**МІНІСТЕРСТВО ОСВІТИ І НАУКИ УКРАЇНИ**

**Чорноморський національний університет імені Петра Могили**

**Факультет комп'ютерних наук**

**Кафедра «Інтелектуальних інформаційних систем»**

Індивідуальна робота

**з вищої математики**

**МКР М.2.3.2**

**Варіант 8**

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Миколаїв – 2020

1

$$z = \frac{y^2}{3x} + \ln(xy)$$

$$\frac{dz}{dx} = U'_x = \left(\frac{y^2}{3} + \frac{1}{x}\right) \cdot x^2 = \frac{x^2y^2}{3} + \frac{x^2}{x}$$

$$\frac{dz}{dy} = U'_y = \left(2x \cdot \frac{y}{3} + \frac{1}{y}\right) \cdot xy = 2x^2y \cdot \frac{xy^2}{3} +$$

$$+ \frac{xy}{y}$$

$$\left(\frac{x^2 \cdot y^2}{3} + x\right) - \left(2x^2y \cdot \frac{xy^2}{3} + x\right) + y^2 =$$

$$= \frac{x^2y^2 + 3x}{3} - \frac{2x^2y \cdot xy^2 + 3x}{3} + \frac{y^2}{3} =$$

$$= \frac{x^2y^2 + 3x - x^3y^3 + 3x + y^2}{3} = \frac{-x^3y^3 + x^2y^2 + y^2 + 6x}{3}$$

$$x^2 \frac{dz}{dx} - xy \frac{dz}{dy} + y^2 \neq 0$$

② $z = f(x,y)$ $\quad P(x,y),\ P_0(x_0,y_0)$

$z = x^2 e^y$, $P(1.94,\ 0.12)$ $P_0(2;0)$

$z(x_0,y_0) = 4\cdot 1 = 4$ $\quad \Delta x = -0.06$ $\Delta y = 0.12$

$$z(x,y) = z(x_0+\Delta x;\ y_0+\Delta y) = z(x_0;y_0) + \frac{dz(x,y)}{dx}\Delta x +$$
$$+\frac{dz(xy)}{dy}\Delta y$$

$$\frac{dz}{dx} = z'_x = 2xe^y$$

$$\frac{dz}{dy} = z'_y = x^2e^y$$

$$z(x,y) = 4 + (2^2\cdot e^0)\cdot(-0.06) + (2^2\cdot e^0)\cdot 0.12 =$$
$$= 4 + 4\cdot(-0.06) + 4\cdot 0.12 = 4.24$$

④ $z = 5\cdot x^2\cdot y - 3\cdot y^2\cdot x + y^4$ $\quad P_0(2;-3)$ $\bar{a} = \{-5; \cancel{12}\}$ 12

$$\text{grad } z = (z'_x, z'_y) = (10xy - 3y^3,\ 5x^2 - 9xy^2 + 4y^3)$$

$$\vec{g} = \text{grab } z(P_0) = (10\cdot 2\cdot(-3) - 3\cdot(-3)^3,\ 5\cdot 2^2 - 9\cdot 2\cdot(-3^2) +$$
$$4\cdot(-3)^3 = (21, -250)$$

$$\frac{dz}{d\bar{a}}(P_0) = \frac{\vec{g}\cdot\vec{a}}{|a|} = \frac{21\cdot(-5) + (-250\cdot 12)}{\sqrt{-5\cdot 2 + 12^2}} = -\frac{3105}{13}$$

(5)

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | 5.3 | 6.1 | 6.8 | 7.5 | 8.5 |

$\sum_{k=1}^{5} x_k = 15$ $\sum y_k = 34.2$ $\sum x_k^2 = 55$

$\sum x_k \cdot y_k = 110.4$ $\begin{cases} 55a + 15b = 110.4 \\ 15a + 5b = 34.2 \end{cases}$

$55a = \frac{552}{5} - 15b$ $a = \frac{\frac{552}{5} - 15b}{55}$

$a = \frac{552}{275} - \frac{3b}{11}$

$15a + 5b = \frac{171}{5}$ $\nearrow \frac{1656}{55}$

$5b + 15\left(\frac{552}{275} - \frac{3b}{11}\right) = \frac{171}{5}$

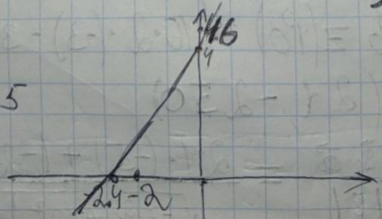
$\frac{10b}{11} = -\frac{1656}{55} + \frac{171}{5}$

