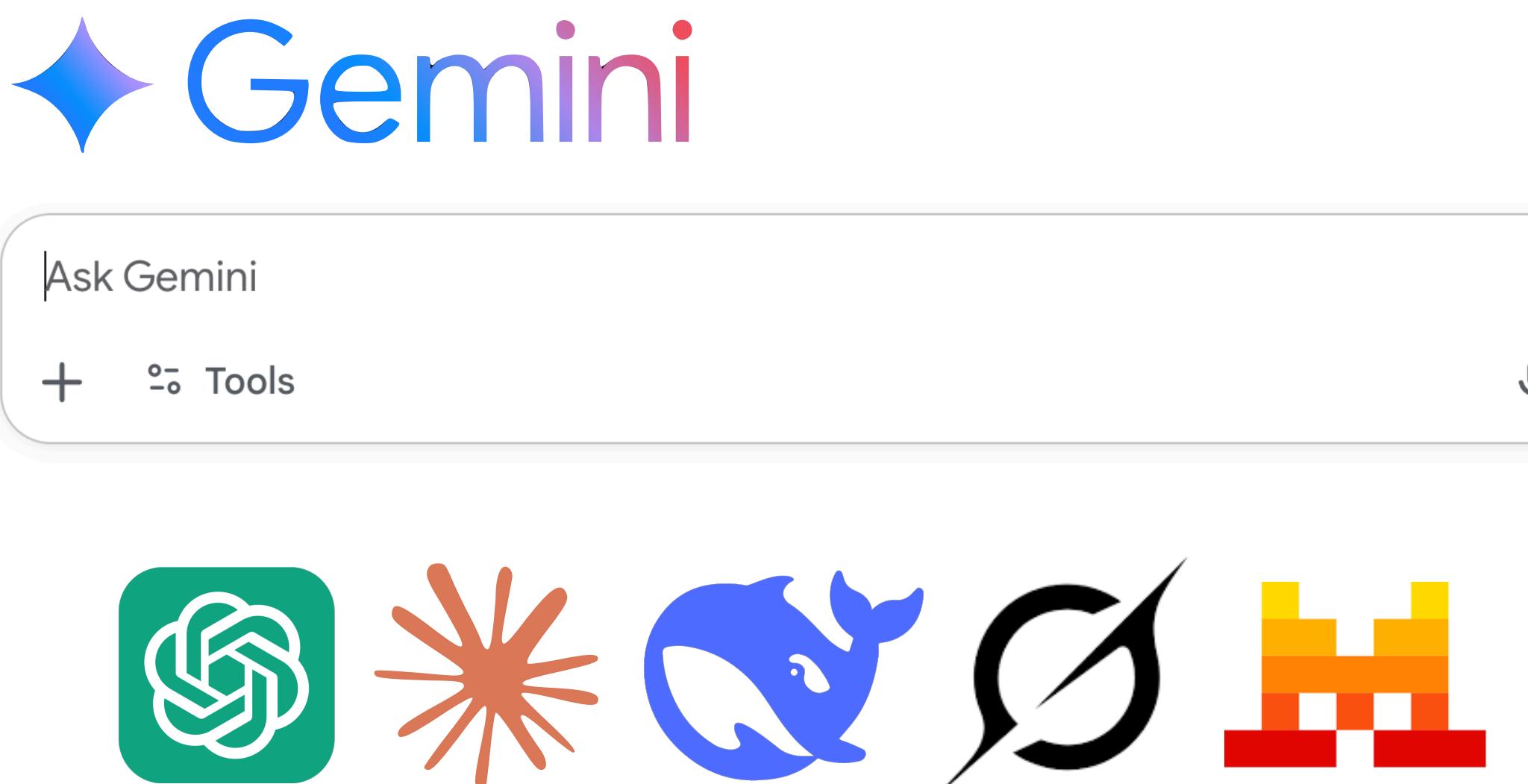


Ph 220: Lecture 12

Generative Quantum Advantage

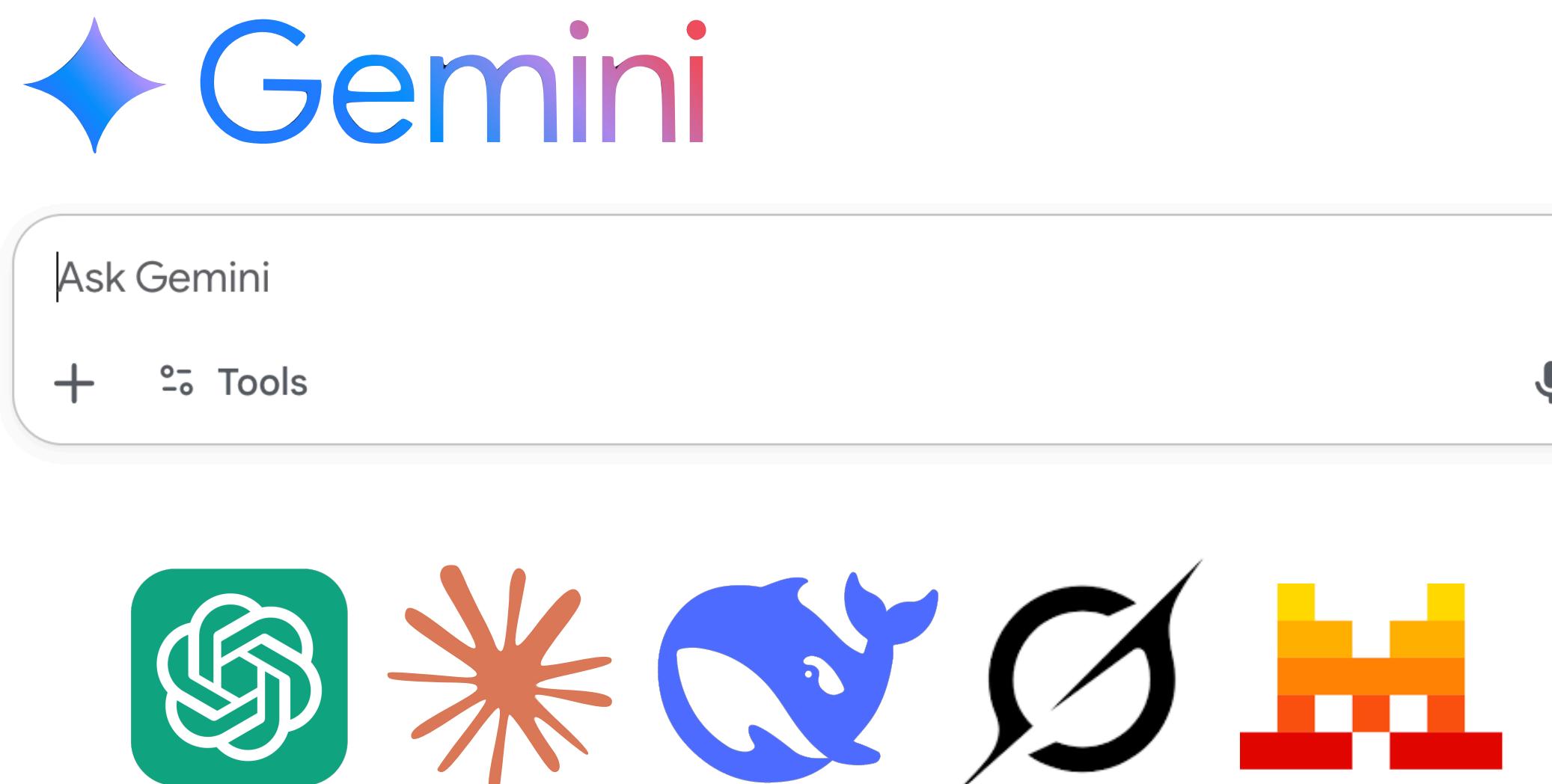
Generative Models

- **Definition (informal): Generative model** — A model that can learn to generate new outputs from some probability distribution.



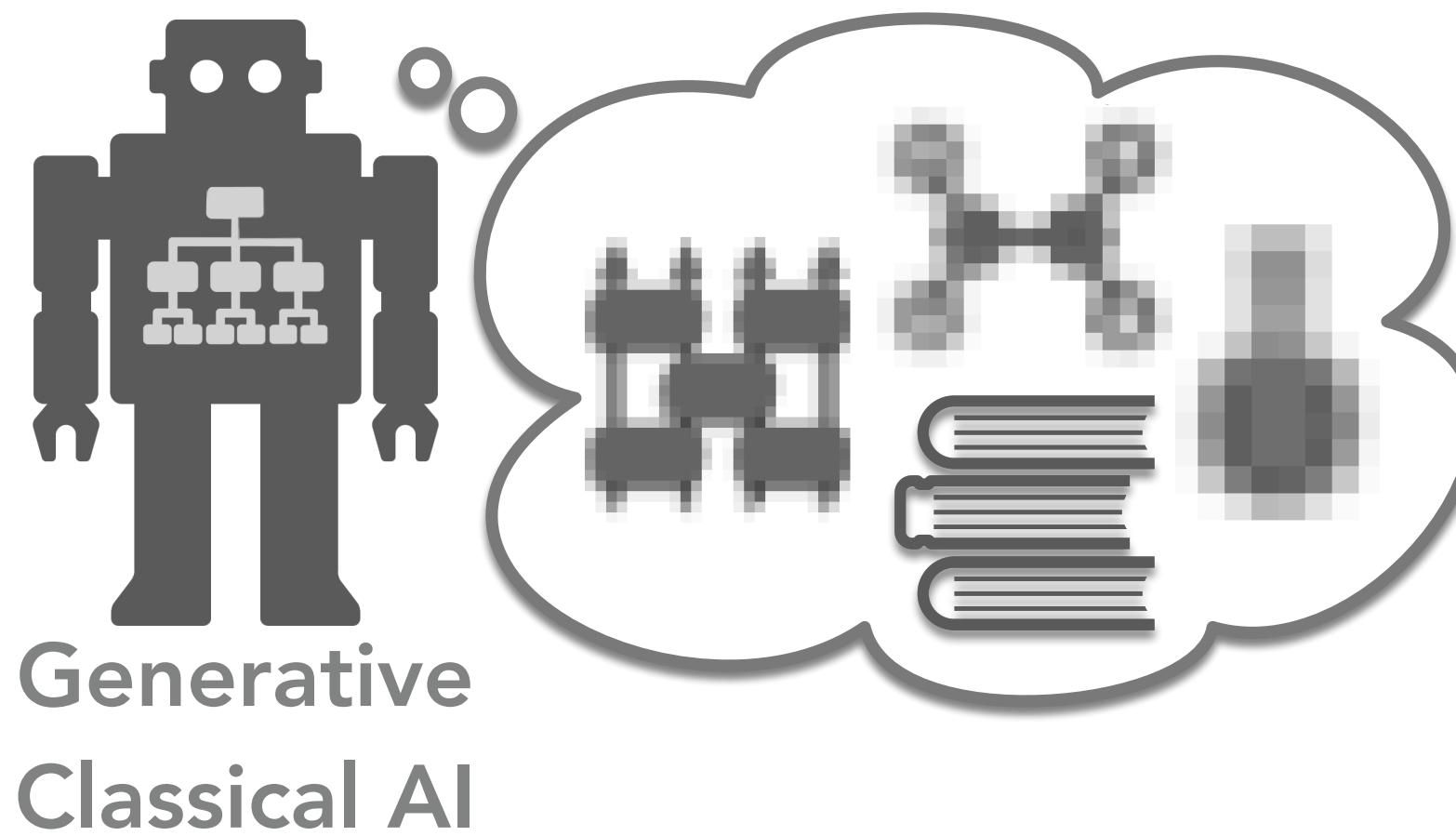
Generative Task

- **Definition:** Given a dataset of (x_i, y_i) sampled from unknown $p(y|x)$, learn to generate **new y** for **any given x** .

