Approximating price vectors for price discrimination over networks

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Motivation

- Humans are social creatures, our behavior influences other behavior
- When our peers engage in a behavior, we are more likely do so as well
- This behavior is widespread from promoting exercise[1], modeling likelihood to smoke[2] or purchase environmentally green products[3]

Motivation

- Provide a mathematical framework that incorporates social influence to provide guidance to marketers and policy designers
- We will use the model first proposed by [4] and later used by [5] to model an individuals utility u

$$u_i(x, p) = ax_i - x_i^2 + 4\rho x_i \sum \frac{G_{ij}}{\|G + G^T\|} x_j - p_i x_i \qquad (1)$$

where a, ρ are constants and p_i is the price user i is charged.

- A manufacturer who can produce goods at unit cost c with $c < a$ wants to maximize profits.
- Use network information to charge influencers less and influencees more.
- The optimal prices to charge each individual when the network is fully known is well understood[4][5]

$$\frac{a+c}{2}\mathbf{1}+\frac{a-c}{2}\frac{\rho}{\|G+G^T\|}(G-G^T)K(G+G^T,\frac{\rho}{\|G+G^T\|}) \quad (2)$$

where $K(X,y)=(I-yX)^{-1}\mathbf{1}$, the Bonacich centrality vector.

- This vector is a weighted sum of walks ending at vertex i.

- But we often don't have ready access to the full network information
- Given partial enough of the network ex. degrees of network what should we do?
- Specifically, we want a way to generate a "good enough" price vector v with respect to this partial information Goal: minimize expected regret

$$\mathbb{E}[1 - \frac{P_G(v)}{Optimal Profit} | \text{Statistic of G}] \tag{3}$$

Where $P_G(v)$ is the profit of our v applied to the real network G.

Degree Seqeunce Information

- Suppose we are given the degree sequence of the network G (directed graph)
- Strategy 1: Make a new graph H with the same degree sequence as G using the configuration model
- Hypothesis: H behaves like G so maybe the optimal price vector of H is close to the optimal price vector of G
- It is not obvious that local properties of the network should strongly impact global properties (optimal profit)

Overview of the results

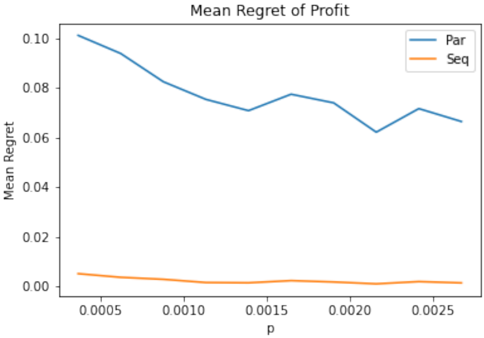
- The answer is yes, this is a strategy to get good price vectors for an unknown graph
- The following results will show this to be the case

Details of Testing

- Generate a graph G with the Erdos-Renyi model with n nodes and link probablity p
- Generate either Same Parameter graphs (i.e. same n and p but no further restriction) or Graphs with the same sequence
- After we have generated a price vector use G to check how close we were.
- For fixed G, get the average regret over several runs

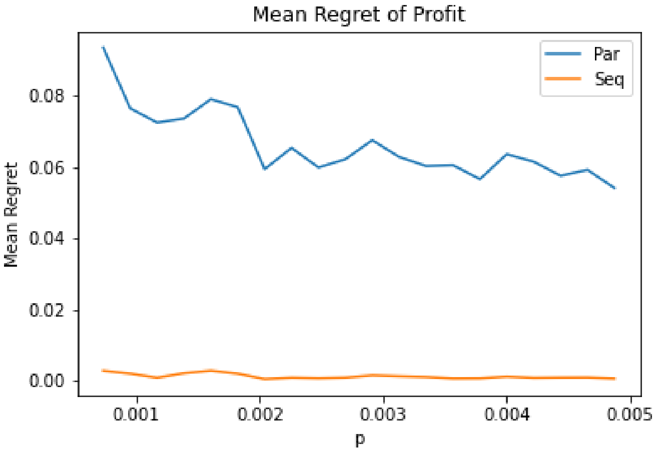
Distribution of profit regret

- How much regret would we have if we used the optimal same sequence price vector or the same parameter price vector. Lower is better
- Using graphs of the same sequence outperforms all Erdos-Renyi Graphs of the same n and p.
- $n = 1500, p \in [\frac{1}{n}, \frac{log(n)}{n}]$



