Verified Security of Hashed Authenticated Key-Exchange Protocols

Work in progress

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Key Exchange Protocols

A pool of n participants try to establish a comon symmetric session key, in the presence of an adversary.

Security Goals

Definition (Key privacy)

No one except the legitimate parties can distinguish the established key from a random key.

Definition (implicit key authentication)

In presence of a passive adversary, two legitimate parties engaged in a session, will compute the same key.

Not to be confused with key confirmation: no one can make a legitimate partner believe he shares a key with a partner, unless this is the case.

Diffie-Hellman

$$
\begin{array}{ccc}
A & & B \\
x \xleftarrow{\$} \mathbb{Z}_q & \xrightarrow{\quad A,\, X = g^x \quad} & \\
 & \xleftarrow{\quad B,\, Y = g^y \quad} & y \xleftarrow{\$} \mathbb{Z}_q
\end{array}
$$

Hashed DH-based key Exchange Protocols

- Participants exchange messages that allow each party to compute at the end of a session a bitstring, called *session string*.
- The session key is derived from the session string by applying a hash function.

Two types of protocols :

- the flow of messages is that of DH; it is the way the session string is computed that guarantees authentication. May suppose public keys, or symmetric keys,... . Matsumoto, Takashima and Imai'86
- the flow of messages is modified modified, e.g., by adding signatures, encryptions, further rounds,.... Bellovin and Merrit'92

HMQV

A variant of MQV.

$$A = g^a,\ B = g^b$$

$$\begin{array}{ccc}
A & & B \\
x \xleftarrow{\$} \mathbb{Z}_q & \xrightarrow{\quad A,\, X = g^x \quad} & \\
 & \xleftarrow{\quad B,\, Y = g^y \quad} & y \xleftarrow{\$} \mathbb{Z}_q
\end{array}$$

$$\textsc{ses}(A, B, g^x, g^y) = g^{(x+H_1(B,X)a)(y+H_1(A,Y)b)}$$

$$K = H(\textsc{ses}(A, B, g^x, g^y))$$

Initializaton:
n participants, for each participants:

$$a \xleftarrow{\$} \mathbb{Z}_q;\ A = g^a$$

$$\begin{array}{ccc} A & & B \\ x \xleftarrow{\$} \{0,1\}^\lambda & \xrightarrow{\quad A, X = g^{H_1(x,a)} \quad} & \\ & \xleftarrow{\quad B, Y = g^{H_1(y,b)} \quad} & y \xleftarrow{\$} \{0,1\}^\lambda \end{array}$$

$$K = H(\textsc{ses}(A, B, X, Y)),\ \text{where}$$
$$\textsc{ses}(A, B, X, Y)) = (Y^a, B^{H_1(x,a)}, Y^{H_1(x,a)}, A, B)$$

Generic Protocol

Initializaton:
n participants, for each participants:

$$a \xleftarrow{\$} \mathsf{PvK};\ A = \mathsf{pv}(a)$$

$$\begin{array}{ccc}
A & & B \\
x \xleftarrow{\$} \mathsf{EphK} & \xrightarrow{\quad A, X = \mathsf{init}(x, a, r) \quad} & r' \xleftarrow{\$} U \\
r \xleftarrow{\$} U & \xleftarrow{\quad B, Y = \mathsf{resp}(y, b, r') \quad} & y \xleftarrow{\$} \mathsf{EphK}
\end{array}$$

$$K = H(\textsc{ses}(A, B, X, Y))$$

$\textsc{ses}(A, B, X, Y))$ is called the session string.

Protocol model

- A protocol is described by two roles:
  1. An initiator with actions defined by oracles
     - $\mathbf{Init}(A, B)$ starts a session (A, B, X), where A is the initiator and B is the responder, and returns X.
     - $\mathbf{Comp}(A, B, X, Y)$ completes the incomple session (A, B, X), if it exists.
  2. A responder with actions defined by oracle $\mathbf{Resp}(A, B, X)$ that completes a session (A, B, X, Y) and returns X.

Security Model

Queries that define the adversary capabilities

- Corruption: $\mathbf{Cor}(A)$ returns the private key of A
- Key reveal: $\mathbf{KeyR}(A, B, X, Y)$ reveals the session key: A checks that she has a *completed* session (A, B, X, Y). If not, she ignores the call. Otherwise, she outputs the session key.
- Ephemeral values reveal: $\mathbf{EphR}(A, B, X, -)$ reveals the ephemeral random values of a session: A checks that she has an open session $(A, B, X, -)$. If not, she ignores the call. Otherwise, she outputs the ephemeral key sampled within the above session.

Security Model

A query for defining the security game

$\mathbf{Test}(A, B, X, Y)$ depending on a randomly chosen bit b, A returns either the actual session key of the session (A, B, X, Y), or a session key drawn randomly from the session key distribution.

Unexposed sessions

Fresh sessions

A completed session (A, B, X, Y) is fresh (clean, unexposed), if the following conditions hold:

- $\mathbf{Cor}(A) \implies \mathbf{EphR}(A, B, X, Y)$ has not been asked.
- $\mathbf{Cor}(B) \implies \mathbf{Hon}(B, Y)$ and $\mathbf{EphR}(B, -, Y, -)$ has not been asked.
- neither $\mathbf{KeyR}(A, B, X, Y)$ nor $\mathbf{KeyR}(B, A, Y, X)$ has been asked.

In order to make the security definition meaningful, the adversary should only run a $\mathbf{Test}$ query on *unexposed* sessions.

Covered Security Properties

- Key-compromise impersonation: the adversary learns a long-term secret key of a party and then impersonates others to this party.
- Weak Perfect Foward Secrecy: Perfect forward secrecy means that established keys remain secret, even when the long-term keys are known after the session is completed. Krawczyk provides a generic attack for 2-message implicitly authenticated DH-baed protocols.
  Weak Perfect Serecy: the same but for passive sessions.
- It covers eCK^{ω} proposed by Cas Cremers.

