Contents


```
clear all;
close all;
figure(1);
figure(2);
figure(3);
% Parameters that are used for the Whipple bicycle model. The model is
% based on the linearized 4th order model and analysis of eigenvalues
% from IEEE CSM (25:4) August 2005 pp 26-47

%   Basic data is given by 26 parameters
g = 9.81;                       % Acceleration of gravity [m/s^2]
b = 1.00;                       % Wheel base [m]
c = 0.08;                       % Trail [m]
Rrw = 0.35; Rfw = 0.35;         % Wheel radii
lambda = pi*70/180;             % Head angle [radians]

% Rear frame mass [kg], center of mass [m], and inertia tensor [kgm^2]
mrf=12;xrf=0.439;zrf=0.579;
Jxxrf=0.475656;Jxzrf=0.273996;Jyyrf=1.033092;Jzzrf=0.527436;
mrf=87;xrf=0.491586;zrf=1.028138;
Jxxrf=3.283666;Jxzrf=0.602765;Jyyrf=3.8795952;Jzzrf=0.565929;

% Front frame mass [kg], center of mass [m], and inertia tensor [kgm^2]
mff=2;xff=0.866;zff=0.676;
Jxxff=0.08;Jxzff=-0.02;Jyyff=0.07;Jzzff=0.02;

% Rear wheel mass [kg], center of mass [m], and inertia tensor [kgm^2]
mrw=1.5;Jxxrw=0.07;Jyyrw=0.14;

% Front wheel mass [kg], center of mass [m], and inertia tensor [kgm^2]
mfw=1.5;Jxxfw=0.07;Jyyfw=0.14;

% Auxiliary variables
xrw=0;zrw=Rrw;xfw=b;zfw=Rfw;
Jzzrw=Jxxrw;Jzzfw=Jxxfw;
mt=mrf+mrw+mff+mfw;
xt=(mrf*xrf+mrw*xrw+mff*xff+mfw*xfw)/mt;
zt=(mrf*zrf+mrw*zrw+mff*zff+mfw*zfw)/mt;
Jxxt=Jxxrf+mrf*zrf^2+Jxxrw+mrw*zrw^2+Jxxff+mff*zff^2+Jxxfw+mfw*zfw^2;
Jxzt=Jxzrf+mrf*xrf*zrf+mrw*xrw*zrw+Jxzff+mff*xff*zff+mfw*xfw*zfw;
Jzzt=Jzzrf+mrf*xrf^2+Jzzrw+mrw*xrw^2+Jzzff+mff*xff^2+Jzzfw+mfw*xfw^2;
mf=mff+mfw;
xf=(mff*xff+mfw*xfw)/mf;zf=(mff*zff+mfw*zfw)/mf;
Jxxf=Jxxff+mff*(zff-zf)^2+Jxxfw+mfw*(zfw-zf)^2;
Jxzf=Jxzff+mff*(xff-xf)*(zff-zf)+mfw*(xfw-xf)*(zfw-zf);
Jzzf=Jzzff+mff*(xff-xf)^2+Jzzfw+mfw*(xfw-xf)^2;
d=(xf-b-c)*sin(lambda)+zf*cos(lambda);
Fll=mf*d^2+Jxxf*cos(lambda)^2+2*Jxzf*sin(lambda)*cos(lambda)+Jzzf*sin(lambda)^2;
Flx=mf*d*zf+Jxxf*cos(lambda)+Jxzf*sin(lambda);
Flz=mf*d*xf+Jxzf*cos(lambda)+Jzzf*sin(lambda);
gamma=c*sin(lambda)/b;
Sr=Jyyrw/Rrw;Sf=Jyyfw/Rfw;St=Sr+Sf;Su=mf*d+gamma*mt*xt;

% Matrices for linearized fourth order model
M=[Jxxt -Flx-gamma*Jxzt;-Flx-gamma*Jxzt Fll+2*gamma*Flz+gamma^2*Jzzt];
```

```
K0=[-mt*g*zt g*Su;g*Su  -g*Su*cos(lambda)];
K2=[0 -(St+mt*zt)*sin(lambda)/b;0 (Su+Sf*cos(lambda))*sin(lambda)/b];
c12=gamma*St+Sf*sin(lambda)+Jxzt*sin(lambda)/b+gamma*mt*zt;
c22=Flz*sin(lambda)/b+gamma*(Su+Jzzt*sin(lambda)/b);
C0=[0 -c12;(gamma*St+Sf*sin(lambda)) c22];
one=diag([1 1]);null=zeros(2,2);

% Nominal velocity
v0=5;
```

