Calvin Sprouse PHYS 475 Homework 4 2024 May 3

1. Consider an infinite square well that runs from 0 to a. As a perturbation, we place a delta-function bump at the center of the well,

$$\hat{H}' = \alpha\delta\left(x - \frac{a}{2}\right),$$

where α is a constant.

(a) Determine the first-order correction to the allowed energies. Why are the energies for even n unperturbed?

(b) Determine the first three non-zero terms in the perturbation expansion,

$$\psi_n^1 = \sum_{m \neq n} \frac{\langle \psi_m^0 | \hat{H}' | \psi_n^0 \rangle}{E_n^0 - E_m^0} \psi_m^0,$$

of the correction to the ground state ψ_1^1.

(a)

2. The allowed energies for the harmonic oscillator are

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega,$$

where $\omega = \sqrt{k/m}$ is the classical frequency and the potential energy is

$$V(x) = \frac{1}{2}kx^2.$$

Suppose the spring constant increases lightly, $k \to (1+\epsilon)\,k$.

(a) Determine the exact new energies, then expand the formula as a power series in ϵ up to second order.

(b) Calculate the first-order perturbation to the energy using

$$E_n^1 = \langle \psi_n^0 | \hat{H}' | \psi_n^0 \rangle.$$

To preform this calculation, it will be necessary to determine what $\hat{H}'$ is in this case. Compare this result with the result from part (a).

(a)