GRATTAN Institute

Closing the gaps

Technical Report

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This technical report was written by Dr Peter Goss, Grattan Institute School Education Program Director, and Dr Cameron Chisholm, Grattan Institute Senior Associate. It was prepared to accompany the Grattan Institute Report, *Closing the gaps*. The purpose is to present the data and methodology used in the analysis, with a discussion exploring robustness and sensitivity.

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A Conceptual framework for translating NAPLAN scale scores into comparative year levels

A.1 Introduction

The report for Grattan Institute *Closing the gaps* seeks to measure student progress on the National Assessment Program – Literacy and Numeracy (NAPLAN) test in a way that is robust, easy to interpret, and comparable across different groups of students. It analyses student-level data to identify some of the factors associated with higher or lower levels of progress, and to quantify the degree of these associations. The analysis does not attempt to quantify the causal impact of these factors, and should not be interpreted as such.

Every year since 2008, the NAPLAN test has been administered to nearly all students in Years 3, 5, 7, and 9.[1] This means that students who were in Year 3 in either 2008 or 2009 have now taken the NAPLAN test across each of the test-taking years. This makes it possible to track how students have improved (as measured by NAPLAN) over a significant proportion of their time spent at school.

These technical report includes four appendices to *Closing the gaps*. Appendix A describes the conceptual framework behind creating a new lens to interpret NAPLAN results. Appendix B describes the data used in the analysis, and discusses some of the data issues. Appendix C outlines the technical detail behind the methodology to convert NAPLAN scale scores onto *comparative year levels*. Finally, Appendix D explains the approach used to track the progress of Victorian students across Year 3 to Year 9.

[1] On average for a given test, about 2 per cent of students are withdrawn, 2 per cent exempt, and about 4 per cent are absent [ACARA (2014)].

A.2 The design of NAPLAN

A.2.1 NAPLAN scale scores

Students that undertake the NAPLAN test receive a score for each assessment domain: reading, writing, language conventions (which include, spelling, grammar and punctuation), and numeracy. This score, called the NAPLAN scale score, is between 0 and 1000. While the scores are used to indicate whether a student is above NAPLAN national minimum standards for each year level, they have no other direct interpretation. The scores are an estimate of student ability, a latent concept – the numbers themselves have no particular meaning. Nor are the scores comparable across assessment domains.

NAPLAN results are also reported using proficiency bands. Section A.5 explains why we do not use these in this report.

A.2.2 Horizontal and vertical equating

The NAPLAN test is designed so that results can be compared between students in different year levels and students taking the test in different years. This means that a student who took the Year 5 NAPLAN test in 2012 and received a scale score of 500 is estimated to be at the equivalent level of a student who took the Year 7 test in 2013 and received the same score. This property of NAPLAN is achieved via a process known as *horizontal* and *vertical equating*.

The horizontal equating process involves a sample of students taking an equating test in addition to the NAPLAN tests. A

scaling process takes place using this equating sample and common items across years on the equating tests. The result is that NAPLAN scale scores are comparable across different years. The vertical equating process involves common test items on the tests administered to different year levels. The results are scaled so that scale scores are comparable across different year levels.[2]

The move to online testing from 2017 is likely to strengthen the equating process.[3] The results presented in this analysis assume that the equating process is robust and reliable for comparing different groups of students.

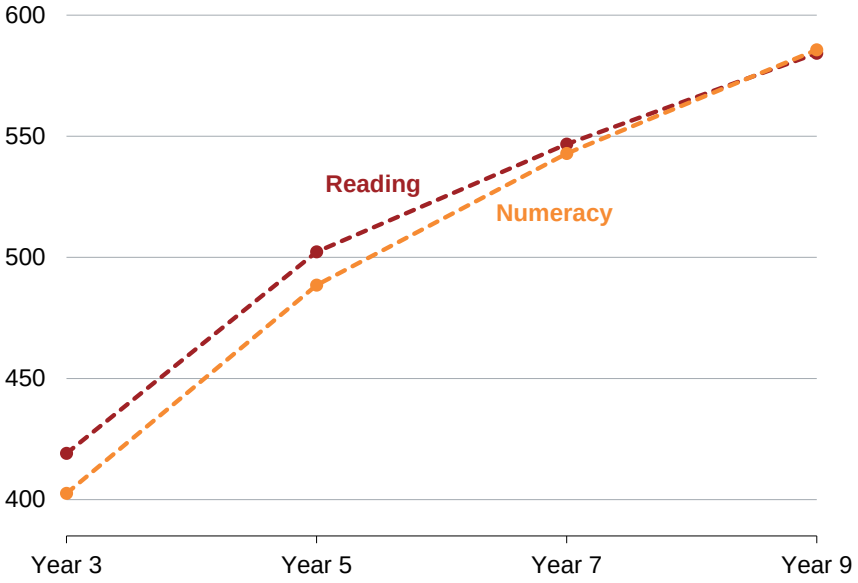
A.3 Looking at progress needs a new lens

A.3.1 Using NAPLAN scale scores to compare student progress is problematic

According to the Rasch model by which NAPLAN scale scores are developed, the estimates of student ability are on an interval scale.[4] This property suggests that student progress can be measured by 'gain scores': the difference between NAPLAN scale scores in two test-taking years. But there are limitations to using this measure, as ACARA notes:

> *It is important to consider that students generally show greater gains in literacy and numeracy in the earlier years than in the later years of schooling, and that students who start with lower NAPLAN scores tend to make greater gains over time than those who start with higher NAPLAN scores.*[5]

Figure A.1: The relationship between NAPLAN scale scores and year level is not linear for the median student
NAPLAN scale score of median student in each year level

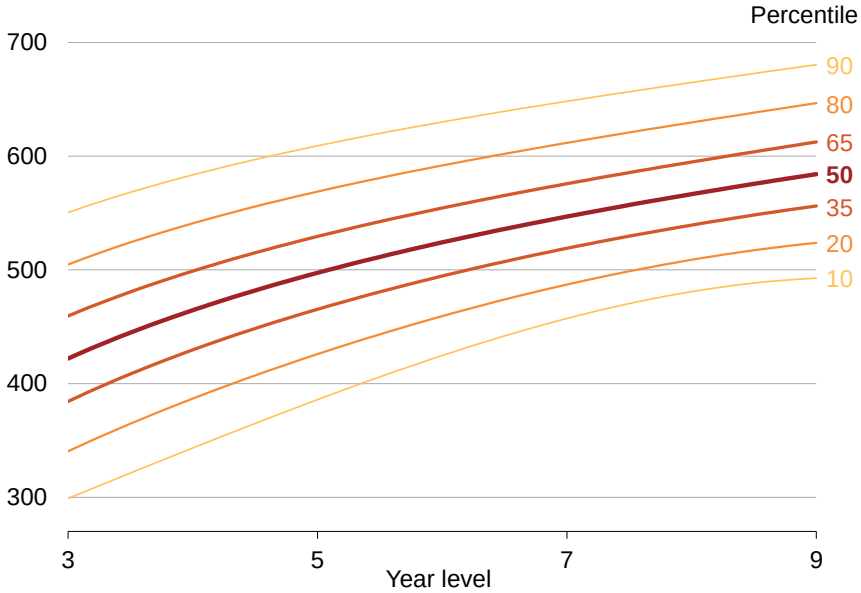


Notes: Based on 2014 and 2012 median scores.
Source: Grattan analysis of ACARA (2014).

That is, the observed "path of progress" across the four NAPLAN test years is not a linear function of the NAPLAN scale score, as shown in Figure A.1. In numeracy, for instance, the median student makes a gain of 86 points between Years 3 and 5 (an average of 43 points each year), 54 points between Years 5 and 7 (an average of 27 points each year), and 43 points between Years 7 and 9 (an average of 21.5 points each year).[6]

[2] See ACARA (2015e), pp. 40–72 for details.
[3] ACARA (2015c).
[4] This means that, in terms of student ability, the difference between a score of 400 and 450 is equivalent to the difference between 600 and 650, for example.
[5] ACARA (2015b).
[6] Grattan analysis of ACARA (2014).

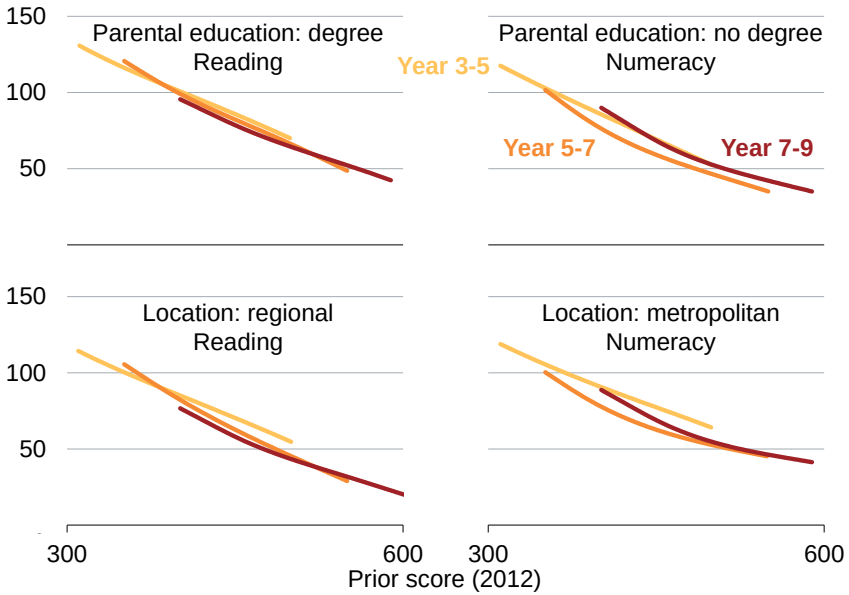
Figure A.2: All percentiles make smaller gain scores at higher year levels
NAPLAN scale score by percentile, reading



Notes: Percentiles defined according to 2014. Each curve is smoothed across four observed points using a third-order polynomial to get a better picture of the relationship. A similar pattern occurs for numeracy.
Source: Grattan analysis of ACARA (2014).

Two competing hypotheses come out from this: either students are making less progress as they get older, or student progress is not linearly related to NAPLAN gain scores. If we accept the first hypothesis, this implies that the education system is failing the average student at later year levels. But the same non-linear pattern holds for students at different percentiles. Figure A.2 suggests that the relationship between NAPLAN scale score and year level is closer to being logarithmic than linear, even as low as the 10th percentile, and as high as the 90th percentile.

Figure A.3: Higher gain scores are observed for lower prior scores, regardless of year level or population sub-group
Median NAPLAN gain score over two years by prior score, 2014



Notes: Similar patterns exist for other sub-groups. Gain scores estimated by a median quantile regression with cubic regression splines.
Source: Grattan analysis of ACARA (2014).

Further analysis suggests that the lower gain scores observed at the top end are only weakly related to year level once prior NAPLAN score (from two years earlier) is taken into account. That is, in *any* year level, students with lower prior NAPLAN scale scores tend to make higher gain scores than those with higher prior scores. This holds for both numeracy and reading and for different sub-groups, with examples shown in Figure A.3.[7] The same relationship is observed in every single Australian jurisdiction.

[7] We have not been able to identify a sub-group of students for which gain score had no apparent relationship with prior score.

Either we must accept that students with higher NAPLAN scale scores are making less progress than those with low scores, or that progress is not linearly related to gain in NAPLAN score. Given that NAPLAN gain is decreasing in prior score for every sub-group we have analysed, Occam's Razor suggests we should accept the second hypothesis.

If we accept that progress is not linearly related to NAPLAN gain scores, we cannot compare the progress of different student groups except in the special case of student groups starting from the same score. To compare students starting from different prior scores requires a new lens.