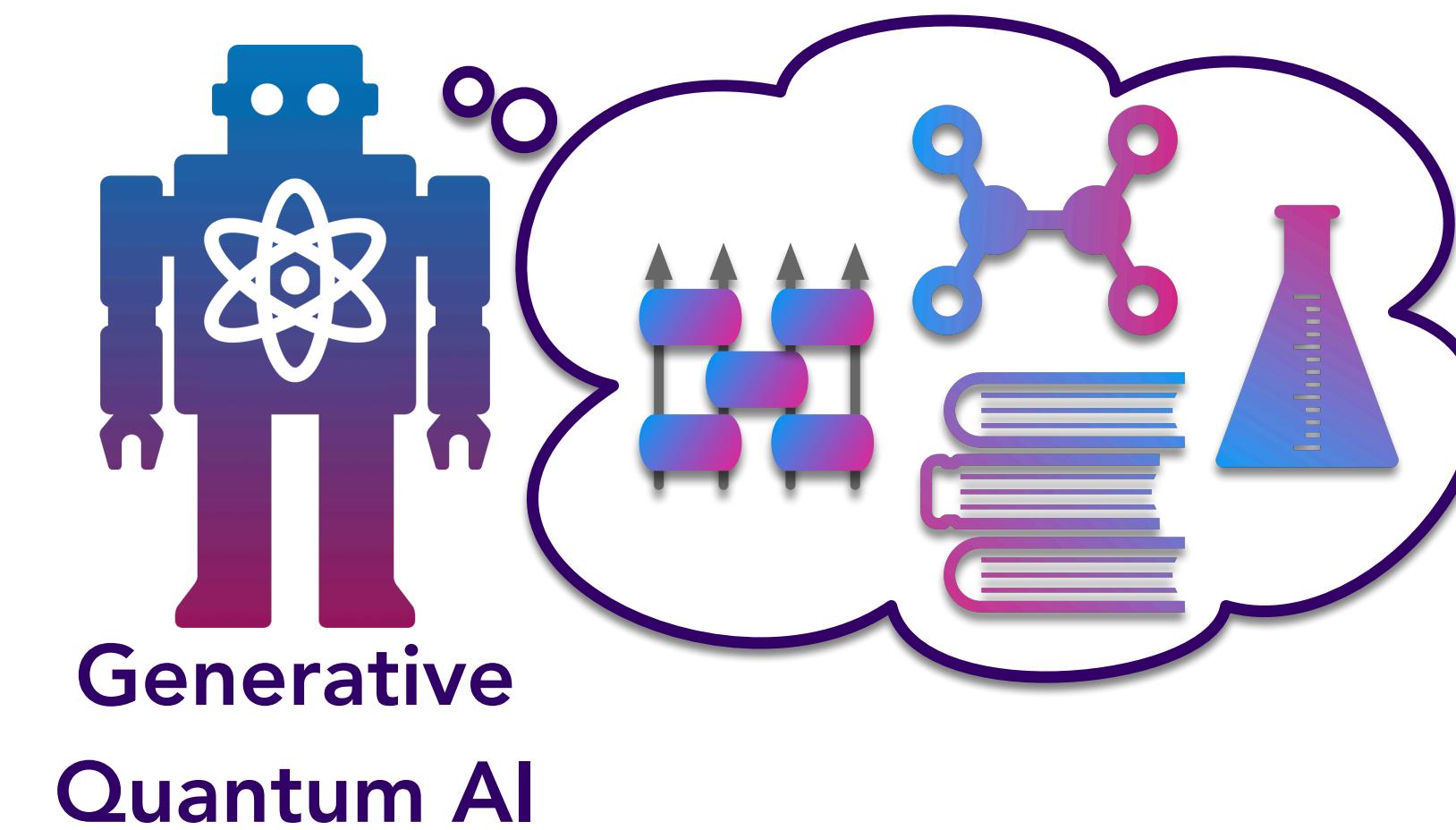


Generative Quantum Advantage

- Definition (informal): **Generative quantum advantage** — A quantum computer can learn to generate the desired outputs

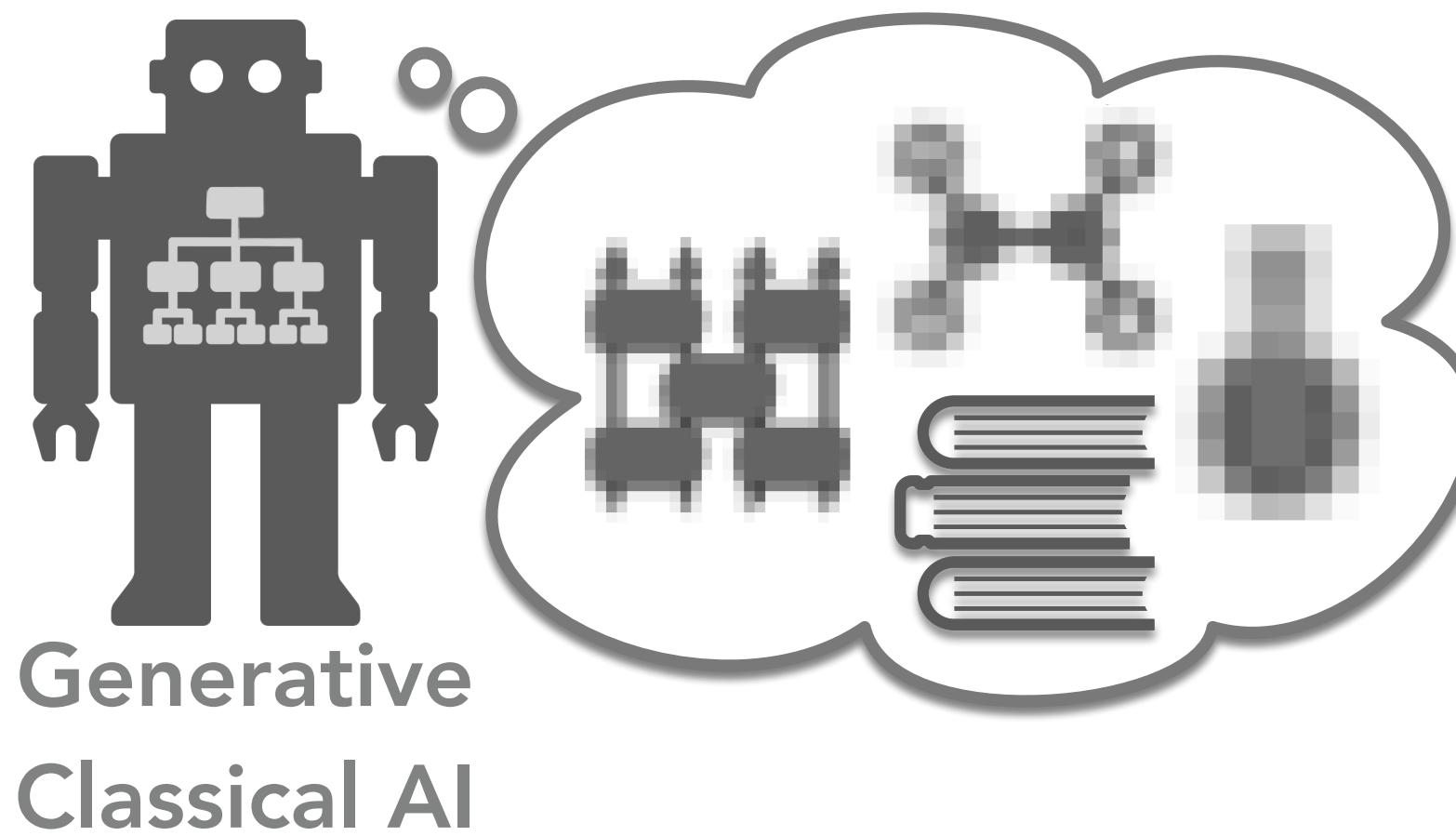


vs

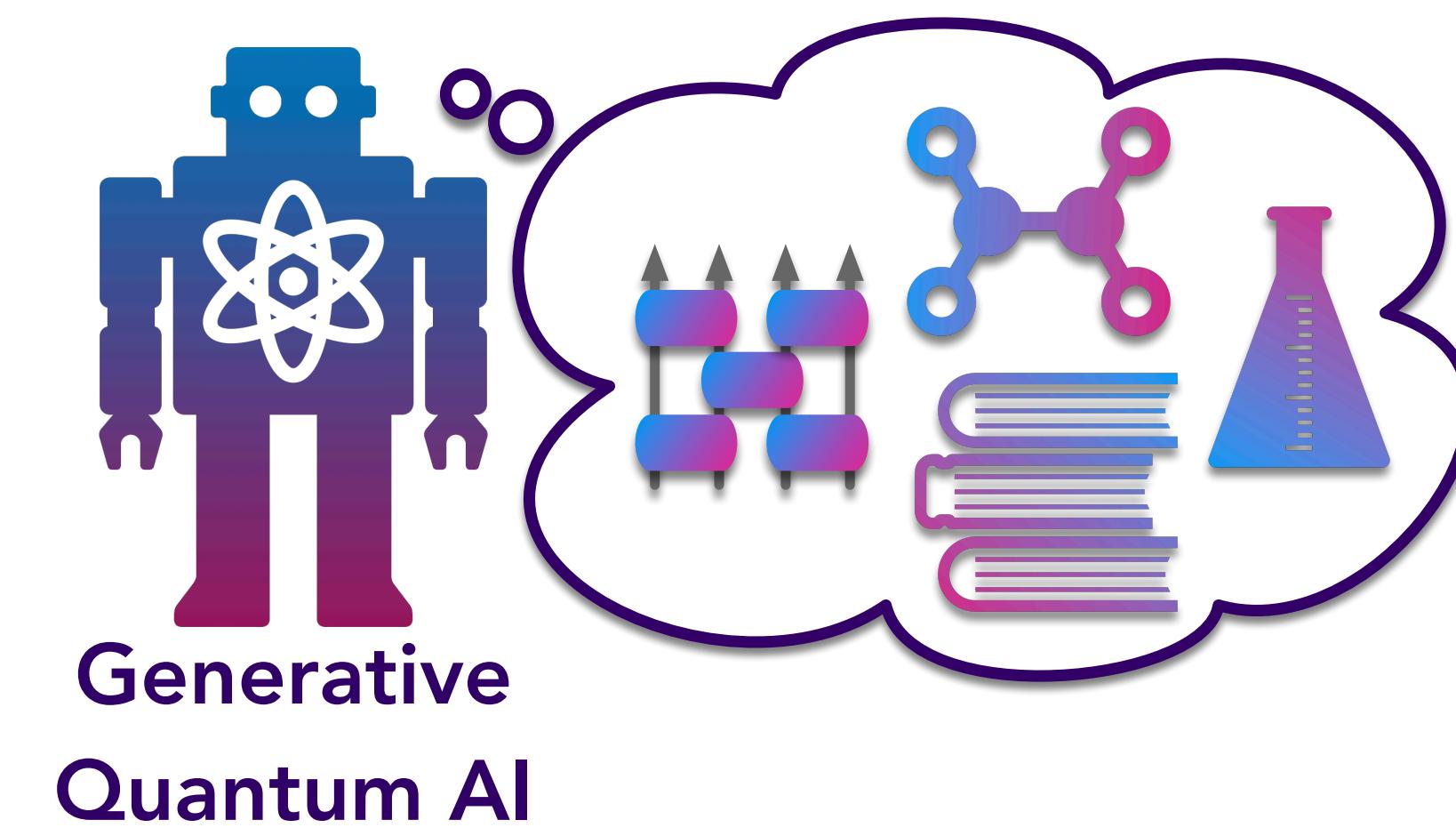


Generative Quantum Advantage

- Definition (informal): **Generative quantum advantage** — A quantum computer can learn to generate the desired outputs with **reduced** sample complexity

