Calvin Sprouse PHYS 475 Midterm Exam #1 2024 April 25

1. A non-relativistic particle with mass m moves in a three-dimensional potential, $V(r)$, which is spherically-symmetric and vanishes as $r \to \infty$. At a certain time, this particle is found in the state

$$\psi(r, \theta, \phi) = C r^{\sqrt{3}} \exp\left[-\alpha r\right] \cos\left[\theta\right],$$

where C and α are constants. We have ignored spin.

(a) What is the orbital angular momentum of this state; that is, what are the quantum numbers l and m_l?

(b) What is the energy, E, of this state? The radial equation, $u = rR$, may be helpful here. Recall, $V(r) \to 0$ as $r \to \infty$.

(c) Now that the energy, E, is known from part (b), what is the potential, $V(r)$?

(a) Since the potential, $V(r)$, is spherically-symmetric, the wavefunction, ψ, may be separated into a radial and angular function:

$$\psi(r, \theta, \phi) = R_{n,\ell}(r) Y_\ell^{m_\ell}(\theta, \phi).$$

Let $C = C_R C_Y$, where C_R is a constant associated with the radial equation and C_Y is a constant associated with the angular equation. The information about orbital angular momentum will come from the radial part of the equation, which will be a spherical harmonic of the form

$$Y_\ell^{m_\ell} = C_Y \cos\left[\theta\right].$$

Griffiths Table 4.3 lists normalized spherical harmonics for particular values of ℓ and m_ℓ. The only spherical harmonic of a related form is given by $\ell = 1$ and $m_\ell = 0$; that is,

$$Y_1^0 = \left(\frac{3}{4\pi}\right)^{1/2} \cos\left[\theta\right].$$

Thus, $\ell = 1$ and $m_\ell = 0$.

(b) Part (b).

(c) Part (c).