PHYS 474 - Quantum Mechanics **Winter 2024**

Homework #2

1. Let the wavefunction, $\Psi(x, t)$, be a solution to the time-dependent Schrödinger equation when the potential energy is given by $V(x)$. What is the solution to the Schrödinger equation if we now consider a potential of $V(x) + V_0$ where V_0 is a real, positive constant. To solve this, you should start with the time-dependent Schrödinger equation and use separation of variables. Compare the results for the wavefunctions obtained in these two cases. You should find that including V_0 introduces a phase factor of the form $e^{-iV_0t/\hbar}$ into the wavefunction.

2. A particle is observed in a quantum state described by the wavefunction,

$$\Psi(x, t) = Ae^{-a\left[\frac{mx^2}{\hbar}+it\right]}.$$

where A and a are positive, real constants.

(a) Normalize Ψ.
(b) What is the potential $V(x)$ that this particle finds itself within?
(c) Determine the expectation values $\langle x\rangle$, $\langle x^2\rangle$, $\langle p\rangle$, and $\langle p^2\rangle$.
(d) Determine the standard deviations for position and momentum, σ_x and σ_p.
(e) Are your values for σ_x and σ_p consistent with the uncertainty principle?

3. An electron is trapped in a harmonic (quadratic) potential. You determine that it has the following expectation value for its position: $\langle x\rangle = \frac{a}{2}\sin(\omega t)$. Here, a is a real, constant with units of length and ω is an angular frequency. What, if anything, can you tell me about the electron's momentum?