1. Let the wavefunction $\Psi(x,t)$ be a solution to the time-dependent Schrödinger equation when the potential energy is given by $V(x)$. What is the solution to the Shrödinger equation if we now consider a potential of $V(x) + V_0$ where V_0 is a real positive constant.

2. A particle is observed in a quantum state described by the wavefunction

$$\Psi(x,t) = A\exp\left(-a\left(\frac{mx^2}{\hbar} + it\right)\right),$$

where A and a are real positive constants.

(a) Normalize Ψ.

As shown in Griffiths, if Ψ is normalized at any time t it is normalized at all times t. We will normalize Ψ at $t = 0$.

$$\begin{aligned}
1 &= \int_{-\infty}^{\infty} \Psi(x, t=0)\, \mathrm{d}x \\
&= \int_{-\infty}^{\infty} A\exp\left(\frac{-amx^2}{\hbar}\right)\, \mathrm{d}x \\
&= A\int_{-\infty}^{\infty} \exp\left(\frac{-amx^2}{\hbar}\right)\, \mathrm{d}x \\
&= A\sqrt{\frac{\pi}{\left(\frac{am}{\hbar}\right)}} \\
&= A\sqrt{\frac{\pi\hbar}{am}}.
\end{aligned}$$

Thus

$$A = \sqrt{\frac{am}{\pi\hbar}}.$$

(b) What is the potential $V(x)$ that this particle finds itself within?

(c) Determine the expectation values $\langle x\rangle$, $\langle x^2\rangle$, $\langle p\rangle$, $\langle p^2\rangle$.

(d) Determine the standard deviations for position, σ_x, and momentum, σ_p.

(e) Are your values for σ_x and σ_p consistent with the uncertainty principle?

3. An electron is trapped in a harmonic quadratic potential. Suppose the expectation value for its position is given by $\langle x \rangle = \frac{a}{2} \sin(\omega t)$. Here, a is a real constant with units of length and ω is an angular frequency. What, if anything, can be concluded about the electron's momentum?