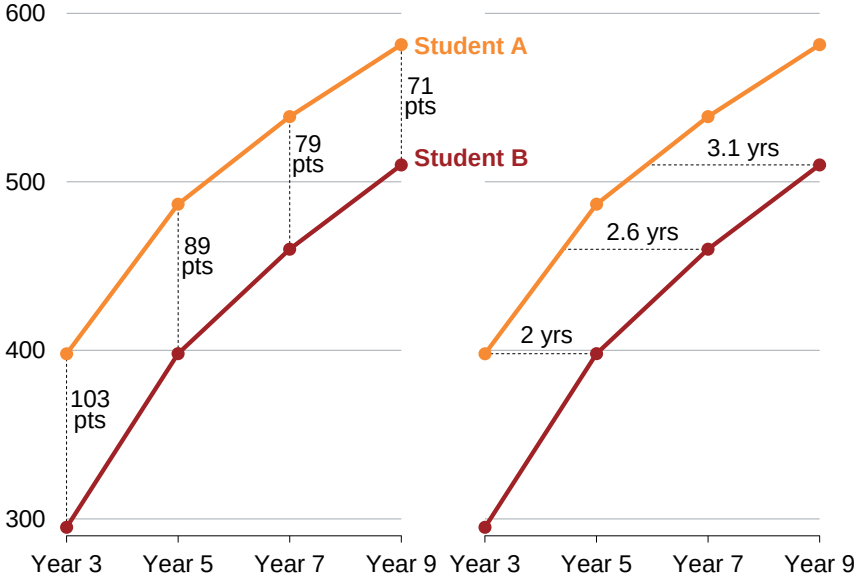
A.3.2 Looking at progress through the lens of time

An alternative measure of student progress is to define a *year of progress* as the improvement expected from a typical student over a year. This measure would take into account that the typical student makes smaller gains in NAPLAN scale scores as they move further up the NAPLAN scale. That is, the NAPLAN gain score required for the typical student to make two *years of progress* between Years 5 and 7 is smaller than that required between Years 3 and 5.

Years of progress is a measure of student *learning* relative to their peers, rather than a measure of their *ability*. This measure, as opposed to using NAPLAN gain scores, gives NAPLAN results new meaning, and can change the interpretation.

Figure A.4 provides an illustration of this for two distinct groups of students: Group A and Group B. The scores displayed on the chart are those of a representative student within each group (the median student): call these students A and B. Student A scores close to the average for numeracy, while Student B is below average, 103 NAPLAN points behind

Figure A.4: Measuring progress in years changes the interpretation of NAPLAN results
NAPLAN scale score



Notes: The points on both charts are identical.
Source: Grattan analysis.

Student A in Year 3. According to the NAPLAN gain scores, Student B reduces the gap every year, as shown on the left chart, suggesting that Group B are catching up to Group A.[8]

Yet the right chart tells a different story. In Year 5, Student B is performing at the level of Student A in Year 3. But by the time they reach Year 9, Student B's score is roughly half way between Student A's scores in Year 5 and Year 7: Student B is performing at about the level of Student A in Year 6. This suggests that Group A has made one more year of progress

[8] This does not account for within-group variation, but it suggests the typical student in Group B is catching up to the typical Student in Group A.

than Group B between Years 5 and 9. Looking at progress through the lens of time suggests that Group B are falling further behind.

A.4 Measuring *Years of Progress*

If we interpret the difference between students A and B according to the right chart of Figure A.4, then Student B makes roughly the same progress over four years (between Year 5 and Year 9) as Student A makes in three years (between Year 3 and Year 6). The difference between the students is defined in terms of Student A's rate of learning, but it could just as easily be defined in terms of Student B's rate of learning: "how long will it take Student B to reach the level of Student A?". While the story – that Student A learns faster than Student B – remains the same regardless of which student is defined as the benchmark, the size of the gap between the two in terms of 'years and months' is different.[9] To consistently compare progress in terms of years and months requires a common benchmark. Given that NAPLAN scores are not linked to absolute curriculum that define the expected capabilities for each year level, the benchmark is necessarily a relative one.

The results presented in *Closing the gaps* use the median or 'typical' student's results as a benchmark for comparing other groups of students. That is, a year of progress is defined according to the gain score expected from the median student at a given level if they were to take the NAPLAN test in one year's time.[10]

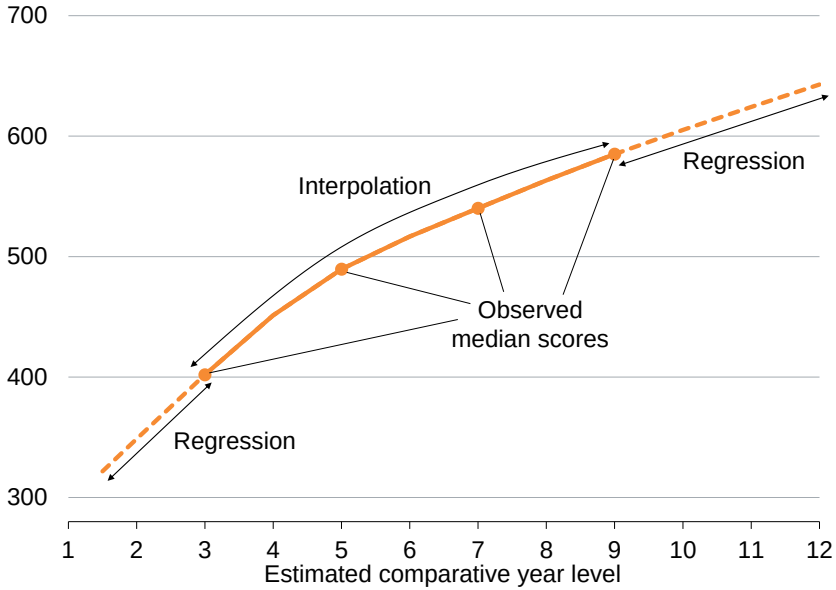
NAPLAN scale scores are mapped onto the path of progress of the typical student across their schooling years. We define the schooling year associated with each NAPLAN score as a *comparative year level*. It is straightforward to estimate the score corresponding to comparative year levels 3, 5, 7, and 9; these are just the observed median scores for each test-taking year. In Year 5 numeracy, for instance, the median NAPLAN scale score is approximately 489 – a student with a numeracy score of 489 in any test-taking year is said to be performing at comparative year level 5, meaning at the same level as a typical Year 5 student.

It is more challenging to estimate the scores corresponding to the non-test-taking years. If we assume that learning is relatively consistent across each two-year period, it is possible to fit a curve through these four points (using a third-order polynomial), giving estimates of the NAPLAN scale score for comparative year levels 4, 6, and 8, along with every month in between. Estimating comparative year levels below Year 3 and above Year 9 involves a regression of student gain scores on scores from the previous test. Figure A.5 shows how these approaches are used to construct a curve that maps NAPLAN scale scores to estimated comparative year levels. This methodology is outlined in more detail in Appendix C on page 23.

[9] In Year 5, for instance, Student B is performing at Student A's level two years earlier, but Student B will take about three years to reach Student A's current level.

[10] Because NAPLAN is taken every two years, it is only possible to observe gain scores over two-year periods. But it is straightforward to interpolate this for a single year of progress.

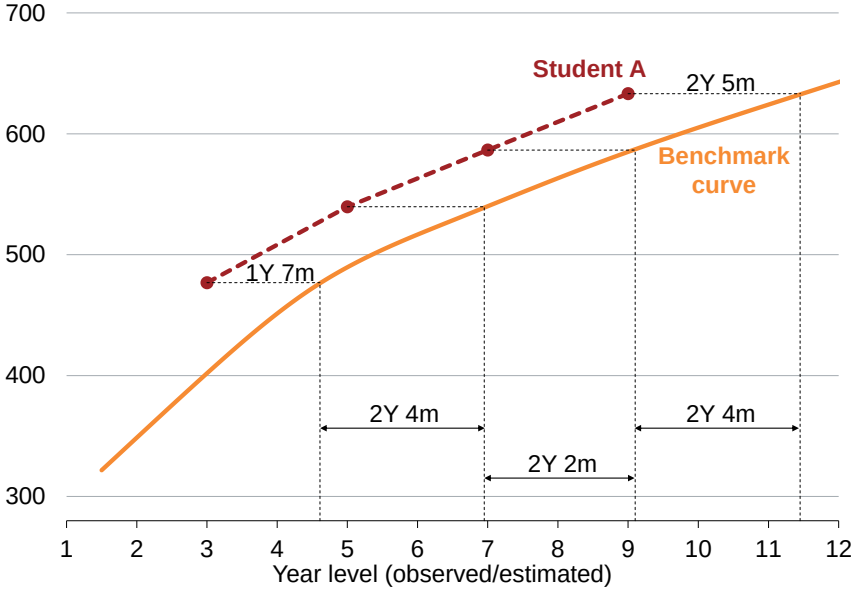
Figure A.5: Estimating comparative year levels involves interpolation and regression
NAPLAN scale score, numeracy



Source: Grattan analysis of ACARA (2014).

Having constructed the benchmark curve, it is possible to track the comparative years of progress made by a given student or a group of students. An example of this is shown in Figure A.6 for an above-average student who is about one year and seven months ahead of the benchmark curve in Year 3. By tracking this student back to the benchmark curve, we can conclude that the student made above-average progress between each NAPLAN test, finishing Year 9 two years and five months ahead of the benchmark.

Figure A.6: Student progress is measured with reference to the benchmark curve
NAPLAN scale score, numeracy



Source: Grattan analysis of VCAA (2014b) and ACARA (2014).

A.5 NAPLAN proficiency bands

In order to simplify the interpretation of NAPLAN scale scores, ACARA also report student achievement using *proficiency bands*. There are ten proficiency bands spanning Year 3 to Year 9, but only six are reported for each year level. With the exception of Band 1 and Band 10, each band spans 52 NAPLAN point scores.[11] As with NAPLAN scale scores, a given proficiency band represents the same level of ability (over a range) regardless of year level. For instance, a Year 3 student in Band 6 is at the same level as a Year 9 student in Band 6.

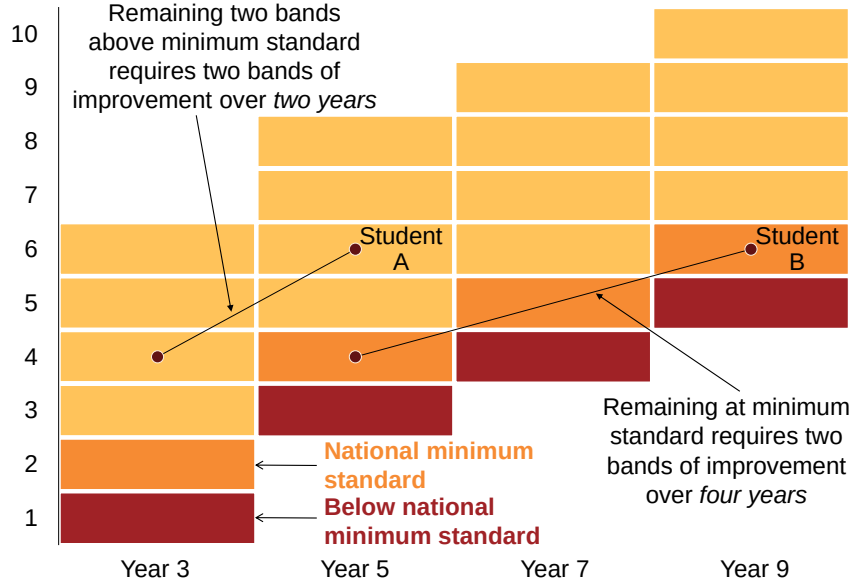
[11] Due to high measurement error in extreme scores in each year level, only six bands are reported for each year level.

The proficiency bands are not used in the analysis of this report. But they can be used to illustrate a limitation in the reporting and interpretation of NAPLAN results to understand student progress. Because there are six bands reported for each year level, a logical way to interpret a student's progress would be to look at their relative position across the six bands over time. For instance, a student performing in the third-highest band in Year 3 (Band 4) who progresses to the third-highest band in Year 5 (Band 6) could be thought to be making an average level of progress. But this interpretation is not consistent for students at different proficiency levels.

The second-lowest of the six proficiency bands at each year level is defined as the national minimum standard. Between Year 3 and Year 5, the minimum standard increases two bands, from Band 2 to Band 4. But it then only increases by one band for every two years after Year 5: the minimum standard in Year 7 is Band 5, and in Year 9 is Band 6. This reflects that students typically make larger gains when starting from a lower score.

