THE EFFECT OF TIME OF USE TARIFFS ON RESIDENTIAL ELECTRICITY CONSUMPTION

CAMERON DAVIES[1], PATRICE MOISAN-ROY, MISHTY RAY, MOHAMAR RIOS FLORES, AND MEREDITH SARGENT[2]

Abstract. We analyze synthetic electrical consumption data obtained from Awesense's Energy Data Model (EDM) for a grid based in Vancouver. We use these analyses and principles of statistics to design Time of Use (ToU) tariffs to incentivize residents to shift their power consumption to times when it will cause less stress on the eletrical grid. We implement and visualize these shifts on the available data.

Introduction

Electrical devices have become a large part of our modern world. A great many things, from lighting to home appliances to electric vehicles are powered by electricity. This is all thanks to modern electric grids. Simply put, an *electric grid* is an interconnected network that transmits electricity from power stations, where electricity is generated from resources (such as wind, solar, hydro, gas, etc.), down to customers. [3] In an electrical grid, the *capacity* refers to the maximum output of electrical power that the power stations on the grid can generate (measured in kilowatts). Conversely, the *demand* on the electrical grid is the amount of energy being used by customers (measured in kilowatts). If the electricity demand exceeds the electricity capacity on a grid, then customers could start experiencing brown-outs or blackouts and equipment could become damaged. As a result, electric grids must be designed so that they always have more than enough energy capacity in order to meet demand [3].

In an ideal world, demand would be constant and one could design a grid where capacity meets demand with very little wasted capacity. However, since many electrical devices don't operate constantly, energy demand fluctuates over time. For instance, when people arrive home from work, they may plug in their electric vehicle and start cooking dinner which will result in an increase in electricity demand during that time of day. Similarly, since people may have different behaviour on different days of the week, we may expect to see fluctuations in demand depending

[1]Supported by Natural Sciences and Engineering Research Council of Canada under a Canada Graduate Scholarship – Doctoral.

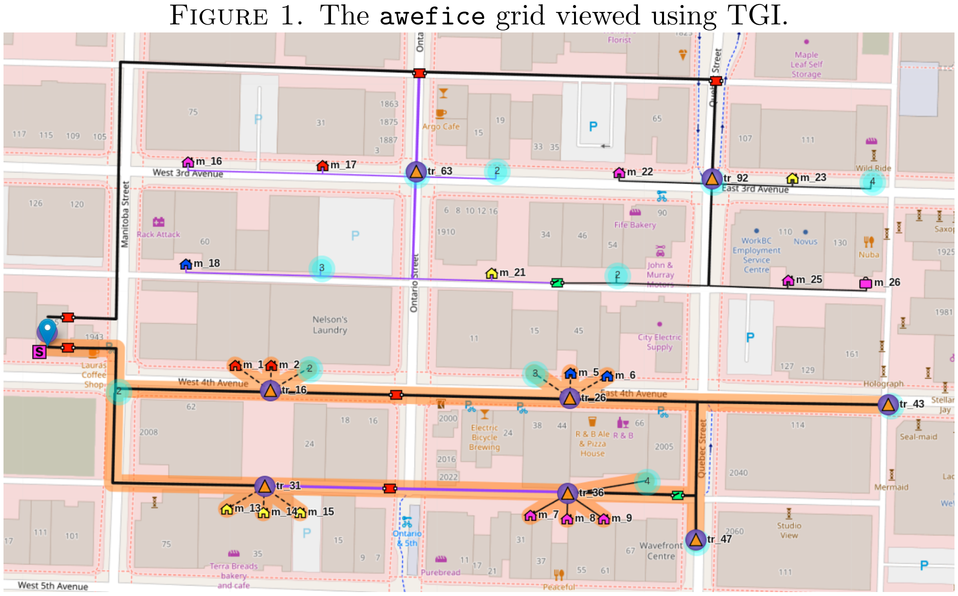
[2]Supported by the Pacific Institute for the Mathematical Sciences (PIMS). The research and findings may not reflect those of the Institute.

[3]In more complex energy grids, containing residential solar panels or significant energy storage capacity, a single customer may both produce and consume electricity, depending on the time of day.

on the day of the week. Moreover, since the operation of some devices, like heaters and air conditioning, is largely dependent on the season, we can expect to see seasonal fluctuations in consumption as well. As a result, a grid must be designed so that it takes into account the worst-case scenario – such as an abnormally cold winter evening, when a single household may be simultaneously heating their home, cooking dinner, doing laundry, and using an electric vehicle (EV) charger. Unfortunately, if the peak energy demand is significantly higher than the mean demand, such a design may be far from optimal. Additionally, while customers may use photovoltaics to reduce their energy demand, it typically does not impact peak demand and the resulting demand graph is called the duck curve [5].

0.1. **The `awefice` Grid.** Awesense provides a platform for analyzing and modeling behaviour of electric grids. Their platform provides users access to a *Digital Twin* of the grid to be analyzed. The grid may be explored using *True Grid Intelligence* (TGI) and its data may be queried directly using an SQL API. In order to protect customer privacy, Awesense provided us access to a digital twin grid generated from synthetic data. This grid, called the `awefice` grid, consists of a few city blocks in Vancouver and includes several residential, industrial, and commercial customers. Each customer has a meter which can be queried through Awesense's API to obtain a sequence of their hourly consumption. Hourly *consumption* is the total amount of demand created by a customer over a period of one hour and is measured in terms of kilowatt hours (kWh). Because our dataset only contains information about hourly consumption, rather than (instantaneous) demand, we will study hourly consumption, as a proxy for demand, throughout this report.

FIGURE 1. The `awefice` grid viewed using TGI.



0.2. **Time of Use Tariffs.** Time of Use (ToU) tariffs are a pricing plan for electricity where the price of electricity varies over the course of a week, depending on the demand anticipated by the utility. Such plans are in place in jurisdictions such as Ontario, which uses two different ToU pricing schemes, depending on the season [2]. Typically, a ToU tariff scheme will increase the price during the hours with peak demand in order to incentivize customers to shift their consumption towards hours with less demand. Traditionally, such schemes were implemented under the assumption that people would manually choose to consume electricity during off-peak hours, but modern smart electrical devices, such as, refrigerators, thermostats, electric vehicle chargers, and others can be configured to avoid using electricity during pre-configured hours.

We now model the effect of Time of Use tariffs on the `awefice` grid.

1. Method 1: Shiftable Consumption Analysis

In a nutshell, this method is based on an in-depth analysis of residential power consumption patterns in the `awefice` grid, which is then used to inform the choice of a ToU pricing regime, as well as a mathematical model for the effects of this regime on electricity consumption. In subsection 1.1, we analyze seasonal, day-of-week, and time-of-day effects on residential consumption. For example, as the `awefice` grid is based on real electricity consumption data in Vancouver, electricity consumption is significantly higher in the winter, when residents warm their houses with electric baseboard heaters, than in the summer, when the weather tends to be temperate and there is little need for power-intensive climate control. We use this conclusion to justify focusing on winter consumption when designing our ToU pricing regime. Following this analysis, in subsection 1.2, we classify power consumption into 'shiftable consumption,' which people could plausibly shift in response to a ToU pricing regime, and 'rigid consumption,' which it is not reasonable to expect people to shift. In subsection 1.3, we design a mathematical model of power consumption where we represent power consumption profiles as measures on a discrete set of hours, and we represent shifts in consumption as maps between such measures which preserve the total measure of the space (*i.e.*, the total consumption). Then, in subsection 1.4, we use assumptions on human behaviour to inform the choice of a realistic map for our model. Finally, in subsection 1.5, we rephrase our model in terms of stochastic (and somewhat sparse) matrices, which allows us to use `numpy` to efficiently find the new, shifted consumption profiles in section 3.1.

1.1. **Identifying Peak Usage.** In this section, we explain how we extract raw data from the grid, obtain a good sample of weekly consumption during winter, and stratify this into peak, mid-peak, and off-peak hours. We use statistical analysis, and in particular the notion of standard deviation, to determine this stratification.

First, we obtain raw data, which is in the form of hourly timestamps between the years 2021 and 2023 across various meters. We filter the data to include only the residential meters of the grid and aggregate the consumption across all such meters. Next, we restrict this data to a desired time period, for example, winter months for the year 2021, *i.e.*, January 2021, February 2021, and December 2021.

We make this assumption because we wish to focus on data from the winter months, where consumption is significantly higher. Of course, our choice "winter months" is specific to the Vancouver grid, and should adjusted based on the characteristics of the grid being studied. The data we extracted from the `awefice` grid has 24 timestamps corresponding to each day during this 3-month winter period. Next, we extract a good sample of a typical day during this time period. This sample should consist of 24 data points, from 0 to 23, where 0 corresponds to the time between 00:00 and 00:59. We also refer to the period between hour h and $h+1$ as "during hour h" or "at hour h".

