IEP report

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91

Resume:

Exercise B: measuring the RC and L/R constant of circuits which has a resistor connected with capacitor and inductor separately.
Exercise C: testing the AC response of the potential divider circuit and investigates both the amplitude and phase response of the circuits across different frequencies.
Exercise B and C are done with PicoScope7 and ADALP 2000 Kit.
These demonstrating the holistic aspect of electrical analysis. (see the result part)

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1.Introduction

Circuits analysis forms the basic parts of electrical engineering; therefore, it is crucial to understand characteristics of electrical at individual and holistic levels. Exercise B is aiming to find the charging behavior of both the capacitor and inductor and then finding the time constant which is τ. This provides the foundation of the Exercise C, which aims at zooming out the performance of the circuit in a holistic to figure out how the potential divider circuits respond to frequency variations in AC circuits.

Some background information of exercise B and C are below:

A capacitor is an electrical device that stores electric energy in an electric field by accumulating electric charges on two closely spaced surfaces that are insulated from each other. It is a passive electronic component with two terminals [1] The capacitance of the capacitor is known as c.
When the capacitor is connected in series with a resistor R and a DC voltage source V_0, the whole circuit will act as a charging circuit [2]. It is called RC circuit (Fig.1).
The voltage across the capacitor as a function of time elapsed is given by:

$$V_c = V_s\left(1 - e^{-\frac{t}{RC}}\right)$$

When t = RC,

$$V_c = V_s\left(1 - \frac{1}{e}\right) \approx 0.63\ V_s$$

This value of $t = RC = \tau_0$ is defined as time constant of the capacitor.

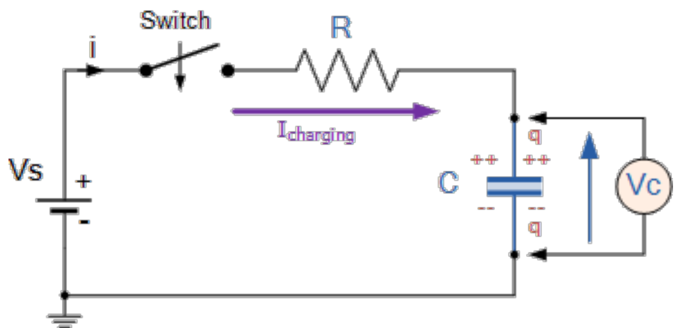


Fig.1: RC circuit, or charging circuit of capacitor [3]

An inductor, also called a coil, choke, or reactor, is a passive two-terminal electrical component that stores energy in a magnetic field when electric current flows through it. An inductor typically consists of an insulated wire around into a coil.

When the inductor is connected in series with a resistor R and a voltage source V 0. It formed an LR (inductor-resistor) circuit. (Fig.2) The current in the circuit can be expressed as a function of time elapsed given below:

$$I_L = \frac{V0}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

When t = RC,

$$I_L = \frac{V0}{R}\left(1 - \frac{1}{e}\right)$$

$$V_R = V_s\left(1 - \frac{1}{e}\right) \approx 0.63\, V_s$$

This value of $t = \frac{L}{R} = \tau_0$ is defined as time constant of the inductors.

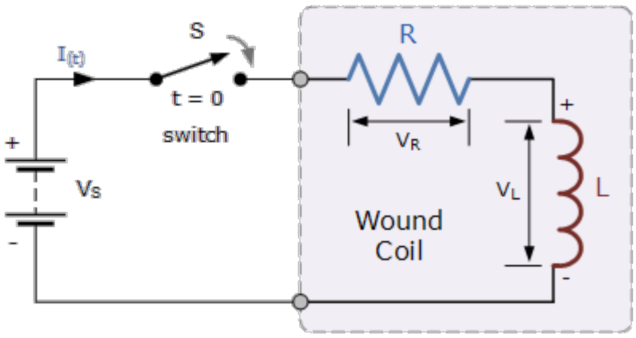


Fig2.LR circuit of inductor [3].