Figure A.7 provides an example of how a standard interpretation of NAPLAN proficiency bands would be internally inconsistent for different students. In this example, Student A moves from Band 4 in Year 3 to Band 6 in Year 5, staying two bands above national minimum standard. Student B performs consistently in the national minimum standard band, moving from Band 4 in Year 5 to Band 6 in Year 9. Both students remain in the same relative proficiency band, which suggests they are learning at the same rate. Yet Student A is seemingly progressing faster since this student makes the same gain over two years as Student B does over four. Both interpretations cannot be correct. This apparent inconsistency is a major reason why proficiency bands are not used in the analysis of this report.

Figure A.7: The level of growth required to remain in the same relative proficiency band changes with year level
NAPLAN proficiency band



Source: ACARA (2015e).

B Data sources and issues

B.1 Student-level NAPLAN datasets used in the report

The analysis in *Closing the gaps* is based on linked student-level NAPLAN records.[12] There are two major datasets used in the analysis:

- NAPLAN results across all four domains and year levels for all Australian students recorded in 2014, linked with their 2012 results where applicable.[13] This dataset contains test scores for more than one million students for each domain in 2014, and more than 700,000 in 2012.[14]
- NAPLAN results across all four domains recorded across 2009 to 2015 for the cohort of Victorian students who began Year 3 in 2009.[15] For each domain, more than 55,000 students have a Year 3 test score and a score from at least one other test year. More than 45,000 students have a test score recorded in all of Years 3, 5, 7, and 9 for both reading and numeracy.

Comparative year levels are estimated using the national dataset to create a national benchmark for student progress. This benchmark is used in analysis of the linked Victorian data, which allows progress of individual students to be tracked from Year 3 to Year 9. In this way, the "years of progress" made by particular groups of Victorian students is relative to the typical Australian student, as opposed to the typical Victorian student.[16]

The data contain a number of student background variables, including gender, parental education and occupation, language background and indigenous status. Some geographic information is available at the school level, including state, and whether the school is located in a metropolitan, regional, or rural area. The Victorian data also include the local government area of the school as well as a measure of school socioeconomic Status (SES): the Index of Community Socio-Educational Advantage (ICSEA).[17] The national dataset contains a randomised school-level indicator – it is possible to identify whether two or more students attend the same school, but not possible to identify schools themselves.

Two additional datasets are used to check the robustness of the analysis across different cohorts – the NAPLAN results across all domains and year levels for all Australian students recorded in 2013, linked with their 2011 results, and the NAPLAN results across all domains recorded across 2008 to 2014 for the cohort of Victorian students who began Year 3 in 2008.[18] Because NAPLAN results vary across cohorts, the

[12] Analysis was carried out for reading and numeracy, but not the other domains.
[13] ACARA (2014).
[14] Only students in Years 5, 7, and 9 in 2014 have a linked record in 2012. Linked records are not available for students in the Northern Territory.
[15] VCAA (2014b).
[16] This allows the analysis to pick up Victorian-specific effects. It should be noted that, on average, Victorian students score higher than most other states. One explanation for this is that Victorian students are, on average, about four months older than their counterparts from other states in the same year level, and are more likely to come from a high SES background [Grattan analysis of ACARA (2014)].
[17] To prevent school identification, this index is grouped into bands of 26 points.
[18] ACARA (2013) and VCAA (2014a).

analysis was rerun with these data. This confirmed that the key findings of the report – in terms of the scale and direction of learning gaps – were not cohort-specific.

B.2 Defining the 'typical' student

The analysis presented in *Closing the gaps* focuses on the 'typical' student, either at the population level or within a particular sub-group of students. As noted in the main report and in Appendix A, for the purposes of measuring *Years of Progress*, the typical student in a given year level is defined as the student with the median NAPLAN scale score. Analysis of particular sub-groups of students (such as those grouped by parental education or school SES) is performed according to the typical student within each sub-group – the sub-group median.

An important advantage of using the median over the mean is that it is not directly affected by outliers. For instance, there may be a number of students who do not care about NAPLAN results who leave questions unanswered on the test instead of attempting them, meaning that their NAPLAN scale scores would not be an accurate estimate of their true ability. These inaccurate results would have a much larger impact on estimates of the mean score and the mean gain score than they would have on the median.[19] NAPLAN scale scores also tend to have a small positive skew (particularly for numeracy), which lifts the mean relative to the median.

B.3 Defining household socioeconomic status

The report analyses how NAPLAN results and progress vary by socioeconomic status (SES), using the Victorian 2009–15 dataset. Variables relating to SES in the data include parental education, parental occupation, indigenous status and geographic location. One option would be to construct an index of household SES based on these variables. But there are issues constructing such an index for Victorian households. Firstly, very few students are indigenous and geographic location is only defined at the school level. Secondly, parental education and occupation are highly correlated – for instance, 85 per cent of households where a parent has a Bachelor's degree are classified as a manager or professional, compared to only 21 per cent where no parent has a degree or diploma.[20]

We instead use a simple proxy for household SES: the highest level of parental education attained. While there is information on the highest schooling year attained, most parents of school-age children in Victoria have completed Year 12; we therefore focus on educational attainment beyond school.[21] Students can be divided into four groups based on the highest level of parental education:

- at or above Bachelor's degree
- diploma
- certificate I to IV
- year 12 or below.

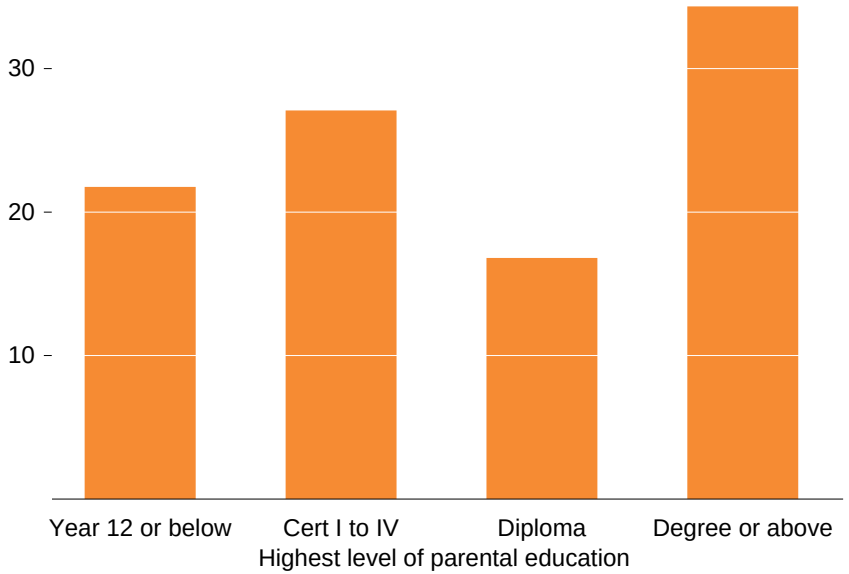
Figure B.1 shows that each of these four categories include at least 15 per cent of all students. Preliminary analysis suggests that the difference in student attainment and progress between

[19] Estimates of the median would only be impacted in this way if a substantial number of students whose true ability is above the median are recorded below the median as a result of leaving questions unanswered.

[20] Grattan analysis of VCAA (2014b). Some studies use a composite measure of parental education and occupation, such as Marks (2015) and Houng and Justman (2014), but a single measure is likely to still provide a reasonable proxy for household SES.

[21] Studies have shown that post-school qualifications are a strong predictor of household income; see, for instance OECD (2015).

Figure B.1: Students are well represented in each category of parental education
Percentage of students, Victoria 2009–15 cohort



Source: Grattan analysis of VCAA (2014b).

the lowest two categories of parental education is small – the report groups these into a single category: 'below diploma'.

B.4 Using ICSEA as a measure of school socioeconomic status

The report analyses how NAPLAN results and progress vary by the Index of Community Socio-Educational Advantage (ICSEA, which is referred to in the report as 'school SES') in the Victorian 2009–15 dataset. ICSEA was developed by ACARA so that NAPLAN results could be compared between schools with similar student backgrounds. The index is based on student-level factors such as parental education and employment, indigenous status, and school-level factors such as remoteness and the proportion of indigenous students.[22] The index is constructed as a linear combination of these SES variables.

To determine the weighting applied to each variable, a regression model is estimated: average NAPLAN score (across all domains) against each SES variable. The estimated parameters of this model determine the weightings – essentially this means that the SES variables are weighted according to how strongly they relate to NAPLAN results. This index is then averaged across all students in each school, and scaled nationally so that the ICSEA distribution has a mean of 1000 and a standard deviation of 100. This methodology provides an estimate of ICSEA for each school, which is adjusted each year.[23]

We use the Victorian linked data to analyse the impact of school ISCEA on student progress. Schools are allocated to one of three ICSEA groups:[24]

- ICSEA greater than 1090 (approximately the top quartile of schools in Victoria)
- ICSEA greater than 970 but less than 1090 (approximately the middle two quartiles of schools in Victoria)
- ICSEA less than 970 (approximately the bottom quartile of schools in Victoria).[25]

[22] Geographic census data are also used in the index calculation.
[23] For more detail, see ACARA (2015a).
[24] Allocation is done for each of 2009, 2011, 2013 and 2015, since schools can change their socio-economic mix and ICSEA is recalculated by ACARA for all schools each year.
[25] This cut points were chosen from the ICSEA bands available to us. It should be noted that the average ICSEA of Victorian schools is higher than the national average.

There is a question as to whether the strong relationship observed between school SES and NAPLAN results is legitimate, or whether it arises as a result of the way ICSEA is constructed. While NAPLAN results are used in the construction of ICSEA, they are not used as an input variable – ICSEA is still entirely a linear function of SES variables. This means that the strong relationship observed between ICSEA and NAPLAN results is driven by SES factors, not by the way the index is constructed.

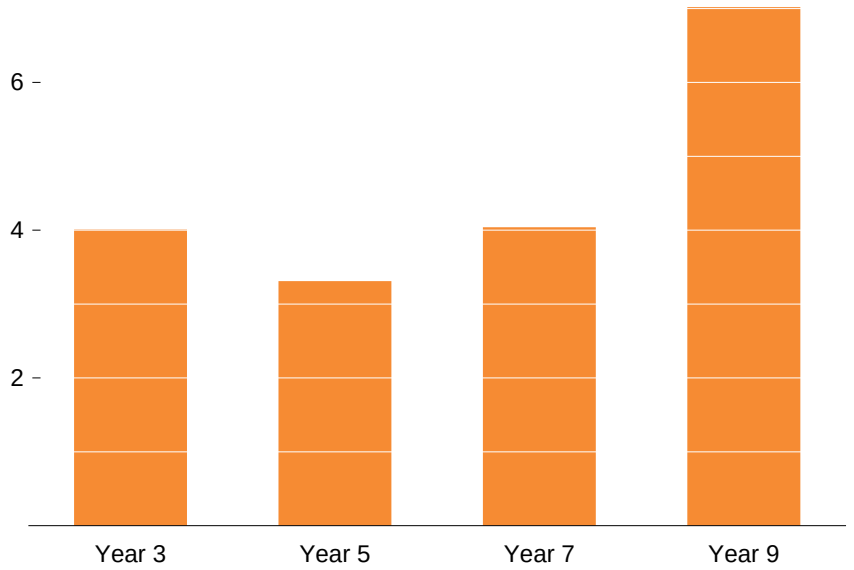
B.5 Missing data

There are two major sources of missing NAPLAN data: non-participation in NAPLAN and results that are not linked for the same student in different years. The non-linkage of results is only an issue for students in the Northern Territory – no linked data are available for Northern Territory in the national dataset.

For any given NAPLAN test, participation rates are high, usually exceeding 90 per cent. The most common reason for non-participation is student absenteeism. This is usually four per cent or less, but rises to seven per cent in Year 9, as shown for numeracy in Figure B.2. A small proportion of students (typically less than two per cent) are given an exemption from taking the NAPLAN test, usually if they have a significant disability or face a major language barrier. Finally, some students are withdrawn from testing by their parent/carer, although this is less than two per cent on almost every test.