Now we compute the mean value of this data. To do so, we let $P(h)$ represent the power consumption during hour h, and let the average consumption be given by

$$\text{mean} = \frac{\sum_{h=0}^{23} P(h)}{24}.$$

The standard deviation is denoted by std and is calculated using

$$\text{std} = \sqrt{\frac{\sum_{h=0}^{23}(P(h) - \text{mean})^2}{24}}.$$

Now we stratify our data as follows.

- Peak hours: We consider the hour h as a peak hour if
$$P(h) - \text{mean} > \text{std},$$
i.e. if that hour's consumption exceeds the mean by more than one standard deviation.
- Mid-peak hours: We consider the hour h a mid-peak hour if
$$0 \leq P(h) - \text{mean} \leq \text{std},$$
i.e. if that hour's consumption exceeds the mean by at most one standard deviation.
- Off-peak hours: We consider the hour h an off-peak hour if
$$P(h) < \text{mean},$$
i.e. if the consumption is below average.

As an example, if you restrict to the winter period mentioned above, we get $\{18, 19, 20, 21\}$ as the set of peak hours. This makes intuitive sense as that is the time when households are likely to perform post work activities, such as making dinner, charging electric vehicles, etc. Given the set of peak hours, we may write down peak, mid-peak, and off-peak indicator functions. A peak indicator function is an array of length 24, where the i-th entry corresponds to the i-th hour of the day. If h is a peak hour, then the entry for $i = h$ is 1, otherwise 0. For our example above, the peak indicator function $f = (f(h))_h$ is

$$f(h) = \begin{cases} 1 & h \in \{18, 19, 20, 21\}, \\ 0 & \text{otherwise.} \end{cases}$$

We may also do this analysis for a given week. We think of a week as a period of 168 hours, starting with hour 0 all the way up to hour 167. Here, hour 0 corresponds to the time period 00:00 to 00:59 on Monday. For hour 0, we take the average of the consumption at the 0-th hour of each week, while varying over all weeks for our chosen time period. Likewise for hour i, we take the average of the consumption at hour i of each week, while varying over all weeks. Now we have 168 data points and can repeat the above process.

For our actual pricing function, we chose an intermediate method, which was based on two key observations. First, non-residential consumption (and, in turn, total consumption) in the `awefice` grid tends to be lower during the weekends. Second, all weekdays have broadly similar power consumption patterns, such as evening peaks. As such, we chose to only apply ToU tariffs to weekdays, and price the entire weekend at the off-peak rate. Moreover, to get the pricing scheme for each weekday, we ran the statistical analysis for an average 24-hour weekday. Thus, for each weekday, we wound up with:

- Peak hours of 18:00-21:59.
- Mid-peak hours of 05:00-06:59, 13:00-17:59, and 22:00-22:59.
- Off-peak hours of 00:00-04:59, 07:00-12:59, and 23:00-23:59.

In the end, for the purposes of simplicity, we followed Ontario's pricing scheme, and chose our prices to be \$0.151 per kWh during peak hours, \$0.102 per kWh during mid-peak hours, and \$0.074 per kWh during off-peak hours [2]. More specifically, we may define the set of peak hours by

$$\mathcal{H}_p := \{24d + h \in \{0, ..., 167\} \mid d \in \{0, 1, 2, 3, 4\}, h \in \{18, 19, 20, 21\}\},$$

the set of mid-peak hours by:

$$\mathcal{H}_m := \{24d + h \in \{0, ..., 167\} \mid d \in \{0, 1, 2, 3, 4\}, h \in \{5, 6, 13, 14, 15, 16, 17, 23\}\},$$

and the set of off-peak hours by

$$\mathcal{H}_o := \{0, ..., 167\} \setminus (\mathcal{H}_p \cup \mathcal{H}_m).$$

Under these definitions, we define our price function $\mathrm{Pr} : \{0, ..., 167\} \to \mathbb{R}$ by

$$\mathrm{Pr}(h) := \begin{cases} 0.151 & \text{if } h \in \mathcal{H}_p \\ 0.102 & \text{if } h \in \mathcal{H}_m \\ 0.074 & \text{if } h \in \mathcal{H}_o \end{cases}.$$

With our pricing scheme set, we now turn our attention to determining what proportion of power consumption consumers will be willing to shift in response to an additional tariff.

1.2. **Shiftable and Rigid Consumption Patterns.** As we discussed in subsection 1.1, there are numerous factors, such as season, time-of-day, and day-of-week which have an effect on power consumption. While it is theoretically appealing to attempt to determine the proportion of shiftable consumption based only on these discernible periodic effects, power consumption in the `awefice` grid exhibits random, idiosyncratic effects which do not appear to obey any identifiable periodic pattern. For example, as discussed earlier, a large proportion of Vancouver's electricity consumption is used for heating, and we expect this consumption to be both rigid and correlated with the largely random temperature outside. As such, during an abnormally cold winter day, we expect there to be very high electricity consumption in Vancouver (and hence the `awefice` grid) and, moreover, we cannot expect consumers to shift much of this consumption to times which are more favourable to the utility company. This means that, at least to some extent, our model should be flexible enough to account for some degree of day-to-day randomness. On the other hand, a model which is too closely dependent on short-duration idiosyncrasies runs the risk of ignoring shiftable power consumption and underselling the potential effects of ToU tariffs. For example, a consumption spike which lasts 2 hours might represent an electric vehicle charging, which could be easily and automatically shifted to off-peak hours if its owner has even a small financial incentive to do so.

To balance these effects, we have opted to use each day's average power consumption as a reference point to classify power consumption into shiftable and rigid parts. As before, let $P(h)$ represent the total power consumption in the `awefice` grid (or a section of this grid) for the hour h, let $\bar{P}(d)$ represent the hourly average consumption during the day d, and define $d(h)$ to be the day in which the hour h occurs. Under these conventions, we define the 'rigid' consumption for each hour as

$$P_r(h) := \min(P(h), \bar{P}(d(h))), \tag{1.1}$$

and the 'shiftable' consumption for each hour to be

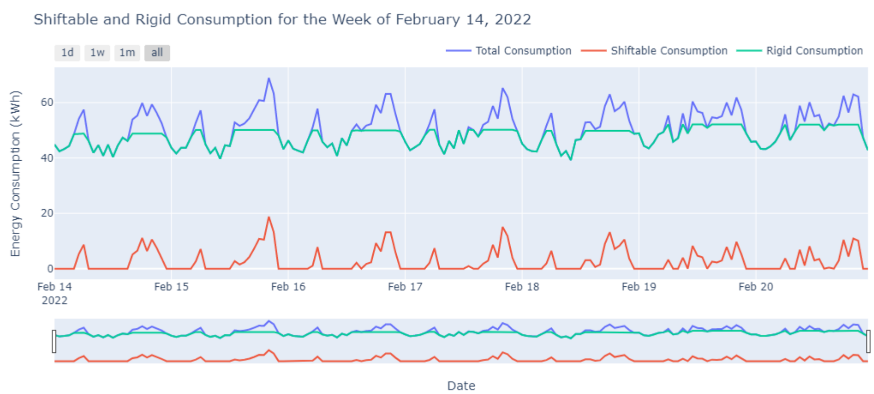
$$P_s(h) := P(h) - P_r(h) = \max(0, P(h) - \bar{P}(d(h))). \tag{1.2}$$

Thus, $P_r(h)$ consists of all power consumption up to that day's hourly average, and $P_s(h)$ consists of power consumption above the average, if any, in the hour h. We provide a visual interpretation of these definitions in Figure 2. When deriving our results, we will assume that ToU pricing only affects the shiftable consumption $P_s(h)$, and not the rigid consumption $P_r(h)$.

1.3. **A Probabilistic Model for Shifting Consumption.** When designing our model, we chose to shift power consumption within each individual week, as we felt that this was the longest time scale over which people are likely to shift their power consumption (*i.e.*, it is reasonable to envision people shifting electricity-intensive chores like laundry from weekdays to weekends if incentivized to do so, but since the tariff structure is the same every week, it is not likely to motivate people to delay consumption by more than a week). As such, we enumerate the 168 hours in a week by the set $H := \{0, 1, 2, ..., 167\}$, where h corresponds to the hth hour of the week, with the convention that the 0th hour of the week is 00:00-00:59 on Monday.

