

Midterm Exam #1

**Instructions:** This is a take-home problem and your solutions must be submitted on or before **Thursday, April 25, 2024 at 11:59 pm**. No late work will be accepted for credit. No resources other than your *own* class notes, *Introduction to Quantum Mechanics* by David J. Griffiths, or an integral table may be used as a reference. Do not consult the internet or use computer programs such as Mathematica, Maple, or other software to solve any of these problems. Do not work together with anyone, and refrain from even discussing the exam with your classmates. Failure to abide by these rules will result in receiving zero credit for the take-home portion of midterm exam #1.

1. (20 points) A non-relativistic particle with mass  $m$  moves in a three-dimensional potential,  $V(r)$ , which is spherically-symmetric and vanishes as  $r \rightarrow \infty$ . At a certain time, this particle is found in the state,

$$\psi(r, \theta, \phi) = Cr^{\sqrt{3}}e^{-\alpha r} \cos \theta,$$

where  $C$  and  $\alpha$  are constants. We have ignored spin.

- (a) What is the orbital angular momentum of this state (*i.e.*, what are  $\ell$  and  $m$ )? If you play your cards right, no calculations are required to answer this question.
- (b) What is the energy of this state,  $E$ ? You may find it helpful to use the radial equation for the variable  $u = rR$  to determine the energy. At the end of your calculation, you will find it helpful to use the property  $V(r) \rightarrow 0$  as  $r \rightarrow \infty$ .
- (c) Now that you know  $E$  from (part (b)), what is the potential,  $V(r)$ ?