Despite a high participation rate on each test, these missing data can potentially reduce the size of the linked samples quite significantly. In the cohort of Victorian students who took the Year 3 test in 2009, only about 72 per cent took all four NAPLAN tests to Year 9 for numeracy and reading. This is because different students missed the test in different years, and also because some students moved out of Victoria before Year 9.[26]

Figure B.2: Students are more likely to be absent from a NAPLAN test in Year 9
Percentage of students that are absent from NAPLAN numeracy test, Victorian 2009-2015 cohort

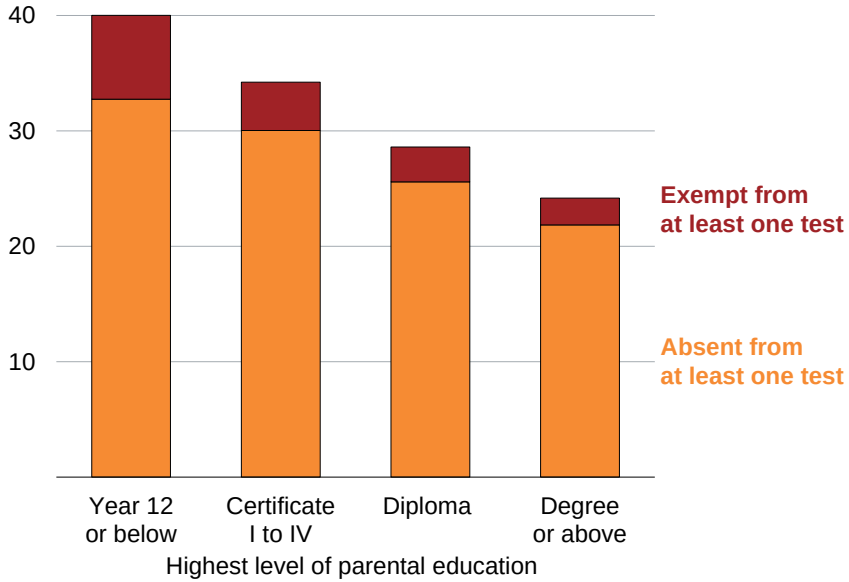


Notes: Does not include students who are exempt, withdrawn or miss a test due to leaving Victoria. Results are similar for reading.
Source: Grattan analysis of VCAA (2014b).

A brief analysis suggests that students are more likely to miss a test due to being absent/withdrawn or an exemption if they are from a low SES background. Figure B.3 shows that of the Victorian cohort of students in Year 3 in 2009, 40 per cent of

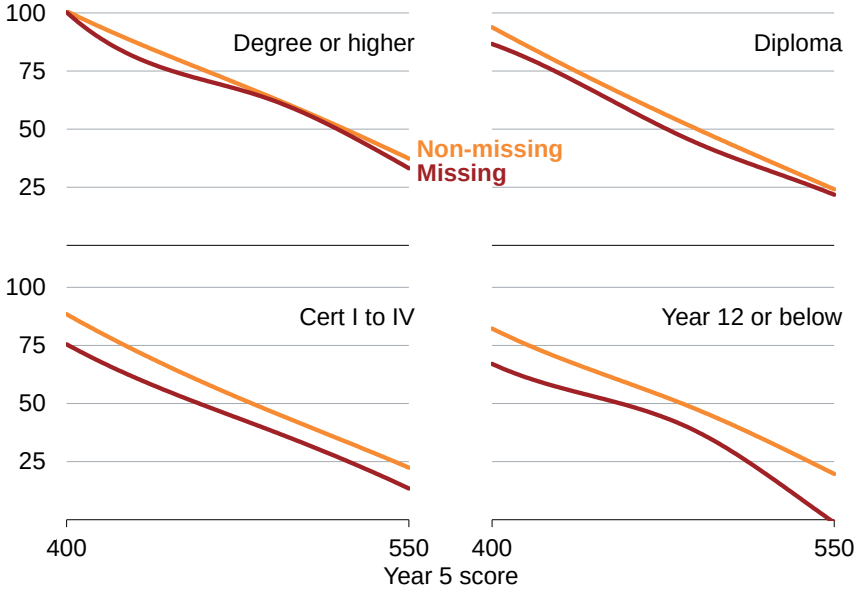
[26] There are also students that accelerate or repeat a year – these students are included in the analysis, although some have not completed Year 9 by 2015.

Figure B.3: Students from lower SES backgrounds are more likely to miss one or more NAPLAN tests
Percentage of students that miss a NAPLAN test, Victoria 2009–15 cohort



Notes: Includes all Victorian students in Year 3 in 2009, and all NAPLAN tests taken up to 2015. 'Absent from at least one test' includes those who were withdrawn, and those not in Victoria in one or more test-taking years after Year 3. Students that have been both absent and exempt from tests are categorised as exempt.
Source: Grattan analysis of VCAA (2014b).

Figure B.4: Missing data have more of an impact on gain scores for students from low SES backgrounds
Median NAPLAN gain score by highest level of parental education, reading, Year 5 to Year 7



Notes: 'Missing' includes all students that were absent/withdrawn from either the Year 3 or Year 9 reading test, but does not include exempt students. 'Non-missing' includes all students that did not miss a single NAPLAN test. A similar pattern exists for numeracy, for other year levels, and for school-level SES.
Source: Grattan analysis of VCAA (2014b).

those whose parents have no tertiary education missed at least one test between Year 3 and Year 9, compared to only 25 per cent of students where a parent has a Bachelor's degree.

Given that students from high SES backgrounds typically score higher and make higher gains from a given starting score than those from low SES backgrounds, the consequence of ignoring missing data is an upwards bias in estimates of the median score and median gain score.[27]

For analysis that tracks the progress of a particular sub-group, such as students that have a parent with a Bachelor's degree, missing data may be associated with lower gain scores from a given starting score. With only two years of linked data it would not be possible to test this. But with four years of linked data,

[27] That is, the estimated median is likely to be above the actual population 50th percentile.

as is available with the Victorian 2009 to 2015 cohort, there are students that have missed a test in one or two years, but for whom we observe NAPLAN scale scores in at least two other years. Figure B.4 shows the estimated median gain score in reading between Year 5 and Year 7 for students that did not miss a test in any year, and for students that missed a test in Year 3, Year 9 or both. Not only are those that missed a test predicted to make smaller gains, but the gap is larger for students from low SES backgrounds (using parental education as a proxy).

This means that estimates of median progress for particular sub-groups are likely to be upwards biased if missing data are ignored. But the bias is likely to be much larger for low SES groups. In turn, this means the gaps in student progress between high and low SES students are likely to be underestimated rather than overestimated.[28]

Our analysis of NAPLAN gain scores does not impute missing results. Students who are given an exemption from one or more tests are excluded from the analysis.[29] When estimating progress for Victorian students, we take an approach that aims to minimise bias – rather than excluding all students that miss a test, we include all students that undertook the Year 3 test and at least one other test. This approach is outlined in more detail in Section D.2.1.

[28] The report shows a very consistent pattern of high SES students outperforming low SES students in Year 3, and this gap growing over time. This is a key finding of the report. Missing data would be more problematic if the consequence was overestimating these gaps.

[29] For the purposes of reporting, ACARA assume exempt students are performing below the national minimum standard. Imputing NAPLAN scale scores for these students would change the sample median, but with so few students exempt it is unlikely the results would change significantly.

B.6 Measurement error and bias

B.6.1 Measurement error at the student level

The NAPLAN scale score that a student receives for a particular test is known as a 'weighted likelihood estimate' (WLE).[30] Two students that answer the same number of correct answers on the same test receive the same WLE.

The score that a student receives on the NAPLAN test provides an estimate of their true current ability in a particular domain, but this is subject to substantial measurement error. The accuracy of the estimate increases with the number of questions asked.[31] Two scores are needed to estimate progress over time, and each is subject to measurement error. It is therefore difficult to accurately estimate the progress of an individual student using NAPLAN results.

NAPLAN results are more accurate for estimating the progress of a sizeable group of students, as measurement error is reduced when results are aggregated across students. But simply aggregating does not solve all of the potential measurement error issues. This section outlines these issues in detail and explains the approach we have taken to mitigate them.[32]

[30] These are also referred to as 'Warm's Estimates'; see Warm (1989).

[31] On the Year 3 numeracy test in 2009, for instance, there are 35 questions, and NAPLAN scale scores are estimated with a standard error between 24 and 35 for the vast majority of students. On the Year 9 numeracy test in 2015, there are 64 questions, and the standard error of NAPLAN scale scores is between 17 and 30 for nearly all students. Extreme scores (nearly all questions correct/incorrect) are estimated with much higher standard errors [ACARA (2015d)].

[32] There may also be measurement error issues in other variables – for instance, parental education may change over the course of a child's schooling years, but this is not recorded. Our analysis assumes that the recording of background variables is accurate.

B.6.2 Using NAPLAN scale scores (WLEs) may result in imprecise estimates of progress

Ability is continuous, but NAPLAN scale scores are discrete

NAPLAN scale scores provide an estimate of student ability, a continuous latent variable. But because there are a finite number of questions on each NAPLAN test, the estimates of student ability (NAPLAN scale scores) have a discrete distribution. This can add greater imprecision in estimating percentiles (including the median) and gain scores.

On the Year 3 numeracy test, for example, there are only 35 questions, meaning that there are only 35 possible NAPLAN scale scores a student can receive. The cohort of students that takes the test in 2014 would receive a different set of scores to the cohort taking the test in 2015, even where there is no significant difference between the two cohorts.[33] Ignoring the discrete nature of the estimates could overstate the difference between two cohorts because of 'edge effects', especially when comparing performance in terms of percentiles, such as the progress or achievement of the median student.

This is best dealt with in two ways. First, adjust for the discrete nature of NAPLAN scale scores (as described in Appendix B.6.3). Second, take care when comparing cohorts across years.[34]

[33] A histogram comparing two cohorts would show a similar overall distribution, but the estimated points on the NAPLAN scale would be different.

[34] In particular, if a particular cohort is used to generate a progress baseline to assess achievement or progress of students from a different cohort, this would increase measurement error.

Regression to the mean

In the context of comparing student progress over two or more NAPLAN tests, *regression to the mean* suggests that an extreme NAPLAN score in one year (either extremely low or high) is likely to be followed by a less extreme score on the following test (two years later). This is not because students at the extremes are making significantly high/low progress, but because the original test score is exaggerated by measurement error. This may lead to learning progress being significantly overstated by gain scores for students who start with a very low score, and understated for students who start with a very high score.[35]

Wu (2005) notes that the average of the WLEs provides an unbiased estimate of the population mean ability, but the sample variance overstates the population variance. This bias disappears as the number of test questions increases. For students who score close to the mean, the bias in the WLE as an estimate of their ability will be small. But for extreme percentiles, the bias can be large.[36]

It is important to note that an extreme score for a particular sub-group might not be an extreme score for another

[35] The data show a systematic pattern of high gain scores for low prior scores and low gain scores for high prior scores; see, for example, Figure A.3 on page 7 and Figure B.4 on page 16. But if this were entirely due to regression to the mean, we would expect the path of progress for the median student from Year 3 to Year 9 to be approximately linear – this is clearly not the case.

[36] A way to think about this is that the effective number of questions declines as student ability moves further from the level at which the test is set. For example, a student at the 90th percentile will find most questions too easy, while a student at the 10th percentile will find most questions too difficult. Only a few questions will be set at an appropriate level for such students. The move to NAPLAN online will allow better targeting of questions, reducing the measurement error at the extremes.

sub-group. For example, the NAPLAN scale score equal to the 95th percentile in Year 7 numeracy for those whose parents have no post-school qualifications is only at the 82nd percentile for those who have a parent with a Bachelor's degree. This means that the regression to the mean between the Year 7 and Year 9 test is likely to be stronger for a high achieving low SES student than it is for a high achieving high SES student.[37]

B.6.3 Approaches to mitigate the impact of measurement error and bias

Simulation approach

All WLEs (NAPLAN scale scores) are point estimates and are associated with a standard error. Warm (1989) shows that these estimates are asymptotically normally distributed. Using this property, we approximate the distribution of student ability, θ, given these estimates:

$$\theta_n \overset{a}{\sim} \mathcal{N}\left(\widehat{\mu}_n, \widehat{\sigma}_n^2\right) \tag{B.1}$$

where n is the number of questions correctly answered, $\widehat{\mu}_n$ is the corresponding WLE, and $\widehat{\sigma}_n^2$ is the variance of the WLE.