The basic idea of our model is to take each hour's shiftable power consumption $P_s(h)$ and, influenced by factors such as price incentives, convenience, and sleep, redistribute this to other hours to get a new consumption profile. To do so, we found it clarifying to use the language of measure theory and probability theory. We do not assume the reader has background in measure theory or probability theory, but refer the curious reader to [4, Chapter 1] for a quick introduction. In

FIGURE 2. Residential consumption in the `awefice` grid for the week of February 14, 2022, decomposed into shiftable (red) and rigid (green) consumption. The green, rigid consumption for each hour represents all consumption in that hour up to that day's hourly average – on a graph, it appears as a truncation of the total power consumption. On the other hand, the red, shiftable consumption for a given hour represents any power consumption in excess of that day's hourly average.



our case, let $\mathcal{M}_+(H)$ denote the space of finite positive measures on H and $\mathcal{P}(H)$ denote the space of probability measures on H. We describe the power consumption profile over a given week by the measure

$$\mu = \sum_{h=0}^{167} P(h)\delta_h = \sum_{h=0}^{167} [P_s(h)\delta_h + P_r(h)\delta_h]. \tag{1.3}$$

Here $\delta_h \in \mathcal{P}(H)$ is the Dirac delta at h, *i.e.*, the probability measure such that $\delta_h(\{h\}) = 1$ and $\delta_h(\{h'\}) = 0$ for $h' \in H \setminus \{h\}$. In addition, recall that the functions P_r and P_s are as defined in equations (1.1) and (1.2), respectively.

To shift consumption, we define a shifting function $\sigma : \mathcal{M}_+(H) \to \mathcal{M}_+(H)$ as follows. For each $h \in H$, let $\sigma(\delta_h) = \sum_{i=0}^{167} \sigma_{ih}\delta_i$, where $\sum_{i=0}^{167} \sigma_{ih} = 1$ and each σ_{ih} is non-negative. For a general measure $\nu \in \mathcal{M}_+(H)$, write $\nu = \sum_{h=0}^{167} \nu_h \delta_h$, and extend the definition of σ by linearity so that

$$\sigma(\nu) = \sum_{h=0}^{167} \nu_h \sum_{i=0}^{167} \sigma_{ih}\delta_i = \sum_{i=0}^{167} \left(\sum_{h=0}^{167} \sigma_{ih}\nu_h \right) \delta_i.$$

In our intended use case, where ν is a power consumption profile, σ_{ih} represents the probability that power consumption at the hour h will be shifted to the hour i, and ν_h represents the original amount of consumption during the hour h. We also note that, by the hypothesis that $\sum_{i=0}^{167} \sigma_{ih} = 1$, the profiles represented by $\sigma(\nu)$ and ν

have the same total power consumption, reflecting our assumption that introducing ToU tariffs will not change the total power consumption.

While, in the interest of maintaining generality, we postpone the definition of a particular choice of σ to subsection 1.4, we note that equation (1.3) suggests the decomposition $\mu = \mu^s + \mu^r$, where $\mu^s_h = P_s(h)$ and $\mu^r_h = P_r(h)$. In other words,

$$\mu^s = \sum_{h=0}^{167} \mu^s_h \delta_h = \sum_{h=0}^{167} P_s(h)\delta_h \qquad \text{and} \qquad \mu^r = \sum_{h=0}^{167} \mu^r_h \delta_h = \sum_{h=0}^{167} P_r(h)\delta_h.$$

Thus, applying the shift σ only to the shiftable part μ^s of μ, we define a shifted power profile $\tilde{\mu} := \sigma(\mu^s) + \mu^r$. After a bit of reindexing, $\tilde{\mu}$ can be written in terms of hourly consumption as

$$\tilde{\mu} = \sum_{h=0}^{167} \left(\sum_{i=0}^{167} \sigma_{hi}\mu^s_i + \mu^r_h \right) \delta_h = \sum_{h=0}^{167} \left(\sigma_{hi} P_s(i) + P_r(h) \right) \delta_h. \tag{1.4}$$

In the next section, we will explicitly explain our choice of shifting function σ, as well as the considerations which led us to this choice.

1.4. **Designing a Shifting Function.** With the basic structure of our model set out in subsection 1.3, we now turn our attention to designing and defining an appropriate shifting function $\sigma : \mathcal{M}_+(H) \to \mathcal{M}_+(H)$. In particular, our model accounts for the following three factors:

- **Cost of Power:** How much more expensive is it to consume power at hour h than it would be during off-peak hours? How will people's behaviour respond to higher prices?
- **Distance of Shift:** How far are people willing to shift their power consumption in order to secure a cheaper rate? In general, we can expect people to be somewhat resistant to shifting their consumption patterns.
- **Sleep (Optional):** When do people sleep? How much consumption can be shifted into sleeping hours without causing inconvenience?

By considering these three factors, we will define a trial shifting function $\tilde{\sigma}_{ih}$ as a product of three functions and then normalize by dividing by $\sum_j \tilde{\sigma}_{jh}$ to get our actual shifting function. More specifically, we take

$$\tilde{\sigma}_{ih} = C(i,h)D(i,h)s(i),$$

where $C(i,h)$ is the cost incentive to move power consumption from hour h to hour i, $D(i,h)$ is the distance penalty for moving consumption from hour h to hour i, and $s(i)$ is the sleep penalty for moving power consumption to hour i. In general, we think of $C(i,h)$ as a baseline financial incentive to shift power consumption, and of $D(i,h)$ and $s(i)$ as competing factors hindering people from shifting their consumption, even if it would be better for their pocketbooks to do so. We now explain our choices of each of these functions:

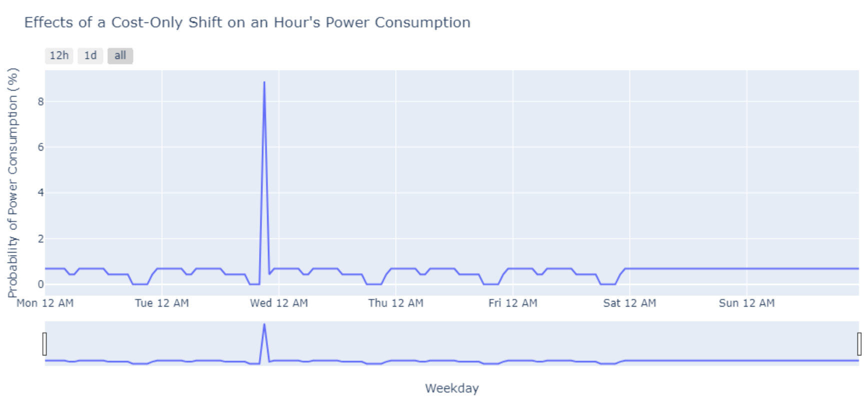
1.4.1. *The Cost Incentive.* Our cost incentive requires the introduction of the tariff scheme defined in subsection 1.1. In brief, the cost incentive function should encourage consumers to move consumption from a given hour into hours with a lower ToU price, and the value of $C(i, h)$ should only depend on the possible savings gained from moving power consumption from hour h to hour i – if the tariff at hour i is not less than the tariff at hour h, then we should have that $C(i, h) = 0$. In addition, in the absence of the distance and sleep penalties discussed below, $C(i, h)$ represents the relative attractiveness of shifting consumption from hour h to hour i. For example, if $C(i, h) = 1$, then after a shift, the hour i will account for the same proportion of shiftable consumption as the original hour h does. This motivates us to write C in the form

$$C(i, h) = c(\max(\Pr(h) - \Pr(i), 0)),$$

where c is a single variable function with $c(0) = 0$.

In particular, to derive our results, we chose $c(t) = t$. In Figure 3, we describe how a cost-only shift redistributes power consumption from the peak hour 21:00-21:59 Tuesday to other hours throughout the week. We notice that there are a couple of

FIGURE 3. Redistribution of power consumption during the hour 21:00-21:59 Tuesday after a cost-only shift. Approximately 8.87% of power consumption remains during this hour, approximately 0.43% gets shifted to each mid-peak hour, and approximately 0.68% gets shifted to each off-peak hour. In other words, power consumption at each mid-peak hour is $0.151 - 0.102 = 0.049$ times the amount of power which remains at the original hour, and power consumption at each off-peak hour is $0.151 - 0.074 = 0.077$ times the amount of consumption which remains at the original hour.



immediate drawbacks with cost-only shifts. First, it assumes that consumers will shift over 90% of their shiftable power consumption to other hours, which we take to be quite unrealistic – it does not take into account factors such as convenience, or resistance to change. Second, no distinction is drawn between nearby off-peak hours and distant off-peak hours. In other words, in this preliminary model, consumers have no preference between shifting their consumption days later, and shifting their consumption by only a couple of hours. These concerns motivate our introduction of a distance penalty in subsubsection 1.4.2.

We conclude our discussion of the cost incentive by remarking that we may tweak parameters in our model to define more general cost incentive functions of the form

$$c_{a_c,n_c,b_c}(t) := a_c t^{n_c} + b_c.$$

Here, $a_c \geq 0$ represents how sensitive people are to saving 1 cent per kWh on electricity. The parameter $n_c \geq 0$ represents how this sensitivity scales with the amount saved – for example, this can reflect how consumers view the difference between saving \$0.10 and saving \$0.11, relative to the difference between paying the same price and saving \$0.01. Finally, the parameter $b_c \geq 0$ represents a 'price offset', which provides an incentive for consumers to shift their power consumption around, even in the absence of price factors.