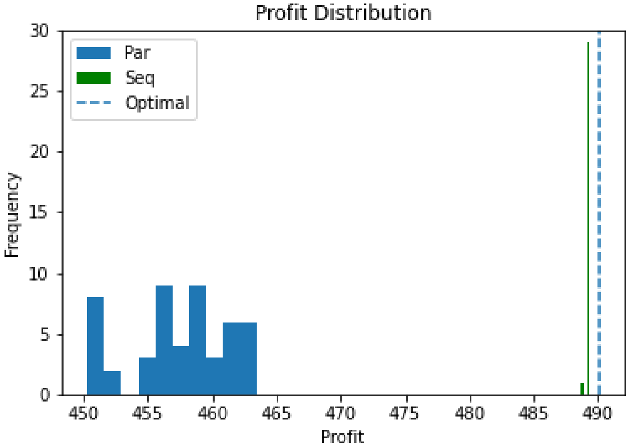
As a function of n

- $n \in [100, 3000], p = \frac{\sqrt{\log(n)}}{n}$



What about the raw profits

- Raw profit holds little meaning as the optimal profit will change a lot with changes to p and n



Statistic of distribution of Profits

- Mean Profit of all same parameter Erdos Renyi graphs: 457.369
- Variance of same parameter case: 14.377
- Mean Profit of same sequence profits 489.190
- Variance of same sequence profits 0.008
- True Optimal Profit 490.030

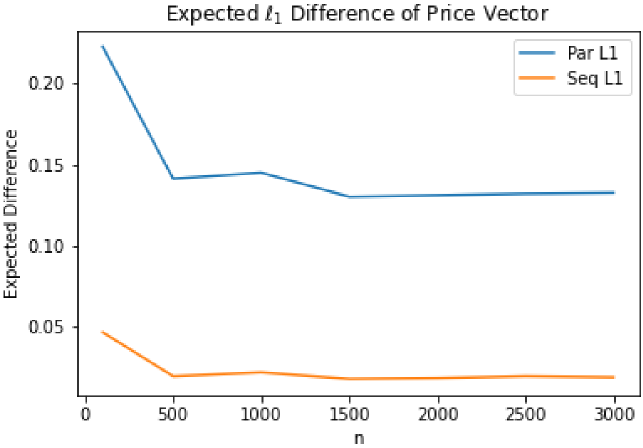
Distribution of Prices

- The next question to ask is do the same sequence price vectors look like the optimal price vector?
- Again the answer is yes

Average differences in price

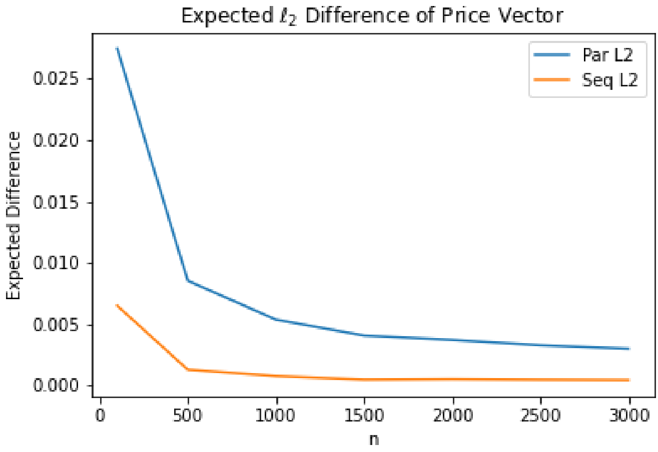
- Generate fixed G and find true optimal price vector v_{opt}
- Generate a guess of a graph and its price vector v_{guess}
- $score = \frac{1}{(\# Trials)*n} \sum_{\# Trials} \|v_{opt} - v_{guess}\|_1$
- Average error in each price.
- Same for ℓ_2

