1. Consider the continuous Gaussian distribution, $\rho(x) = Ae^{-\lambda(x-a)^2}$, where A, a, and λ are positive, real constants. Note that this is not a wavefunction, but rather a distribution.

   (a) Normalize the distribution to determine A.

$$\begin{aligned}
1 &= \int_{-\infty}^{\infty} \rho(x)\,\mathrm{d}x \\
&= \int_{-\infty}^{\infty} Ae^{-\lambda(x-a)^2}\,\mathrm{d}x \\
&= A\int_{-\infty}^{\infty} e^{-\lambda(x-a)^2}\,\mathrm{d}x \\
&= A\int_{-\infty}^{\infty} e^{-(\lambda x^2-2\lambda a x+\lambda a^2)}\,\mathrm{d}x \\
&= A\sqrt{\frac{\pi}{\lambda}}\exp\left(\frac{(-2\lambda a)^2-4\lambda^2 a^2}{4\lambda}\right) \\
&= A\sqrt{\frac{\pi}{\lambda}} \\
\sqrt{\frac{\lambda}{\pi}} &= A.
\end{aligned}$$

   (b) Determine $\langle x\rangle$, $\langle x^2\rangle$, and σ.

The average value of x is given by

$$\langle x\rangle = \int_{-\infty}^{\infty} x\rho(x)\,\mathrm{d}x$$

The average of the squares of x is given by

$$\langle x^2\rangle = \int_{-\infty}^{\infty} x^2\rho(x)\,\mathrm{d}x$$

The standard deviation, σ, of ρ is given by

$$\sigma = \sqrt{\langle x^2\rangle - \langle x\rangle^2}$$

2. At time $t = 0\,\mathrm{s}$, an electron is represented by the wave function,

$$\Psi(x,0) = \begin{cases} A\frac{x}{a}, & 0 \leq x \leq a \\ A\frac{(b-x)}{(b-a)}, & a \leq x \leq b \\ 0, & \text{otherwise} \end{cases}$$

where A, a, and b are constants.

(a) Normalize Ψ.

(b) Sketch $\Psi(x,0)$ as a function of x.

(c) Where is the electron most likely to be found at $t = 0\,\mathrm{s}$?

(d) What is the probability the electron will be found in the region $x \leq a$? Check your result in the limiting case where $b = a$ and $b = 2a$.

(e) Determine $\langle x \rangle$.