In practice, we anticipate that realistic models will have $a_c > 0$, $n_c \in (0, 1]$, and $b_c = 0$, but to determine or estimate precise values for such parameters would require novel techniques such as machine learning methods or sociological surveys.

1.4.2. *The Distance Penalty.* The distance penalty is designed to reflect the fact that people tend to be comfortable with their current consumption patterns and as such are hesitant to shift power consumption, especially by large amounts of time. In particular, the distance incentive should depend on the distance between the source hour h and the target hour i. We take this distance to be of the form

$$\min(|h - i|, 168 - |h - i|)$$

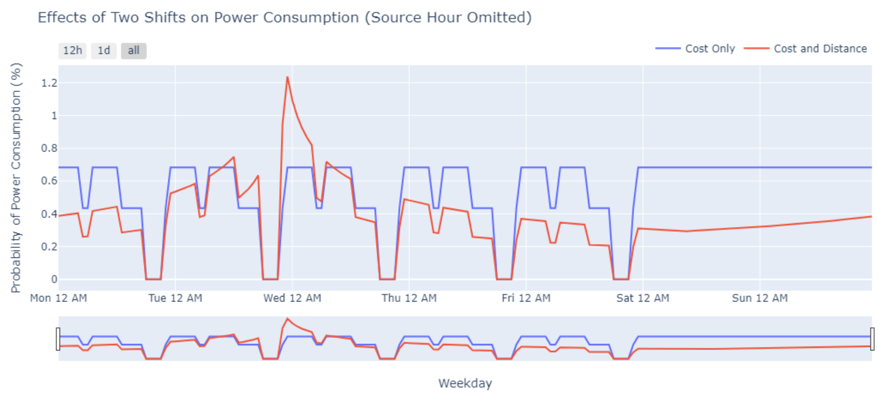
to account for the fact that it doesn't matter whether consumers shift their power consumption forward in time or backwards in time (for example, Monday's laundry might get shifted backward to Sunday, whereas Friday's laundry might get shifted forward to Saturday). As such, we write

$$D(i, h) = d(\min(|h - i|, 168 - |h - i|))$$

for some single variable function d. In particular, as $d(t)$ represents a relative penalty against shifting power by t hours, we should have that $d(0) = 1$, and that d is a decreasing, non-negative function of t.

For our model, we chose $d(t) = (t^{0.5} + 1)^{-1}$, as we felt that this scaling represented a plausible model of people's resistance to moving their power consumption. We examine the effects of the introduction of a distance penalty in figure 4. In particular, this factor promotes consumption during the original hour and nearby hours, while penalizing consumption which occurs well before or well after the original hour of consumption. For example, it shows that the most favourable place to shift power consumption from 21:00-21:59 Tuesday to is later that night. The only immediate downside to this model is that it does not account for consumers being reluctant or unable to shift power consumption into sleeping hours. We will account for this shortcoming in subsubsection 1.4.3.

FIGURE 4. Comparison of distributions of power consumption during the hour 21:00-21:59 Tuesday after a cost-only shift and after a shift accounting for cost and distance. For the purpose of graphical exposition, we have omitted the source hour 21:00-21:59 Tuesday, but we note that 8.87% of consumption remains in this hour after a cost-only shift, and 38.80% of consumption remains in this hour after a shift which accounts for cost and distance.



Again, our model allows us to tweak some parameters to use more general distance incentive functions of the form

$$d_{a_d,n_d,b_d}(t) = (a_d t^{n_d} + b_d)^{-1}.$$

Here, a_d captures how reluctant people are to shift power consumption by 1 hour. In turn, n_d governs how this reluctance scales with the distance by which people are shifting power. Finally, b_d is a distance offset, which we included for technical reasons to ensure that $d(0) < \infty$.

Notice that, in order to satisfy the constraint that $d(0) = 1$, we must have that $b_d = 1$. We should also choose $a_d \geq 0$ and $n_d \geq 0$ to ensure that d_{a_d,n_d,b_d} is decreasing and non-negative. We see no other obvious constraints on these values, but note that, as in the case of cost incentives, it is likely possible to use a combination of machine learning and social scientific study to determine realistic values of these parameters.

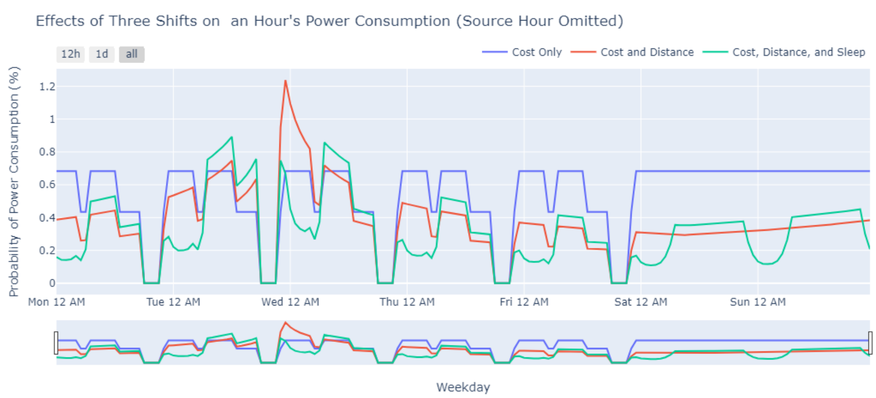
1.4.3. *The Sleep Penalty.* The last piece of our model is the sleep penalty, which penalizes consumption shifts into sleeping hours while not affecting waking hours. This portion of the model accounts for two key factors – what are people's sleeping hours, and how much less likely are they to shift power consumption into these hours. Our model, while rudimentary, defines a sleeping period during which sleep effects occur, estimates how much less likely people are to consume electricity during the centre of this sleeping period, and estimates how this likelihood scales towards the beginning and end of the sleeping period.

We wound up using the sleeping penalty function

$$s(i) = \min\left(1, 0.7\left(\frac{\min(|(i \mod 24) - 2|, |(i \mod 24) - 26|}{5}\right)^3 + 0.3\right).$$

As we see in figure 5, by incorporating a penalty against shifting into sleeping hours, consumers tend to shift power that would have been consumed from 21:00-21:59 Tuesday into the preceding day and the following morning. Accounting for sleep

FIGURE 5. Comparison of distributions of power consumption during the hour 21:00-21:59 Tuesday after a cost-only shift, after a shift accounting for cost and distance, and after a shift accounting for cost, distance, and sleep. For the purpose of graphical exposition, we have again omitted the source hour 21:00-21:59 Tuesday, but we note that 8.87% of shiftable consumption remains in this hour after a cost-only shift, 38.80% of shiftable consumption remains in this hour after a shift which accounts for cost and distance, and 46.39% of shiftable consumption remains in this hour after a shift accounting for all three factors.



also increases the amount of consumption which does not get shifted from 38.80% to 46.39% – in general, it makes shifting power a less attractive option. We also remark that it would be possible to use an identical approach to penalize shifting into working hours (or any other set of hours of the reader's choice), but we have avoided doing so for the sake of keeping our model relatively simple.

As with the other incentives and penalties, we can introduce additional parameters in our model to define sleep penalties of the form:

$$s_{a_s,c_s,\ell_s,n_s}(i) = \min\left(1, (1-a_s)\left(\frac{\min(|(i \mod 24) - c_s|, |(i \mod 24) - 24 - c_s|}{0.5\ell_s}\right)^{n_s} + a_s\right).$$

Here, a_s represents how much less likely consumers are to shift electricity to the centre of the sleep period. We took a_s to be 0.3, as a result of our assumption that 30% of consumption can be automatically shifted into the overnight hours, and 70% cannot. Next, the parameter c_s represents the centre of the sleep period (which we took to be 2:00 AM), and ℓ_s represents the length of the sleeping period (which we took to be 10 hours). Finally, n_s determines how power consumption scales between the centre and the edges of the sleeping period. We found that $n_s = 3$ led to a relatively sharp drop-off at the edges of the sleep period, followed by more modest decreases approaching the centre of the sleep period, which we took as a realistic depiction of the fact that most people tend to sleep at roughly the same time.

We conclude this discussion by reiterating that this version of the sleep penalty function should be thought of as a proof-of-concept – this is one of the parts of the model which could be best informed by additional information, in particular information about people's sleeping patterns, and a more stratified breakdown of electricity consumption into tasks which can be automated to happen overnight (*e.g.* EV charging and dishwashers) and those which largely cannot (*e.g.* laundry).

