

1. In the context of exercise 5 (iii) on page 31:

- a. Give two examples of open sets that are not \mathbb{R} or \emptyset . Use at least one complete sentence to explain why the given sets are open.

τ_3 consists of \mathbb{R} , \emptyset , and every interval $[n, \infty)$, for $n \in \mathbb{R}^+$.

The sets $[1, \infty)$ and $[2, \infty)$ are open to τ_3 . Either sets compliment is an interval from $-\infty$ to the start-point exclusive. Since the compliment of the sets is not in τ_3 , the sets are open to τ .

- b. Give two examples of closed sets that are not \mathbb{R} or \emptyset . Use at least one complete sentence to explain why the given sets are closed.

The sets $(-\infty, 1)$ and $(-\infty, 2)$ are closed to τ_3 . These are, in fact, the compliments to the sets defined above. Since the compliment of either set is in τ_3 , these sets are closed to τ_3 .

2. In the context of exercise 6 (ii) on page 31:

- a. Give two examples of open sets that are not \mathbb{N} or \emptyset . Use at least one complete sentence to explain why the given sets are open.

τ_2 consists of \mathbb{N} , \emptyset , and every set $\{n, n+1, \dots\}$, for $n \in \mathbb{Z}^+$. This is called the final segment topology.

The sets $\{2, 3, 4, \dots\}$ and $\{3, 4, 5, \dots\}$ are in τ_2 and are thus open sets.

- b. Give two examples of closed sets that are not \mathbb{N} or \emptyset . Use at least one complete sentence to explain why the given sets are closed.

The sets $\{1\}$ and $\{1, 2\}$ are closed sets to τ_2 . These sets are the compliments to the sets define above over the positive integers, \mathbb{N} , and are thus closed sets to τ_2 .

3. Exercise 1.2: #2 (page 36). Let (X, τ) be a topological space with the property that every subset is closed. Prove that this is a discrete space.

Proof.

□