Statistical Modelling of Radioactive Decay of Cs-137 Source

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Abstract

The experiment investigates the statistical properties of radioactive decay using a Geiger counter and a Cs-137 source. By recording the number of decay events over fixed time intervals and measuring the time between successive decays, the distribution of the event decays is observed to transition from a Poisson to a Gaussian form as the count rate increases. Data collection was conducted at varying source distances to simulate different activity levels, revealing that at low count rates, the Poisson nature of decay is distinctly observable, while at higher rates, the distribution approximates a Gaussian form. Using python scripts and real-time data acquisition, statitical test, in particular the chi-square test, confirmed the theoretical predictions regarding the radioactive decay distributions. The results undergird the fundamental statistical principles of probabilistic modelling that underlay random processes in nuclear physics and highlight the .

Contents

1 Introduction 1

2 Experimental Methods 2

3 Results 3

4 Discussion 6

5 Conclusion 6

1 Introduction

This experiment aims to analyze the statistical governance of the physical process of radioactive decay. Radioactive decay - a prototypical example used to demonstrate a random process - at its core is a counting process. In particular, the number of radioactive decay events detected in some fixed time interval will fluctuate around a well-defined mean. If this measurement is repeated many times, a distribution of the event counts can be obtained. Such a process is naturally described by the Poisson distribution, where the probability of detecting n decays in any specified time interval is

$$\mathbb{P}(n) = \frac{\lambda^n}{n!}e^{-\lambda} \tag{1}$$

Where λ represents the expected number of decay events per time interval. The general shape of the Poisson curve can also grant some physical insight into the decay process. For instance, for small λ, the Poisson curve is highly asymmetric and skewed peak at low values and a long tail toward higher counts. This behavior is indicative when the mean count rate is small for the radiation source, in other words individual decay events are rare.

Conversely, although radioactive decay events are inherently discrete and random, as the expected count rate increases, it is expected that the Poisson distribution gradually approaches a Gaussian form. This occurs because the relative fluctuations decrease as λ increases, leading to a smoother, more symmetric distribution [1]. The Gaussian, or normal distribution is given by:

$$\mathbb{P}(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{x-\mu}{2\sigma^2}\right) \tag{2}$$

Where μ is the mean, and σ^2 is the variance. Both the Poisson and Gaussian distributions will be used as tests to determine which is a more suitable model for the data. Sec. 2 will detail the data collection methods, while Sec. 3 and Sec. 4 will elaborate on the rationale for the selection of the above distributions to statistically model radioactive decay.

2 Experimental Methods

The experiment was conducted using a Geiger counter to each decay event from a sealed Cs-137 radioactive source. The Geiger counter was connected to an Arduino microcontroller, which processed the detection events and transmitted the data to a computer using the Arduino Integrated Development Environment (IDE). A schematic of the experimental setup is shown in Fig. 1.

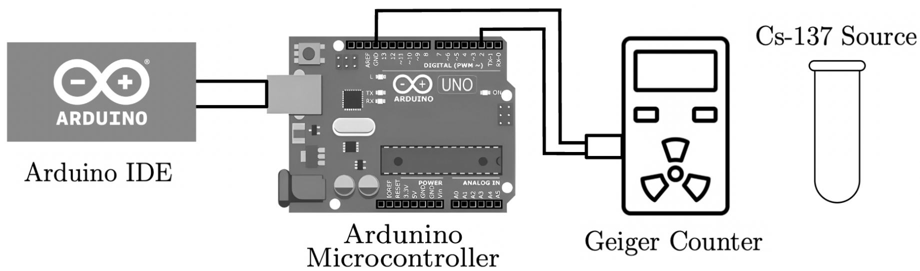

Figure 1: The experimental setup in which the Cs-137 source is placed at a distance from the Geiger counter that yielded the desired amount of counts. The Geiger counter is connected to the Arduino microcontroller, which in turn, is connected to the Arduino IDE where the data can be collected and processed.