1.4.4. *Combined Cost Incentive.* We now summarize the discussion in the earlier parts of this subsection by defining our choice of σ_{ih}, both in the case where we account for sleeping effects, and in the case where we do not. In the former, when there are no sleeping effects, we define a non-normalized shifting function by

$$\tilde{\sigma}_{ih} = \frac{\max(\Pr(h) - \Pr(i), 0)}{(\min(|h-i|, 168 - |h-i|))^{0.5} + 1} + \mathrm{Id}_{ih},$$

where Id is the identity matrix. The entries of $\tilde{\sigma}_{ih}$ represent the probability that power consumption will be shifted from hour h to hour i, relative to the probability that power consumption remains at hour h (for example, $\tilde{\sigma}_{ih} = 0.1$ means that, after the shift, the consumption at hour i will only increase by 10% of the shiftable consumption that remains during hour h. To get our actual shifting function, we normalize to define

$$\sigma_{ih} = \frac{\tilde{\sigma}_{ih}}{\sum_{j=0}^{167} \tilde{\sigma}_{jh}}$$

In the case of sleeping effects, we can define

$$\tilde{\sigma}^s_{ih} = \tilde{\sigma}_{ih} \min\left(1, 0.7\left(\frac{\min(|(i \bmod 24) - 2|, |(i \bmod 24) - 26|}{5}\right)^3 + 0.3\right),$$

and again renormalize to take

$$\sigma^s_{ih} = \frac{\tilde{\sigma}^s_{ih}}{\sum_{j=0}^{167} \tilde{\sigma}^s_{jh}}.$$

For reference, in the case where we account for sleeping effects, we will refer to our shift as 'sleepy'.

More generally, our model can be tweaked by adjusting ten parameters in the definition of `shift_by_tariff_and_dist` function in the companion Jupyter notebook. We provide these parameters, the variables used to denote them in this paper, the

Python variables used to denote them in the companion notebook, and summaries of their effects in the following table:

Parameter	**Symbol**	**Python Variable**	**Effect**
cost scaling	a_c	`price_scaling`	Determines how strongly consumers react to price increases in general. We expect $a_c \geq 0$
cost power	n_c	`price_power`	Determines how consumer reactions scale with the size of the price increase. We expect $n_c \in (0, 1]$.
cost offset	b_c	`price_offset`	Causes consumers to shift consumption to other hours with the same cost of electricity. We expect $b_c = 0$
distance scaling	a_d	`dist_scaling`	Measures how reluctant consumers are to shifting power consumption in general. We expect $a_d \geq 0$.
distance power	n_d	`dist_power`	Measures how consumer reluctance scales with the distance by which consumption is shifted. We expect $n_d \geq 0$.
distance offset	b_d	`dist_offset`	Measures how likely people are to want to keep power consumption the same, in the absence of price constraints. We expect $b_d = 1$.
sleep minimum	a_s	`sleep_min`	Represents the penalty against power consumption in the centre of the sleeping period. We require $a_s \in [0, 1]$.
sleep centre	c_s	`sleep_centre`	Represents the centre of the sleeping period, in 24-hour time. We expect that $c_s \in [1, 4]$.
sleep length	ℓ_s	`sleep_length`	Represents the length of the sleeping period, in hours. We expect that $\ell_s \in [6, 10]$.
sleep power	n_s	`sleep_power`	Represents how power consumption scales up to full while approaching the edges of the sleeping period. We expect $n_s \geq 0$.

1.5. **Implementing a Shift.** As the astute reader has likely noticed by now, the coefficients σ_{ih} describing the shifting function $\sigma : \mathcal{M}_+(H) \to \mathcal{M}_+(H)$ can be encoded in a 168-by-168 matrix $\Sigma := (\sigma_{ih})_{i,h=0}^{167}$ representing how electricity consumption shifts from any hour in the week to any other hour in the week. Moreover, by assumption the rows of Σ sum to 1. Hence Σ is considered to be a 'stochastic matrix', as discussed in [1, Chapter 1]. While we do not leverage this property much in our model, we remark that it may allow the application of tools from probability theory and optimal transportation to be applied in the future. In addition, if h is an off peak hour, then there is no incentive to shift power from h to any other hour i, so $\sigma_{ih} = \mathrm{Id}_{ih}$. Since, in our regime, $108/168 \approx 64.2\%$ of our hours are off-peak, this implies that $\frac{167\cdot 108}{168\cdot 168} \approx 63.9\%$ of the entries of Σ are 0, and it may be possible to take advantage of this sparseness to perform efficient calculations.

In any case, to properly implement our model in Python, we encode the original shiftable consumption profile μ^s as the column vector $M^s = (\mu_h^s)_{h=0}^{167}$ and the original rigid consumption profile μ^r as the column vector $M^r = (\mu_h^r)_{h=0}^{167}$. Under this convention, we can rewrite equation (1.4) for the shifted consumption profile as

$$\tilde{\mu} = \sum_{h=0}^{167} (\Sigma \cdot M^s + M^r)_h \, \delta_h. \tag{1.5}$$

We can also reinterpret the shifted consumption profile as a column vector $\tilde{M}$ of length 168 with $\tilde{M}_h := (\Sigma M^s + M^r)_h$ for $h = 0, ..., 167$. This formulation has a couple of computational advantages. First, matrix multiplication and vector addition are relatively fast operations for a computer to perform, especially when using `numpy ndarrays`. Second, this allows us to shift a large number of weeks of consumption by using matrix multiplication. That is, if we replace M^s and M^r by $168 \times W$ matrices representing the shiftable and rigid consumption over a period of W weeks, then $(\sigma \cdot M^s + M^r)_{hw}$ represents the shifted consumption at hour h of week w. In Section 3.1, we will explain what results we derive from this method but for now, with our first method fully described, we turn our attention to describing an alternative method based on frequency analysis.

2. Method 2: Fourier Transform and Frequencies

Short of having the weekly consumption data on each and every appliance in the grid, we can still use harmonic analysis to decompose the consumption signal based on its frequency spectrum. If the hourly consumption for a given week is modeled by a function $\varphi(h)$, then we may write

$$\begin{aligned}\varphi(h) &= \frac{1}{168}\sum_{f=0}^{167}\hat{\varphi}(f)\ e^{2\pi i f h/168} \\ &= \frac{1}{168}\sum_{f=-83}^{84}\hat{\varphi}(f)\ e^{2\pi i f h/168} \\ &= \frac{1}{168}\left(\hat{\varphi}(0)+\hat{\varphi}(84)\ e^{\pi i h}+\sum_{f=1}^{83}\left(\hat{\varphi}(f)\ e^{2\pi i f h/168}+\hat{\varphi}(-f)\ e^{-2\pi i f h/168}\right)\right),\end{aligned}$$

where

$$\hat{\varphi}(f)=\sum_{h=0}^{167}\varphi(h)\ e^{-2\pi i f h/168}$$

is the (discrete) Fourier transform of φ. Mathematically, adding up the terms for a frequency and its negative gives us a real-valued trigonometric function. Heuristically, an event happening $168-f$ times in a week is the same as having this event *not* happen exactly f times in that week. Keeping in mind the need to work with frequencies and their negatives in the same way, one can then decide how to individually shift the contributions of each different frequency to the overall signal on any given week. All we need is a family of shifting matrices, one for each non-negative frequency. If $\mathrm{Sh}_{|f|}$, for $0\le|f|\le 84$ is such a family, then we may define the overall shift by

$$\mathrm{SpSh}(\varphi)=\frac{1}{168}\sum_{f=-83}^{84}\left(\hat{\varphi}(f)\ e^{2\pi i f h/168}\right)_{0\le h\le 167}\cdot \mathrm{Sh}_{|f|}.$$

$\mathrm{SpSh}(\varphi)=v$ is then a row vector whose entries v_h define a new function Φ for the hourly shifted consumption in the given week.

2.1. **Distributions on a per-frequency basis.** The advantage of frequency analysis is that we may choose the support of our distributions based on the frequency we are working with. For example, an event happening with frequency 0 is something that is never happening, or equivalently, whose complementary event is happening 168 hours per week, *i.e.*, all the time. This is a constant (flat) signal, that should intuitively remain exactly the same after a change in ToU tariff. Using the same heuristic, a frequency $f=7$ event needs to happen every single day (*e.g.* cooking dinner) and should never be shifted more than $24=168/7$ hours away, with 24 already being quite generous. As such we can define the distribution probability of change from hour h_0 to hour h so as to be completely supported within the interval $[-168/f+h_0,\ h_0+168/f]$. Recall that we are considering here each week as a circle of 168 hours, and that shifting only occurs within a given week, so by interval here we mean an interval on the circle. There is a lot of freedom as to how one chooses to define these distributions, but ultimately, the matrix $\mathrm{Sh}_{|f|}$ whose h_0'th row is the distribution described above ought to only depend on the ToU tariff choice, and of course on $|f|$.

