

# The Frank-Hertz Experiment: An Experimental Measurement of Planck's Constant using Electrons in Argon

PHYS 333 Experimental Physics

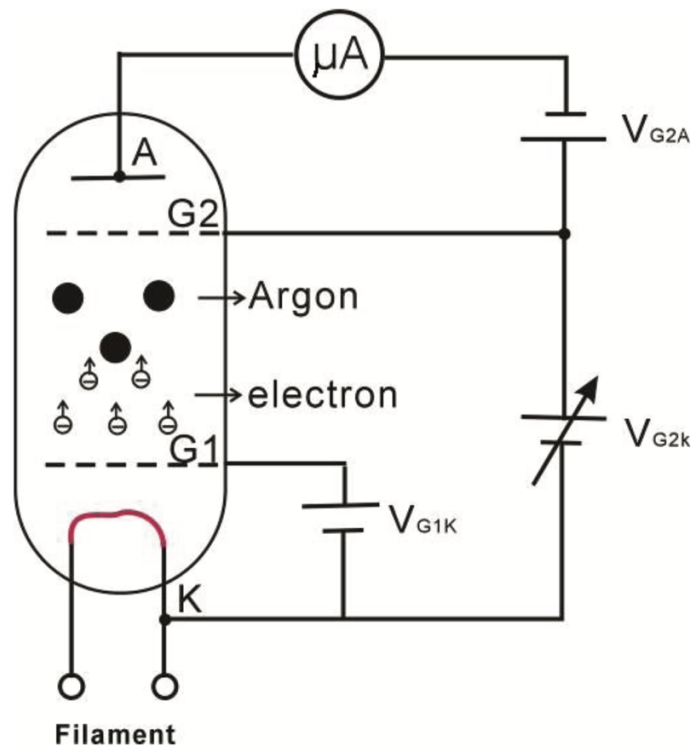
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## Introduction and Background

The Frank-Hertz experiment is a foundational experiment to modern physics. While originally an investigation seeking to uncover a more complete model of electron drift behavior in gasses, the Frank-Hertz experimental results demonstrate the quantization of atomic energy levels [1]. This report will explore two elements of the Frank-Hertz experiment: the experimental results and analysis, and a justification for a simple model behavior of the electrons in Argon gas.



*Figure 1:* Schematic diagram of the Frank-Hertz experiment [2]. Argon gas, the gas under investigation, fills the glass tube. Electrons are emitted from the filament and accelerated by a variable potential  $V_{G2k}$  through the Argon gas. The opposing potential  $V_{G2A}$  prohibits electrons from reaching the collector without first having reached a minimum energy level  $eV_{G2A}$ . Electrons with enough energy to overcome the opposing potential come into contact with the collector and are recorded as current through an ammeter.

Figure 1 shows the Frank-Hertz apparatus used in this experiment. The following simple model is proposed to explain the behavior of electrons in this experiment:

1. Electrons can undergo inelastic collisions with Argon after obtaining kinetic energy greater than or equal to the energy difference of the first excited state and ground state of Argon.
2. An inelastic collision with Argon results in the electron losing kinetic energy equal to the first excitation energy of Argon.
3. Spectral analysis of the Argon gas while electrons are accelerated through reveals an emission line given as  $\lambda = 108.1 \text{ nm}$  corresponding to the excitation energy of Argon [2, 5].

