1. Use
$$\vec{B}_{\text{int}} = \frac{1}{4\pi\epsilon_0}\frac{e}{m_e c^2 r^3}\vec{L},$$
where $e$ is the elementary charge, to estimate the internal magnetic field in a hydrogen atom. This value characterizes the boundary between the strong and weak field limit.

The magnitude of the internal magnetic field is given by
$$\vec{B}_{\text{int}} \cdot \vec{B}_{\text{int}} = \left|\vec{B}_{\text{int}}\right| = \left(\frac{1}{4\pi\epsilon_0}\frac{e}{m_e c^2}\right)^2 \left(\frac{1}{r^3}\right)^2 \left|\vec{L}^2\right|.$$
Let $\beta$ be defined as the pre-factors,
$$\beta \equiv \frac{1}{4\pi\epsilon_0}\frac{e}{m_e c^2}.$$
Then, the magnitude of the internal magnetic field may be represented in terms of operators
$$\left|\vec{B}_{\text{int}}\right| = \hat{B}_{\text{int}} = \beta^2 \frac{1}{\hat{r}^3}\frac{1}{\hat{r}^3}\hat{L}^2.$$
The average internal magnetic field can then be found as
$$\langle \Psi | \hat{B}_{\text{int}} | \Psi \rangle = \beta^2 \langle \Psi | \left(\frac{1}{\hat{r}^3}\frac{1}{\hat{r}^3}\hat{L}^2\right) | \Psi \rangle = \beta^2 \left\langle \frac{1}{r^3}\frac{1}{r^3} L^2 \right\rangle.$$
We may insert the identity matrix between each of the three terms of the expectation value. The identity matrix may be expressed as the outer-product of wave-functions. This allows us to express the expectation value of the internal magnetic field as
$$\langle B_{\text{int}} \rangle = \beta^2 \left\langle \frac{1}{r^3} \right\rangle \left\langle \frac{1}{r^3} \right\rangle \left\langle L^2 \right\rangle.$$
These expectation values are known for our chosen basis:
$$\left\langle \frac{1}{r^3} \right\rangle = \frac{1}{l\left(1+\frac{1}{2}\right)(l+1)n^3 a^3}, \quad \left\langle L^2 \right\rangle = \hbar^2 l(l+1).$$
Then,
$$\langle B_{\text{int}} \rangle = \beta^2 \frac{\hbar^2 l(l+1)}{\left[l\left(l+\frac{1}{2}\right)(l+1)n^3 a^3\right]^2} = \beta^2 \frac{\hbar^2}{l(l+1)\left[\left(l+\frac{1}{2}\right)n^3 a^3\right]^2}.$$

This implies that the strength of the internal magnetic field is strongest at low values of $n$ and $l$, which makes sense by the classical picture. Let $n = 1$. Then, $l \in \{0, 1\}$. While $l = 0$ appears problematic, we recall that for $l = 0$ $\langle L^2 \rangle = 0$ and thus $\langle B_{\text{int}} \rangle = 0$. Therefore, let $l = 1$. Then, substituting known values,

$$\langle B_{\text{int}} \rangle \approx 34\,\text{T}.$$

Therefore, $|B_{\text{ext}}| \ll 30\,\text{T}$ is a weak field while $|B_{\text{ext}}| \gg 30\,\text{T}$ is a strong field.

2. Consider the eight $n = 2$ states for the hydrogen atom, $\langle 2, l, j, m_j|$. Determine the energy of each state under weak-field Zeeman splitting and construct a diagram like the one in Figure 6.11 of Griffiths to show how the energies evolve as a function of $B_{\text{ext}}$. Label each line clearly and indicate the slope of each line on the graph.