Pricing on Social Networks with Partial Information

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Motivation

- Many modern companies make use of discounts and specials in an attempt to optimize profits

Social Platforms

Social Influence Measurements

Companies Using Social Information

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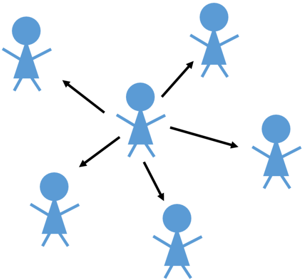
Companies Using Social Information

- The goal is influential people are offered discounts to buy more and their use of the product incentives others to buy.

Introducing the Model

Consider a network G with:

- N individuals as nodes
- $G_{ij} \geq 0$ is the influence j has on i
- Each Person chooses an amount of a divisible good to buy that maximizes their utility
- This depends (positively) on how much the people around them consume.



Utility

We model utility of the ith person as

$$u_i = a * x_i - p_i x_i - x_i^2 + 4\rho x_i \sum_{i \neq j} \frac{G_{ij}}{\|G + G^T\|} x_j$$

Where:

- u_i is utility
- x_i amount purchased
- p_i price charged
- $a > 0, 0 < \rho < 1$ constants.

Note that aside from position in the network, everyone has the same utility function. This model of utility has been previously studied. [HMW22], [CBO12] [FG16] [BQ13]

Consumption

Each individual will consume an amount that maximizes their utility
Unique Equilibrium Consumption $x(\mathbf{p})$ is

$$x(\mathbf{p}) = \frac{1}{2}\left(I - 2\frac{\rho}{\|G + G^T\|}G\right)^{-1} * (a * \mathbf{1} - \mathbf{p})$$

[CBO12]

Profits

The goal of the firm is to charge each consumer a different amount to maximize profit based on the network

$$max_p(p - c\mathbf{1})^T\mathbf{x}(p)$$

Where $a > c > 0$ is the unit cost to produce one unit of the good.

Optimal Prices

The firm will attempt to maximize profits by strategically choosing prices. If they are unrestricted then the optimal price vector is

$$\begin{aligned} p^* &= \frac{a+c}{2}\mathbf{1} && \text{(Constant)} \\ &+ \frac{a-c}{2}\frac{\rho}{\|G+G^T\|}G * \mathcal{K}(G+G^T, \frac{\rho}{\|G+G^T\|})\mathbf{1} && \text{(Markup)} \\ &- \frac{a-c}{2}\frac{\rho}{\|G+G^T\|}G^T * \mathcal{K}(G+G^T, \frac{\rho}{\|G+G^T\|})\mathbf{1} && \text{(Discount)} \end{aligned}$$

[CBO12]
Where $\mathcal{K}(G,\alpha) = (I-\alpha G)^{-1} = \sum_{i=0}^{\infty}(\alpha G)^i$ is the Bonacich centrality vector
and profit
$\pi^* = \frac{1}{8}(a-c)^2\mathbf{1}^T(I - \frac{\rho}{\|G+G^T\|}(G+G^T))^{-1}\mathbf{1}$[HMW22]

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New viewpoint: Under optimal pricing the consumption is proportional to $\mathcal{K}(G+G^T, \frac{\rho}{\|G+G^T\|})$

Uniform Price

If we require that we charge one price instead then

1. the optimal uniform price $\mathbf{p}_0$ is $\mathbf{p}_0 = \frac{a+c}{2}\mathbf{1}$
2. the profit π_0 is $\pi_0 = \frac{1}{8}(a-c)^2\mathbf{1}(I - \frac{2\rho}{\|G+G^T\|}G)^{-1}\mathbf{1}$

Regret for Random Graphs

Over Erdos-Renyi Graphs and Power-law graphs with $\alpha > 2$ with n nodes.

Theorem

(Uniform Regret) $\lim_{n\to\infty} E[1 - \frac{\pi_0}{\pi^*}] = 0$

[HMW22]

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Theorem

(Uniform Regret) $\lim_{n\to\infty} E[1 - \frac{\pi_0}{\pi^*}] = 0$

[HMW22] This means for large Erdos-Renyi and powerlaw Graphs there is no benefit using optimal prices instead of uniform prices.

Is there is a class of random graphs with an asymptotically positive value of regret?

