Bits and Bytes

Chapter 2 of B&O

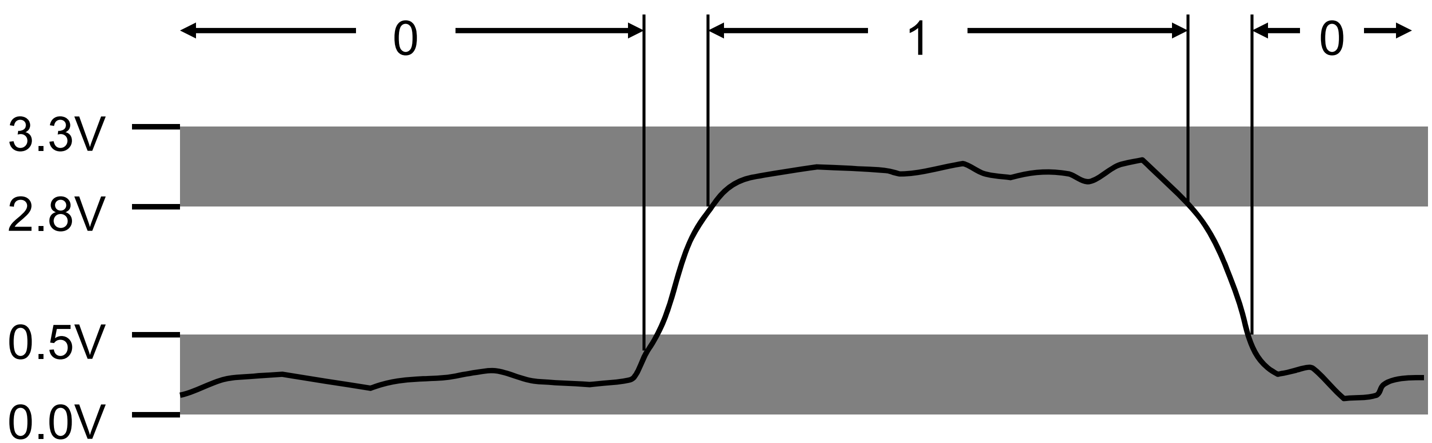
Some notes adopted from Bryant and O'Hallaron

Why Don't Computers Use Base 10?

- Base 10 Number Representation
  - That's why fingers are known as "digits"
  - Natural representation for financial transactions
    - Floating point number cannot exactly represent $1.20
  - Even carries through in scientific notation
    - 1.5213 X 104 This means 10^1 + 10^2 + 10^3 …. multiplied by the actual digit in that place. This is called decimal
- Implementing Electronically
  - Hard to store
    - ENIAC (First electronic computer) used 10 vacuum tubes / digit
  - Hard to transmit
    - Need high precision to encode 10 signal levels on single wire
  - Messy to implement digital logic functions
    - Addition, multiplication, etc.

Binary Representations

- Base 2 Number Representation
  - Represent 15213_{10} as 11101101101101_2
  - That's $1 \times 2^0 + 0 \times 2^1 + 1 \times 2^2 + 1 \times 2^3 + 0 \times 2^4 + \ldots$
- Electronic Implementation
  - Easy to store with bistable elements
  - Reliably transmitted on noisy and inaccurate wires



Encoding Byte Values

- Byte = 8 bits
  - Binary 00000000_2 to 11111111_2
  - Decimal: 0_{10} to 255_{10}
  - Hexadecimal 00_{16} to FF_{16}
    - Base 16 number representation
    - Use characters ‘0’ to ‘9’ and ‘A’ to ‘F’
    - Write $FA1D37B_{16}$ in C as `0xFA1D37B`
      - Or `0xfa1d37b`

It is really long, so we group them into 4's for hexidecimal

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Byte-Oriented Memory Organization

