Statistical Methods for Discrete Response, Time Series, and Panel Data (W271): Lab 2

(3 points) Task 0a: Introduction

The Keeling Curve

In the 1950s, the geochemist Charles David Keeling observed a seasonal pattern in the amount of carbon dioxide present in air samples collected over the course of several years. He was able to attribute this pattern to the variation in global rates of photosynthesis throughout the year, caused by the difference in land area and vegetation cover between the Earth's northern and southern hemispheres.

In 1958 Keeling began continuous monitoring of atmospheric carbon dioxide concentrations from the Mauna Loa Observatory in Hawaii. Mauna Loa was chosen as a long-term monitoring site due to its remote location far from continents and its lack of vegetation. Keeling soon observed a trend increase in carbon dioxide (CO2) levels in addition to the seasonal cycle. He was able to attribute this trend increase to growth in global rates of fossil fuel combustion.

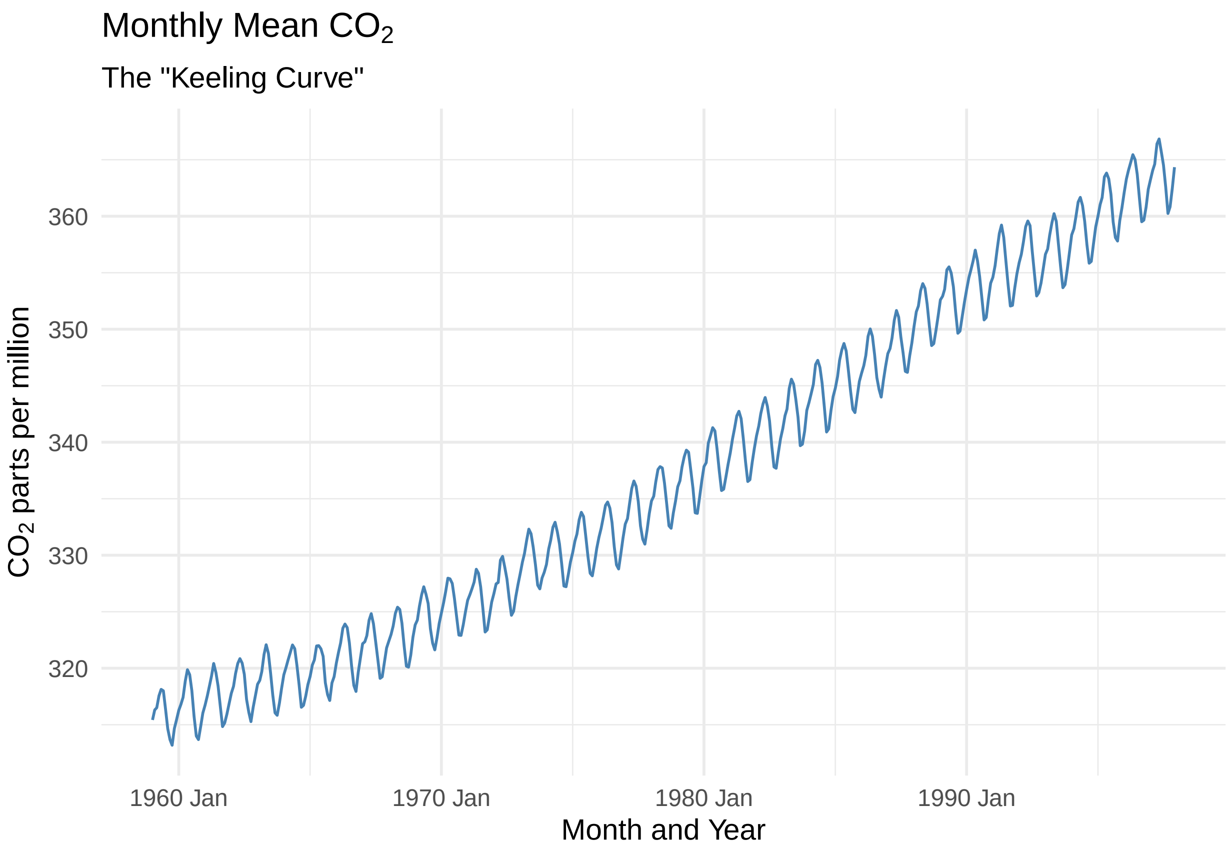
Since CO2 is a greenhouse gas, higher levels of CO2 can lead to a warmer planet, causing climate change. Climate change can result in more frequent and severe weather events such as floods, cyclones, typhoons, and wildfires. It can also lead to a decrease in crop yields and put many animal species at risk of extinction.

Our team will analyze the CO2 average parts per million recorded from Manua Loa since 1957 to the present (1997).

The apparent increasing trend could be dismissed as a transient stochastic phenomenon, if so, it should be possible to model the trend using stochastic functions and without the use of deterministic functions.

Our team will fit the data using stochastic ARIMA models, and deterministic Linear Time Trend models and and use diagnostics to determine which one best fits the data. Our team will also forecast what the expected CO2 levels wil be in the 2020 and 2022, if the same trends continue to hold.

Our exercise will aid the scientific community to decide if the increasing trend can be dismissed or should further research be made. Furthermore, our forecasts will aid policy makers to take the steps to prevent the increasing trend of CO2 levels in the atmosphere.

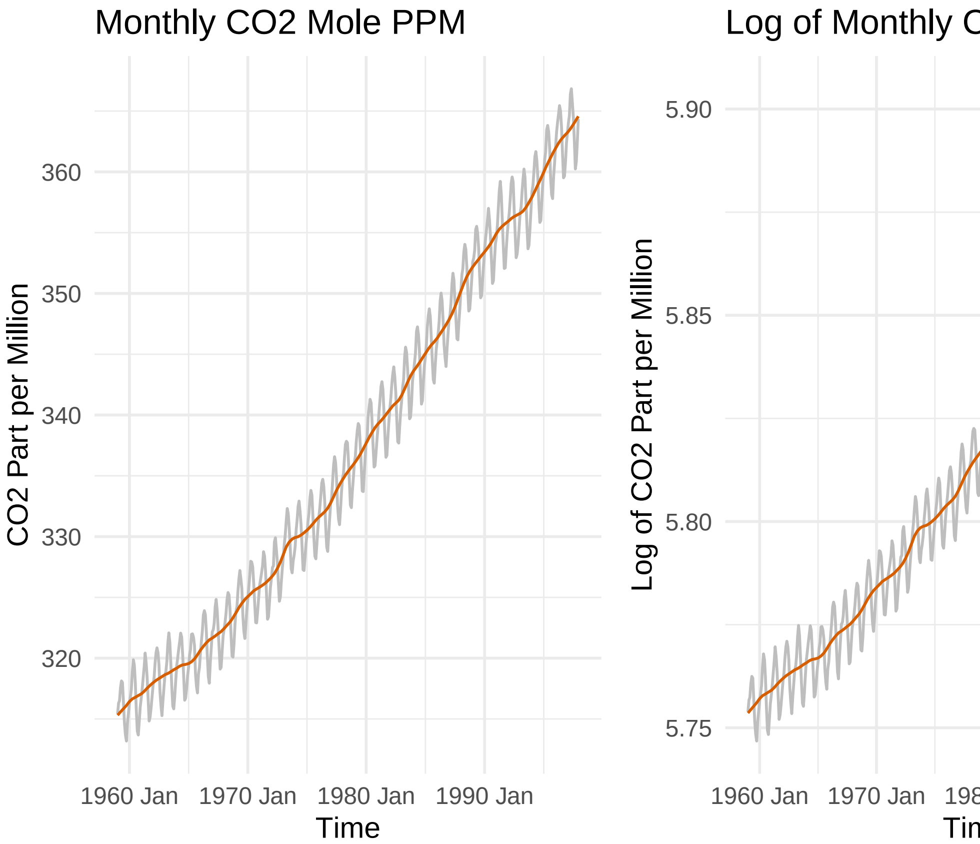

Monthly Mean CO2
The "Keeling Curve"
360
350
340
330
320
CO2 parts per million
1960 Jan
1970 Jan
1980 Jan
1990 Jan
Month and Year

(3 points) Task 1a: CO2 data

Background and Exploratory Data Analysis (EDA)

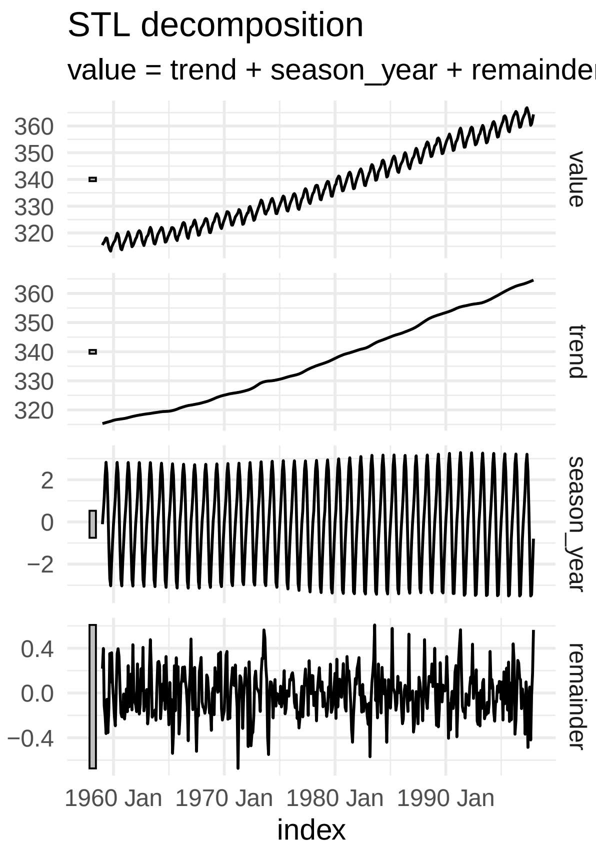
Background The data is the accumulation of carbon dioxide in the Earth's atmosphere based on continuous measurements taken at the Mauna Loa Observatory on the island of Hawaii from 1959 to the present day. Since local emmission of CO2 from local vegetation would create a local trend, the summit of Mauna Loa was chosen as a long-term monitoring site due to its remote location far from continents and its lack of vegetation. The altitude (3,400 meter) of the site is well situated to measure representative air masses for a large area. Furthermore Keeling and his collaborators measured the incoming ocean breeze above the thermal inversion layer to minimize local contamination from volcanic vents. The data iss normalized to remove any influence from local contamination.

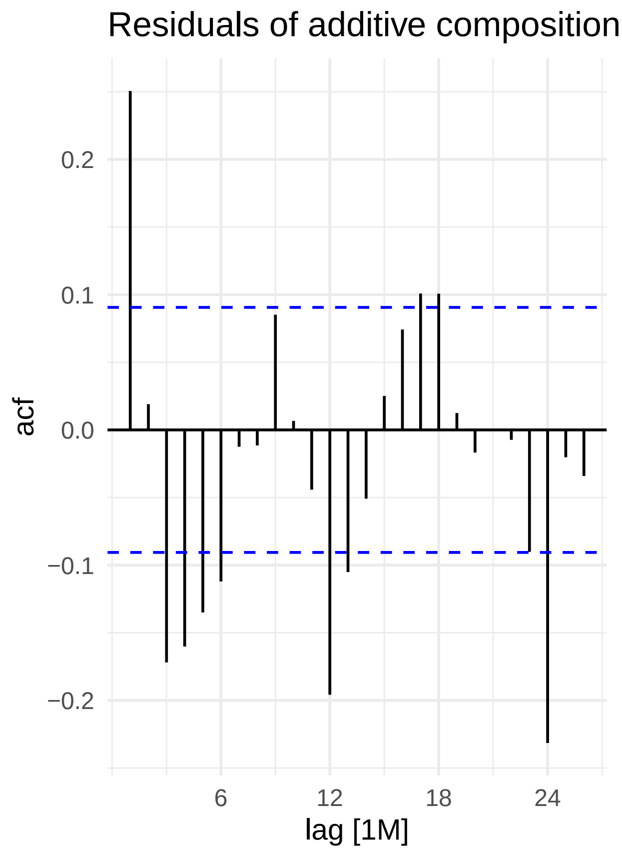
The CO2 data is collected by a CO2 analyzer that uses a technique called "Cavity Ring-Down Spectroscopy (CRDS). The measurements are made with an infrared spectrophotometer known as a nondispersive infrared sensor, that is calibrated using World Meteorological Organization standards. The site has use the same sensor type since 1959. In addition, the CO2 measurements are compared with other individual measurements to ensure the accuracy of the measure data.



