The Value of Community Information for Pricing under Network Externalities

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Motivation

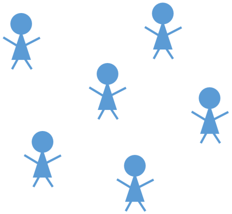
The use of social network information in marketing has become prevalent

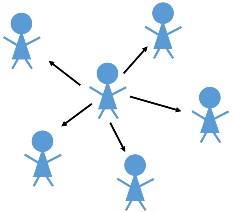
- Many e-commerce websites use some form of social information to provide personalization — offering personalized products, prices, etc

Social Platforms	Social Influence Measurements	Companies Using Social Information
Instagram	PeerIndex	GILT GROUPE
twitter	KLOUT	PALMS
	Kr .Kred	

Fundamental questions: Is it really worthwhile to seek and use network information for pricing? What information is useful and how to use partial information?

Deterministic Model



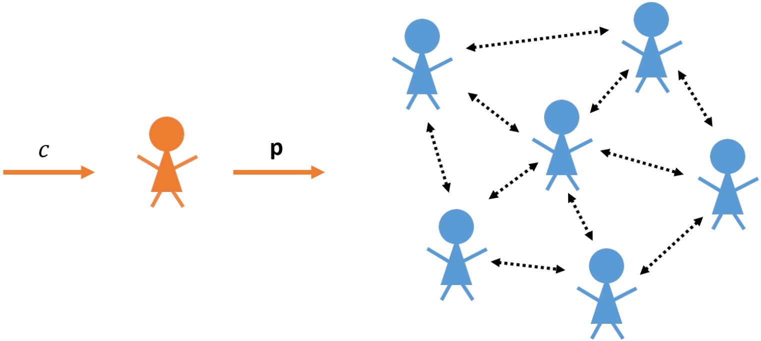


- Product is divisible
- n independent customers
- $u_i = ax_i - x_i^2 - p_i x_i$
- $x_i^*(p_i) = \frac{a-p_i}{2}$

- n customers in binary network G
- $u_i = ax_i - x_i^2 + \sum_{j \neq i} \frac{4\rho x_i G_{ij} x_j}{\|G+G^T\|} - p_i x_i$[1]
- Externalities coefficient: $\rho \in (0, 1)$
- $\mathbf{x}^*(\mathbf{p}) = \frac{1}{2}\left(I - \frac{2\rho G}{\|G+G^T\|}\right)^{-1}(a\mathbf{1} - \mathbf{p})$

[1] Candogan et. al. 2012, Bloch and Querou 2013, Fainmesser 2015

Deterministic Model



Monopolist: $\max_{\mathbf{p}} \; (\mathbf{p} - c\mathbf{1})^T \mathbf{x}^*(\mathbf{p})$

Consumption Equilibria and Optimal Prices

- Consumption equilibrium:
  $\mathbf{x}^*(\mathbf{p}) = \frac{1}{2}\left(I - \frac{2\rho}{\|G+G^T\|}G\right)^{-1}(a\mathbf{1} - \mathbf{p})$
- Optimal price vector:

$$\begin{aligned}
\mathbf{p}^* &= \left(\frac{a+c}{2}\right)\mathbf{1} && \text{Common} \\
&+ \left(\frac{a-c}{2}\right)\frac{\rho G}{\|G+G^T\|}\mathcal{K}\left(\frac{2\rho}{\|G+G^T\|}\left(G+G^T\right), \frac{1}{2}\right) && \text{Markup} \\
&- \left(\frac{a-c}{2}\right)\frac{\rho G^T}{\|G+G^T\|}\mathcal{K}\left(\frac{2\rho}{\|G+G^T\|}\left(G+G^T\right), \frac{1}{2}\right) && \text{Discount}
\end{aligned}$$

  where $\mathcal{K}(G, \alpha) = (I - \alpha G)^{-1}\mathbf{1} = \sum_{i=0}^{\infty}(\alpha G)^i\mathbf{1}$ is the Bonacich centrality vector.
- Optimal profit of the monopolist:
  $\pi^* = \frac{1}{2}\left(\frac{a-c}{2}\right)^2\mathbf{1}^T\left(I - \frac{\rho}{\|G+G^T\|}(G+G^T)\right)^{-1}\mathbf{1}$
  - Proportional to weighted sum of number of walks of different lengths in multigraph $G + G^T$

