1. Use

$$\vec{B}_{\text{int}} = \frac{1}{4\pi\epsilon_0}\frac{e}{m_e c^2 r^3}\vec{L},$$

where e is the elementary charge, to estimate the internal magnetic field in a hydrogen atom. This value characterizes the boundary between the strong and weak field limit.

The angular momentum vector may be expanded in the typical form,

$$\vec{L} = L_x\vec{x} + L_y\vec{y} + L_z\vec{z},$$

where $\vec{x}$ represents the unit vector of the x-axis. Then, invoking the (I THINK) Virial theorem allows $\vec{L}$ to be expressed in terms of quantum expectation values,

$$\vec{L} = \langle L_x\rangle\,\vec{x} + \langle L_y\rangle\,\vec{y} + \langle L_z\rangle\,\vec{z}.$$

Then, a particular component can be found by (NAME OF THEOREM OR EQUATION)

$$\langle L_x\rangle = \langle n, l, m_l, m_s|\hat{L}_x|n, l, m_l, m_s\rangle,$$

where the wavefunction is expressed in terms of the appropriate basis.

2. Consider the eight $n = 2$ states for the hydrogen atom, $\langle 2, l, j, m_j|$. Determine the energy of each state under weak-field Zeeman splitting and construct a diagram like the one in Figure 6.11 of Griffiths to show how the energies evolve as a function of B_{ext}. Label each line clearly and indicate the slope of each line on the graph.