Data acquisition was performed by specifying a fixed time interval, in this case 1000 ms, and counting the "clicks", observed by the Geiger counter (denoted as decay events) within this time interval. Arduino scripts were employed through the Arduino IDE to communicate through the serial ports, and python scripts were used to log each observed count and save them into a text file for later analysis.

The Cs-137 source was placed at various distances from the Geiger counter to produce a range of average click counts per time interval. In particular, measurements were taken at three distinct ranges: a low activity range (∼1-3 clicks per time interval), a medium activity range (∼7-10 clicks per time interval), and a high activity range (∼10-13 clicks per time interval). The motivation behind this method of data collection was to investigate how the statistical model changes in order to appropriately describe the data for varying simulations of activity (represented by the Geiger counts). For instance, using the ansätze described in Sec. 1, a low activity radiation source is expected to be governed by a skewed, asymmetric Poisson distribution, while a more active source will approach the Gaussian description.

3 Results

We gathered and independently analysed two long-run data series, with the source and Geiger counter positioned differently. In each case, we performed analysis as follows:

1. We divided the data into a number of replicas (35 for Fig. 2 and 29 for Fig. 3, chosen as to balance the number of replicas and the number of data points within each replica). Replicas were taken from contiguous ranges of data points, and any remaining data points beyond what was divisible by the number of replicas were discarded.

2. For each possible number of counts per time interval, we calculated the mean and standard deviation across all replicas, and used this to estimate $1-\sigma$ and $2-\sigma$ confidence intervals for the mean.

3. We then fit both a Gaussian and Poisson distribution to the data, where optimal parameters (μ, σ, and a scaling factor for the Gaussian, λ and a scaling factor for the Poisson) were found by a minimization algorithm, `scipy.optimize.curve_fit`, trying to predict the bin-means given bin-uncertainties.

4. We then measured the χ^2 of the best fitting Gaussian and Poisson distributions, relative to the uncertainty derived from the spread of the replicas.

The results of this analysis are shown in Fig. 2 and Fig. 3.

We then plotted the χ^2 of the best fitting Gaussian and Poisson distributions across various replica sizes, on the dataset with 7560 counts averaging 12 clicks per second. The results are shown in Fig. 4.