Stochastic Block Model

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1) Build a truly random A from sampling each edge independently according to P
2) Build a matrix that reflects the average density i.e. $\bar{A}_{ij} = P_{C(i),C(j)}$ where $C(i)$ is the index for the community that i is a part of

Example

If $P = \begin{bmatrix} 0.25 & 0.5 \\ 1 & 0.75 \end{bmatrix}$ If there are 2 individuals in each community then

$$\bar{A} = \begin{bmatrix} 0.25 & 0.25 & 0.5 & 0.5 \\ 0.25 & 0.25 & 0.5 & 0.5 \\ 1 & 1 & 0.75 & 0.75 \\ 1 & 1 & 0.75 & 0.75 \end{bmatrix}$$

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And a sample A could look like $A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$

Performance

In practice we find that only knowing the community level information is sufficient

Specifically let $Regret(v) = 1 - \frac{\text{Profit(v)}}{\pi*}$ where

- v is the price vector we derive from $\bar{A}$
- $\pi*$ optimal profit of A

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The first part says the limited information is sufficient to get good performance and the second says there is a loss if you don't use graph information.

Blocking

One way to use this idea is given a partition of the network ask that everyone in each block of the partition is charged the same price. How do we do we determine prices?

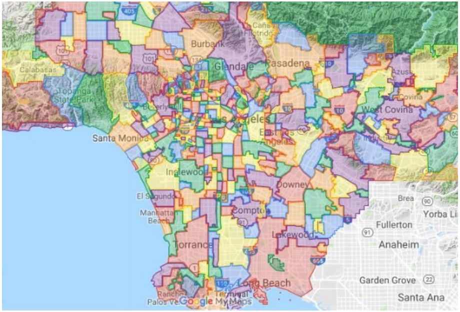

Figure: Neighborhoods of Los Angeles. Instead of being able to charge individuals prices a realistic constraint is that we set the price in each neighborhood.

Partitioning

- Letting $B = \frac{1}{2}(I - 2\rho G)^{-1}$ one can express the prices p to charge is $(B + B^T)v = (aB\mathbf{1} + c * B^T\mathbf{1})$
- We adapt this to a given partition with q blocks.

  Let $\mathcal{P}$ be a $q \times \underbrace{mn}_{\text{Size of network}}$ matrix where $\mathcal{P}_{ij} = \begin{cases} 1 & \text{If j } \in \text{ Block } i \\ 0 & \text{Else} \end{cases}$
- Then the optimal prices to charge in each block are $\mathcal{P}(B + B^T)\mathcal{P}^T v = a * \mathcal{P}B\mathbf{1} + c * \mathcal{P}B^T\mathbf{1}$
- Denote by q the price vector from such a partitioning scheme.

Losses

- Let p^* optimal price vector, q prices calculated from partition
- We conjecture based on experiment that the loss of using q instead of p^* is

$$Profit(p^*) - Profit(q) = (p^*)^T Bp^* - q^T Bq$$

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- This is not true for general prices.
- The set of prices that satisfy this is not linear subspace.

Examples

With $a = 6, c = 4, \rho = 0.9$

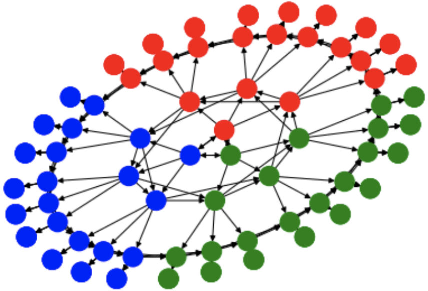

Figure: Bad Partition of 3 blocks. The three colors blocks are identical. Under this restriction of prices the graph acts like a symmetric graph which has no benefit to price discrimination. The optimal price here is the uniform price $\frac{a+c}{2} = 5$ and the profit is 166.73

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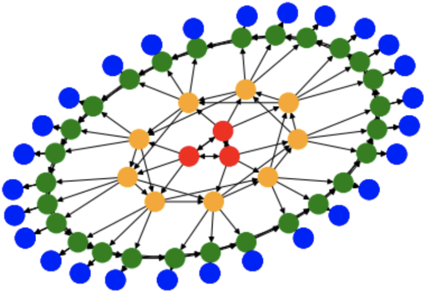

Figure: A good Partition using 4 blocks. Under this restriction of prices the graph acts like a directed line graph which does benefit to price discrimination. The profit is 295.14 and is the true optimal. This is 77% increase in profit over the previous partition.

Thank you

Questions?

[BQ13] Francis Bloch and Nicolas Quérou. “Pricing in social networks”. In: Games and economic behavior 80 (2013), pp. 243–261.

[CBO12] Ozan Candogan, Kostas Bimpikis, and Asuman Ozdaglar. “Optimal pricing in networks with externalities”. In: Operations Research 60.4 (2012), pp. 883–905.

[FG16] Itay P Fainmesser and Andrea Galeotti. “Pricing network effects”. In: The Review of Economic Studies 83.1 (2016), pp. 165–198.

[HMW22] Jiali Huang, Ankur Mani, and Zizhuo Wang. “The value of price discrimination in large social networks”. In: Management Science 68.6 (2022), pp. 4454–4477.