Two coupled first order ODE's:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = 2x + 2y,$$
$$\frac{\mathrm{d}y}{\mathrm{d}t} = 2x - y,$$

can be written in matrix form:

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} f_1(t, u_1, u_2) \\ f_2(t, u_1, u_2) \end{bmatrix},$$

where $u_1 = x$, $u_2 = y$, $f_1(t, u_1, u_2) = 2x + 2y$, and $f_2(t, u_1, u_2) = 2x - y$. The code for a function defining the two 1st order ODE's in matrix form looks like:

```
% Note: Have to do t then u for MATLAB built-in functions like ode45
function dudt = f(t,u)

    dudt = zeros(2,1);
    dudt(1) = 2*u(1)+2*u(2);
    dudt(2) = 2*u(1)-u(2);

end
```

The general form is

$$\frac{\mathrm{d}\vec{u}}{\mathrm{d}t} = \vec{F}(t, \vec{u}),$$

where $\vec{u}$ and $\frac{\mathrm{d}\vec{u}}{\mathrm{d}t}$ are arrays.

Write a `dudt` function similar to the one above that could be used with MATLAB's built in solvers to solve the following two coupled ODE's:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = x - yt,$$
$$\frac{\mathrm{d}y}{\mathrm{d}t} = t + y.$$

```
function dudt = f(t,u)
    dudt = zeros(2, 1);
    dudt(1) = u(1) - u(2)*t;
    dudt(2) = t + u(2);
end
```

Write pseudo code to define the `dudt` function for the following two coupled first order differential equations:

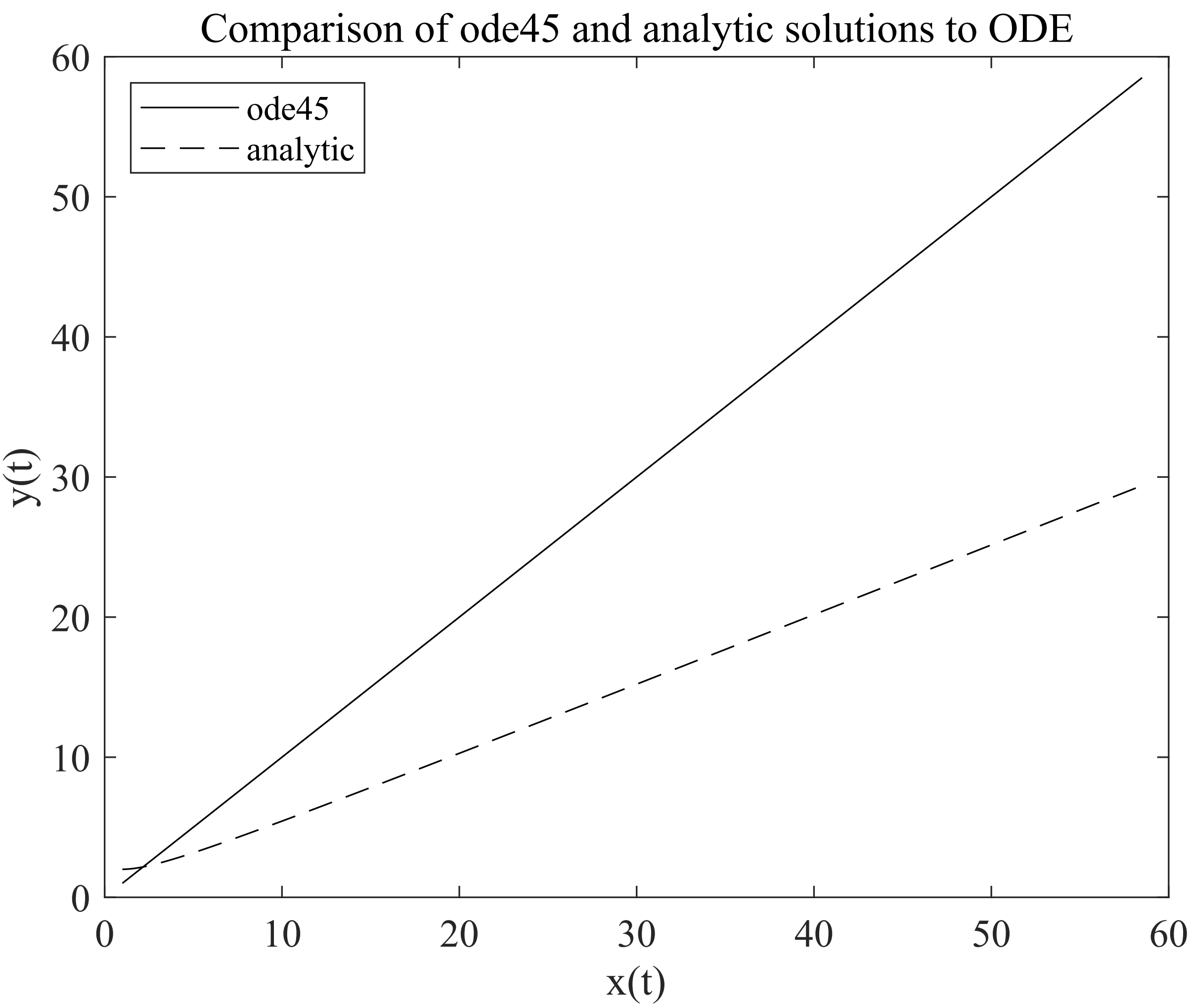
$$\frac{\mathrm{d}x}{\mathrm{d}t} = 2x + 2y,$$
$$\frac{\mathrm{d}y}{\mathrm{d}t} = 2x - y.$$

Since these equations are no more complicated than above, I can just write the code.

```
function dudt = f(t,u)
    dudt = zeros(2,1);
    dudt(1) = 2*u(1) + 2*u(2);
    dudt(2) = 2*u(1) - u(2);
end
```

Write code to solve the ODE's in the previous task from $t = 0$ to $t = 1.2$ with $x(0) = 1$ and $y(0) = 2$. Using a time step of $h = 0.4$. Compare the results with the exact solution:

$$x = \frac{e^{-2t}\left(8e^{5t} - 3\right)}{5},$$
$$y = \frac{2e^{-2t}\left(2e^{5t} + 3\right)}{5}.$$



Comparison of ode45 and analytic solutions to ODE
ode45
analytic
y(t)
x(t)
0
10
20
30
40
50
60