Objectives

Kudla and Paterson reduce the security of an AKE protocol to the security of a similar protocol without $\mathbf{KeyR}$ and $\mathbf{Test}$ and where the adversary has to guess a session string. The reduction uses Gap DH.

Our objective:

1. Develop an automated proof of an extended version of the Kudla and Paterson reduction, in EasyCrypt. The reduction is in an extended model of CK (including $\mathbf{EphR}$), does not rely on Gap DH by construction, removes $\mathbf{KeyR}$, $\mathbf{Test}$ and $\mathbf{EphR}$, and reduces to three attacks.
2. Apply the reduction to prove security of Naxos, HMQV, etc...

A sample attack

Consider the following oracles:

1. $\mathbf{SameSeS}$: given two sessions, $\mathbf{SameSeS}$ decides whether they have the same session string
2. $\mathbf{EqS}$: given a session and a bitstring, $\mathbf{EqS}$ decides whether the latter is the session string of the former.

1. The adversay is given:
   - the long-term key a of A,
   - a message X computed with a and an ephemral value x,
   - access to oracles run by a party B,
   - $\mathbf{SameSeS}$ and $\mathbf{EqS}$.

   He wins, if he can provide a message β and a bitstring bs such that bs is the session string of (A, B, X, β).

EasyCrypt

A proof is a sequence of games related by **claims**.
Automatic or at least tool-supported verification such claims.

Rationale

- Claims are based on probabilistic statements that have a direct translation to relational Hoare judgments
- Verification of Hoare judgments reduces to the validity of formulae - verification conditions. Such verification conditions are automatically computed by EasyCrypt and submitted to SMT solvers.
- Invariant generation for oracles.
- Code-based sound transformations, e.g., eager sampling, ...

```
KG() : public_key
Init(A : part, B: part) : msg optio
Respond(B : part, A : part, X : msg) : msg option
Complete(A:part, B:part, X:msg, Y:msg) : unit
```

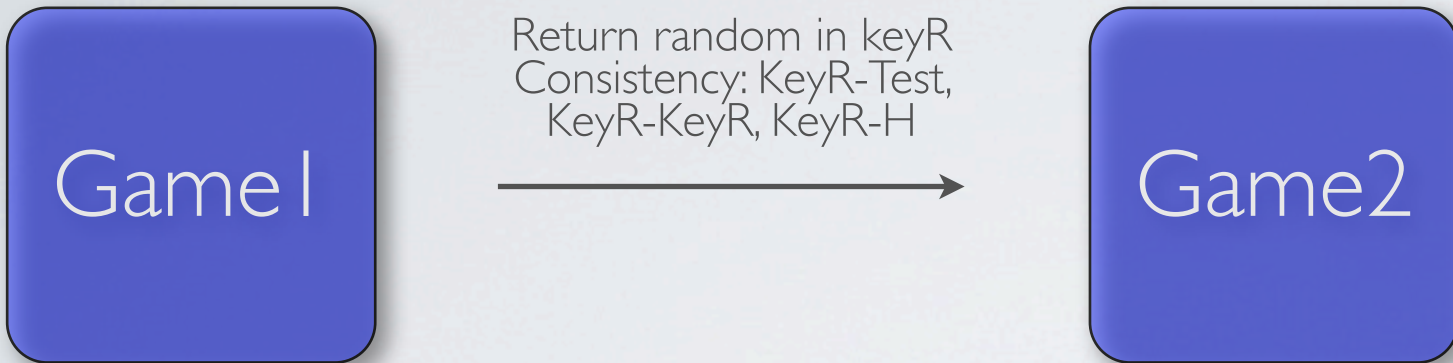
```
Hash(str : session_string) : session_key
Corrupt(A : part) : private_key option
EphKeyReveal(A : part, X : msg) :  eph_key option
KeyReveal(s : session_id) : session_key
Test(s : session_id) : session_key
```

```
fun Main () : bool = {
 var b' : bool;
 var tt : unit;
 complete_sessions = empty_map;  incomplete_sessions = empty_map;
 corrupt = empty_map;  pkey = empty_map;
 skey = empty_map;  LH  = empty_map;
 LHT  = empty_map;  seed  = empty_map;
 tested_session = empty_map;  G = empty_map;
 b = {0,1};
 b' = A();
 return (b = b');
}
```

GAME SEQUENCE

Game 1

GAME SEQUENCE



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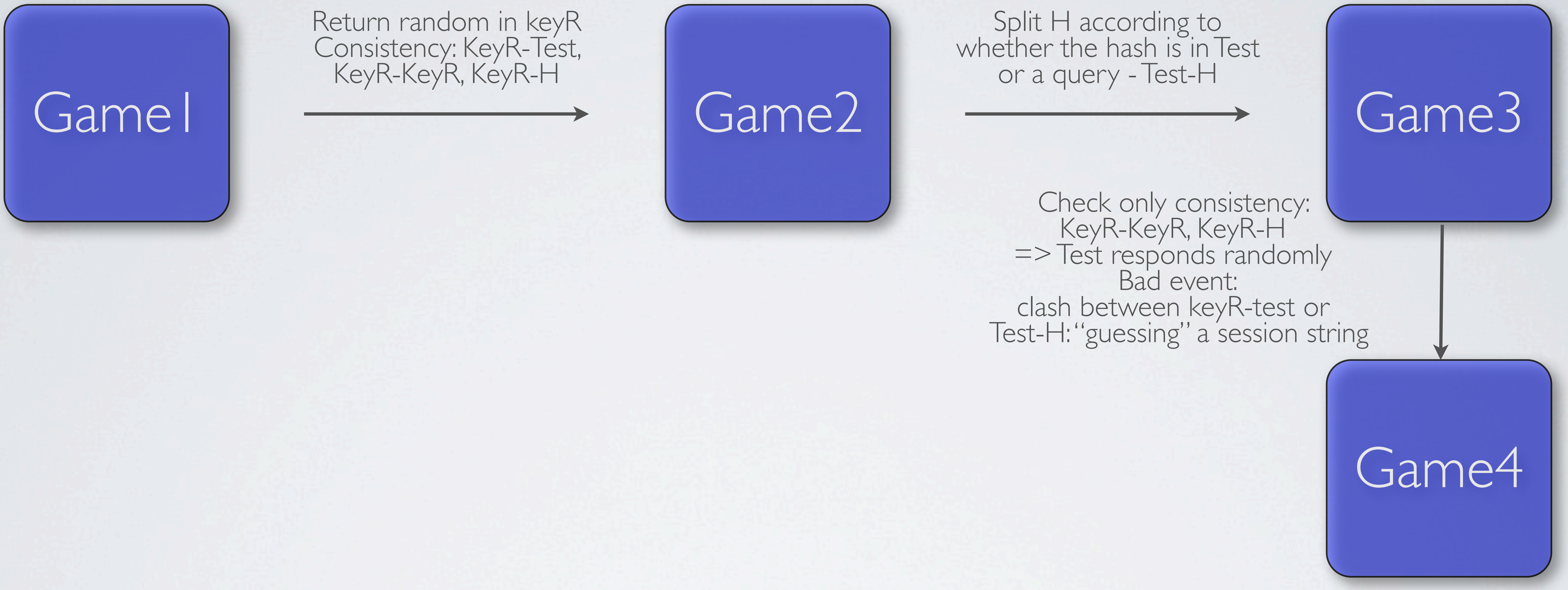

