Optimizing Beamforming Vectors for Uniform Linear Arrays

This presentation explores methods for calculating optimal beamforming vectors for uniform linear arrays.

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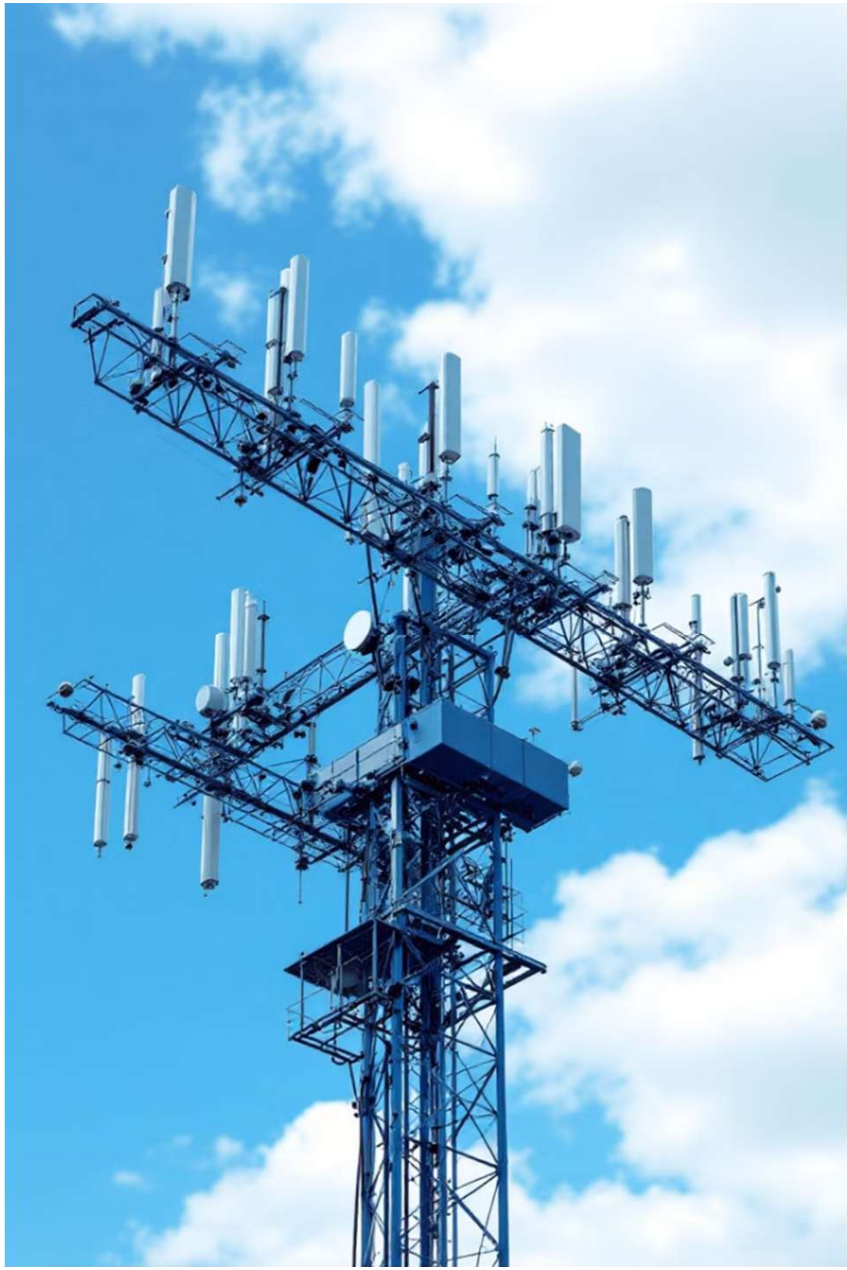
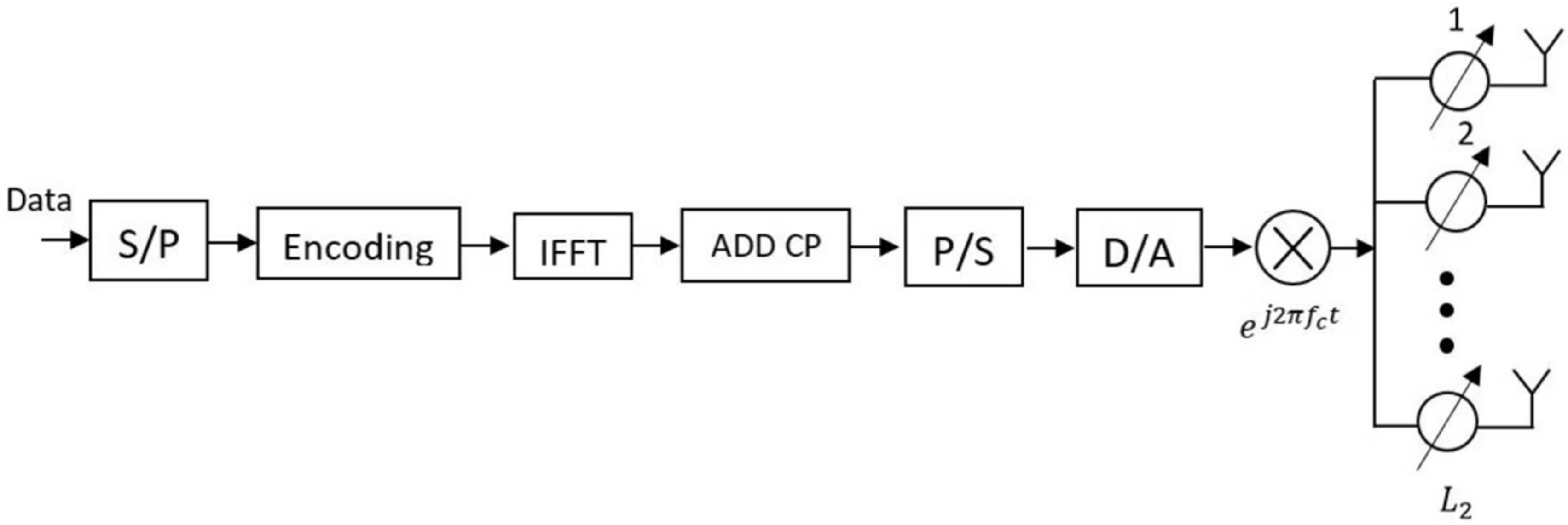