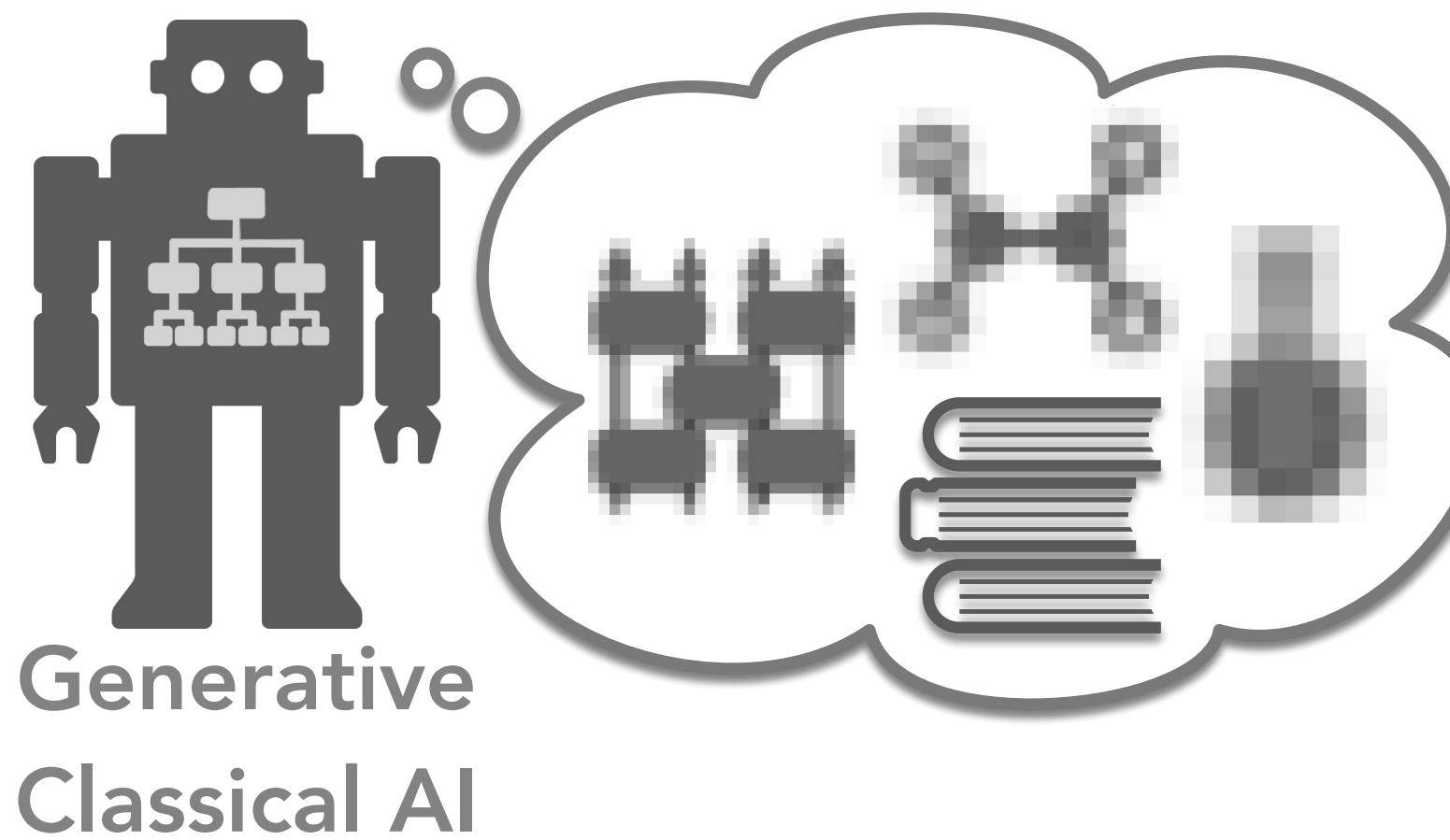


vs

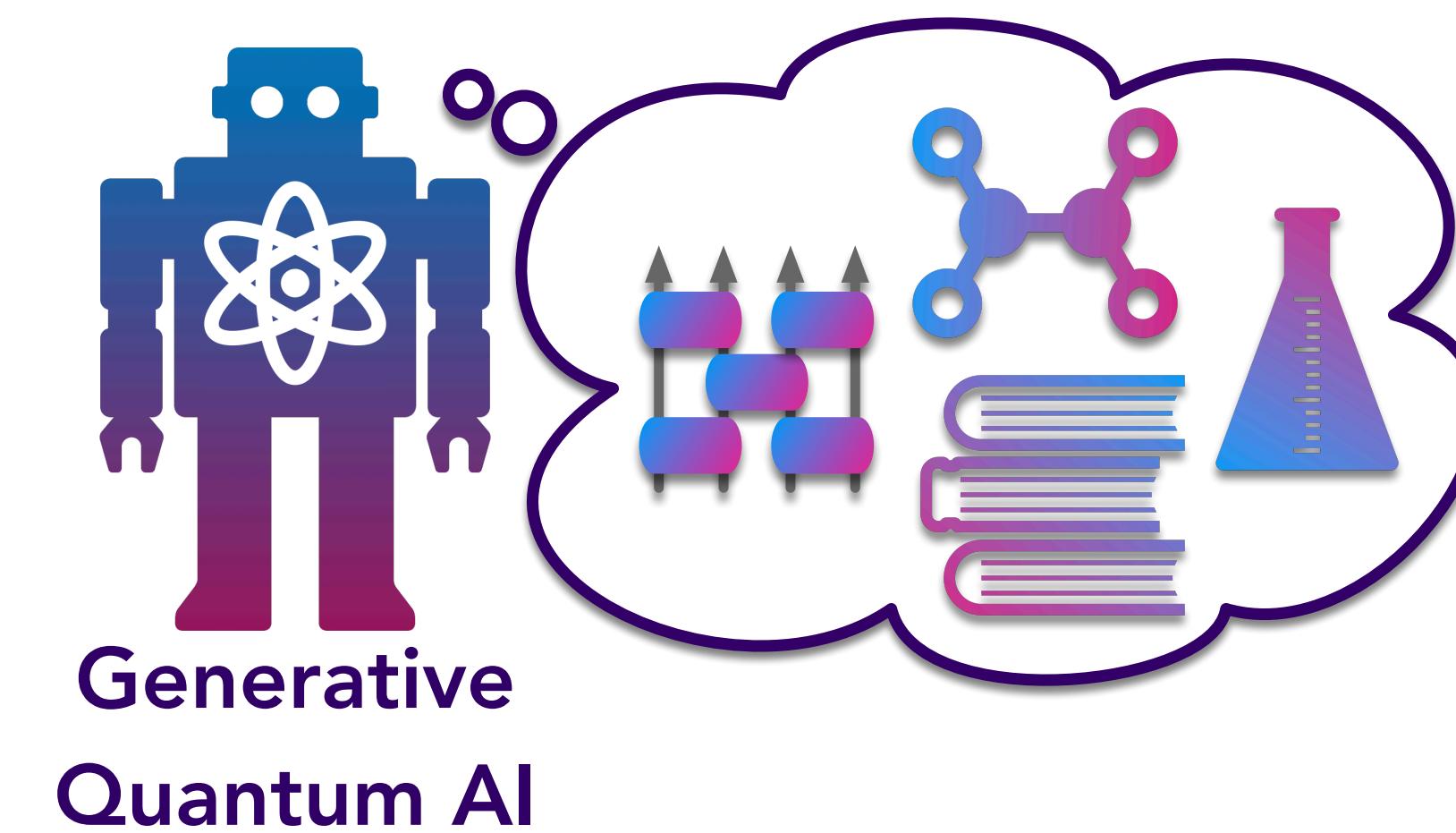


Generative Quantum Advantage

- Definition (informal): **Generative quantum advantage** — A quantum computer can learn to generate the desired outputs with **reduced sample complexity, higher accuracy**,