Eclectic current flows in two ways which is known as DC current and AC current. Fig1 and Fig2 illustrate the pattern for DC current. However, in AC circuit, current keeps changing its direction back and forward with a present frequency f. Capacitor and Inductor has impedance:

$$Z_c = \frac{1}{j\omega C} \text{ and } Z_L = j\omega L \quad (\omega = 2\pi f)$$

$$V_{out} = V_{in}\left(\frac{Z}{R + Z}\right)$$

The circuit is a potential divider, and the voltage transfer function can be derived (often referred to as voltage gain or attenuation)

The objectives of the overall experiments [4]

the aims of the overall experiment from these two sets of measurements are:
• To compare the measurements with the expected performance based on the theory presented in the Physical Principles of Electronics (using the nominal value of the RC time constant expected from the components)

• To quantify the asymptotic behavior (roll-off) of a low-pass filter in terms of attenuation versus frequency in terms of dB/decade

• To understand the impact of the resistor tolerance (5%) and the capacity tolerance (20%) on the value of RC time constant that might be expected to be realized in practice

• To determine if a value of the RC time constant within the expected tolerance gives better agreement with the experimental values measured in the time and frequency domain.

2.Methodology (exercise B).

For RC circuit, the steps are as follows:

The resistor is selected with 100kΩ resistance and capacitor with 1μF capacitance.

1. Setting the USB power supply V_{USB} to be 5V with probe A set to measure DC with the amplitude set to be +/- 10V.
2. On the PicoScope, the Probe A is set to x10 and the PicoScope software Probe A is set to x10, with the Resolution Enhancement being 10 bits. The trigger is set to single because just one single period set by the Timebase value needed to be measured. Therefore, the Timebase is set to 200 msec/div and the Trigger Threshold value is 1V.

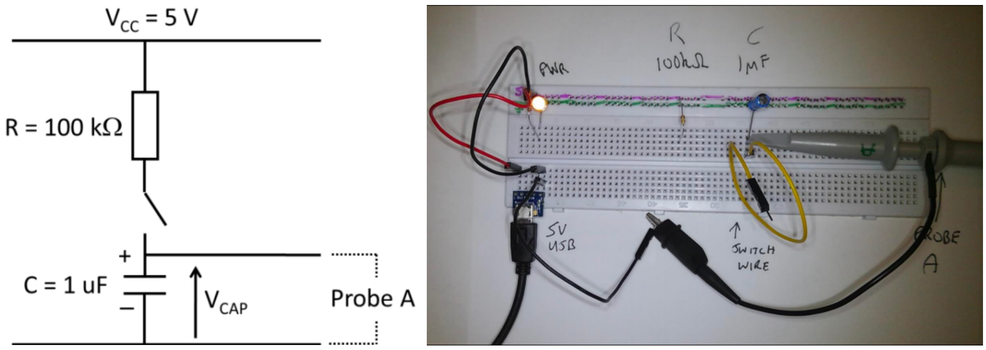


Fig3.The connection of RC circuit in Exercise B

For LR circuits, the steps are as follows:

the resistance is 2.2 kΩ and the self-inductance 10mH.

1. Set the Probe A and software to x1. Under the setting for Probe A, change the Resolution Enhancement to 10 bits. On the PicoScope, set the Amplitude of probe A to +/-10 V and the Trigger to Single. Set the Timebase to 10 sec/div. Also set the Trigger Threshold value to 1 V.
2. Start the trace running.

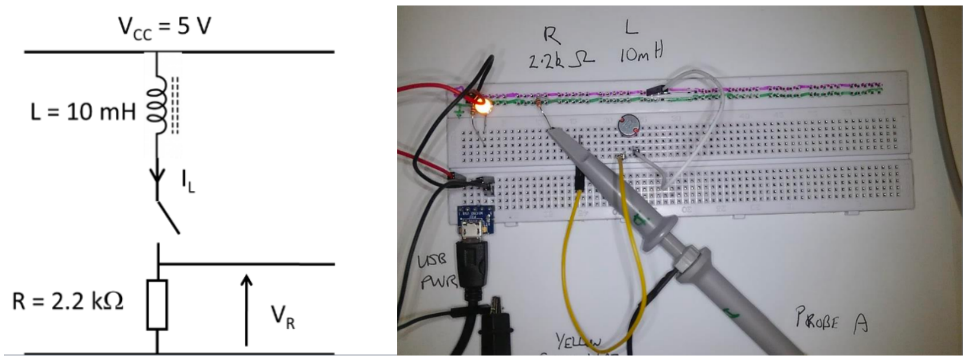


Fig4. The connection of LR circuit in Exercise B

Methodology (exercise C).

