

1. Consider the continuous Gaussian distribution,  $\rho(x) = Ae^{-\lambda(x-a)^2}$ , where  $A$ ,  $a$ , and  $\lambda$  are positive, real constants. Note that this is not a wavefunction, but rather a distribution.

(a) Normalize the distribution to determine  $A$ .

$$\begin{aligned} 1 &= \int_{-\infty}^{\infty} \rho(x) \, dx \\ &= \int_{-\infty}^{\infty} Ae^{-\lambda(x-a)^2} \, dx \\ &= A \int_{-\infty}^{\infty} e^{-\lambda(x-a)^2} \, dx \\ &= A \int_{-\infty}^{\infty} e^{-(\lambda x^2 - 2\lambda ax + \lambda a^2)} \, dx \\ &= A \sqrt{\frac{\pi}{\lambda}} \exp\left(\frac{(-2\lambda a)^2 - 4\lambda^2}{4\lambda}\right) \\ &= A \sqrt{\frac{\pi}{\lambda}} \exp(\lambda a^2 - \lambda) \end{aligned}$$

$$\sqrt{\frac{\lambda}{\pi}} \exp(-\lambda a^2 + \lambda) = A$$

(b) Determine  $\langle x \rangle$ ,  $\langle x^2 \rangle$ , and  $\sigma$ .

2. At time  $t = 0$  s, an electron is represented by the wave function,

$$\Psi(x, 0) = \begin{cases} A\frac{x}{a}, & 0 \leq x \leq a \\ A\frac{(b-x)}{(b-a)}, & a \leq x \leq b \\ 0, & \text{otherwise} \end{cases}$$

where  $A$ ,  $a$ , and  $b$  are constants.

- (a) Normalize  $\Psi$ .
- (b) Sketch  $\Psi(x, 0)$  as a function of  $x$ .
- (c) Where is the electron most likely to be found at  $t = 0$  s?
- (d) What is the probability the electron will be found in the region  $x \leq a$ ? Check your result in the limiting case where  $b = a$  and  $b = 2a$ .
- (e) Determine  $\langle x \rangle$ .