Experimental Investigation of Polarization: Verifying Malus's Law and Measuring Brewster's Angle

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Abstract

This experiment explored fundamental polarization phenomena in optics: Malus' Law and Brewster's angle. Using a photodiode and a rotating polarizer, the transmitted light intensity as a function of the polarizer's angle was measured, confirming the sinusoidal dependence predicted by Malus' Law. In addition, Brewster's angle was investigated by measuring the transmittance of a diode laser passing through a glass-air followed by an glass-air interface at varying angles of incidence. By fitting the transmittance data to Fresnel's equations, the refractive index of the glass slide was be experimentally determined as $n = 1.60 \pm 0.05$ with a corresponding Brewster's angle of $57.6° \pm 0.9°$. While the theoretical models showed strong agreement with experimental data, a extraordinarily large χ^2 result suggested the presence of small systematic errors in the measurements, replicated extremely consistently across repeated runs. This resulting in an incorrectly low estimation of measurement uncertainty.

Contents

1 Introduction

This experiment aims to investigate two key optical phenomena, namely Brewster's angle and Malus' law. Leveraging a common experimental setup involving a laser, a photo-diode, and a stepper motor where the glass side and polarizer can be affixed to, the intensity readings from the photodiode can aid in determining the relationship between the laser beam intensity and the angle of incidence or the polarization.

Brewster's angle is given as the angle of incidence such that any unpolarized electromagnetic wave is linearly polarized upon reflection [1]. Or rather, the reflected light is S-polarized, which the light is polarized in a direction perpendicular to the plane of the polarizer, in this case, the glass slide [2]. Quantitatively, the Brewster's angle, θ_p, is given as

$$\tan\theta_p = \frac{n_2}{n_1}, \tag{1}$$

where n_1 and n_2 are the refractive indices of the two media the light reflects, in the case of the experiment, n_1 is the refractive index air, and n_2 is the refractive index of the glass slide. Moreover, intensity measurements of the transmitted beam of laser can be used to determine the refractive index of the glass slide used. This setup also behaves in a way that satisfies Fresnel's equation for transmittance:

$$R_s = \left|\frac{n_1\cos\theta_i - n_2\cos\theta_t}{n_1\cos\theta_t + n_2\cos\theta_t}\right|^2 \qquad R_p = \left|\frac{n_1\cos\theta_t - n_2\cos\theta_i}{n_1\cos\theta_t + n_2\cos\theta_t}\right|^2, \tag{2}$$

where R_s and R_p give the reflectance for S-polarized and P-polarized light, respectively, and θ_i and θ_t are the incident, and transmitted angles, respectively. The transmittance is then found by taking the product of the complements of reflectance across the two boundaries:

$$T_{A\to B} = 1 - (R_{s,A\to B})\cdot(R_{p,A\to B}) \tag{3}$$

$$T = (T_{\text{glass}\to\text{air}}) \cdot (T_{\text{air}\to\text{glass}}) \tag{4}$$

In addition, this experiment investigates Malus' law, which described how the intensity of linearly polarized light passing through a polarizing filter is dependent on the relative angle θ between the light's polarization direction and the axis of the polarizing filter [3]. The transmitted intensity I, observed at the photodiode is expected to vary sinusoidally with the relative angle θ by:

$$I(\theta) = I_0 \cos^2 \theta, \tag{5}$$

where I_0 is the intensity of the laser. Similarly to the determination of Brewster's angle, the polarizer can be rotated via the stepper motor and the intensity measurements can be observed to affirm the sinusoidal variation as prescribed above.

