Liquid Solid Hybrid Rocket Engine

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CHIMERA HADES Engine Software Model

Contents

0.1 Purpose:

Based on Rhode's First Principle model, we hoped to develop a model with higher creative freedom to start designing the liquid-solid hybrid rocket engine HADES.

0.2 The Model:

Leaving Rhode's First Principles, we moved forward with the first principles such as Bernoulli's Compressible flow and the ideal gas law while incorporating new models throughout the stages of the rocket.

0.2.1 Blowdown Oxidizer Tank Model:

The governing equations for the blowdown model include ...

1. Ideal Gas Law
2. Conservation of Mass
3. Bernoulli's Compressible Flow

The assumptions for the blowdown model include...

- Single Phase injector
- Pressure of the entire tank is determined by the ullage gas
- Constant temperature (walls of the tank are adiabatic)
- Liquid Nitrous Oxide is only in the liquid state and acts as an incompressible gas
- The Ullage gas will act as an Ideal Gas
- There is no leakage, all mass flow goes into the drain port

The important equations include

$$\dot{m}_{ox} = C_{d_{inj}} \cdot A_{inj} * \sqrt{2 \cdot \rho_{ox} \cdot (P_{tank} - P_{chmb})} \tag{1}$$

This equation calculates the oxidizer mass flow for a single-phase injector. One thing to note is that in the code, from the perspective of the combustion chamber, this is a positive value. However, from a holistic view, this should be negative as it is being expelled from the engine.

$$\frac{\partial V_u}{\partial t} = -\left(\frac{\partial V_{ox}}{\partial t}\right) = \frac{-\dot{m}_{ox}}{\rho_{ox}} \tag{2}$$

This equation calculates the change in volume of the ullage gas with respect to time and is used to find V_u at any time step.

$$\frac{\partial P_{tank}}{\partial t} = \frac{P_{u_i} \cdot V_{u_i}}{(V_u)^2} \cdot C_D \cdot A_{inj} \sqrt{\frac{2}{\rho_{ox}}(P_{ox} - P_{chmb})} \tag{3}$$

This equation calculates the change in oxidizer tank pressure with respect to time. For more information, please look at the appendix Appendix.A.

0.2.2 Combustion Chamber Model:

The governing equations for the combustion chamber model include ...

1. Conservation of Mass
2. Ideal Gas Law

The assumptions for the combustion chamber model include ...

- Combustion chamber is adiabatic, there is no heat loss to the environment
- Choked Flow of Nozzle inlet
- Flow through the combustion chamber is uniform and non-rotational
- Complete combustion of oxidizer and fuel, only gaseous products contribute to the pressure change
- Combustion is uniform across the fuel grain
- Fuel grain burnings occur normal to the axial direction in a uniform matter
- Pressure is uniform across the entire combustion chamber
- Temperature is uniform across the entire combustion chamber

Fuel Regression Model:

The fuel regression model to be used is called the Marxmann and Gilberts Law. It is the following...

$$\dot{r} = a \cdot G_{ox}^n \tag{4}$$

Where

- $\dot{r}$ is the burn rate in in/s or mm/s
- a is a property of the fuel for its burn coefficient
- G_{ox} is the mass flux of the liquid oxidizer. This can also be viewed as

$$G_{ox} = \frac{\dot{m}_{ox}}{A_{port}} \tag{5}$$

where the A_{port} is the combustion port cross-sectional area.

One thing to note is the following:

$$ConversionFactor = (0.0254^{(1+2n)}) \cdot (0.453592^n) \tag{6}$$

Based on$n$ value which is always less than 1, this conversion factor is necessary to correct a to have the correct units.

Combustion Model:

The important equations include

$$A_{port} = \pi \cdot r^2 \tag{7}$$

This equation calculates the combustion port cross-sectional area at a time step based on the radius of the fuel grain r.

$$A_b = 2 * \pi * r * L \tag{8}$$

This equation calculates the current burnable surface area of the fuel grain at a time step based on the radius of the fuel grain r.

$$V_{chmb} = \pi \cdot r^2 \cdot L \tag{9}$$

This equation calculates the current gaseous volume of the chamber at a time step based on the radius of the fuel grain r.

$$\rho_{chmb} = \frac{P_{chmb}}{R' \cdot T_{chmb}} \tag{10}$$

This equation calculates the current gaseous density of the chamber at a time step. This is a form of $PV = nRT$ or the Ideal gas law.