• A potential divider circuit (Fig.5) two resistors are connected in series and V $_{out}$, which is the voltage across the resistor R2 is measured. Hence V $_{out}$ can be derived.

$$V_{out} = V\frac{R_2}{R_1 + R_2}$$

• If R2 is replaced by a capacitor, a similar expression can be found using impedances with Z_1 = R and $Z_2 = 1 / \ j\omega C$ to give the below formula and its form of attenuation in decibels

$$\frac{v_{out}}{v_{in}} = \frac{Z_2}{Z_1 + Z_2} = \frac{1}{1 + j\omega RC} = \frac{1 - j\omega RC}{1 + (\omega RC)^2} = \frac{1}{\sqrt{1 + (\omega RC)^2}}\exp[-j\tan^{-1}(\omega RC)]$$

$$20\log_{10}\left(\frac{V_o}{V_i}\right) = 20\log_{10}\left[\frac{1}{\sqrt{1 + (\omega RC)^2}}\right] = -10\log_{10}[1 + (\omega RC)^2]$$

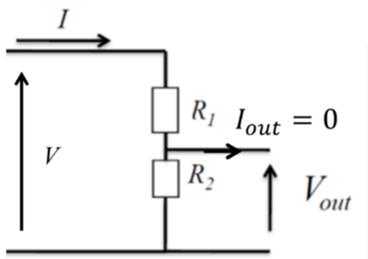
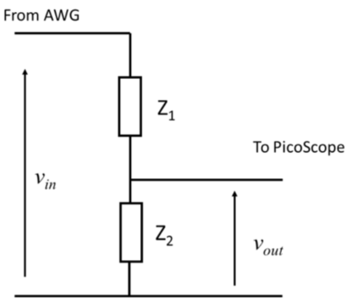


Fig 5 The connection of a potential divider circuit (Exercise C)

V $_{out}$ = 0 when t < 0 is because no current flows through the circuit so there is no potential difference between R_2

$$v_{out}(t) = \begin{cases} 0 & t < 0 \\ V_{USB}\left[1 - \exp\left(-\frac{t}{\tau_{RC}}\right)\right] & t \geq 0 \end{cases}$$

• Constructing the circuit below (Fig.6) using Z_1 = 100 kΩ and Z_2 = 10 kΩ.
• Set the AWG signal
• measure the peak voltage values of *vin* with Probe A and V $_{out}$ with Probe B along with the phase between them and then calculate the attenuation.



$$v_{out} = v_{in}\frac{Z_2}{Z_1 + Z_2}$$

Fig6. Considering the impedances of electric components

3.results

3.1exercise B results

3.11. From exercise B, the time constant of the RC circuit can be found by measuring the readings on PicoScope7, which is 101.5ms.
The figures below illustrate the statics from PicoScope7 and Excel