Exploratory Data Analysis (EDA)

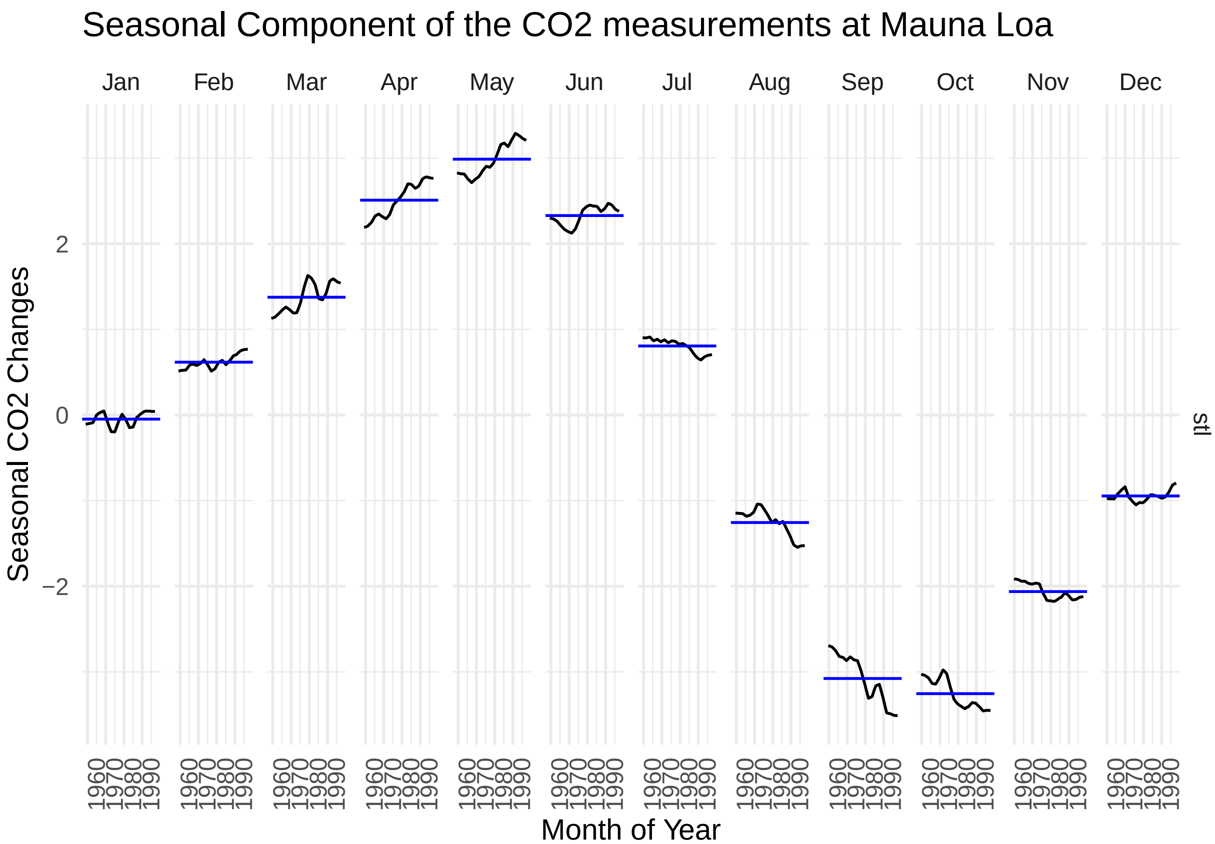
The team performed exploratory data analysis (EDA) to understand the timeseries data of CO2 mole fraction (part per million - ppm) at Mauna Loa. In the above, the left plot shows the trend between the CO2 measurements with time and on the right the logarithm of the CO2 measurements with time. The trend lines were generated using additive (left) and multiplicative (right) decomposition methods. The left plot suggests an approximate constant growth rate, and thus an additive decomposition method may be a sufficient for this problem. However, the data was fit with both methods and used the diagnostic statistics to determine which method is a more appropriate.



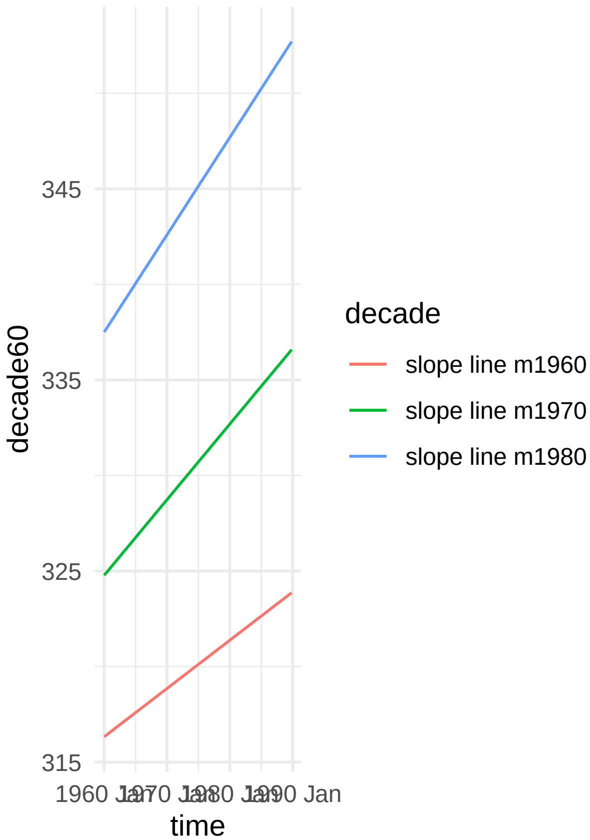


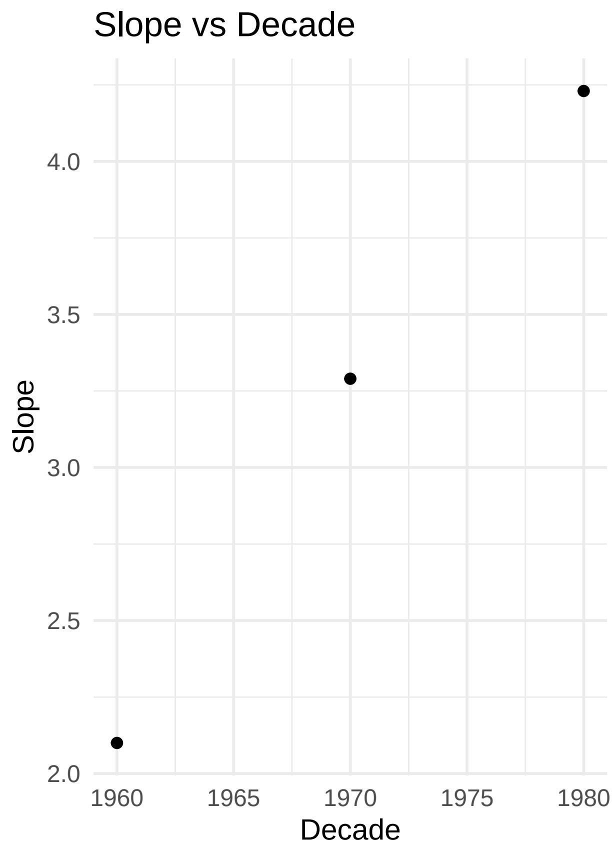
The second plot above shows the actual decomposition using the additive method, the autocorrelation (ACF) plot of the residuals. The linear trend portion suggests that from 1959 to 1997, the CO2 level increases about 40 ppms, the seasonality portion suggests that the CO2 mole fraction increases/ decreases by 2 ppm depending on season. The ACF plots show that although the residuals seem to be stationary, they do not follow a white noise signal (there are significant lags within the plot), suggesting a linear time and seasonal model may not be sufficient to fit the data. The plots of the multiplicative method show the the same results, therefore they are omitted.

As noted above the seasonal component appears to have a mode and antimode of -2 ppm to 2 ppm, with a slight increase in amplitude over time. This increase is an interesting finding that we will explore next.

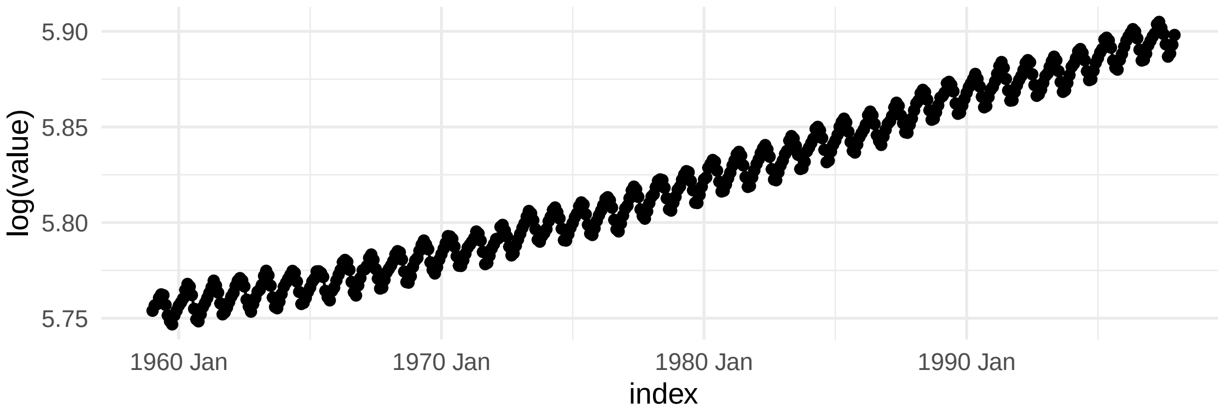


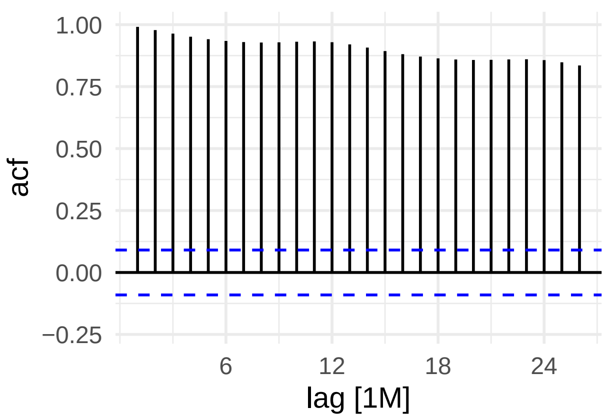
A seasonal subseries plot (above) facets the time series by each season in the seasonal period. This function is particularly useful in identifying changes in the seasonal pattern over time. From the plot above we can see that from 1960 to 1990 the mode and antimode have increased over the years, this is particulary visible in the month of May, August, September and October. We will explore if this increase in amplitude affects the overall growth trend.

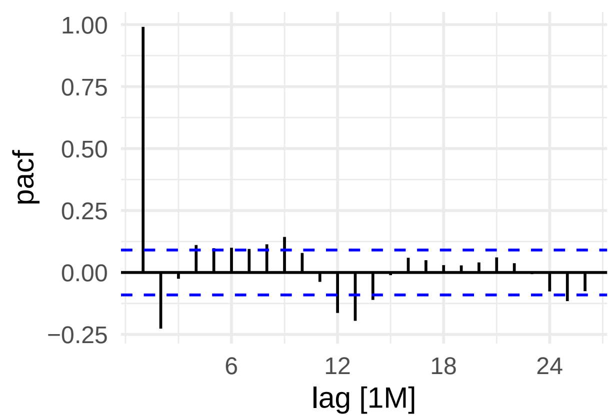




The plot on the left is each decade fitted to a linear model. We have complete decades from 1960 up to the decade of 1980. From the plot we can see that the slope is increasing. On the right is the scatter plot of the slope for each decade. The rate of growth increased from an average of 2.2 PPM to 4.4 PPM over three decades. We expect an exponential model to better fit the data. The CO2 trend not only includes a seasonal component with an increasing amplitude but also an increasing slope over time. It would be satisfying to find out if those two components are correlated. That is beyond the scope of this analysis.