$$R' = R_u/M \tag{11}$$

This equation calculates the R' gas constant of the combustion chamber gas that is created. This is found from inputs from GDL ProPrep or NASA CEARUN.

$$\dot{m}_f = \rho_f \cdot A_b \cdot \dot{r} \tag{12}$$

This equation calculates the mass flow of the fuel grain at a time step based on the radius of the fuel grain.

$$\frac{\partial P_{chmb}}{\partial t} = \frac{R' \cdot T_{chmb}}{V_{chmb}} \cdot \left[A_b \dot{r}(\rho_f - \rho_{chmb}) + \dot{m}_{ox} - P_{chmb} \cdot A_t \sqrt{\frac{\gamma}{R' T_{chmb}}} \left(\frac{2}{\gamma + 1} \right)^{\left(\frac{\gamma+1}{2(\gamma-1)}\right)} \right] \tag{13}$$

This equation calculates the change in combustion chamber pressure with respect to time.

0.2.3 Nozzle and Exit Velocity Model:

The governing equations for the nozzle model include ...

1. F = ma Newton Equations of Motion
2. Bernoulli's Compressible Flow
3. Conservation of Mass
4. Isentropic Flow Relationships

The assumptions for the nozzle model include ...

- Combustion gases can be treated as calorically perfect gases
- One-dimensional flow
- Compressible flow
- Perfectly expanded nozzle ($P_e = P_{amb}$
- Choked Flow through the Nozzle

The important equations include ...

$$F = A_t \cdot P_{chmb} \sqrt{\left(\frac{2 \cdot \gamma^2}{\gamma - 1}\right) \cdot \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{\gamma-1}} \cdot \left[1 - (\frac{P_e}{P_{chmb}})^{\frac{\gamma-1}{\gamma}}\right]} \tag{14}$$

This equation calculates the force of thrust created based on the chamber pressure at a time step. For more information please look at the appendix in Appendix.B.

0.3 Inputs:

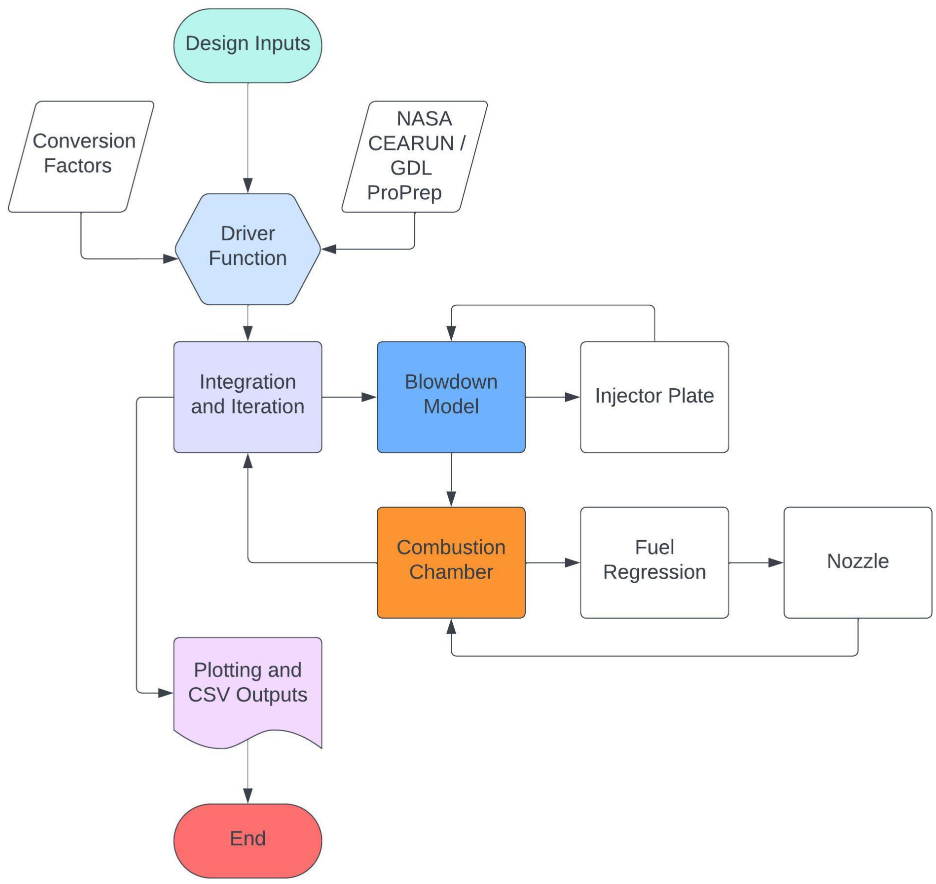
This section tabulates the inputs to the model. The base model is built on ballpark estimates and are tabulated below.