2.2. **Using multiple tariffs.** As we observed, the linear transformation SpSh needs to depend on the choice of ToU tariff function, *i.e.*, the function which describes the hour-by-hour princing over a week. Thus it can be thought of as a function from the space of tariff functions, to the space of 168-by-168 matrices. We should then write $\mathrm{SpSh}(\varphi) = \mathrm{SpSh}(\mathrm{tariff}, \varphi)$, and

$$\mathrm{SpSh}(\mathrm{tariff}, \varphi) = \frac{1}{168} \sum_{f=-83}^{84} \left(\hat{\varphi}(f)\ e^{2\pi i f h/168}\right)_{0 \le h \le 167} \mathrm{Sh}(\mathrm{tariff}, |f|).$$

By identifying φ (resp. $\hat{\varphi}$) with the row vector of its values, we may think of the discrete Fourier transform and its inverse as 168-by-168 matrices. That is we can write $\hat{\varphi} = \varphi \cdot \mathrm{DFT}$, where

$$\mathrm{DFT} = \left(e^{-2\pi i f h/168}\right)_{0 \le f,h \le 167} \ ,$$

and $\varphi = \hat{\varphi} \cdot \mathrm{IDFT}$, where

$$\mathrm{IDFT} = \left(\frac{1}{168}\ e^{2\pi i f h/168}\right)_{0 \le f,h \le 167} \ .$$

Then, we have

$$\mathrm{SpSh}(\mathrm{tariff}, \varphi) = \sum_{f=-83}^{84} \hat{\varphi}(f) \cdot \mathrm{IDFT} \cdot \mathrm{Sh}(\mathrm{tariff}, |f|)$$

and computing the matrix ShM(tariff) whose rows are

$$\mathrm{ShM}(\mathrm{tariff})_f = \mathrm{IDFT} \cdot \mathrm{Sh}(\mathrm{tariff}, |f|)$$

for $-83 \le f \le 84$, independently of φ, allows us to write

$$\mathrm{SpSh}(\mathrm{tariff}, \varphi) = \sum_{\mathrm{rows}} \Big(\mathrm{diag}\left(\varphi \cdot \mathrm{DFT}\right) \cdot \mathrm{ShM}(\mathrm{tariff})\Big).$$

This is a linear transformation in φ, that is a 168-by-168 matrix, which outputs the shift of the consumption φ for a week, according to a fixed tariff function, while still allowing us to handle each frequency individually. Shifting the consumption data of multiple weeks then simply boils down to computing the matrices ShM(tariff) for each different tariff we want to use, and then shifting each week by the appropriate matrix for that tariff.

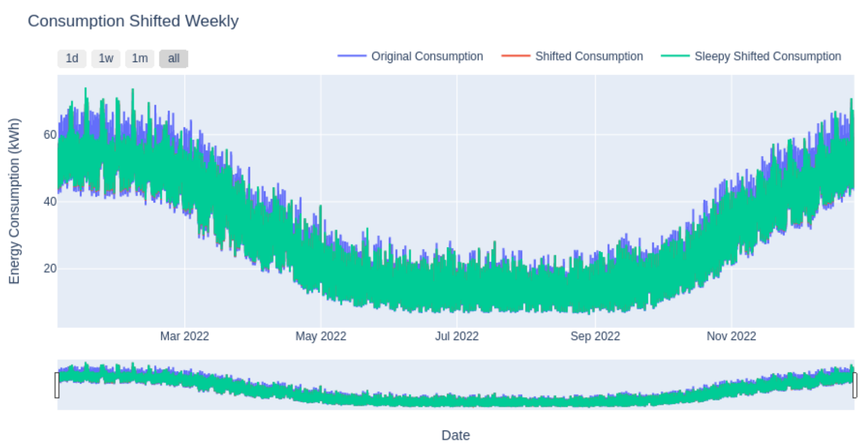
3. Results

Both of our models show that Time of Use tariffs can provide noticeable improvements to stress on the total grid due to peak hours, despite our conservative assumptions. Each method yielded different results, so we present them separately, but the results are over the same period of time: January 3, 2022 through December 25, 2022 (every full Sunday-Monday week in 2022).

3.1. **Method 1: Shiftable Percentages.** Because our Time of Use scheme is focused around residential users, we'll first consider just the effect on peaks in residential consumption, and then how those peaks fit into the consumption data for the entire grid.

3.1.1. *The Residential Data.* In Figure 6, we show the aggregated residential consumption for each hour in our time period before and after the two variations on our shift described in Section 1.5.

FIGURE 6. Residential consumption Jan 3, 2022 through Dec 25, 2022. The original consumption is in blue, the shifted consumption is in red and the sleepy shifted consumption is in green.



It is also enlightening to consider the way that consumption shifts over different time scales. For example, in (Figure 7), the graph of weekly consumption before and after shifting suggests how consumption may change on the consumer level.

Examining the residential data before and after a shift, we see that in some places (particularly in the winter months, see Figure 8) the peak consumption increases after a shift. However, the hour-by-hour data in Figure 8 contains unnecessary information about hourly variation, which threatens to obscure understanding of the peaks, especially when studying longer time scales.

Instead, we consider the daily maximum consumption for days in our window (Figures 9 and 10). These take the maximum of the consumption for each day instead of considering the consumption per hour and make it apparent that the peak consumption on certain days increases after the shift.

Over the period from Jan 3, 2022 to Dec 25, 2022, the peak residential consumption in the raw data was 72.77 kWh. After our Time of Use shift, this increases to 73.93 kWh, and after a sleepy shift the peak consumption is 74.12 kWh. This increase is because our Time of Use scheme incentivizes residential users to shift some of their consumption to the weekends, as our ToU scheme lists the entirety

FIGURE 7. Residential consumption for Monday, Feb 14, 2022 through Sunday, Feb 20, 2022. The original consumption is in blue, the shifted consumption is in red and the sleepy shifted consumption is in green.

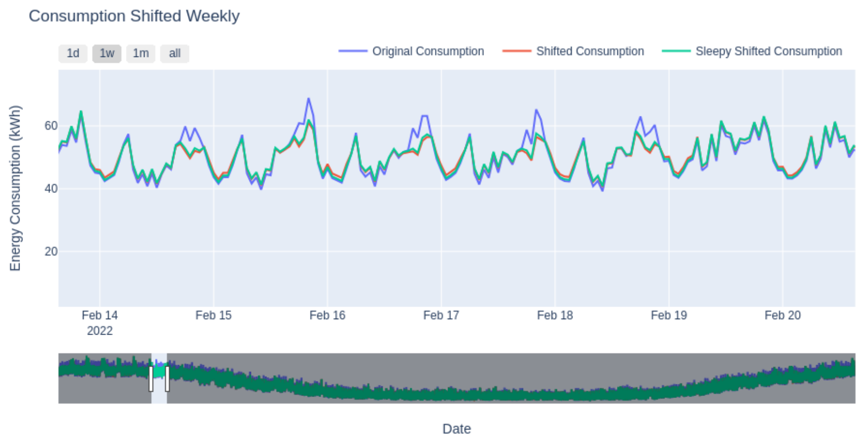
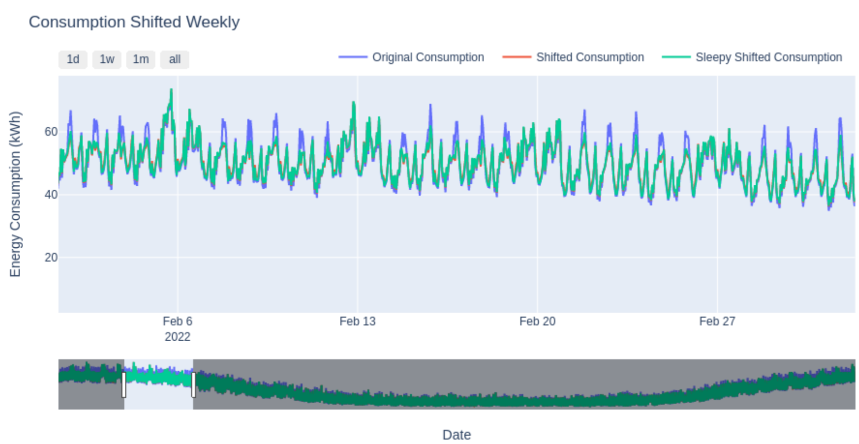


FIGURE 8. Residential consumption before and after a shift and a sleepy shift, restricted to February 2022. Notice that the maximum is higher after the shift than in the raw data.



of weekends as off-peak hours. This is a common practice in jurisdictions such as Ontario which use ToU pricing, as commercial and industrial consumption tend to be significantly lower on weekends [2]. Likewise, in the `awefice` grid, this increase

FIGURE 9. The daily maximum residential consumption before an after a shift and a sleepy shift.

