1. Use separation of variables in cartesian coordinates to solve the infinite cubical well. The potential of the infinite cubical well is

$$V(x,y,z) = \begin{cases} 0, & \text{if } x, y, z \text{ are all between 0 and } a, \\ \infty, & \text{otherwise.} \end{cases}$$

   (a.) Find the stationary states and their corresponding energies. Apply boundary conditions at the boundaries of the well. Note that solutions should depend on three distinct quantum numbers.

   (b.) Call the distinct energies E_1, E_2, E_3, etc., in order of increasing energy. Note that the subscripted integer is not a quantum number here; instead, these number simply represent the lowest energy, E_1, second lowest energy, E_2, etc. Determine what E_1, E_2, E_3, E_4, E_5, and E_6 are in terms of $\hbar, m, \pi$, and a. Determine the degeneracy of each of these energies.

   (c.) What is the degeneracy of E_{14}, and what is unusual about this case?

2. Construct the spherical harmonics $Y_0^0(\theta,\phi)$ and $Y_2^1(\theta,\phi)$ using the formula for spherical harmonics,

$$Y_\ell^m(\theta,\phi) = \epsilon\sqrt{\frac{(2\ell+1)}{4\pi}\frac{(\ell-|m|)!}{(\ell+|m|)!}}\exp(im\phi)P_\ell^m\left(\cos(\theta)\right),$$

where the constant ϵ is defined to be

$$\epsilon = \begin{cases} (-1)^m, & m \geq 0, \\ 1, & m \leq 0, \end{cases}$$

the Rodrigues formula for the Legendre polynomial,

$$P_\ell^m(x) \equiv \left(1-x^2\right)^{|m|/2}\frac{\mathrm{d}^{|m|}}{\mathrm{d}x^{|m|}}P_\ell(x).$$

Check that solutions for $Y_0^0(\theta,\phi)$ and $Y_2^1(\theta,\phi)$ are normalized and orthogonal.