Optimal Uniform Prices

- Optimal uniform price vector: $\mathbf{p}_0 = \frac{a+c}{2}\mathbf{1}$
- Profit of the monopolist under $\mathbf{p}_0$:
  $\pi_0 = \frac{1}{2}\left(\frac{a-c}{2}\right)^2 \mathbf{1}^T \left(I - \frac{2\rho}{\|G+G^T\|} G\right)^{-1} \mathbf{1}$
  - Proportional to weighted sum of number of walks of different lengths in graph G
- The optimal uniform price vector doesn't depend on the network information.
  - Network effects still play a role in determining the consumption equilibrium and the profit.

The Value of Price Discrimination

- Monopolist's regret under optimal uniform pricing, i.e. -

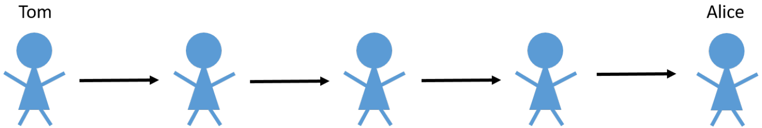
$$R(\mathbf{p}_0) = \frac{1}{2}\left(\frac{a-c}{2}\right)^2 \left[\mathbf{1}^T \left(I - \frac{\rho}{\|G+G^T\|}\left(G+G^T\right)\right)^{-1}\mathbf{1} - \mathbf{1}^T\left(I - \frac{2\rho}{\|G+G^T\|}G\right)^{-1}\mathbf{1}\right].$$

- Monopolist's fractional regret under optimal uniform pricing, i.e. -

$$R_F(\mathbf{p}_0) = 1 - \frac{\mathbf{1}^T\left(I - \frac{2\rho}{\|G+G^T\|}G\right)^{-1}\mathbf{1}}{\mathbf{1}^T\left(I - \frac{\rho}{\|G+G^T\|}\left(G+G^T\right)\right)^{-1}\mathbf{1}}.$$

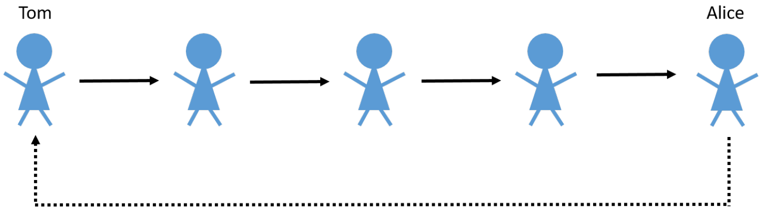
- Equivalent to value of network information

Is There Any Value of Price Discrimination?



Unbalanced graph: Yes

Is There Any Value of Price Discrimination?



Balanced graph: No

Random Networks

Now we move to random networks. We consider a sequence of networks $G(n)$ indexed by the number of consumers in the network

- Utility for consumer i in the n_{th} network

$$u_i(n) = ax_i - x_i^2 + 4\rho x_i \sum_{j\neq i} \frac{G_{ij}(n)}{\|G(n) + G(n)^T\|} x_j - p_i x_i$$

Goal: Evaluate the asymptotic value of price discrimination for a sequence of random networks

Directed Erdös-Renyi Networks

There has been focus on a special class of random networks — directed Erdös-Renyi (E-R) networks

- n nodes in the network
- For each i and j, $g_{ij} = 1$ with probability $p(n)$ and 0 with probability $1 - p(n)$
- Usually not symmetric

Results: Huang et. al. 2022

Huang et.al. obtained the following results for E-R networks

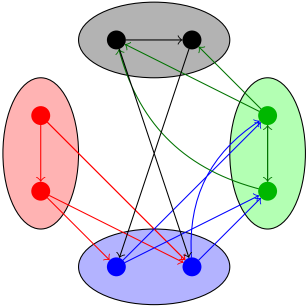
Network Density	Expected Regret	Expected Fractional Regret
$O\left(n^{-(1+\epsilon)}\right)$	$\Theta\left(n^2 p(n)\right)$	$\Theta\left(np(n)\right)$
$\Theta\left(n^{-1}\right)$	$\Theta\left(\frac{\log \log n}{\log n} n\right)$	$\Theta\left(\frac{\log \log n}{\log n}\right)$
$\omega\left(\frac{\log n}{n}\right)$	$\Theta\left(p(n)^{-1}\right)$	$\Theta\left(n^{-1} p(n)^{-1}\right)$

