

1. Consider a particle interacting with the potential barrier:

$$V(x) = \begin{cases} 0, & x < 0, \\ V_0, & x > 0. \end{cases}$$

What kinds of wavefunction solutions are possible?

Free particle wavefunctions are possible solutions to this potential. In the region $x < 0$ we have the typical free particle condition $V(x) = 0$. At the barrier there exist reflected and transmitted components of the free particle solution. In the region $x > 0$ free particle solutions can exist only if the particle energy is greater than the potential.

2. We calculated the even bound-state wavefunctions for the finite-square well in class. Now, perform this calculation for the odd wavefunction solutions. By determining the wavefunction and apply appropriate boundary conditions, calculate the transcendental equation that will allow you to determine the bound-state energies. Use $Z = \ell a$, and $Z_0 = \frac{a}{\hbar} \sqrt{2mV_0}$ to express your transcendental equation more compactly. Plot the left- and right-hand sides of the equation as a function of Z for $Z_0 = 2$. How many bound states with odd wavefunctions are there for $Z_0 = 2$?

3. Consider a particle interacting with the following potential energy landscape:

$$V(x) = \begin{cases} \infty, & x < 0, \\ -V_0, & 0 < x < a, \\ 0, & x > a. \end{cases}$$

- (a.) Determine the bound-state wavefunctions in three regions: (i.) $x < 0$, (ii.) $0 < x < a$, and (iii.) $x > a$. Note that this potential energy is not symmetric, so you cannot assume solutions will be alternately even and odd; instead, you must use the most general solution for the middle region's wavefunction.
- (b.) Apply appropriate boundary conditions at $x = 0$ and $x = a$. Combine the results to determine the transcendental equation that governs the bound-state energies. Does it look familiar?