PHYS 475 - Quantum Mechanics **Spring 2024**

Homework #1

1. Use separation of variables in cartesian coordinates to solve the infinite cubical well (this three-dimensional analogue of the infinite-square well is also called the "particle in a box"). The potential of the infinite cubical well is

$$V(x, y, z) = \begin{cases} 0, & \text{if } x, y, z \text{ are all between 0 and } a \\ \infty, & \text{otherwise} \end{cases}$$

(a) Find the stationary states (you don't need to normalize them) and their corresponding energies. You should use separation of variables to convert the three-dimensional time-independent Schrödinger equation (a partial differential equation) into three recognizable ordinary differential equations. You will also need to apply boundary conditions at the boundaries of the well. Note that your solutions should depend on three distinct quantum numbers.

(b) You should call the distinct energies E_1, E_2, E_3, etc, in order of increasing energy. Note that the subscripted integer is <u>not</u> a quantum number here (remember you have three quantum numbers characterizing your states); instead, these numbers simply represent the lowest energy (E_1), second lowest energy (E_2), etc. Determine what E_1, E_2, E_3, E_4, E_5, and E_6 are in terms of $\hbar, m, \pi$, and a. Determine the degeneracy of each of these energies (*how many distinct states have these energies*).

(c) What is the degeneracy of E_{14}, and what is unusual about this case?

2. Construct the spherical harmonics $Y_0^0(\theta, \phi)$ and $Y_2^1(\theta, \phi)$ using the formula for spherical harmonics,

$$Y_\ell^m(\theta, \phi) = \epsilon \sqrt{\frac{(2\ell + 1)}{4\pi} \frac{(\ell - |m|)!}{(\ell + |m|)!}} e^{im\phi} P_\ell^m(\cos\theta),$$

(where the constant $\epsilon = (-1)^m$ for $m \geq 0$ and $\epsilon = 1$ for $m \leq 0$), the Rodrigues formula for the Legendre polynomial,

$$P_\ell(x) \equiv \frac{1}{2^\ell \ell!} \left(\frac{d}{dx}\right)^\ell (x^2 - 1)^\ell,$$

and the definition of the associated Legendre function,

$$P_\ell^m(x) \equiv (1 - x^2)^{|m|/2} \left(\frac{d}{dx}\right)^{|m|} P_\ell(x).$$

Check that your solutions for $Y_0^0(\theta, \phi)$ and $Y_2^1(\theta, \phi)$ are normalized and orthogonal.