1. **Consider the word `unusual`. How many unique subsets of 5 letters (of the 7) exist? How many different strings could be made from 5 of those 7 letters?**

$$\binom{7}{5} = 21 \text{ different strings}$$

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2. **Using a standard deck of playing cards, how many ways are to form a 5-card hand with 2 pairs (i.e. pair of one value, a pair of a different value, and a fifth card of some other value)?**

Given that there are 13 unique cards for any given set within a standard deck and we're choosing for 2 pairs:

$$\binom{13}{2} = 78$$

Since there are 4 sets of cards in a deck and we're choosing for each pair:

$$\binom{4}{2} = 6$$

After each pair is chosen, there is $(11) * (4)$ cards left.

Meaning the total number of ways is:

$$\binom{13}{2} \times \binom{4}{2}^2 \times (11) \times (4) = 78 \times (6)^2 \times (44) = 123,552 \text{ ways}$$

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3. **A violinist serenades couples at a romantic restaurant. She will play 16 songs in an hour and there are 7 couples. One couple is having a fight and will allow at most 1 song to be played to them before they ask the violist not to return to their table. If we care only about the number of songs each couple receives, how many ways can the songs be distributed amongst the couples.**

There are 2 cases dependent on if the fighting couple:

1. Fighting Couple receives 0 songs, meaning there are 16 songs to distribute among 6 couples:

$$\binom{16}{6}$$

2. Fighting Couple receives 1 song, meaning there are 15 songs to distribute among the remaining 6 couples:

$$\binom{15}{6}$$

Meaning the total number of ways is:

$$\binom{16}{6} + \binom{15}{6} = 13,013 \text{ ways}$$

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4. **There is a Binary Search Tree with 12 nodes. Each node has a distinct value between 1 and 12. The root has value 3, and its right child has value 9. How many possible Binary Search Trees could this be? Tip: _Try to define how many ways there are to form a BST of 2 nodes. Then try to define how many ways there are to form a BST of 3 nodes (think about the possible insertion order based on rank: smallest, medium, largest) in terms of 2 node trees for certain cases. Continue to do this for 4 node trees (in terms of 3- and 2-node trees for various cases of insertion ordering based on rank) and 5 node trees.**

    1. `![[bst.excalidraw]]`

$$52,024 \text{ ways}$$

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5. **10 friends arrive to get their COVID vaccine during a particular time slot. During that time slot there are 4 identical nurses administering shots, but 1 of the nurses may (or may not) be scheduled for a break during the time slot in which the friends arrive. Also, how long it takes the nurses to administer a shot varies wildly, so the nurses working during the time slot are guaranteed to serve at least 1 person, but how many additional people they are able to serve is arbitrary. How many different combinations are there for the number of patients served by the nurses?**

Case 1 (4 Nurses): Assume that each nurse is able to help at least one person, leaving 6 left for the others to handle and since all nurses are identical:

Nurse 1	Nurse 2	Nurse 3	Nurse 4
6	0	0	0
5	1	0	0
4	1	1	0
3	1	1	1
2	2	1	1
1	2	2	1

Nurse 1	Nurse 2	Nurse 3	Nurse 4
4	2	0	0
3	3	0	0
2	2	2	0

This leads to 9 different ways.

Case 2 (3 Nurses): Assume that each nurse is able to help at least one person, leaving 7 left for the others to handle and since all nurses are identical:

Nurse 1	Nurse 2	Nurse 3
7	0	0
6	1	0
5	2	0
5	1	1
4	2	1
4	3	0
3	3	1
3	2	2

This leads to 8 different ways.

$$8 + 9 = 17 \text{ Different Ways}$$