Part A

M th_dd + C v0 th_d + (K0 + K2*v0) th = [0; T]; where th = [gamma; delta] -> angles of bicycle dynamics let x1 = gamma, x2 = gamma_d, x3 = delta, x4 = delta_d thus, x1_d = x2, x2_d = gamma_dd x3_d = x4, x4_d = delta_dd expand matrix equations for 2 eom in terms of state-variables

```
syms x1 x2 x3 x4 x1_d x2_d x3_d x4_d T real
% get coefficents from matrices
m1 = M(1,1); m2 = M(1,2); m3=M(2,1); m4=M(2,2);
c1 = C0(1,1); c2=C0(1,2); c3=C0(2,1); c4=C0(2,2);
K = [K0 + K2*v0^2];
k1 = K(1,1); k2=K(1,2); k3=K(2,1); k4=K(2,2);

% form equations of motion using state-variables

%x2_d = (-m2/m1)*x4_d - (c1*v0/m1)*x2 - (c2*v0/m1)*x4-(k1/m1)*x1-(k2/m1)*x3;
%x4_d = (-m3/m4)*x2_d - (c3*v0/m4)*x2 - (c4*v0/m4)*x4-(k3/m4)*x1 - (k4/m4)*x3 + T/m4;

% rearrange and substitute expression for x2_d into eom of x4_d; eom2
x2_d_eqn = (-m2/m1)*x4_d - (c1*v0/m1)*x2 - (c2*v0/m1)*x4-(k1/m1)*x1-(k2/m1)*x3;
eom2 = (-m3/m4)*(x2_d_eqn) - (c3*v0/m4)*x2 - (c4*v0/m4)*x4-(k3/m4)*x1 - (k4/m4)*x3 + (T/m4) - x4_d;

% solve for x4_d and x2_d, solution denoted with _s
x4_d_s = collect(solve(eom2 == 0, x4_d), [x1; x2; x3; x4; T]);
x2_d_s = collect(subs(x2_d_eqn, x4_d, x4_d_s), [x1; x2; x3; x4; T]);
x1_d_s = x2;
x3_d_s = x4;

% construct state-space presentation
A = [0 1 0 0;...
     0 0 0 0;...
     0 0 0 1;...
     0 0 0 0];

 B = [0; 0; 0; 0];

 x = [x1; x2; x3; x4];

 % put the coeffients from the solution to x2_d, into A,
 % and input term into B
```

```
[C_, T_] = coeffs(vpa(x2_d_s));

for i = 1:size(T_,2)
    if T_(i) == x1
        A(2, 1) = C_(i);
    elseif T_(i) == x2
        A(2, 2) = C_(i);
    elseif T_(i) == x3
        A(2, 3) = C_(i);
    elseif T_(i) == x4
        A(2, 4) = C_(i);
    elseif T_(i) == T
        B(2) = C_(i);
    end
end

% put the coeffients from x4_d into A, and input term into B
[C_, T_] = coeffs(vpa(x4_d_s));

for i = 1:size(T_,2)
    if T_(i) == x1
        A(4, 1) = C_(i);
    elseif T_(i) == x2
        A(4, 2) = C_(i);
    elseif T_(i) == x3
        A(4, 3) = C_(i);
    elseif T_(i) == x4
        A(4, 4) = C_(i);
    elseif T_(i) == T
        B(4) = C_(i);
    end
end

% view poles of the state-space model, in Hz
P_A = eig(A)/2/pi;

% findings: v0=5 yields two unstable poles, whilst v0=10 is stable
%
%
% x1_d  =         0    1.0000         0         0   * x1   +       0    * T
% x2_d       8.7611   -0.6370   23.2100    2.1486     x2      0.2922
% x3_d            0         0         0    1.0000     x3           0
% x4_d     -14.9477  -17.2465   29.1529  -12.9089     x4      7.9109
```