MODEL	VARIABLE	DESCRIPTION	UNITS	VALUE
Ambient	-	-	-	-
	P_{amb} T_0 R_u	Ambient Pressure Ambient Temperature Universal Gas Constant	[Pa] [K] $[J/(mol * K)]$	$1.026e + 5$ 298 8.3145
Blowdown	-	-	-	-
	V_{tank} P_{tank} u ρ_{ox} A_{inj} $C_{D_{inj}}$	Oxidizer Tank Volume Initial Oxidizer Tank Pressure Fraction of Ullage/Oxidizer Liquid Nitrous Oxide Density Area of Injector Coefficient of Discharge of Injector	$[m^3]$ [Pa] [%] $[kg/m^3]$ $[m^2]$ [-]	0.003 $2.068e + 7$ 20 1226 $9.4e - 6$ 0.4
Combustion Chamber	-	-	-	-
	$Fuel_{OD}$ $Fuel_{ID}$ L	Fuel Grain Outer Diameter Fuel Grain Inner Diameter Length of Fuel Grain	[m] [m] [m]	0.0343 0.0429 0.03048
Fuel Regression	-	-	-	-
	a n ρ_{fuel}	Burn Rate Coefficient Burn Rate Coefficient Fuel Grain Density	[-] [-] $[kg/m^3]$	$2.85e - 5$ 0.681 919
Combustion Gas	-	-	-	-
	M γ T_{chmb}	Combustion Gas Molecular Density Combustion Gas Ratio of Specific Heats Chamber Temperature	$[g/mol]$ [-] [K]	27.442 1.2448 3474
Nozzle	-	-	-	-
	D_t	Diameter of Nozzle Throat	$[m^2]$	0.0127

Table 1: Input Data for Each Part of the Model - Metric

0.4 Conclusion:

0.4.1 The Code Flow:



The image above outlines the current high-level tasks the software code undergoes. The heart of the equations we discussed is found in the integration and iteration loop. Here, the inputs and outputs are fed into each other over time until termination events occur. The most common, and should be the most common, termination event is when fuel runs out. After this, all of the outputs will be plotted and fed into CSV files.

0.4.2 Room for Improvement - Refining the Model for the Future:

Here are some possible leads that can be investigated further for implementing the model. There is room for improvement but it is a trade on man-hours versus uncertainty with calculations.

Blowdown Model

- Increase the credibility of the injector and mass flow by implementing a two-phase injector like the Dyer Model
- Consider the effect of not just gaseous ullage and liquid nitrous oxide but the vaporous nitrous oxide that might occur in phase equilibrium, models may include the Peng-Robinson equation of state or the Zilliac-Karaberough Model.

Combustion Chamber Model

- In the pressure chamber, there are three states of pressure. Start-up, Steady-state, and Tail-off. In this model, a steady state was implemented. Start-up and Tail-off implementations may improve the model
- Marxman's Fuel regression uses Mass Flux in the combustion chamber rather than chamber pressure. This is shown with $\dot{r} = a \cdot G^n$. Exploring different types of fuel regression may improve the burn rate calculations.
- GDL / CEARUN (NASA) can be used to find the combustion gas properties like M, T_{chmb} and γ. After our first iteration of general thrust, the OF ratio can be used to find the combustion gas properties throughout the entire flight. (Gather benchmark points and polyfit, then call the polyfit rather than constant inputs). Maybe implementing a call for a polynomial equation for combustion gas properties as a function of oxidizer to fuel ratio.

Nozzle Model

- More iterations on the Nozzle, since it is one big optimization problem. Just do more optimization. The current lead is the information and design methodology that Professor Nabity has given us.