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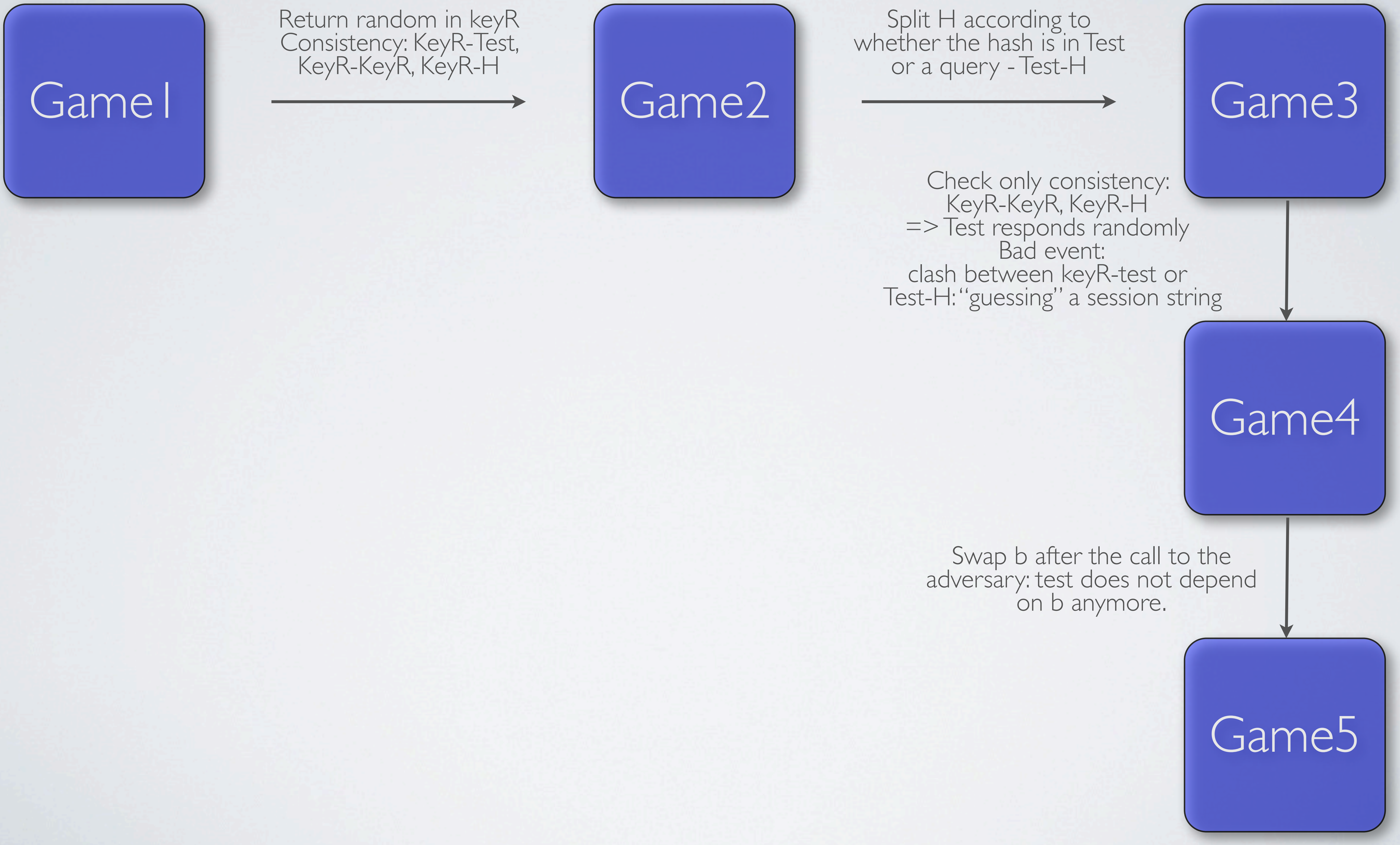

