learn a shallow QNN by optimizing local inversions of the QNN



Optimization Performance





Provably Efficient Learning

Theorem

Any n-qubit shallow QNN can be learned to ε error in **poly**$(n, 1/\varepsilon)$ **time**.