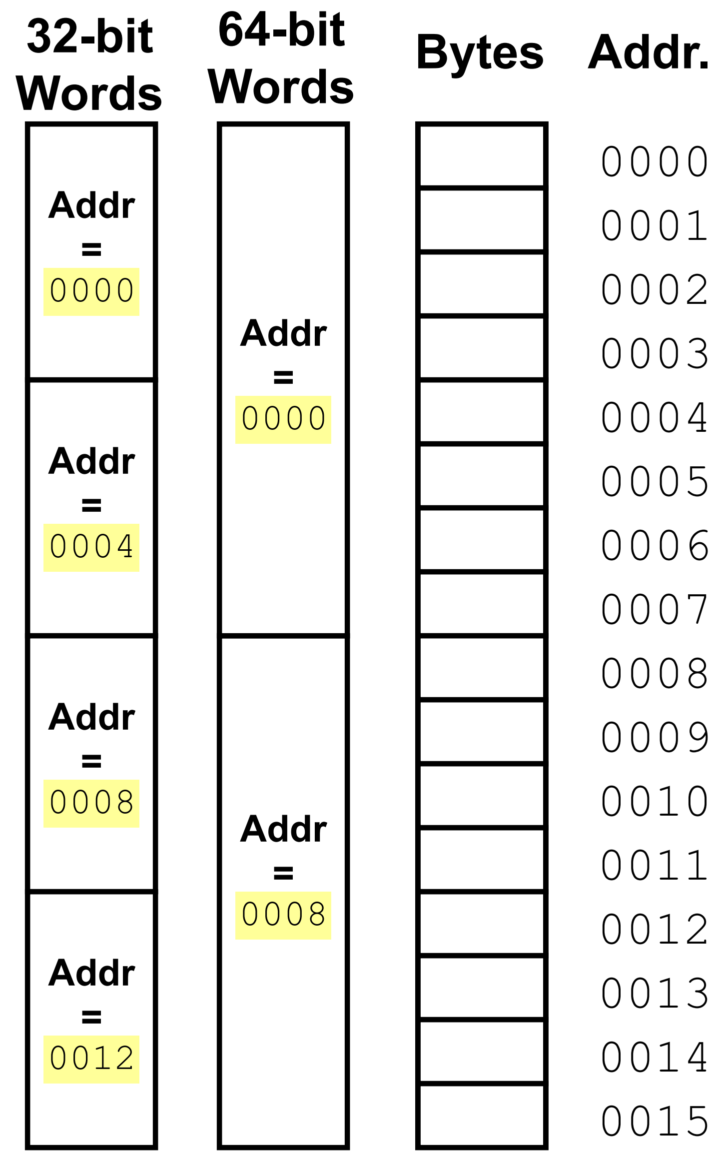
- Programs Refer to Virtual Addresses
  - Conceptually very large array of bytes
  - Actually implemented with hierarchy of different memory types
  - System provides address space private to particular “process”
    - Program being executed
    - Program can clobber its own data, but not that of others
- Compiler + Run-Time System Control Allocation
  - Where different program objects should be stored
  - In any case, all allocation within single virtual address space

Machine Words

- Machine Has "Word Size"
  - Nominal size of integer-valued data
    - Including addresses
  - 32-bit Systems (4 bytes) 4 bytes by 8 bits
    - Limits addresses to 4GB
    - Becoming too small for memory-intensive applications
  - 64-bit Systems (8 bytes)
    - Potentially address $\approx 1.8 \times 10^{19}$ bytes
    - x86-64 machines support 48-bit addresses: 256 Terabytes
  - Machines support multiple data formats
    - Fractions or multiples of word size
    - Always integral number of bytes

Word-Oriented Memory Organization

- Words are chunks of bits
- Addresses Specify Byte Locations
  - Address of first byte in word
  - Addresses of successive words differ by 4 (32-bit) or 8 (64-bit)



Stack

Heap
Global
Text

Data Representations

- Sizes of C Objects (in Bytes)

C Data Type	Typical 32-bit	IA32	x86-64
char	1	1	1
short	2	2	2
int	4	4	4
long	4	4	8
long long	8	8	8
float	4	4	4
double	8	8	8
long double	8	10/12	10/16
char*	4	4	8

  – Or any other pointer

```
#include <stdio.h>

int main() {
  printf ("%lu %lu %lu\n", sizeof(int), sizeof(long), sizeof(char*));
}
```