2 Experimental Methods

The experimental setup consists of a diode laser, a stepper motor from which the glass slide or the polarizing filter can be mounted to, and a photodiode where intensity measurements are captured. Furthermore, the photodiode, connected to an Arduino via a biasing circuit, which records the transmitted intensity. An image of the experimental setup is shown below:

To investigate Brewster's angle, the laser is aligned such that it struck the glass plate at varying angles of incidence controlled by the stepper motor. The stepper motor ensures precise control of the angle, in increments of exactly 400 steps for one full 360° rotation. We determined the motor to be accurate enough for this task by aligning the laser in-line with the plain of the glass slide, rotating for $400 \times 100 = 40,000$ steps, and confirming that there was no visible drift from the aligned position after the 100 cycles had completed — meaning any drift was less than $1/100^{\text{th}}$ of what would be visually detectable.

For the Malus' Law experiment, a polarizing filter is mounted on the stepper motor. The laser beam passes through the filter, and the photodiode records intensity values at various angles of the polarizer relative to the initial laser polarization.

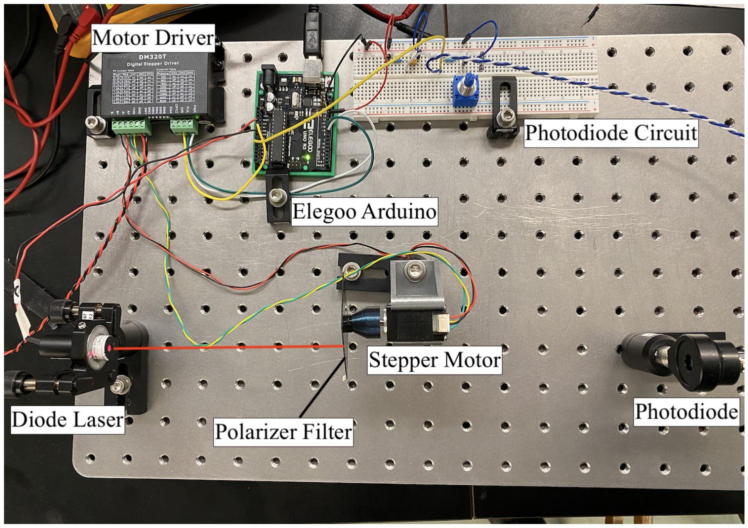

Figure 1: Experimental apparatus. Pictured here is the polarizing filter mounted on the stepper motor for Malus's Law determination. For Brewster's angle, the glass slide is affixed instead. The VDC power supply for the motor driver is also present, though not visible in this image.

For both the Brewster's and Malus' Law experiments, data series were recorded across five identical runs, with a delay of 10000 μs between motor steps and photo-diode recordings. These were used to produce a central estimate and uncertainty for the total transmission at each angle. This uncertainty estimate is discussed further in *Analysis*.

3 Results

For the Brewster's angle determination, Fig. 2 illustrates the measured photodiode intensity as a function of the incident angle between the laser beam and the normal to the glass slide. Here, the $0°$ position is where the glass slide plane is perpendicular to the direction of the laser (in other words, the laser travels along the normal of the glass slide). The superimposed orange curve gives the best fit for total transmittance, as predicted by Fresnel equations for light though an A-B then subsequent B-A material interface. The possibility of multiple internal reflections was left out of the model.

In the calculation of total transmittance, partial results predicting S-polarized and P-polarized component to the reflected light are calculated. These have been scaled up, and plotted along side the overall fit. The vertical dashed line marks the predicted Brewster