Part B

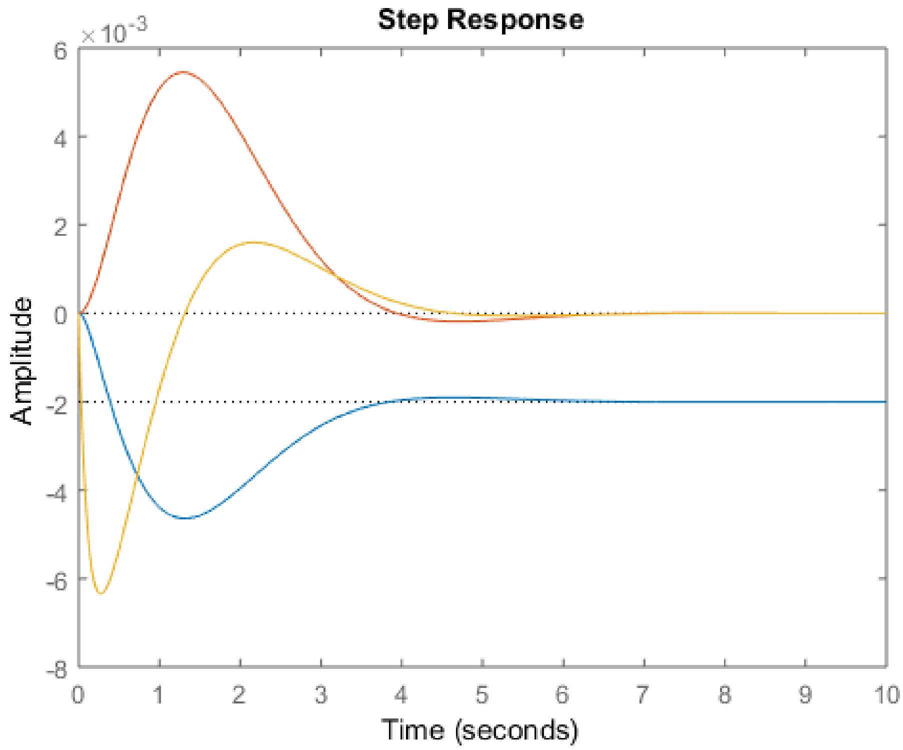
u = -kx, x_d = Ax + Bu, y = Cx -> x_d = (A-Bk)x poles of closed loop system = eig(A-Bk) desired poles:

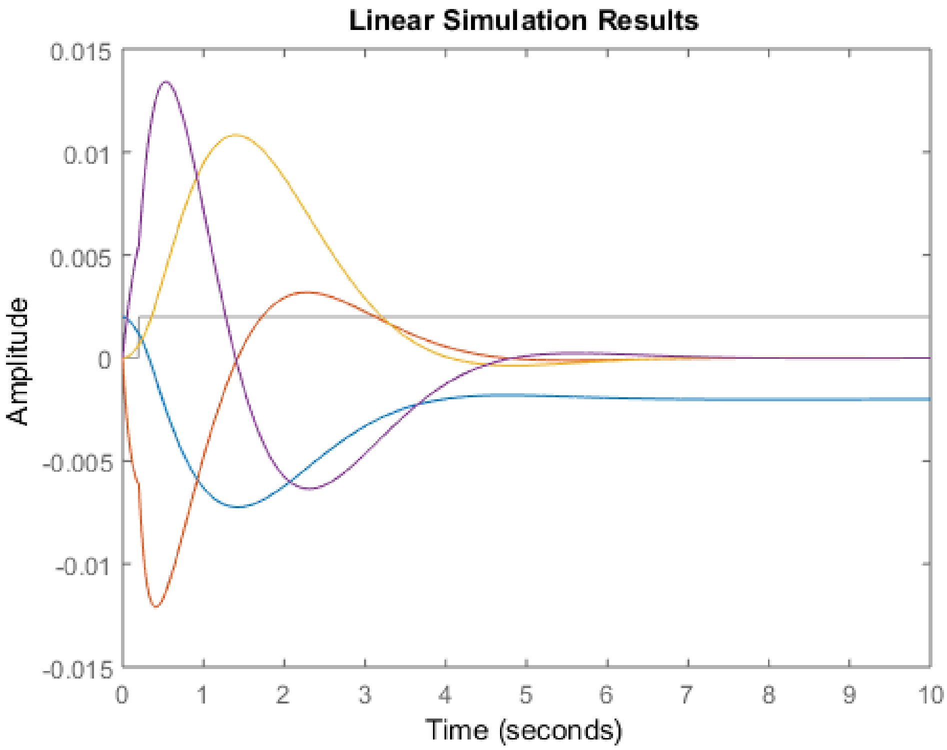
```
Pol = [-2 -10 -1+1i -1-1i];
% required feeback gain, k:
k = acker(A, B, Pol);

%k =
%    -1.2565    5.2073  -32.5252   -0.4145
clp = eig(A-B*k);
```

