Many semiconductor devices are designed to confine electrons within a thin layer that is only a few nanometers thick. When a potential difference is applied across such a layer, the electrons respond as though they are trapped within a microscopic capacitor. If our "capacitor" plates are separated by a distance, L, that is of the order of the de Broglie wavelength of a trapped electron, we must apply a quantum treatment to the study of its behavior. We can treat this "capacitor" like an infinite square well,

$$V(x) = \begin{cases} 0, & 0 \leq x \leq L, \\ \infty, & \text{otherwise.} \end{cases}$$

Applying a potential difference of ΔV_0 across it introduces an additional potential energy of,

$$V'(x) = \frac{e\Delta V_0}{L} x.$$

Use perturbation theory to determine the first-order correction to the energy of the nth eigenstate associated with the perturbation.

The perturbation hamiltonian, $\hat{H}'$, is simply the perturbation potential,

$$\hat{H}' = \frac{e}{L}\Delta V_0 \hat{x}.$$

Then, the first-order energy correction is given by Griffiths Equation 7.9 to be

$$E_n^1 = \left\langle \psi_n^0 \middle| \hat{H}' \middle| \psi_n^0 \right\rangle,$$

where the unperturbed stationary states, ψ_n^0, are given by Griffiths Equation 2.31 to be

$$\psi_n^0 = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a} x\right),$$

where $a = L$. The first-order energy correction may be found by evaluating the integral corresponding to Equation 7.9. Then,

$$E_n^1 = \int_0^L \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L} x\right) \frac{e\Delta V_0}{L} x \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L} x\right) \, \mathrm{d}x,$$

where we have recognized that the only non-zero terms in the integral are in the region $0 \leq x \leq L$. Let a be a constant defined as

$$a \equiv \frac{n\pi}{L}.$$

Then,

$$E_n^1 = \frac{2e\Delta V_0}{L^2} \int_0^L x \sin^2(ax)\, \mathrm{d}x.$$

This integral is given by Equation 17 on the class Table of Integrals:

$$\int x \sin^2(ax)\, \mathrm{d}x = \frac{x^2}{4} - \frac{x\sin(2ax)}{4a} - \frac{\cos(2ax)}{8a^2}.$$

Then,

$$E_n^1 = \frac{2e\Delta V_0}{L^2}\left[\frac{L^2}{4} - \frac{L^2\sin(2n\pi)}{8(n\pi)^2} - \frac{L^2\cos(2n\pi)}{8(n\pi)^2} - 0 + 0 + \frac{L^2}{8\left(n\pi\right)^2}\right],$$

where trivial trigonometric functions have been simplified. We recognize that $\sin(2\pi n) = 0$ and $\cos(2\pi n) = 1$ for $n \in \mathbb{Z}^+$. Thus,

$$E_n^1 = \frac{e\Delta V_0}{2}.$$