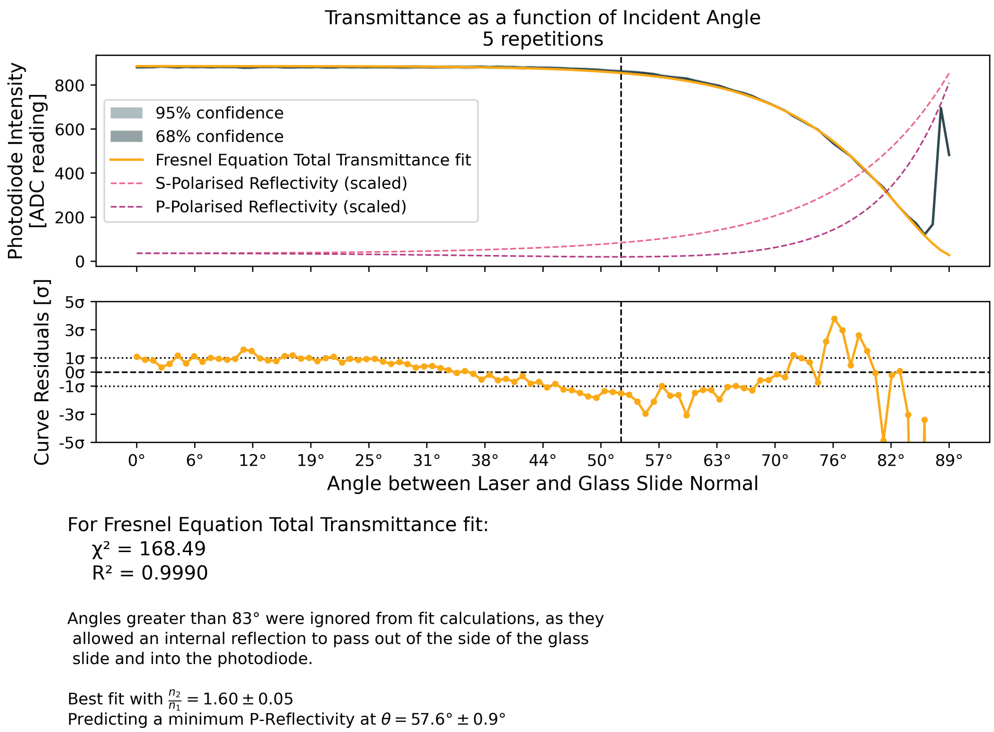

Figure 2: Photodiode intensity values plotted as a function of the angle of incidence of the laser on the glass slide, for 5 repeated trials.

angle, where the reflectance of P-polarized light is minimized.

The uncertainties on the predicted values ($\frac{n_1}{n_2}$ and θ) are derived the uncertainty in the fit of the model to the data in each parameter respectively, and the uncertainty of the minimum in the resulting P-reflectivity curve — and as such are largely unaffected by likely underestimation of uncertainty in the underlying data.

For the Malus' law determination, Fig. 3 shows the measured photodiode intensity as a function of the polariser's orientation angle. The characteristic $a + b\cos^2(\theta - \theta_0)$ fit is superimposed, as motivated by Eq. (5)