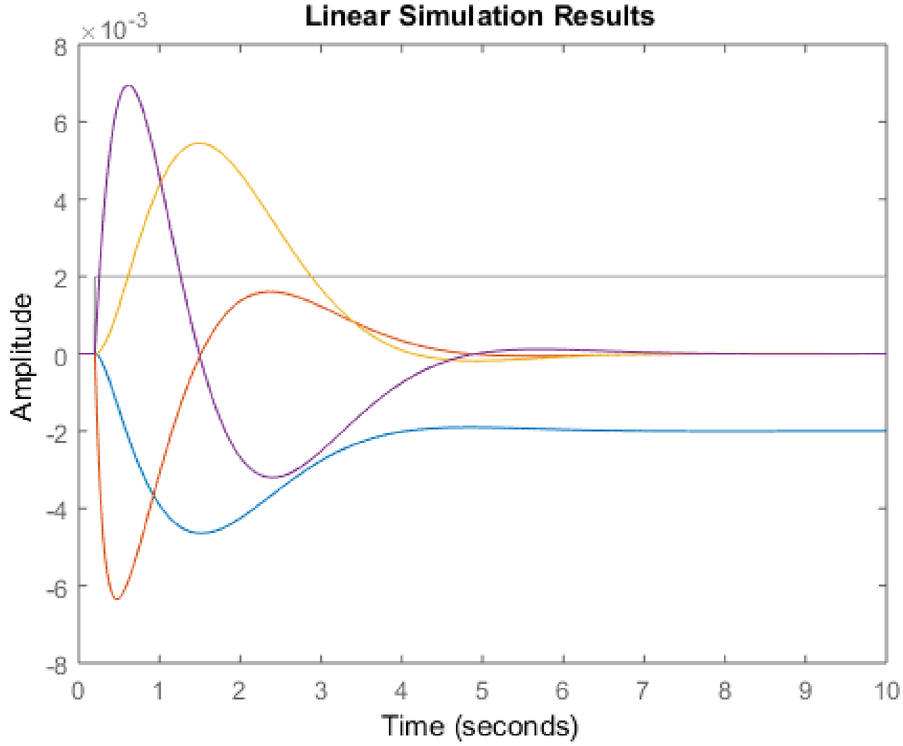
Part C

```
%                    .
%      u        +     x  /----------\  x
%      --> B_f ->() ---->| integral |-----> y=Cx ---> y
%               +|        \---------/  |
%                |                     |
%                |<---(A_f = A-Bk)<----|
%
```

```
% with state-feedback and input change in delta
% x_d = (A - Bk) [x1 ; x2 ; (x3 + u) ; x4]
% x_d = (A - Bk) x + (A-Bk)[0 ; 0 ; 1 ; 0] u
% x_d = A_f * x + B_f * u

sr = [0;0;1;0];                        % selected state for step input
A_f = A-B*k;                           % state matrix with state-feedback
B_f = A_f*sr;                          % construct B matrix
C = [0;0;1;0]';                        % view only delta on the output
D = 0;
sys_delta = ss(A_f,B_f,C,D);

% apply a step input with size of 0.002, plot response of each state
% using the step() function
opt = stepDataOptions;
opt.StepAmplitude = 0.002;             % step amplitude set
t = (0:0.005:10);                      % time set
figure(1); hold on;
step(sys_delta, t, opt);               % response on delta
sys_gamma = ss(A_f,B_f,[1,0,0,0],D);    % response on gamma
sys_gamma_d = ss(A_f,B_f,[0,1,0,0],D); % response on gamma_d
step(sys_gamma, t, opt);
sys_delta_d = ss(A_f,B_f,[0,0,0,1],D); % response on delta_d
step(sys_delta_d, t, opt);
hold off;

% model using lsim, providing an initial condition for delta = 0.002

% construct a step input, u
 u = [];
 for n = 1:size(t,2)
     if t(n) < 0.2
         u(n) = 0;
     else
         u(n) = 0.002;
     end
 end

 figure(2); hold on;
 lsim(sys_delta, u, t, [0;0;0.002;0]);
 lsim(sys_delta_d, u, t, [0;0;0.002;0]);
 lsim(sys_gamma, u, t, [0;0;0.002;0]);
 lsim(sys_gamma_d, u, t, [0;0;0.002;0]);
 hold off;

 % without the initial condition, can be seen that the same as
 % using the step function
 figure(3); hold on;
 lsim(sys_delta, u, t, [0;0;0;0]);
 lsim(sys_delta_d, u, t, [0;0;0;0]);
 lsim(sys_gamma, u, t, [0;0;0;0]);
 lsim(sys_gamma_d, u, t, [0;0;0;0]);
% lsim(syslqrcl, u, t);
 hold off;
```