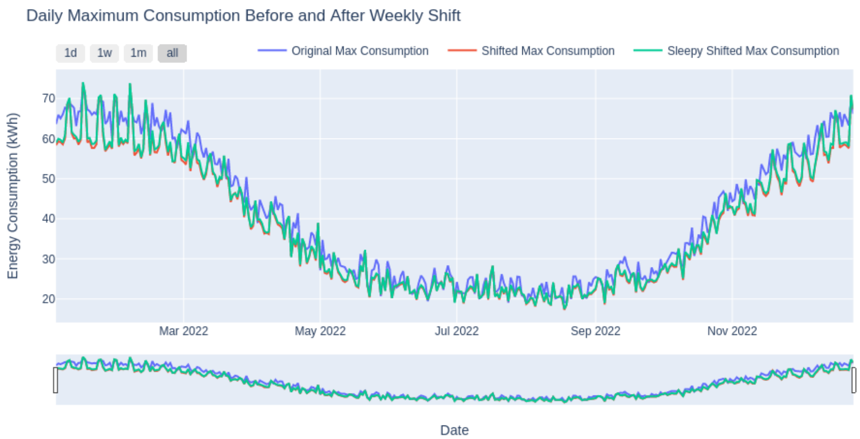
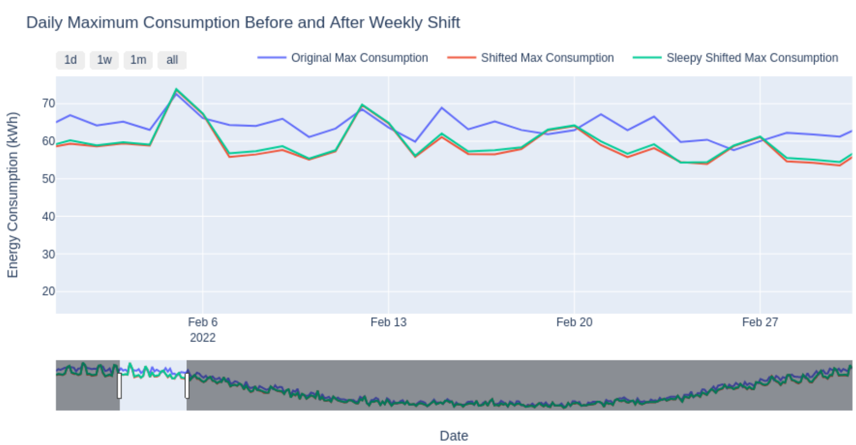


FIGURE 10. The daily maximum residential consumption before and after a shift and a sleepy shift, restricted to February 2022. Notice that the maximum is higher after the shift than in the raw data.



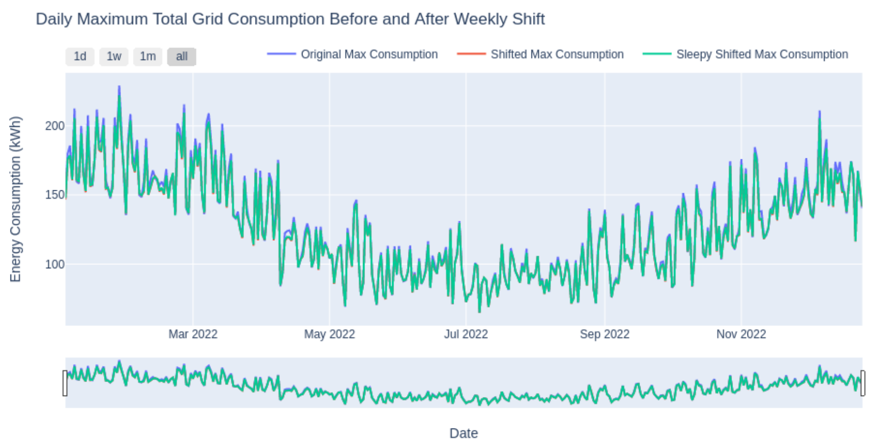
in residential use on the weekends is more than compensated for when we factor in commercial and industrial use.

3.1.2. *The Total Grid.* Here we add the (unchanged) commercial and industrial consumption to the unshifted, shifted, and sleepy shifted residential consumption.

Similarly to above, we find it most helpful to consider the daily maximum consumption instead of the hour by hour consumption because any peak consumption will be captured in the daily maximum.

Compare Figure 12 to the residential consumption in Figure 10 and note that the increase in the peak of the residential consumption is compensated for by the decrease in commercial and industrial consumption, which has the effect of flattening of the overall peak. For the total grid, the pre-shift peak is 229.20 kWh. After a shift of the residential consumption, this is reduced to 221.21 kWh (a decrease of 3.5%), and the sleepy shift reduces the peak to 222.19 (a decrease of 3%).

FIGURE 11. The daily maximum consumption for the entire `awefice` grid before an after a shift and a sleepy shift.



3.2. **Method 2: Frequencies.** For consistency, we examine the same time period as above: Jan 3, 2022 to Dec 25, 2022.

3.2.1. *The Residential Data.* As with the shiftable percentages method described in Section 3.1.1, we first examine only the aggregated residential consumption during our testing period. Figure 13 shows the hourly residential consumption on the `awefice` grid before and after using the Fourier transform and frequencies method described in section 2 and Figure 14 shows the consumption over a single week.

Again it is helpful to consider just the daily maximum consumption, since we're interested in flattening peaks. Also, as above, the peaks occur in the winter months, so in Figure 15, we restrict to February 2022.

Over the period from Jan 3, 2022 to Dec 25, 2022, the peak residential consumption in the raw data was 72.77 kWh. After a frequency shift, the peak residential consumption over that period was 62.57 kWh (a decrease of 14%).

FIGURE 12. The daily maximum consumption for the entire `awefice` grid before and after a shift and a sleepy shift, restricted to February 2022.

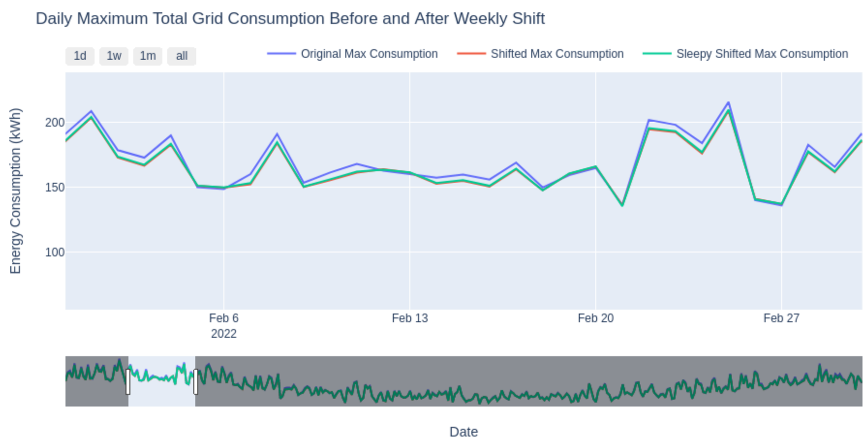
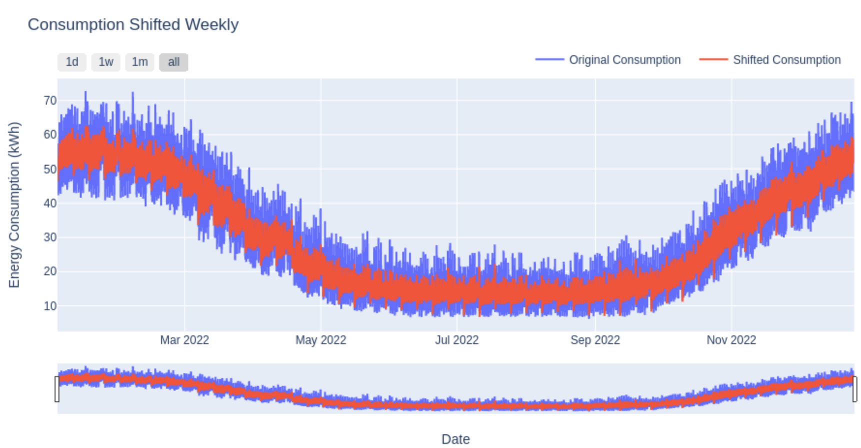


FIGURE 13. The hourly residential consumption before an after frequency shift with the original use in blue and the shifted use in red.



3.2.2. *The Total Grid.* We are interested in the impact of Time of Use on the total grid, so as before, we can combine our shifted residential consumption with the unchanged commercial and industrial consumption to see the total effect on the grid. Figure 16 shows the hourly consumption, and Figure 17 shows the daily maximum consumption restricted to February 2022.