Byte Ordering

- How should bytes within multi-byte word be ordered in memory?
- Conventions
  - Older Sun's, PowerPC's are "Big Endian" machines
    - Least significant byte has highest address
  - x86's are "Little Endian" machines
    - Least significant byte has lowest address
  - SPARC V9, MIPS, Alpha are "Bi-Endian" machines
    - Can appear to be configurable as Big or Little Endian
      - Internals may still be one way or the other though

Byte Ordering Example

- Big Endian ("natural language", M68k, IBM Power)
  - Least significant byte has highest address
  - "decrease significance with increasing address"
- Little Endian ("arithmetic language", x86)
  - Least significant byte has lowest address
  - "increase significance with increasing address"
- Example
  - Variable `x` has 4-byte representation `0x01234567`
  - Address given by `&x` is `0x100`

Significant is always on the left

0x means you are dealing with hexidecimal

short 2 bytes, 16 bits needed to hold the memory

Big Endian

		0x100	0x101	0x102	0x103		
		01	23	45	67		

Little Endian

		0x100	0x101	0x102	0x103		
		67	45	23	01		

0xAABB is really 10x16^3 + 10 x 16^2 + 11 x 16 + 11x 16^0 Big Endian
0xBBAA Little Endian
Little Endian is the most common in this class8

Big Endian and Little Endian is only when you put 2 bytes together like 6D and 4F

Reading Byte-Reversed Listings

- Disassembly
  - Text representation of binary machine code
  - Generated by program that reads the machine code
- Example Fragment

Address	Instruction Code	Assembly Rendition
`8048365:`	`5b`	`pop     %ebx`
`8048366:`	`81 c3 ab 12 00 00`	`add     $0x12ab,%ebx`
`804836c:`	`83 bb 28 00 00 00 00`	`cmpl    $0x0,0x28(%ebx)`

- Deciphering Numbers
  - Value: `0x12ab`
  - Pad to 4 bytes: `0x000012ab`
  - Split into bytes: `00 00 12 ab`
  - Reverse: `ab 12 00 00`

Examining Data Representations

- Code to Print Byte Representation of Data
  - Casting pointer to unsigned char* creates byte array

```
typedef unsigned char* pointer;

void show_bytes(pointer start, int len)
{
  int i;
  for (i = 0; i < len; i++)
    printf("0x%p\t0x%.2x\n",
           start+i, start[i]);
  printf("\n");
}
```

Printf directives:

`%p`: **Print pointer**

`%x`: **Print Hexadecimal**

show_bytes Execution Example

```
int a = 15213;
printf("int a = 15213;\n");
show_bytes((pointer) &a, sizeof(int));
```