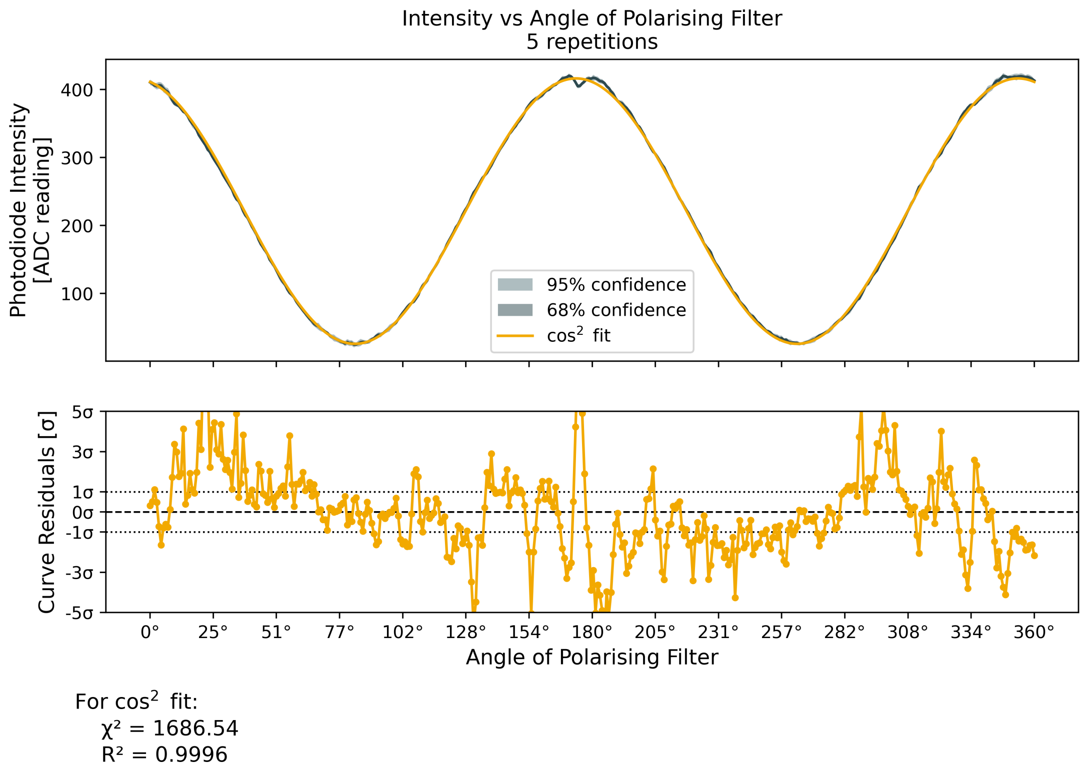

Figure 3: Photodiode intensity as a function of the polarizing filter angle for 5 trial runs.

We additionally used the experimental procedure that generated Figure-3 to conduct an analysis on the sensitivity of our results to the introduction (or removal) of a delay between sending a "step" signal to the motor, and recording the value of the Photodiode. If there were to be any issue with very rapid recording *(e.g. vibrations in the system)*, we would expect the best-fit parameters and R^2 of the fit to vary in the low step-time regime, then stabilize as we drive the motor slower.

4 Discussion

The experimental for the Brewster's angle results confirm the expected behaviour of light transmission and reflection at interfaces. According to Fresnel's equations, P-polarized light experiences zero reflectance at the Brewster's angle, allowing for maximum transmittance into the second medium. The experimental fit closely aligns with this theoretical expectation,

