

1. Let the wavefunction $\Psi(x, t)$ be a solution to the time-dependent Schrödinger equation when the potential energy is given by $V(x)$. What is the solution to the Schrödinger equation if we now consider a potential of $V(x) + V_0$ where V_0 is a real positive constant.

2. A particle is observed in a quantum state described by the wavefunction

$$\Psi(x, t) = A \exp \left(-a \left(\frac{mx^2}{\hbar} + it \right) \right),$$

where A and a are real positive constants.

- (a) Normalize Ψ .
- (b) What is the potential $V(x)$ that this particle finds itself within?
- (c) Determine the expectation values $\langle x \rangle$, $\langle x^2 \rangle$, $\langle p \rangle$, $\langle p^2 \rangle$.
- (d) Determine the standard deviations for position, σ_x , and momentum, σ_p .
- (e) Are your values for σ_x and σ_p consistent with the uncertainty principle?

3. An electron is trapped in a harmonic quadratic potential. Suppose the expectation value for its position is given by $\langle x \rangle = \frac{a}{2} \sin(\omega t)$. Here, a is a real constant with units of length and ω is an angular frequency. What, if anything, can be concluded about the electron's momentum?