Common Parallel Computing Scenarios



- Many parallel threads need to generate a single result
 → Reduce
- Many parallel threads need to partition data→ Split
- Many parallel threads produce variable output / thread
 → Compact / Expand



- Partition data to operate in well-sized blocks
 - Small enough to be staged in shared memory
 - Assign each data partition to a thread block
 - No different from cache blocking!
- Provides several performance benefits
 - Have enough blocks to keep processors busy
 - Working in shared memory cuts memory latency dramatically
 - Likely to have coherent access patterns on load/store to shared memory

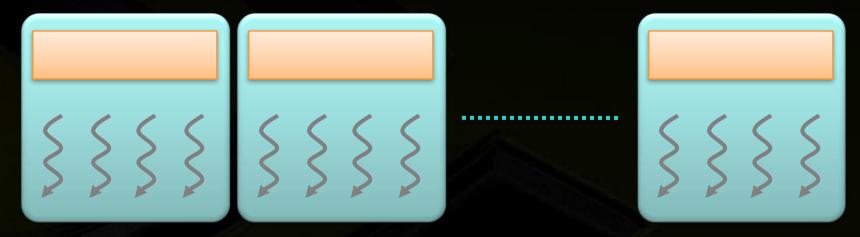




Partition data into subsets that fit into shared memory

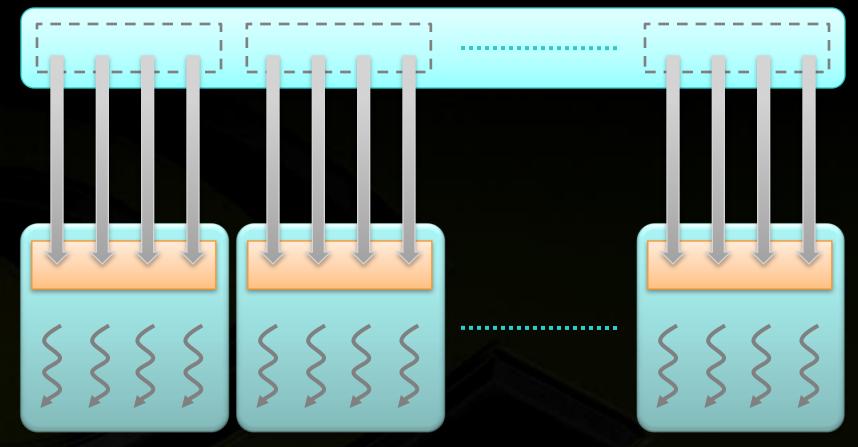






Handle each data subset with one thread block

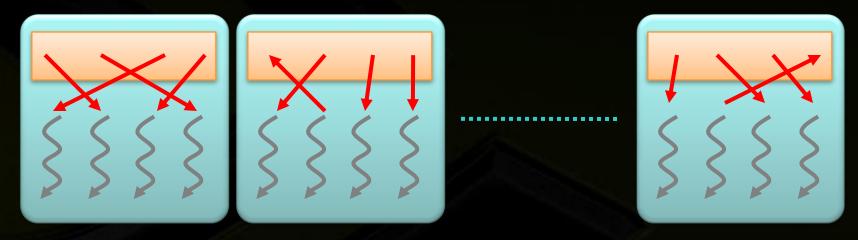




Load the subset from global memory to shared memory, using multiple threads to exploit memorylevel parallelism

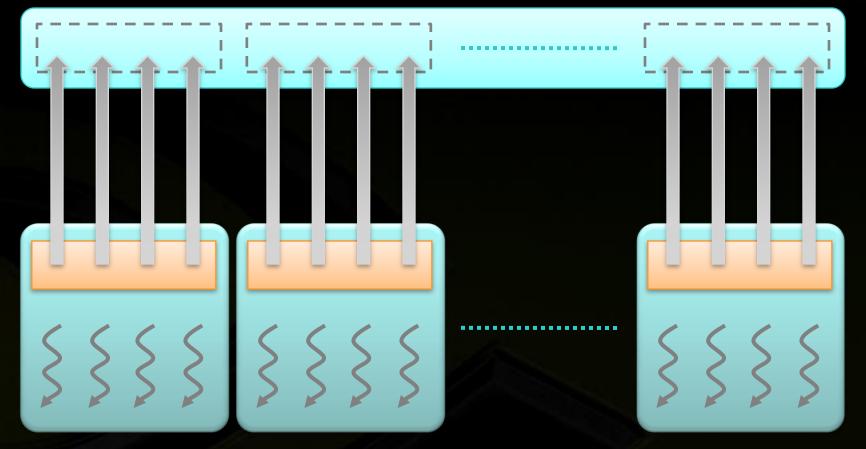






Perform the computation on the subset from shared memory





Copy the result from shared memory back to global memory



- All CUDA kernels are built this way
 - Blocking may not matter for a particular problem, but you're still forced to think about it
 - Not all kernels require shared memory
 - All kernels do require registers
- All of the parallel patterns we'll discuss have CUDA implementations that exploit blocking in some fashion

