Homework 3

1.) $\underset{3\times 2}{\begin{bmatrix} -4 & 2 \\ 1 & 6 \\ 0 & 1 \end{bmatrix}} \underset{3\times 1}{\begin{bmatrix} 3 \\ 1 \\ 7 \end{bmatrix}}$ For matrix multiplication, the rows of the second matrix must "match" the columns of the first. Therefore, the system cannot be defined. $\boxed{\text{Undefined}}$

3.) $\underset{3\times 2}{\begin{bmatrix} 6 & 5 \\ -4 & -3 \\ 7 & 6 \end{bmatrix}} \underset{2\times 1}{\begin{bmatrix} 1 \\ -3 \end{bmatrix}} = \begin{bmatrix} 6(1) + 5(-3) \\ -4(1) + -3(-3) \\ 7(1) + 6(-3) \end{bmatrix} = \underset{3\times 1}{\begin{bmatrix} -9 \\ 5 \\ -11 \end{bmatrix}}$

$$\begin{bmatrix} 6 & 5 \\ -4 & -3 \\ 7 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ -3 \end{bmatrix} = 1\begin{bmatrix} 6 \\ -4 \\ 7 \end{bmatrix} + -3\begin{bmatrix} 5 \\ -3 \\ 6 \end{bmatrix} = \begin{bmatrix} 6 \\ -4 \\ 7 \end{bmatrix} + \begin{bmatrix} -15 \\ 9 \\ -18 \end{bmatrix} = \begin{bmatrix} -9 \\ 5 \\ -11 \end{bmatrix}$$

11.) $\begin{bmatrix} 1 & 2 & 4 & : & -2 \\ 0 & 1 & 5 & : & 2 \\ -2 & -4 & -3 & : & 9 \end{bmatrix}$ $R3' = R3 + 2R1$ $\begin{bmatrix} 1 & 2 & 4 & : & -2 \\ 0 & 1 & 5 & : & 2 \\ 0 & 0 & 5 & : & 5 \end{bmatrix}$

$x_1 + 2x_2 + 4x_3 = -2$ $\quad x_1 = 0$

$x_2 + 5x_3 = 2$ $\quad x_2 = -3$

$5x_3 = 5$ $\quad x_3 = 1$

$\boxed{\begin{bmatrix} 0 \\ -3 \\ 1 \end{bmatrix}}$

12.) $\begin{bmatrix} 1 & 2 & 1 & : & 0 \\ -3 & -1 & 2 & : & 1 \\ 0 & 5 & 3 & : & -1 \end{bmatrix}$ $R2' = R2 + 3R1$ $\begin{bmatrix} 1 & 2 & 1 & : & 0 \\ 0 & 5 & 5 & : & 1 \\ 0 & 5 & 3 & : & -1 \end{bmatrix}$

$R3' = R3 - R2$ $\begin{bmatrix} 1 & 2 & 1 & : & 0 \\ 0 & 5 & 5 & : & 1 \\ 0 & 0 & -2 & : & -2 \end{bmatrix}$ $\quad x_1 + 2x_2 + x_3 = 0$

$5x_2 + 5x_3 = 1$

$-2x_3 = -2$

$x_1 = -\frac{3}{5}$

$x_2 = -\frac{4}{5}$

$x_3 = 1$

$= \boxed{\begin{bmatrix} -\frac{3}{5} \\ -\frac{4}{5} \\ 1 \end{bmatrix}}$

13.) Does $u = \begin{bmatrix} 0 \\ 4 \\ 4 \end{bmatrix}$ lie in Span$\left\{ \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} -5 \\ 6 \\ 1 \end{bmatrix} \right\}$

$\begin{bmatrix} 3 & -5 & : & 0 \\ -2 & 6 & : & 4 \\ 1 & 1 & : & 4 \end{bmatrix}$ $R2' = \frac{2}{3}R1 + R2$, $R3' = -\frac{1}{3}R1 + R3$ $\begin{bmatrix} 3 & -5 & : & 0 \\ 0 & 8/3 & : & 4 \\ 0 & 8/3 & : & 4 \end{bmatrix}$ $R3' = R3 - R2$

$\begin{bmatrix} 3 & -5 & : & 0 \\ 0 & 8/3 & : & 4 \\ 0 & 0 & : & 0 \end{bmatrix}$ The system is consistent, so u is in the Span of A.

