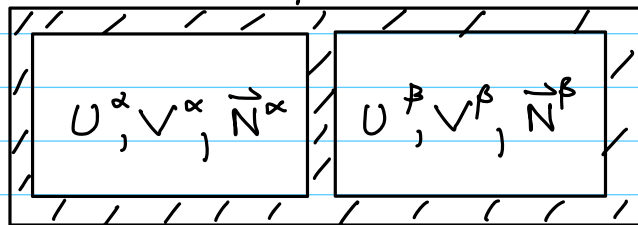


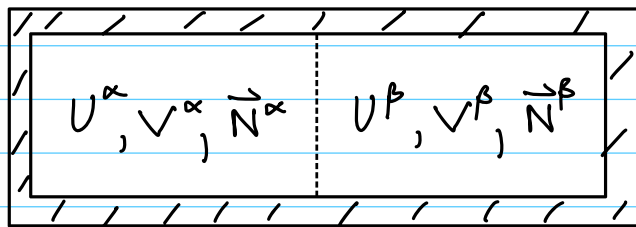
Math Thermo  
class # 07  
02/10/2026

## Equilibrium with a Heat Bath

Consider two isolated systems that are initially isolated from each other.



Now, let us remove the isolating wall so that energy, volume, and mass may be exchanged



The entropy will attain its maximum at equilibrium and

$$\begin{aligned} T^\alpha &= T^\beta \\ P^\alpha &= P^\beta \\ \mu_i^\alpha &= \mu_i^\beta, \quad i = 1, \dots, r. \end{aligned}$$

Now, suppose that instead of isolating from the universe, our composite system is put into contact with a heat bath.

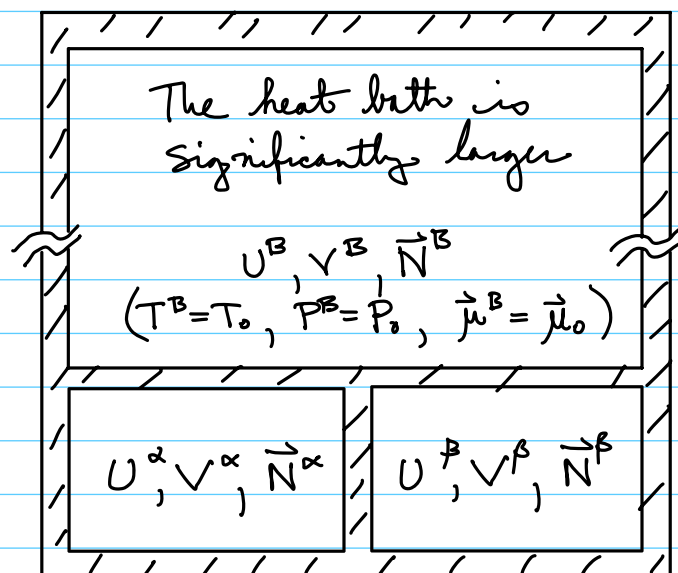
Definition (7.1): A heat bath is an otherwise-isolated thermodynamic system that is so large that, when it exchanges a finite amount of energy with an ordinary, otherwise-isolated, compound system its temperature, pressure, and chemical potential changes are so small as to be negligible.

Similarly, we have the following

heat-pressure bath: finite exchanges of energy and volume lead to negligible changes in  $T$ ,  $P$ , and  $\mu$  in bath.

heat-pressure-chemical bath: finite exchanges of energy, volume, and matter lead to negligible changes in  $T$ ,  $P$ , and  $\mu$  in the bath.

let us consider a picture for the case of the heat bath.



let us recall the "integrated form" of the entropy functions.

$$\begin{aligned}\tilde{S}^\square &= \tilde{S}^\square(U^\square, V^\square, \vec{N}^\square) \\ &= \frac{1}{T^\square} U^\square + \frac{P^\square}{T^\square} V^\square - \frac{\vec{\mu}^\square}{T^\square} \cdot \vec{N}^\square\end{aligned}$$

