Homework 8

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4.

$$A=\begin{bmatrix}2&1&-7\\-2&-3&5\\1&2&4\end{bmatrix} \updownarrow \begin{bmatrix}1&2&4\\-2&-3&5\\2&1&-7\end{bmatrix} \begin{matrix}R2'=R2+2R1\\R3'=R3-2R1\end{matrix} \begin{bmatrix}1&2&4\\0&1&13\\0&-3&-15\end{bmatrix}$$

$$R3'=R3+3R2 \begin{bmatrix}1&2&4\\0&1&13\\0&0&24\end{bmatrix}$$

The set is linearly independent and the vectors span $\mathbb{R}^3$, so this set is a basis for $\mathbb{R}^3$.

8.

$$A=\begin{bmatrix}1&0&3&0\\-4&3&-5&2\\3&-1&4&-2\end{bmatrix} \begin{matrix}R2'=R2+4R1\\R3'=R3-3R1\end{matrix} \begin{bmatrix}1&0&3&0\\0&3&7&2\\0&-1&-5&-2\end{bmatrix} \updownarrow \begin{bmatrix}1&0&3&0\\0&-1&-5&-2\\0&3&7&2\end{bmatrix}$$

$$R3'=R3+3R2 \begin{bmatrix}1&0&3&0\\0&-1&-5&-2\\0&0&-8&-4\end{bmatrix}$$

The set not linearly independent and the vectors span $\mathbb{R}^3$, so this set is not a basis for $\mathbb{R}^3$.

10.

$$\left[\begin{array}{ccccc|c}1&0&-5&1&4&0\\-2&1&6&-2&-2&0\\0&2&-8&1&9&0\end{array}\right] R2'=R2+2R1 \left[\begin{array}{ccccc|c}1&0&-5&1&4&0\\0&1&-4&0&6&0\\0&2&-8&1&9&0\end{array}\right] R3'=R3-2R2$$

$$\left[\begin{array}{ccccc|c}1&0&-5&1&4&0\\0&1&-4&0&6&0\\0&0&0&1&-3&0\end{array}\right] R1'=R1-R3 \left[\begin{array}{ccccc|c}1&0&-5&0&7&0\\0&1&-4&0&6&0\\0&0&0&1&-3&0\end{array}\right]$$

$x_1=-5x_3+7x_5$
$x_2=-4x_3+6x_5$
$x_4=-3x_5$
$x_3, x_5 =$ free

$$x_3\begin{bmatrix}-5\\-4\\1\\0\\0\end{bmatrix}+x_5\begin{bmatrix}7\\6\\0\\-3\\1\end{bmatrix}$$

$\Rightarrow$ these 2 vectors are linearly independent, so they are the basis for Nul A.

15.

$$\begin{bmatrix}1&0&-3&1&2\\0&1&-4&-3&1\\-3&2&1&-8&-6\\2&-3&6&7&9\end{bmatrix} \begin{matrix}R3'=R3+3R1\\R4'=R4-2R1\end{matrix} \begin{bmatrix}1&0&-3&1&2\\0&1&-4&-3&1\\0&2&-8&-5&0\\0&-3&12&5&5\end{bmatrix} \begin{matrix}R3'=R3-2R2\\R4'=R4+3R2\end{matrix}$$

$$\begin{bmatrix}1&0&-3&1&2\\0&1&-4&-3&1\\0&0&0&1&-2\\0&0&0&-4&8\end{bmatrix} R4'=R4+4R3 \begin{bmatrix}1&0&-3&1&2\\0&1&-4&-3&1\\0&0&0&1&-2\\0&0&0&0&0\end{bmatrix}$$

$\{v_1, v_2, v_4\}$ is the basis $\left\{\begin{bmatrix}1\\0\\-3\\2\end{bmatrix}, \begin{bmatrix}0\\1\\2\\-3\end{bmatrix}, \begin{bmatrix}1\\-3\\-8\\7\end{bmatrix}\right\}$

22.) False - for a basis to exist, it must be linearly independent and span the subspace, and this only satisfies independence

30.) True - The pivot colums of B show that the corresponding columns in A are the basis.