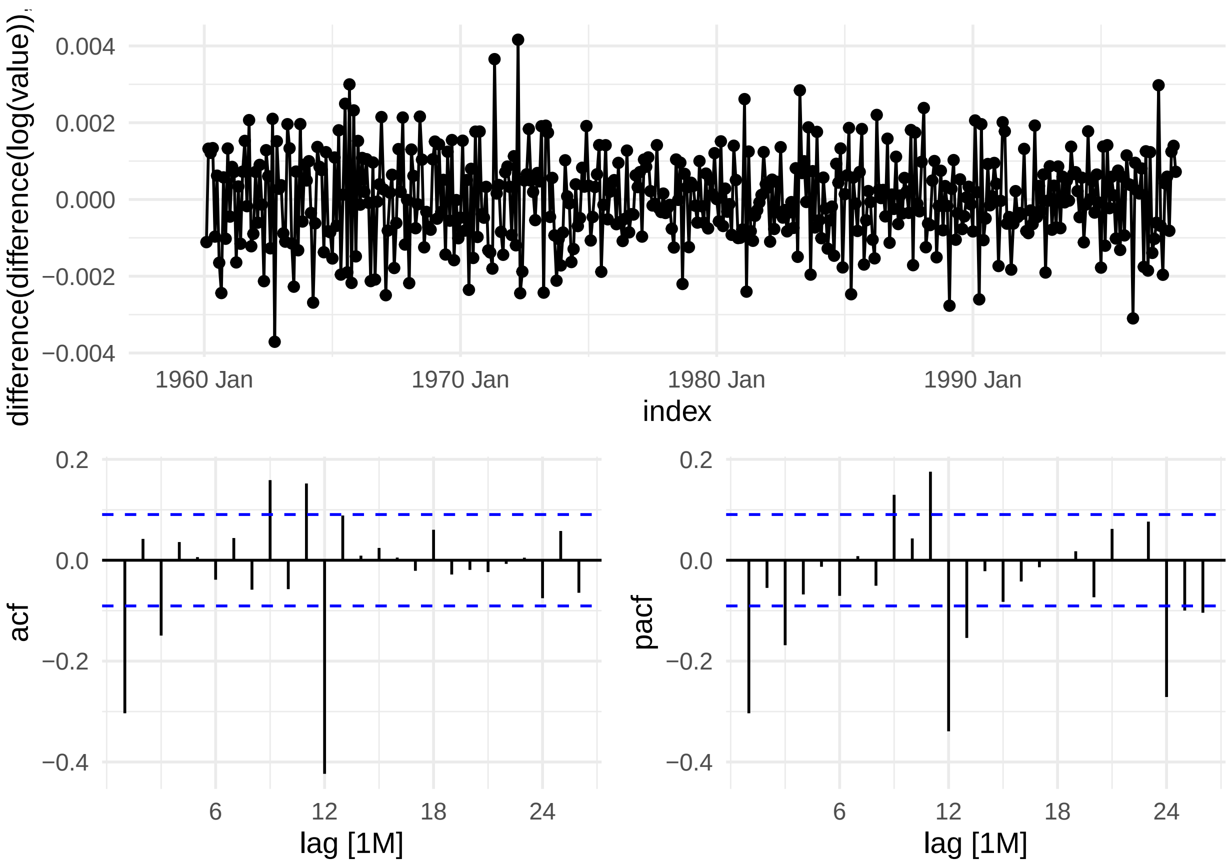




The first plot shows the ACF and PACF plots of the raw data after its log transformation. The ACF portion of this plot shows a strong trend. The PACF plot shows seasonality every 12th lag, and some cyclical components.

```
## Warning: Removed 13 rows containing missing values (`geom_line()`).
```

```
## Warning: Removed 13 rows containing missing values (`geom_point()`).
```



The team performed a series of transformation steps to remove the trend and seasonality component from the CO2 timeseries data. The team took a difference of the CO2 measurements in order to remove the trend component. The ACF and PACF portion of the first order differencing of the CO2 measurements showed that a seasonality component remained at every 12 lags and the ACF portion was very distinct from white noise. To remove the seasonality component, the team performed a further differencing at every 12 lags of the dataset. The plot above shows that the dataset after two differencing is closer to random white noise. The PACF portion shows that the autoregressive term at lag 1 and lag 12 is significant but the level of significance is lower compared to the moving average term at lag 1 and 12, this suggests that we can use a ma1 and seasonal ma1 model to fit the data. This information will be using when fitting the ARIMA model. The team performed the analysis on the additive and multiplicative model, but we are showing the results for the multiplicative model only, as the additive model results are exactly the same.

(3 points) Task 2a: Linear time trend model

Fitting the Data to an LTTM

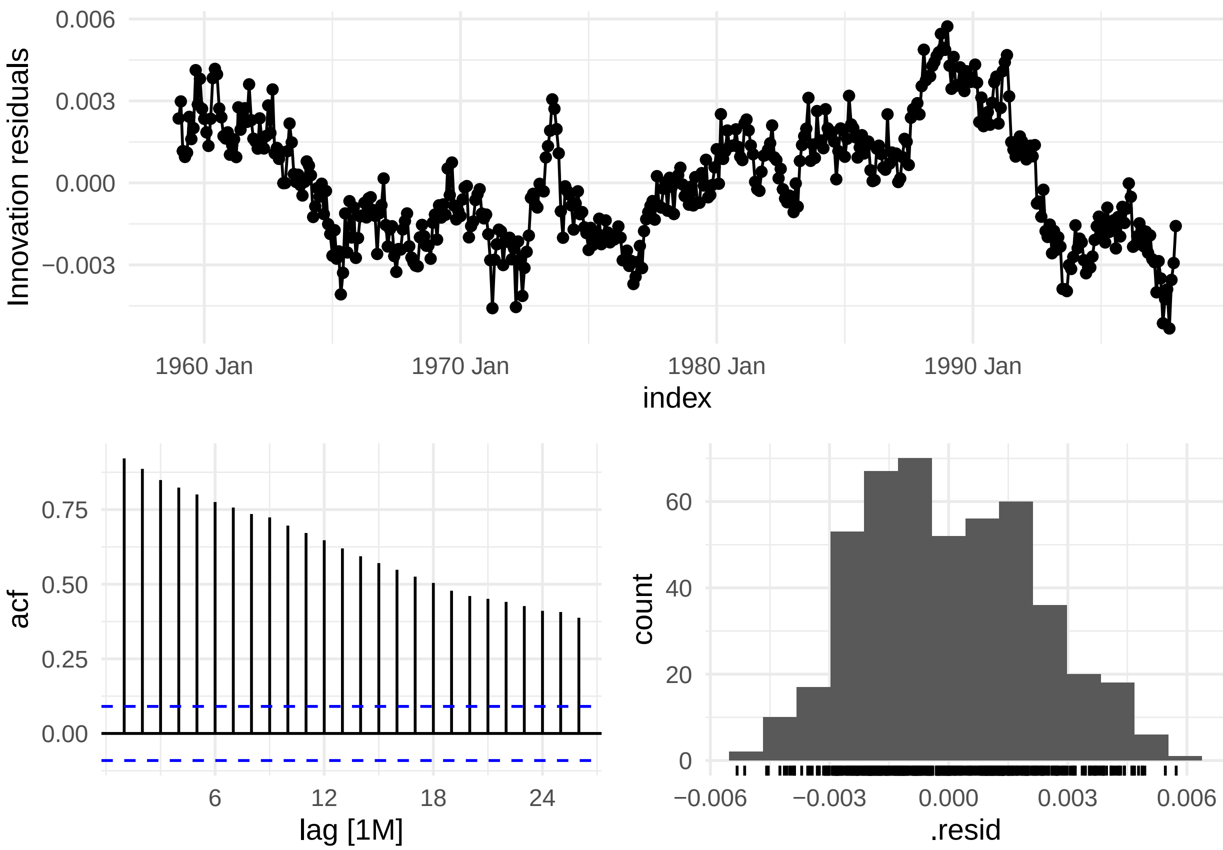
```
## # A tibble: 4 x 15
##   .model    r_squared adj_r_squared sigma2 statistic p_value    df log_lik   AIC
##   <chr>         <dbl>         <dbl>  <dbl>     <dbl>   <dbl> <int>   <dbl> <dbl>
## 1 add_lin       0.969         0.969  6.85     14795.       0     2  -1113.  905.
## 2 add_quad      0.979         0.979  4.76     10750.       0     3  -1028.  735.
## 3 add_lin_~     0.988         0.988  2.68      3218.       0    13   -888.  476.
## 4 add_quad~     0.998         0.998  0.524    15315.       0    14   -506. -286.
## # i 6 more variables: AICc <dbl>, BIC <dbl>, CV <dbl>, deviance <dbl>,
## #   df.residual <int>, rank <int>
```

```
## # A tibble: 4 x 15
##   .model  r_squared adj_r_squared  sigma2 statistic p_value    df log_lik    AIC
```

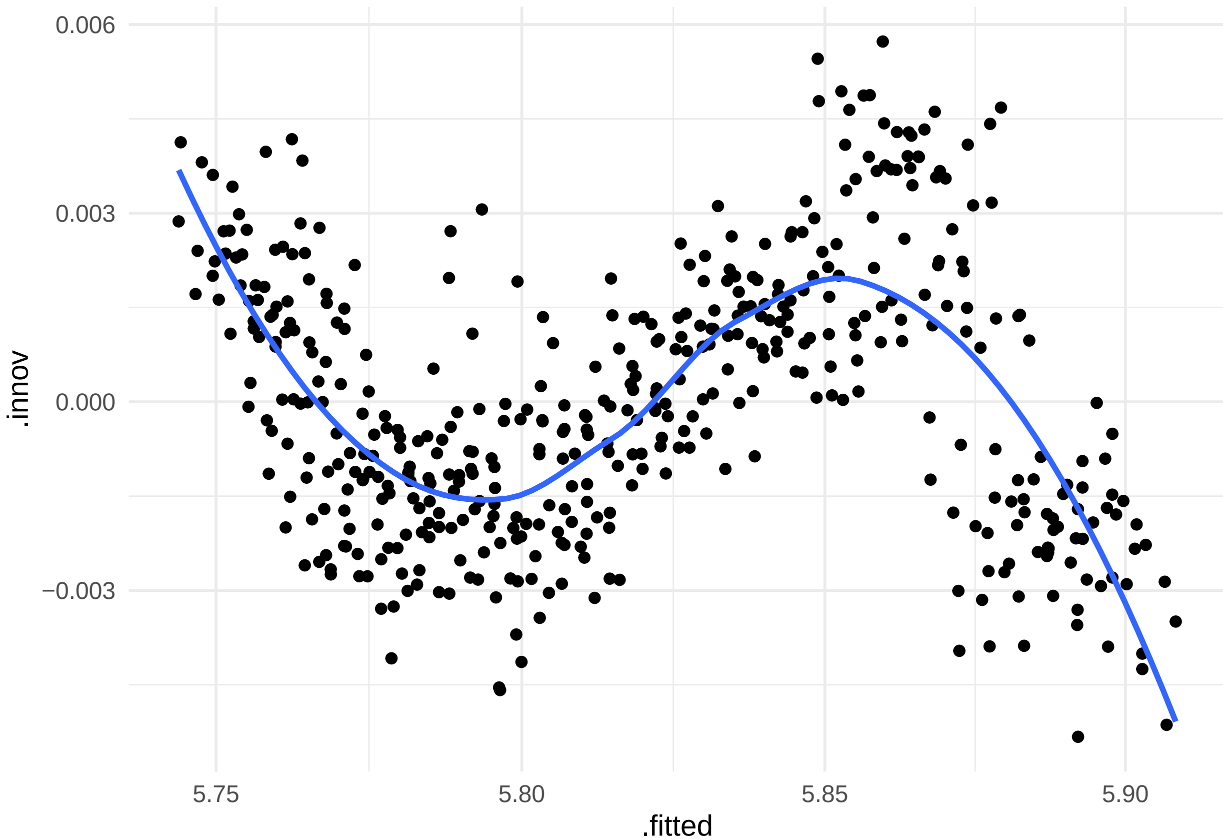
```
##   <chr>       <dbl>         <dbl>   <dbl>     <dbl>   <dbl> <int>   <dbl>  <dbl>
## 1 mul_lin     0.972         0.972 5.44e-5    16325.       0     2   1635. -4591.
## 2 mul_qu~     0.979         0.978 4.21e-5    10609.       0     3   1695. -4710.
## 3 mul_li~     0.991         0.991 1.75e-5     4316.       0    13   1906. -5112.
## 4 mul_qu~     0.998         0.998 4.82e-6    14529.       0    14   2208. -5713.
## # i 6 more variables: AICc <dbl>, BIC <dbl>, CV <dbl>, deviance <dbl>,
## #   df.residual <int>, rank <int>
```

```
## Series: log_value
## Model: TSLM
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -0.0053270 -0.0017362 -0.0001774  0.0015139  0.0057292
##
## Coefficients:
##                  Estimate Std. Error   t value Pr(>|t|)
## (Intercept)     5.751e+00  4.533e-04 12687.967  < 2e-16 ***
## trend()         2.223e-04  3.012e-06    73.817  < 2e-16 ***
## I(trend()^2)    2.150e-07  6.219e-09    34.566  < 2e-16 ***
## season()year2   1.969e-03  4.974e-04     3.959 8.73e-05 ***
## season()year3   4.163e-03  4.974e-04     8.371 7.16e-16 ***
## season()year4   7.498e-03  4.974e-04    15.075  < 2e-16 ***
## season()year5   8.911e-03  4.974e-04    17.916  < 2e-16 ***
## season()year6   6.965e-03  4.974e-04    14.004  < 2e-16 ***
## season()year7   2.480e-03  4.974e-04     4.986 8.78e-07 ***
## season()year8  -3.662e-03  4.974e-04    -7.362 8.61e-13 ***
## season()year9  -9.098e-03  4.974e-04   -18.290  < 2e-16 ***
## season()year10 -9.661e-03  4.974e-04   -19.423  < 2e-16 ***
## season()year11 -6.113e-03  4.974e-04   -12.290  < 2e-16 ***
## season()year12 -2.799e-03  4.974e-04    -5.627 3.21e-08 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.002196 on 454 degrees of freedom
## Multiple R-squared: 0.9976,  Adjusted R-squared: 0.9975
## F-statistic: 1.453e+04 on 13 and 454 DF, p-value: < 2.22e-16
```

