An electron is sent from $x \to -\infty$ towards a potential barrier,

$$V(x) = \begin{cases} 0, & x < 0, \\ V_0, & x > 0. \end{cases}$$

The electron's energy is $E > V_0$.

(a.) (*5 points*) Solve the time-independent Schrödinger equation for $\psi_{\text{I}}(x)$ and $\psi_{\text{II}}(x)$, which are solutions for $x < 0$ and $x > 0$ respectively. Like we did in class for finite square-wells, combine the collection of constants, $\hbar, m, V_0$, and E, into real quantities $k, \ell \in \mathbb{R}$.

(b.) (*5 points*) Apply boundary conditions at $x = 0$ and solve for the reflection and transmission coefficients R and T. Remember that these coefficients are defined

$$R \equiv \frac{|B|^2}{|A|^2}, \quad T \equiv \frac{|F|^2}{|A|^2},$$

where A is the incident amplitude, B is the reflected amplitude, and F is the transmitted amplitude.

(c.) (*2 points*) Using your results from part (b.), calculate $R+T$. What do you expect $R+T$ should equal? Do you have any ideas for why $R+T$ gives an unexpected answer in this case?

Given that R and T are interpreted as probabilities, and that a particle can only be reflected or transmitted at a barrier, we should expect $R + T = 1$.

(d.) (*5 points*) Calculate a quantity called the probability current,

$$j(x) \equiv \frac{i\hbar}{2m}\left[\psi\frac{\partial\psi^*}{\partial x} - \psi^*\frac{\partial\psi}{\partial x}\right],$$

on both sides of the barrier. Let $j_{\text{I}}(x)$ be the probability current for $x < 0$ and $j_{\text{II}}(x)$ be the probability current for $x > 0$. Evaluate them at $x = 0$; that is, $j_{\text{I}}(0)$ and $j_{\text{II}}(0)$. Don't forget that A, B, and F could be complex.

(e.) (*3 points*) It must be true that $j_{\text{I}}(0) = j_{\text{II}}(0)$. Construct an equation using this conservation rule and your answers from part (d.). Divide this equation by $|A|^2$ and rearrange it so that you determine what linear combination of R and T sums to 1.