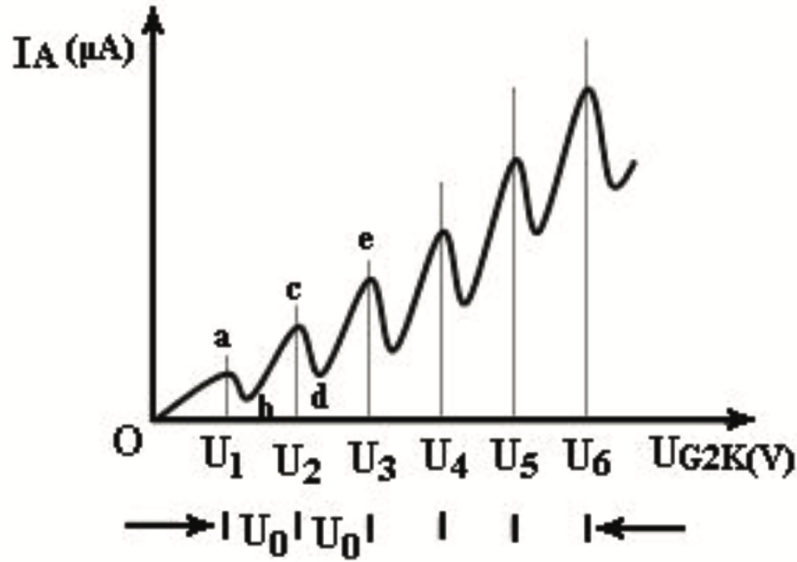


Figure 2: A graph of the expected relationship of the accelerating voltage and collector current. Local maximums are denoted with a, c, and e; local minimums are denoted with b, and d. Local maximum, and local minimum, are separated on the horizontal axis by  $U_0$  where  $eU_0$  is the excitation energy of Argon in electron volts and  $e$  is the electron charge [2].

From the analysis of Figure 2 and the relationship between photon energy and wavelength the following equation for Planck's constant can be obtained:

$$h = \frac{eU_0\lambda}{c} \quad \text{Equation 1}$$

where  $eU_0$  is the excitation energy of Argon found from analyzing the voltage and collector current relationship,  $\lambda$  is the emission wavelength, and  $c$  is the speed of light in a vacuum.

This simple model of the Frank-Hertz experiment intentionally discounts significant underlying physical phenomenon. Item 1 of the simple model restricts electrons to inelastic collisions and does not consider the possibility of an elastic collision with Argon. Under this behavioral model

electrons travel in a beam from the emitter towards the collector only reducing in speed, but remaining beam like while undergoing inelastic collisions. Elastic collisions, however, occur between electrons and Argon gas and can be described by the application of kinetic gas theory and the Boltzmann kinetic equation [3]. This type of kinetic gas analysis utilizes a momentum transfer cross section: a description of the momentum transfer behavior between elastically colliding objects [4]. The first assumption in the simple model, that electrons only interact with inelastic collisions, can be equivalently stated as the momentum transfer cross section is zero. A full kinetic analysis of electrons in a Mercury gas is used as an analogue to the Argon gas and its results are as follows [2]:

1. The momentum transfer cross section significantly impacts the amplitude of the collector current accelerating voltage relationship shown in Figure 2.
2. The momentum transfer cross section has negligible impact on the period of oscillations under investigation in this experiment.

While the momentum transfer cross section does have a significant impact on the measurements made in this experiment it does not have a significant impact on the quantity used for analysis: the period. Because of this, the first assumption of the simple model is justified and setting the momentum transfer cross section to zero does not significantly impact the results.

No measurement was done to verify the emission wavelength provided by the experimental procedure. The first excitation energy of Argon is, however, reported to be 11.6 electron-volts [5]. This value corresponds to a wavelength of 107 nanometers, in agreement with the provided value of 108.1 nanometers. The given value, 108.1 nanometers, will be used for this analysis.

The methods and results of the following sections of this report investigate the application of the simple model and subsequent analysis to the apparatus described in Figure 1. The application of Equation 1 will be used to measure Planck's constant using results in the form of Figure 2.

### **Methods and Procedures**

A Frank-Hertz tube, Pasco model SE-9650, is assembled as shown in Figure 1. A Pasco Tunable DC Power Supply I model SE-6615 is connected across the filament using its 0 to 6.3 volt output and  $V_{G1k}$  using its -4.5 to 30 volt output. A Pasco Tunable DC Power Supply II model SE-9644 is connected across  $V_{G2A}$  using its 0 to 12 volt output and across  $V_{G2k}$  using its 0 to 100 volt output. A Pasco DC Current Amplifier model SE-6621 is connected to the collector and set to the  $10^{-10}$  Amp current range. The data outputs of the DC Current Amplifier and 0 to 100 volt output of Power Supply II are connected to a Pasco 850 Universal Interface model UI-5000 which is connected to a computer running Pasco Capstone Software version 2.5.1.