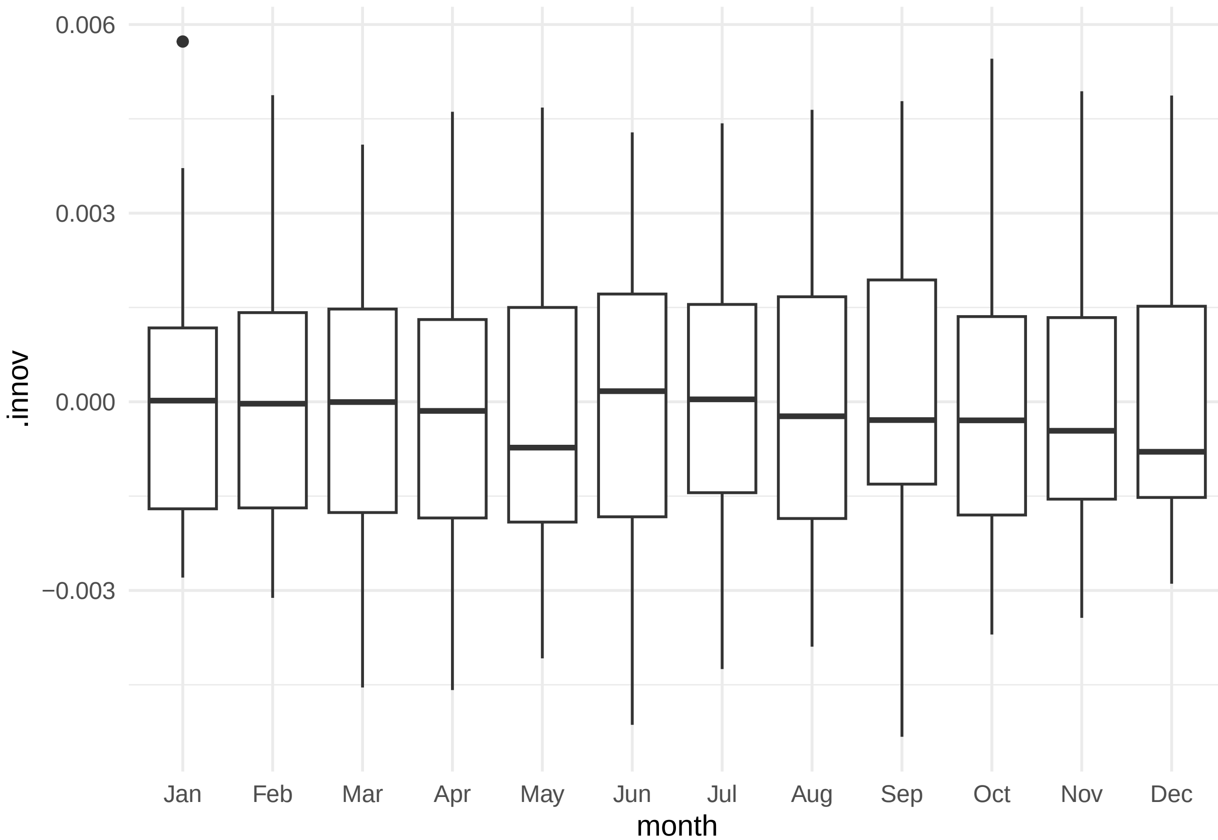
The team fit the data using four modeling combinations: linear trend only, linear with quadratic trend, linear trend with seasonal, and linear trend with quadratic trend and seasonal. For each modeling combination, the team used both decomposition methods to fit the data. The team selected the model with the lowest corrected Akaike information criterion (AIC). Based on the information presented in the above tables, and as expected, the model using multiplicative decomposition methodology and including the linear trend, quadratic trend and seasonal terms is the best model out of the eight models. The summary table shows that all coefficients were statistically different from 0. The adjusted R square of the model is 0.9976, which indicates that more than 99% of the response variance is explained in this model.




```
## `geom_smooth()` using method = 'loess' and formula = 'y ~ x'
```

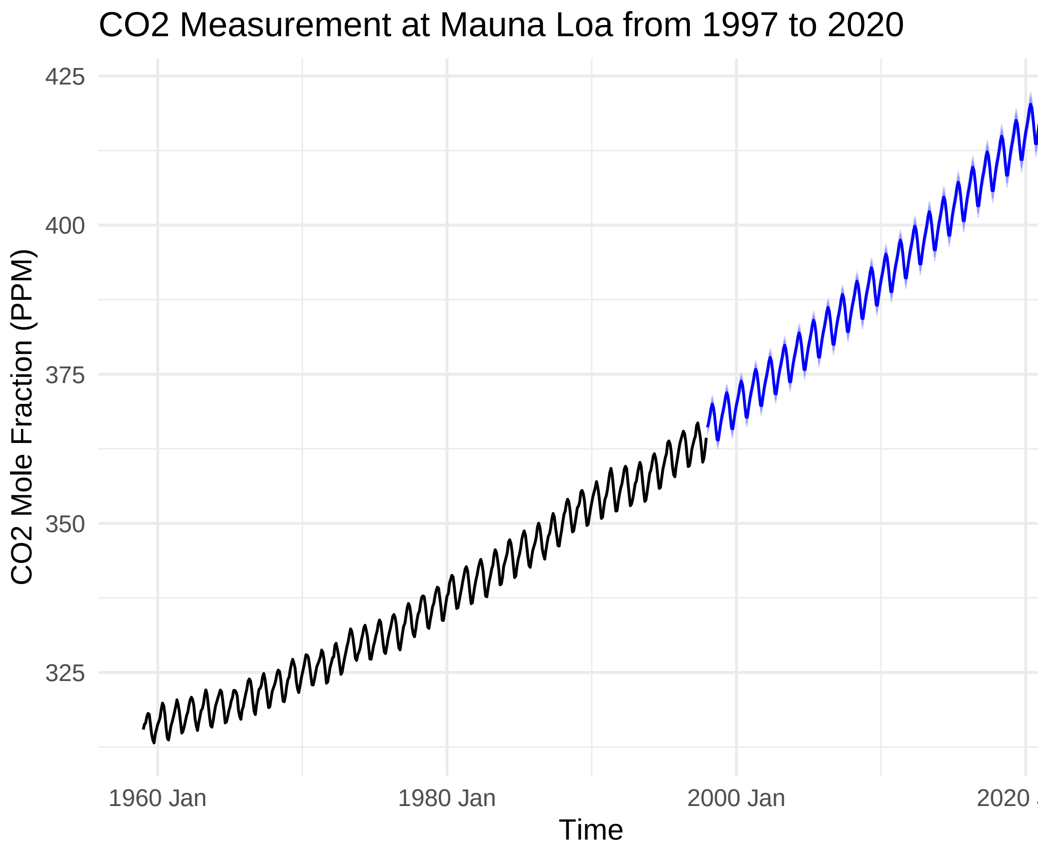


The team also conducted a residual analysis on the selected LTTM. The residual ACF plot shows that the residuals do not follow a white noise signal and is not a stationary timeseries. The residual against fitted value plot also shows that there is a relationship between the model residuals and the fitted value, which indicates that there are confounding variables.




```
## # A tibble: 1 x 3
##   .model      lb_stat lb_pvalue
##   <chr>         <dbl>     <dbl>
## 1 trend_model  10741.         0
```

Moreover, the ljung-box test (p-value less than 0.05) also shows that the residuals of the model exhibit a serial correlation. The diagnostics indicate that structure remains in the residual data, however a model with such residuals can be tolerated if the model provides useful forecasts. The team decided not to add more variables because of concerns with overfitting and as a consequence poor forecasting.



Forecasting with a LTT model

```
## # A tsibble: 6 x 5 [1M]
## # Key:       .model [1]
##   .model         index      log_value .mean                  `90%`
##   <chr>          <mth>         <dist> <dbl>                 <hilo>
## 1 trend_model 2020 Jul lN(6, 7.3e-06)  418. [416.1660, 419.8688]90
## 2 trend_model 2020 Aug lN(6, 7.3e-06)  416. [413.8391, 417.5258]90
## 3 trend_model 2020 Sep lN(6, 7.3e-06)  414. [411.8158, 415.4893]90
## 4 trend_model 2020 Oct lN(6, 7.3e-06)  414. [411.8042, 415.4823]90
## 5 trend_model 2020 Nov lN(6, 7.3e-06)  415. [413.4894, 417.1873]90
## 6 trend_model 2020 Dec lN(6, 7.3e-06)  417. [415.0845, 418.8014]90
```

The team used the selected model to generate CO2 mole fraction from December 1997 to December 2020 (as shown in the above graph). The generated forecast indicates a similar pattern of growth for the CO2 level in the atmosphere. By 2020, the CO2 mole fraction will reach approximately 417 ppm with a 90% confidence interval (CI) between 415 and 419 ppm.

(3 points) Task 3a: ARIMA times series model

Fitting the Data to an ARIMA model We will now choose an ARIMA model to fit to the time series.

```
## # A tibble: 1 x 1
##   ndiffs
##    <int>
## 1      1
```

```
## # A tibble: 1 x 1
##   nsdiffs
##     <int>
## 1       1
```

```
## # A tibble: 1 x 2
##   kpss_stat kpss_pvalue
##       <dbl>       <dbl>
## 1    0.0115         0.1
```

Following the initial analysis done in the EDA section, the team used a unit root test to determine the number of differences (both seasonal and non-seasonal) to make the CO2 timeseries stationary. The unit root test results show that it requires one seasonal and one non-seasonal difference to make the timeseries stationary. These results are in agreement with our findings in the explanatory data analysis section.

The KPSS test of the CO2 timeseries after applying the differences (at lag 1 and lag 12) shows a p-value of 0.1, which is greater than 0.05, therefore we fail to reject the null hypothesis that the time series is stationary after two differencing.

The ACF portion in the EDA showed that we can expect to use multiple ma1 non-seasonal terms and a seasonal ma1 model to fit the data, the EDA did not indicate a need for an AR term.

