Thermodynamics as a theory of decision-making with information processing cost

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Central Washington University, February 2024

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Thermodynamics

- State x: A particular configuration or outcome.
- Entropy S: A measure of how disorderly a system is. Isolated systems tend towards higher entropy.
- Free Energy F: A measure of the available resources used to process information.
- Boltzmann Distribution (Initial Information State):

$$p_0(x) = \frac{1}{Z_0} e^{-\alpha \phi_0(x)}$$

where $Z_0 = \sum_{x'} e^{-\alpha \phi_0(x')}$, α indicates inverse temperature, and $\phi_0(x)$ represents information about the state x.

Bayes' Theorem

Bayes' Theorem states that

$$P(H|E) = \frac{P(E|H)P(H)}{P(E)}$$

where

- H is the hypothesis
- E is a statistical model for observed data
- $P(H)$ represents the prior probability, which is the probability of H before E is observed
- $P(H|E)$ represents the posterior probability, which is the probability of H given E (after E is observed)
- $P(E|H)$ is called the likelihood, and is the probability of observing E given H

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What a decision maker does

- Recall $\phi_0(x)$ represents the initial information of a state x: a prior.
- A decision maker expends energy on ϕ_0 to obtain ϕ: a posterior.
- The difference in potential between ϕ and ϕ_0 is the free energy.
- We relate ϕ to utility $\mathcal{U}$ by $\mathcal{U} = -\Delta\phi$.
- We quantify the resources or energy to modify ϕ_0 by β.
- $p(x) = \frac{1}{Z} p_0(x) e^{\beta \mathcal{U}(x)}$ describes the probability of actions.
- A perfectly rational decision maker, $\beta \to \infty$, transforms p_0 to a Dirac delta function; that is, they identify, with no ambiguity, a single preferred decision.
- A non-preferential decision maker, $\beta = 0$, makes no change to the prior action distribution, $p(x) = p_0(x)$.

What a decision maker is

- Successfully models a *bounded rational* decision-maker (one that has limited information or is impacted by their environment, leading the decision maker to not always make the "best" decision).
- Quantifies a decision makers rationality with a temperature-like parameter β which is related to the computational resources available to the decision maker.
- The rationality of the decision maker determines the amount of energy available to effect the posterior distribution and effectively "search" for the best decision.
- This model of decision making is able to recover expected utility theory when computational costs are ignored; that is, $\beta \to \infty$.

Ellsberg Paradox Resolution

Consider an urn containing 90 balls, 30 of which are red, and the remaining 60 are black or yellow in unknown proportion.

- Gamble 1: receive 100$ if red is drawn
- Gamble 2: receive 100$ if black is drawn
- Gamble 3: receive 100$ if red or yellow is drawn
- Gamble 4: receive 100$ if black or yellow is drawn

Preference of 1 over 2 and 4 over 3 is a contradiction on the supposed proportion of black and yellow balls.
Thermodynamic resolution: the environment has rationality $\beta < 0$ which limits the decision makers computational resources.

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Research Questions

- What are the limitations of this model of decision making?
- What other paradoxes are resolved under this decision making model?
    - The paper details a resolution to Allais' paradox.
- How do we connect this abstract notion of limited resources β with observable limitations such as time.
    - The paper hints at how this might be done by considering a problem in which a decision maker must sample an urn of balls with replacement and using a limited number of draws determine an optimal gamble much like the Ellsberg paradox.