Inverted Indexes

Compressed Inverted Indexes

Compressed Inverted Indexes

- It is possible to combine index compression and text compression without any complication
  - In fact, in all the construction algorithms mentioned, compression can be added as a final step
- In a full-text inverted index, the lists of text positions or file identifiers are in ascending order
- Therefore, they can be represented as sequences of **gaps** between consecutive numbers
  - Notice that these gaps are small for frequent words and large for infrequent words
  - Thus, compression can be obtained by encoding small values with shorter codes

Compressed Inverted Indexes

- A coding scheme for this case is the **unary code**
  - In this method, each integer $x > 0$ is coded as $(x-1)$ 1-bits followed by a 0-bit
- A better scheme is the Elias-γ code, which represents a number $x > 0$ by a concatenation of two parts:
  1. a unary code for $1 + \lfloor \log_2 x \rfloor$
  2. a code of $\lfloor \log_2 x \rfloor$ bits that represents the number $x - 2^{\lfloor \log_2 x \rfloor}$ in binary
- Another coding scheme is the Elias-δ code
- Elias-δ concatenates parts (1) and (2) as above, yet part (1) is not represented in unary but using Elias-γ instead

Compressed Inverted Indexes

- Example codes for integers

Gap x	Unary	Elias-γ	Elias-δ	Golomb ($b = 3$)
1	0	0	0	00
2	10	100	1000	010
3	110	101	1001	011
4	1110	11000	10100	100
5	11110	11001	10101	1010
6	111110	11010	10110	1011
7	1111110	11011	10111	1100
8	11111110	1110000	11000000	11010
9	111111110	1110001	11000001	11011
10	1111111110	1110010	11000010	11100

Note: Golomb codes will be explained later

Compressed Inverted Indexes

- In general,
  - Elias-γ for an arbitrary integer $x > 0$ requires $1 + 2\lfloor \log_2 x \rfloor$ bits
  - Elias-δ requires $1 + 2\lfloor \log_2 \log_2 2x \rfloor + \lfloor \log_2 x \rfloor$ bits
- For small values of x Elias-γ codes are shorter than Elias-δ codes, and the situation is reversed as x grows
- Thus the choice depends on which values we expect to encode

Compressed Inverted Indexes

- Golomb presented another coding method that can be parametrized to fit smaller or larger gaps
- For some parameter b, let q and r be the quotient and remainder, respectively, of dividing $x - 1$ by b
  - I.e., $q = \lfloor (x-1)/b \rfloor$ and $r = (x-1) - q \cdot b$
- Then x is coded by concatenating
  - the unary representation of $q + 1$
  - the binary representation of r, using either $\lfloor \log_2 b \rfloor$ or $\lceil \log_2 b \rceil$ bits

Compressed Inverted Indexes

- If $r < 2^{\lfloor \log_2 b \rfloor - 1}$ then r uses $\lfloor \log_2 b \rfloor$ bits, and the representation always starts with a 0-bit
- Otherwise it uses $\lceil \log_2 b \rceil$ bits where the first bit is 1 and the remaining bits encode the value $r - 2^{\lfloor \log_2 b \rfloor - 1}$ in $\lfloor \log_2 b \rfloor$ binary digits
- For example,
  - For $b = 3$ there are three possible remainders, and those are coded as 0, 10, and 11, for $r = 0$, $r = 1$, and $r = 2$, respectively
  - For $b = 5$ there are five possible remainders r, 0 through 4, and these are assigned the codes 00, 01, 100, 101, and 110

Compressed Inverted Indexes

- To encode the lists of occurrences using Golomb codes, we must define the parameter b for each list
- Golomb codes usually give better compression than either Elias-γ or Elias-δ
  - However they need two passes to be generated as well as information on terms statistics over the whole document collection
- For example, in the TREC-3 collection, the average number of bits per list entry for each method is
  - Golomb = 5.73
  - Elias-δ = 6.19
  - Elias-γ = 6.43
- This represents a five-fold reduction in space compared to a plain inverted index representation

Compressed Inverted Indexes

- Let us now consider inverted indexes for ranked search
  - In this case the documents are sorted by decreasing frequency of the term or other similar type of weight
- Documents that share the same frequency can be sorted in increasing order of identifiers
- This will permit the use of gap encoding to compress most of each list