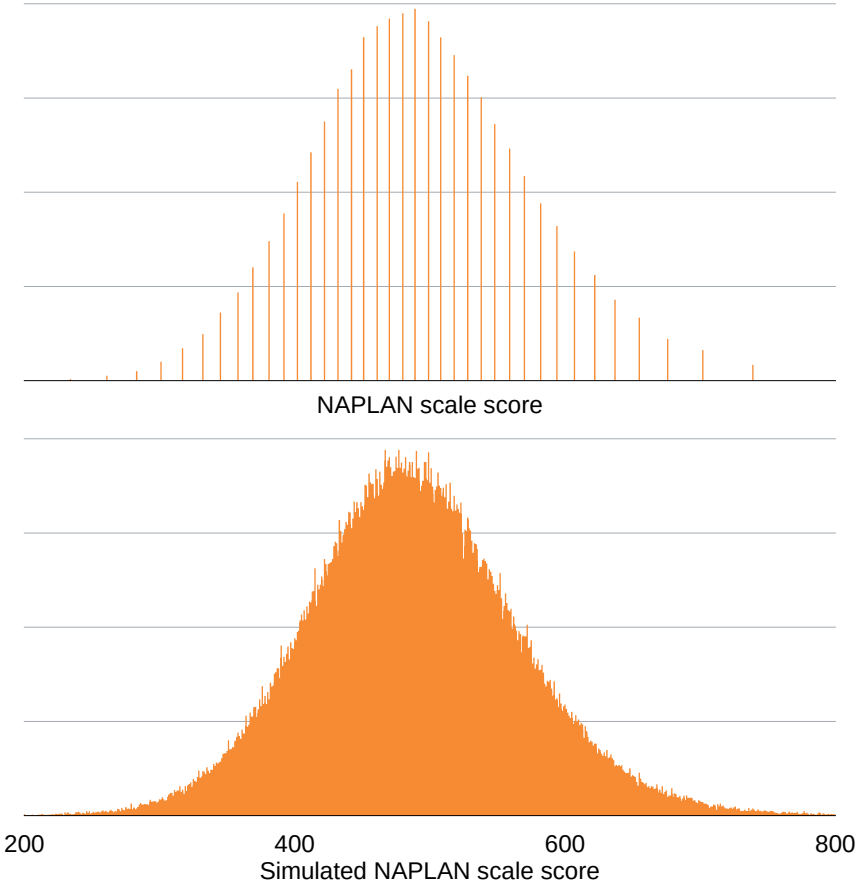
For each student, we simulate a NAPLAN scale score (an ability level) as a random draw from this distribution.[38] This creates a sample that has the properties of a continuous distribution, allowing for more accurate estimates of percentiles.

[37] This does not mean that all high NAPLAN scale scores for low SES students are overstating their true ability. But when we compare a low SES and a high SES student with the same high score, the low SES student is more likely to have had a particularly good test day than the high SES student.

[38] This is performed for each year in the Victorian cohort and each year in the national dataset, using the standard errors reported by ACARA (2015d).

Figure B.5: The simulation approach solves the issues of discrete NAPLAN scale scores

Histogram of Year 5 NAPLAN scale score, numeracy



Notes: Frequency is not shown on Y-axes, but scaled so that both charts can be compared. Bin width = 0.5.
Source: Grattan analysis of ACARA (2014).

While this approach does not remove measurement error at the individual student level, it takes into account that measurement error varies across students with different scores. Figure B.5 compares a histogram of discrete NAPLAN scale scores to a histogram of simulated NAPLAN scale scores.

Use of sub-groups with large samples

Simulating NAPLAN scale scores does not remove measurement error at the individual student level. In fact, it increases the standard error associated with an individual student estimate and gain score.[39] We keep this measurement error to a minimum by aggregating students into sub-groups that have large samples, and calculating our results based on five sets of random draws.[40]

`insert table showing sample sizes for different sub-groups`

Avoiding extreme percentiles

There is no straightforward way to estimate the magnitude of the bias in the WLEs for different percentiles. But it is well known that the magnitude of the bias due to regression to the mean is largest for extreme percentiles, and that the bias is small for percentiles close to the median. The impact of regression to the mean is also larger when the correlation between two measurements (such as test scores) is weak. In our sample, the correlation between NAPLAN test scores across two test-taking years for a given domain is between 0.75 and 0.8 – this strong correlation suggests regression to the mean will have only a small impact for most percentiles.

Nonetheless, our analysis aims to avoid estimating NAPLAN scale scores and gain scores for students at extreme percentiles, and most analysis is focused around the median student. We use a rule of thumb to minimise bias due to regression to the mean – no analysis is based on the estimated NAPLAN scale score or gain score of students below the 10th percentile or above the 90th percentile.[41]

In constructing the benchmark curve to estimate comparative year levels (outlined in Appendix C on page 23), it is necessary to estimate the median gain score of below-average students from Years 3 to 5, and above-average students from Years 7 to 9. It is possible to estimate the NAPLAN scale score for a student as low as six months below Year 2 level, and as high as Year 12 level without using extreme percentiles.

For the analysis of progress using Victorian data, we track low, medium, and high achieving students based on their percentile at Year 3 – the 20th, 50th, and 80th at the population level. But these percentiles can be more extreme when analysing sub-groups. In Year 3 numeracy, for example, the 20th percentile across the population is equal to the 12th percentile for students who have a parent with a Bachelor's degree, and the 80th percentile at the population level is the 87th percentile when the highest level of parental education is below a diploma. Table B.1 shows the within-group percentiles for different levels of parental education – none of these are more extreme than the 10th or 90th percentiles.

[39] This approach would be inappropriate for reporting individual student results.

[40] The standard error due to measurement in a sub-group is proportional to $\sqrt{n}$, the square root of the sub-group sample size. For a sub-group with 10,000 people, the standard error will be 100 times smaller than it will be for an individual student.

[41] These extreme percentiles are avoided both for the overall population, and for particular sub-groups.

Table B.1: Analysis of particular sub-groups does not extend beyond the 10th or 90th percentiles within each group
Within-group percentile in Year 3 numeracy by parental education

	Percentile		
Population	*20*	*50*	*80*
Degree or above	11.9	37.1	70.8
Diploma	19.7	50.1	81.5
Below diploma	26.3	59.5	86.7

Source: Grattan analysis of VCAA (2014b).

Nonetheless, the gaps in progress between high and low SES may still be overstated due to regression to the mean, particular when starting from either the 20th or the 80th percentile in Year 3. This is explore more in Section D.4.

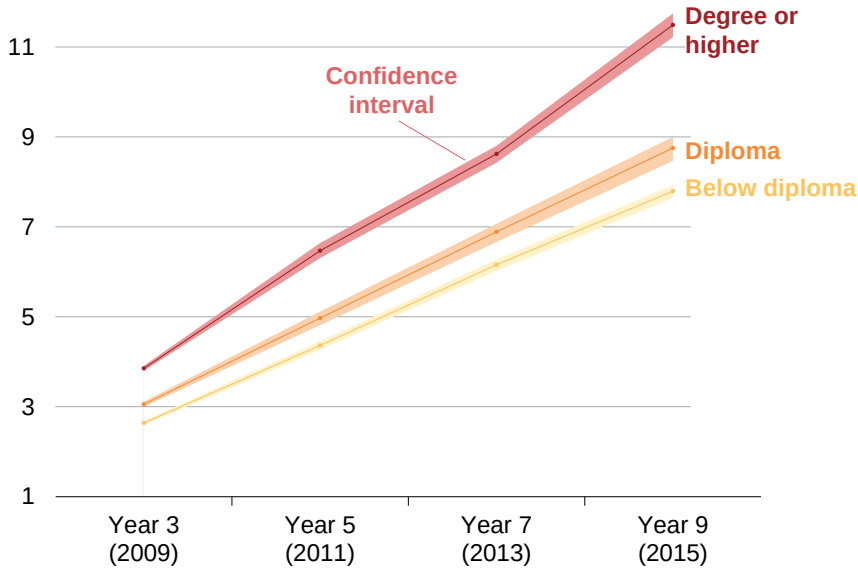
Reporting of results and standard errors

To simplify the presentation of our findings, the report does not show standard errors or confidence bounds on point estimates of NAPLAN scale scores or comparative year levels. But confidence bounds are estimated to ensure the significance of reported results. We calculate 99 per cent confidence intervals using a bootstrap approach with 200 replications, each with a different set of random draws.[42] Separate bootstrap simulations are run for estimation of the benchmark curve with the national dataset and for estimation of student progress using the Victorian dataset.

We estimate a confidence interval for the benchmark comparative year level curve, as well as confidence intervals for the analysis of progress using the Victorian cohort. For results

[42] The lower bound of each confidence interval is estimated as the average of the two smallest bootstrap point estimates, while the upper bound is estimated as the average of the two largest bootstrap point estimates.

Figure B.6: Most points are estimated with narrow confidence bounds
Estimated comparative year level by highest level of parental education



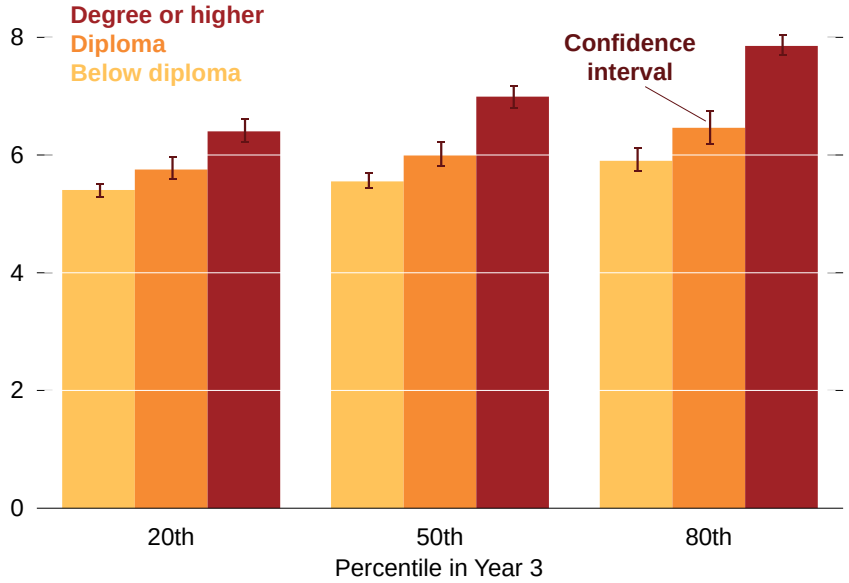
Notes: Chart shows 99 per cent confidence interval
Source: Grattan analysis of VCAA (2014b) and ACARA (2014).

that are reported in terms of comparative year levels or years of progress, these confidence intervals are calculated using both bootstrap simulations.[43]

The confidence bounds are used to validate our analysis for robustness – we do not draw conclusions from any results that are not statistically significant (including gaps in progress between different groups). The report notes any case in which

[43] Each replication from one simulation is linked to a replication from the other. This approach takes into account the measurement error in the Victorian cohort, as well as the measurement error in the estimation of comparative year levels.

Figure B.7: Confidence bounds are larger for years of progress conditional on starting score
Estimated years of progress by parental education



Notes: Chart shows 99 per cent confidence interval
Source: Grattan analysis of VCAA (2014b) and ACARA (2014).

a comparative year level is estimated with a confidence interval larger than six months.