Step Response
× 10^-3
6
4
2
0
-2
-4
-6
-8
Amplitude
0 1 2 3 4 5 6 7 8 9 10
Time (seconds)


Linear Simulation Results
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
Amplitude
0 1 2 3 4 5 6 7 8 9 10
Time (seconds)



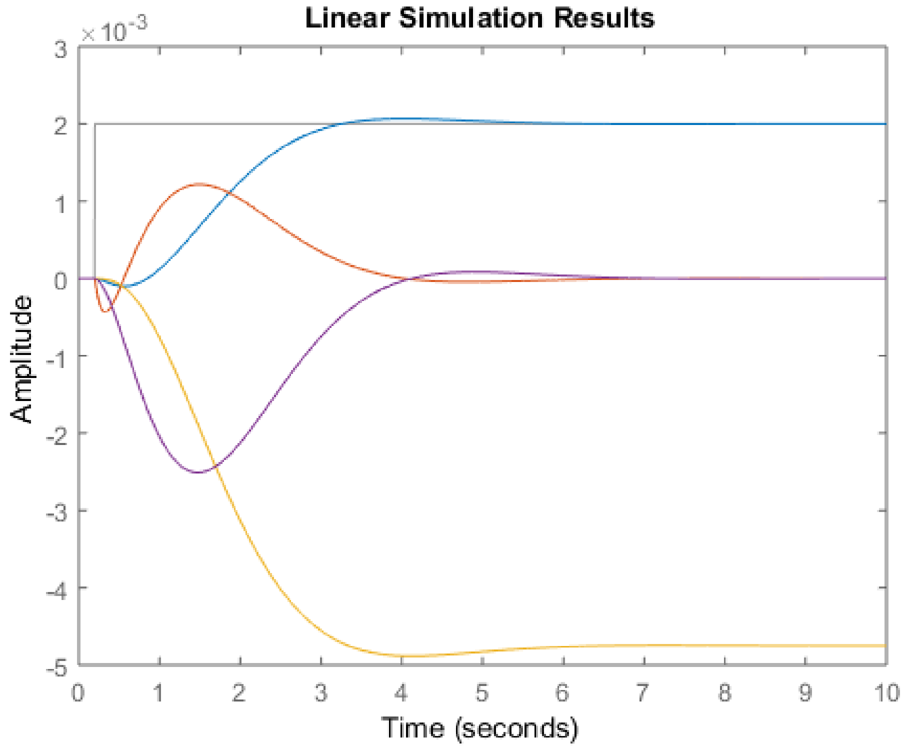
Part C - 2.0

```
                     /---------\
    r         +     u  | .         |
    ---> kr -->() ---->| X=Ax+Bu |--> y=Cx ---> y
              -|       \---------/
               |              |
               |<---- k <----|
```

insert a reference for state (delta)