14. $\left[\begin{array}{ccc|c} 5 & 8 & 7 & 2 \\ 0 & 1 & -1 & -3 \\ 1 & 3 & 0 & 2 \end{array}\right] \leftrightarrow \left[\begin{array}{ccc|c} 1 & 3 & 0 & 2 \\ 0 & 1 & -1 & -3 \\ 5 & 8 & 7 & 2 \end{array}\right]$ $R3' = -5R1 + R3$

$\left[\begin{array}{ccc|c} 1 & 3 & 0 & 2 \\ 0 & 1 & -1 & -3 \\ 0 & -7 & 7 & -8 \end{array}\right]$ $R3' = 7R2 + R3$ $\left[\begin{array}{ccc|c} 1 & 3 & 0 & 2 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 0 & -29 \end{array}\right]$

$0 \neq -29$ so u is not in the subset of $\mathbb{R}^3$.

16. $\left[\begin{array}{ccc|c} 1 & -3 & -4 & b_1 \\ -3 & 2 & 6 & b_2 \\ 5 & 1 & -8 & b_3 \end{array}\right]$ $R2' = 3R1 + R2$, $R3' = R3 - 5R1$ $\left[\begin{array}{ccc|c} 1 & -3 & -4 & b_1 \\ 0 & -7 & -6 & 3b_1 + b_2 \\ 0 & -14 & 12 & -5b_1 + b_3 \end{array}\right]$

$R3' = R3 - 2R2$ $\left[\begin{array}{ccc|c} 1 & -3 & -4 & b_1 \\ 0 & -7 & -6 & 3b_1 + b_2 \\ 0 & 0 & 0 & 5b_1 - 2b_2 + b_3 \end{array}\right]$

$x_1 - 3x_2 - 4x_3 = b_1$
$-7x_2 - 6x_3 = 3b_1 + b_2$
$0 = 5b_1 - 2b_2 + b_3$

For the set to have a solution, $5b_1 - 2b_2 + b_3$ must $= 0$.

21. $\begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 1 & -1 \end{bmatrix}$ $R3' = R3 + R1$ $\begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix}$

$R4' = R4 + R2$ $\begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix}$ $R4' = R4 + R3$ $\begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

The set of vectors span $\mathbb{R}^3$ not $\mathbb{R}^4$, because the last row cannot have a leading entry.

25. True, for $Ax = b$ to be consistent, b must be in the set.

34. False, a pivot position in each row means no free variables, therefore there will be a consistent solution.

1.
$$\left[\begin{array}{ccc|c} 2 & -5 & 8 & 0\\ -2 & -7 & 1 & 0\\ 4 & 2 & 7 & 0\end{array}\right] \quad \begin{array}{l} R2' = R2 + R1 \\ R3' = R3 - 2R1 \end{array} \quad \left[\begin{array}{ccc|c} 2 & -5 & 8 & 0\\ 0 & -12 & 9 & 0\\ 0 & 12 & -9 & 0\end{array}\right]$$

$$R3' = R3 + R2 \quad \left[\begin{array}{ccc|c} 2 & -5 & 8 & 0\\ 0 & -12 & 9 & 0\\ 0 & 0 & 0 & 0\end{array}\right] \qquad x_3 = \text{free}$$

Since x_3 is a free variable, there will be a nontrivial solution

2.
$$\left[\begin{array}{ccc|c} 1 & -3 & 7 & 0\\ -2 & 1 & -4 & 0\\ 1 & 2 & 9 & 0\end{array}\right] \quad \begin{array}{l} R2' = 2R1 + R2 \\ R3' = R3 - R1 \end{array} \quad \left[\begin{array}{ccc|c} 1 & -3 & 7 & 0\\ 0 & -5 & 10 & 0\\ 0 & 5 & 2 & 0\end{array}\right]$$

$$R3' = R3 + R2 \quad \left[\begin{array}{ccc|c} 1 & -3 & 7 & 0\\ 0 & -5 & 10 & 0\\ 0 & 0 & 12 & 0\end{array}\right] \qquad \begin{array}{l} x_1 = 0 \\ x_2 = 0 \\ x_3 = 0 \end{array}$$

There is no nontrivial solution as x_1, x_2, and x_3 are 0.