Text Compression

Text Compression

- A representation of text using less space
- Attractive option to reduce costs associated with
  - space requirements
  - input/output (I/O) overhead
  - communication delays
- Becoming an important issue for IR systems
- *Trade-off:* time to encode versus time to decode text

Text Compression

- Our focus are compression methods that
  - allow **random access** to text
  - do not require decoding the entire text
- Important: compression and decompression speed
  - In many situations, decompression speed is more important than compression speed
  - For instance, in textual databases in which texts are compressed once and read many times from disk
- Also important: possibility of searching text without decompressing
  - faster because much less text has to be scanned

Compression Methods

- Two general approaches
  - **statistical** text compression
  - **dictionary** based text compression
- **Statistical methods**
  - Estimate the probability of a symbol to appear next in the text
  - **Symbol**: a character, a text word, a fixed number of chars
  - **Alphabet**: set of all possible symbols in the text
  - **Modeling**: task of estimating probabilities of a symbol
  - **Coding** or **encoding**: process of converting symbols into binary digits using the estimated probabilities

Compression Methods

- **Dictionary methods**
  - Identify a set of sequences that can be referenced
  - Sequences are often called **phrases**
  - Set of phrases is called the **dictionary**
  - Phrases in the text are replaced by pointers to dictionary entries

Statistical Methods

- Defined by the combination of two tasks
  - the **modeling task** estimates a probability for each next symbol
  - the **coding task** encodes the next symbol as a function of the probability assigned to it by the model
- A **code** establishes the representation (**codeword**) for each source symbol
- The entropy E is a **lower bound** on compression, measured in bits per symbol

Statistical Methods

- Golden rule

  Shorter codewords should be assigned to more frequent symbols to achieve higher compression

- If probability p_c of a symbol c is much higher than others, then $\log_2 \frac{1}{p_c}$ will be small

- To achieve good compression
  - Modeler must provide good estimation of probability p of symbol occurrences
  - Encoder must assign codewords of length close to $\log_2 \frac{1}{p}$

Statistical Methods: Modeling

- Compression models can be
  - **adaptive**, **static**, or **semi-static**
  - **character-based** or **word-based**
- *Adaptive models:*
  - start with no information about the text
  - progressively learn the statistical text distribution
  - need only one pass over the text
  - store no additional information apart from the compressed text
- Adaptive models provide an inadequate alternative for full-text retrieval
  - decompression has to start from the beginning of text

Static models

- **Static models**
  - assume an average distribution for all input texts
  - modeling phase is done only once for all texts
  - achieve poor compression ratios when data deviate from initial statistical assumptions
    - a model that is adequate for English literary texts will probably perform poorly for financial texts

Semi-static models

- **Semi-static models**
  - Do not assume any distribution on the data
  - Learn data distribution (fixed code) in a first pass
  - Text compressed in a second pass using fixed code from first pass
  - Information on data distribution sent to decoder before transmitting encoded symbols
  - Advantage in IR contexts: <u>direct access</u>
    - Same model used at every point in compressed file

Semi-static models

- Simple semi-static model: use **global frequency** information
- Let f_c be the frequency of symbol c in the text $T = t_1 t_2 \ldots t_n$
- The corresponding entropy is

$$E = \sum_{c \in \Sigma} \frac{f_c}{n} \log_2 \frac{n}{f_c}$$

- This simple modeling may not capture the redundancies of the text

Semi-static models

- In the 2 gigabyte TREC-3 collection:
  - Entropy under this simple model: 4.5 bits per character
  - Compression ratio cannot be lower than 55%
  - But, state-of-the-art compressors achieve compression ratios between 20% and 40%

Semi-static models

- **Order** k of a model
  - Number of symbols used to estimate probability of next symbol
  - Zero-order model: computed independently of context
  - Compression improves with higher-order models
    - Model of order 3 in TREC-3 collection
      - compression ratios of 30%
      - handling about 1.3 million frequencies
    - Model of order 4 in TREC-3 collection
      - compression ratio of 25%
      - handling about 6 million frequencies
- In adaptive compression, a higher-order modeler requires much more memory to run

Word-based Modeling

- **Word-based modeling** uses zero-order modeling over a sequence of words
- Good reasons to use word-based models in IR
  - Distribution of words more skewed than that of individual chars
  - Number of different words is not as large as text size
  - Words are the atoms on which most IR systems are built
  - Word frequencies are useful in answering queries