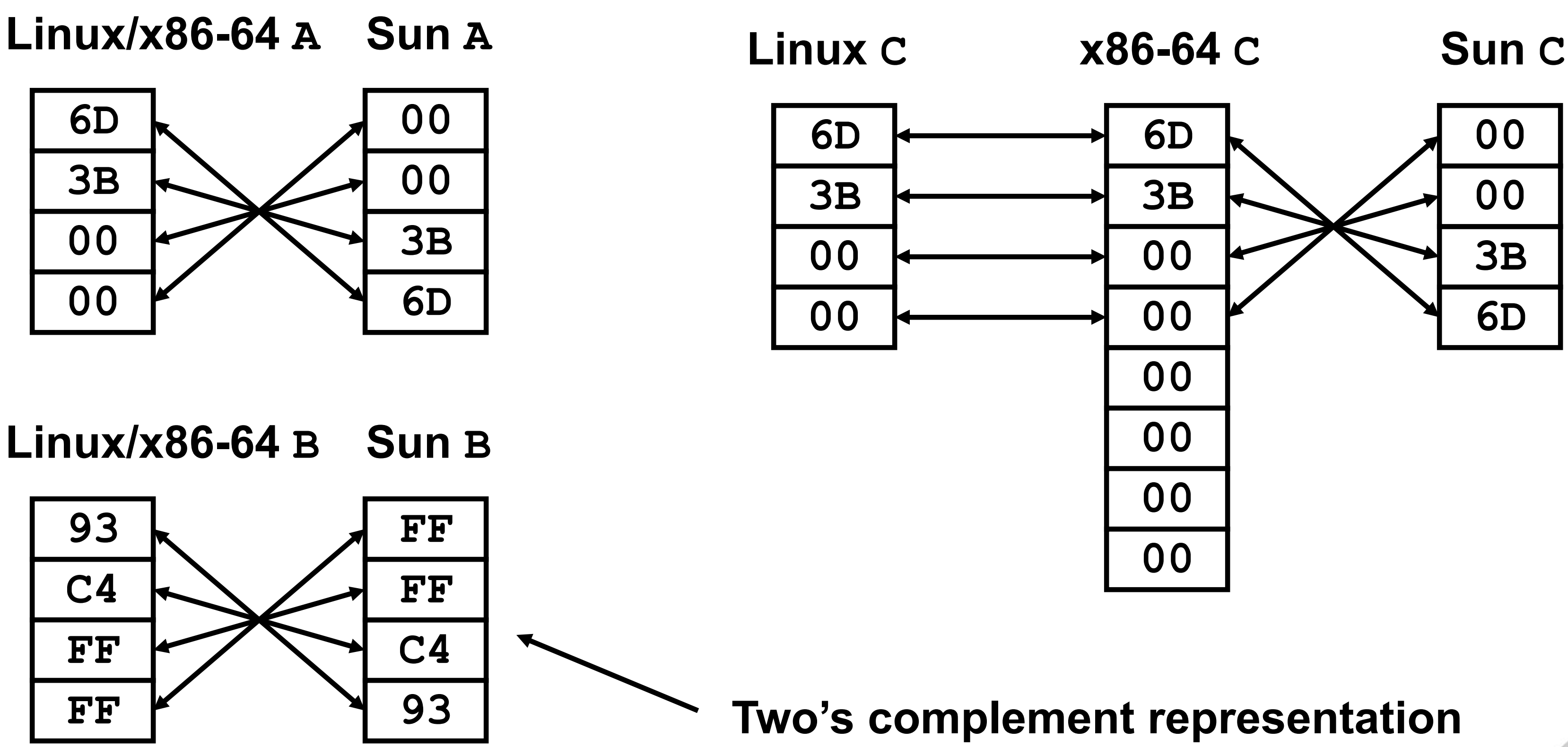
Result (Linux):

```
int a = 15213;
0x11ffffcb8   0x6d
0x11ffffcb9   0x3b
0x11ffffcba   0x00
0x11ffffcbb   0x00
```

Representing Integers

- `int A = 15213;`
- `int B = -15213;`
- `long int C = 15213;`

Decimal:	15213			
Binary:	0011	1011	0110	1101
Hex:	3	B	6	D



Two's complement representation
(Covered next lecture)

Rev erse the num bers and add 1 to the byte

Representing Pointers

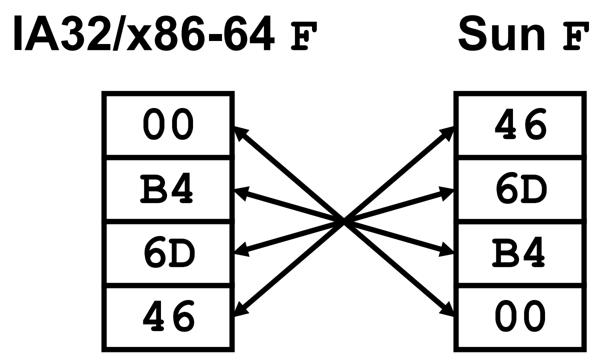
- `int B = -15213;`
- `int *P = &B;`

Sun P	IA32 P	x86-64 P
EF	D4	A0
FF	F8	FC
FB	FF	FF
2C	BF	FF
		01
		00
		00
		00

Different compilers & machines assign different locations to objects

Representing Floats

- `Float F = 15213.0;`



IEEE Single Precision Floating Point Representation

Hex:	4	6	6	D	B	4	0	0
Binary:	0100	0110	0110	1101	1011	0100	0000	0000
15213:			1110	1101	1011	01		