Optimal Beamforming Vector Calculation



Objective

Maximize signal strength in desired direction

Minimize interference from other directions

Constraints

Multi-dimensional problem

Non-convex, no closed-form solution

Solutions

Problem Formulation

$$\max_{u_l \forall l} \sum_{n=1}^{N} \sum_{k=1}^{K} \log_2 \left(1 + \frac{\left|\mathbf{u}^T \mathbf{h}_n(k)\right|^2}{\xi_1}\right)$$

$$S.T. \quad |u_l| = 1,$$

We're maximizing Sound-to-Noise ratio of signal using b-bit phase shifters with phases u from P:

$$\mathcal{P} = \left\{0,\ \frac{2\pi}{2^b},\ 2 \times \frac{2\pi}{2^b},\ 3 \times \frac{2\pi}{2^b}, \cdots,\ (2^b - 1) \times \frac{2\pi}{2^b}\right\}.$$

Successive Convex Optimization

$$\max_{u_l \forall l} \sum_{n=1}^{N} \sum_{k=1}^{K} \log_2 \left(1 + c_n(k)\right)$$

$$S.T.\ C_1 : |u_l| \leq 1 \quad \forall l$$

$$C_2 : a_n(k) = \Re\left\{\frac{\mathbf{u}^T \mathbf{h}_n(k)}{\xi_1}\right\} \quad \forall n$$

$$C_3 : b_n(k) = \Im\left\{\frac{\mathbf{u}^T \mathbf{h}_n(k)}{\xi_1}\right\} \quad \forall n$$

$$C_4 : c_n(k) \leq f(a_n(k), b_n(k)) \quad \forall n,$$

$$f(a_n(k), b_n(k)) = \tilde{a}_n(k)^2 + \tilde{b}_n(k)^2 + 2\tilde{a}_n(k)\left(a_n(k) - \tilde{a}_n(k)\right) + 2\tilde{b}_n(k)\left(b_n(k) - \tilde{b}_n(k)\right),$$