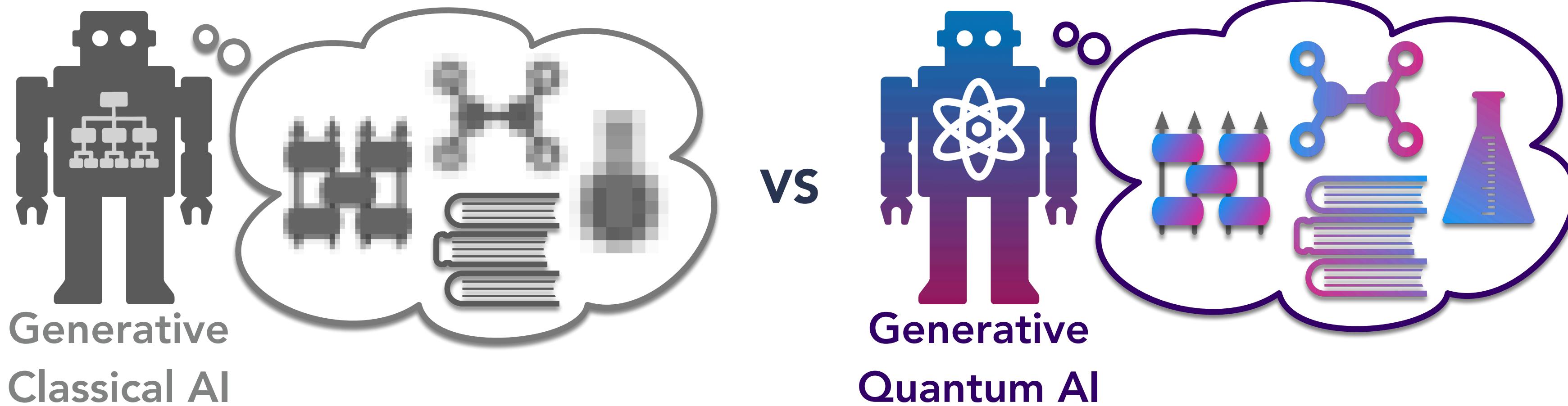


vs



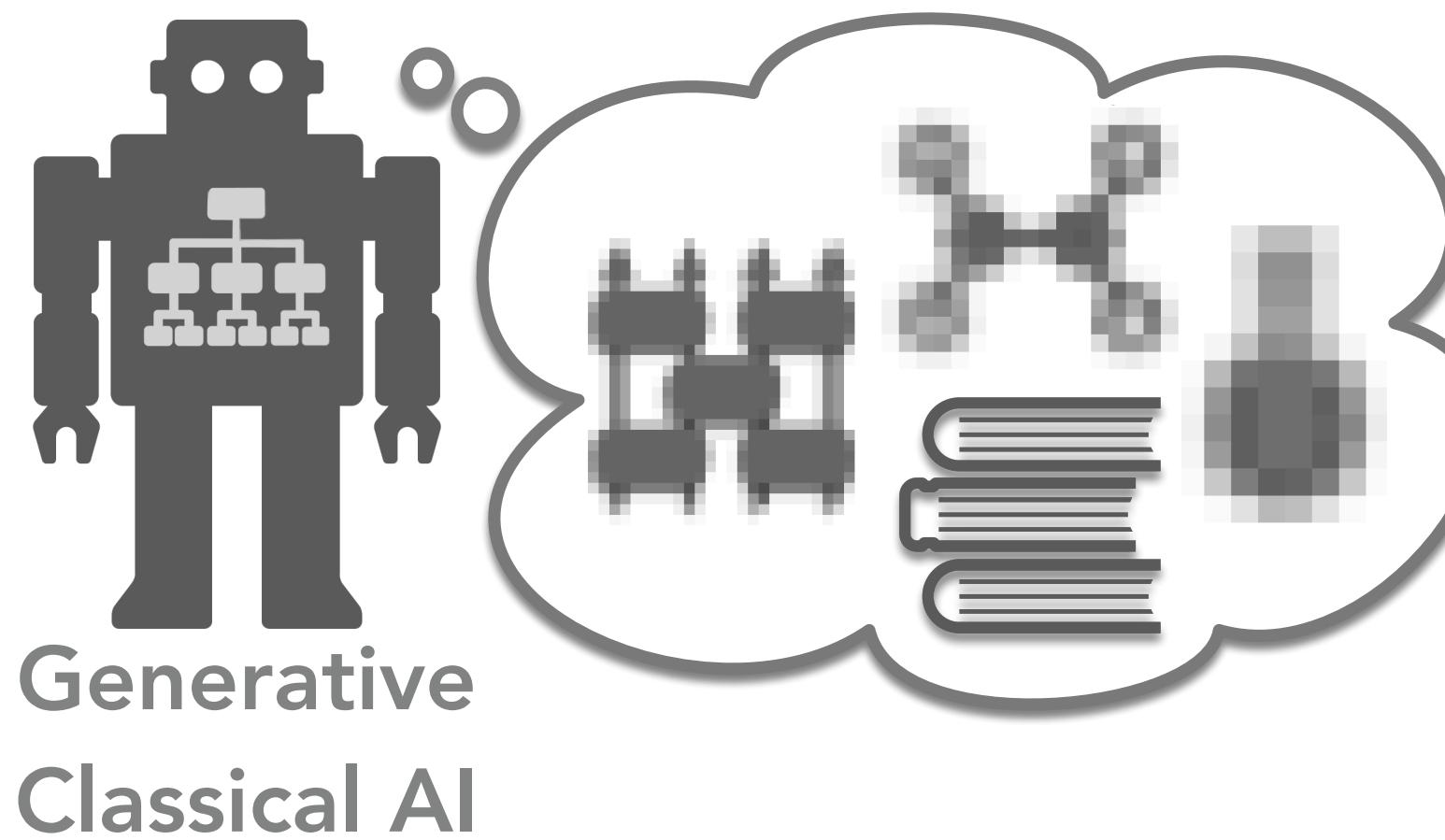
Generative Quantum Advantage

- Definition (informal): **Generative quantum advantage** — A quantum computer can learn to generate the desired outputs with **reduced** sample complexity, **higher** accuracy, **faster** learning and/or generation time,

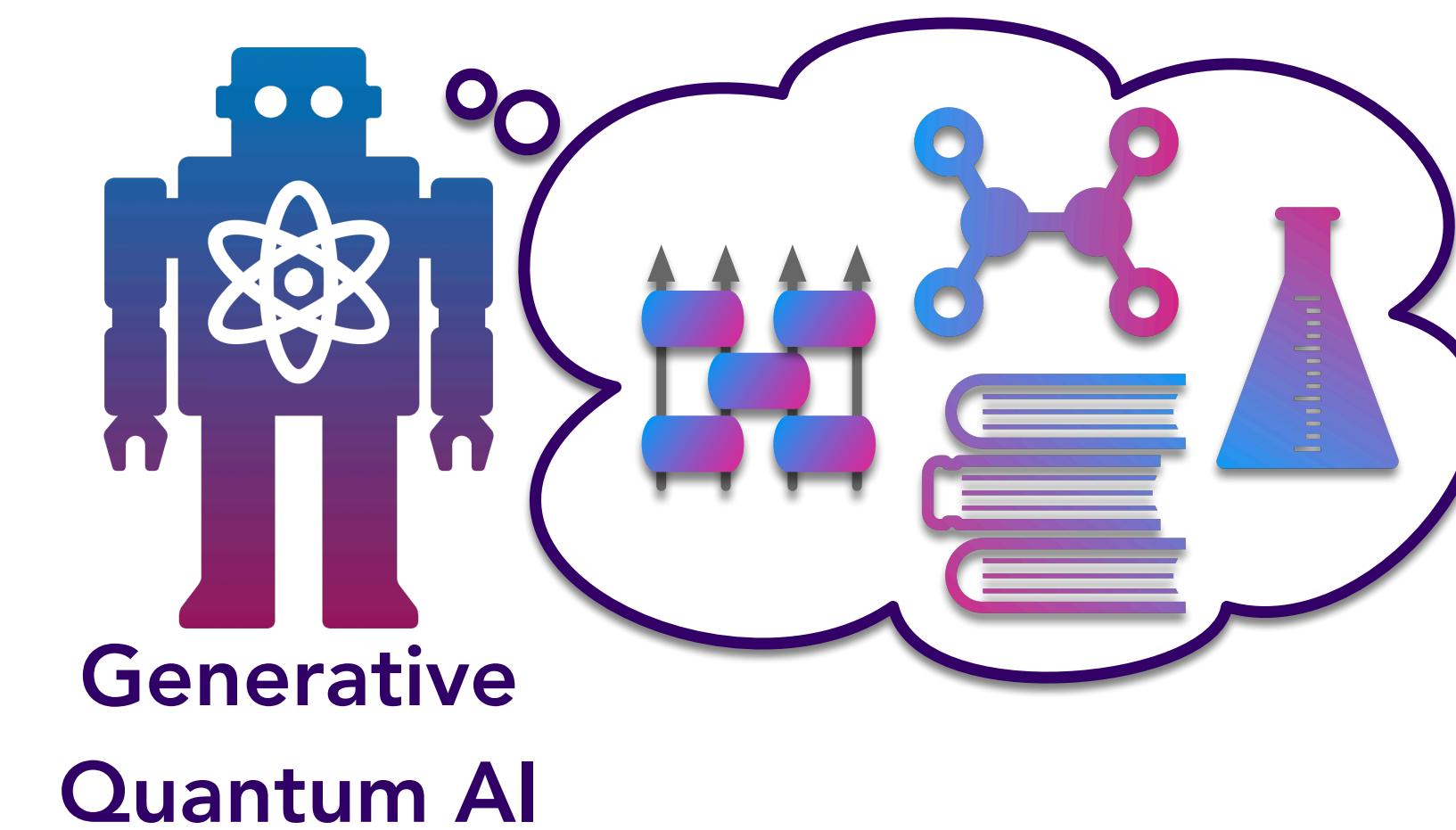


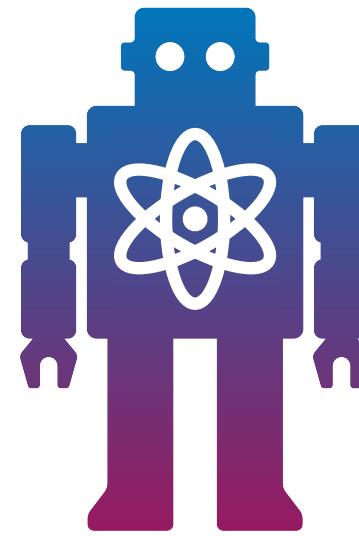
Generative Quantum Advantage

- Definition (informal): **Generative quantum advantage** — A quantum computer can learn to generate the desired outputs with **reduced** sample complexity, **higher** accuracy, **faster** learning and/or generation time, or outputs infeasible for classical computers.



vs

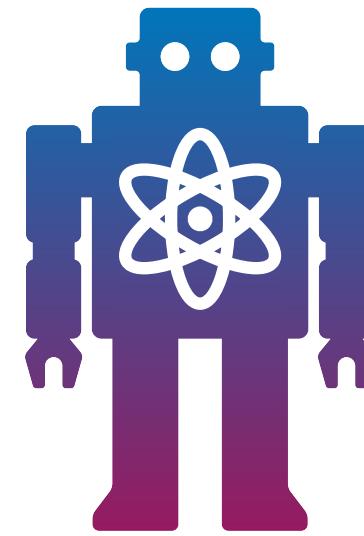




Main Question

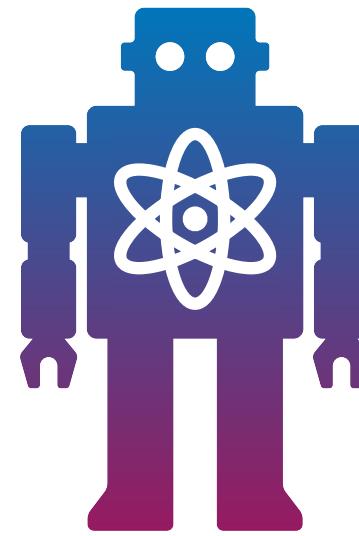
Are there families of unknown distributions $p(y|x)$ mapping classical inputs to classical outputs such that:

- Quantum computers can **efficiently learn** from few samples;
- Quantum computers can **efficiently generate** new outputs;
- Classical computers **cannot efficiently generate** new outputs?



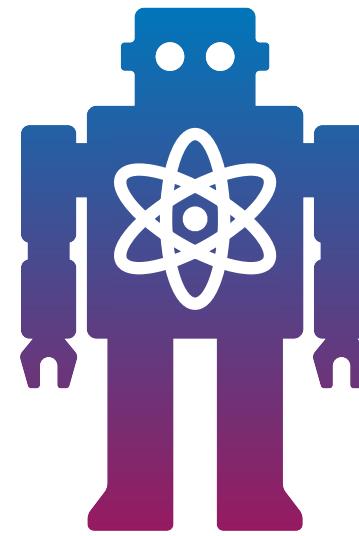
Generative QNNs

- **Task:** Given a dataset of (x_i, y_i) sampled from unknown $p(y|x)$, learn to generate **new y** for **any given x** .



Generative QNNs

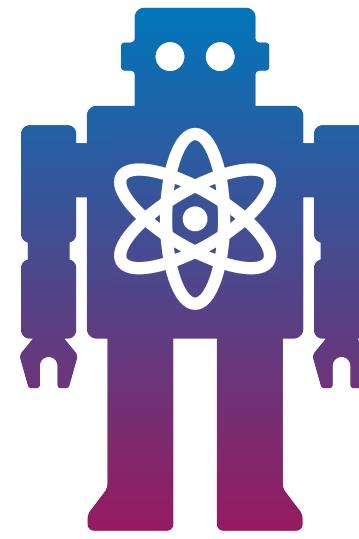
- **Task:** Given a dataset of (x_i, y_i) sampled from unknown $p(y|x)$, learn to generate **new y** for **any given x** .
- **Generative Quantum Neural Networks (QNNs):**
 1. Encode x into quantum state $|\psi_x\rangle$ and measurement basis M_x .
 2. Apply trainable quantum circuit C_β to the state $|\psi_x\rangle$.
 3. Measure $C_\beta|\psi_x\rangle$ in the basis M_x to sample y from $p(y|x; \beta)$.