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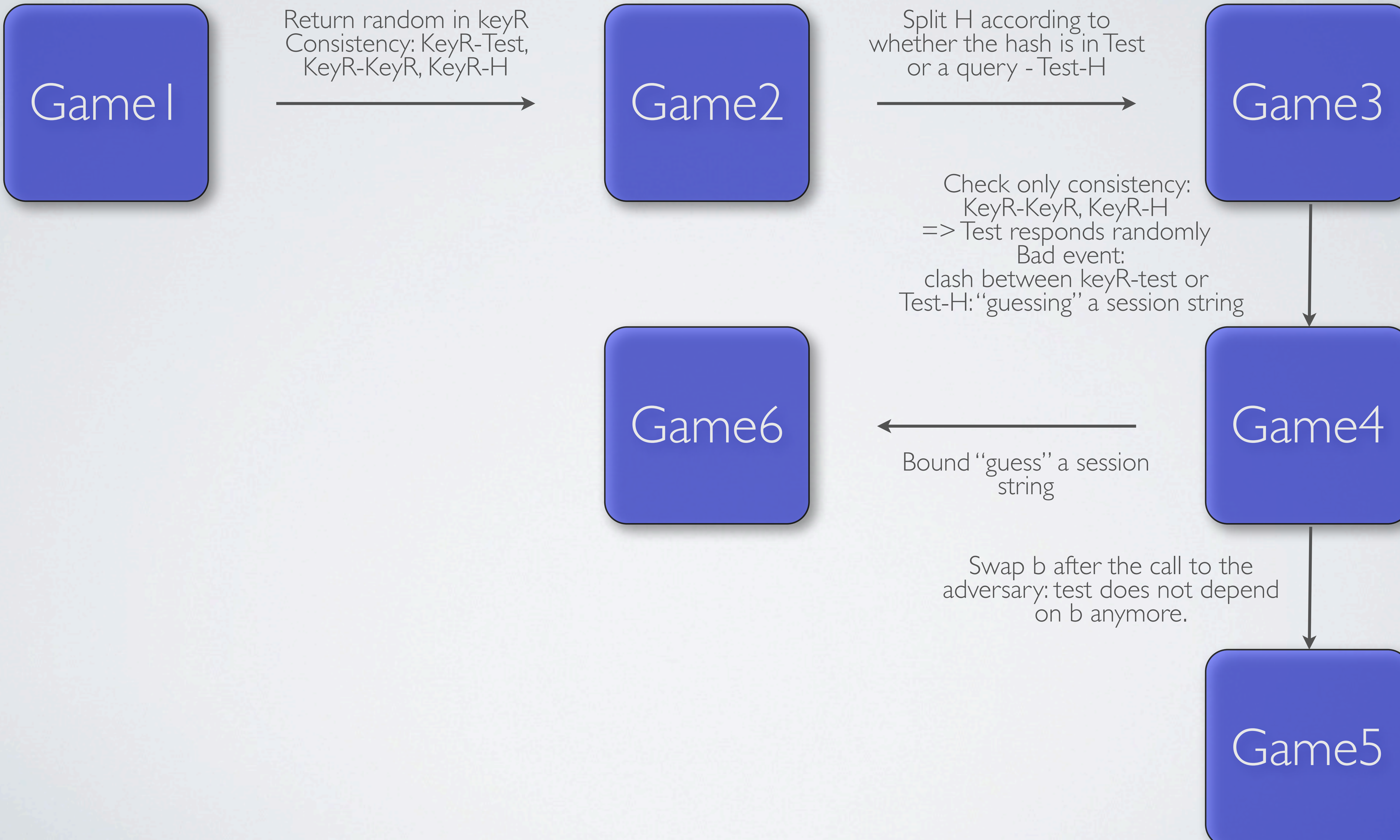

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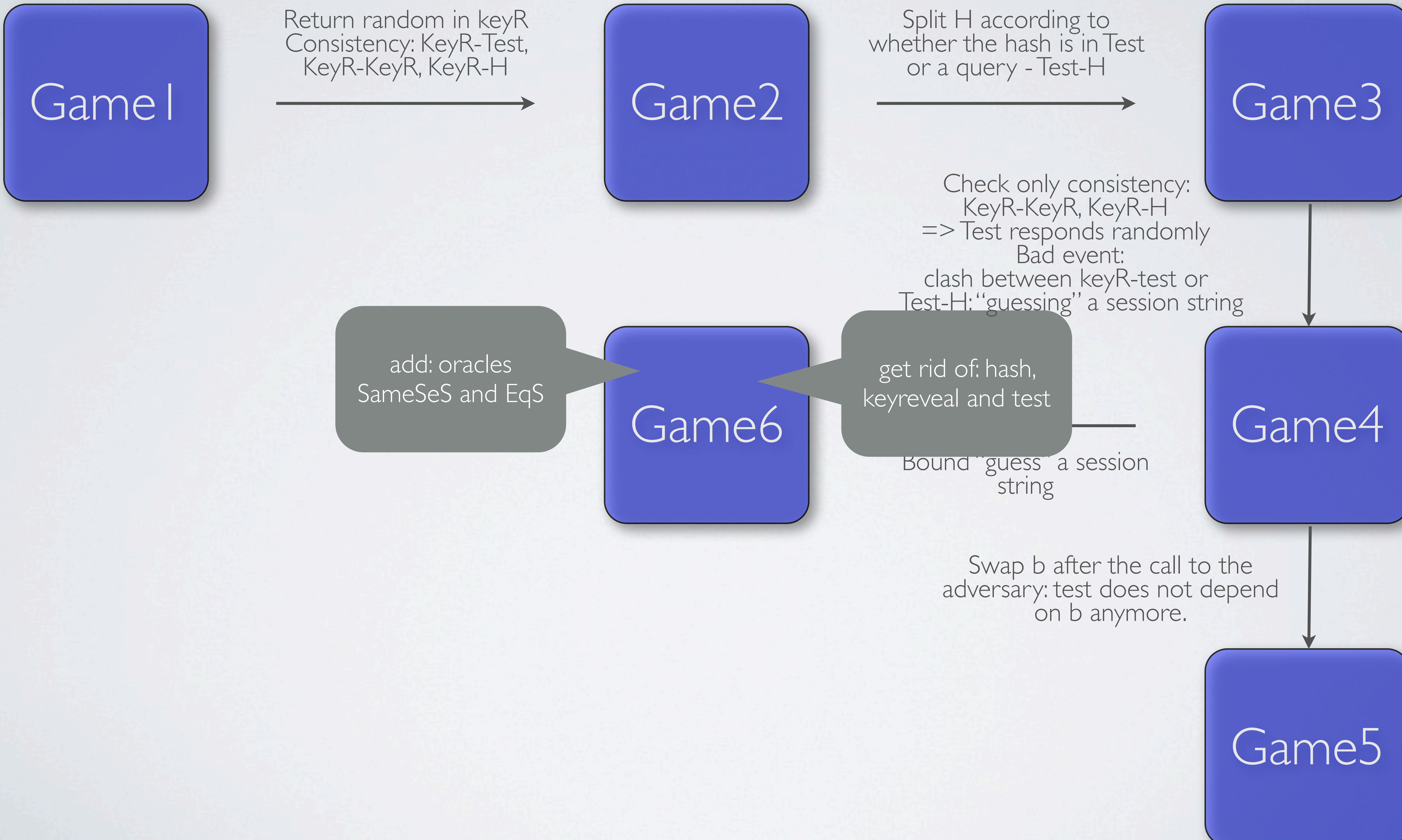

