PHYS 475 - Quantum Mechanics **Spring 2024**

Homework #5

1. Consider a quantum system that includes three distinct states; its Hamiltonian is

$$\hat{H} = V_0 \begin{bmatrix} (1-\epsilon) & 0 & 0 \\ 0 & 1 & \epsilon \\ 0 & \epsilon & 2 \end{bmatrix}$$

where V_0 is a constant and ϵ is a small unitless number ($\epsilon << 1$).

(a) Determine the eigenvectors and eigenvalues of the unperturbed Hamiltonian, $\hat{H}^0$ (just let $\epsilon = 0$ to access the unperturbed Hamiltonian).

(b) Determine the exact eigenvalues of $\hat{H}$, then expand each as a power series in ϵ up to second order. You may find it useful to use the binomial theorem to expand two of the eigenvalues in a power series.

(c) Use first- and second-order non-degenerate perturbation theory to determine the approximate eigenvalue for the state that grows out of the non-degenerate eigenvector of $\hat{H}^0$. Compare this with the exact result from part (b).

(d) Use degenerate perturbation theory to determine the first-order correction to the two initially degenerate eigenvalues. Compare this with the exact results.