Main Question

Are there families of unknown distributions $p(y|x)$ mapping classical inputs to classical outputs such that:

- Quantum computers can **efficiently learn** from few samples;
- Quantum computers can **efficiently generate** new outputs;
- Classical computers **cannot efficiently generate** new outputs?



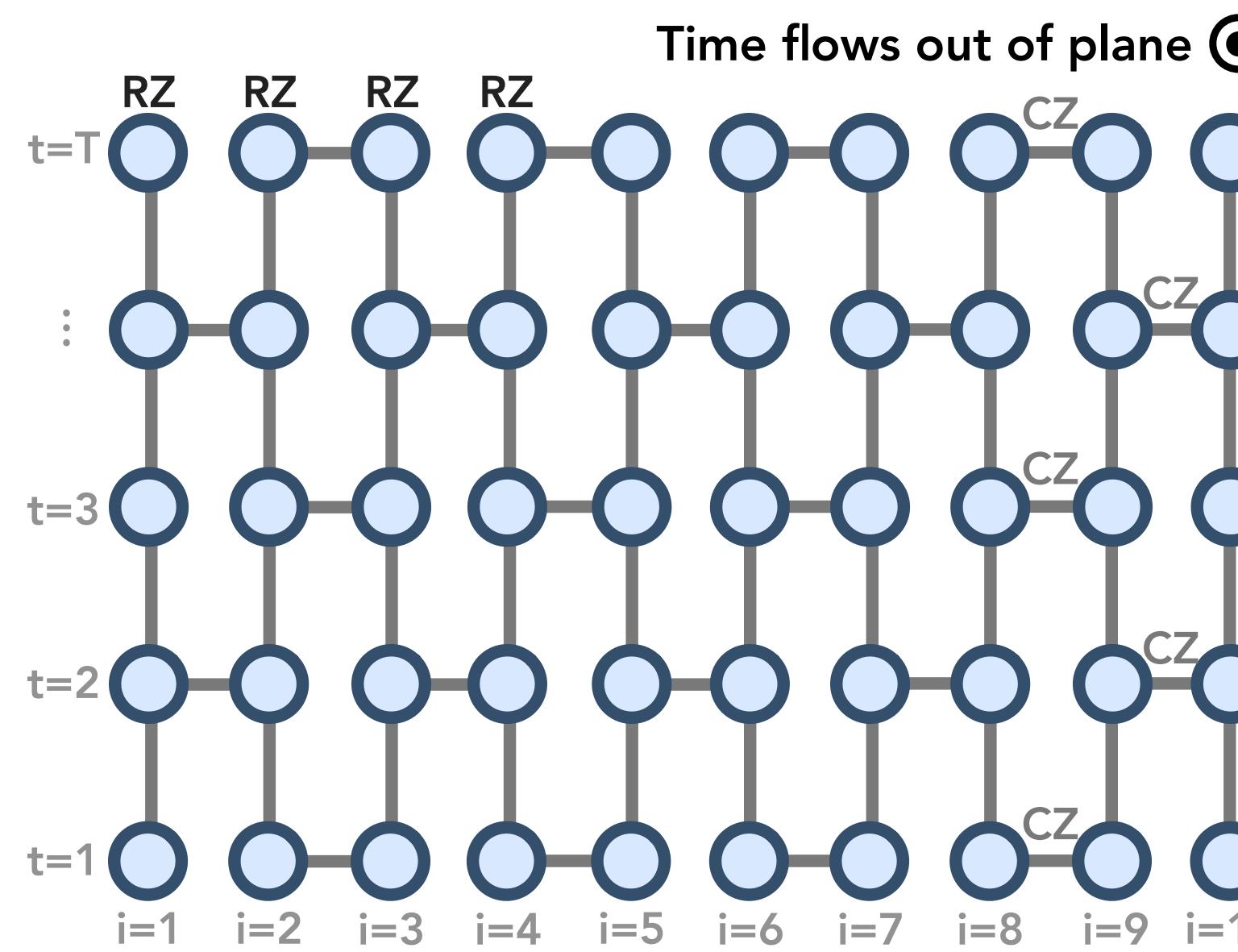
Main Question

Are there families of unknown distributions $p(y|x)$ mapping classical inputs to classical outputs such that:

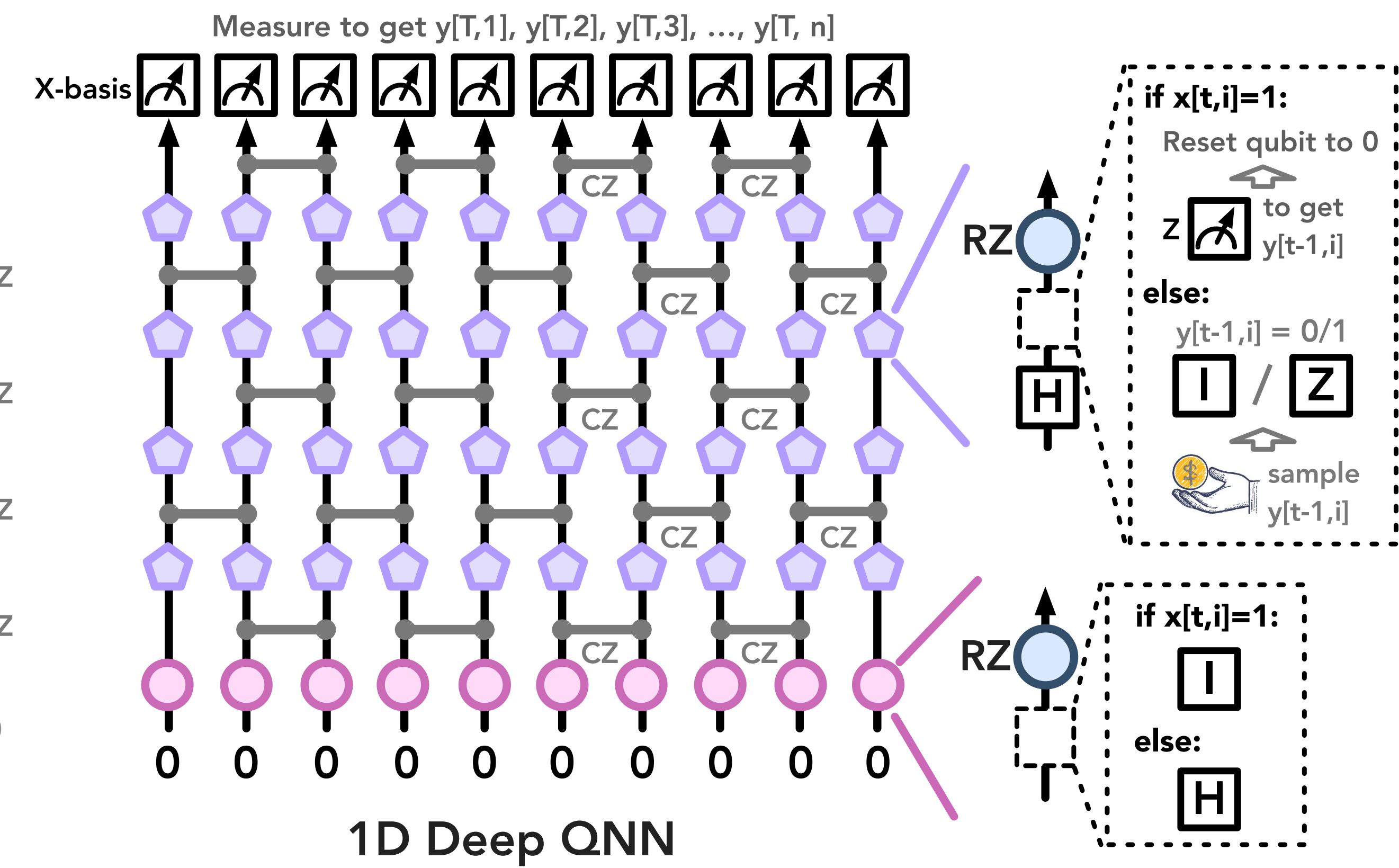
- Quantum computers can **efficiently learn** from few samples;
-  Quantum computers can **efficiently generate** new outputs;
- Classical computers **cannot efficiently generate** new outputs?

Computational Power

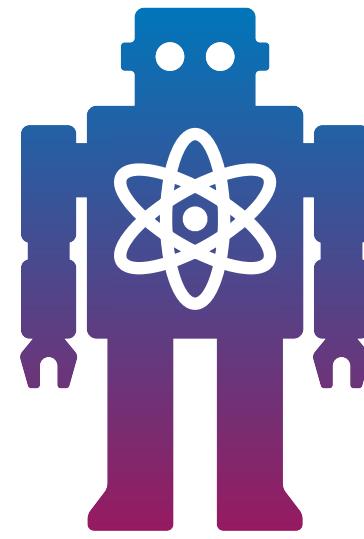
- Under standard conjectures, there are **shallow QNNs** that can generate distributions **no** poly-time classical algorithm \mathcal{A} can.



2D Shallow QNN



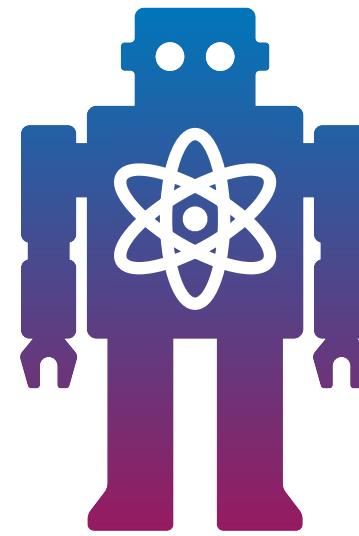
1D Deep QNN



Main Question

Are there families of unknown distributions $p(y|x)$ mapping classical inputs to classical outputs such that:

- Quantum computers can **efficiently learn** from few samples;
-  Quantum computers can **efficiently generate** new outputs;
- Classical computers **cannot efficiently generate** new outputs?



