1. A decision maker is described by the utility function $u(w) = w^{1/3}$. She is given the choice between two random amounts X_1 and X_2, in exchange for her entire present wealth w_0. Suppose that

$$X_1 = \begin{cases} 8 & \text{with probability } 0.5 \\ 27 & \text{with probability } 0.5 \end{cases}$$

and

$$X_2 = \begin{cases} 1 & \text{with probability } 0.6 \\ 64 & \text{with probability } 0.4 \end{cases}$$

   (a) Show that she prefers X_1 to X_2.

   (b) Determine for what values of w_0 she should decline the offer.

   (c) Give an example of a utility function in which she would prefer X_2 to X_1.

2. Recall that the iso-elastic property says that for any $k > 0$, $u(kw) = f(k)u(w) + g(k)$ for some $f(k)$ and $g(k)$.

   (a) Identify the functions $f(k)$ and $g(k)$ in the case of $u(w) = \ln(w)$.

   (b) Identify the functions $f(k)$ and $g(k)$ in the case of $u(w) = \frac{w^\lambda - 1}{\lambda}$.

3. Recall that the Arrow-Pratt absolute risk aversion function is given by

$$A(w) = -\frac{\frac{\mathrm{d}^2 u(w)}{\mathrm{d}w^2}}{\frac{\mathrm{d}u(w)}{\mathrm{d}w}}.$$

   (a) Compute $A(w)$ in the case of $u(w) = \ln(w)$. Is $A(w)$ non-increasing?

$$\begin{aligned} A(w) &= -\frac{\frac{\mathrm{d}^2 u(w)}{\mathrm{d}w^2}}{\frac{\mathrm{d}u(w)}{\mathrm{d}w}} \\ &= -\frac{\frac{\mathrm{d}^2 \ln(w)}{\mathrm{d}w^2}}{\frac{\mathrm{d}\ln(w)}{\mathrm{d}w}} \\ &= \end{aligned}$$

   (b) Compute $A(w)$ in the case of $u(w) = \frac{w^\lambda - 1}{\lambda}$. Is $A(w)$ non-increasing.