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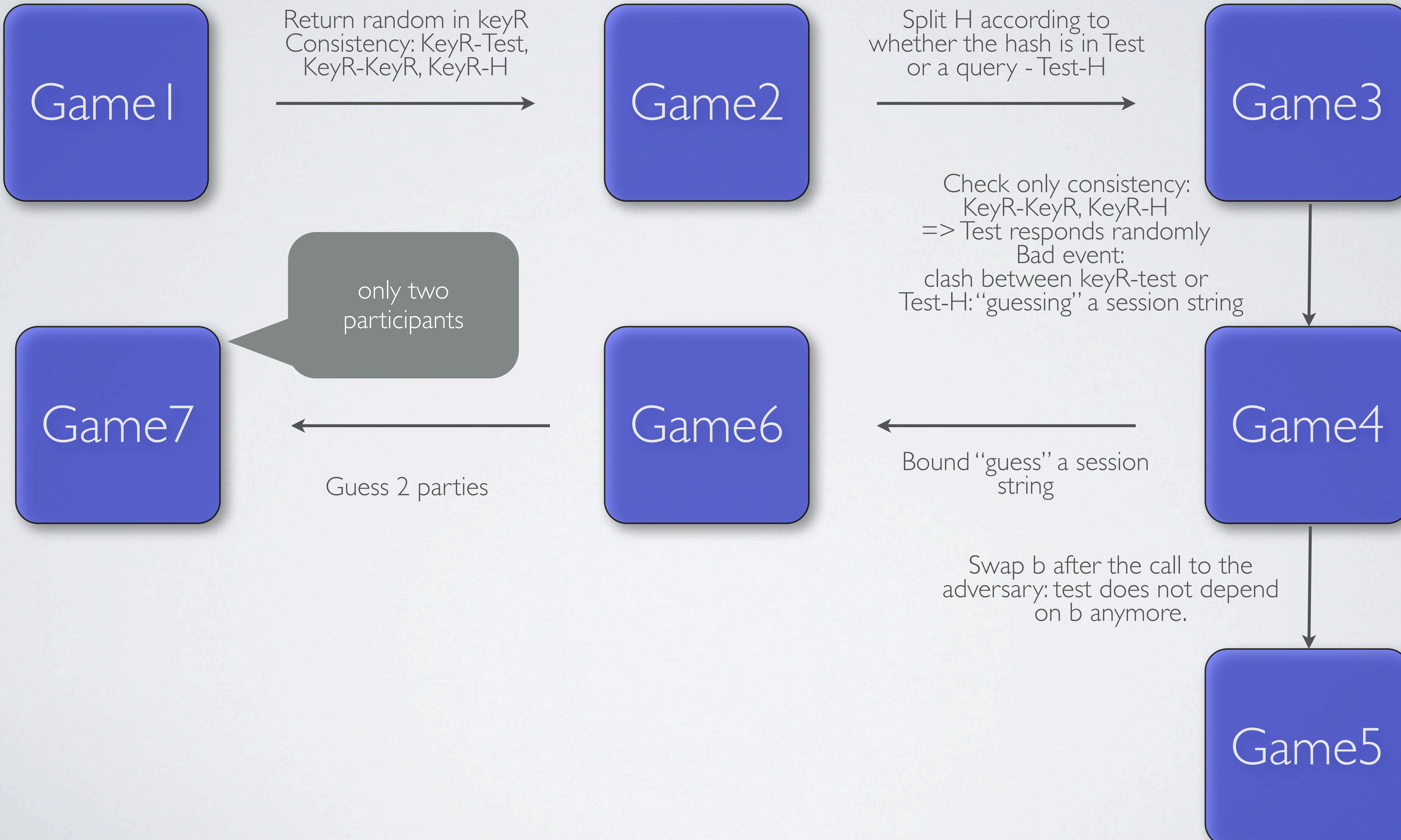