Word-based Modeling

- Two different alphabets can be used
  - one for words
  - one for separators
- In TREC-3, 70% – 80% of separators are spaces
- Good properties of word-based models stem from well-known statistical rules:
  - **Heaps' law**: $V = O(n^\beta)$,
  - **Zipf's law**: the i-th most frequent word occurs $O(n/i^\alpha)$ times

Statistical Methods: Coding

- **Codeword:** representation of a symbol according to a model
- **Encoders:** generate the codeword of a symbol (coding)
  - assign short codewords to frequent symbols
  - assign long codewords to infrequent ones
  - entropy of probability distribution is lower bound on average length of a codeword
- **Decoders:** obtain the symbol corresponding to a codeword (decoding)
- Speed of encoder and decoder is important

Statistical Methods: Coding

- **Symbol code:** an assignment of a codeword to each symbol
- The least we can expect from a code is that it be **uniquely decodable**
- Consider three source symbols A, B, and C
  - Symbol code: $A \rightarrow \texttt{0}$, $B \rightarrow \texttt{1}$, $C \rightarrow \texttt{01}$
  - Then, compressed text `011` corresponds to ABB or CB?

Statistical Methods: Coding

- Consider again the three source symbols A, B, and C
  - Symbol code: $A \rightarrow$ `00`, $B \rightarrow$ `11`, $C \rightarrow$ `110`
  - This symbol code is uniquely decodable
  - However, for the compressed text `110000000`
    - we must count total number of zeros to determine whether first symbol is B or C
- A code is said to be **instantaneous** if every codeword can be decoded immediately after reading its last bit
- **Prefix-free** or **prefix** codes: no codeword should be a prefix of another