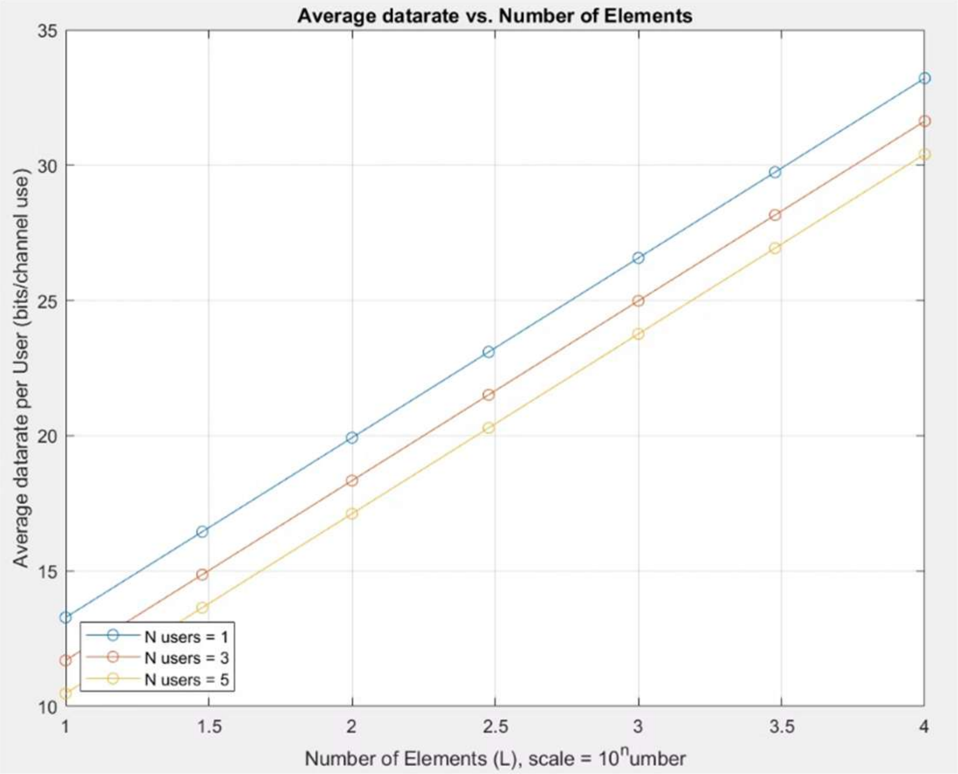
Brute-force search

We have finite number of possible beamforming vectors for b-bit phase shifters, so we can use brute-force search using cupy

$$\max_{u_l \forall l} \sum_{n=1}^{N} \sum_{k=1}^{K} \log_2 \left(1 + \frac{\left|\mathbf{u}^T \mathbf{h}_n(k)\right|^2}{\xi_1}\right)$$

$$S.T. \quad |u_l| = 1,$$

Convex Optimization Approach



I've used MATLAB CVX library for convex optimization

Computation time: 13882 s

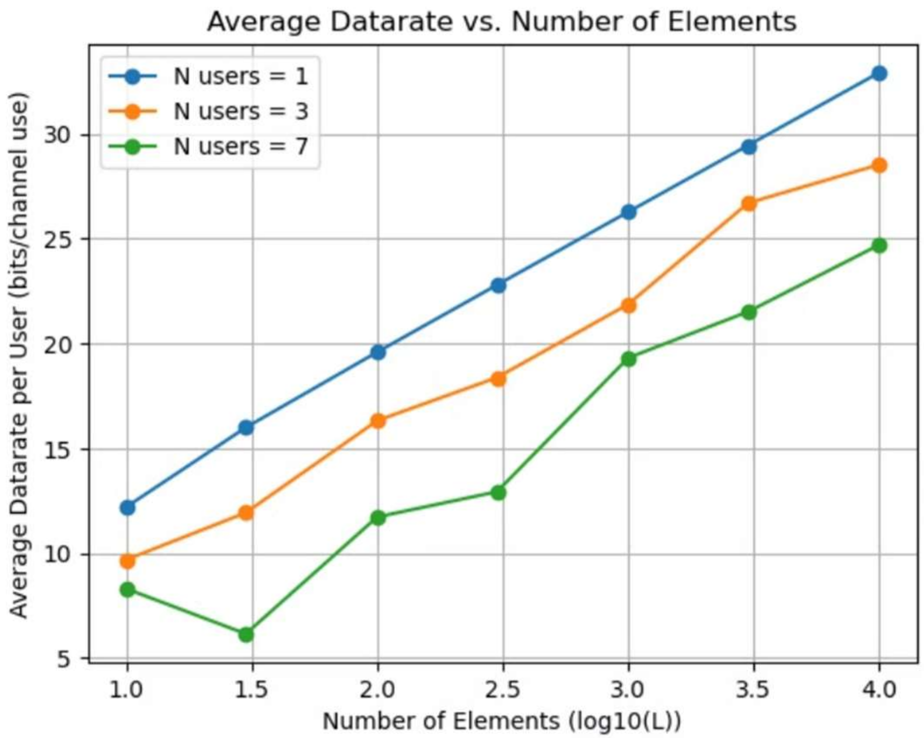
Computation complexity: O(K^4.5; N^3.5; L)

