

1. (a) Normalize $R_{2,0}$ for hydrogen and construct the wavefunction $\psi_{2,0,0}$.
- (b) Normalize $R_{2,1}$ for hydrogen and construct the wavefunction $\psi_{2,1,1}$, $\psi_{2,1,0}$, and $\psi_{2,1,-1}$.

2. (a) Determine $\langle r \rangle$ and $\langle r^2 \rangle$ for an electron in the ground state of the hydrogen atom. Express solutions in terms of the Bohr radius, a .

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- (b) Determine $\langle x \rangle$ and $\langle x^2 \rangle$ for an electron in the ground state of the hydrogen atom. If the symmetry of the ground state is exploited, there will not be any new integration for this calculation.
- (c) Determine $\langle x^2 \rangle$ for an electron in a hydrogen atom in the state $n = 2$, $l = 1$, $m = 1$. It is helpful to use the fact that $x = r \sin(\theta) \cos(\phi)$.

3. (a) Starting with $[r_i, p_j] = -[p_i, r_j] = i\hbar\delta_{ij}$ and $[r_i, r_j] = [p_i, p_j] = 0$, where the index i stands for x, y , or z , and $r_x = x, r_y = y, r_z = z$, work out the following commutator relations:

$$\begin{aligned} [L_z, x] &= i\hbar y, & [L_z, y] &= i\hbar x, & [L_z, z] &= 0, \\ [L_z, p_x] &= i\hbar p_y, & [L_z, p_y] &= i\hbar p_x, & [L_z, p_z] &= 0. \end{aligned}$$

- (b) Use the results from part (a) and the definitions

$$L_x = yp_z - zp_y,$$

$$L_y = zp_x - xp_z,$$

$$L_z = xp_y - yp_x,$$

to obtain $[L_z, L_x] = i\hbar L_y$.

- (c) Evaluate the commutators $[L_z, r^2]$ and $[L_z, p^2]$, where $r^2 = x^2 + y^2 + z^2$ and $p^2 = p_x^2 + p_y^2 + p_z^2$.

- (d) Show that the Hamiltonian,

$$\hat{H} = \frac{\hat{p}^2}{2m} + \hat{V},$$

commutes with all three components of $\hat{\vec{L}}$ if \hat{V} depends only on r .