PHYS 474 - Quantum Mechanics **Winter 2024**

Homework #3

1. A particle is trapped in an infinite square well with a width of a. We know that at $t = 0$, it is equally probable for the particle to be found anywhere on the left-side of the well and impossible for it to be found on the right-side of the well.

   (a) What is the normalized wavefunction, $\Psi(x, 0)$ that represents this initial state? Note that you'll have to break one of our fundamental rules about wavefunctions for you to construct this state.
   (b) What is the probability that you would measure an energy of $E = \frac{4\pi^2\hbar^2}{2ma^2}$ at $t = 0$?

2. Consider the standard infinite square well with width a. The stationary state solutions are $\psi_n(x)$.

   (a) Compute $\langle x\rangle$ and $\langle x^2\rangle$ for $\psi_n(x)$.
   (b) Compute $\langle p\rangle$ and $\langle p^2\rangle$ for $\psi_n(x)$.
   (c) Compute σ_x and σ_p and confirm that the uncertainty principle is satisfied for any allowed value of n.

3. A particle in an infinite square well with width a is initially observed in a quantum state described by the wavefunction,

$$\Psi(x, 0) = A\left[\psi_1(x) + \psi_3(x)\right].$$

   where A is a positive, real constant and $\psi_1(x)$ and $\psi_3(x)$ are solutions to the time-independent Schrödinger equation for $n = 1$ and $n = 3$, respectively.

   (a) Normalize $\Psi(x, 0)$ assuming that $\psi_1(x)$ and $\psi_3(x)$ are both separately normalized.
   (b) Compute $|\Psi(x, t)|^2$ and simplify as much as possible.
   (c) If you measured the particle's energy, what value(s) might you possibly obtain and what is the probability of measuring them?