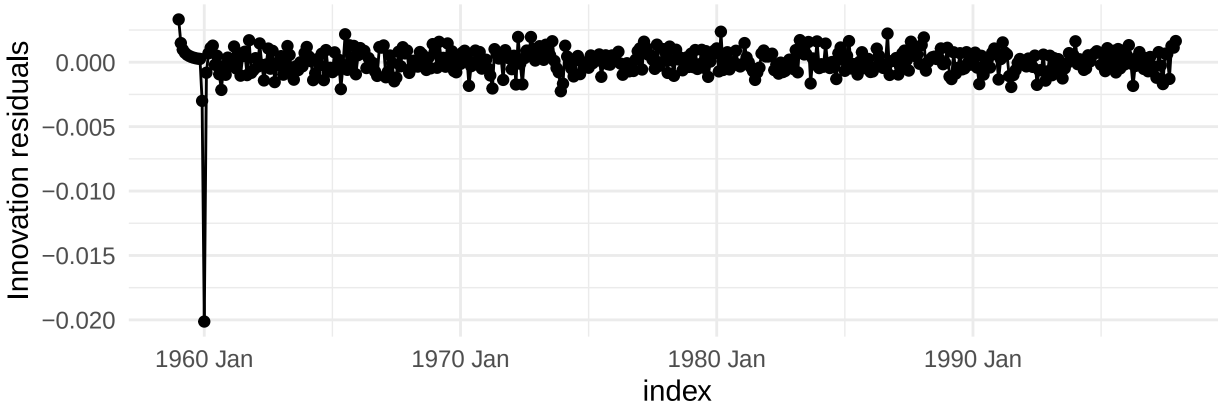
```
ari.fit <- df %>%
  model(ARIMA111111 = ARIMA(log(value) ~ pdq(1,1,1) + PDQ(1,1,1)),
        ARIMA111211 = ARIMA(log(value)  ~ pdq(1,1,1) + PDQ(2,1,1)),
        ARIMA211211 = ARIMA(log(value)  ~ pdq(2,1,1) + PDQ(2,1,1)),
        ARIMA112112 = ARIMA(log(value)  ~ pdq(1,1,2) + PDQ(1,1,2)),
        ARIMA112111 = ARIMA(log(value)  ~ pdq(1,1,2) + PDQ(1,1,1)),
        ARIMA111112 = ARIMA(log(value)  ~ pdq(1,1,1) + PDQ(1,1,2)),
        ARIMA013011 = ARIMA(log(value)  ~ pdq(0,1,3) + PDQ(0,1,1)),
        auto = ARIMA(log(value), stepwise = FALSE, approx = FALSE)
        )

ari.fit %>% pivot_longer(everything(), names_to = "Model name", values_to = "Orders")
```

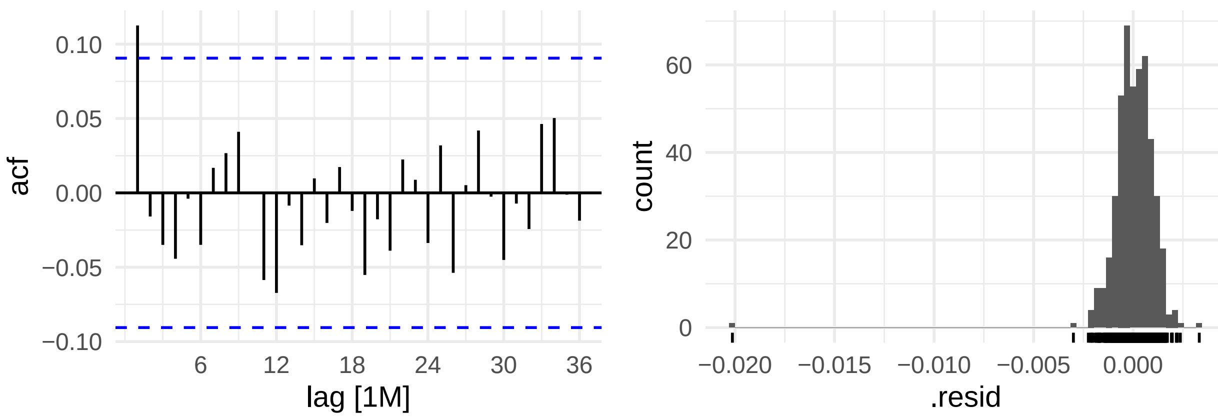
```
## # A mable: 8 x 2
## # Key:     Model name [8]
##   `Model name`                    Orders
##   <chr>                          <model>
## 1 ARIMA111111  <ARIMA(1,1,1)(1,1,1)[12]>
## 2 ARIMA111211  <ARIMA(1,1,1)(2,1,1)[12]>
## 3 ARIMA211211  <ARIMA(2,1,1)(2,1,1)[12]>
## 4 ARIMA112112  <ARIMA(1,1,2)(1,1,2)[12]>
## 5 ARIMA112111  <ARIMA(1,1,2)(1,1,1)[12]>
## 6 ARIMA111112  <ARIMA(1,1,1)(1,1,2)[12]>
## 7 ARIMA013011  <ARIMA(0,1,3)(0,1,1)[12]>
## 8 auto         <ARIMA(1,1,0)(2,1,2)[12]>
```

```
glance(ari.fit) |> arrange(AIC) |> select(.model:AIC)
```

```
## # A tibble: 8 x 4
##   .model          sigma2 log_lik    AIC
##   <chr>            <dbl>   <dbl>  <dbl>
## 1 ARIMA013011 0.00000165   2572. -5133.
## 2 ARIMA111211 0.00000165   2571. -5131.
## 3 ARIMA211211 0.00000165   2572. -5131.
## 4 ARIMA111111 0.00000165   2570. -5130.
## 5 ARIMA112111 0.00000165   2571. -5130.
## 6 ARIMA111112 0.00000165   2571. -5130.
## 7 ARIMA112112 0.00000166   2571. -5129.
## 8 auto        0.00000166   2569. -5125.
```

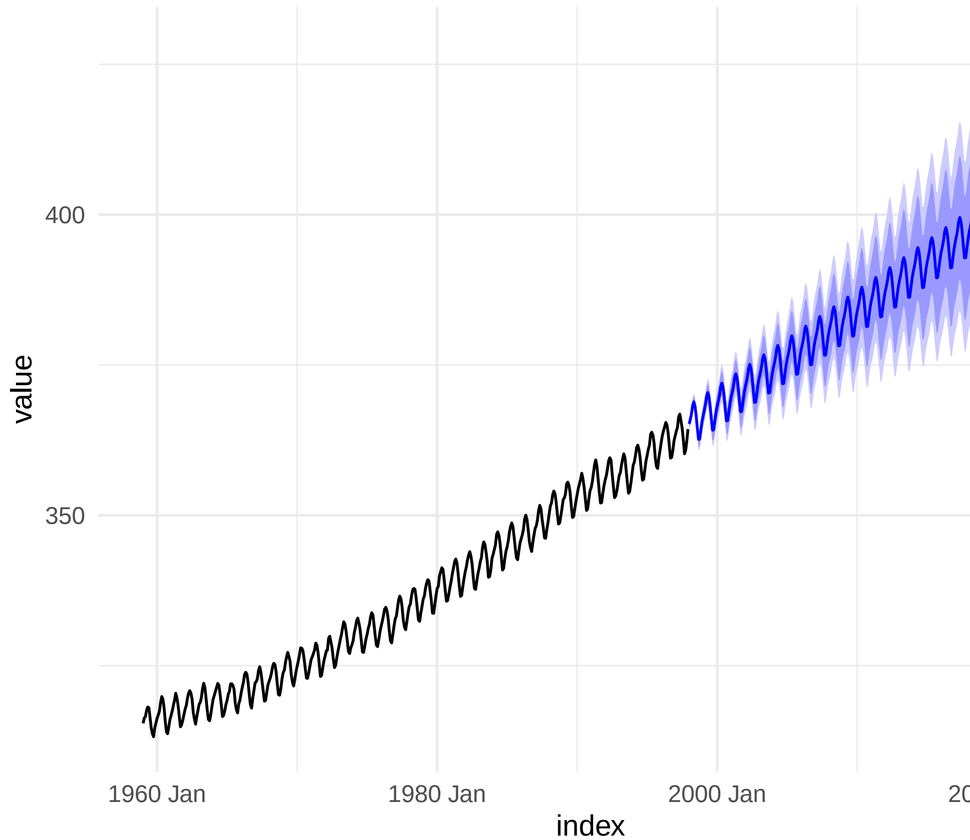
Based on the results of testing out different model combination, the SARIMA model with one seasonal and one non-seasonal differencing, three non-seasonal moving average terms and one seasonal moving average term seems to be the model with the best performance (lowest AIC) and its consistent with our EDA analysis.






```
## # A tibble: 8 x 3
##   .model       lb_stat lb_pvalue
##   <chr>          <dbl>     <dbl>
## 1 ARIMA013011     122.      1.00
## 2 ARIMA111111     124.      1.00
## 3 ARIMA111112     124.      1.00
## 4 ARIMA111211     122.      1.00
## 5 ARIMA112111     125.      1.00
## 6 ARIMA112112     124.      1.00
## 7 ARIMA211211     123.      1.00
## 8 auto            122.      1.00
```

The residual analysis of the selected model shows that the residuals timeseries is white noise signal. The ljung-box test result shows that all selected models have p-value greater than 0.05, which suggests that the residuals of the selected model is stationary. The ARIMA model accounts for the structure of the data better than our LTTM model. ARIMA models are more flexible then LTTM as they can capture more complex patterns in the data.

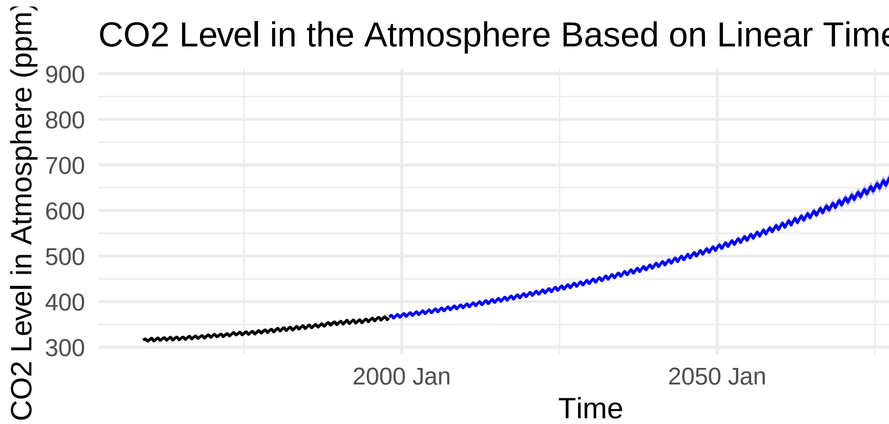


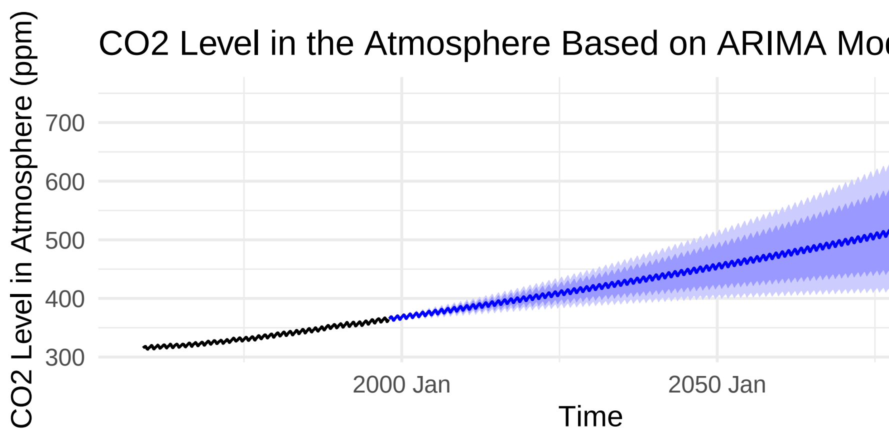
Forecasting with an ARIMA model

```
## # A tsibble: 6 x 5 [1M]
## # Key:       .model [1]
##   .model         index             value .mean                  `90%`
##   <chr>          <mth>            <dist> <dbl>                 <hilo>
## 1 ARIMA013011 2022 Jul t(N(6, 0.00066))  406. [388.7085, 422.9186]90
## 2 ARIMA013011 2022 Aug t(N(6, 0.00066))  403. [386.4154, 420.5499]90
## 3 ARIMA013011 2022 Sep t(N(6, 0.00067))  401. [384.3505, 418.4278]90
## 4 ARIMA013011 2022 Oct t(N(6, 0.00067))  401. [384.3485, 418.5505]90
## 5 ARIMA013011 2022 Nov t(N(6, 0.00068))  403. [385.8291, 420.2877]90
## 6 ARIMA013011 2022 Dec t(N(6, 0.00068))  404. [387.2602, 421.9715]90
```

The team used the selected model to generate CO2 level in the atmosphere to December 2022. Based on the generated forecast, in December 2022, the forecasted CO2 level will be approximately 402 ppm with a 90% CI between 389 and 416 ppm. This result is lower than the forecast using the LTTM model which already forecasts 417 ppm by 2020, two years before.