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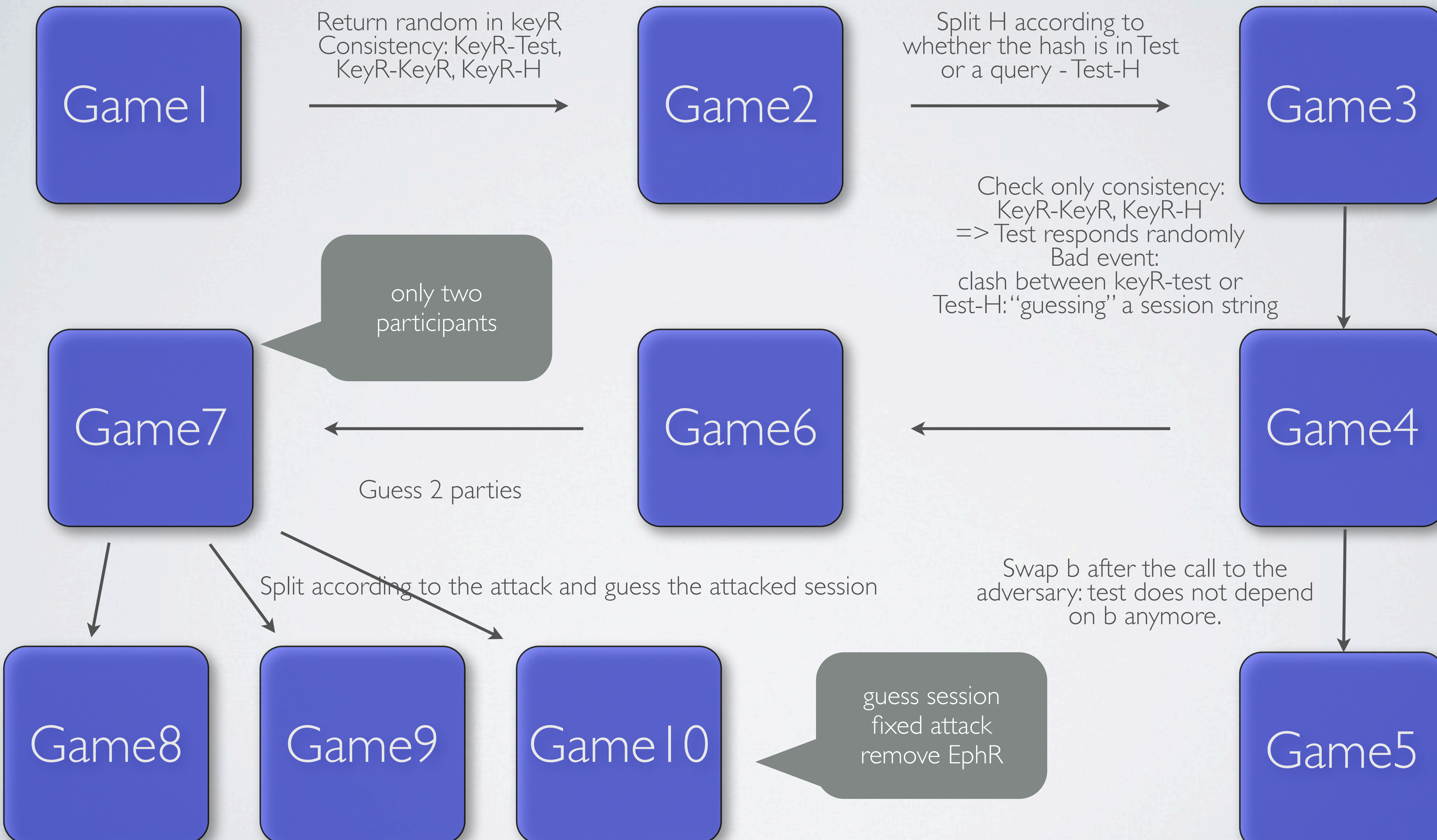

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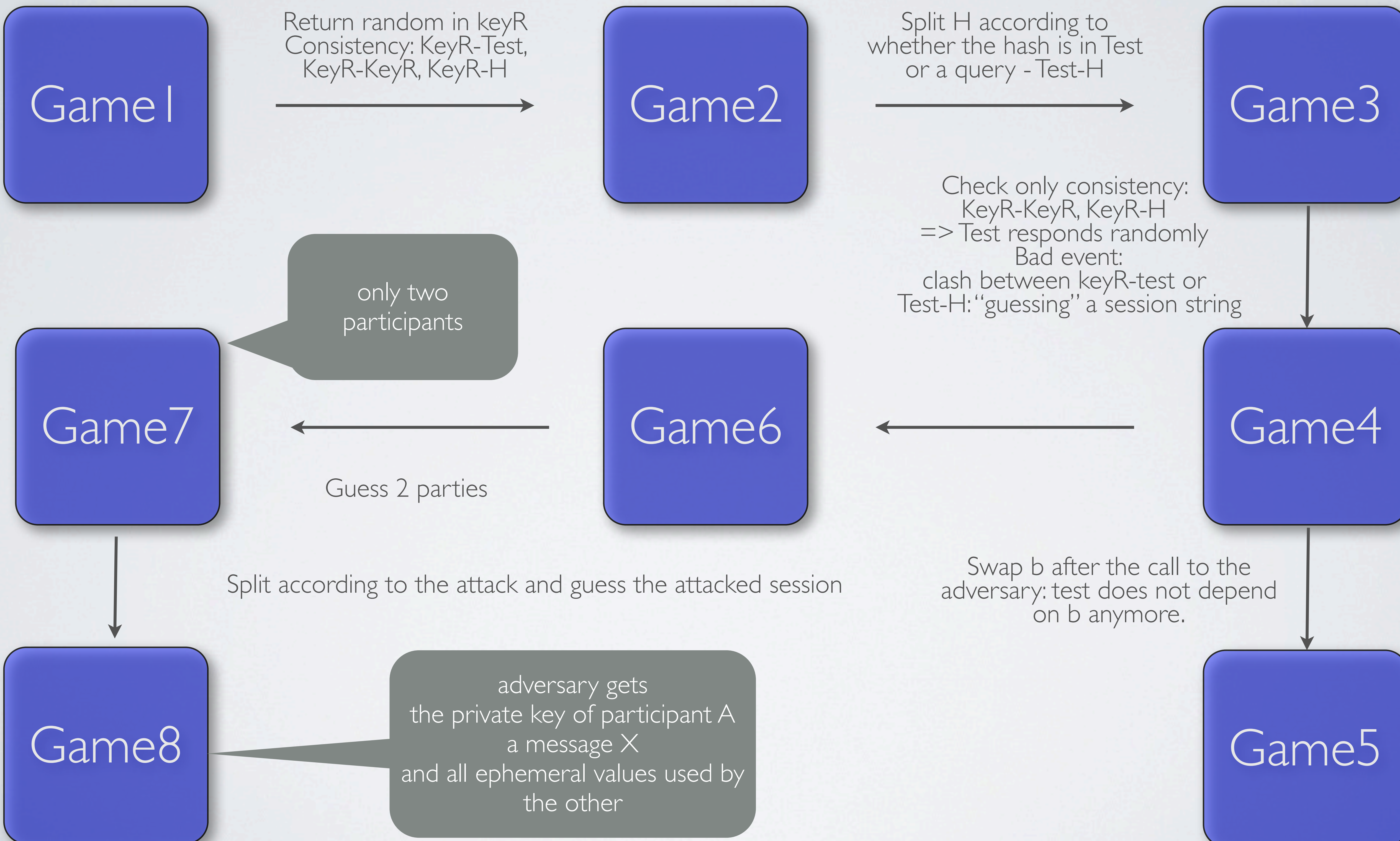

