1. Using the trial wavefunction

$$\psi(x) = A \exp\left[-bx^2\right],$$

obtain the lowest upper-bound you can on the ground-state energies associated with the following one-dimensional potential energies.

(a) $V(x) = \alpha|x|$.

(b) $V(x) = \alpha x^4$.

(a) The expectation value of total energy is given by Griffiths Equation 8.4,

$$\langle H \rangle = \langle T \rangle + \langle V \rangle .$$

The kinetic energy expectation value, $\langle T \rangle$, is independent of the potential energy function and is given by

$$\langle T \rangle = \int_{-\infty}^{\infty} \psi^*(x) \left(-\frac{\hbar^2}{2m}\frac{\mathrm{d}^2}{\mathrm{d}x^2}\right) \psi(x)\,\mathrm{d}x = -\frac{\hbar^2}{2m}\int_{-\infty}^{\infty} \psi^*(x)\frac{\mathrm{d}^2}{\mathrm{d}x^2}\psi(x)\,\mathrm{d}x.$$

2. Use the WKB approximation to determine the allowed energies, E_n, of an infinite-square well with a “shelf” of height V_0 extending half-way across,

$$V(x) = \begin{cases} V_0, & 0 < x < a/2, \\ 0, & a/2 < x < a, \\ \infty, & \text{else.} \end{cases}$$

Express the solution in terms of V_0 and E_n^0, the energy for the nth state of the infinite-square well. Assume that $E_1^0 > V_0$, but do not assume $E_n \gg V_0$.

Response.

Use the WKB approximation result for transmission probability,

$$T \approx e^{-2\gamma},$$

where

$$\gamma \equiv \frac{1}{\hbar} \int_0^a |p(x)| \, \mathrm{d}x,$$

to compute the approximate transmission probability for a particle with energy E that encounters a finite-square barrier with height $V_0 > E$ and width $2a$. Compare the solution to the exact result,

$$T^{-1} = 1 + \frac{V_0^2}{4E(V_0 - E)} \sinh^2 \left(\frac{2a}{\hbar} \sqrt{2m(V_0 - E)} \right),$$

which should reduce to the solution in the WKB regime where $T \ll 1$.

Response.