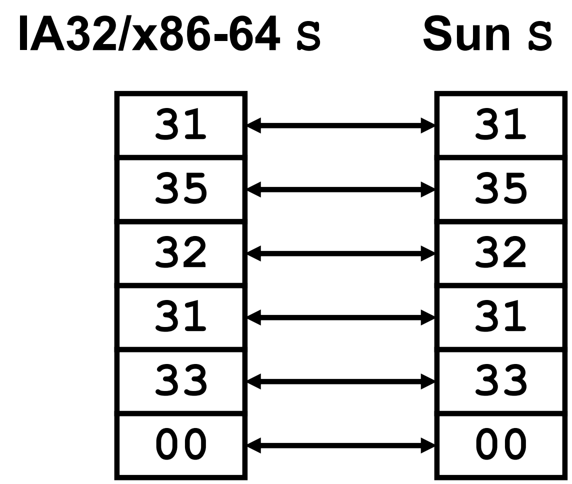
Not same as integer representation, but consistent across machines

Can see some relation to integer representation, but not obvious

Representing Strings

- Strings in C
  - Represented by array of characters
  - Each character encoded in ASCII format
    - Standard 7-bit encoding of character set
    - Other encodings exist, but uncommon
    - Character “0” has code 0x30
      - Digit i has code 0x30+i
  - String should be null-terminated
    - Final character = 0

- `char S[6] = "15213";`



- Compatibility
  - Byte ordering not an issue
    - Data are single byte quantities
  - Text files generally platform independent
    - Except for different conventions of line termination character(s)!

Machine-Level Code Representation

- Encode Program as Sequence of Instructions
  - Each simple operation
    - Arithmetic operation
    - Read or write memory
    - Conditional branch
  - Instructions encoded as bytes
    - Alpha's, Sun's, Mac's use 4 byte instructions
      - Reduced Instruction Set Computer (RISC)
    - PC's use variable length instructions
      - Complex Instruction Set Computer (CISC)
  - Different instruction types and encodings for different machines
  - Most code not binary compatible
- Programs are Byte Sequences Too!

Representing Instructions

```
int sum(int x, int y)
{
    return x+y;
}
```

- For this example, Alpha & Sun use two 4-byte instructions
  - Use differing numbers of instructions in other cases
- PC uses 7 instructions with lengths 1, 2, and 3 bytes

Alpha sum	Sun sum	PC sum
00	81	55
00	C3	89
30	E0	E5
42	08	8B
01	90	45
80	02	0C
FA	00	03
6B	09	45
		08
		89
		EC
		5D
		C3

Different machines use totally different instructions and encodings

Boolean Algebra

- Developed by George Boole in 19th Century
  - Algebraic representation of logic
    - Encode "True" as 1 and "False" as 0

- And
  - A&B = 1 when both A=1 and B=1

&	0	1
0	0	0
1	0	1

- Or
  - A|B = 1 when either A=1 or B=1

<table>
<tr><th>|</th><th>0</th><th>1</th></tr>
<tr><td>0</td><td>0</td><td>1</td></tr>
<tr><td>1</td><td>1</td><td>1</td></tr>
</table>

- Not
  - ~A = 1 when A=0

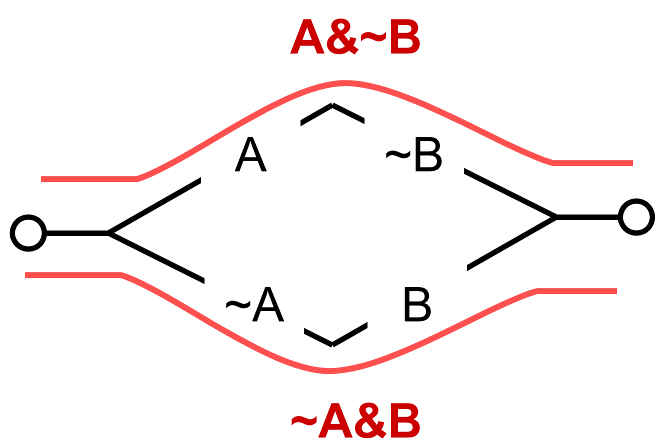
~	
0	1
1	0

- Exclusive-Or (Xor)
  - A^B = 1 when either A=1 or B=1, but not both

^	0	1
0	0	1
1	1	0

Application of Boolean Algebra

- Applied to Digital Systems by Claude Shannon
  - 1937 MIT Master's Thesis
  - Reason about networks of relay switches
    - Encode closed switch as 1, open switch as 0