where N is the number of users,

K is the number of OFDMA sub-carriers per user

L is the number of antennas

Brute-force search



I've used Python with cupy library

Computation time: **1034 s**

Computation complexity: O(K; N; L, 2^b)

where N is the number of users,

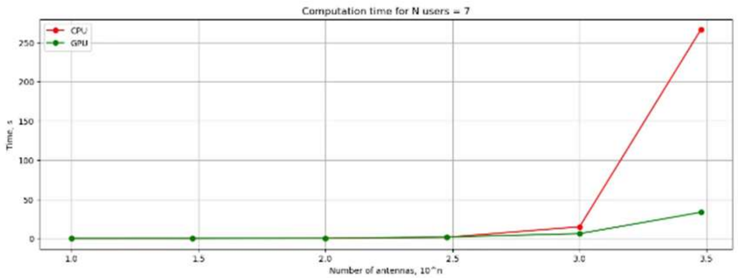
K is the number of OFDMA sub-carriers per user

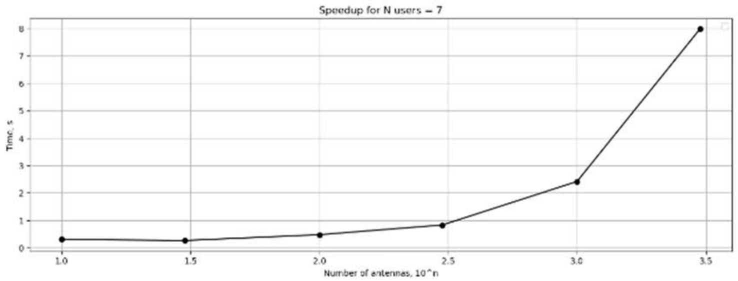
L is the number of antennas

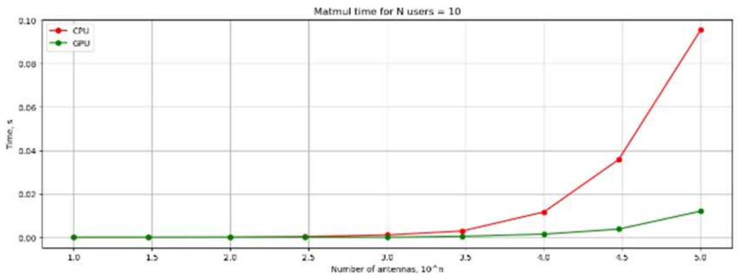
2^b is the number of possible phase-shifts

Results are almost the same as with CVX

Results Comparison







Comparing njit and cupy,

Brute-force search is 8 times faster using GPU for large antenna arrays (L = 10^4). It's because most computation time is matrix multiplication, which is 8 times faster.

Comparing cupy and CVX,

cupy is 13 times faster for large antenna arrays (L = 10^4)

Method	Computation Time (s)
njit	9231 s
cupy	1034 s
CVX	13882 s

Conclusion

Convex Optimization

Effective for calculating optimal beamforming vectors for small antenna arrays, provides optimal solution for continious phase-shifts

Cupy

Accelerates computation and improves scalability for big antenna arrays, provides optimal solution for discrete phase-shifts

Future Work

Explore combination of CVX and GPU-accelerated computations