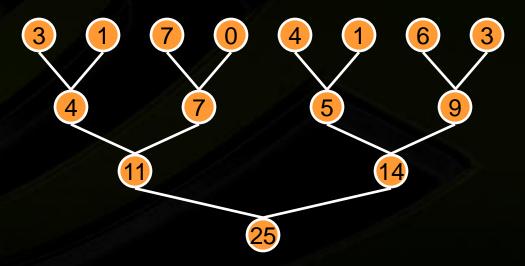
Reduction



- Reduce vector to a single value
 - Via an associative operator (+, *, min/max, AND/OR, ...)
 - CPU: sequential implementation

for (int
$$i = 0$$
, $i < n$, ++i) ...

GPU: "tree"-based implementation



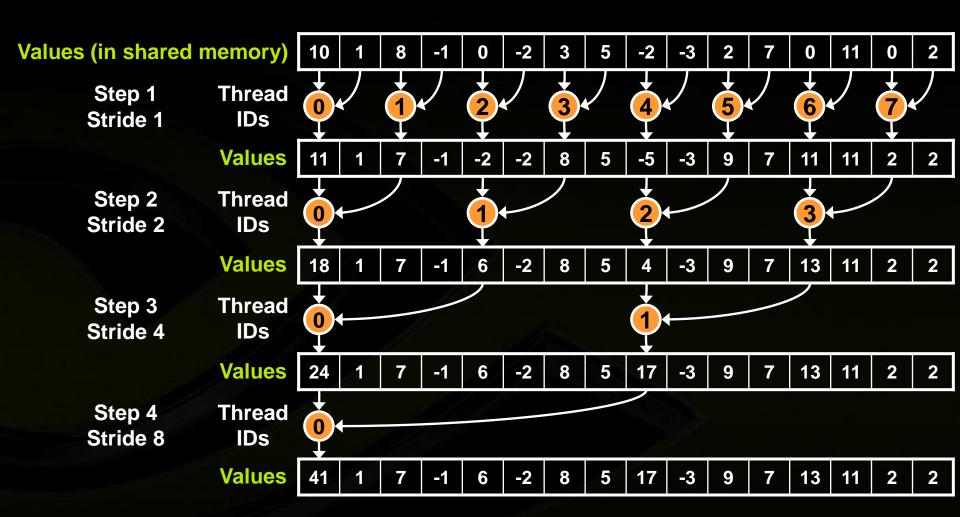
Serial Reduction



```
// reduction via serial iteration
float sum(float *data, int n)
{
 float result = 0;
 for (int i = 0; i < n; ++i)
    result += data[i];
  return result;
```