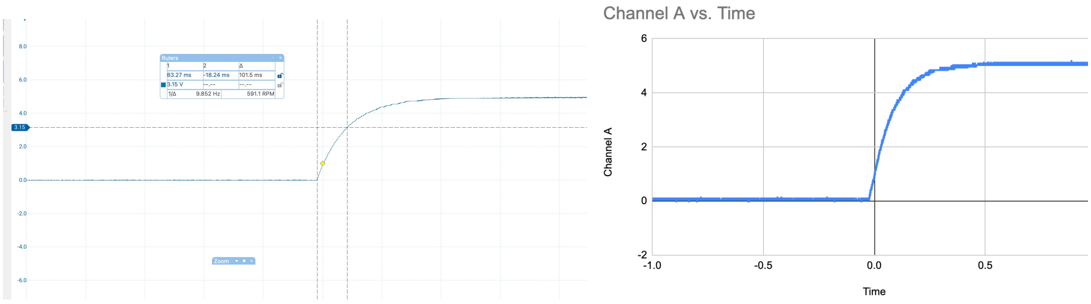


Fig7. RC-circuit measurement

3.12. For LR circuit, the time constant measured from PicoScope graph is 4.598μs

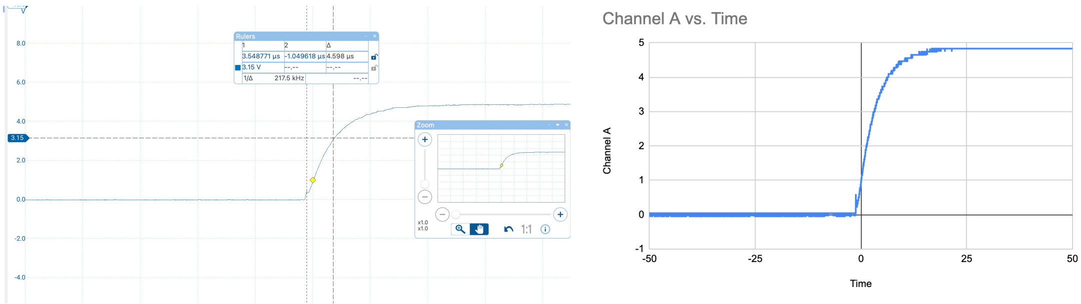


Fig8. LR-circuit measurements

3.2 exercise C results

3.21. V $_{out}$ measured from the 2.2Ω resistor and the attenuation between V_1 and V_2 can be calculated through the readings on the graph.

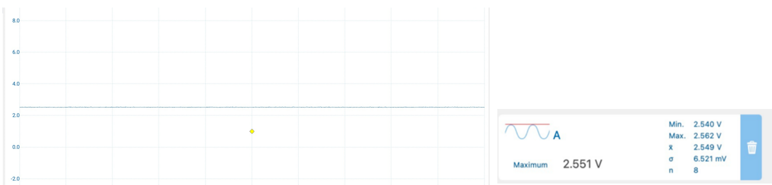


Fig9. Readings through 2.2kΩ resistor

3.22. Let AWG provide the voltage, which is shown as a sine wave. (Fig.10) The attenuation calculated from the graph is 0.1900V /1.992V = 0.0954

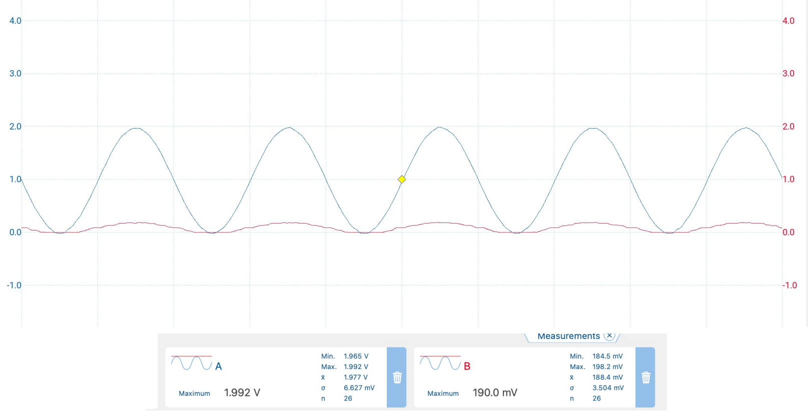


Fig10. 100KΩ and 10KΩ resistor in series with an AC voltage input

3.23. The graph (Fig 12.) is plotted according to the measurements from in Excel. It illustrates the measured and theoretical gain of the circuit, where the measured value is gained from exercise C (Fig11.)

Raw Experimental Data							Gain and phase (from experiment and theory)			
Frequency (Hz)	RMS A (mV)	RMS B (mV)	Time delay between A and B (ms)	Setting Probe B Amplitude	Coupling for A & B	Setting for Time base	Measured gain (dB)	Theoretical gain with RC=0.1 s (dB)	Measured phase (degrees)	Theoretical phase with RC=0.1 s (degrees)
0.1	733.1	722	187.000	±5V	DC	2s	−0.13	−0.02	−6.73	−3.60
0.2	720.3	703.9	139.000	±5V	DC	1s	−0.20	−0.07	−10.01	−7.16
0.5	697.4	652.4	101.100	±5V	DC	500ms	−0.58	−0.41	−18.20	−17.44
1	703.5	582.3	94.010	±5V	DC	200ms	−1.64	−1.45	−33.84	−32.14
2	696	423.3	70.730	±1V	AC	100ms	−4.32	−4.11	−50.93	−51.49
5	688.1	207.2	40.730	±500mV	AC	50ms	−10.43	−10.36	−73.31	−72.34
10	699.3	108.1	21.950	±500mV	AC	20ms	−16.22	−16.07	−79.02	−80.96
20	699	54.87	11.270	±500mV	AC	10ms	−22.10	−22.01	−81.14	−85.45
50	698.4	23.96	5.094	±500mV	AC	5ms	−29.29	−29.95	−91.69	−88.18
100	699.5	11.43	2.634	±500mV	AC	2ms	−35.73	−35.96	−94.82	−89.09
RC time constant (s)	0.10									

