

# CÁLCULO INTEGRAL

Febrero 10/2013

## 1 Sustitución.

$$\int f(x)dx \rightarrow g(t) = x \rightarrow \int f(g(t)) \times g'(t)dt = \quad (1)$$

$$\int f(g(t)) \times g'(t)dt \rightarrow x = g(t) \rightarrow \int f(x)dx \quad (2)$$

### 1.1 Ejemplo 1.

$$\int \sqrt{(a^2 - x^2)} \rightarrow x = a \operatorname{sen}(t) \rightarrow \int \sqrt{(a^2 - a^2 \operatorname{sen}^2 t)} \times a \cos(t) dt = \quad (3)$$

$$\int \sqrt{(a^2 \times (1 - \operatorname{sen}^2 t))} \times a \cos(t) dt = \quad (4)$$

$$\int a \times \cos(t) \times a \cos(t) dt = \quad (5)$$

$$\int a^2 \cos^2 t dt = \quad (6)$$

$$\int a^2 \times \frac{1 + \cos(2t)}{2} dt = \quad (7)$$

$$\int \frac{a^2}{2} + \frac{a^2 \cos(2t)}{2} dt = \quad (8)$$

$$\frac{a^2 t}{2} + \frac{a^2 \operatorname{sen}(2t)}{4} + c \quad (9)$$

### 1.2 Ejemplo 2.

$$\int \sqrt{\frac{(9 - x^2)}{x^2}} \rightarrow x = 3 \operatorname{sen}(t) \rightarrow \int \sqrt{\frac{(9 - 9 \operatorname{sen}^2 t)}{9 \operatorname{sen}^2 t}} \times 3 \cos t dt = \quad (10)$$

$$\int \sqrt{\frac{(9 \times (1 - \operatorname{sen}^2 t))}{9 \operatorname{sen}^2 t}} \times 3 \cos t dt = \quad (11)$$

$$\int \frac{3 \cos(t) \times 3 \cos(t)}{9 \operatorname{sen}^2 t} dt = \quad (12)$$

$$\int \cot^2 t dt = \quad (13)$$

$$-\cot(t) - t + c \quad (14)$$

### 1.3 Ejemplo 3.

$$\sqrt{(x^2 + a^2)} \rightarrow x = a \tan(t) \rightarrow \sqrt{(a^2 + a^2 \tan^2 t)} = \quad (15)$$

$$\sqrt{a^2 \times (\tan^2 t + 1)} = \quad (16)$$

$$\sqrt{a^2 \times \sec^2 t} = \quad (17)$$

$$a \times \sec(t) \quad (18)$$

#### 1.4 Ejemplo 4.

$$\int \frac{dx}{x^2 \times \sqrt{x^2 + 4}} \rightarrow x = 2\tan(t) \rightarrow \int \frac{2\sec^2 t}{4\tan^2 t \times 2\sec(t)} dt = \quad (19)$$

$$\frac{1}{4} \int \frac{\sec(t)}{\tan^2 t} dt = \quad (20)$$

$$u = \sec(t), du = \sec(t)\tan(t)dt \leftarrow \frac{1}{4} \int \frac{\sec(t)}{\tan^2 t} dt = \quad (21)$$

$$\frac{1}{4} \int \frac{du}{u^2} = \quad (22)$$

$$\frac{-1}{4u} = \quad (23)$$

$$\frac{-1}{4\sec(t)} = \quad (24)$$

$$\frac{-\csc(t)}{4} \quad (25)$$

#### 1.5 Ejemplo 5.

$$\sqrt{(x^2 - a^2)} \rightarrow x = a\sec(t) \rightarrow \sqrt{(a^2\sec^2 t) - a^2} = \quad (26)$$

$$\sqrt{a^2 \times (\sec^2 t - 1)} = \quad (27)$$

$$\sqrt{a^2 \times \tan^2 t} = \quad (28)$$

$$a \times \tan(t) \quad (29)$$

#### 1.6 Ejemplo 6.

$$\int \frac{dx}{\sqrt{x^2 - a^2}} \rightarrow x = a\sec(t) \rightarrow \int \frac{a\sec(t) \times \tan(t)}{a\tan(t)} dt = \quad (30)$$

$$\int \sec(t) dt = \quad (31)$$

$$\ln(\sec(t) + \tan(t)) \quad (32)$$