Full results with confidence bounds are available for download.[44]

Plausible values

The best approach that could be taken to reduce the impact of measurement error would be to use plausible values. Like the simulation approach outlined above, this approach would simulate a NAPLAN scale score from a continuous distribution for each student, including imputing values for missing data. But plausible values are simulated from a distribution that takes into account student and school background factors.[45] NAPLAN reports produced by ACARA are based on analysis using plausible values.[46]

When simulated correctly, plausible values are able to produce unbiased estimates of percentiles and gain scores for each sub-group.[47] Plausible values were, unfortunately, only available for the 2014 test year in the national dataset, but not for the 2012 results or the Victorian 2009–15 cohort. This means we did not have the data to use plausible values to analyse progress.[48]

We do, however, utilise the 2014 plausible values (generated by ACARA) for estimating the population distribution of results for each year level. These estimates therefore take missing data and measurement error into account.

[44] http://www.grattan.edu.au/

[45] In theory these could also take into account NAPLAN scores in other year levels.

[46] ACARA (2015e), p. 22.

[47] Wu (2005).

[48] In any case, the 2014 plausible values are, to the best of our knowledge, generated independently of prior test scores. Analysing student progress would ideally be done using plausible values simulated from a distribution that takes both prior and subsequent test scores into account.

C Methodology for mapping NAPLAN scale scores to comparative year levels

C.1 Introduction

The NAPLAN scale is designed to be independent of year level – a student should receive the same score on average regardless of whether they take a test normally administered to Year 3, Year 5, Year 7 or Year 9 students.[49] This property makes it possible to compare students in different test-taking year levels. For example, a Year 5 student is predicted to be reading above the typical Year 7 level if they score higher than the typical Year 7 student in NAPLAN reading. But because NAPLAN tests are only administered to students in four different year levels, it is not possible to compare students to those outside these year levels without further assumptions.

Closing the gaps presents a new framework from which to interpret NAPLAN results. NAPLAN scale scores are mapped onto a new measure, *comparative year levels*. The NAPLAN scale score corresponding to the comparative year level 4, for example, is the median score expected from students if they took an age-appropriate NAPLAN test when they were in Year 4.[50]

This appendix outlines the theoretical framework for mapping NAPLAN scale scores onto comparative year levels and the methodology and assumptions used to estimate this relationship.

[49] A student's NAPLAN scale score will generally be a more precise estimate of their true ability when they are administered an age-appropriate test. Giving a typical Year 3 student a test meant for Year 9 students is likely to produce a NAPLAN scale score with a large standard error.

[50] To be precise, in May of the year they were in Year 4, as this is when the NAPLAN test is taken.

C.2 Theoretical framework for mapping

Let X_j ($X_j \in \mathbb{R}$) be a random variable denoting student ability (as estimated by NAPLAN scale scores) in domain j ($j =$ reading, numeracy), and Y be a variable denoting schooling year level, continuous over the range of schooling years, (y_{min}, y_{max}).[51]

We assume that median student ability increases monotonically as students progress through school. We define a function $f_j(Y)$ as the median of X_j conditional on Y:

$$\begin{aligned} f_j(Y) &= Q_{50}\left[X_j \mid Y\right] \\ y_1 < y_2 &\implies f_j(y_1) < f_j(y_2) \\ f_j(Y) &\in f_j\left[y_{\min}, y_{\max}\right] \end{aligned} \tag{C.1}$$

That is, $f_j(Y)$ is the median NAPLAN scale score in domain j of students taking a NAPLAN test in year level Y. For every schooling level there is a corresponding median NAPLAN scale score (for each domain). We also assume that $f_j(Y)$ is continuous and monotonically increasing – at the population level, median student ability increases steadily over time.[52]

Following this, we propose that a given NAPLAN scale score corresponds to a median schooling year – the point in time in

[51] Lower case letters are used to denote realisations of these random variables. This report's analysis focuses on reading and numeracy only, but it would be possible to apply the same analysis to the other assessment domains.

[52] For example, if NAPLAN tests were taken every month, we would expect the median score to improve with every test. This may not hold for individual students, but should hold at the population level.

the median student's path of progress (in terms of year level and months) at which their ability is equal to that score. We define this schooling year as a *comparative year level*, denoted as Y^*:

$$Y^* = f_j^{-1}(X_j) \tag{C.2}$$

All NAPLAN scale scores in the range $(f_j[y_{min}], f_j[y_{max}])$ therefore correspond to a *comparative year level*.

C.3 Estimating comparative year levels

This methodology aims to estimate $f_j(Y)$ for reading and numeracy at each schooling year level, $Y = 1, 2, ..., 12$, then interpolate over these points to construct a smooth curve. If the NAPLAN tests were administered to students in every year level (from Year 1 to Year 12), this would be straightforward – $\widehat{f}_j(Y)$ would just be the sample median from each year level. But with the tests only administered in four year levels, we must make further assumptions to estimate $f_j(Y)$.

The report estimates $f_j(Y)$ (the median NAPLAN scale scores corresponding to a given year level) using the simulated NAPLAN results (see Section B.6.3) of all Australian students in 2014 linked to their 2012 simulated results (where applicable). It is possible to apply this methodology to NAPLAN results in other years, provided linked data are available.

Step 1: Estimate the corresponding NAPLAN scale scores for comparative year levels 3, 5, 7, and 9

These are estimated as the sample median scores in those year levels:

$$\begin{aligned} \widehat{f}_j(3) &= \tilde{x}_{j,3} \\ \widehat{f}_j(5) &= \tilde{x}_{j,5} \\ \widehat{f}_j(7) &= \tilde{x}_{j,7} \\ \widehat{f}_j(9) &= \tilde{x}_{j,9} \end{aligned} \tag{C.3}$$

where $\tilde{x}_{j,y}$ is the sample median NAPLAN scale score in year level y.[53]

Step 2: Interpolate between Year 3 and Year 9

Using a third-order polynomial, fit a smooth curve through the four data points, $([Y, \widehat{f}_j(Y)], Y = 3, 5, 7, 9)$, to estimate $f_j(Y)$ between Year 3 and Year 9, as shown in Figure C.1.

This interpolation could be extrapolated to year levels below 3 and above 9, but this is unlikely to provide the best estimate of $f_j(Y)$ at such points. Instead, our approach takes into account the gain in NAPLAN scores as students progress through school.

We denote a function, $g_{j,Y}(X_{j,Y-2})$, equal to the median gain score conditional on year level and NAPLAN scale score from two years earlier:

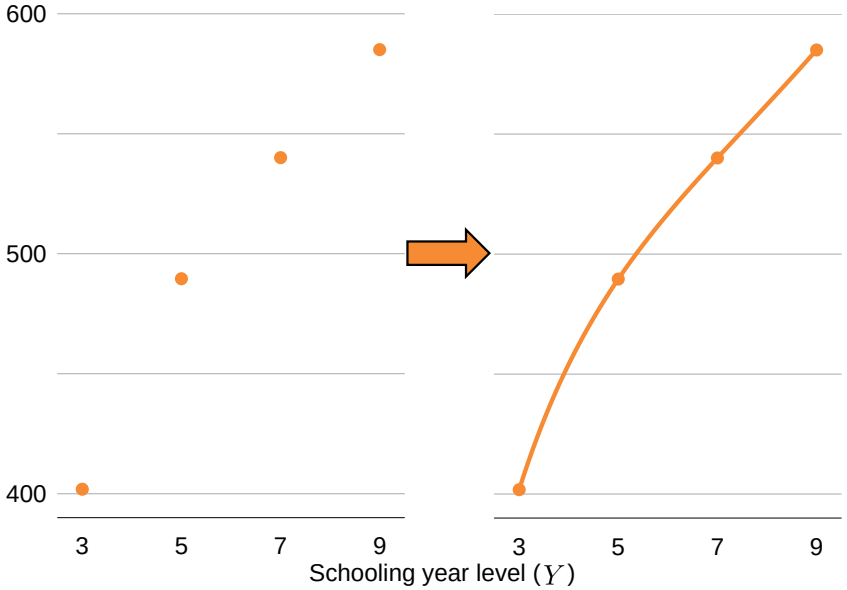
$$g_{j,Y}(X_{j,Y-2}) = Q_{50}[X_{j,Y} - X_{j,Y-2} | Y, X_{j,Y-2}] \tag{C.4}$$

where $X_{j,Y}$ denotes NAPLAN scale score in domain j in school year Y. For students that scored $x_{j,3}$ in Year 3 reading, for

[53] For Years 3, 5, and 7, we estimated the corresponding NAPLAN scale score, $\widehat{f}_j(Y)$, as the average of the medians in 2012 and 2014.

Figure C.1: A third-order polynomial is used to interpolate between Year 3 and Year 9

Estimated median NAPLAN scale score, $\widehat{f}_j(Y)$, numeracy



Source: Grattan analysis of ACARA (2014).

example, $g_{j,5}(x_{j,3})$ is the median gain score these students will make to Year 5.[54]

From eqs. (C.1) and (C.4), it follows that:

$$g_{j,Y}\left[f_j(Y-2)\right] = f_j(Y) - f_j(Y-2) \tag{C.5}$$

That is, the difference between the median scores two years apart is equal to the median gain made from the same starting score.

[54] The function $g_{j,Y}$ can only be empirically estimated for $Y = 5$, 7 and 9, corresponding to gain scores from Years 3 to 5, Years 5 to 7, and Years 7 to 9 respectively.

Step 3: Estimate the median gain score curves for Years 3 to 5 and Years 7 to 9

To estimate $g_{j,Y}$ for $Y = 5$ and $Y = 9$ first requires parameterising the functions. We allow for non-linearity in $g_{j,Y}$ by using restricted cubic regression splines, meaning that $g_{j,Y}$ can be written as a linear function:

$$\begin{aligned} g_{j,Y}(X_{j,Y-2}) = \beta_0 + \beta_1 X_{j,Y-2} + \beta_2 S_2(X_{j,Y-2}) \\ +\beta_3 S_3(X_{j,Y-2}) + \beta_4 S_4(X_{j,Y-2}) \end{aligned} \tag{C.6}$$

where S_2, S_3 and S_4 are functions that create spline variables.[55] Alternatively, this function could be specified with quadratic or higher order polynomial terms.

Given $g_{j,Y}$ represents a conditional median gain score, eq. (C.6) can be thought of as a quantile regression model at the median. This can be estimated using least absolute deviations.[56]

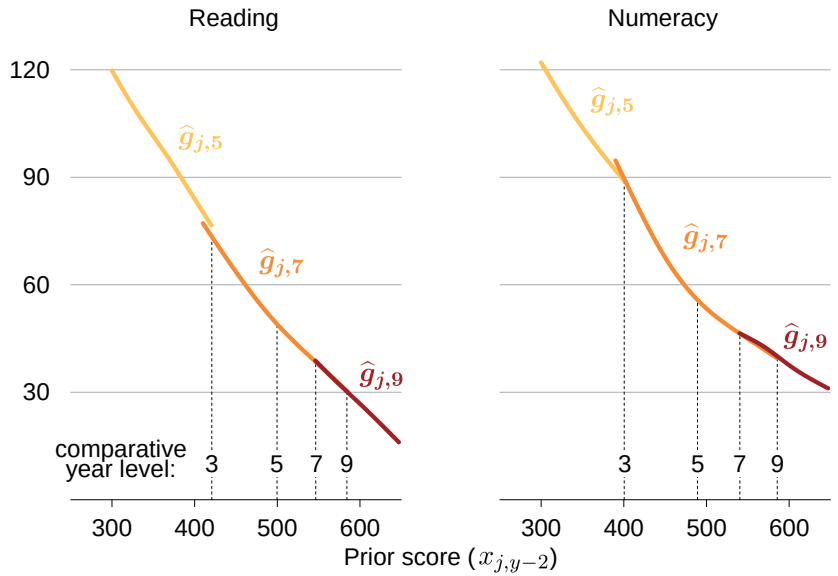
Figure C.2 plots the estimated functions, $\widehat{g}_{j,y}\left(x_{j,y-2}\right)$, for $y = 5, 7$ and 9 for both reading and numeracy. Predicted median NAPLAN gain scores are much higher for lower prior scores, but year level appears to have little effect on gain scores once prior scores are controlled for. For instance, when evaluated at the NAPLAN score for comparative year level 3, $\widehat{f}_j\left(3\right)$, the functions $\widehat{g}_{j,5}$ and $\widehat{g}_{j,7}$ are very close. Similarly, when evaluated at comparative year level 7, $\widehat{f}_j\left(7\right)$, the functions $\widehat{g}_{j,9}$ and $\widehat{g}_{j,7}$ are very close.[57] That is, expected NAPLAN gain from a given starting point is similar for students that are two year levels apart.