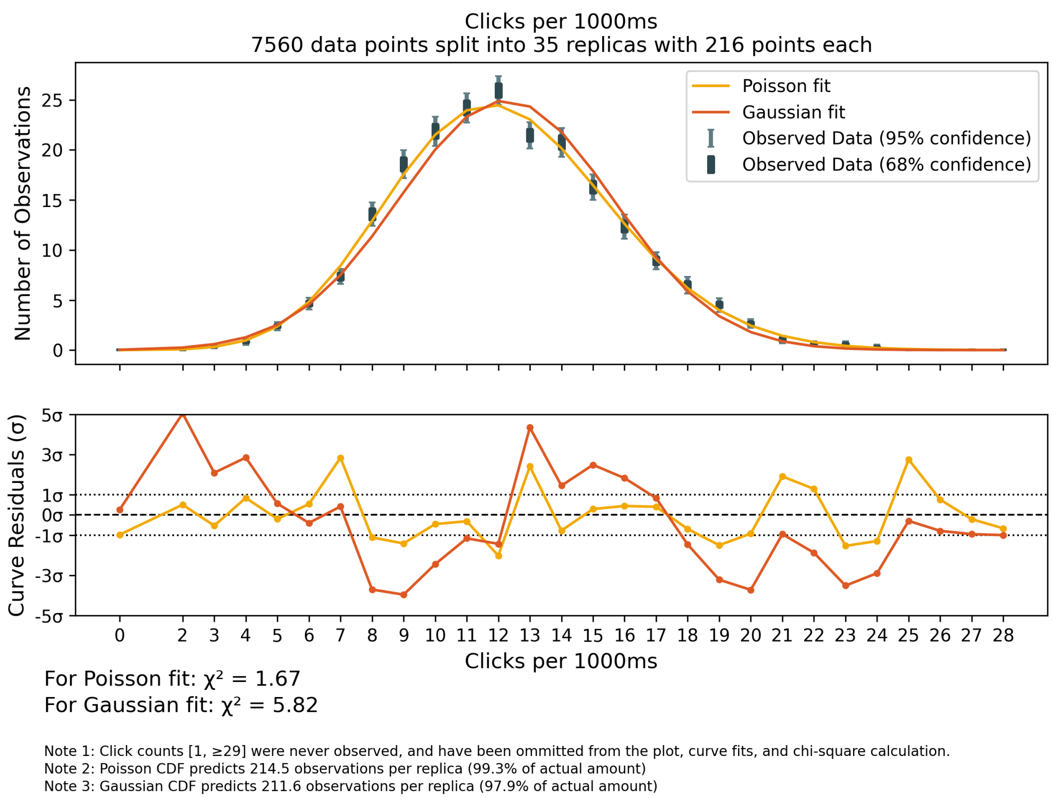

Figure 2: Source and Geiger counter positioned to register an average of 12 clicks per second

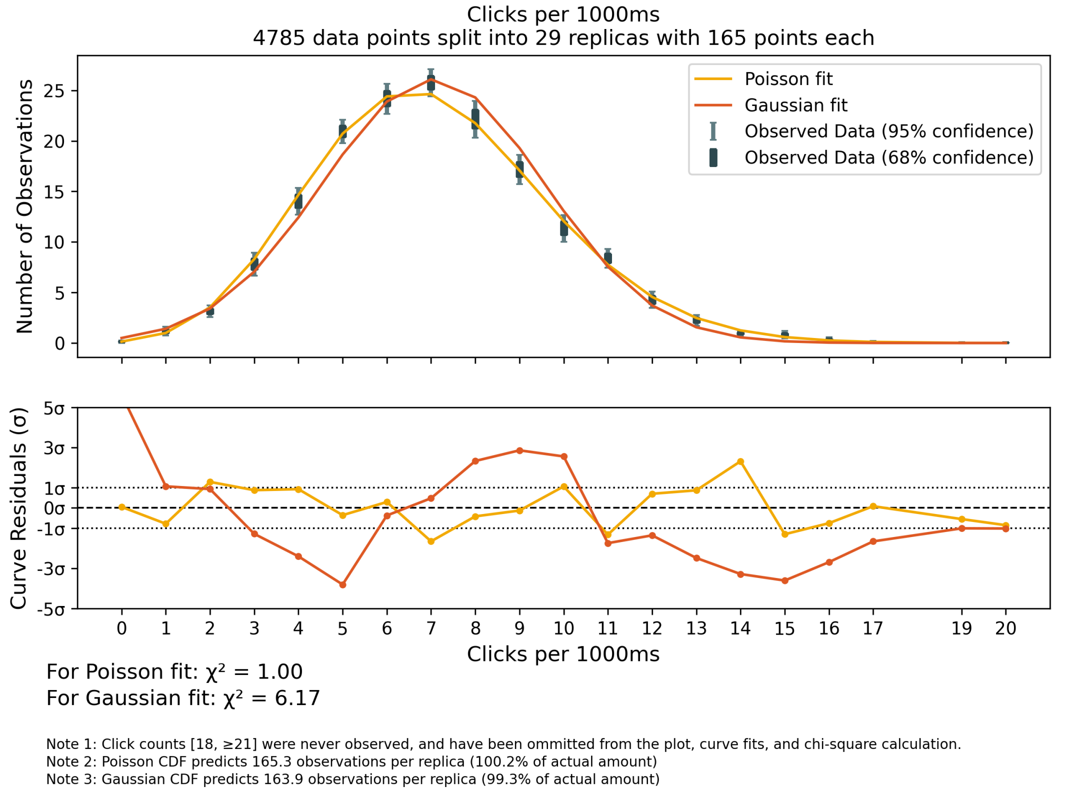

Figure 3: Source and Geiger counter positioned to register an average of 7 clicks per second