$\frac{10b}{11} = \frac{45}{11}$ $b = \frac{9}{2}$ $a = \frac{552}{275} - \frac{27}{22}$

$b \approx 4.5$ $a = \frac{39}{50} \approx 1.86$

$y = ax + b$

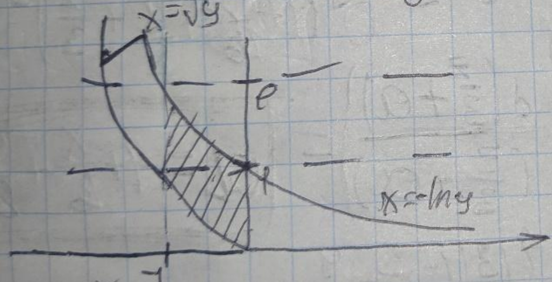
$y = 1.86x + 4.5$



Частина II (1)

$$\int_0^1 dy \cdot \int_{-\sqrt{y}}^{0} f\,dx + \int_1^e dy \cdot \int_{-1}^{-\ln y} f\,dx$$

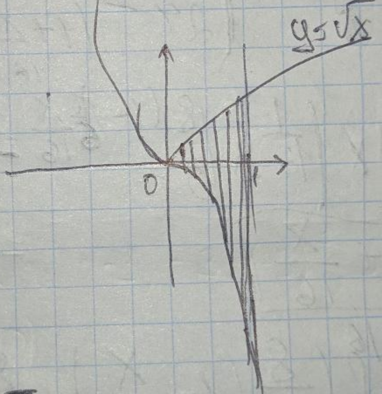
$D_1 \begin{cases} 0 \le y \le 1 \\ -\sqrt{y} \le x \le 0 \end{cases}$ $\quad D_2 \begin{cases} 1 \le y \le e \\ -1 \le x \le -\ln y \end{cases}$




$$\int_{-1}^{0} dx \int_{x^2}^{e^{-x}} dy$$

(2) $\iint\limits_D (27x^2y^2 + 48x^3y^3)\,dx\,dy$

$D = \begin{cases} x = 1 \\ y = \sqrt{x} \\ y = -x^3 \end{cases}$




$$\int_0^1 dx \int_{-x^3}^{\sqrt{x}} (27x^2y^2 + 48x^3y^3)\,dy =$$

$$= \int_0^1 dx\,(9x^2y^3 + 12x^3y^4)\Big|_{-x^3}^{\sqrt{x}} =$$

$$= \int_0^1 (9x^3\sqrt{x} + 9x^{11} + 12x^5 - 12x^{15})\,dx = \left(2x^{\frac{9}{2}} + \frac{3}{4}x^{12} + 2x^6 - \frac{3}{4}x^{16}\right)\Big|_0^1 = 2 + 2 = 4$$

$$= -\frac{3}{4}\int_0^{16}\left(\frac{1}{2}-\frac{x}{32}+\frac{8}{3}\left(\frac{1}{8}-\frac{1}{(1+\frac{x}{16})^3}\right)\right)dx = -\frac{3}{4}\left(\left(\frac{5}{6}x-\frac{x^2}{64}\right)\Big|_0^{16} -\right.$$

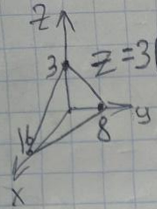
$$\left.-\frac{8}{3}\int_0^{16}\frac{dx}{(1+\frac{x}{16})^3}\right) = -\frac{3}{4}\left(\frac{80}{6}-4-\frac{8\cdot 16}{3}\int_0^{16}\frac{d(1+\frac{x}{16})}{(1+\frac{x}{16})^3}\right) =$$

$$= -\frac{3}{4}\left(\frac{40}{3}-4-\frac{128}{3}\left(-\frac{1}{2}\cdot\frac{1}{(1+\frac{x}{16})^2}\Big|_0^{16}\right)\right) =$$