0.5 Appendix:

0.5.1 Appendix.A: Oxidizer Tank Pressure

Starting with the Ideal Gas Law, we can see the following.

$$P_{u_i} \cdot V_{u_i} = n \cdot R_u \cdot T_{tank} \tag{15}$$

The initial pressure and volume of the ullage gas can be found with the number of moles (n), universal gas constant, and the temperature of the tank. Furthermore, assumptions such as the pressure of the ullage is equivalent to the pressure of the oxidizer tank means that $P_u = P_{tank}$. This is further backed up as the liquid nitrous oxide acts as an incompressible fluid.

$$P_{u_i} \cdot V_{u_i} = P_u \cdot V_u \tag{16}$$

The temperature of the oxidizer tank decreases over time as the liquid nitrous leaves the tank and the ullage gas expands. However, due to the length of the rocket burn, the temperature change is negligible. Therefore, the assumption is that the tank is at a constant temperature and the control volume is adiabatic with respect to the tank walls. The entire system can be seen to have constant temperature. This allows us to use Boyle's Law such that $P_1V_1 = P_2V_2$.

Solving the equation such that the pressure can be defined by any volume state at any time, differentiating the equation allows us to find the change in oxidizer tank pressure with the change in ullage gas volume.

$$\begin{aligned} P(V_u) &= \frac{P_{u_i} \cdot V_{u_i}}{V_u(t)} \\ \frac{\partial P_{ox}}{\partial V_u} &= \frac{\partial}{\partial t}(\frac{1}{V_u(t)}) \cdot P_{u_i} \cdot V_{u_i} \\ \frac{\partial P_{ox}}{\partial V_u} &= \frac{-P_{u_i} \cdot V_{u_i}}{(V_u(t))^2} \end{aligned}$$

The last differential equation to find is the change in volume with respect to the change in time. This is found using Bernoulli's compressible flow equation. There are two versions of the equation and both work. To go in between the two flow rate equations just multiply by density such as $m = \rho \cdot V$.

$$\begin{aligned} Q = \frac{\partial V_{ox}}{\partial t} &= C_D \cdot A_{inj}\sqrt{\frac{2}{\rho_{ox}}(P_{ox} - P_{chmb})} \\ \dot{m}_{ox} = \frac{\partial m_{ox}}{\partial t} &= C_D \cdot A_{inj}\sqrt{2 \cdot \rho_{ox}(P_{ox} - P_{chmb})} \end{aligned}$$

The first equation is the volumetric flow rate of the liquid nitrous oxide across the injector while the second equation is the mass flow rate of the liquid nitrous oxide across the injector. Through some manipulation. We can see that the volume flow of the liquid oxidizer will be equal to the volume gain of the ullage gas.

$$\frac{\partial V_u}{\partial t} = -\left(\frac{\partial V_{ox}}{\partial t}\right) = -C_D \cdot A_{inj}\sqrt{\frac{2}{\rho_{ox}}(P_{ox} - P_{chmb})} \tag{17}$$

Now with the two differential equations, we can solve the chain rule for the change in oxidizer tank pressure with respect to time.

$$\begin{aligned} \frac{\partial P_{ox}}{\partial t} &= \frac{\partial P_{ox}}{\partial V_u} \cdot \frac{\partial V_u}{\partial t} \\ \frac{\partial P_{ox}}{\partial t} &= \frac{-P_{u_i} \cdot V_{u_i}}{(V_u(t))^2} \cdot -C_D \cdot A_{inj}\sqrt{\frac{2}{\rho_{ox}}(P_{ox} - P_{chmb})} \\ &= \frac{P_{u_i} \cdot V_{u_i}}{(V_u)^2} \cdot C_D \cdot A_{inj}\sqrt{\frac{2}{\rho_{ox}}(P_{ox} - P_{chmb})} \end{aligned}$$

0.5.2 Appendix.B: Chamber Pressure

Starting with the conservation of mass. It is evident that all of the mass that enters and exits throughout the combustion chamber must be equal. Another way to view this is the rate of combustion product generation is equal to the rate of consumption of the propellant grain. Therefore, we can simplify the mass flow into four main parts

$$\dot{m}_g + \dot{m}_{ox} = \frac{\partial M_s}{\partial t} + \dot{m}_{noz} \tag{18}$$

What enters the system, the left-hand side is the mass flow generated due to the burning fuel from the fuel regression model and the mass of the oxidizer that comes from Bernoulli's compressible flow from the blowdown model. What exits the system, on the right-hand side is the mass due to the change in gas volume from propellant consumption and the mass flow through the nozzle.