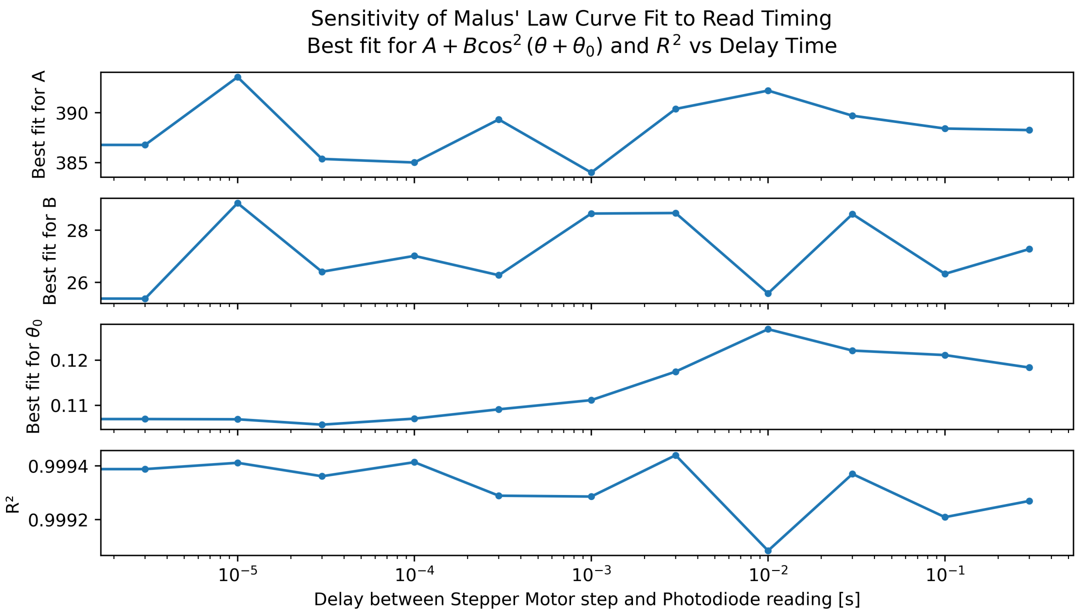

Figure 4: Sensitivity Analysis to delay between Stepper Motor Step and Photodiode Reading

with the minimum reflectance occurring at $57.6° \pm 0.9°$, from Eq. (1) this consistent with the predicted Brewster angle for a material with a refractive index ratio $\frac{n_2}{n_1} = 1.60 \pm 0.05$, which is itself consistent with a refractive index ratio we would expect from a glass-air interface.

For both the Brewster's angle and Malus' law experiments, the χ^2 values are absurdly large given the quality of fit and physical intuition. This can be attributed to the presence of systematic uncertainties replicated across each repeated trial, and hence not accounted for in our model of error. This is evidenced by the fact that both the transmittance and $\cos^2$ curves for the Brewster and Malus experiments respectively produce a near perfect fit ($R^2 = 0.9990$ and 0.9996 respectively), and can be observed visually; in fig-3 there is a notable dip in the data at $\theta \approx 170°$, perfectly repeated across runs, which looks to be indicative of something physical, such as a scratch in the lens.

The chi-square statistic assumes that the uncertainty in each data point follows a purely random Gaussian distribution. If there is a systematic uncertainty that affects all measurements in the same way, the model fit will appear to deviate far than it actually does, leading to an inflated χ^2 value.

Since neither the best-fit parameters nor the R^2 of the best fit varied by much in fig-4, and since none of the variance seems to be systemic, we can determine that a long delay between motor steps and photodiode readings would be largely unnecessary.

A key limitation of the Brewster's angle measurement is the reliance on transmitted intensity data rather than directly measuring the reflected intensity of p-polarized light. A more accurate method would involve placing a detector at the reflection angle and observing the point at which the reflected p-polarized component is minimized directly, instead of inferring this through a fit to Fresnel's equations.

By combining the two experimental setups, we were able to approximately determine the absolute polarisation of the polarising filter, and hence the absolute polarisation of the laser. By angling the glass slide obliquely to the direction of the laser, the reflected beam can be viewed through the polarizing filter. The polarizing filter was then by rotated until the reflected laser beam vanishes. When the glass slide is oriented at the Brewster's angle, this reflected light is known to be vertically polarized. In our case, this happened with the filter at positions $\theta \approx 45°$ and $\theta \approx 135°$, in the same reference frame as used in fig-3. Since direct laser observation in fig-3 is at maximum intensity when the filter is 173°, this means our laser has an absolute polarisation of $(-(173° - (45° + 90)°))\%180° =\sim 142°$ degrees to the vertical, taken clockwise when looking towards the beam emitting side of the laser.

It should be noted that was done as a quick exercise, involving manually rotating the polarizing filter. As such, the approximate angle is likely somewhat imprecise, and an exact answer would require a more sophisticated setup.

5 Conclusion

The results of this experiment affirm the expected optical phenomenon regarding the polarization behaviour of light as predicted by Malus' Law and Brewster's angle. The observed sinusoidal dependence of transmitted intensity through a polarizer yielded results congruent with Malus' law. Moreover, the analysis of Brewster's angle gave expected qualitative results as prescribed by Fresnel's Equations. However, our chi-square analysis indicated systematic uncertainties underlaid both experiments, and is something that motivates improving the experimental apparatus for future refinements. In spite of this, this experiment provided an insightful understanding of polarization effects in optics, and an instructive survey into Brewster's angle and Malus' law.

References

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