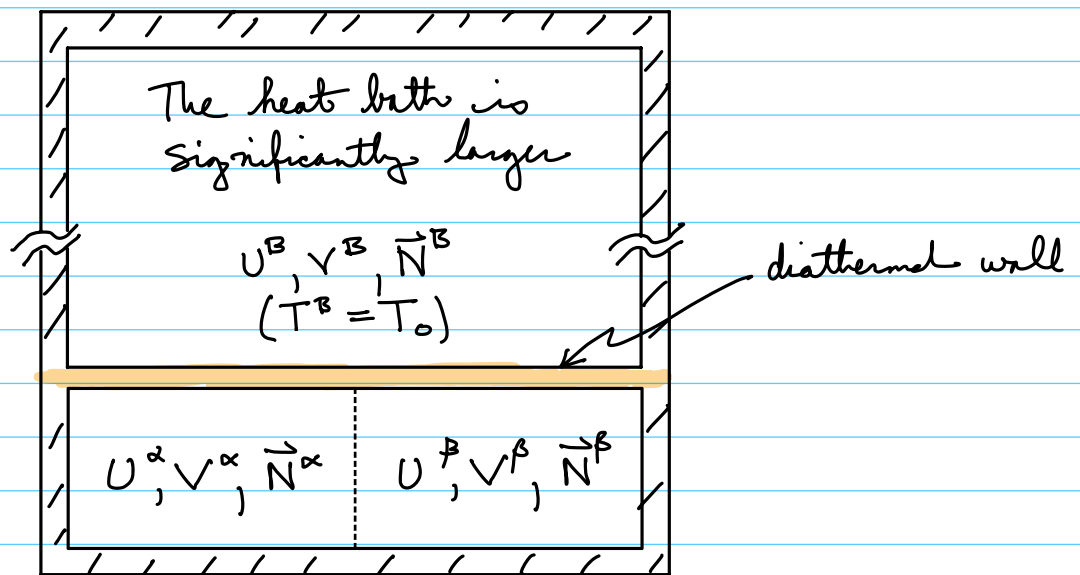
where

$$T^\square = T^\square(U^\square, V^\square, \vec{N}^\square)$$

et cetera. We will write

$$\begin{aligned}S^{\text{Tot}} &= \tilde{S}^B + \tilde{S}^\alpha + \tilde{S}^\beta \\ &= \tilde{S}^B + \tilde{S}^{\alpha+\beta}\end{aligned}$$

Now, our system will not exchange volume or mass with the bath, only energy, because we will replace the horizontal isolation wall with a diathermal wall.



We know that, at equilibrium

$$T^\alpha = T^\beta = T^B = T_0.$$

Recall the deviation of  $T^B$  from  $T_0$  at equilibrium is assumed negligible.

Systems  $\alpha$ ,  $\beta$ , and  $B$  can exchange energy. Systems  $\alpha$  and  $\beta$  can exchange volume and mass with each other but not with the bath.

Thus,

$$S^{\text{Tot,eq}} = \max_C \{ \tilde{S}^B + \tilde{S}^{\alpha+\beta} \}$$

where

$$C := \begin{cases} U^\alpha + U^\beta + U^B = U_0, \\ V^\alpha + V^\beta = V_0, \\ N_i^\alpha + N_i^\beta = N_{i,0}. \end{cases}$$

Now

$$\begin{aligned} \tilde{S}^B &= \frac{1}{T_0} U^B + \underbrace{\frac{P_0}{T_0} V^B + \frac{\vec{\mu}_0}{T_0} \cdot \vec{N}^B}_{= \text{const}, C_1} \\ &= \frac{1}{T_0} U^B + C_1. \end{aligned}$$

The change in  $T^B$ ,  $P^B$ ,  $\vec{\mu}^B$  from  $T_0$ ,  $P_0$ ,  $\vec{\mu}_0$  is negligible because the bath is so large.

Writing

$$U^B = U_0 - U^\alpha - U^\beta$$

$$\begin{aligned}\tilde{S}^B &= -\frac{U^\alpha}{T_0} - \frac{U^\beta}{T_0} + \frac{U_0}{T_0} + C_1 \\ &= -\frac{U^\alpha}{T_0} - \frac{U^\beta}{T_0} + C_2.\end{aligned}$$

Thus,

$$\begin{aligned}J^{\text{Tot,eq}} &= \max_C \left\{ -\frac{U^\alpha}{T_0} - \frac{U^\beta}{T_0} + C_2 + \tilde{S}^{\alpha+\beta} \right\} \\ &= C_2 + \max_C \left\{ -\frac{(U^\alpha + U^\beta)}{T_0} + \tilde{S}^\alpha + \tilde{S}^\beta \right\} \\ &= C_2 + \max_C \left\{ -\frac{(F^\alpha + F^\beta)}{T_0} \right\} \\ &= C_2 - \frac{1}{T_0} \min_{\substack{V^\alpha + V^\beta = V_0 \\ N_i^\alpha + N_i^\beta = N_{i,0}}} \left\{ F^\alpha(T_0, V^\alpha, \vec{N}^\alpha) + F^\beta(T_0, V^\beta, \vec{N}^\beta) \right\}\end{aligned}$$

Postulate VI: Consider a compound, otherwise-isolated thermodynamic system in contact with an otherwise-isolated heat bath at temperature  $T_0$ . Equilibrium of the compound system is the state satisfying the isothermal condition

$$T^\alpha = T^\beta = T_0$$

which minimizes

$$F^\alpha(T_0, V^\alpha, \vec{N}^\alpha) + F^\beta(T_0, V^\beta, \vec{N}^\beta)$$

Subject to the constraints

$$V^\alpha + V^\beta = V_0, \quad (\text{volume conservation})$$

$$N_i^\alpha + N_i^\beta = N_{i,0}. \quad (\text{mass conservation})$$

Remark: Since the system  $\alpha + \beta$  exchanges energy with the heat bath, energy is not conserved in  $\alpha + \beta$ !

Energy is conserved in the system  $B + \alpha + \beta$ .