Furthermore, given any sequence of network densities, $p(n)$, for the sequence of Erdös-Renyi random networks, the expected regret $\mathbf{E}_G[R(\mathbf{p}_0)] = o(n)$, and the expected fractional regret $\mathbf{E}_G[R_F(\mathbf{p}_0)] = o(1)$.

When Does Network Information Matter?

Consider a network with communities (Stochastic Blockmodel)

- m blocks, $B \in [0,1]^{m \times m}$, B is not price discrimination free.
  B_{kl} is the probability of link from block k to block l
- n members in each block,
  $p(n) \in (0,1)$ is the decay factor If i in community k and j in community l then
  $P(G_{ij} = 1) = p(n) B_{kl}$
  $p(n) = \omega(\frac{log(n)}{n})$



When Does Network Information Matter?

Optimal price with community level price discrimination

$$\mathbf{p}^* = \left(\frac{a+c}{2}\right)\mathbf{1} + \left(\frac{a-c}{2}\right)\frac{\rho B}{\|B+B^T\|}\mathcal{K}\left(\frac{1}{\|B+B^T\|}\left(B+B^T\right),\rho\right)$$
$$-\left(\frac{a-c}{2}\right)\frac{\rho B^T}{\|B+B^T\|}\mathcal{K}\left(\frac{1}{\|B+B^T\|}\left(B+B^T\right),\rho\right)$$

where $\mathcal{K}(B,\alpha) = (I-\alpha B)^{-1}\mathbf{1} = \sum_{i=0}^{\infty}(\alpha B)^i\,\mathbf{1}$ is the Bonacich centrality vector.

Information requirement: $\Theta\left(\underbrace{mn\log(m)}_{\text{Community identity}} + \underbrace{m^2}_{\text{Matrix B}}\right)$

Result

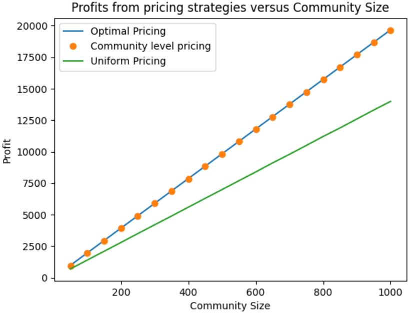
A directed stochastic block model has expected fractional regret:

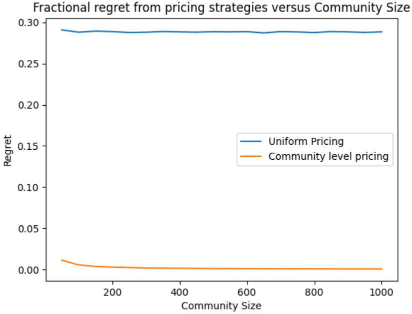
Community Level Pricing	Uniform Pricing
$\Theta\left(n^{-1}p(n)^{-1}\right)$	$\Theta\left(1\right)$

Fractional regret of community level pricing is $o(1)$ and for uniform pricing is constant.

When Does Network Information Matter?

Consider a network with communities (Stochastic Blockmodel) (4 communities, a = 6, c = 4, p(n) = 0.9)





Application

Consider pricing across neighborhoods in California

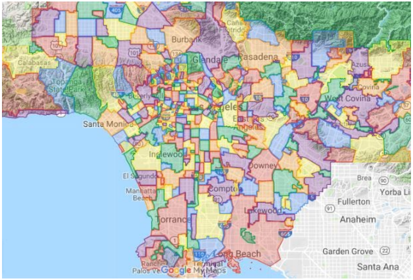


Figure: Neighborhoods of Los Angeles. Using block prices we set a price for each neighborhood.

Conclusions

- For a large class of random network models, the asymptotic fractional regret going to 0.
- For stochastic blockmodels, network information is useful but knowledge of community membership and community influence structure is sufficient.

Thank You!