PHYS 475 - Quantum Mechanics **Spring 2024**

Homework #2

1. (a) Normalize R_{20} for hydrogen and construct the wave function ψ_{200}.

   (b) Normalize R_{21} for hydrogen and construct the wave functions ψ_{211}, ψ_{210}, and ψ_{21-1}.

2. (a) Determine $\langle r \rangle$ and $\langle r^2 \rangle$ for an electron in the ground state of the hydrogen atom. Express your answers in terms of the Bohr radius.

   (b) Determine $\langle x \rangle$ and $\langle x^2 \rangle$ for an electron in the ground state of the hydrogen atom. If you exploit the symmetry of the ground state, you will not need to perform any new integration here.

   (c) Determine $\langle x^2 \rangle$ for an electron in a hydrogen atom in the state $n = 2$, $\ell = 1$, $m = 1$. It is helpful to use the fact that $x = r \sin\theta \cos\phi$.

3. (a) Starting with $[r_i, p_j] = -[p_i, r_j] = i\hbar\delta_{ij}$ and $[r_i, r_j] = [p_i, p_j] = 0$ where the index i stands for x, y, or z and $r_x = x, r_y = y, r_z = z$, work out the following commutator relations:

$$
\begin{array}{lll}
[L_z, x] = i\hbar y & [L_z, y] = -i\hbar x & [L_z, z] = 0 \\
[L_z, p_x] = i\hbar p_y & [L_z, p_y] = -i\hbar p_x & [L_z, p_z] = 0
\end{array}
$$

   (b) Use the results from part (a) and the definitions $L_x = yp_z - zp_y$, $L_y = zp_x - xp_z$, $L_z = xp_y - yp_x$ to obtain $[L_z, L_x] = i\hbar L_y$.

   (c) Evaluate the commutators $[L_z, r^2]$ and $[L_z, p^2]$ where $r^2 = x^2 + y^2 + z^2$ and $p^2 = p_x^2 + p_y^2 + p_z^2$.

   (d) Show that the Hamiltonian $\hat{H} = \frac{\hat{p}^2}{2m} + \hat{V}$ commutes with all three components of $\hat{\vec{L}}$ if $\hat{V}$ depends only on r.