[55] More spline variables can be included, if desired.

[56] It is only necessary to estimate $g_{j,5}$ for $x_{j,3} \leq \widehat{f}_j(3)$ and $g_{j,9}$ for $x_{j,7} \geq \widehat{f}_j(7)$.

[57] These functions are also very close when evaluated at comparative year level 9.

Figure C.2: The estimated median gain score is strongly related to prior score, but only weakly related to year level
Two-year median NAPLAN gain score, $\widehat{g}_{j,y}(x_{j,y-2})$



Source: Grattan analysis of ACARA (2014).

Setting $Y = 10$ and re-arranging eq. (C.5) gives:

$$f_j(10) = f_j(8) + g_{j,10}[f_j(8)] \tag{C.7}$$

The point $f_j(8)$ was estimated in Step 2, but it is not possible to estimate $g_{j,10}$ without NAPLAN data for Year 10 students (linked to Year 8 results). But given that year level has little effect on gain scores once prior scores are controlled for, we can assume:

$$g_{j,10}[f_j(8)] \approx g_{j,9}[f_j(8)] \tag{C.8}$$

That is, our estimate of $g_{j,9}$ can be used as a proxy for $g_{j,10}$. In other words, we assume that a student in Year 8 performing at the median Year 8 level will make a similar gain over two years as a Year 7 student performing at the median Year 8 level.

Similarly, we can use our estimate of $g_{j,5}$ as a proxy for $g_{j,4}$ by assuming:

$$g_{j,4}[f_j(2)] \approx g_{j,5}[f_j(2)] \tag{C.9}$$

That is, a Year 2 student performing at the median Year 2 level is assumed to make a similar gain over two years as a Year 3 student performing at the median Year 2 level.

These approximations can be extended to further year levels outside the range of Year 3 to Year 9. For instance, $g_{j,9}$ can also be used to approximate $g_{j,11}$ and $g_{j,12}$.[58]

Step 4: Estimate the corresponding NAPLAN scale scores for comparative year levels 10, 11, and 12

Using the assumption made in eq. (C.8) and its extensions, $f_j(10), f_j(11)$ and $f_j(12)$ are estimated using the following:

$$\begin{aligned}\widehat{f}_j(10) &= \widehat{f}_j(8) + \widehat{g}_{j,9}\left[\widehat{f}_j(8)\right] \\ \widehat{f}_j(11) &= \widehat{f}_j(9) + \widehat{g}_{j,9}\left[\widehat{f}_j(9)\right] \\ \widehat{f}_j(12) &= \widehat{f}_j(10) + \widehat{g}_{j,9}\left[\widehat{f}_j(10)\right]\end{aligned} \tag{C.10}$$

where, for example, $\widehat{f}_j(8)$ is the estimated median NAPLAN scale score for Year 8 students, calculated in Step 2, and $\widehat{g}_{j,9}$ is the estimated median NAPLAN gain score from Year 7 to Year 9, calculated in Step 3. It is necessary to calculate $\widehat{f}_j(10)$ before calculating $\widehat{f}_j(12)$.

Step 5: Estimate the corresponding NAPLAN scale scores for comparative year levels 1.5, 2, and 2.5

Using the assumption made in eq. (C.9) and its extensions, $f_j(1.5), f_j(2)$ and $f_j(2.5)$ are estimated by solving the following

[58] The further away from the Year 3 to Year 9 range, the less accurate these approximations are likely to be. For instance, $\widehat{g}_{j,9}$ is probably not going to be a good proxy for $\widehat{g}_{j,13}$.

equations for $\widehat{f}_j(Y)$:

$$\begin{aligned}\widehat{f}_j(1.5) &= \widehat{f}_j(3.5) - \widehat{g}_{j,5}[f_j(1.5)] \\ \widehat{f}_j(2) &= \widehat{f}_j(4) - \widehat{g}_{j,5}[f_j(2)] \\ \widehat{f}_j(2.5) &= \widehat{f}_j(4.5) - \widehat{g}_{j,5}[f_j(2.5)]\end{aligned} \quad \text{(C.11)}$$

where, for example, $\widehat{f}_j(3.5)$ is the estimated median NAPLAN scale score for Year 3 students, six months after the NAPLAN test (November), and $\widehat{g}_{j,5}$ is the estimated median gain score from Year 3 to Year 5, calculated in Step 3. The points are estimated closer together because $f_j(Y)$ has a larger gradient for lower values of Y.

Step 6: Interpolate over estimated points

Using a range of estimated points for $\left[Y, \widehat{f}_j(Y)\right]$ (for example, use Y = 1.5, 2, 2.5, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12), construct a smooth curve for $\widehat{f}_j(Y)$ using interpolation.[59] We extrapolate our curve so that $y_{min} = 1$ and $y_{max} = 13$ (reported as 'above Year 12'), although our analysis aims to avoid these extremes as much as possible given the high standard errors associated with these estimates.[60]

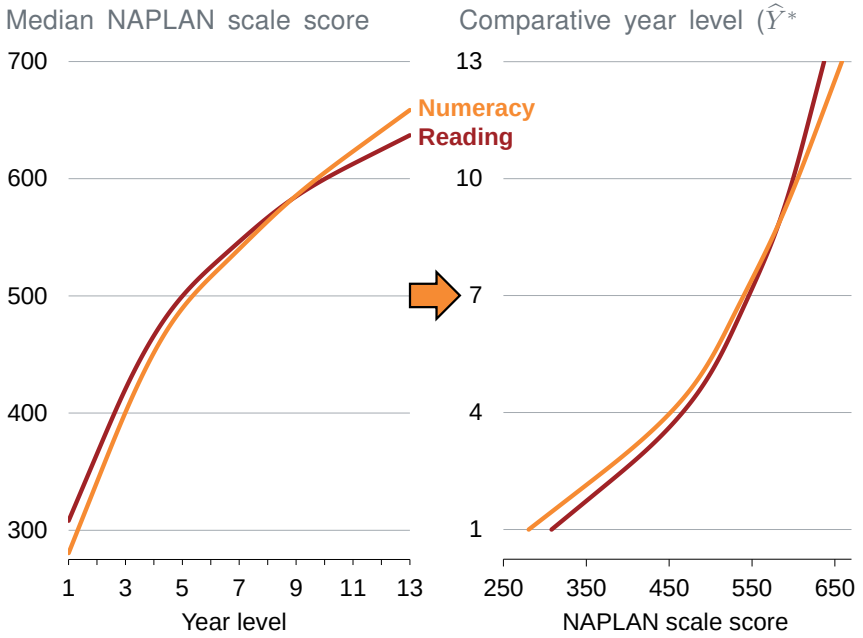
We now have a curve that estimates the median NAPLAN scale score for each schooling year level: $\widehat{f}_j(Y)$. The inverse of this curve is used to estimate the comparative year level, Y^*, corresponding to any given NAPLAN scale score, X_j:

$$\widehat{Y}^* = \widehat{f}_j^{-1}(X_j) \quad \text{(C.12)}$$

[59] Our methodology fits a curve using a regression with restricted cubic splines – some of the points already estimated for $f_j(Y)$ shift slightly as a result.

[60] Given that NAPLAN is designed for students between Year 3 and Year 9, not only are estimates less reliable outside this range, but the interpretation of comparative year levels becomes more difficult as we move further outside this range.

Figure C.3: All NAPLAN scale scores correspond to a comparative year level



Notes: Left chart shows estimated function $\widehat{f}_j(Y)$, while right chart shows its inverse, $\widehat{f}_j^{-1}(X_j)$. The left chart can be interpreted as the estimated median NAPLAN scale score for a given year level, whereas the right chart can be interpreted as the estimated comparative year level for a given NAPLAN scale score.
Source: Grattan analysis of ACARA (2014).

Figure C.3 shows this curve for reading and numeracy, both in terms of $\widehat{f}_j(Y)$ and in terms of its inverse, $\widehat{f}_j^{-1}(X_j)$. As the right chart shows, every NAPLAN score (within the range of the curve) can be mapped to a comparative year level. A score of 500 in numeracy, for instance, corresponds to a comparative year level of 5 years and 4 months – a student at this level can be interpreted as performing four months ahead of the typical

(median) Year 5 student at the time of the Year 5 NAPLAN test.[61]

These curves can be used to compare different cohorts or sub-groups of students in terms of differences in their achievement, and to track student progress relative to the median student. Years of progress is simply calculated as the difference in comparative year levels between two points in time. If, for example, a student makes 2 years and 6 months of progress over a two-year period, they have made the same amount of progress as the typical (median) student is expected to make over 2 years and 6 months, starting from the same point. This student could be said to be learning 25 per cent faster than the typical student.

C.4 Robustness of comparative year level estimates

There are a number of questions that may arise in relation to the methodology used to estimate comparative year levels. For instance:

- what is the standard error around point estimates?
- how accurate are estimates beyond Year 3 and Year 9?
- how do the estimates change with different assumptions?
- are the results robust to the cohort used?

It is worth investigating each of these questions in detail to ensure that the methodology and the results are robust.

C.4.1 Standard errors around point estimates

There are two sources of error that the standard error accounts for: sample size and measurement error. But the comparative year level curve is calculated from a very large sample, meaning that standard errors due to both sources are naturally small.

In reporting, we prefer using confidence intervals to standard errors, since comparative year levels are asymmetrically distributed around NAPLAN scale scores. We calculate a 99 per cent confidence interval at each point along the curve, $\widehat{f}_j(Y)$, between $Y = 1$ and $Y = 13$. This is based on a bootstrap simulation with 200 replications.[62]

Between Year 3 and Year 9, comparative year levels are estimated within a month of learning. As the curve is flatter in Year 9 than it is in Year 3, the confidence interval around Year 9 is wider. The width of the confidence interval naturally increases as you go below Year 3 or above Year 9. But the interval is widest when estimating comparative year level 1: three months for numeracy, and six months for reading, as shown in Figure C.4. At comparative year level 13, the width of the confidence interval is two months for numeracy, and three months for reading, reflecting that there are still a significant number of students who reach this level by Year 9.

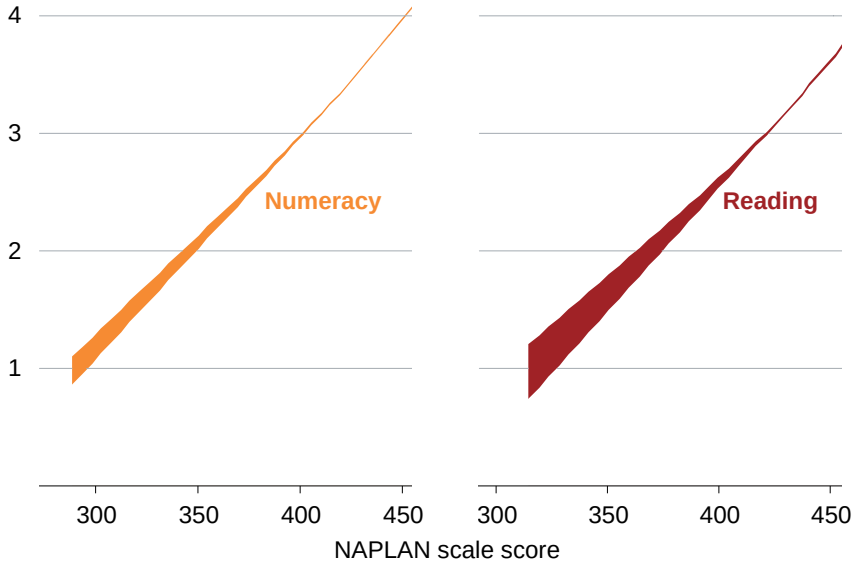
The confidence intervals for each comparative year level are displayed in Table C.1.