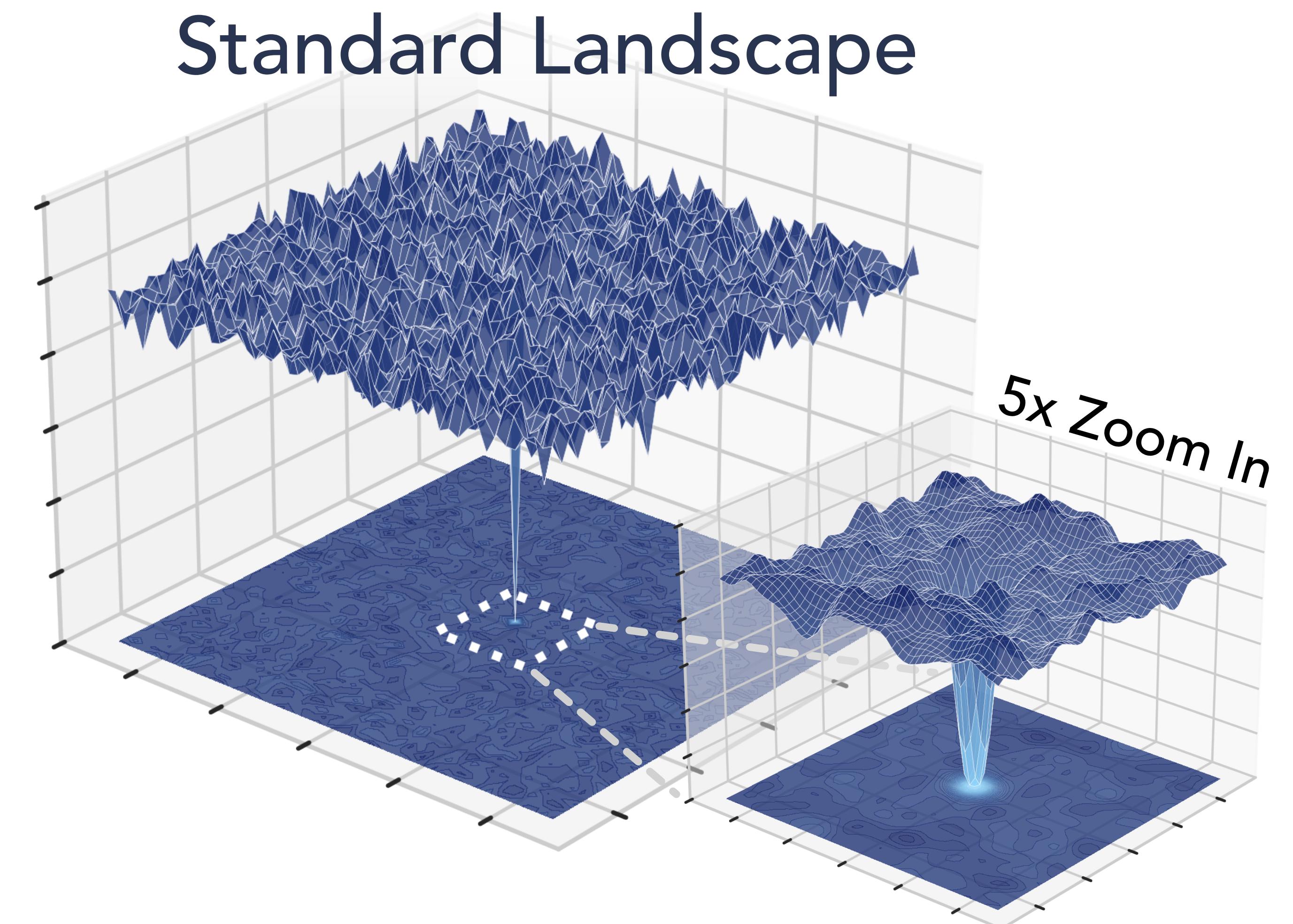
Main Question

Are there families of unknown distributions $p(y|x)$ mapping classical inputs to classical outputs such that:

- Quantum computers can **efficiently learn** from few samples;
- Quantum computers can **efficiently generate** new outputs;
- Classical computers **cannot efficiently generate** new outputs?

Bad Loss Landscape

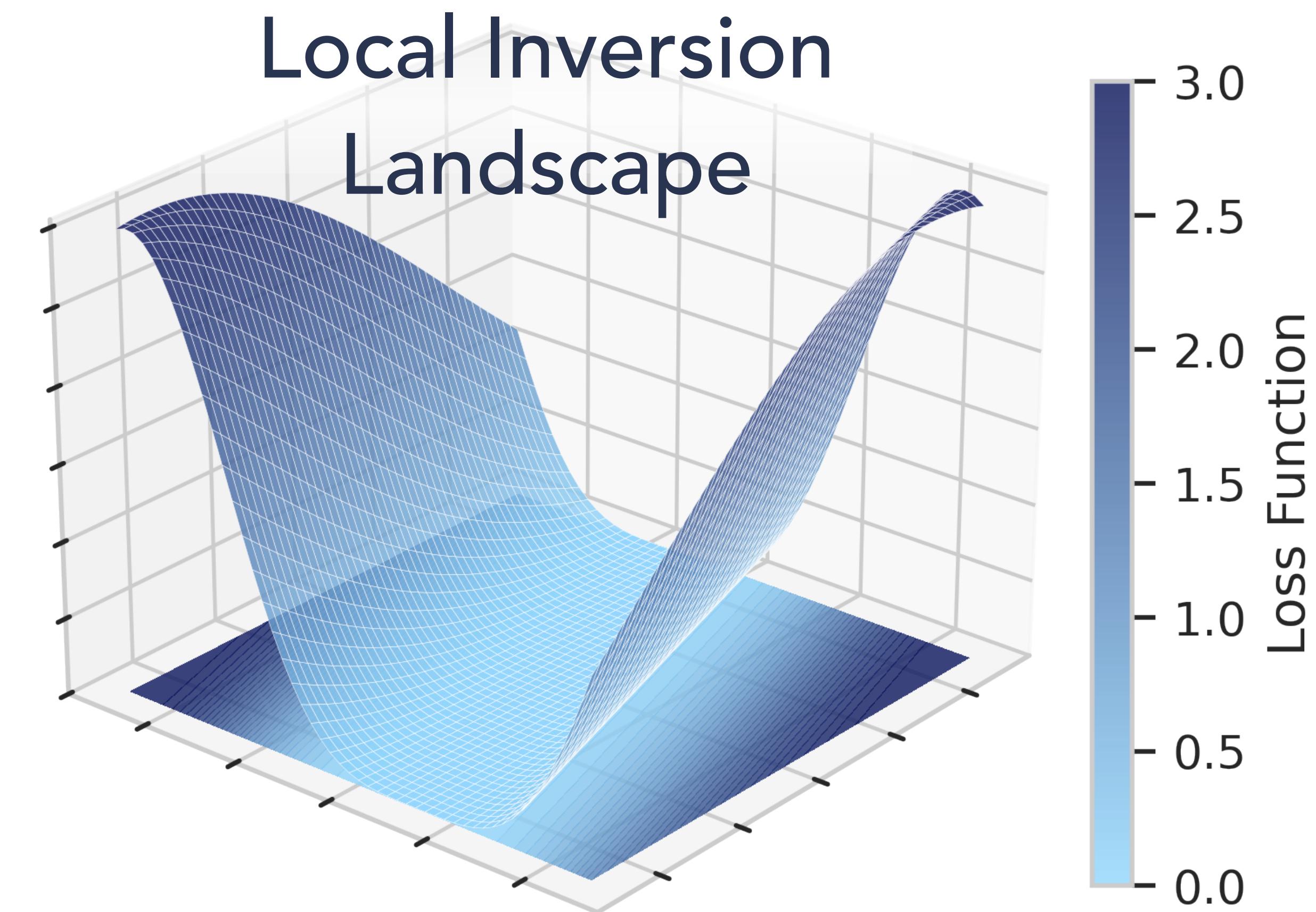
Shallow QNNs
have **extremely**
bad landscape.

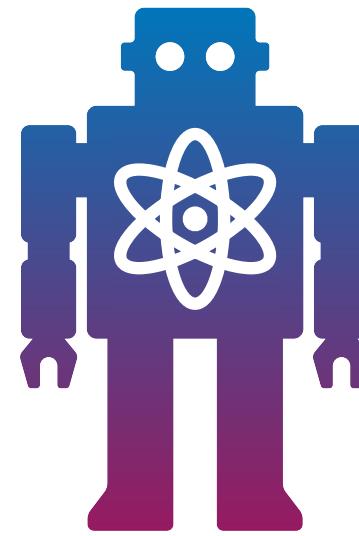


Provably Efficient Learning

Theorem

Any n -qubit shallow QNN
can be learned to ε error
in $\text{poly}(n, 1/\varepsilon)$ time.

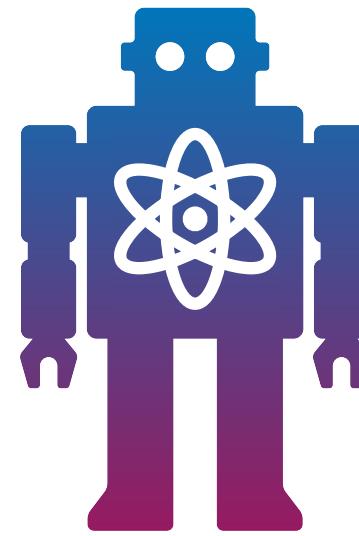




Main Question

Are there families of unknown distributions $p(y|x)$ mapping classical inputs to classical outputs such that:

- Quantum computers can **efficiently learn** from few samples;
- Quantum computers can **efficiently generate** new outputs;
- Classical computers **cannot efficiently generate** new outputs?

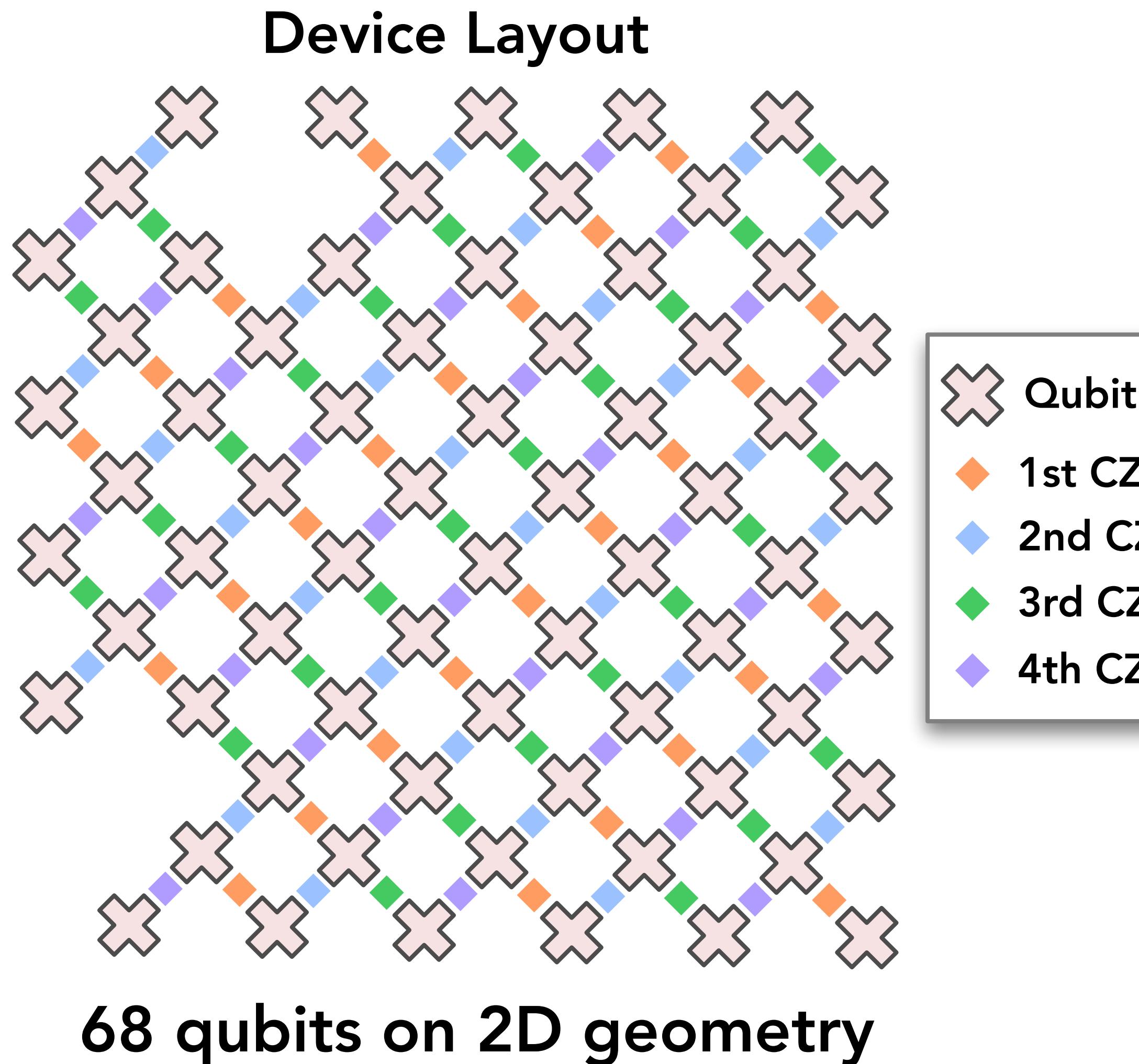


Main Question

Are there families of unknown distributions $p(y|x)$ mapping classical inputs to classical outputs such that:

