Methodology, Modelling and Consultancy Skills

Assesssment 1

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Problem 1

To start we write the vectors (in transpose for ease of formatting)

$$\begin{aligned}
X^T &= (\text{Brass Bronze Duralumin}) \\
M^T &= (70000\ 100000\ 90000) \\
\text{Sale}^T &= (8\ 16\ 14) \\
Y^T &= (\text{Zinc Tin Aluminium}) \\
Q^T &= (100000\ 75000\ 50000) \\
\text{Cost}^T &= (5\ 2\ 8) \\
C^T &= (C_1\ C_2\ C_3)
\end{aligned}$$

Denoting the products, max production of each product, sale price of each product, the non-copper materials, the maximum quantity of each material we can buy, the costs of the non-copper materials, and the copper materials allotted to each alloy respectively. We should also note the maximum amount of copper we can use, $M_C = 80000$.
Let p_i represent the percentage of copper in the alloy x_i and let y_i denote the mass of the material in Y used

$$\frac{C_i}{C_i + y_i} = p_i$$

We can rewrite this so that the mass of each material used when blending our alloy is in terms of the mass of copper used.

$$y_i = \frac{1 - p_i}{p_i} \cdot C_i$$

This is useful as we can write our product masses in terms of the amount of copper used and the proportion of the alloy that will be copper.

$$\begin{aligned}
x_i &= 0.9 \cdot (y_i + C_i) \\
x_i &= 0.9 \cdot (\frac{(1 - p_i}{p_i} \cdot C_i + C_i) \\
x_i &= 0.9 \cdot (\frac{(1 - p_i}{p_i} + 1) \cdot C_i \\
x_i &= 0.9 \cdot \frac{1}{p_i} \cdot C_i
\end{aligned}$$

We should now look at the profit per product (ppp) of copper of each alloy. Let Sale(i) denote the

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ith entry of the Sale vector.

$$\begin{aligned}
\text{ppp}(i) &= \text{Sale}(i) \cdot x_i - \text{Cost}(i) \cdot y_i - 12 \cdot C_i \\
&= \text{Sale}(i) \cdot 0.9 \cdot \frac{1}{p_i} \cdot C_i - \text{Cost}(i) \cdot \frac{(1-p_i)}{p_i} \cdot C_i - 12 \cdot C_i \\
&= C_i(\frac{9 \cdot \text{Sale}(i)}{10 \cdot p_i} + \frac{p_i \cdot \text{Cost}(i) - 12 \cdot p_i - \text{Cost}(i)}{p_i}) \\
&= C_i(\frac{9 \cdot \text{Sale}(i) + 10 \cdot p_i \cdot \text{Cost}(i) - 120 \cdot p_i - 10 \cdot \text{Cost}(i)}{10 \cdot p_i})
\end{aligned}$$

$\frac{\text{ppp}(i)}{C_i}$ is our profit per kilogram of copper. In fact we have our linear programming problem we want to maximise now.

$$\sum_{i=1}^{3} \text{ppp}(i)$$

We want to maximise $\frac{\text{ppp}(i)}{C_i}$ for maximal profit per kilogram of copper. $\frac{\text{ppp}(i)}{C_i}$ is decreasing on all the following intervals of p_i

$$\begin{aligned}
0.2 \leq & p_1 \leq 0.5 \\
0.6 \leq & p_2 \leq 0.8 \\
0.1 \leq & p_3 \leq 0.5
\end{aligned}$$

As such we want to minimise p_i in each case. To show this is a linear optimisation problem we want to write the objective function in the form

$$S^T C$$

which is rather easy as we saw above the obvious choice for S_i is

$$s_i = \frac{\text{ppp}(i)}{C_i}$$

as such we have

$$S^T = (4 \ \frac{32}{3} \ 42)$$

From here we just need to add our constraints.

$$\begin{aligned}
\sum_{i=1}^{3} C_i &\leq 80000 = M_C \\
x_i = \frac{9 \cdot C_i}{10 \cdot p_i} &\leq M_i \\
y_i = \frac{(1-p_i)}{p_i} \cdot C_i &\leq Q_i
\end{aligned}$$

$$\max\{\sum_{i=1}^{3} S_i C_i | \sum_{i=1}^{3} C_i \leq M_C, \ x_i = \frac{9 \cdot C_i}{10 \cdot p_i} \leq M_i, \ y_i = \frac{(1-p_i)}{p_i} \cdot C_i \leq Q_i\}$$