We can expand each of these four mass flows into their respective parts as follows.

$$\begin{aligned}
\dot{m}_g &= A_b \cdot \rho_f \cdot \dot{r} \\
\dot{m}_{ox} &= C_D \cdot A_{inj}\sqrt{2 \cdot \rho_{ox}(P_{ox} - P_{chmb})} \\
\frac{\partial M_s}{\partial t} &= A_b \cdot \rho_{chmb} \cdot \dot{r} + V_{chmb}\frac{\partial \rho_{chmb}}{\partial t} \\
\dot{m}_{noz} &= P_{chmb} \cdot A_t\sqrt{\frac{\gamma}{R'T_{chmb}}}\left(\frac{2}{\gamma+1}\right)^{\left(\frac{\gamma+1}{2(\gamma-1)}\right)}
\end{aligned}$$

Now we can substitute these into the conservation of mass with the addition of the fuel regression model. One thing to note is that $\dot{m}_{ox}$ will not be substituted because it is in its most simplified form.

$$A_b \cdot \rho_f \cdot \dot{r} + \dot{m}_{ox} = A_b \cdot \rho_{chmb} \cdot \dot{r} + V_{chmb}\frac{\partial \rho_{chmb}}{\partial t} + P_{chmb} \cdot A_t\sqrt{\frac{\gamma}{R'T_{chmb}}}\left(\frac{2}{\gamma+1}\right)^{\left(\frac{\gamma+1}{2(\gamma-1)}\right)} \tag{19}$$

We would like to highlight the density of the chamber differential with respect to the change in time. From this and the ideal gas relationship, we are able to isolate the change in combustion chamber pressure with respect to time. Let's walk through that now.

$$\begin{aligned}
PV &= nRT \\
P &= \frac{n}{V} \cdot RT \\
P &= \rho RT \\
\rho &= \frac{P}{RT} \\
&\longrightarrow \\
\frac{\partial \rho_{chmb}}{\partial t} &= \frac{1}{R' \cdot T_{chmb}} \cdot \frac{\partial P_{chmb}}{\partial t} \\
&\longrightarrow \\
V_{chmb}\frac{\partial \rho_{chmb}}{\partial t} &= \frac{V_{chmb}}{R' \cdot T_{chmb}} \cdot \frac{\partial P_{chmb}}{\partial t}
\end{aligned}$$

Substituting this in, we can then start the simplification and combination of liked terms to isolate the change in combustion pressure with respect to time.

$$\begin{aligned}
A_b \cdot \rho_f \cdot \dot{r} + \dot{m}_{ox} &= A_b \cdot \rho_{chmb} \cdot \dot{r} + V_{chmb}\frac{\partial \rho_{chmb}}{\partial t} + P_{chmb} \cdot A_t\sqrt{\frac{\gamma}{R'T_{chmb}}}\left(\frac{2}{\gamma+1}\right)^{\left(\frac{\gamma+1}{2(\gamma-1)}\right)} \\
\frac{V_{chmb}}{R' \cdot T_{chmb}} \cdot \frac{\partial P_{chmb}}{\partial t} &= A_b \cdot \rho_f \cdot \dot{r} - A_b \cdot \rho_{chmb} \cdot \dot{r} + \dot{m}_{ox} - P_{chmb} \cdot A_t\sqrt{\frac{\gamma}{R'T_{chmb}}}\left(\frac{2}{\gamma+1}\right)^{\left(\frac{\gamma+1}{2(\gamma-1)}\right)} \\
\frac{V_{chmb}}{R' \cdot T_{chmb}} \cdot \frac{\partial P_{chmb}}{\partial t} &= A_b\dot{r}(\rho_f - \rho_{chmb}) + \dot{m}_{ox} - P_{chmb} \cdot A_t\sqrt{\frac{\gamma}{R'T_{chmb}}}\left(\frac{2}{\gamma+1}\right)^{\left(\frac{\gamma+1}{2(\gamma-1)}\right)} \\
\frac{\partial P_{chmb}}{\partial t} &= \frac{R' \cdot T_{chmb}}{V_{chmb}} \cdot \left[A_b\dot{r}(\rho_f - \rho_{chmb}) + \dot{m}_{ox} - P_{chmb} \cdot A_t\sqrt{\frac{\gamma}{R'T_{chmb}}}\left(\frac{2}{\gamma+1}\right)^{\left(\frac{\gamma+1}{2(\gamma-1)}\right)}\right]
\end{aligned}$$

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