Approximating price vectors

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Introduction

- Humans are social creatures, our behavior influences other behavior
- When our peers engage in a behavior, we are more likely do so as well
- CITE SOME OF THE RELEVANT EXAMPLES INCLUDING SOME SEEN IN CLASS

- How to analyze the effect of the social network on consumption
- We will use the model first proposed by [1] and later used by [2] to model an individuals utility u

$$u_i(x, p) = ax_i - x_i^2 + 4\rho x_i \sum \frac{G_{ij}}{\|G + G^T\|} x_j - p_i x_i \quad (1)$$

where a, ρ are constants and p_i is the price user i is charged.

- A manufacturer who can produce goods at unit price c with $c < a$ wants to maximize profits.
- Use network information to charge influencers less and influencees more.
- The optimal prices to charge each individual is well understood[1][2]

$$\frac{a+c}{2}\mathbf{1}+\frac{a-c}{2}\frac{\rho}{\|G+G^T\|}(G-G^T)K(G+G^T,\frac{\rho}{\|G+G^T\|}) \tag{2}$$

where $K(X,y) = (I - yX)^{-1}\mathbf{1}$, the bonanich centrality vector.

- But we often don't have ready access to the full network information
- Given partial enough of the network ex. degrees of network should we do?
- Specifically, we want a way to generate a "good enough" price vector v with respect to this partial information
  Goal: minimize expected regret

$$\mathbb{E}[1 - \frac{P_G(v)}{Optimal - Profit} | \text{Statistic of G}] \tag{3}$$

Where $P_G(v)$ is the profit of our v on the real network G.

Degree Seqeunce Information

- Suppose we are given the degree sequence of the network G(directed graph)
- Strategy 1: Make a new graph H with the same degree sequence as G using the configuration model
- Hypothesis: H behaves like G so maybe the optimal price vector of H is close to the optimal price vector of G
- It is not obvious that local properties of the network should strongly impact global properties(optimal profit)

Results

- The answer is yes, this is a strategy to get good price vectors for an unknown graph
- The following results will show this to be the case

Details of Testing

- Generate a graph G with the Erdos-Renyi model with n nodes and link probablity p
- Generate either Same Parameter graphs(i.e. same n and p but no further restriction) or Graphs with the same sequence
- After we have generated a price vector use G to check how close we were.
- Examine experiment various properties of these samples

Distribution of profits

- How much more money do we make applying a Same-Sequence price vector than a Same-Parameter to the true graph

Statistic of distribution of Profits

fdafd

Distribution of Prices

- The next question to ask is do the same sequence price vectors look like the optimal price vector?
- Again the answer is yes

Statistics of the Prices

fada

A second strategy

- Above we have shown that the price vector of generated graphs is on average like the optimal price vector

Strategy 2 The averaged price vector is even more like the optimal price vector

$$v = \frac{1}{\#Trials} \sum Profit_G(\text{Guessed vector}) \tag{4}$$

$$v = Profit_G(\frac{1}{\#Trials} \sum \text{Guessed vector}) \tag{5}$$

Results

Other Directions

- If knowing the degrees is good then maybe knowing $[|N(v)|, |N(N(v)) \setminus (N(v) \cup v)|$ is better
- i.e. how many nodes can v reach in 1 or 2 steps
- What about k steps?

Results for limited walk information

Future Work

- Mathematical Assurances of error in either strategy
- Other Network distributions other than Erdos Renyi


O. Candogan, K. Bimpikis, and A. Ozdaglar, "Optimal pricing in networks with externalities," *Operations Research*, vol. 60, no. 4, pp. 883–905, 2012.


J. Huang, A. Mani, and Z. Wang, "The value of price discrimination in large social networks," *Management Science, Forthcoming*, 2021.