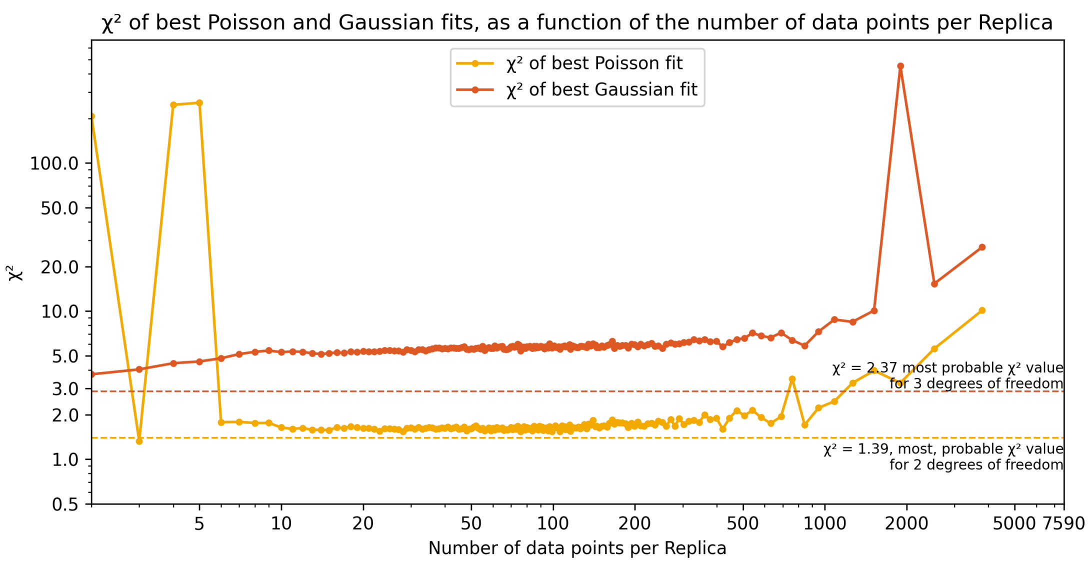

Figure 4: The χ^2 of the best fitting Gaussian and Poisson distributions for various sizes of replicas. Also shown, the most probable value of χ^2 for both two and three degrees of freedom, assuming the distributions were to have generated the data.

4 Discussion

Across a reasonable range of replica sizes, the χ^2 of the best fitting Poisson distribution was consistent with what would be expected were the data to have been generated by a Poisson process. Conversely, the χ^2 of the best fitting Gaussian was consistency much higher than what was probable for Gaussian distributed data. For Fig. 2, with the 3 degrees of freedom in our Gaussian fit, we would only expect a χ^2 as high as the observed 5.82 about 12% of the time. For Fig. 3, the number lowers to 10%.

Analysing Fig. 4, we see that it becomes increasingly easy to distinguish between the two generating distributions as the replica size increases, but even at small replica sizes (down to $n = 6$ samples per replica) the Poisson distribution is clearly more probable. For very large and very small replicas, the uncertainty estimation and curve fitting algorithm struggle to reach a low value, and the χ^2 values balloon.

5 Conclusion

Taken together, this experiment provided a clear demonstration of how statistical methods can be applies to physical radioactive decay. By systematically varying the distance between the source and the detector, it was possible to investigate how different count rates influence the observed statistical distributions. At low counts rates in particular, a Poisson fit suitably describes the discrete and probabilistic nature of radioactive decay. As the count rate increased, the distribution gradually approaches a Gaussian form. Through the data collection and statistical fitting, it was confirmed that the process of radioactive decay is a memoryless process, with the analysis, supported by a chi-squared test, validated the theoretical expectations regarding the transition between the Poisson and Gaussian behavior.

References

[1] W. M. Hubbard, "The approximation of a Poisson distribution by a Gaussian distribution," *proceedings of the IEEE*, vol. 58, no. 30, pp. 1374–1375, 1970. [Online].

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