1. (a) Normalize $R_{2,0}$ for hydrogen and construct the wavefunction $\psi_{2,0,0}$.

   (b) Normalize $R_{2,1}$ for hydrogen and construct the wavefunction $\psi_{2,1,1}, \psi_{2,1,0}$, and $\psi_{2,1,-1}$.

2. (a) Determine $\langle r \rangle$ and $\langle r^2 \rangle$ for an electron in the ground state of the hydrogen atom. Express solutions in terms of the Bohr radius, a.

   (b) Determine $\langle x \rangle$ and $\langle x^2 \rangle$ for an electron in the ground state of the hydrogen atom. If the symmetry of the ground state is exploited, there will not be any new integration for this calculation.

   (c) Determine $\langle x^2 \rangle$ for an electron in a hydrogen atom in the state $n = 2$, $l = 1$, $m = 1$. It is helpful to use the fact that $x = r\sin(\theta)\cos(\phi)$.

3. (a) Starting with $[r_i, p_j] = -[p_i, r_j] = i\hbar\delta_{ij}$ and $[r_i, r_j] = [p_i, p_j] = 0$, where the index i stands for x, y, or z, and $r_x = x$, $r_y = y$, $r_z = z$, work out the following commutator relations:

$$[L_z, x] = i\hbar y, \qquad [L_z, y] = i\hbar x, \qquad [L_z, z] = 0,$$
$$[L_z, p_x] = i\hbar p_y, \qquad [L_z, p_y] = i\hbar p_x, \qquad [L_z, p_z] = 0.$$

(b) Use the results from part (a) and the definitions

$$L_x = yp_z - zp_y,$$
$$L_y = zp_x - xp_z,$$
$$L_z = xp_y - yp_x,$$

to obtain $[L_z, L_x] = i\hbar L_y$.

(c) Evaluate the commutators $[L_z, r^2]$ and $[L_z, p^2]$, where $r^2 = x^2 + y^2 + z^2$ and $p^2 = p_x^2 + p_y^2 + p_z^2$.

(d) Show that the Hamiltonian,

$$\hat{H} = \frac{\hat{p}^2}{2m} + \hat{V},$$

commutes with all three components of $\hat{\vec{L}}$ if $\hat{V}$ depends only on r.