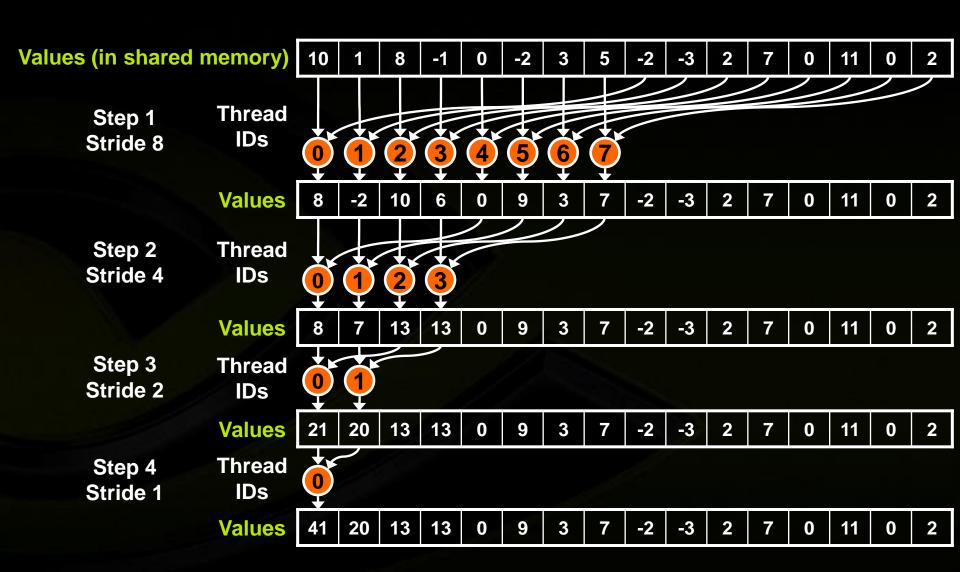
Parallel Reduction – Interleaved





Parallel Reduction – Contiguous







```
global void block sum (float *input,
                         float *results,
                         size t n)
 extern shared float sdata[];
 int i = ..., int tx = threadIdx.x;
 // load input into shared memory
 float x = 0;
 if(i < n)
   x = input[i];
 sdata[tx] = x;
 syncthreads();
```



```
// block-wide reduction in shared
                                       mem
for(int offset = blockDim.x / 2;
    offset > 0;
    offset >>= 1)
 if(tx < offset)</pre>
  {
    // add a partial sum upstream to our own
    sdata[tx] += sdata[tx + offset];
  }
  syncthreads();
```



```
// finally, thread 0 writes the result
if(threadIdx.x == 0)
{
    // note that the result is per-block
    // not per-thread
    results[blockIdx.x] = sdata[0];
}
```

An Aside



```
// is this barrier divergent?
for(int offset = blockDim.x / 2;
    offset > 0;
    offset >>= 1)
{
        ...
        syncthreads();
}
```

An Aside



```
// what about this one?
 global void do i halt(int *input)
 int i = ...
 if(input[i])
     syncthreads(); // a divergent barrier
                     // hangs the machine
```



```
// global sum via per-block reductions
float sum(float *d input, size t n)
{
 size t block size = ..., num blocks = ...;
  // allocate per-block partial sums
  // plus a final total sum
  float *d sums = 0;
  cudaMalloc((void**)&d sums,
   sizeof(float) * (num blocks + 1));
```



```
// reduce per-block partial sums
int smem sz = block size*sizeof(float);
block sum<<<num blocks,block size,smem sz>>>
  (d input, d sums, n);
// reduce partial sums to a total sum
block sum<<<1,block size,smem sz>>>
  d sums, d sums + num blocks, num blocks);
// copy result to host
float result = 0;
cudaMemcpy(&result, d sums+num blocks, ...);
return result;
```

Split Operation



Given:array of true and false elements (and payloads)



Return an array with all true elements at the beginning

T	T	T	F	F	F	F	F
3	0	6	1	7	4	1	3

Examples: sorting, building trees

Variable Output Per Thread: Compact



Remove null elements

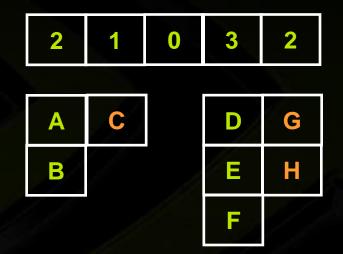


Example: collision detection

Variable Output Per Thread: General Case



Reserve Variable Storage Per Thread



Example: binning

Split, Compact, Expand



Each thread must answer a simple question:

"Where do I write my output?"

- The answer depends on what other threads write!
- Scan provides an efficient parallel answer

Scan (a.k.a. Parallel Prefix Sum)



Given an array $A = [a_0, a_1, ..., a_{\underline{n}-1}]$ and a binary associative operator \oplus with identity I,

$$scan(A) = [I, a_0, (a_0 \oplus a_1), ..., (a_0 \oplus a_1 \oplus ... \oplus a_{n-2})]$$

lacksquare Prefix sum: if \oplus is addition, then scan on the series

returns the series

0 3 4 11 11 15 16 2	2
---------------------	---

Applications of Scan



- Scan is a simple and useful parallel building block for many parallel algorithms:
 - Radix sort
 - Quicksort (seg. scan)
 - String comparison
 - Lexical analysis
 - Stream compaction
 - Run-length encoding

- Polynomial evaluation
- Solving recurrences
- Tree operations
- Histograms
- Allocation
- Etc.
- Fascinating, since scan is unnecessary in sequential computing!

