Calculus 3 Chapter 13 Test Fall 2009 Name _____
Show all work.

1. A particle moves by $\mathbf{r}(t)=\frac{1}{9}t^3\mathbf{i}+\ln(2t-5)\mathbf{j}+2(t+1)^{5/2}\mathbf{k}$. Find the angle between the velocity vector and the acceleration vector at $t=3$. (10 points)

2. Find the equation of the tangent line to $\mathbf{r}(t)=e^{2t-8}\mathbf{i}+\ln(3t-11)\mathbf{j}+t^2\mathbf{k}$. at $t=4$. (9 points)

3. The velocity vector of a particle is given by $\boldsymbol{v}(t)=42t^5\mathbf{i}+10\mathbf{j}+12t^3\mathbf{k}$. If $\mathbf{r}(1)=2\mathbf{i}+3\mathbf{j}+7\mathbf{k}$, find the position vector at any time t. (9 points)

4. A particle moves by $\mathbf{r}(t)=t^3\mathbf{i}+(3t^2+t)\mathbf{j}+(2t+t^2)\mathbf{k}$. Find the tangential and normal components of acceleration at $t= -2$. (12 points)

5. Find the distance traveled from $t=0$ to $t=1$ by the particle with position vector function $\mathbf{r}(t)=t^6\mathbf{i}+t^4\mathbf{j}+2t^4\mathbf{k}$. (10 points)

6. Curvature can be found by $\frac{\left\|\vec{\boldsymbol{v}}(t)\times\vec{\boldsymbol{a}}(t)\right\|}{\left\|\vec{\boldsymbol{v}}(t)\right\|^3}$. Find the curvature when $t= 0$ for $\mathbf{r}(t)=e^{4t}\mathbf{i}+\cos(2t)\mathbf{j}+(3t^2+t)\mathbf{k}$. (10 points)

7. A book is thrown from a height of 250 meters at an angle of 30 degrees above horizontal with an initial speed of 150 meters per second on a planet with no air resistance and where acceleration due to gravity is 50 meters per second per second.

a) Write the acceleration, velocity, and position vector functions for the motion of the book. Assume that the origin is at ground level directly below the point where the book is released. (6 points)

b) Find the horizontal ground distance covered before impact. (3 points)

c) Find the maximal height of the book. (3 points)

8. If $\mathbf{G}(t)= (7t\text{-}1)\mathbf{i}+\sin(3t)\mathbf{j}+e^{4t}\,\mathbf{k}$ and $\mathbf{H}(t)=e^{6t}\mathbf{i}+\cos(5t)\mathbf{j}+(2t\text{-}8)\,\mathbf{k}$, find $\frac{d}{dt}\left(\mathbf{G}(t)\times\mathbf{H}(t)\right)$ evaluated at t=0. (10 points)

9. If $\mathbf{r}(t)=\left(\frac{7t^2+9t}{5t}\right)\mathbf{i}+\left(\frac{\sin(8t)}{3t}\right)\mathbf{j}+\left(\frac{e^{11t}-1}{6t}\right)\boldsymbol{\kappa}$, find $\lim_{t\to 0}\mathbf{r}(t)$. (9 points)

10. A particle moves in a circle of radius 13, taking $2\pi/5$ seconds to make one rotation.

a) Write an equation for the position vector at any time t. (Pick anywhere you want for the position at t=0. It may go in either the clockwise or counterclockwise direction.) (4 points)

b) Using part a, show that the speed is constant. (5 points)