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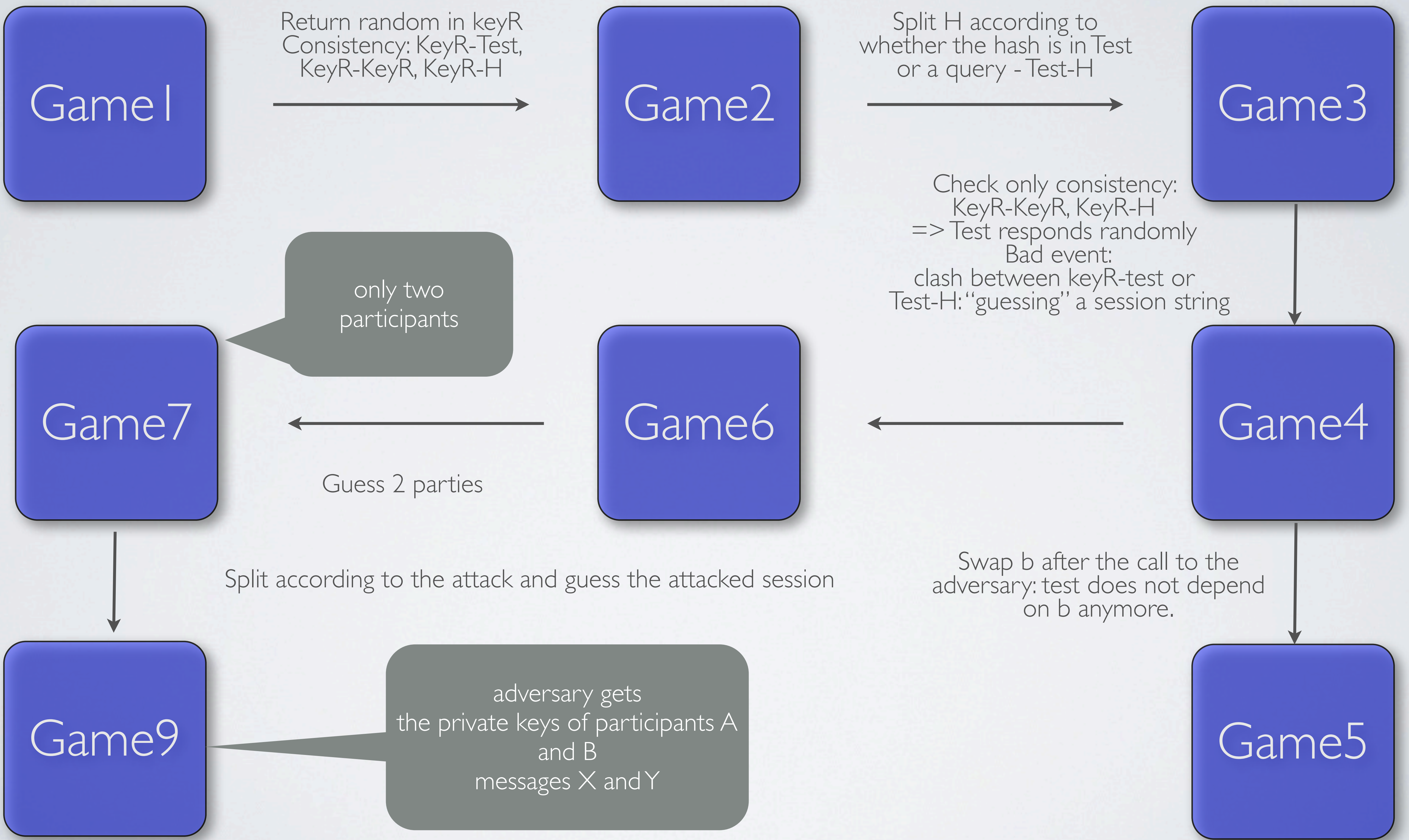

