1. Let Y be the set of all living people, and let the binary relation be "is married to" (assuming monogamy throughout society).

   (a) Identify the meaning of cases (1) through (4) as found in slide 5/14.

Case (1). (xRy, yRx). x is married to y and y is married to x.

Case (2). $(xRy, \text{not } yRx)$. x is married to y and y is not married to x.

Case (3). $(\text{not } xRy, yRx)$. x is not married to y and y is married to x.

Case (4). $(\text{not } xRy, \text{not } yRx)$. x is not married to y and y is not married to x.

   (b) For each property p1 through p9, as found in slide 6/14, state whether or not the property is satisfied and why.

p1. Reflexive. x is married to x. This is not satisfied; society does not recognize marriage to the self.

p2. Irreflexive. x is not married to x. This is satisfied; society does not recognize marriage to the self.

p3. Symmetric. If x is married to y then y is married to x. This is satisfied; two people are mutually in marriage to one another.

p4. Asymmetric. If x is married to y then y is not married to x. This is not satisfied; two people are mutually in marriage to one another as a pair.

p5. Antisymmetric. If x is married to y and y is married to x then x is y. This is not satisfied; a marriage exists between two unique people.

p6. Transitive. If x is married to y and z is married to x then x is married to z. This is not satisfied; a person cannot be married to two people so person x cannot be married to both y and z by assumption of monogamy.

p7. Negatively transitive. If x is not married to y and y is not married to z then x is not married to z. This is not satisfied; if x were married to z it could still be said that x is not married to y and y is not married to z.

p8. Connected. x is married to y and y is married to x for all people x and y. This is not satisfied; there are people who are not married.

p9. Weakly connected. If x and y are different people then x is married to y or y is married to x. This is not satisfied; there are people who are not married.

2. Show that asymmetry (p4) and negative transitivity (p7) imply transitivity (p6).

Proof. Let Y be some set and R be some binary operation on Y with the properties asymmetry and negative transitivity. Let $x, y, z \in Y$. Suppose xRy and yRz.

By asymmetry $xRy \to$ not yRx.

By asymmetry $yRz \to$ not zRy.

By negative transitivity (not zRy, not yRx) $\to$ not zRx.

Then by asymmetry, not $zRx \to$ not not $xRz \equiv xRz$.

Thus by asymmetry and negative transitivity we have $(xRy, yRz) \to xRz$ which is the property transitivity. $\square$