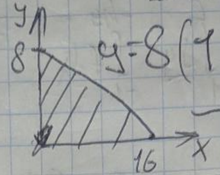
$$= -\frac{3}{4}\left(\frac{40}{3}-4+\frac{64}{3}\left(\frac{1}{4}-1\right)\right) = -\frac{3}{4}\left(\frac{40}{3}-4+\frac{64}{3}\left(-\frac{3}{4}\right)\right) =$$

$$= -\frac{3}{4}\left(\frac{40}{3}-20\right) = -10+15 = 5$$

$V: \frac{x}{16}+\frac{y}{5}+\frac{z}{3}=1,\ x=0,\ y=0,\ z=0$ $\quad V \begin{cases} 0\le x\le 16 \\ 0\le y\le 8\left(1-\frac{x}{16}\right) \\ 0\le z\le 3\left(1-\frac{x}{16}-\frac{y}{8}\right) \end{cases}$




$z=3\left(1-\frac{x}{16}-\frac{y}{8}\right)$




$\boxed{y=8\left(1-\frac{x}{16}\right)}$

$$\iiint \frac{dx\,dy\,dz}{\left(1+\frac{x}{16}+\frac{y}{8}+\frac{z}{3}\right)^5}=\int_0^{16}dx\int_0^{8\left(1-\frac{x}{16}\right)}dy\int_0^{3\left(1-\frac{x}{16}-\frac{y}{8}\right)}\frac{dz}{\left(1+\frac{x}{16}+\frac{y}{8}+\frac{z}{3}\right)^5}=$$

$$=3\int_0^{16}dx\int^{8\left(1-\frac{x}{16}\right)}dy\int^{3\left(1-\frac{x}{16}-\frac{y}{8}\right)}\frac{d\left(\frac{z}{3}+a\right)}{\left(\frac{z}{3}+a\right)}=\boxed{a=1+\frac{x}{16}+\frac{y}{8}}$$

$$=3\int_0^{16}dx\int_0^{8\left(1-\frac{x}{16}\right)}dy\ \frac{\left(\frac{z}{3}+a\right)^{-4}}{-4}\Bigg|_0^{3\left(1-\frac{x}{16}-\frac{y}{8}\right)}=$$

$$=-\frac{3}{4}\int_0^{16}dx\int_0^{8\left(1-\frac{x}{16}\right)}\frac{1}{\left(1+\frac{x}{16}+\frac{y}{8}+\frac{z}{3}\right)^4}\Bigg|_0^{3\left(1-\frac{x}{16}-\frac{y}{8}\right)}dy=$$

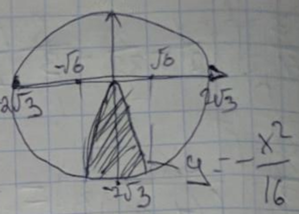
$$=-\frac{3}{4}\int_0^{16}dx\int_0^{8\left(1-\frac{x}{16}\right)}\left(\frac{1}{2^4}-\frac{1}{\left(1+\frac{x}{16}+\frac{y}{8}\right)^4}\right)dy=$$

$$=-\frac{3}{4}\int_0^{16}dx\left(\frac{1}{16}y\Big|_0^{8\left(1-\frac{x}{16}\right)}-8\int_0^{8\left(1-\frac{x}{16}\right)}\frac{d\left(\frac{y}{8}+b\right)}{\left(\frac{y}{8}+b\right)^4}\right.=$$

$$\boxed{b=1+\frac{x}{16}}$$

$$=-\frac{3}{4}\int_0^{16}\left(\frac{8}{16}\left(1-\frac{x}{16}\right)+\frac{8}{3}\frac{1}{\left(\frac{y}{8}+1+\frac{x}{16}\right)^3}\Bigg|_0^{8\left(1-\frac{x}{16}\right)}\right)dx=$$

6) $x^2+y^2=12$    $S - ?$




$-\sqrt{6}y = x^2 \quad y \le 0$

$$\begin{cases} x^2+y^2=12 \\ x^2=-\sqrt{6}y \end{cases} \quad y^2=12+\sqrt{6}y$$