(3 points) Task 4a: Forecast atmospheric CO2 growth





Comparing LTTM and ARIMA Forecasts

The team generated forecast from 1998 to 2100. Based of the generated forecast, for the linear model, the CO2 level at Mauna Loa is expected to reach 420 ppm the first time in 18383 and last time in 19297. However, based on the 90% CI band, we may observe the CO2 level to reach 420 ppm as early as in 18353 and may see it again the last time in 19631. Also, according to the linear model, the CO2 level at Mauna Loa is expected to reach 500 ppm the first time in 27453 and last time in 28032. But based on its 90% CI, we may observe the CO2 level to reach 500 ppm as early as 27088 and may see it again the last time in 28397.

For the ARIMA model, the CO2 level is expected to reach 420 ppm the first time in 21670 and last time in 23284. Based on its 90% CI, we may observe the CO2 level to reach 420 ppm as early as in 18383 and may see it again the last time in 36798. The CO2 level is expected to reach 500 ppm the first time in 36615 and last time in 37894. Based on its 90% CI, we may observe the CO2 level to reach 500 ppm as early as 27088 and may come back again to this level after the last time step of the forecast horizon, which is 47816.

In 2100, based on the linear model, the CO2 level at Mauno Loa is expected to reach 853.374051902969 ppm for the linear model and 567.154144578856 ppm for the ARIMA model. The ARIMA model suggests a slower growth of CO2 level and also has a wider 90% CI, than the predictions from the linear model.

Athough it is unlikely both models would provide accurate CO2 levels in 2100 (given they do not account for changing external trends), one model can be closer in its prediction than the other model. ARIMA models are stochastic models and by their essence are generally inadequate for long-term forecasting, such as more than a few months ahead. This fact is reflected by its growing band of confidence interval with time. The LTTM is a deterministic model that depends heavily on an underlying process and if the process generating the data changes, the model will continue to forecast based on the original process. If the underlying reason for CO2 emissions is deterministic, then our team does not reasonably expect the trend to change drastically and thus we would have more confidence in the LTTM prediction. We can only determine which model is better, and also, if the CO2 emissions trend is a stochastic or deterministic, by measuring their forecasting. The statical diagnostics in our analysis lean towards favoring the ARIMA model and hence a stochastic trend.

Report from the Point of View of the Present

One of the very interesting features of Keeling and colleagues' research is that they were able to evaluate, and re-evaluate the data as new series of measurements were released. This permitted the evaluation of previous models' performance and a much more difficult question: If their models' predictions were "off" was this the result of a failure of the model, or a change in the system?

(1 point) Task 0b: Introduction

In this introduction, you can assume that your reader will have **just** read your 1997 report. In this introduction, **very** briefly pose the question that you are evaluating, and describe what (if anything) has changed in the data generating process between 1997 and the present.

The 1997 CO2 forecast provided CO2 level forecast to 2100. Reasonably, the further the forecast horizon is, the more uncertain the forecast becomes. In the following sections, the team decided to evaluate and update our forecast conducted in 1997 with more recent data. In the process of this work, we also attempted to identify any systematic changes in the way CO2 level grows at Mauna Loa.

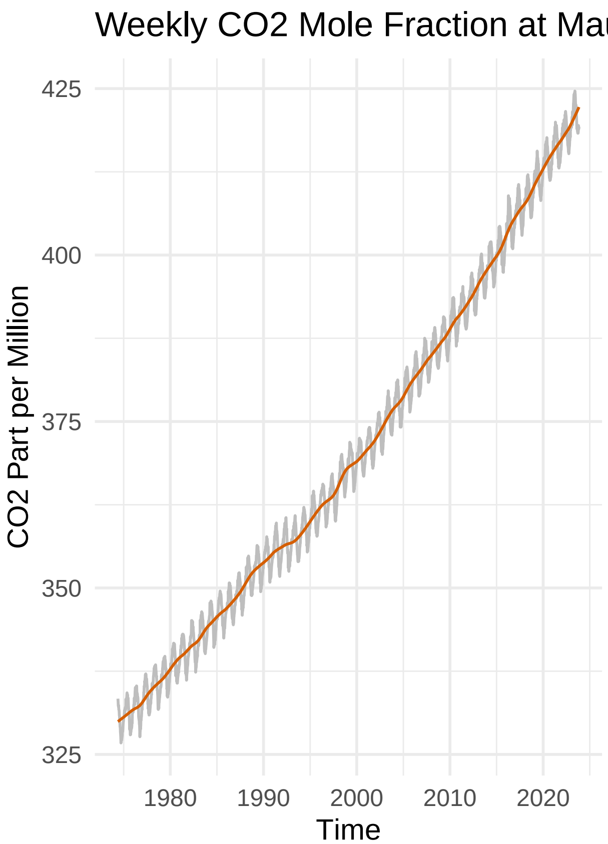
(3 points) Task 1b: Create a modern data pipeline for Mona Loa CO2 data.

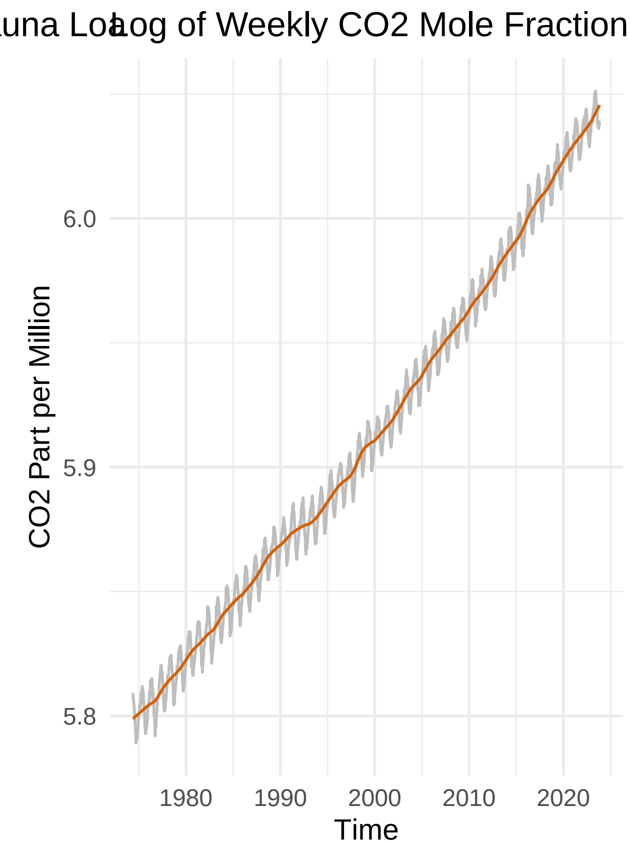
The most current data is provided by the United States' National Oceanic and Atmospheric Administration, on a data page [here]. Gather the most recent weekly data from this page. (A group that is interested in even more data management might choose to work with the hourly data.)

Create a data pipeline that starts by reading from the appropriate URL, and ends by saving an object called `co2_present` that is a suitable time series object.

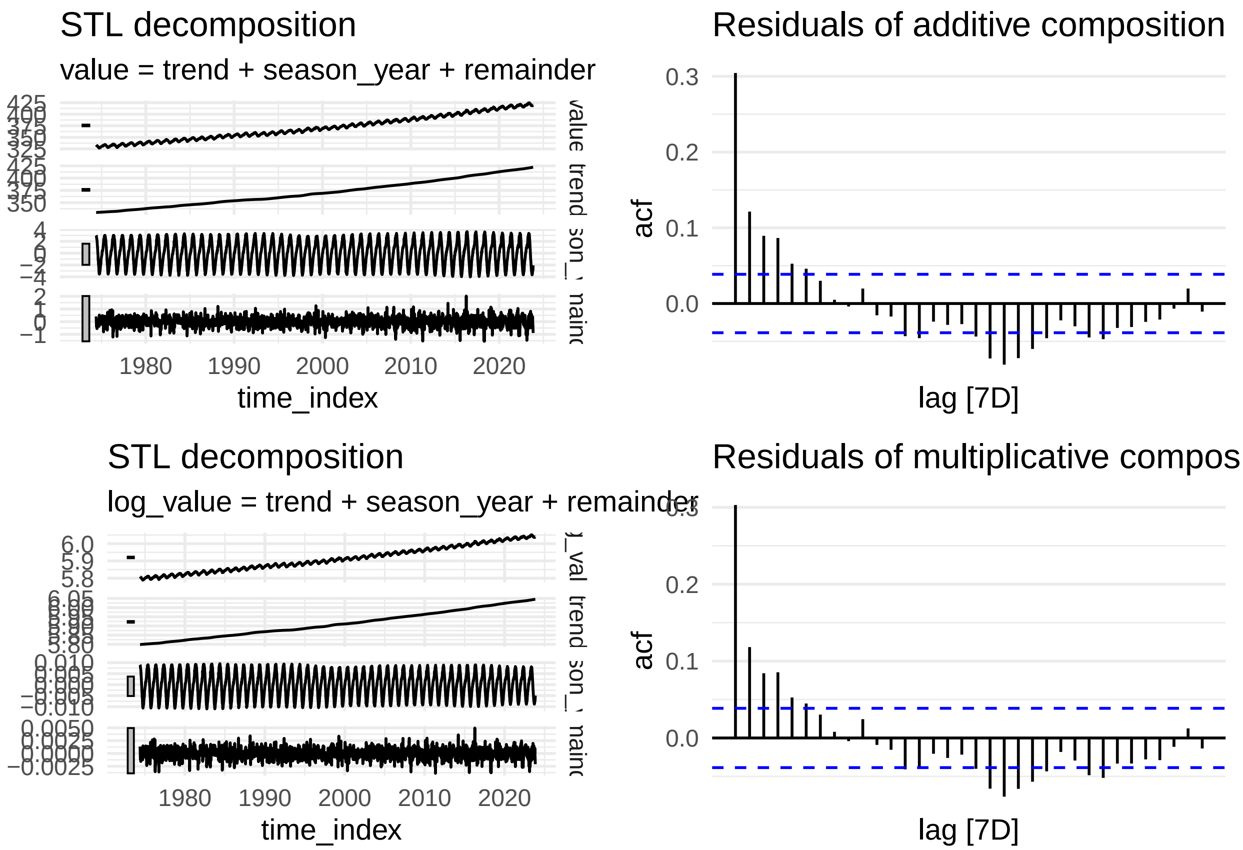
Conduct the same EDA on this data. Describe how the Keeling Curve evolved from 1997 to the present, noting where the series seems to be following similar trends to the series that you "evaluated in 1997" and where the series seems to be following different trends. This EDA can use the same, or very similar tools and views as you provided in your 1997 report.

```
## Rows: 2582 Columns: 9
## -- Column specification --------------------------------------------------------
## Delimiter: ","
## dbl (9): year, month, day, decimal, average, ndays, 1 year ago, 10 years ago...
##
## i Use `spec()` to retrieve the full column specification for this data.
## i Specify the column types or set `show_col_types = FALSE` to quiet this message.
```