With all devices turned on the DC Current Amplifier is first calibrated to read 0 Amps. The 0 to 6.3 volt output of Power Supply I is set such that the voltmeter reads 3.0 Volts, and the -4.5 to 30 volt output is adjusted until the voltmeter reads 1.5 Volts. The 0 to 12 volt output of Power Supply II is set such that the voltmeter reads 10.0 Volts, and the 0 to 100 volt output is set such that the voltmeter reads 0 Volts. The Frank-Hertz tube is kept at this configuration for 15 minutes to allow the gas to warm before collecting measurements.

Using the Pasco Capstone Software, a sweep of accelerating voltages can be performed by slowly increasing the 0 to 100 volt output of Power Supply II. The Pasco Capstone Software records 20 times per second the value of the 0 to 100 volt output of Power Supply II and the value of the DC Current Amplifier on the x and y axis of a graph respectively. The sweep is anticipated to take a form similar to Figure 2 with periodic behavior as the accelerating voltage increases.

From a graph resembling Figure 2 the potentials  $V_a, V_c, V_e$  refer to the local maximums in ascending order and  $V_b, V_d$  refer to the local minimums in ascending order. From these potentials the first excitation value can be found by calculating the average distance between sequential local maximums or minimums.

### **Results and Analysis**

The analysis of the collected data was done in an *Excel* spreadsheet using an imported CSV file from the Pasco Capstone Software. For each trial the local maximums and minimums were recorded in ascending order. From this the average of each ordered local maximum and minimum was calculated. Each trial was plotted on Figure 3 with the average of each local maximum and minimum denoted by a vertical dotted line.