$y^2-\sqrt{6}y-12=0 \quad D=(3\sqrt{6})^2 \quad y_1=2\sqrt{6}, \; y_2=-\sqrt{6} \qquad y_2=-\sqrt{6} \quad x_{1,2}=\pm\sqrt{6}$

$$S=\int_{-\sqrt{6}}^{\sqrt{6}} dx \int_{-\sqrt{12-x^2}}^{-\frac{x^2}{\sqrt{6}}} dy = \int_{-\sqrt{6}}^{\sqrt{6}} dx\left(-\frac{x^2}{\sqrt{6}}+\sqrt{12-x^2}\right) = -\frac{x^3}{3\sqrt{6}}\Big|_{-\sqrt{6}}^{\sqrt{6}} +$$

$$+\int_{-\sqrt{6}}^{\sqrt{6}} \sqrt{12-x^2}\,dx = -2-2+\left| \begin{array}{l} x=2\sqrt{3}\sin t \\ dx=2\sqrt{3}\cdot\cos t\,dt \\ a=\frac{\pi}{4} \\ b=\frac{\pi}{4} \end{array} \right| +$$

$$+\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sqrt{12-12\cdot\sin^2 t}\cdot 2\sqrt{3}\cdot\cos t\,dt = -4+\frac{12}{2}\cdot\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$$

$$(1+\cos 2t)\,dt = -4+6\left(t+\sin\frac{2t}{2}\right)\Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} =$$

$$= -4+\frac{6\pi}{2}+\frac{6}{2}+\frac{6}{2} = 3\pi+2$$

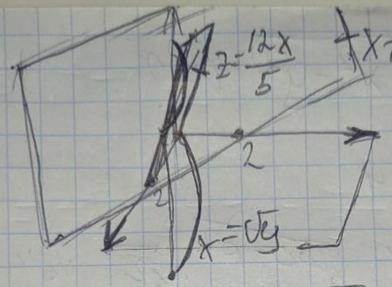
⑩ $x+y=2$

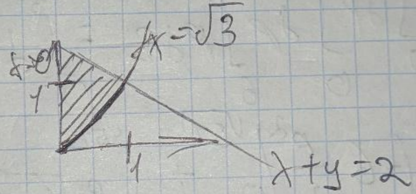
$x=\sqrt{y}$

$z=\dfrac{12x}{5}$

$z=0$

$V-?$



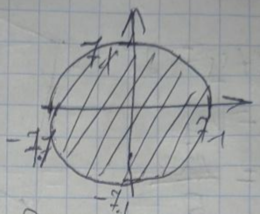




$$V=\int_0^1 dx\int_{x^2}^{-x+2} dy\int_0^{\frac{12x}{5}} dz=\int_0^1 dx\int_{x^2}^{-x+2}\frac{12x}{5}\,dy=\int_0^1\frac{12x}{5}\left(-x+2-x^2\right)dx=$$

$$=\int_0^1\left(-\frac{12x^2}{5}+\frac{24x}{5}-\frac{12x^3}{5}\right)dx=\left(-\frac{4x^3}{5}+\frac{12x^2}{5}-\frac{3x^4}{5}\right)\Big|_0^1=$$

$$=-\frac{4}{5}+\frac{12}{5}-\frac{3}{5}=1$$

⑬ $\begin{cases} z=\sqrt{100-x^2-y^2} \\ z=6 \\ x^2+y^2=54 \end{cases}$ $V-?$




$$V=\int_0^{2\pi} d\varphi\int_0^{\sqrt{51}} d\rho\int_6^{\sqrt{100-\rho^2}}\rho\,dz=\int_0^{2\pi} d\varphi\int_0^{\sqrt{51}}\rho\sqrt{100-\rho^2}-6\rho\,d\rho=$$

$$\int_0^{2\pi} d\varphi\left(-6\frac{\rho^2}{2}\Big|_0^{\sqrt{51}}+\int_0^{\sqrt{51}}\rho\sqrt{100-\rho^2}\,d\rho\right)=$$

$$=\int_0^{2\pi} d\varphi\left(\frac{1}{2}\int_0^{5}\sqrt{100-\rho^2}\,d(100-\rho^2)-6\cdot\frac{51}{2}\right)=$$

$$=\int_0^{2\pi} d\varphi\left(\frac{1}{2}\cdot 2\,\frac{\sqrt{(100-\rho^2)^3}}{3}\Big|_0^{\sqrt{51}}-\frac{6\cdot 54}{2}\right)=$$

$$=\int_0^{2\pi} 219\cdot 153\,d\varphi=66\cdot 2\pi=132\pi$$