Calvin Sprouse MATH407 HW2 Due 2024 Jan 19

1. Show that the converse of p4 is not true. That is, prove a counterexample of an asymmetric binary relation R with elements x and y such that $\neg xRy$ does not imply yRx.

2. Consider the set of all triples where each component is a real number, $\mathbb{R}^3$. Let $x = (x_1, x_2, x_3)$ and $y = (y_1, y_2, y_3)$ and define the weak preference $x \preccurlyeq^* y$ if $x_i \leq y_i$ for at least two out of the three components, $i = 1, 2, 3$.

   (a) Show $\preccurlyeq^*$ is connected.

   (b) Show $\preccurlyeq^*$ is not transitive by providing a counterexample.

   (c) Define the strict preference $x \prec^* y$ by $x \preccurlyeq^* y$ but not $y \preccurlyeq^* x$.

      i. Explain why it is equivalent to say $x \prec^* y$ if $x_i < y_i$ for at least two out of the three components, $i = 1, 2, 3$.

      ii. Prove or give a counterexample: $\prec^*$ is symmetric.

      iii. Prove or give a counterexample: $\prec^*$ is negatively transitive.