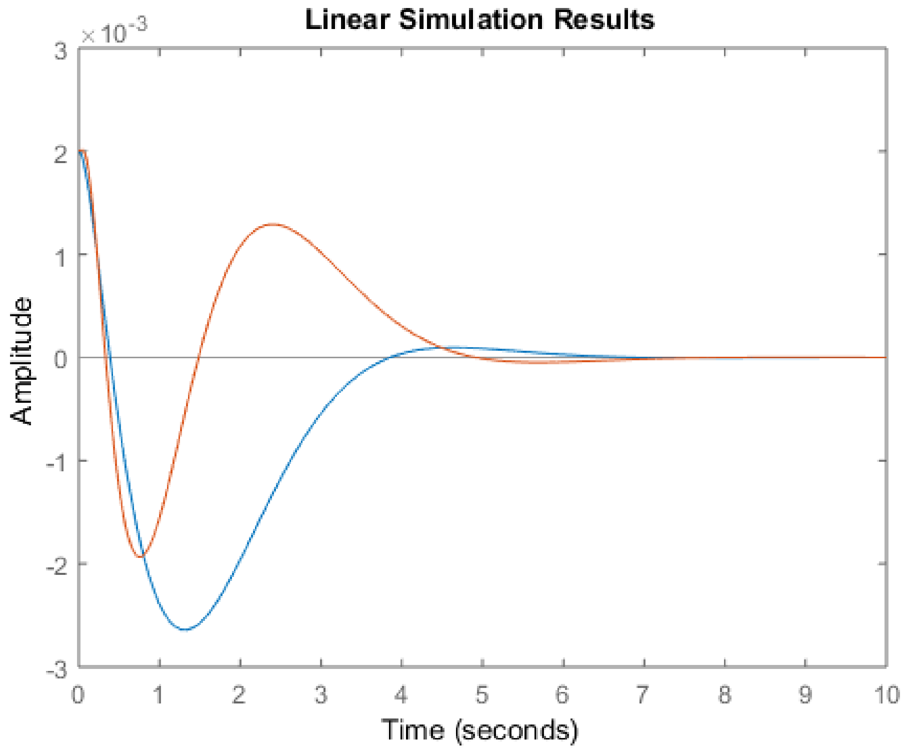
```
 % x_d = (A-Bk)*x + (kr*B)*r
 % x_d = A_star*x + B_star*r
 % where r is a reference step input for delta

 A_star = A-B*k;
 C = [0,0,1,0];                              %design kr for only delta on output
 D = 0;
 kr = -(C*(A-B*k)^-1 *B)^-1;%-(rscale(ss(A-B*k,B,C,D),k) - 1.68);     %equal to -0.541 (rscale calculates Kr)
 B_star = kr*B;

% apply a step input with size of 0.002, plot response of each state
% using the step() function
t = (0:0.005:10);                           % time set
figure(6); hold on;
sys_delta = ss(A_star,B_star,C,D);
sys_gamma = ss(A_star,B_star,[1,0,0,0],D);    % response on gamma
sys_gamma_d = ss(A_star,B_star,[0,1,0,0],D);  % response on gamma_d
sys_delta_d = ss(A_star,B_star,[0,0,0,1],D);  % response on delta_d

lsim(sys_delta, u, t);
lsim(sys_delta_d, u, t);
lsim(sys_gamma, u, t);
lsim(sys_gamma_d, u, t);
% lsim(syslqrcl, u, t);

hold off;
%Nbar = rscale(ss(A-B*k,B,C,D),k)
```

```
%aa = ss(A-B*k, -(Nbar-1.68)*B, C, D)
%figure(7); lsim(aa, u, t)
```



Part D

```
P = [-8 -16 -4+1i -4-1i];           % pole locations for observer
L = place(A', C', P)';              % poles at eig(A-LC);

% state space model for complete state-feeback and observer

A_p = [A -B*k; L*C A-B*k-L*C];
B_p = [B*kr; B*kr];
C_p = [C zeros(size(C))];
C_p_hat = [zeros(size(C)) C];

sys_p = ss(A_star, B_star, C, D);           % system which outputs plant delta
sys_p_hat = ss(A_p, B_p, C_p, 0);       % system which outputs observer delta

figure(8); hold on;
no_input = zeros(size(t));              % no step input

%show response for initial disturbance on plant delta
lsim(sys_p, no_input, t, [0, 0, 0.002, 0]);
lsim(sys_p_hat, no_input, t, [0, 0, 0.002, 0, zeros(1,4)]);
hold off;
```