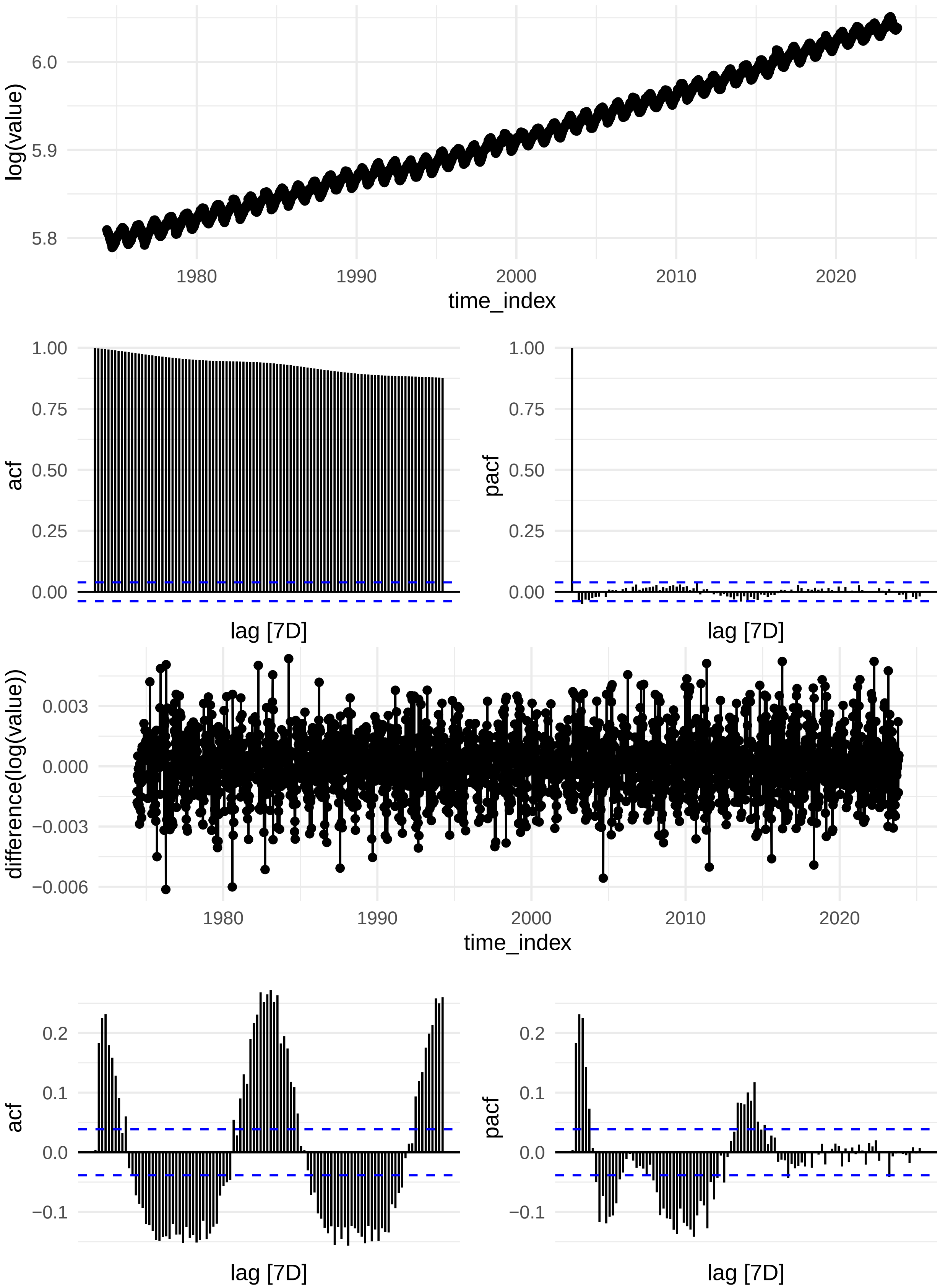


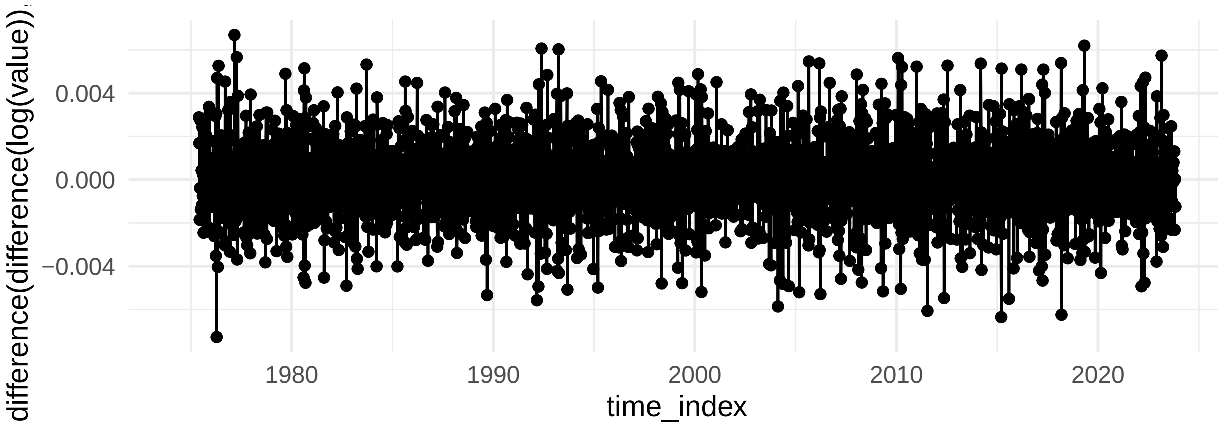


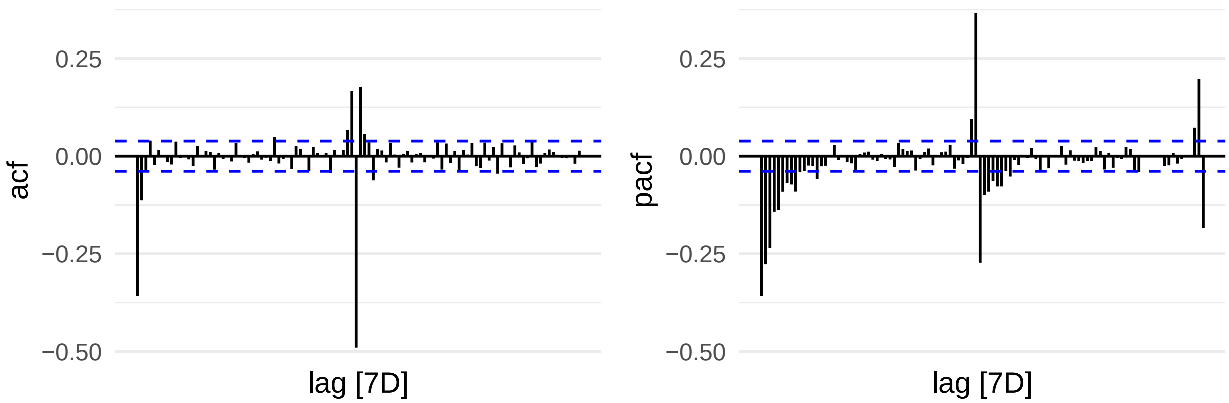
The team conducted the evaluation using a similar approach that we did for the 1997 CO2 dataset. We performed explanatory data analysis on the updated CO2 dataset. Different from the 1997 study, we worked with the weekly CO2 data in the current study. We processed to remove faulty values within the dataset (negative values or suspicious outliers). The plot above presents the weekly CO2 level at Mauna Loa from May 1974 to October 2023. The graph on the left presents the CO2 level and the graph on the right presents the logarithmic version of the CO2 level. The plot shows a more obvious non-linear trend than the 1997 study, which indicates that the multiplicative decomposition method is a more appropriate method.



The above plot shows the actual decomposition using the additive and multiplicative method along with the ACF plots of the residuals (after the trend and the seasonal component were removed from the series). The ACF plots show that, at the weekly granularity, the residuals timeseries does not seem to be stationary, suggesting that further differencing transformation may be needed.


log(value)
time_index
acf
pacf
lag [7D]
difference(log(value))
time_index
acf
pacf
lag [7D]

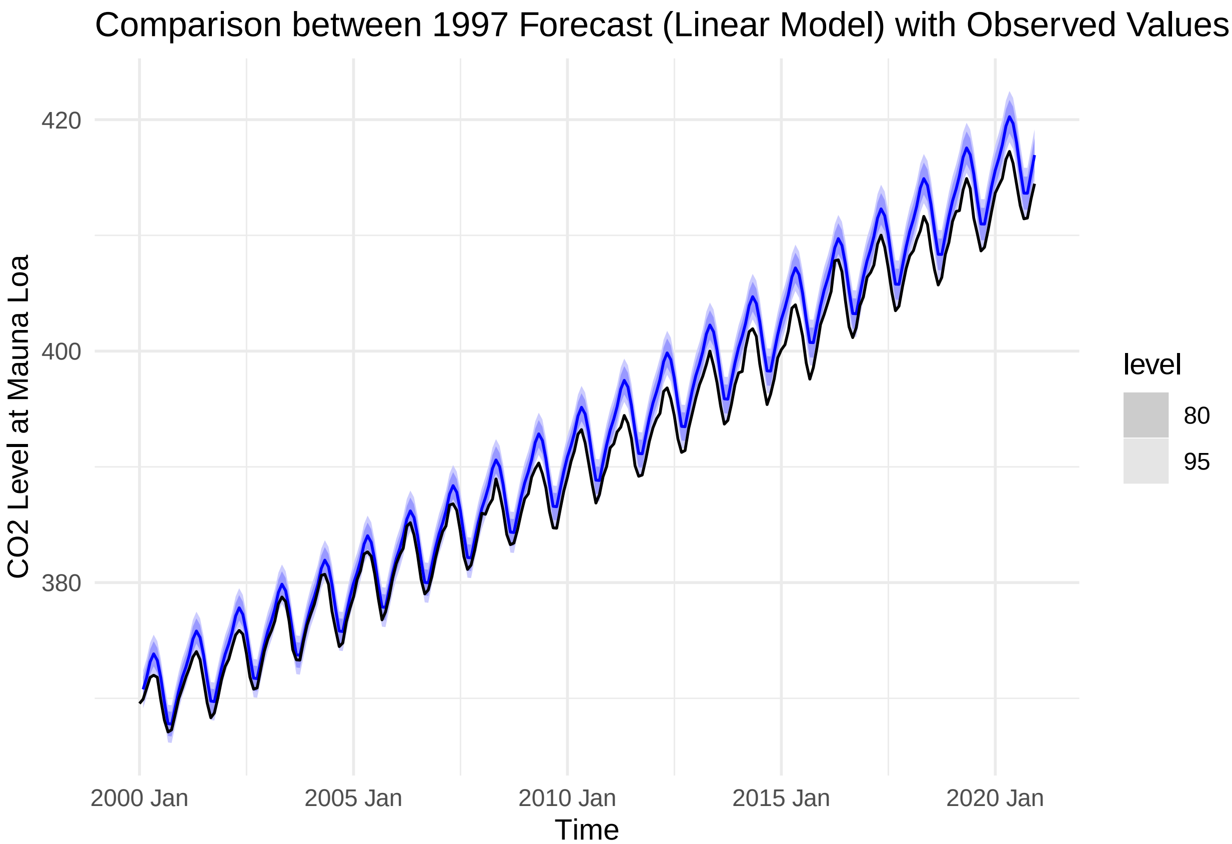




The team performed a series of differencing transformation on the updated dataset. Similarly to the analysis included in the 1997 study, we generated a CO2 timeseries with the differencing at the subsequent lag and every 52 lags to remove the trend and the seasonality component from the data. The above plots show the timeseries with differencing transformation and their associated ACF and PACF plots. The third plot, which shows the timeseries after the trend and seasonal differencing, indicates that there is no trend or seasonality within the series and suggests an ARIMA model with both seasonal and non-seasonal moving average and autoregressive terms may be a good candidate to model the updated dataset.