ℓ_1 Difference



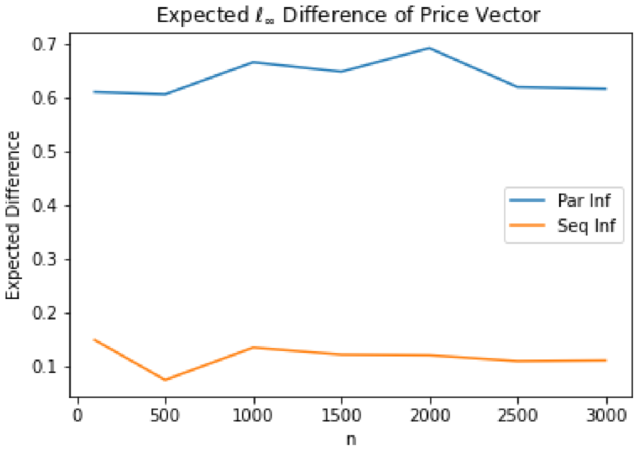
$$p = \frac{\sqrt{\log(n)}}{n}$$

ℓ_2 Difference



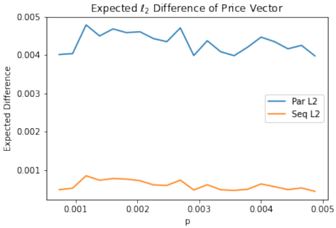
ℓ_∞ Difference

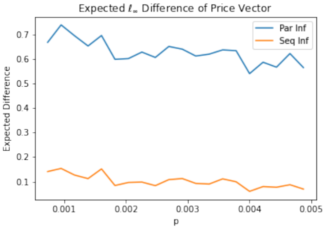
Simpler formula $\frac{1}{\# Trials} \sum_{Trials} \left\| v_{true} - v_{guess} \right\|_\infty$



Effect of p

- $n = 1500, p \in [\frac{1}{n}, \frac{\log(n)}{n}]$
- No clear effect of p on error in price





Differences in Price Vector

- The price vector from the same sequence graphs is always notably closer than for the same parameter graph
- For the same parameter graphs ℓ_2 norm appears to converge but ℓ_∞ appears stagnant regardless of the size of the network.

A second strategy

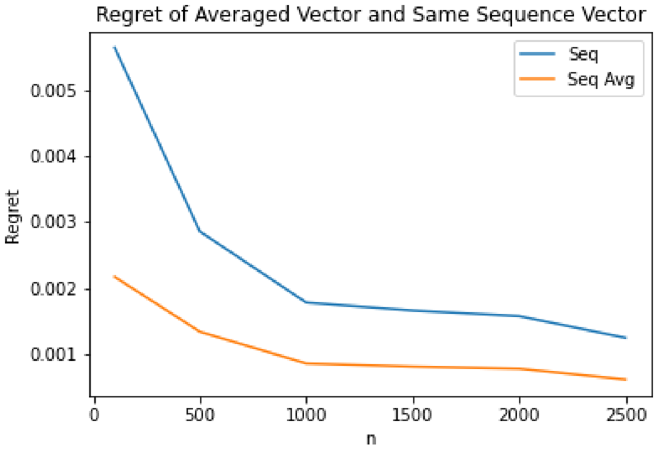
- Above we have shown that the price vector of generated graphs is on average like the optimal price vector
- Strategy 2 The averaged price vector is even close to the optimal price vector

$$v = \frac{1}{\# Trials} \sum Profit_G(\text{Guessed vector}) \tag{4}$$

$$v = Profit_G(\frac{1}{\# Trials} \sum \text{Guessed vector}) \tag{5}$$

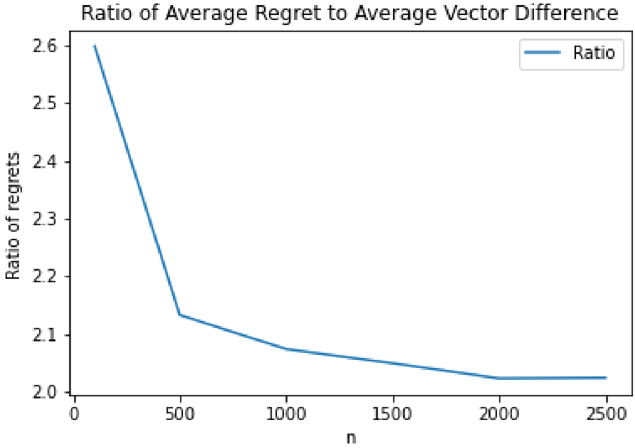
Results

Comparison between these two strategies: $p = \frac{\sqrt{\log(n)}}{n}$



Results

Conjecture ratio of these two regrets appear to approach Strategy 2 is twice as good as Strategy 1



Other Directions

- If knowing the degrees is good then maybe knowing $[|N(v)|, |N(N(v)) \setminus (N(v) \cup v)|]$ is better
- i.e. how many nodes can v reach in 1 or 2 steps
- What about k steps?

Walk Test

- Instead of calculating $\left(I - \frac{\rho}{\|G+G^T\|}(G + G^T)\right)^{-1}\mathbf{1}$ we calculate $\sum_{i=1}^{k}\left(\frac{\rho}{\|G+G^T\|}(G + G^T)\right)^k$
- I.e. only walks of length k or less have any bearing on the price vector

Results

The loss from number of steps decays very quickly at first then marginal utility. n=1000

Next Steps

- Derive mathematical bounds
- Explore other network configurations other than Erdos-Renyi
- Possibility of more sophisticated pricing vectors

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