FIGURE 14. Residential consumption for Monday, Feb 14, 2022 through Sunday, Feb 20, 2022. The original consumption is in blue and the shifted consumption is in red.

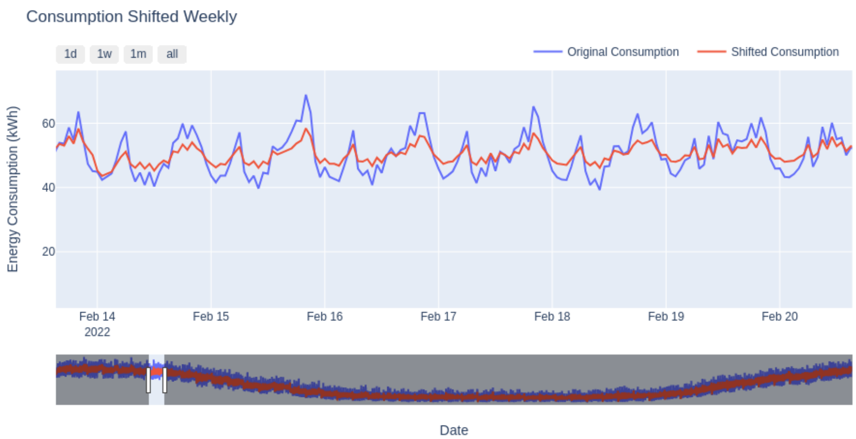
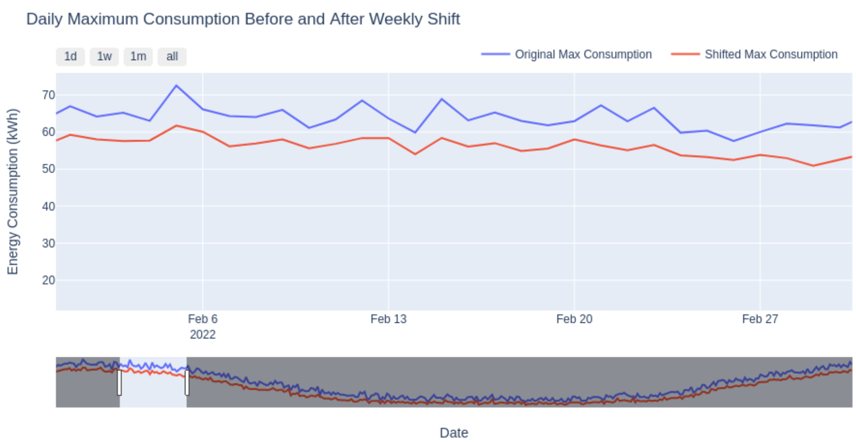


FIGURE 15. The daily maximum residential consumption before and after a frequency shift, restricted to February 2022.



The consumption for the total `awefice` grid over this time period was 229.20 kWh, and after shifting the residential consumption the peak for the total grid is 226.31 kWh (a decrease of 1.3%).

Figure 16. The hourly consumption for the entire `awefice` grid before an after a frequency shift.

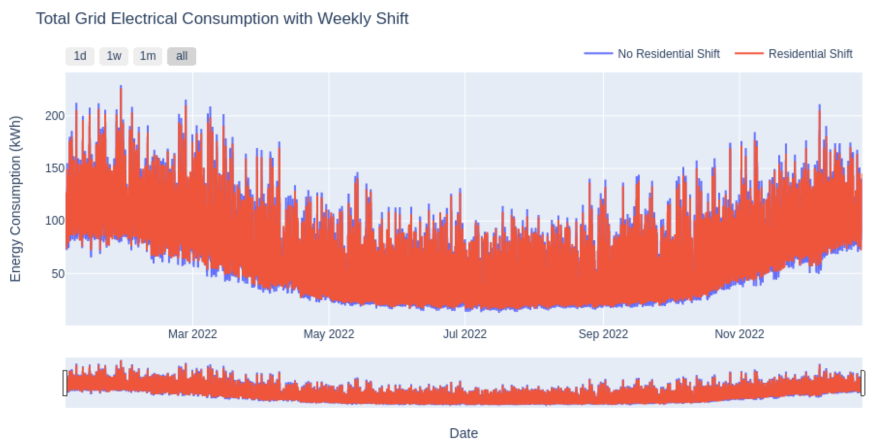
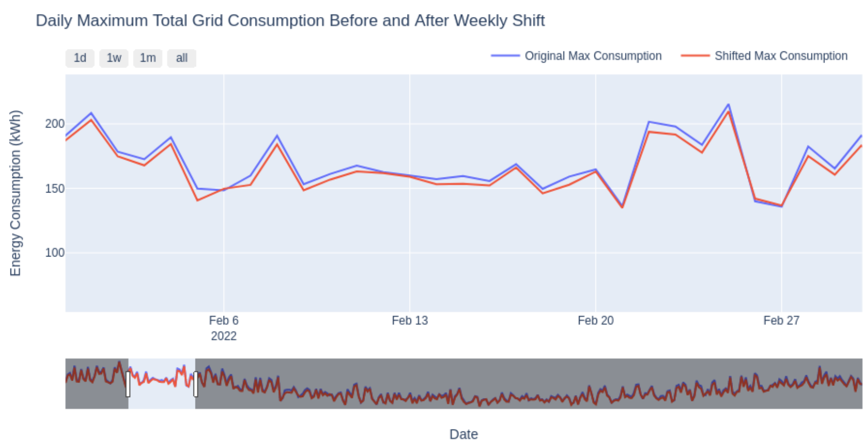


Figure 17. The daily maximum consumption for the entire `awefice` grid before and after a frequency shift, restricted to February 2022.



4. Conclusion and Future Work

Our work shows that Time of Use tariffs could be effective at lowering the peak consumption of the overall grid. The shiftable percentages model shows a 3.5% decrease in the total peak, and the Fourier transform method shows a 1.3% decrease in the total peak. Each model is based on conservative assumptions, so it is quite possible that real-world ToU pricing will have a larger impact than either. Additionally, neither method explicity considers electric vehicle chargers which could be programmed to run only overnight, and neither considers photovoltaics. Consumers who have photovoltaic panels on their homes may choose to shift some of their consumption to bright daytime hours instead of overnight so they can take advantage of the electricity generated by the panels.

In future work, we would incorporate these elements, as well as potentially different behaviour in the summer vs the winter. Awesense's simulated `awefice` grid is set in Vancouver where summer consumption is lower, so in this case, we deemed it unnecessary to create a different ToU regime for the summer months. However, in a warmer climate where air conditioning is more common, and especially in a continental climate which experiences hot summers and cold winters, it may be useful to include multiple tariff schemes which depend on the season. Either of our current models could easily be adjusted for this by constructing a different tariff function representing a different Time of Use billing scheme for summer months and then shifting seasons separately.

Finally, another element that could be considered is commercial use and different pricing schemes for commercial and industrial Time of Use. We assumed that most industrial consumption cannot be shifted, but the Awesense simulated *commercial* data implies that some commercial consumption could be shifted with the correct incentive. In particular, the `awefice` grid contains a bus depot where electric busses are charged. Similarly to residential EV chargers, these chargers could be programmed to charge overnight and this represents a significant potential decrease in consumption during peak hours. Not all commercial EV chargers would have this behaviour (as some might be in use by employees parked at work), but one could use the detailed Awesense EV charger data to explore this further.

5. Acknowledgements

The authors would like to thank the Pacific Institute for the Mathematical Sciences and the Math to Power Industry program for giving us the opportunity to work on this project. We are also grateful to the team at Awesense, in particular Aviv Fried and Elena Popovici for their guidance and insight on this project, and Dr. John Sang Jin Kang for his support.

References

1. Søren Asmussen, *Applied Probability and Queues*, Springer, 2003.
2. Ontario Energy Board, *Electricity rates*, oeb.ca https://www.oeb.ca/consumer-information-and-protection/electricity-rates, May 2023.
3. Natural Resources Canada, *About electricity*, https://natural-resources.canada.ca/our-natural-resources/energy-sources-distribution/electricity-infrastructure/about-electricity/7359#reliability, June 2020.
4. Rick Durrett, *Probability: Theory and Examples, 5th Ed.*, Cambridge University Press, 2019.
5. Jordan Wirfs-Brock, *IE Questions: Why Is California Trying To Behead The Duck?*, InsideEnergy.org, https://insideenergy.org/2014/10/02/ie-questions-why-is-california-trying-to-behead-the-duck/, October 2014.

University of Toronto
E-mail address: cameron.davies@mail.utoronto.ca

University of Toronto
E-mail address: patmroy@math.utoronto.ca

University of Calgary
E-mail address: mishty.ray@ucalgary.ca

University of Calgary
E-mail address: mohamar.rios@gmail.com

University of Manitoba
E-mail address: meredithsargent@gmail.com