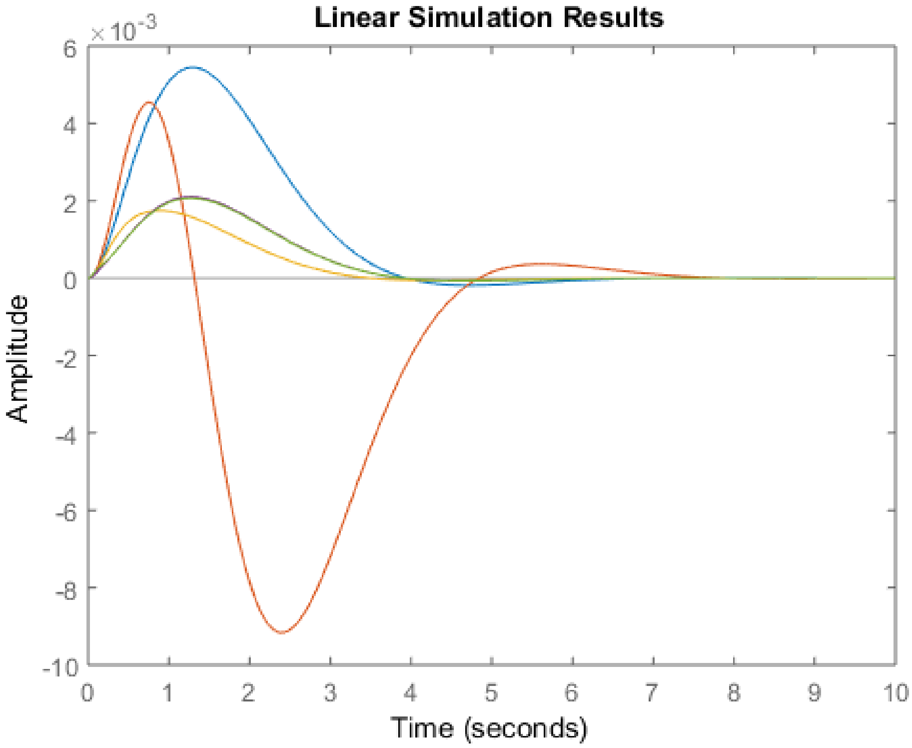


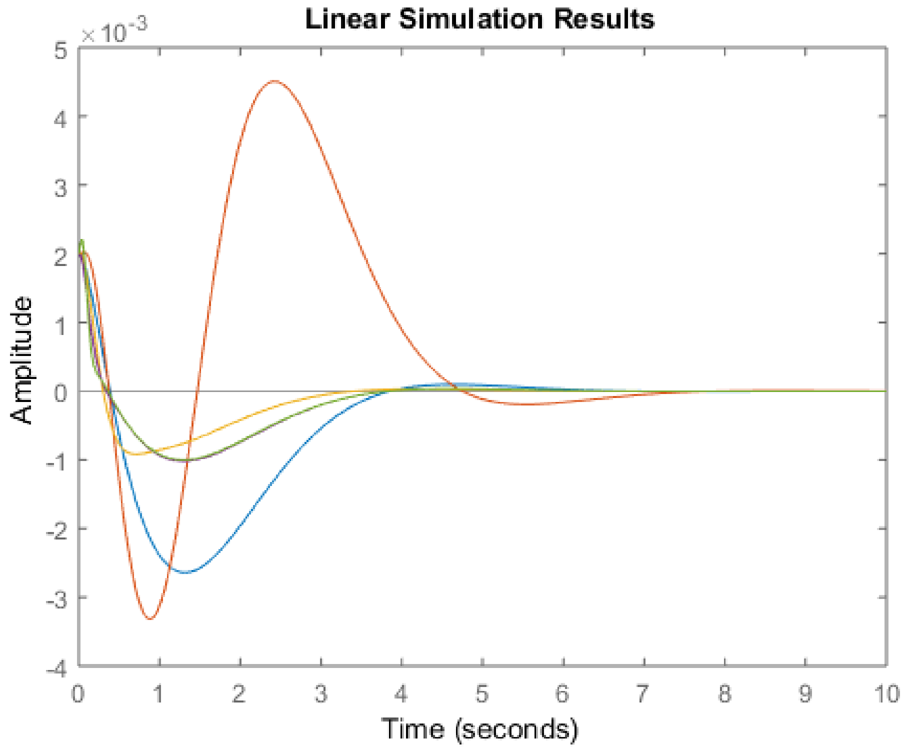
Part E

to show the effect of observer poles on replicating the plant Pol is a matrix containing the poles of the state-feedback component of the plant

