phys 218 ch 6.1 reading take 2

Ψ is the wave-function that is dependant on the potential field that allows us to calculate the probable values of the particles position, energy, and momentum

- cannot find exact -

NO solve for x, E, p

Solve instead for $\langle x \rangle$, $\langle E \rangle$, $\langle p \rangle$

Time dependant schrodinger eq describes a particle moving through a field in 1 dimension

$$\underbrace{\underbrace{i\hbar \frac{\partial \Psi(x,t)}{\partial t}}_{\text{the time?}} = \underbrace{-\frac{\hbar^2}{2m} \frac{\partial^2 \Psi(x,t)}{\partial x^2}}_{\text{the position?}} + \underbrace{V\Psi(x,t)}_{\text{the potential } \Psi}}$$

** not related

the classical wave equation:

$$\frac{\partial^2 \Psi(x,t)}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 \Psi(x,t)}{\partial t^2}$$

schroedinger eq has no derivative but does describe reality

the wave function must be linear such that it can interfere with itself (sum) and form wave packets:

$$\Psi(x,t) = a\,\Psi_1(x,t) + b\,\Psi_2(x,t)$$

a and b are constants

two waves satisfying schrodinger eq

1 SQUARE = ____________

Example 1

can we prove $\Psi(x,t) = a\Psi_1(x,0) + b\Psi_2(x,0)$?

first take the derivatives:

$$\frac{\partial \Psi(x,t)}{\partial t} = a\frac{\partial \Psi_1}{\partial t} + b\frac{\partial \Psi_2}{\partial t} \qquad \frac{\partial}{\partial t} \text{ time 1}$$

$$\frac{\partial \Psi(x,t)}{\partial x} = a\frac{\partial \Psi_1}{\partial x} + b\frac{\partial \Psi_2}{\partial x} \qquad \frac{\partial}{\partial x} \text{ pos 1}$$

this was only written to get this one

$$\frac{\partial^2 \Psi(x,t)}{\partial x^2} = a\frac{\partial^2 \Psi_1}{\partial x^2} + b\frac{\partial^2 \Psi_2}{\partial x^2} \qquad \frac{\partial^2}{\partial x^2} \text{ pos 2}$$

time 1 and pos 2 are in schrodinger eq

$$i\hbar\frac{\partial \Psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi(x,t)}{\partial x^2} + V\Psi(x,t)$$

then combine the derivatives

$V(a\Psi_1 + b\Psi_2)$

$$i\hbar\left(a\frac{\partial \Psi_1}{\partial t} + b\frac{\partial \Psi_2}{\partial t}\right) = -\frac{\hbar^2}{2m}\left(a\frac{\partial^2 \Psi_1}{\partial x^2} + b\frac{\partial^2 \Psi_2}{\partial x^2}\right) + \cancel{V\Psi(x,t)}$$

$$i\hbar a\frac{\partial \Psi_1}{\partial t} + i\hbar b\frac{\partial \Psi_2}{\partial t} = -\frac{\hbar^2}{2m}a\frac{\partial^2 \Psi_1}{\partial x^2} - \frac{\hbar^2}{2m}b\frac{\partial^2 \Psi_2}{\partial x^2} + Va\Psi_1 + Vb\Psi_2$$

$$i\hbar a\frac{\partial \Psi_1}{\partial t} + \frac{\hbar^2}{2m}a\frac{\partial^2 \Psi_1}{\partial x^2} - Va\Psi_1 = -i\hbar b\frac{\partial \Psi_2}{\partial t} - \frac{\hbar^2}{2m}b\frac{\partial^2 \Psi_2}{\partial x^2} + Vb\Psi_2$$

$$a\left(i\hbar\frac{\partial \Psi_1}{\partial t} + \frac{\hbar^2}{2m}\frac{\partial^2 \Psi_1}{\partial x^2} - V\Psi_1\right) = -b\left(i\hbar\frac{\partial \Psi_2}{\partial t} + \frac{\hbar^2}{2m}\frac{\partial^2 \Psi_2}{\partial x^2} - V\Psi_2\right)$$

they're of the same form so they work

1 SQUARE = ____________

for a wave of wave num K and angular frequency w moving +x
the wave function is
$$\psi(x,t) = A\sin(Kx - wt + \phi)$$
note that this is not the most general form because our function may have sin and cosine and need not be real

$$\psi(x,t) = Ae^{i(Kx-wt)} = A\left[\cos(Kx-wt) + i\sin(Kx-wt)\right]$$

A may also be complex

example 2
if $\psi(x,t) = Ae^{i(Kx-wt)}$ can we show $Ae^{i(Kx-wt)}$ satisfies schroedinger

$$\frac{\partial \psi(x,t)}{\partial t} = -iwAe^{i(Kx-wt)} = -iw\psi$$

$$\frac{\partial \psi(x,t)}{\partial x} = iKAe^{i(Kx-wt)} = iK\psi$$

$$\frac{\partial^2 \psi(x,t)}{\partial x^2} = i^2K^2\psi = -K^2\psi$$

$$i\hbar(-iw\psi) = -\frac{\hbar^2}{2m}(-K^2\psi) + V\psi$$

$$\left(\hbar w - \frac{\hbar^2K^2}{2m} - V\right)\psi = 0 \quad \longleftrightarrow$$