It should be noted that these confidence intervals are calculated assuming that the methodology is correct. If we

[61] Given that NAPLAN is administered in May of each year, another interpretation is to say that this student is performing at the level we'd expect of the typical Year 5 student in September.

[62] Each replication uses a different set of random draws. The lower bound at each point is the average of the two lowest simulated points, while the upper bound at each point is the average of the two highest simulated points.

Figure C.4: Confidence intervals are widest for low NAPLAN scale scores
Comparative year level with 99 per cent confidence interval



Notes: Confidence intervals become wider for comparative year levels greater than Year 9, but are not as wide as they are for low year levels.
Source: Grattan analysis of ACARA (2014).

were to account for uncertain assumptions, the intervals would be wider.[63]

C.4.2 Accuracy of estimates beyond Year 3 and Year 9

Without students taking a NAPLAN test outside of the test-taking years, it is impossible to validate whether our estimates of the median NAPLAN scale score in Years 1, 2, 10, 11, and 12 reflect how the median student would actually perform in those year levels. But it is possible to use a similar methodology to predict the median score in Year 3 and Year 9 without using data from Year 3 and Year 9.

Using data for students in Year 7 linked to their Year 5 results, Figure C.5 shows that the methodology does a reasonable job of estimating the curve outside the years that data are available. Interestingly, the curve estimated with only Years 5 and 7 overestimates the median score at both the bottom and the top of the curve.[64]

[63] The narrow confidence intervals tell us that error due to measurement and sample size is very small. They do not tell us whether or not the methodology is appropriate.

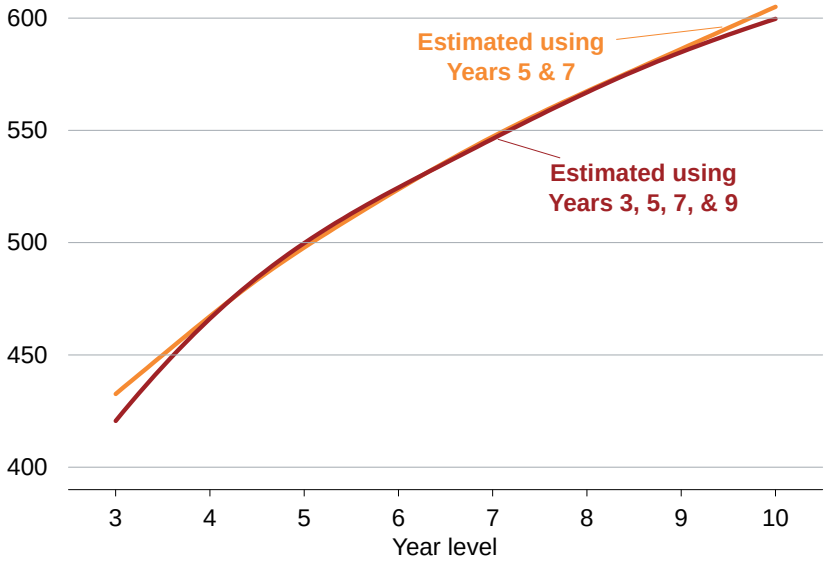
Table C.1: Estimated comparative year levels with 99 per cent confidence interval

Comp. year level ($\widehat{Y}^*$)	Numeracy		Reading	
	$\widehat{f}_j(Y)$	Interval	$\widehat{f}_j(Y)$	Interval
1	288.5	(0.86,1.1)	314.8	(0.74,1.21)
2	345.4	(1.93,2.05)	368.7	(1.88,2.1)
3	401.4	(2.98,3.01)	421.1	(2.98,3.01)
4	451.4	(3.99,4.01)	466.0	(3.98,4.01)
5	489.4	(4.99,5.02)	499.9	(4.99,5.02)
6	516.2	(5.98,6.02)	524.6	(5.98,6.02)
7	540.0	(6.98,7.02)	546.2	(6.98,7.02)
8	563.9	(7.97,8.02)	566.7	(7.98,8.03)
9	586.0	(8.96,9.02)	584.6	(8.97,9.06)
10	605.6	(9.94,10.03)	599.4	(9.97,10.08)
11	623.8	(10.93,11.04)	612.4	(10.95,11.09)
12	641.4	(11.92,12.05)	624.8	(11.92,12.11)
13	659.0	(12.9,13.06)	637.1	(12.88,13.13)

Notes: Parentheses show upper and lower bounds of 99 per cent confidence interval for estimated comparative year levels. This is estimated by a bootstrap simulation with 200 replications.
Source: Grattan analysis of ACARA (2014).

[64] This is the opposite of what is expected due to regression to the mean.

Figure C.5: Data from Years 5 and 7 students will slightly overestimate the median score for year levels outside the range
Estimated median NAPLAN scale score, reading



Notes: Overestimating the median score at high and low year levels will lead to conservative estimates of the gap between high and low achievers in terms of years of progress. A similar pattern holds for numeracy.
Source: Grattan analysis of ACARA (2014).

C.4.3 How do estimates change with different assumptions?

to be completed

C.4.4 How robust are estimates to different cohorts

2013 data linked to 2011 --
numeracy curve is flatter at the top

C.5 How comparative year levels could be implemented as part of NAPLAN reporting

- inital curve could be based on multiple cohorts (to improve robustness, since there are cohort specific effects)
- having calculated an initial curve, this should be fixed over time to track system-level improvements
- curves could be calculated using plausible values
- for individuals or schools, could track progress relative to a different percentile

D Tracking student progress using linked NAPLAN data

D.1 Introduction

The Victorian cohort that sat NAPLAN in Year 3 in 2009 did Year 9 NAPLAN in 2015. They provide a rich source of data to track progress over six years of schooling. Our methodology analyses this cohort in two different ways:

- by student background (household SES, school SES, geolocation of school)
- by NAPLAN score in Year 3.[65]

When tracking results and progress, we report the progress made by the median student within each SES sub-group, or for the median student starting from a given percentile. While results are reported in terms of our own measure, *comparative year levels* and *years of progress*, the analysis takes place using NAPLAN scale scores and gain scores; at the very last step these results are converted to comparative year levels.

D.2 Estimating median NAPLAN scale scores

D.2.1 SES sub-groups

For each SES sub-group, we estimate the NAPLAN scale score (for each of numeracy and reading) and the corresponding comparative year level of the median student in Years 3, 5, 7, and 9. The obvious way to do this is via the sample median in each year level, but this approach could lead to progress being overstated for some sub-groups. This is because there are more missing data in higher year levels, due to greater levels of absenteeism and withdrawal, as well as students who leave Victoria. As Section B.5 shows, students who miss one or more tests are more likely to come from lower SES backgrounds, and typically make smaller gains than other students even after controlling for SES background and prior NAPLAN score. This means the median student that sat NAPLAN in Year 9 is likely to have been above the median student in Year 3.[66]

It is difficult to account for all the bias due to missing data, but we take an approach to estimating median scores that aims to reduce this bias. The sample median of each sub-group is used to estimate the population sub-group median in Year 3:

$$\widehat{Q_{50}\left[X_{j,3}|s\right]} = \tilde{x}_{j,3,s} \tag{D.1}$$

Where s is an indicator of sub-group.[67] This is likely to be an overestimate of the population median for Year 3, given the patterns of missing data. But the proportion of missing data in Year 3 is relatively small, meaning that the bias is likely to be small.

For Years 5, 7, and 9, we define a function for the median sub-group NAPLAN score conditional on Year 3 score:

$$\begin{gathered} Q_{50}\left[X_{j,Y}|s, X_{j,3}\right] = h_{j,Y,s}\left(X_{j,3}\right) \\ Y = 5, 7, 9 \end{gathered} \tag{D.2}$$

The functions $h_{j,Y,s}$ are estimated for $j =$ reading and numeracy, $Y = 5, 7, 9$, and for each subgroup using least

[65] We classify students according to the 20th, 50th, and 80th percentiles of Victorian performance, which we refer to as 'low, medium, and high' achievers respectively.

[66] If students are tracked consistently, then the median Year 3 student is expected to match up to the median Year 9 student.

[67] See Appendix C on page 23 for explanation of notation.

absolute deviations. Restricted cubic regression splines are used to allow for non-linearity in $h_{j,Y,s}$. These functions are evaluated at the estimated Year 3 sample median for each sub-group, $\tilde{x}_{j,3,s}$, to estimate each sub-group population medians for Years 5, 7, and 9:

$$\widehat{Q_{50}[X_{j,Y}|s]} = \widehat{h}_{j,y,s}\left(\tilde{x}_{j,3,s}\right) \quad Y = 5,7,9 \tag{D.3}$$

These estimates are typically lower than the sample medians at higher year levels, suggesting that this approach reduces some of the bias due to missing data.[68]

D.2.2 Estimating percentiles

We estimate the 20th, 50th, and 80th percentiles for the population in Year 3:

$$\begin{aligned}\widehat{Q_{20}[X_{j,3}]} &= \tilde{x}_{j,3}^{(20)} \\ \widehat{Q_{50}[X_{j,3}]} &= \tilde{x}_{j,3}^{(50)} \\ \widehat{Q_{80}[X_{j,3}]} &= \tilde{x}_{j,3}^{(80)}\end{aligned} \tag{D.4}$$

These are used to track progress within each sub-group (and for the population) for a given level of ability in Year 3. We estimate the median NAPLAN score in Years 5, 7, and 9 conditional on sub-group *and* Year 3 percentile:

$$\widehat{Q_{50}[X_{j,Y}|s, X_{j,3}]} = \widehat{h}_{j,y,s}\left(\tilde{x}_{j,3}^{(P)}\right) \quad Y = 5,7,9 \tag{D.5}$$

where P represents the Year 3 percentile, and $\widehat{h}_{j,y,s}$ has been estimated separately for each year level and sub-group.[69]

[68] This approach is still likely to overestimate the sub-group medians, since excluding missing data is likely to overstate gain scores, as evidenced by Figure B.3 on page 16.

[69] While $\widehat{h}_{j,y,s}$ is estimated separately for different sub-groups, it is not estimated separately for different percentiles.

This means that for every SES sub-group, we have estimated median NAPLAN scale scores in Years 3, 5, 7, and 9, both for the median of the sub-group, and conditional on the Year 3 percentile for the Victorian population. Table D.1 shows these results for students who do not have a parent with a degree or diploma. Given this is a low SES sub-group, the group median results are, unsurprisingly, lower than the results for those at the 50th percentile of the Victorian population in Year 3.

Table D.1: For each sub-group we estimate both group medians and the medians conditional on Year 3 percentile
Estimated median NAPLAN scale score, parental education below diploma, Victorian 2009–15 cohort

	Group median	Year 3 percentile (Victorian population)		
Year level	(below diploma)	20th	50th	80th
Year 3	390.7	344.5	408.9	476.9
Year 5	477.0	452.4	487.1	526.2
Year 7	520.8	496.9	530.4	570.6
Year 9	570.6	549.4	579.3	615.3

Source: Grattan analysis of VCAA (2014b).

D.3 Converting NAPLAN scale scores to comparative year levels

Having estimated a range of NAPLAN scale scores for SES sub-groups, it is then possible to convert these to comparative year levels. As outlined in Section C.2, every NAPLAN scale score within the range of the median student between Year 1 and Year 13 has a corresponding comparative year level. Having estimated $\widehat{f}_j(Y)$, it is straightforward to find the value of Y that corresponds to a given $\widehat{f}_j(Y)$. The reported comparative

year level includes both the schooling year and any additional months of learning.

Years of progress between Years 3 and 9 is then calculated as the difference in comparative year levels between Years 3 and 9. This is also reported in terms of years and months. The median student is expected to make six years of progress over this time.

D.4 Robustness of student progress results

- results with confidence bounds (examples). Note that all results with confidence bounds are available online
- results using 2008–14 cohort

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