Connection when

A&~B | ~A&B

= A^B

Integer Algebra

- Integer Arithmetic

  $\langle Z, +, *, -, 0, 1 \rangle$ forms a ring

  Addition is "sum" operation

  Multiplication is "product" operation

  – is additive inverse

  0 is identity for sum

  1 is identity for product

Boolean Algebra

- Boolean Algebra

  $\langle\{0,1\}, |, \&, \sim, 0, 1\rangle$ forms a "Boolean algebra"

  Or is "sum" operation

  And is "product" operation

  ~ is "complement" operation (not additive inverse)

  0 is identity for sum

  1 is identity for product

Boolean Algebra ≈ Integer Ring

- *Commutativity*
  - $A \mid B = B \mid A$ — $A + B = B + A$
  - $A \,\&\, B = B \,\&\, A$ — $A * B = B * A$
- *Associativity*
  - $(A \mid B) \mid C = A \mid (B \mid C)$ — $(A + B) + C = A + (B + C)$
  - $(A \,\&\, B) \,\&\, C = A \,\&\, (B \,\&\, C)$ — $(A * B) * C = A * (B * C)$
- *Product distributes over sum*
  - $A \,\&\, (B \mid C) = (A \,\&\, B) \mid (A \,\&\, C)$ — $A * (B + C) = A * B + B * C$
- *Sum and product identities*
  - $A \mid 0 = A$ — $A + 0 = A$
  - $A \,\&\, 1 = A$ — $A * 1 = A$
- *Zero is product annihilator*
  - $A \,\&\, 0 = 0$ — $A * 0 = 0$
- *Cancellation of negation*
  - $\sim(\sim A) = A$ — $-(-A) = A$

Boolean Algebra ≠ Integer Ring

- Boolean: *Sum distributes over product*

$A \mid (B \,\&\, C) = (A \mid B) \,\&\, (A \mid C)$	$A + (B * C) \neq (A + B) * (B + C)$

- Boolean: *Idempotency*

$A \mid A = A$ $\qquad\qquad$ $A + A \neq A$

  - "A is true" or "A is true" = "A is true"

$A \,\&\, A = A$ $\qquad\qquad$ $A * A \neq A$

- Boolean: *Absorption*

$A \mid (A \,\&\, B) = A$ $\qquad\qquad$ $A + (A * B) \neq A$

  - "A is true" or "A is true and B is true" = "A is true"

$A \,\&\, (A \mid B) = A$ $\qquad\qquad$ $A * (A + B) \neq A$

- Boolean: *Laws of Complements*

$A \mid {\sim}A = 1$ $\qquad\qquad$ $A + -A \neq 1$

  - "A is true" or "A is false"

- Ring: *Every element has additive inverse*

$A \mid {\sim}A \neq 0$ $\qquad\qquad$ $A + -A = 0$

Properties of AND and OR

- Boolean Ring
  - ⟨{0,1}, ^, &, I, 0, 1⟩
  - Identical to integers mod 2
  - I is identity operation: I(A) = A

    A ^ A = 0

Property	Boolean Ring
– Commutative sum	A ^ B = B ^ A
– Commutative product	A & B = B & A
– Associative sum	(A ^ B) ^ C = A ^ (B ^ C)
– Associative product	(A & B) & C = A & (B & C)
– Prod. over sum	A & (B ^ C) = (A & B) ^ (B & C)
– 0 is sum identity	A ^ 0 = A
– 1 is prod. identity	A & 1 = A
– 0 is product annihilator	A & 0 = 0
– Additive inverse	A ^ A = 0