43.) Neither polynomial can be multiplied to get the other, so it is linearly independent

7.) $\left[\begin{array}{ccc|c} 1 & -3 & 2 & 8 \\ -1 & 4 & -2 & -9 \\ -3 & 9 & 4 & 6 \end{array}\right]$ $R2' = R2 + R1$, $R3' = R3 + 3R1$ $\left[\begin{array}{ccc|c} 1 & -3 & 2 & 8 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 10 & 30 \end{array}\right]$

$x_1 - 3x_2 + 2x_3 = 8$ $\quad x_1 = -1$

$x_2 = -1$ $\quad x_2 = -1$

$10x_3 = 30$ $\quad x_3 = 3$

$\boxed{[x]_B = \begin{bmatrix} -1 \\ -1 \\ 3 \end{bmatrix}}$

12.) $B = \begin{bmatrix} 4 & 6 \\ 5 & 7 \end{bmatrix}$ $x = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$ $B^{-1} = \frac{1}{28-30} \begin{bmatrix} 7 & -6 \\ -5 & 4 \end{bmatrix}$

$= \begin{bmatrix} -7/2 & 3 \\ 5/2 & -2 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix} = \boxed{[x]_B = \begin{bmatrix} -7 \\ 5 \end{bmatrix}}$

13.) $x_1(1+t^2) + x_2(t+t^2) + x_3(1+2t+t^2) = 1+4t+7t^2$

$x_1 + x_3 = 1$ $\quad x_2 + 1 = 7$ $\quad x_1 = 2$

$x_2 + 2x_3 = 4$ $\quad 2x_3 + 6 = 4$ $\quad x_2 = 6$ $\quad = \begin{bmatrix} 2 \\ 6 \\ -1 \end{bmatrix}$

$x_1 + x_2 + x_3 = 7$ $\quad x_1 - 1 = 1$ $\quad x_3 = -1$

16.) True - by definition of a coordinate matrix

17.) False - $x = P_B[x]_B$ means $[x]_B = P_B^{-1}x$

36a.) $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & -3 & -3 \end{bmatrix}$ $R3' = R3 - R1$ $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & -3 & -4 \end{bmatrix}$ $R3' = 3R2 + R3$

$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{bmatrix}$ The polynomials form a basis to $\mathbb{P}_2$ because they are linearly independent and spans $\mathbb{P}_2$.

36b. $q(t) = -p_1(t) + p_2(t) + 2p_3(t)$

$= -(1+t^2) + (t-3t^2) + 2(1+t-3t^2)$

$\Rightarrow \boxed{1 + 3t - 10t^2}$

5a. $a\begin{bmatrix}1\\2\\-1\\3\end{bmatrix} + b\begin{bmatrix}-4\\5\\0\\7\end{bmatrix} + c\begin{bmatrix}-2\\-4\\2\\6\end{bmatrix} = \begin{bmatrix}1 & -4 & -2\\2 & 5 & -4\\-1 & 0 & 2\\3 & 7 & 6\end{bmatrix}$ Since the third vector is dependent

Basis $= \begin{bmatrix}1\\2\\-1\\3\end{bmatrix}, \begin{bmatrix}-4\\5\\0\\7\end{bmatrix}$

5b. Since there are 2 vectors in the basis, $\boxed{\dim = 2}$

9. $\begin{bmatrix}1 & 3 & 9 & -7\\0 & 1 & 4 & -3\\2 & 1 & -2 & 1\end{bmatrix}$ $R3 = R3 - 2R1$ $\begin{bmatrix}1 & 3 & 9 & -7\\0 & 1 & 4 & -3\\0 & -5 & -20 & 15\end{bmatrix}$ $R3' = R3 + 5R2$

$\begin{bmatrix}1 & 3 & 9 & -7\\0 & 1 & 4 & -3\\0 & 0 & 0 & 0\end{bmatrix}$ There are 2 pivot positions, so the $\boxed{\dim = 2}$

12. There are 3 pivot positions, so $\boxed{\dim \text{Col } A = 3}$

$\dim \text{Col } A = \dim \text{Row } A$, so $\boxed{\dim \text{Row } A = 3}$

There are 6 colums, so $6 - 3 = \boxed{\dim \text{Nul } A = 3}$

14. $A = \begin{bmatrix}3 & 4\\-6 & 10\end{bmatrix}$ $R2 = R2 + 2R1$ $\begin{bmatrix}3 & 4\\0 & 18\end{bmatrix}$

There are 2 pivot positions, so $\boxed{\dim \text{Col } A = 2}$

$\dim \text{Col } A = \dim \text{Row } A$, so $\boxed{\dim \text{Row } A = 2}$

There are no nonpivot colums, so $\boxed{\dim \text{Nul } A = 0}$

18. False - The number of free variables in $Ax=0$ determine Nul A

20. False - $\mathbb{P}_4$ can have less than 4 vectors

35. $\text{Col } A = \mathbb{R}^4$ and $\text{Nul } A = \mathbb{R}^3$

There are 4 pivot columns, which makes $\text{Col } A = \mathbb{R}^4$ true.

$\text{Nul } A \neq \mathbb{R}^3$ because the vectors in Nul A have 7 entries.