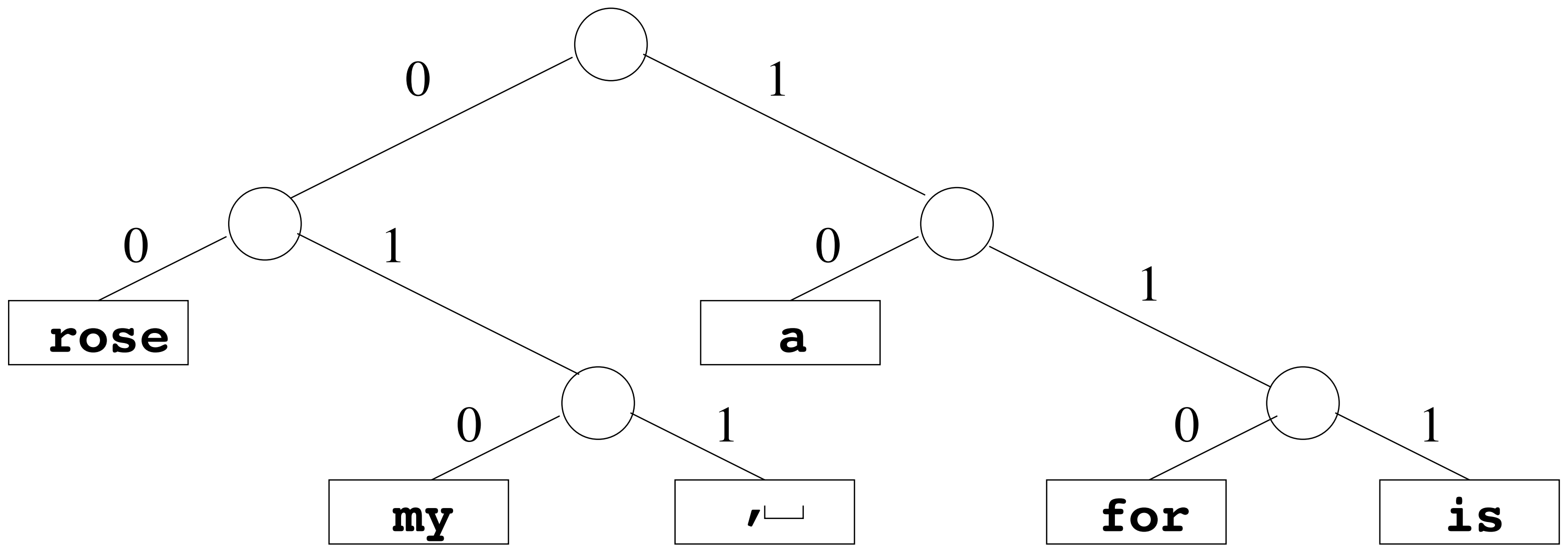
Statistical Methods: Coding

Huffman coding

- a method to find the best prefix code for a probability distribution
- Let $\{p_c\}$ be a set of probabilities for the symbols $c \in \Sigma$
  - Huffman method assigns to each c a codeword of length ℓ_c
  - Idea: minimize $\sum_{c \in \Sigma} p_c \cdot \ell_c$
- In a first pass, the modeler of a semi-static Huffman-based compression method:
  - determines the probability distribution of the symbols
  - builds a coding tree according to this distribution
- In a second pass, each text symbol is encoded according to the coding tree

Huffman Codes

- Figure below presents an example of Huffman compression



Original text: **for my rose, a rose is a rose**

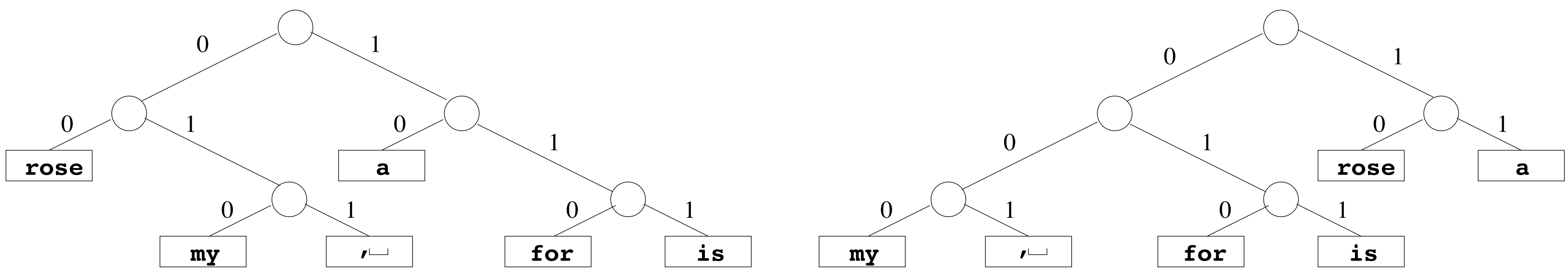
Compressed text: **110 010 00 011 10 00 111 10 00**

Huffman Codes

- Given V symbols and their frequencies in the text, the algorithm builds the Huffman tree in $O(V \log V)$ time
- Decompression is accomplished as follows
  - Stream of bits in file is traversed from left to right
  - Sequence of bits read is used to also traverse the Huffman coding tree, starting at the root
  - Whenever a leaf node is reached, the corresponding word or separator is printed out and the tree traversal is restarted
- In our example, the presence of the codeword `110` in the compressed file leads to the symbol `for`

Huffman Codes

- The Huffman tree for a given probability distribution is not unique



Original text:	`for my rose, a rose is a rose`	`for my rose, a rose is a rose`
Compressed text:	`110 010 00 011 10 00 111 10 00`	`010 000 10 001 11 10 011 11 10`

- **Canonical tree**
  - right subtree of no node can be taller than its left subtree
  - can be stored very efficiently
  - allows faster decoding

Byte-Huffman Codes

- Original Huffman method leads to binary coding trees
- However, we can make the code assign a sequence of whole bytes to each symbol
  - As a result, Huffman tree has degree 256 instead of 2
  - This word-based model degrades compression ratios to around 30%
  - In exchange, decompression of byte-Huffman code is much faster than for binary Huffman code

Byte-Huffman Codes

- In byte-Huffman coding, direct searching on compressed text is simpler
- To search for a word in the compressed text
  - first find it in the vocabulary
    - for TREC-3, vocabulary requires just 5 megabytes
  - mark the corresponding leaf in the tree
  - proceed over text as if decompressing, except that no symbol is output
  - instead, report occurrences when visiting marked leaves

Byte-Huffman Codes

- Process is simple and fast: only 30% of the I/O is necessary
- Assume we wish to search for a complex pattern including ranges of characters or a regular expression
  - just apply the algorithm over the vocabulary
  - for each match of a whole vocabulary word, mark the word
  - done only on the vocabulary (much smaller than whole text)
  - once relevant words are marked, run simple byte-scanning algorithm over the compressed text

