PHYS 474 - Quantum Mechanics **Winter 2024**

Homework #7

1. Determine the eigenfunctions and eigenvalues of the operator $\hat{Q} = \frac{d^2}{d\phi^2}$ where ϕ is the angle in polar coordinates. Due to the rotational symmetry of the problem, your eigenfunctions should satisfy the boundary condition $f(\phi) = f(\phi + 2\pi)$. Is the spectrum of eigenvalues degenerate or non-degenerate?

2. (a) Suppose that $f(x)$ and $g(x)$ are both eigenfunctions of an operator $\hat{Q}$. The spectrum is degenerate such that both $f(x)$ and $g(x)$ have the same eigenvalue, q. Prove that <u>any</u> linear combination of $f(x)$ and $g(x)$ is also an eigenfunction of $\hat{Q}$. What is its eigenvalue?

   (b) An *anti*-hermitian operator obeys the following condition: $\hat{Q}^\dagger = -\hat{Q}$. Show that the expectation value of an anti-hermitian operator is imaginary.

   (c) Show that the commutator of two hermitian operators is anti-hermitian.

   (d) Show that the commutator of two anti-hermitian operators is also anti-hermitian.

3. We have two operators $\hat{A}$ and $\hat{B}$ with two eigenstates each. The eigenstates and corresponding eigenvalues are characterized by the equations:

$$\hat{A}\psi_1 = a_1\psi_1$$
$$\hat{A}\psi_2 = a_2\psi_2$$
$$\hat{B}\phi_1 = b_1\phi_1$$
$$\hat{B}\phi_2 = b_2\phi_2$$

   Let's say we also know the that eigenstates for each operator are related by:

$$\psi_1 = \frac{1}{5}(3\phi_1 + 4\phi_2)$$
$$\psi_2 = \frac{1}{5}(4\phi_1 - 3\phi_2)$$

   (a) If observable A is measured and we obtain a value of a_1, what is the state of the system in the instant after the measurement was made?

   (b) If B is now measured (following the measurement in part (a)), what are the possible results and what are their associated probabilities?

   (c) If we measure A again immediately following the measurement of B in part (b), what is the probability of obtaining a_1? This is tricky because we do not know what value of B we obtained in part (b).