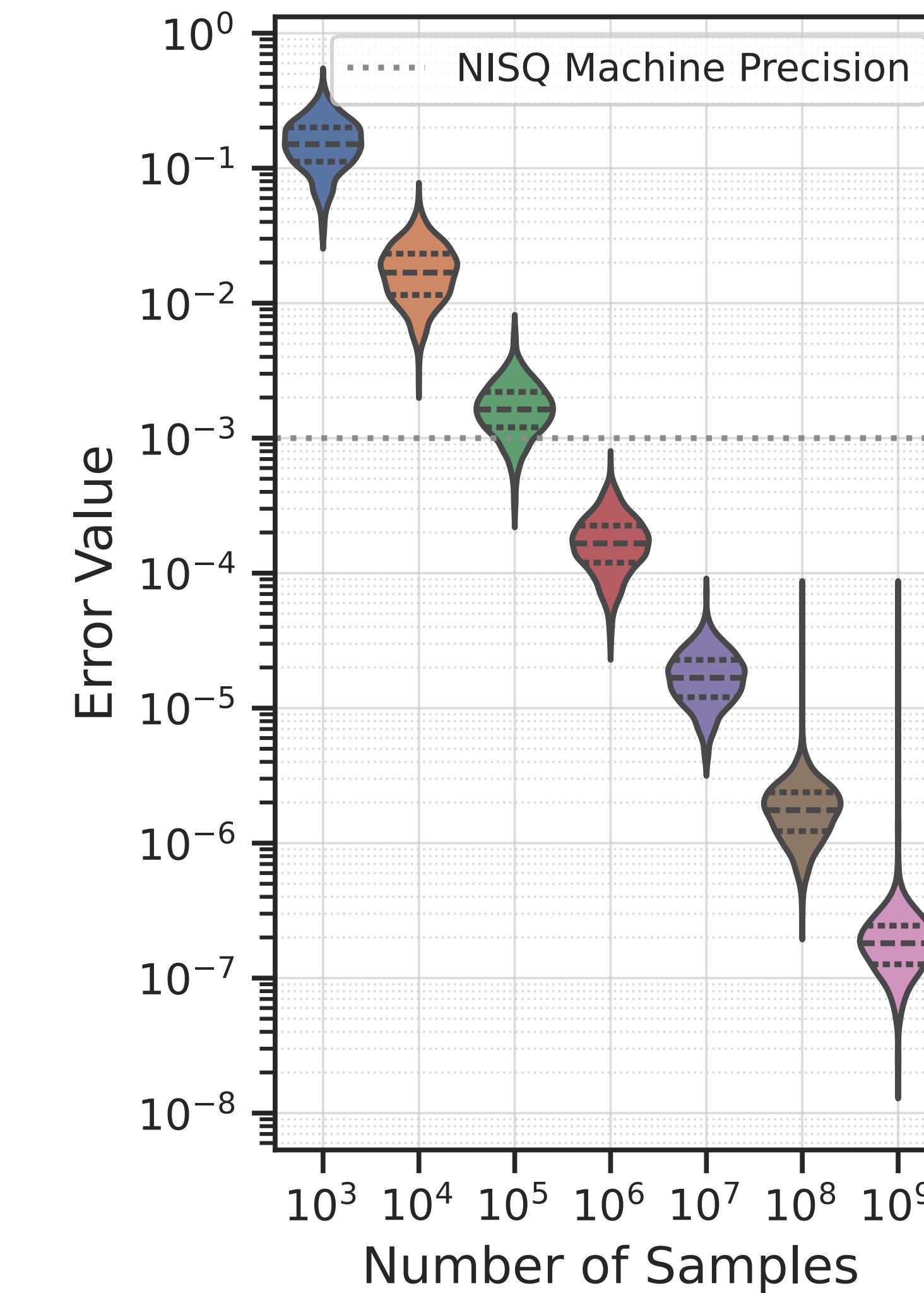
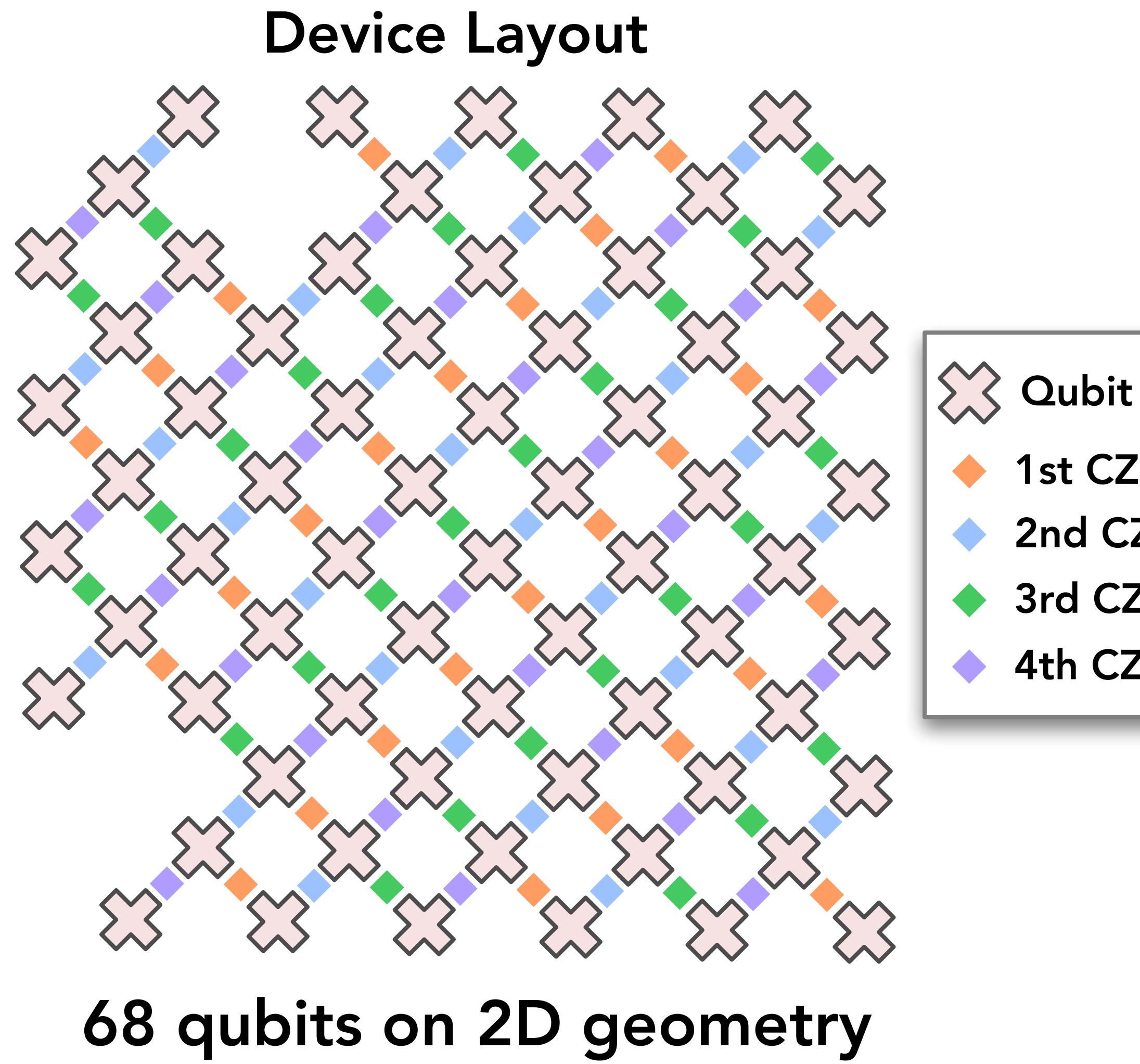
- ✓ Quantum computers can **efficiently learn** from few samples;
- ✓ Quantum computers can **efficiently generate** new outputs;
- ✓ Classical computers **cannot efficiently generate** new outputs?

Experimental Demonstration



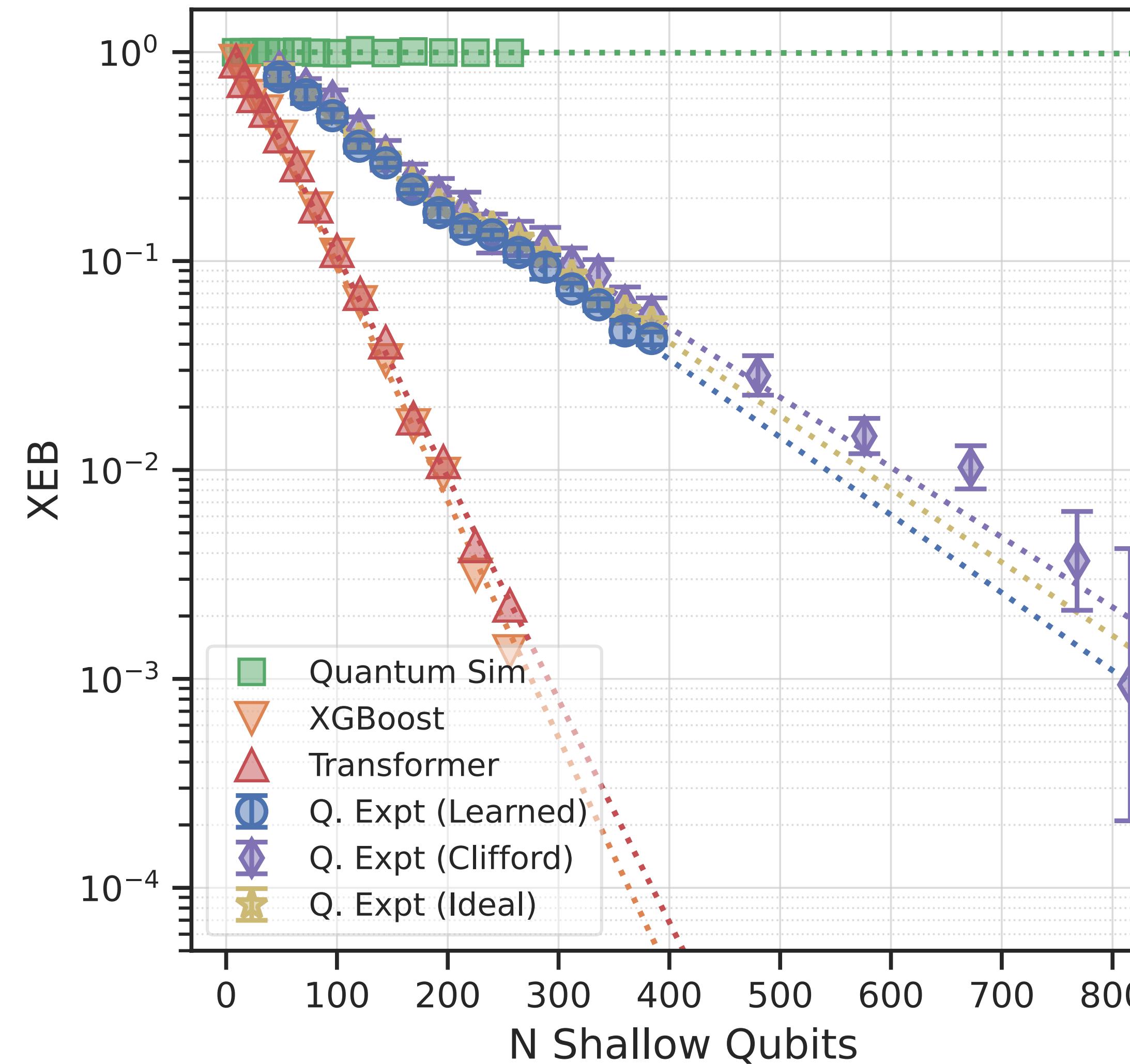
- Train **3D** shallow QNN.
- Map **3D** shallow QNN to **2D** deep QNN.
- Generate new output y using **2D** deep QNN.

