1. In the context of exercise 5 (iii) on page 31:

    a. Give two examples of open sets that are not $\mathbb{R}$ or $\emptyset$. Use at least one complete sentence to explain why the given sets are open.

    τ_3 consists of $\mathbb{R}$, $\emptyset$, and every interval $[n, \infty)$, for $n \in \mathbb{R}^+$.

    The sets $[1, \infty)$ and $[2, \infty)$ are open to τ_3 by definition.

    b. Give two examples of closed sets that are not $\mathbb{R}$ or $\emptyset$. Use at least one complete sentence to explain why the given sets are closed.

    The sets $(-\infty, 1)$ and $(-\infty, 2)$ are closed to τ_3. These are, in fact, the compliments to the sets defined above. Since the compliment of either set is in τ_3, these sets are closed to τ_3.

2. In the context of exercise 6 (ii) on page 31:

    a. Give two examples of open sets that are not $\mathbb{N}$ or $\emptyset$. Use at least one complete sentence to explain why the given sets are open.

    τ_2 consists of $\mathbb{N}$, $\emptyset$, and every set $\{n, n+1, \dots\}$, for $n \in \mathbb{Z}^+$. This is called the final segment topology.

    The sets $\{2, 3, 4, \dots\}$ and $\{3, 4, 5, \dots\}$ are in τ_2 and are thus open sets.

    b. Give two examples of closed sets that are not $\mathbb{N}$ or $\emptyset$. Use at least one complete sentence to explain why the given sets are closed.

    The sets $\{1\}$ and $\{1, 2\}$ are closed sets to τ_2. These sets are the compliments to the sets define above over the positive integers, $\mathbb{N}$, and are thus closed sets to τ_2.

3. Exercise 1.2: #2 (page 36). Let (X, τ) be a topological space with the property that every subset is closed. Prove that this is a discrete space.

   Proof. Let (X, τ) be a topological space with the property that every subset is closed. Then, the subset S, $S \subseteq X$, is a closed set. The compliment of S in X, $X \backslash S$, is open in (X, τ) by Definition 1.2.4. Furthermore, the compliment of S in X is a subset of X, $(X \backslash S) \subseteq X$. Thus, the compliment of S in X is also closed in (X, τ). Since the compliment of S in X is closed, its compliment in X, the original set S, is open. Therefore, both S and it's compliment in X are clopen sets in (X, τ). Since every subset of X is clopen in X, τ, the topological space (X, τ) is the discrete space by the extended Definition 1.2.6. $\square$