7.
$$\begin{bmatrix} 1 & 3 & -3 & 7 \\ 0 & 1 & -4 & 5 \end{bmatrix} \qquad \begin{array}{l} x_1 + 3x_2 - 3x_3 + 7x_4 = 0 \\ x_2 - 4x_3 + 5x_4 = 0 \end{array} \quad \Bigg| \quad \begin{array}{l} x_1 = -9x_3 + 8x_4 \\ x_2 = 4x_3 - 5x_4 \end{array}$$

$\boxed{x_3, x_4 = \text{free}}$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -9x_3 + 8x_4 \\ 4x_3 - 5x_4 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -9x_3 \\ 4x_3 \\ x_3 \\ 0 \end{bmatrix} + \begin{bmatrix} 8x_4 \\ -5x_4 \\ 0 \\ x_4 \end{bmatrix} = \boxed{x_3 \begin{bmatrix} -9 \\ 4 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 8 \\ -5 \\ 0 \\ 1 \end{bmatrix}} \; ☆$$

11.
$$\begin{bmatrix} 1 & -4 & -2 & 0 & 3 & -5 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{array}{l} x_1 - 4x_2 - 2x_3 + 3x_5 - 5x_6 = 0 \\ x_3 - x_6 = 0 \\ x_5 - 4x_6 = 0 \end{array}$$

$\boxed{x_2, x_4, x_6 = \text{free}}$

$x_1 - 4x_2 - 2(x_6) + 3(4x_6) - 5x_6 = 0$

$x_1 = 4x_2 - 5x_6$

$x_3 = x_6$

$x_5 = 4x_6$

$$= \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 4x_2 - 5x_6 \\ x_2 \\ x_6 \\ x_4 \\ 4x_6 \\ x_6 \end{bmatrix} = \begin{bmatrix} 4x_2 \\ x_2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -5x_6 \\ 0 \\ x_6 \\ 0 \\ 4x_6 \\ x_6 \end{bmatrix} =$$

$$☆ \quad \boxed{x_2 \begin{bmatrix} 4 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_6 \begin{bmatrix} -5 \\ 0 \\ 1 \\ 0 \\ 4 \\ 1 \end{bmatrix}}$$

19. $\left[\begin{array}{ccc:c} 1 & 3 & 1 & 1 \\ -4 & -9 & 2 & -1 \\ 0 & -3 & -6 & -3 \end{array}\right]$ $R2' = R2 + 4R1$ $\left[\begin{array}{ccc:c} 1 & 3 & 1 & 1 \\ 0 & 3 & 6 & 3 \\ 0 & -3 & -6 & -3 \end{array}\right]$

$R3' = R3 + R2$ $\left[\begin{array}{ccc:c} 1 & 3 & 1 & 1 \\ 0 & 3 & 6 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$x_1 + 3x_2 + x_3 = 1$
$3x_2 + 6x_3 = 3$
$x_3 = \text{free}$

$x_1 = 5x_3 - 2$
$x_2 = -2x_3 + 1$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5x_3 - 2 \\ -2x_3 + 1 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5x_3 \\ -2x_3 \\ x_3 \end{bmatrix} + \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$$

$$= x_3 \underbrace{\begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix}}_{v} + \underbrace{\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}}_{p} \qquad x = p + tv$$

$Ax = b$ solution goes through $p = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$, and solution is parallel to $Ax = 0$.

25. $x = p + t(q - p)$

$$x = \begin{bmatrix} 2 \\ -5 \end{bmatrix} + t\left\{\begin{bmatrix} -3 \\ 1 \end{bmatrix} - \begin{bmatrix} 2 \\ -5 \end{bmatrix}\right\} = \boxed{x = \begin{bmatrix} 2 \\ -5 \end{bmatrix} + t\begin{bmatrix} -5 \\ 6 \end{bmatrix}}$$

28. False, a nontrivial solution of $Ax=0$ occurs when a nonzero vector satisfies $Ax=0$. Therefore, x can have some zero entries but not all zeros.

31. False, If $Ax=0$ is homogeneous, then the there is a trivial solution even if the equation doesn't have a free variable.