1. A particle is trapped in a harmonic oscillator potential. We know that at $t = 0$, the particle can be represented by the wavefunction

$$\Psi(x,0) = A\left(2\psi_0(x) + 5\psi_2(x)\right),$$

where ψ_0 and ψ_2 are the stationary-state solutions for $n = 0$ and $n = 0$, respectively.

(a) Normalize $\Psi(x,0)$.

$$\begin{aligned}1 &= \langle \Psi(x,0) \mid \Psi x, 0\rangle \\ &= A^2\left(4\langle\psi_0 \mid \psi_0\rangle + 25\langle\psi_2 \mid \psi_2\rangle + 20\langle\psi_0 \mid \psi_2\rangle\right) \\ &= 29A^2.\end{aligned}$$

Thus, $A = \sqrt{1/29}$.

(b) Construct $\Psi(x,t)$ and then determine $|\Psi(x,t)|^2$. Will $\langle x\rangle$ depend on time?

The complete wavefunction is given by

$$\Psi(x,t) = \sqrt{\frac{1}{29}}\left(2\Psi_0 + 5\Psi_2\right),$$

where Ψ_n represents $\psi_n(x)\phi_n(t)$. Thus,

$$\begin{aligned}|\Psi(x,t)|^2 &= \frac{1}{29}\left(2\Psi_0^* + 5\Psi_2^*\right)\left(2\Psi_0 + 5\Psi_2\right) \\ &= \frac{1}{29}\left(4\left|\Psi_0\right|^2 + 2\Psi_0^*\Psi_2 + 5\Psi_2^*\Psi_0 + 25\left|\Psi_2\right|^2\right) \\ &= \frac{1}{29}\left(4\left|\psi_0\right|^2 + 20\psi_0\psi_2\left(e^{-2i\omega t} + e^{2i\omega t}\right) + 25\left|\psi_2\right|^2\right) \\ &= \frac{1}{29}\left(4\left|\psi_0\right|^2 + 10\psi_0\psi_2\cosh(2i\omega t) + 25\left|\psi_2\right|^2\right)\end{aligned}$$

2. Consider the stationary states of the harmonic oscillator. As straightforwardly as possible, compute the following quantities for the nth stationary state $\psi_n(x)$.

   (a) $\langle x \rangle$

   (b) $\langle x^2 \rangle$

   (c) $\langle p \rangle$

   (d) $\langle p^2 \rangle$

   (e) $\langle T \rangle$

   (f) Is the Heisenberg uncertainty principle satisfied for all values of n?

3. A particle in a harmonic oscillator potential is described by the normalized wave-function

$$|\Psi(x,0)\rangle = \frac{1}{\sqrt{5}}|1\rangle + \frac{2}{\sqrt{5}}|2\rangle,$$

where $|n\rangle$ represents the nth stationary state.

(a) What is $|\Psi(x,t)\rangle$?

(b) What is the expectation value for energy?

(c) What is $\langle x(t)\rangle$?