**PHYS 474 - Quantum Mechanics** **Winter 2024**

# Homework #4

1. A particle is trapped in a harmonic oscillator potential. We know that at $t = 0$, the particle can be represented by the wavefunction, $\Psi(x, 0) = A\,[2\psi_0(x) + 5\psi_2(x)]$, where $\psi_0$ and $\psi_2$ are the stationary-state solutions for $n = 0$ and $n = 2$, respectively.

   (a) Normalize $\Psi(x, 0)$.
   (b) Construct $\Psi(x, t)$ and then determine $|\Psi(x, t)|^2$. Will $\langle x\rangle$ depend on time?

2. Consider the stationary states of the harmonic oscillator. As straightforwardly as possible, compute the following quantities for the $n^{\text{th}}$ stationary state, $\psi_n(x)$:

   (a) $\langle x\rangle$
   (b) $\langle x^2\rangle$
   (c) $\langle p\rangle$
   (d) $\langle p^2\rangle$
   (e) $\langle T\rangle$
   (f) Is the Heisenberg uncertainty principle satisfied for all values of $n$?

3. A particle in a harmonic oscillator potential is described by the normalized wavefunction $|\Psi(x, 0)\rangle = \frac{1}{\sqrt{5}}|1\rangle + \frac{2}{\sqrt{5}}|2\rangle$ where $|n\rangle$ represents the $n^{\text{th}}$ stationary state.

   (a) What is $|\Psi(x, t)\rangle$?
   (b) What is the expectation value for energy?
   (c) What is $\langle x(t)\rangle$?