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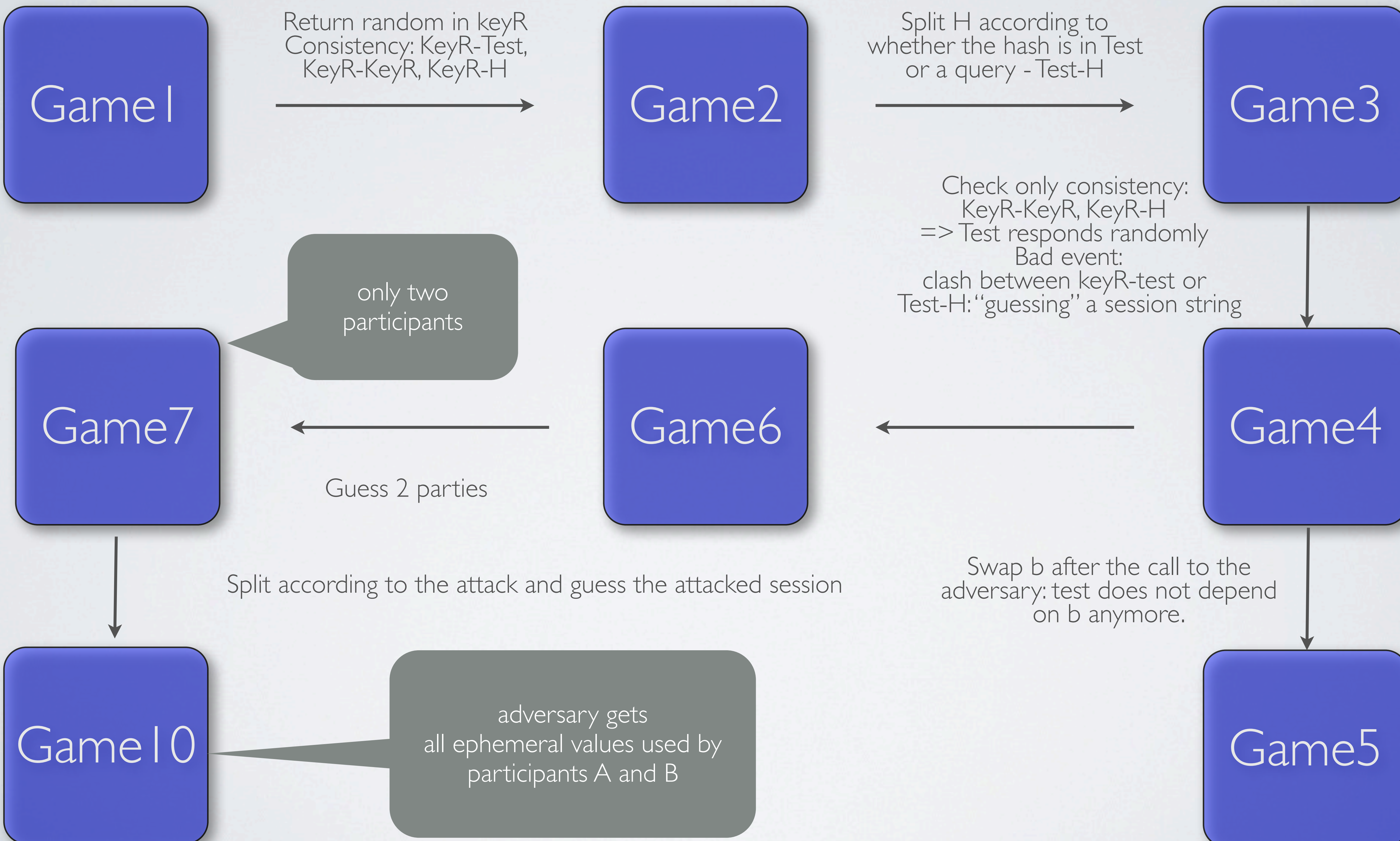

Remarks about the model and the proof

- In order to reason about a class of protocols, our model contains abstract data types and abstract operations that are specified by axioms. Notice that abstract operations are stateless.
- Thus, the proved statement is a universal quantification over all implementations of such abstract data types.

$$
\begin{array}{l}
\forall \vec{t} \forall \vec{op},\ \mathbf{Axiom}(\vec{op}, \vec{t}) \implies \\
\quad \forall A \exists B_1 \exists B_2 \exists B_3, \\
\Pr[G_1(A, \vec{t}, \vec{op}) : b = b'] - \frac{1}{2} \leq \\
\qquad \Pr[G_8(B_1, \vec{t}, \vec{op}) : E_1] + \Pr[G_9(B_2, \vec{t}, \vec{op}) : E_2] + \\
\qquad \Pr[G_9(B_3 \vec{t}, \vec{op}) : E_3]
\end{array}
$$

- About 11 Kloc - model + invariants + proof

Initializaton:
n participants, for each participants:

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$$\begin{array}{ccc} A & & B \\ x \xleftarrow{\$} \{0,1\}^\lambda & \xrightarrow{\quad A, X = g^{H_1(x,a)} \quad} & \\ & \xleftarrow{\quad B, Y = g^{H_1(y,b)} \quad} & y \xleftarrow{\$} \{0,1\}^\lambda \end{array}$$

$$K = H(\textsc{ses}(A, B, X, Y)),\ \text{where}$$
$$\textsc{ses}(A, B, X, Y)) = (Y^a, B^{H_1(x,a)}, Y^{H_1(x,a)}, A, B)$$

$\textsc{Naxos}$ uses an extra hash function H_1. Which is not part of the generic model.

Solution: reduce security of $\textsc{Naxos}$ to security of $\textsc{Naxos}$' by "internalizing" H_1.

$$
\begin{array}{ccc}
A & & B \\
x \xleftarrow{\$} \{0,1\}^\lambda & \xrightarrow{\quad A, X = g^r \quad} & r' \xleftarrow{\$} \mathbb{Z}_q \\
r \xleftarrow{\$} \mathbb{Z}_q & \xleftarrow{\quad B, Y = g^{r'} \quad} & y \xleftarrow{\$} \{0,1\}^\lambda
\end{array}
$$

$$K = H(\textsc{ses}(A, B, X, Y)), \text{ where}$$

$$\textsc{ses}(A, B, X, Y)) = (Y^a, B^r, Y^r, A, B)$$

For all adversaries A there is an adversary B s.t.

$$
\begin{aligned}
&\Pr[G_1'(A, \textsc{Naxos}) : b' = b] = \\
&\quad \Pr[G_1(B, \textsc{Naxos}')b' = b] + \Pr[G_1(B, \textsc{Naxos}) : \text{ Guess fresh } a \text{ or } x]
\end{aligned}
$$

Using the same generic proof, we can bound the probability of

$$\Pr[G_1(B, \textsc{Naxos}) : \text{ Guess fresh } a \text{ or } x]$$

with respect to Gap Discrete Log.
Gap Discrete Log:
Given g^a, compute a while having access to a restricted DDH oracle:

$$(g^x, z) : \; z \stackrel{?}{=} g^{ax}$$

Conclusion

- An automated proof in EasyCrypt of a modular reduction of the key security of hashed key exchange protocols to simplified session string unforgery.
- Applications: HMQV, NAXOS, a new famity of protocols based on twin DH we designed.

Several challenges for EasyCrypt

- Introduction of an instantiation mechanism.
- Automatic invariant generation.
- Better handling of verification conditions.
- Improvement of the undelying logic - exploiting CIL (Computational Indistinguishability Logic)