1. **A professor has 15 students and during lecture will (uniformly) at random choose a student to answer a question. The professor asks 8 questions during the lecture. What is the probability no student will have to answer more than one question?**

   If the order of questions matters then the probability is:

$$15^8$$

   If the order of questions doesn't matter and no student will have to answer more than one question

$$15!$$

   The total probability is

$$\frac{\frac{15!}{7!}}{15^8} = 0.101$$

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2. **An integer from the range 00000 - 99999 is generated uniformly at random. We are interested only in even integers that start with 2 odd digits where all digits are unique. If we randomly generate 8 of these numbers in succession, what is the probability we get exactly 5 numbers that meet our criteria?**

Odd	Odd	Remaining	Remaining	Remaining
1	1	7 Remaining Options	6 Remaining Options	5 Remaining
3	3			
5	5			
7	7			
9	9			

For the first integer, whichever out of the 5 options, there will leave 4 options for the next one left. Since this takes out 2 options, the remaining unique integers are 7, and then 6, and then 5 respectively

$$5 * 4 * 7 * 6 * 5 = 4200$$

Since there are 100,000 different combinations that can be generated

$$\frac{4200}{100,000} = 0.042$$

3. **You roll 3 six-sided, fair dice. Let A be the event that at least 2 dice show 4 or above. Let B be the event that all 3 dice show the same value. Are A and B independent?**

    1. Yes, they are independent because the likelihood of B happening is not dependent on A since it is possible for all three dice to show the same value but not have two of them be 4 or above.

4. In poker, a flush is any 5-card hand where all the cards of the same suit. For this problem we will not distinguish between an ordinary flush and special flushes (like straight and royal flushes), meaning we will call any hand that has all 5 cards from the same suit a flush. Poker-player Paul loves a flush. What is the expected number of hands of poker he has to play to get a flush. (We assume each hand is dealt from a new deck containing of randomly ordered cards).

    If the order matters, then the total number of dealings is

$$\frac{52!}{47!} = 311875200$$

    Since you need 5 cards in any given suit (13 cards) to get a flush and there are 4 suits, then the number of flushes is

$$4 \times \frac{13!}{8!} = 617760$$

    Therefore the probability of a flush is

$$\frac{4 \times \frac{13!}{8!}}{\frac{52!}{47!}} = \frac{617760}{311875200} = 0.00198$$

    The expected number of hands is

$$\frac{1}{0.00198} = 505 \text{ hands}$$

5. **A basketball team has a superstar. When their superstar plays, they win 70% of the time. When their superstar does not play they win 50% of the time. Entering a 5 game stretch, the superstar had been recovering from an injury and said the chance they would play the next 5 games was 75%. You go on a trip to the jungle (no internet access). When you return you find out the team won 4 of the 5 games. What is the probability the superstar played those 5 games? You may assume the superstar doesn't get injured during those games (either they play all or none of the 5).**

E = Team wins $\frac{4}{5}$ games
F = Superstar Plays
F^c = Superstar doesn't play

$P(F)$ = $\frac{3}{4} = 0.75$
$P(F^c)$ = $1 - P(F) = 0.25$

$P\frac{E}{F} = \binom{5}{4}(0.7)^4 \times (0.3)^1 = 0.36015$
$P\frac{E}{F^c} = \binom{5}{4}(0.5)^4 \times (0.5)^1 = 0.15625$

Given that the team won $\frac{4}{5}$ games

$$P\frac{E}{F} = \frac{P\frac{E}{F} \times P(F)}{P\frac{E}{F} \times P(F) \times P\frac{E}{F^c} \times P(F^c)} = \frac{0.36015 \times 0.75}{0.36015 \times 0.75 + 0.15625 \times 0.25} = 0.873655697$$