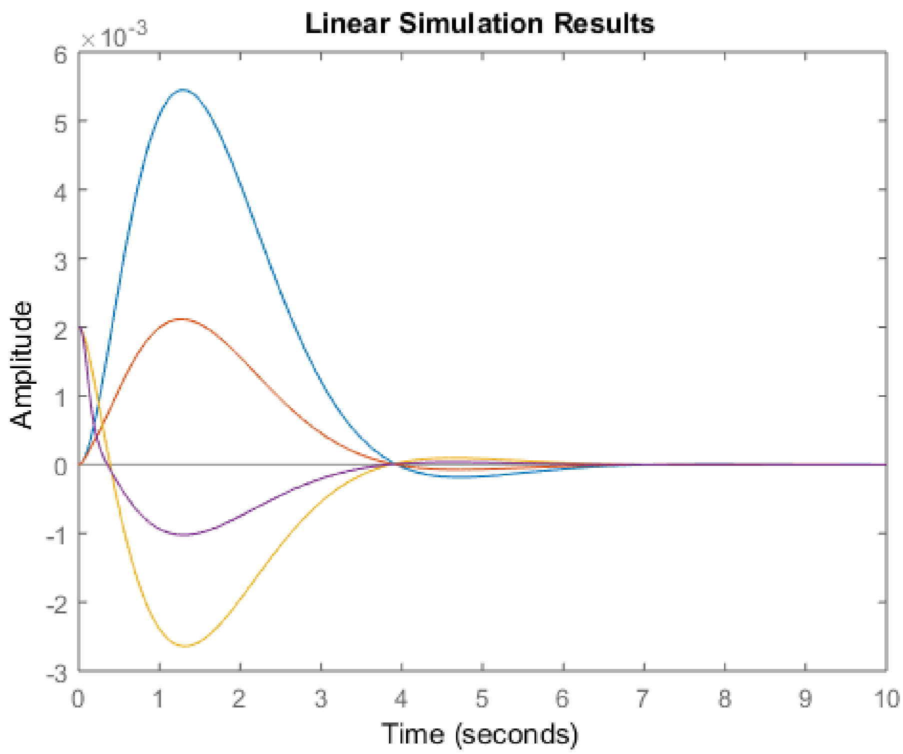
```
% make the observer poles a multiple of the plant observes, my multiplying
% Pol by an element in co before evaluating L.
co = [2, 5, 10, 20];

% For each multiple of Pol, plot the response against the plant for an
% initial state offset -> delta by 0.002

for i = 1:4
    % outputing delta
    C = [0,0,1,0];
    % place poles of observer at a multiple of Pol
    L = place(A', C', co(i)*Pol)';              % poles at eig(A-LC);
    A_p = [A -B*k; L*C A-B*k-L*C];              % observer A
    B_p = [B*kr; B*kr];                         % observer B
    C_p = [C zeros(size(C))];                   % observer C

    % retrieve the state-space model for an output of delta only
    if i == 1
        sys_delta = ss(A_star, B_star, C, D);        % system which outputs plant delta
    end
    sys_delta_hat = ss(A_p, B_p, C_p, 0);        % system which outputs observer delta
    % retrieve the state-space model for an output of phi
    C = [1,0,0,0];
    C_p_hat = [ C, zeros(size(C))];
    if i == 1
        sys_phi = ss(A_star, B_star, C, D);        % system which outputs plant delta
    end
    sys_phi_hat = ss(A_p, B_p, C_p_hat, 0);        % system which outputs observer delta
    figure(9); hold on;
    % plot response on phi
    if i == 1
        % plot the plant response, only once
        lsim(sys_phi, no_input, t, [0, 0, 0.002, 0]);
    end
    lsim(sys_phi_hat, no_input, t, [0, 0, 0.002, 0, zeros(1,4)]);
```

```
    hold off;
    figure(10); hold on;
    % plot reponse on delta
    if i == 1
        % plot the plant response, only once
        lsim(sys_delta, no_input, t, [0, 0, 0.002, 0]);
    end
    lsim(sys_delta_hat, no_input, t, [0, 0, 0.002, 0, zeros(1,4)]);
    hold off;

end
```





```
i = 1;
    % outputing delta
    C = [0,0,1,0];
    % place poles of observer at a multiple of Pol
    L = place(A', C', [-15+7.5i -15-7.5i -20+10i -20-10i])';              % poles at eig(A-LC);
    A_p = [A -B*k; L*C A-B*k-L*C];              % observer A
    B_p = [B*kr; B*kr];                         % observer B
    C_p = [C zeros(size(C))];                   % observer C

    % retrieve the state-space model for an output of delta only
    if i == 1
        sys_delta = ss(A_star, B_star, C, D);        % system which outputs plant delta
    end
    sys_delta_hat = ss(A_p, B_p, C_p, 0);        % system which outputs observer delta
    % retrieve the state-space model for an output of phi
    C = [1,0,0,0];
    C_p_hat = [ C, zeros(size(C))];
    if i == 1
        sys_phi = ss(A_star, B_star, C, D);        % system which outputs plant delta
    end
    sys_phi_hat = ss(A_p, B_p, C_p_hat, 0);        % system which outputs observer delta
    figure(11); hold on;
    % plot response on phi
    if i == 1
        % plot the plant response, only once
        lsim(sys_phi, no_input, t, [0, 0, 0.002, 0]);
    end
    lsim(sys_phi_hat, no_input, t, [0, 0, 0.002, 0, zeros(1,4)]);
    hold off;
    figure(11); hold on;
    % plot reponse on delta
    if i == 1
        % plot the plant response, only once
        lsim(sys_delta, no_input, t, [0, 0, 0.002, 0]);
    end
    lsim(sys_delta_hat, no_input, t, [0, 0, 0.002, 0, zeros(1,4)]);
    hold off;
```



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