Serial Scan



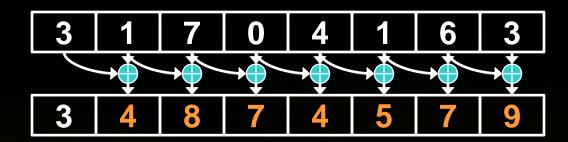
```
int input[8] = \{3, 1, 7, 0, 4, 1, 6, 3\};
int result[8];
int running sum = 0;
for (int i = 0; i < 8; ++i)
  result[i] = running sum;
  running sum += input[i];
// \text{ result} = \{0, 3, 4, 11, 11, 15, 16, 22\}
```



3 1 7 0 4 1 6 3

Assume array is already in shared memory



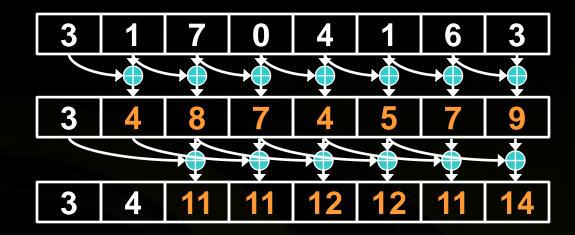


Iteration 0, *n-1* threads

Each \bigoplus corresponds to a single thread.

Iterate log(n) times. Each thread adds value stride elements away to its own value



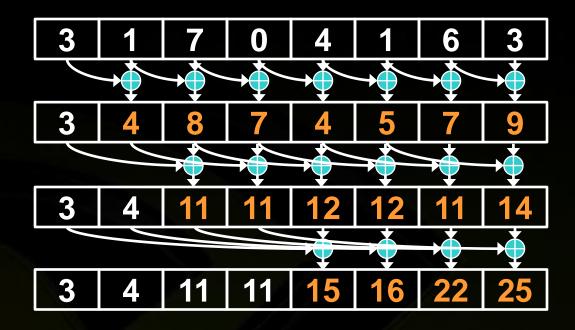


Iteration 1, *n-2* threads

Each \bigoplus corresponds to a single thread.

Iterate log(n) times. Each thread adds value offset elements away to its own value





Iteration *i*, *n-2ⁱ* threads

Each corresponds to a single thread.

Iterate log(n) times. Each thread adds value offset elements away to its own value.

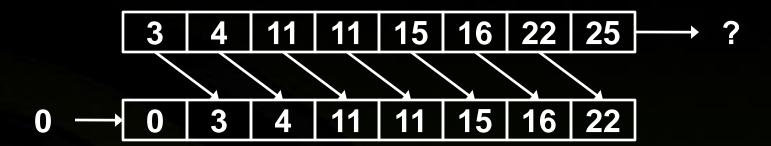
Note that this algorithm operates in-place: no need for double buffering



3 4 11 11 15 16 22 25

We have an inclusive scan result





- For an exclusive scan, right-shift through shared memory
- Note that the unused final element is also the sum of the entire array
 - Often called the "carry"
 - Scan & reduce in one pass

CUDA Block-wise Inclusive Scan



```
global void inclusive scan(int *data)
 extern shared int sdata[];
 unsigned int i = ...
  // load input into shared memory
 int sum = input[i];
 sdata[threadIdx.x] = sum;
 syncthreads();
```

CUDA Block-wise Inclusive Scan



```
for (int o = 1; o < blockDim.x; o <<= 1)
 if(threadIdx.x >= o)
    sum += sdata[threadIdx.x - o];
  // wait on reads
 syncthreads();
  // write my partial sum
  sdata[threadIdx.x] = sum;
  // wait on writes
  syncthreads();
```

CUDA Block-wise Inclusive Scan



```
// we're done!
// each thread writes out its result
result[i] = sdata[threadIdx.x];
```

Results are Local to Each Block



Block 0

Input:

5 5 4 4 5 4 0 0 4 2 5 5 1 3 1 5

Result:

5 10 14 18 23 27 27 27 31 33 38 43 44 47 48 53

Block 1

Input:

1 2 3 0 3 0 2 3 4 4 3 2 2 5 5 0

Result:

1 3 6 6 9 9 11 14 18 22 25 27 29 34 39 39

Results are Local to Each Block



- Need to propagate results from each block to all subsequent blocks
- 2-phase scan
 - 1. Per-block scan & reduce
 - 2. Scan per-block sums
- Final update propagates phase 2 data and transforms to exclusive scan result
- Details in MP3