Relations Between Operations

- DeMorgan's Laws
  - Express & in terms of |, and vice-versa
    - A & B = ~(~A | ~B)
      - A and B are true if and only if neither A nor B is false
    - A | B = ~(~A & ~B)
      - A or B are true if and only if A and B are not both false
- Exclusive-Or using Inclusive Or
    - A ^ B = (~A & B) | (A & ~B)
      - Exactly one of A and B is true
    - A ^ B = (A | B) & ~(A & B)
      - Either A is true, or B is true, but not both

General Boolean Algebras

- Operate on Bit Vectors
  - Operations applied bitwise

```
  01101001     01101001     01101001
& 01010101   | 01010101   ^ 01010101   ~ 01010101
  01000001     01111101     00111100     10101010
```

- All of the Properties of Boolean Algebra Apply

Representing & Manipulating Sets

- Representation
  - Width w bit vector represents subsets of {0, …, w–1}
  - $a_j = 1$ if $j \in A$

    01101001 { 0, 3, 5, 6 }

    76543210

    01010101 { 0, 2, 4, 6 }

    76543210

- Operations

– &	Intersection	01000001	{ 0, 6 }
– |	Union	01111101	{ 0, 2, 3, 4, 5, 6 }
– ^	Symmetric difference	00111100	{ 2, 3, 4, 5 }
– ~	Complement	10101010	{ 1, 3, 5, 7 }

Bit-Level Operations in C

- Operations &, |, ~, ^ Available in C
  - Apply to any "integral" data type
    - long, int, short, char, unsigned
  - View arguments as bit vectors
  - Arguments applied bit-wise
- Examples (Char data type)
  - ~0x41 --> 0xBE
    - ~01000001_2 --> 10111110_2
  - ~0x00 --> 0xFF
    - ~00000000_2 --> 11111111_2
  - 0x69 & 0x55 --> 0x41
    - 01101001_2 & 01010101_2 --> 01000001_2
  - 0x69 | 0x55 --> 0x7D
    - 01101001_2 | 01010101_2 --> 01111101_2

Contrast: Logic Operations in C

- Contrast to Logical Operators
  - &&, ||, !
    - View 0 as "False"
    - Anything nonzero as "True"
    - Always return 0 or 1
    - Early termination
- Examples (char data type)
  - !0x41 --> 0x00
  - !0x00 --> 0x01
  - !!0x41 --> 0x01

  - 0x69 && 0x55 --> 0x01
  - 0x69 || 0x55 --> 0x01
  - p && *p (avoids null pointer access)

Shift Operations

- Left Shift: x << y
  - Shift bit-vector x left y positions
    - Throw away extra bits on left
    - Fill with 0's on right
- Right Shift: x >> y
  - Shift bit-vector x right y positions
    - Throw away extra bits on right
  - Logical shift
    - Fill with 0's on left
  - Arithmetic shift
    - Replicate most significant bit on right
    - Useful with two's complement integer representation
- Undefined Behavior
  - Shift amount < 0 or ≥ word size

Argument x	01100010
<< 3	00010*000*
Log. >> 2	*00*011000
Arith. >> 2	*00*011000

Argument x	10100010
<< 3	00010*000*
Log. >> 2	*00*101000
Arith. >> 2	*11*101000

Cool Stuff with Xor

- Bitwise Xor is form of addition
- With extra property that every value is its own additive inverse

  A ^ A = 0

```
void funny(int *x, int *y)
{
   *x = *x ^ *y;     /* #1 */
   *y = *x ^ *y;     /* #2 */
   *x = *x ^ *y;     /* #3 */
}
```

	*x	*y
Begin	A	B
1	A^B	B
2	A^B	(A^B)^B = A
3	(A^B)^A = B	A
End	B	A

Main Points

- It's All About Bits & Bytes
  - Numbers
  - Programs
  - Text
- Different Machines Follow Different Conventions
  - Word size
  - Byte ordering
  - Representations
- Boolean Algebra is Mathematical Basis
  - Basic form encodes "false" as 0, "true" as 1
  - General form like bit-level operations in C
    - Good for representing & manipulating sets