$E = hf = \hbar w$
$p = \hbar K$

$$\left(E - \frac{p^2}{2m} - V\right) = 0$$

because $E = K + V = \frac{p^2}{2m} + V$ is true Non Relativistic

this shows $\psi(x,t) = Ae^{i(Kx-wt)}$ is an acceptable form of the schroedinger eq

1 SQUARE = ____________

this has demonstrated acceptable form of the time dependent schrodinger eq $\Psi(x,t)$

Now for normalization and probability

Probability interpretation: the probability $P(x)dx$ of finding a particle between x and $x+dx$

$$P(x)\,dx = \Psi(x,t)^{*}\,\Psi(x,t)\,dx$$

$$P = \int_{x_1}^{x_2} \Psi^{*}\Psi\,dx \leftarrow \text{Probability}$$

~~the complex conjugate of $a+bi = a-bi$ thus $(a+bi)(a-bi)$ $a^2 - abi$~~

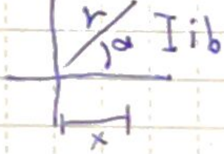
$$\int_{-\infty}^{\infty} \Psi^{*}(x,t)\,\Psi(x,t)\,dx = 1 \leftarrow \text{Normalizes}$$

because the particle must exist

- Complex interuption -

if z is a complex number $z = a+bi$ then the complex conjugate of

z, $\bar{z} = a - bi$ $\quad z = a+bi = re^{i\alpha}$ (euler) same complex # vector



$z\bar{z} = (a+bi)(a-bi) = a^2 - abi + abi + b^2i^2 = a^2 + 0 + (-1)b^2$

$a^2 - b^2 = z\bar{z}$

- resume -

Factoring $\quad z^{*}z = (a+bi)(a-bi)$ when z is real $z = a - 0i$

so $z^{*}z = (a+0)(a+0) = a^2 = z^2$ sure!

1 SQUARE =

there was normalization and probability for the time dependent schrodinger eq.

time independent schrodinger eq

most potentials will not depend on the time, thus time and pos could be separated

$$\Psi(x,t) = \psi(x) f(t)$$

(insert into schrodinger)

$$i\hbar\, \psi(x) \frac{\partial f(t)}{\partial t} = -\frac{\hbar^2 f(t)}{2m} \frac{\partial^2 \psi(x)}{\partial x^2} + V(x)\psi(x) f(t)$$

divide by $\psi(x) f(t)$

$$\underbrace{i\hbar \frac{1}{f(t)} \frac{\partial f(t)}{dt}}_{\text{the left side depends exclusively on time}} = \underbrace{-\frac{\hbar^2}{2m} \frac{1}{\psi(x)} \frac{d^2\psi(x)}{dx^2} + V(x)}_{\text{the right side depends exclusively on pos}}$$

Since each side is in one variable they must also be equal to a constant (labeled B) and we solve for it?

time: $i\hbar \frac{1}{f(t)} \frac{df(t)}{dt} = B \rightarrow i\hbar \frac{1}{f}\frac{df}{dt} = B \rightarrow i\hbar \frac{df}{f} = B\,dt$

$i\hbar \int \frac{df}{f} = \int B\,dt \rightarrow i\hbar \ln f = Bt + C \leftarrow$ we can think of C as 0

thus

$$\ln f = \frac{Bt}{i\hbar}$$

thus $\rightarrow f(t) = e^{Bt/i\hbar} = e^{-iBt/\hbar}$

1 SQUARE =

compare to $f(t)$ for time dependence $(e^{-i\omega t})$

$$B = \hbar\omega = E$$
$$B = E$$

$$i\hbar \frac{1}{f(t)} \frac{df(t)}{dt} = E$$

Most important

thus

$$\boxed{-\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)}$$ this is the time-independent schroedinger wave equation

$$f(t) = e^{-iBt/\hbar} = e^{-i\omega t}$$ thus $\Psi(x,t) = \psi(x) e^{-i\omega t}$

the probability density $\Psi^*\Psi = \psi^2(x)(e^{i\omega t} e^{-i\omega t}) = \psi^2(x)$
where the potential is not time dependent the probability distro is constant in time and depends only on position