Byte-Huffman Codes

- All complexity of the search is encapsulated in the vocabulary scanning
- For this reason, searching the compressed text is
  - up to 8 times faster when complex patterns are involved
  - about 3 times faster when simple patterns are involved

Byte-Huffman Codes

- Although the search technique is simple and uniform, one could do better especially for single-word queries
- Concatenation of two codewords might contain a third codeword
  - Consider the code: $A \rightarrow \texttt{0}$, $B \rightarrow \texttt{10}$, $C \rightarrow \texttt{110}$, $D \rightarrow \texttt{111}$
    - DB would be coded as `11110`
    - If we search for C, we would incorrectly report a spurious occurrence spanning the codewords of DB
    - To check if the occurrence is spurious or not, rescan all text from the beginning

Dense Codes

- An alternative coding simpler than byte-Huffman is **dense coding**
- Dense codes arrange the symbols in decreasing frequency order
  - Codeword assigned to the i-th most frequent symbol is, essentially, the number $i-1$
  - number is written in a variable length sequence of bytes
  - 7 bits of each byte are used to encode the number
  - highest bit is reserved to signal the last byte of the codeword

Dense Codes

- Codewords of symbols ranked 1 to 128 are 0 to 127
  - they receive one-byte codewords
  - highest bit is set to 1 to indicate last byte (that is, we add 128 to all codewords)
  - symbol ranked 1 receives codeword $\langle 128 \rangle = \langle 0 + 128 \rangle$
  - symbol ranked 2 receives codeword $\langle 129 \rangle = \langle 1 + 128 \rangle$
  - symbol ranked 128 receives codeword $\langle 255 \rangle$
- Symbols ranked from 129 to 16,512 (i.e., $128 + 128^2$) are assigned two-byte codewords $\langle 0, 128 \rangle$ to $\langle 127, 255 \rangle$

Dense Codes

- **Stoppers**
  - these are those bytes with their highest bit set
  - they indicate the end of the codeword
- **Continuers**
  - these are the bytes other than *stoppers*
- Text vocabularies are rarely large enough to require 4-byte codewords

Dense Codes

- Figure below illustrates an encoding with dense codes

Word rank	Codeword	Bytes	# of words
1	$\langle 128 \rangle$	1	128
2	$\langle 129 \rangle$	1	
...	...		
128	$\langle 255 \rangle$	1	
129	$\langle 0, 128 \rangle$	2	128^2
130	$\langle 0, 129 \rangle$	2	
...	...		
256	$\langle 0, 255 \rangle$	2	
257	$\langle 1, 128 \rangle$	2	
...	...		
16,512	$\langle 127, 255 \rangle$	2	
16,513	$\langle 0, 0, 128 \rangle$	3	128^3
...	...		
2,113,664	$\langle 127, 127, 255 \rangle$	3	

Dense Codes

- Highest bit signals the end of a codeword
  - a dense code is automatically a prefix code
- **Self-synchronization**
  - Dense codes are **self-synchronizing**
    - Given any position in the compressed text, it is very easy to determine the next or previous codeword beginning
    - decompression can start from any position, be it a codeword beginning or not
  - Huffman-encoding is **not self-synchronizing**
    - not possible to decode starting from an **arbitrary position** in the compressed text
    - notice that it is possible to decode starting at an *arbitrary codeword beginning*

Dense Codes

- Self-synchronization allows faster search
- To search for a single word we can
  - obtain its codeword
  - search for the codeword in the compressed text using **any** string matching algorithm
- This does not work over byte-Huffman coding

Dense Codes

- An *spurious occurrence* is a codeword that is a suffix of another codeword
  - assume we look for codeword `a b` $\overline{\texttt{c}}$, where we have overlined the stopper byte
  - there could be a codeword `d a b` $\overline{\texttt{c}}$ in the code, so that we could find our codeword in the text `...e f` $\overline{\texttt{g}}$ `d a b` $\overline{\texttt{c}}$`...`
  - yet, it is sufficient to access the text position preceding the candidate occurrence, `'d'`, to see that it is not a stopper
- Such a fast and simple check is not possible with Huffman coding
- To search for phrases
  - concatenate the codewords
  - search for the concatenation