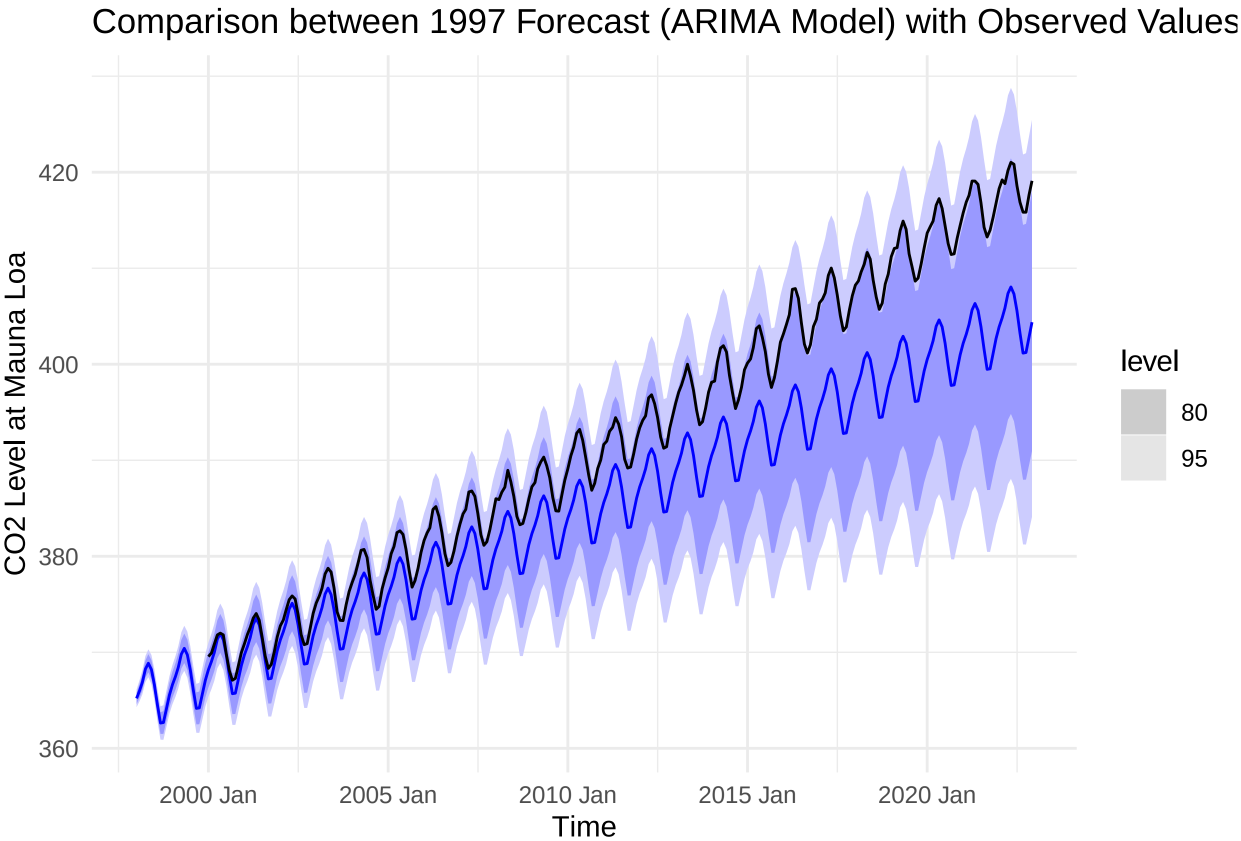
(1 point) Task 2b: Compare linear model forecasts against realized CO2

Descriptively compare realized atmospheric CO2 levels to those predicted by your forecast from a linear time model in 1997 (i.e. "Task 2a"). (You do not need to run any formal tests for this task.)



To compare the forecast from our 1997 linear model to the observed CO2 level, we aggregated the weekly actual CO2 data to monthly data so that the dataset has the same time index as the forecast values. Based on the comparison in the above plot (black line shows the observed CO2 level and blue line shows the forecast CO2 level), the 1997 linear model overforecast the CO2 level at Mauna Loa from 2000 to 2020.

(1 point) Task 3b: Compare ARIMA models forecasts against realized CO2



The plot above shows the comparison between observed CO2 level and the forecast from the 1997 ARIMA model. The observed CO2 levels are closer to the upper bound of the 95% CI of the forecast. The forecast from 1997 ARIMA model shows that the expected forecast is lower than the observed CO2 values at a more significant degree than that of the linear model. However, the observed CO2 seems to be closer to the upper bound of the 95% CI of the ARIMA model.

Descriptively compare realized atmospheric CO2 levels to those predicted by your forecast from the ARIMA model that you fitted in 1997 (i.e. "Task 3a"). Describe how the Keeling Curve evolved from 1997 to the present.

(3 points) Task 4b: Evaluate the performance of 1997 linear and ARIMA models

In 1997 you made predictions about the first time that CO2 would cross 420 ppm. How close were your models to the truth?

After reflecting on your performance on this threshold-prediction task, continue to use the weekly data to generate a month-average series from 1997 to the present, and compare the overall forecasting performance of your models from Parts 2a and 3b over the entire period. (You should conduct formal tests for this task.)

The observed CO2 values indicate that the CO2 level reached 420 ppm the first time in 19078. This is earlier than the prediction from the linear model and later than the prediction from the ARIMA model. The linear model (with a tight CI band) predicted that the CO2 level would reach 420 ppm as early as in 18353 and the ARIMA model predicted that the CO2 level would reach 420 ppm as early as in 18383. Based on their prediction, the ARIMA model provided a better expectation than the linear model.

```
## # A tibble: 6 x 10
##   .model       .type    ME  RMSE   MAE    MPE  MAPE  MASE RMSSE  ACF1
##   <chr>        <chr> <dbl> <dbl> <dbl>  <dbl> <dbl> <dbl> <dbl> <dbl>
```

```
## 1 ARIMA013011 Test   6.19  7.49  6.19  1.54  1.54   4.20  4.76 0.984
## 2 ARIMA211211 Test   6.03  7.30  6.03  1.50  1.50   4.09  4.64 0.984
## 3 ARIMA111211 Test   5.93  7.19  5.93  1.48  1.48   4.02  4.57 0.984
## 4 ARIMA111111 Test   6.17  7.48  6.17  1.54  1.54   4.18  4.76 0.984
## 5 ARIMA112111 Test   6.26  7.57  6.26  1.56  1.56   4.24  4.82 0.985
## 6 trend_model Test  -1.72  1.97  1.74 -0.437 0.443  1.18  1.25 0.890
```

The team calculated the performance metrics in order to evaluate the performance of the models that we developed in 1997. The performance metrics include but are not limited to root mean square error , mean absolute error and mean absolute percentage error. Based on the results of the metrics table, the linear model with trend (linear and quadratic terms) and seasonal terms was the best model with the lowest score in all metrics.

(4 points) Task 5b: Train best models on present data

Seasonally adjust the weekly NOAA data, and split both seasonally-adjusted (SA) and non-seasonally-adjusted (NSA) series into training and test sets, using the last two years of observations as the test sets. For both SA and NSA series, fit ARIMA models using all appropriate steps. Measure and discuss how your models perform in-sample and (psuedo-) out-of-sample, comparing candidate models and explaining your choice. In addition, fit a polynomial time-trend model to the seasonally-adjusted series and compare its performance to that of your ARIMA model.

```
## # A mable: 8 x 2
## # Key:     Model name [8]
##   `Model name`                               Orders
##   <chr>                                     <model>
## 1 ARIMA111101  <ARIMA(1,1,1)(1,0,1)[12] w/ drift>
## 2 ARIMA111201  <ARIMA(1,1,1)(2,0,1)[12] w/ drift>
## 3 ARIMA211201  <ARIMA(2,1,1)(2,0,1)[12] w/ drift>
## 4 ARIMA112102                          <NULL model>
## 5 ARIMA112101                          <NULL model>
## 6 ARIMA111102  <ARIMA(1,1,1)(1,0,2)[12] w/ drift>
## 7 ARIMA013001  <ARIMA(0,1,3)(0,0,1)[12] w/ drift>
## 8 auto                  <ARIMA(0,2,2)(2,0,0)[12]>
```

```
## # A tibble: 6 x 5
##   .model           sigma2 log_lik    AIC   AICc
##   <chr>             <dbl>   <dbl>  <dbl>  <dbl>
## 1 ARIMA111201 0.000000525   3347. -6679. -6679.
## 2 ARIMA211201 0.000000526   3347. -6678. -6678.
## 3 ARIMA111102 0.000000528   3345. -6676. -6676.
## 4 ARIMA111101 0.000000532   3342. -6672. -6672.
## 5 auto        0.000000612   3323. -6636. -6636.
## 6 ARIMA013001 0.000000569   3322. -6632. -6632.
```

```
## Series: season_adjust
## Model: TSLM
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -0.0047119 -0.0015646 -0.0001864  0.0014537  0.0059227
##
## Coefficients:
##               Estimate Std. Error  t value Pr(>|t|)
## (Intercept)  5.801e+00  2.555e-04 22699.16   <2e-16 ***
## trend()      3.083e-04  2.063e-06   149.40   <2e-16 ***
```

```
## I(trend()^2) 1.724e-07  3.493e-09    49.37   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.002028 on 568 degrees of freedom
## Multiple R-squared: 0.9991,  Adjusted R-squared: 0.9991
## F-statistic: 3.134e+05 on 2 and 568 DF, p-value: < 2.22e-16
```

```
## # A tibble: 6 x 10
##   .model      .type         ME    RMSE     MAE      MPE   MAPE  MASE RMSSE  ACF1
##   <chr>       <chr>      <dbl>   <dbl>   <dbl>    <dbl>  <dbl> <dbl> <dbl> <dbl>
## 1 ARIMA111201 Test     2.17e-3 2.68e-3 2.27e-3  3.60e-2 0.0375   NaN   NaN 0.746
## 2 ARIMA211201 Test     2.15e-3 2.66e-3 2.25e-3  3.56e-2 0.0372   NaN   NaN 0.746
## 3 ARIMA111102 Test     2.16e-3 2.67e-3 2.26e-3  3.58e-2 0.0374   NaN   NaN 0.751
## 4 ARIMA111101 Test     2.11e-3 2.62e-3 2.21e-3  3.50e-2 0.0366   NaN   NaN 0.755
## 5 ARIMA013001 Test     6.55e-4 1.29e-3 1.05e-3  1.08e-2 0.0174   NaN   NaN 0.581
## 6 trend_model Test    -3.58e-6 9.62e-4 8.04e-4 -6.05e-5 0.0133   NaN   NaN 0.351
```

```
## # A mable: 8 x 2
## # Key:     Model name [8]
##   `Model name`                     Orders
##   <chr>                           <model>
## 1 ARIMA111101  <ARIMA(1,1,1)(1,1,1)[12]>
## 2 ARIMA111201  <ARIMA(1,1,1)(2,1,1)[12]>
## 3 ARIMA211201                <NULL model>
## 4 ARIMA112102  <ARIMA(1,1,2)(1,1,2)[12]>
## 5 ARIMA112101  <ARIMA(1,1,2)(1,1,1)[12]>
## 6 ARIMA111102  <ARIMA(1,1,1)(1,1,2)[12]>
## 7 ARIMA013001  <ARIMA(0,1,3)(0,1,1)[12]>
## 8 auto         <ARIMA(0,1,1)(0,1,1)[12]>
```

```
## # A tibble: 7 x 5
##   .model          sigma2 log_lik    AIC   AICc
##   <chr>            <dbl>   <dbl>  <dbl>  <dbl>
## 1 auto        0.00000154   3133. -6259. -6259.
## 2 ARIMA112101 0.00000154   3135. -6258. -6257.
## 3 ARIMA111101 0.00000155   3133. -6257. -6257.
## 4 ARIMA013001 0.00000155   3133. -6256. -6255.
## 5 ARIMA111201 0.00000155   3133. -6255. -6255.
## 6 ARIMA111102 0.00000155   3133. -6255. -6255.
## 7 ARIMA112102 0.00000155   3133. -6253. -6253.
```

(3 points) Task Part 6b: How bad could it get?

With the non-seasonally adjusted data series, generate predictions for when atmospheric CO2 is expected to be at 420 ppm and 500 ppm levels for the first and final times (consider prediction intervals as well as point estimates in your answer). Generate a prediction for atmospheric CO2 levels in the year 2122. How confident are you that these will be accurate predictions?