1 SQUARE =

expectation values

holy moly this is chl day ! please god help me

the expectation value is the avg value of a property (pos, momentum, E) in a wave function taken over many measurements

the equation for average where N_i is the number of times particle x was observed at pos i

$$\bar{x} = \frac{N_1x_1 + N_2x_2 + \cdots + N_ix_i}{N_1 + N_2 + \cdots + N_i} = \frac{\sum_i N_i x_i}{\sum_i N_i}$$

these variable are discrete but our function is continuous

$$\downarrow$$

$$\bar{x} = \frac{\int_{-\infty}^{\infty} x P(x)\,dx}{\int_{-\infty}^{\infty} P(x)\,dx}$$

$$P(x)dx = \Psi^*(x,t)\,\Psi(x,t)\,dx$$

$$\langle x \rangle = \frac{\int_{-\infty}^{\infty} x\,\Psi^*(x,t)\,\Psi(x,t)\,dx}{\int_{-\infty}^{\infty} \cancel{P(x)dx}\,\Psi^*(x,t)\,dx\,\Psi(x,t)}$$

this is the normalization equation and is = 1

If the wave function is not normalized use this

$$\downarrow$$

$$\langle x \rangle = \int_{-\infty}^{\infty} x\,\Psi^*(x,t)\,\Psi(x,t)\,dx$$

this procedure can be used to find the expectation value for a function g(x) for a normalized wave function

$$\langle g(x) \rangle = \int_{-\infty}^{\infty} \Psi^*(x,t)\,g(x)\,\Psi(x,t)\,dx$$

the wave function can only give the expectation value for a function g that is written in terms of x ie cannot give individual measurements consider the wave function of the free particle $\Psi(x,t) = e^{i(kx-wt)}$

$$\frac{\partial \Psi}{\partial x} = \frac{\partial}{\partial x} e^{i(kx-wt)} = ike^{i(kx-wt)} = ik\Psi$$ and known: $k = p/\hbar$

$$\frac{\partial \Psi}{\partial x} = i\frac{p}{\hbar}\Psi$$

re-arrangement

$$p\left[\Psi(x,t)\right] = -i\hbar\frac{\partial \Psi(x,t)}{\partial x}$$

operator is a function that transforms one into another three $\hat{Q}$ to be an operator

$$\hat{Q}\, f(x) = g(x)$$

p is an operator: $-i\hbar\frac{\partial}{\partial x}$ is an operator on the function $\Psi(x,t)$

$\hat{p}$ is the momentum operator $-i\hbar\frac{\partial}{\partial x}$ and $-i\hbar\frac{\partial}{\partial x}\Psi(x,t) = \hat{p}\Psi(x,t)$

every physical observable has an operator used to compute the expectation value

$\hat{Q}$ must operate on $\Psi(x,t)$

$$\langle Q \rangle = \int_{-\infty}^{\infty} \Psi^*(x,t)\,\hat{Q}\,\Psi(x,t)\,dx$$

the expectation value of Q ($\langle Q \rangle$) is the integral where the wave function is operated on by the operator of Q $\hat{Q}$

1 SQUARE = __________

$z = a + bi$ a and b are constant real numbers

$\bar{z} = a - bi$

$z\bar{z} = (a+bi)(a-bi) = a^2 + abi - abi + i^2b^2 = a^2 + 0 + (-1)b^2 = a^2 - b^2$

$z^*z = (a+bi)(a-bi) = z\bar{z} = a^2 - b^2$

Dear Ayten

$K = a + bi = 8 + (0)i$

$K^*K = (8+0i)(8-0i) = 8 \times 8 = 8^2 = K^2$

So the expectation value of p is

$$\langle p \rangle = -i\hbar \int_{-\infty}^{\infty} \psi^*(x,t) \frac{\partial \psi(x,t)}{\partial x} dx$$

take the time derivative

$$\frac{\partial \psi}{\partial t} = \frac{\partial}{\partial t} e^{i(kx-\omega t)} = -i\omega e^{i(kx-\omega t)} = -i\omega \psi$$

$\omega = E/\hbar$ because $E = \omega\hbar$ $\hat{E}$ = energy operator $= i\hbar \frac{\partial}{\partial t}$

$$E[\psi(x,t)] = i\hbar \frac{\partial \psi(x,t)}{\partial t}$$

$$\langle E \rangle = i\hbar \int_{-\infty}^{\infty} \psi^*(x,t) \frac{\partial \psi(x,t)}{\partial t} dx$$