PHYS 475 - Quantum Mechanics **Spring 2024**

Homework #7

1. Using a trial wavefunction,

$$\psi(x) = Ae^{-bx^2},$$

obtain the lowest upper bound you can on the ground state energies associated the following one-dimensional potential energies.

(a) $V(x) = \alpha|x|$.

(b) $V(x) = \alpha x^4$.

2. Use the WKB approximation to determine the allowed energies, E_n, of an infinite-square well with a "shelf" of height V_0 extending half-way across,

$$V(x) = \begin{cases} V_0, & 0 < x < a/2 \\ 0, & a/2 < x < a \\ \infty, & \text{otherwise} \end{cases}$$

Express your answer in terms of V_0 and $E_n^0 \equiv (n\pi\hbar)^2/2ma^2$ (the energy for the n^{th} state of the infinite-square well without the "shelf"). Assume that $E_1^0 > V_0$, but do not assume that $E_n >> V_0$.

3. Use the WKB approximation result for transmission probability,

$$T \simeq e^{-2\gamma} \text{ where } \gamma \equiv \frac{1}{\hbar}\int_0^a |p(x)|dx,$$

.

to compute the approximate transmission probability for a particle with energy E that encounters a finite square barrier with height $V_0 > E$ and width $2a$. Compare your answer to the exact result,

$$T^{-1} = 1 + \frac{V_0^2}{4E(V_0 - E)} \sinh^2\left(\frac{2a}{\hbar}\sqrt{2m(V_0 - E)}\right),$$

which should reduce to your answer in the WKB regime where $T << 1$.