PHYS 475 - Quantum Mechanics **Spring 2024**

Homework #4

1. Consider an infinite square well that runs from 0 to a. As a perturbation, we place a delta-function bump at the center of the well, $\hat{H}' = \alpha\delta(x - a/2)$ where α is a constant.

   (a) Determine the first-order correction to the allowed energies. Why are the energies for even n unperturbed?

   (b) Determine the first three non-zero terms in the perturbation expansion, $\psi_n^1 = \sum_{m \neq n} \frac{\langle \psi_m^0 | \hat{H}' | \psi_n^0 \rangle}{(E_n^0 - E_m^0)} \psi_m^0$, of the correction to the ground state ψ_1^1.

2. The allowed energies for the harmonic oscillator are $E_n = (n + 1/2)\hbar\omega$ where $\omega = \sqrt{k/m}$ is the classical frequency and the potential energy is $V(x) = (1/2)kx^2$. Suppose the spring constant increases slightly, $k \to (1 + \epsilon)k$.

   (a) Determine the exact new energies, then expand your formula as a power series in ϵ up to second order.

   (b) Calculate the first-order perturbation to the energy using $E_n^1 = \langle \psi_n^0 | \hat{H}' | \psi_n^0 \rangle$. To perform this calculation, you will need to determine what $\hat{H}'$ is in this case. Compare you result with your result from part (a).