# Periodic Behavior of Electron Motion Through Argon Gas Under an Accelerating Voltage

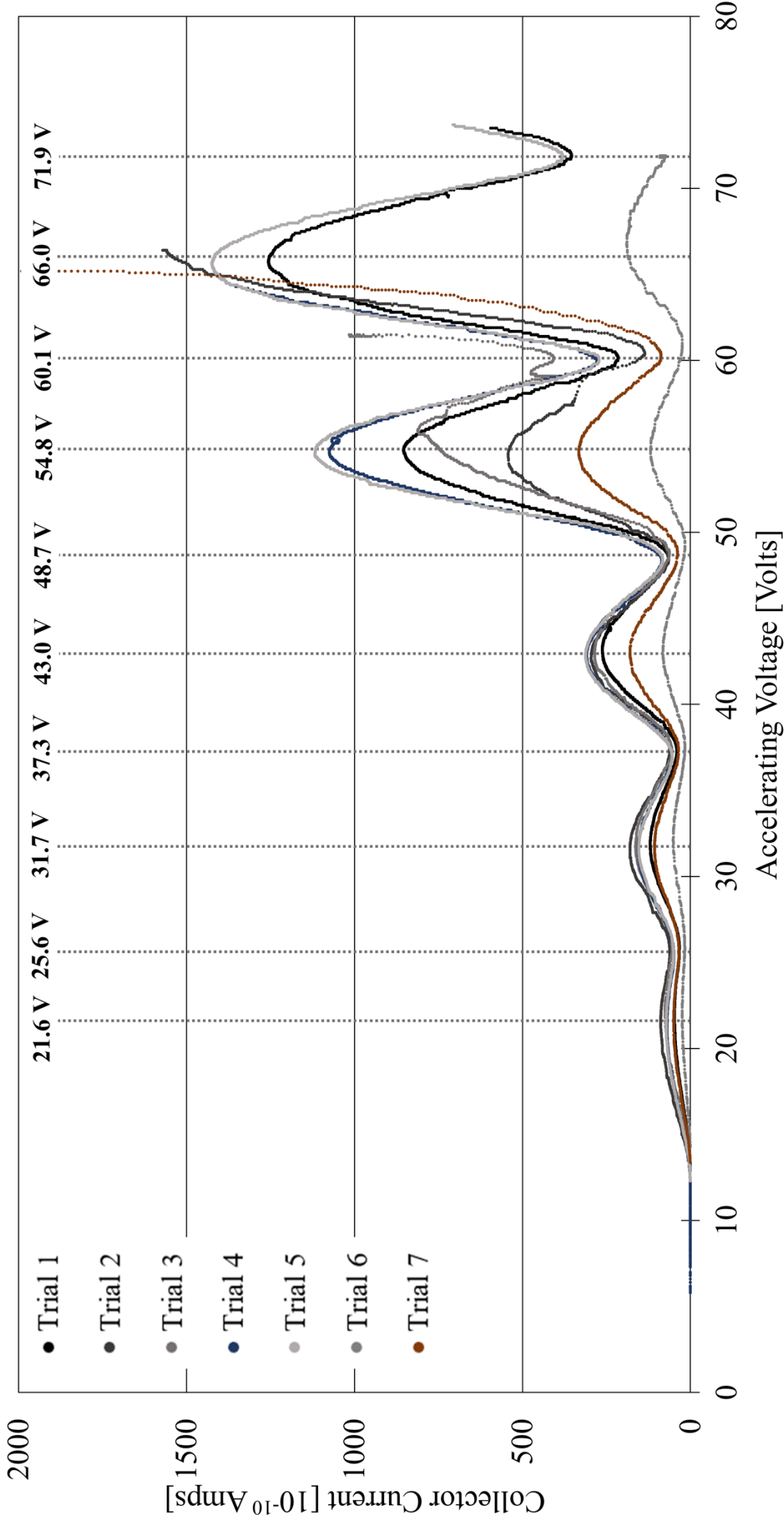


Figure 3: A graph of 7 trials of the Frank-Hertz experiment. Vertical dotted lines are the average of each local maximum and minimum across trials. Some trials were ended early to protect equipment from taking too much current and do not contribute to the calculation of average local maximum and minimum beyond their termination.



A qualitative analysis of Figure 3 reveals, as expected, close agreement between each trials local maximum and minimums accelerating voltage. While the collector current varies between trials the exact value is not relevant to the analysis, only the location of the local maximums and minimums on the accelerating voltage axis. Some trials were terminated early to prevent damage to the equipment and do not contribute to local maximum or minimums after their termination point.

index	Average Max [Volts]	Average Min [Volts]
1	21.6 Volts	25.6 Volts
2	31.7 Volts	37.3 Volts
3	43.0 Volts	48.7 Volts
4	54.8 Volts	60.1 Volts
5	66.0 Volts	71.9 Volts

*Table 1:* The average local maximums and minimums for each nth order peak and valley.

Finding the average separation between local maximums and minimums two values of the first excited state of Argon are found: 11.1 Volts and 11.6 Volts respectively with uncertainties not evaluated or estimated. Calculating Planck's constant using Equation 1 and the previous values two experimental measurements of Planck's constant are found:  $6.41\text{E-}34$  and  $6.68\text{E-}34$  respectively. These two values correspond to a 3.19 and 0.740 % difference from the accepted value of Planck's constant. The uncertainties associated with the measurements reported were not evaluated or estimated.

## **Conclusion**

Further experimentation is needed with this model. A full kinetic analysis for electrons in an Argon gas has yet to be published, as of 2014 [3]. A direct measurement of the emission spectrum, with the expectation being a wavelength at or near 107 nanometers, is needed to turn this indirect measurement into a direct measurement. An uncertainty analysis of measurements is needed to determine the precision of this apparatus and the simple model. The results, a precise measurement of Planck's constant, however, suggest the simple model proposed is a valid description of the electron behavior in this experimental context of measuring periodic structures.

## **References**

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