Experimental Demonstration

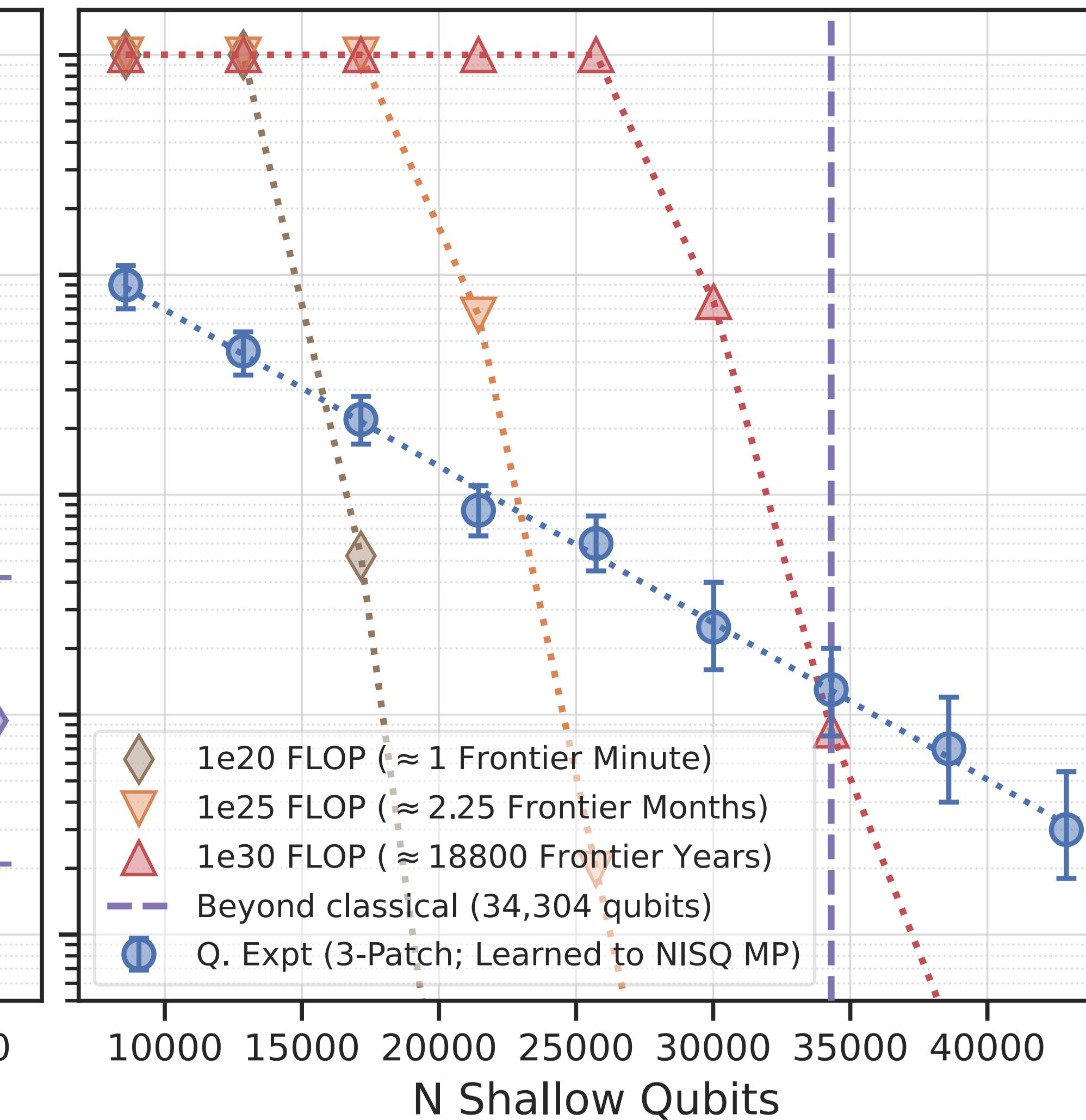


Experimental Demonstration

New data collected for smaller shallow QNNs

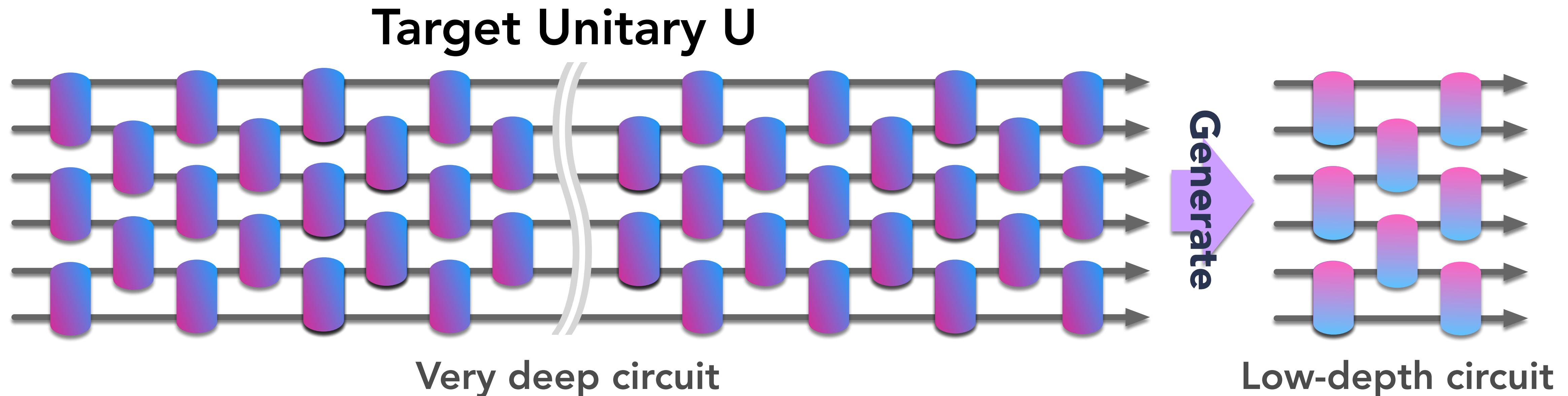


Data repurposed from Morvan et al., *Nature* (2024)

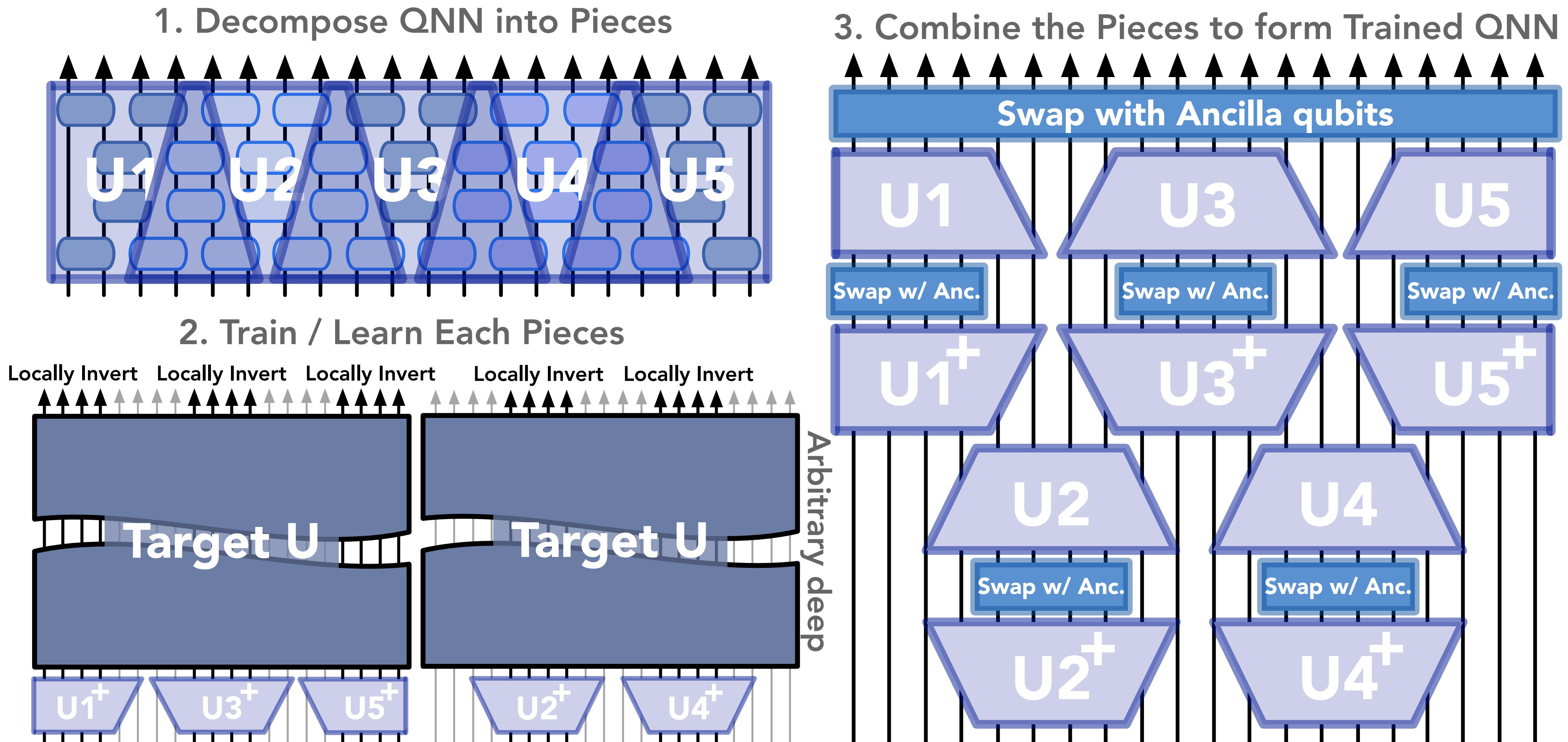


Application: Compressing Dynamics

- We also established generative quantum advantage for the task of compressing physical dynamics.



Application: Compressing Dynamics



Application: Compressing Dynamics

