Math 407 Homework 2

Due by 5pm on **Friday, Jan. 19**

Neatly write (or better yet, type up) your answers to the following questions and submit a pdf copy.

1. Show that the converse of p4 is not true. That is, provide a counterexample of an asymmetric binary relation R with elements x and y such that (not xRy) does not imply yRx.

2. Consider the set of all triples where each component is a real number (that is, $\mathbb{R}^3$). Let $x = (x_1, x_2, x_3)$ and $y = (y_1, y_2, y_3)$ and define the weak preference $x \preccurlyeq^* y$ if $x_i \leq y_i$ for at least two out of the three components, $i = 1, 2, 3$.

   (a) Show that (or clearly explain why) $\preccurlyeq^*$ is connected.

   (b) Show that $\preccurlyeq^*$ not transitive by providing a counterexample.

   (c) Define the strict preference $x \prec^* y$ by $x \preccurlyeq^* y$ but not $y \preccurlyeq^* x$.

      i. Explain why it is equivalent to say $x \prec^* y$ if $x_i < y_i$ for at least two out of the three components, $i = 1, 2, 3$.

      ii. Is $\prec^*$ asymmetric? Provide a short proof or give a counterexample.

      iii. Is $\prec^*$ negatively transitive? Provide a short proof or give a counterexample.