Fig11. The measured rms of A and B and phase difference between them

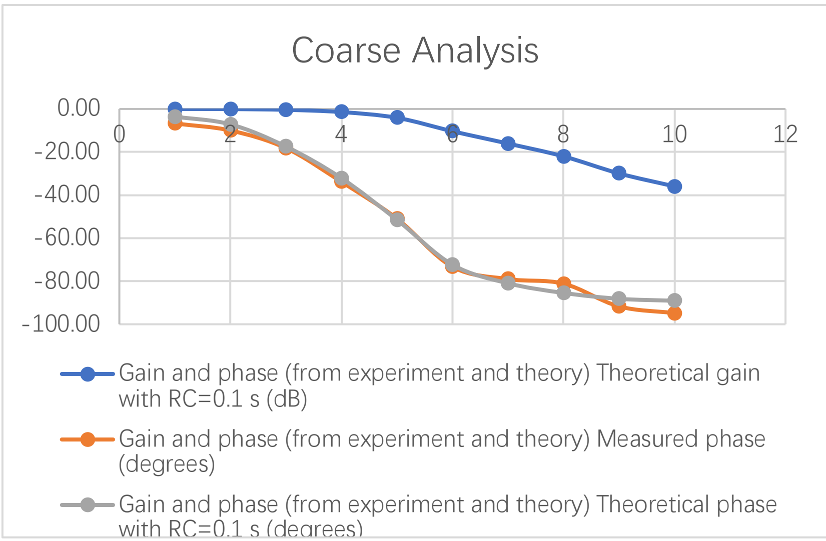


Fig.12 The Coarse Analysis graph of the low pass filter from Excel

4.Discussion of results

1. From 3.11 and 3.12, the deduced value of L / R and RC constant are 101.5ms and $4.598\mu s$ separately. However, the precise value from calculation is $100 \times 10^3 \times 1 \times 10^{-6} = 100ms$ and $4.55\mu s$. Where a 1.5% and a 1.05% error exist.
   RC constant error exists because of the tolerance of the resistor tolerance of capacitor.
   The maximum tolerance of resistor and capacitor are 5% and 20% each, in this situation, the total tolerance of RC is nearly 1.5%.

2. The inaccurate of the reading can also be caused from the finite probe resistance. Which means that the measured voltage of the voltage source is not as same as 5V.

3. The asymptotic behavior or roll-off of a low-pass filter refers to how quickly the filter attenuates or decreases the amplitude of higher-frequency components beyond a certain cut off frequency.
   The ideal situation of the attenuation according to different frequency of the AWG source id shown in Fig12. It is found that the measured value is almost the same as the theoretical value.

5.Conclusion

From the IEP exercise B, the time independent behavior of a capacitor and inductor is deduced when they are connected to a DC voltage via a resistance R. The plotted graph from measurements provided the values of RC and L/R constant, which are approved to be the nearly the same as theory of the behaviors of those two components. From exercise C. The attenuation and phase difference of the circuit are deduced by changing the frequency of AWG input. From the measurement, the phase difference is decreasing with the increase of the AWG input. Overall, the attenuation and phase difference provide key features of a low pass filter.

Reference

[1] https://en.wikipedia.org/wiki/Capacitor

[3] Electronics Tutorials / RC Charging Circuits

https://www.electronics-tutorials.ws/rc/rc_1.html

[4] Time and frequency domain analysis of an electronic low-pass filter/

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https://www.vle.cam.ac.uk