# Hypergraphs: Algorithms, Implementations, and Applications

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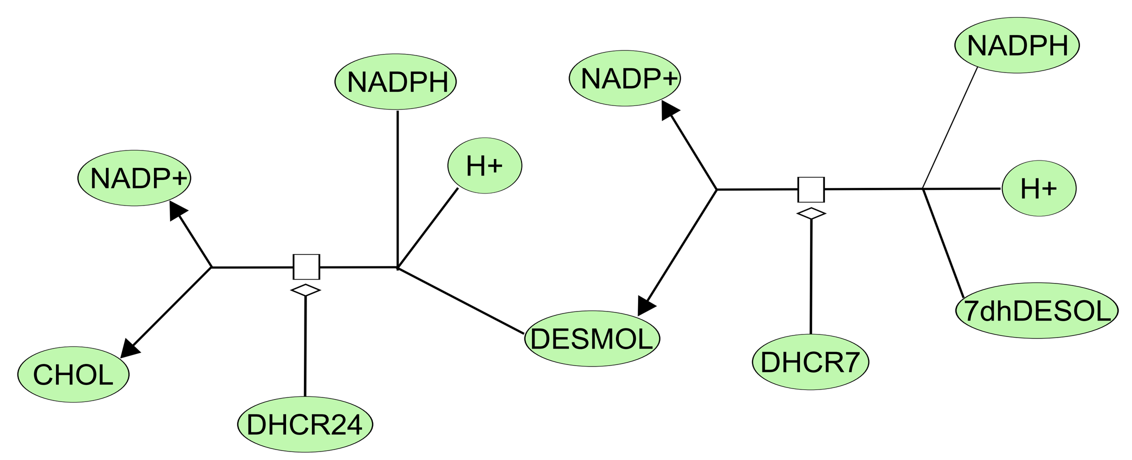
## Motivation

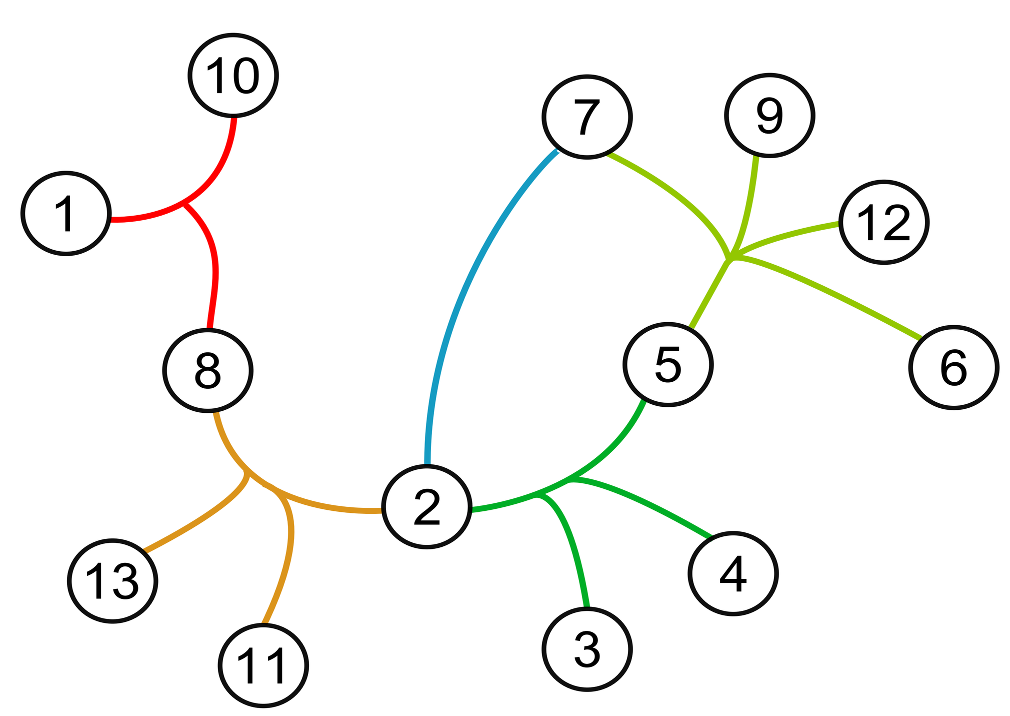
- Networks are ubiquitous in modern science, engineering, and the humanities
- Networks modeled with graphs can only represent pairwise interactions, not higher-order relationships




- Many software libraries for graphs exist: NetworkX, Boost, JUNG, etc.
- **Challenge: no libraries for hypergraphs exist that have both data structures and algorithms. This leaves a lot of work to do!**

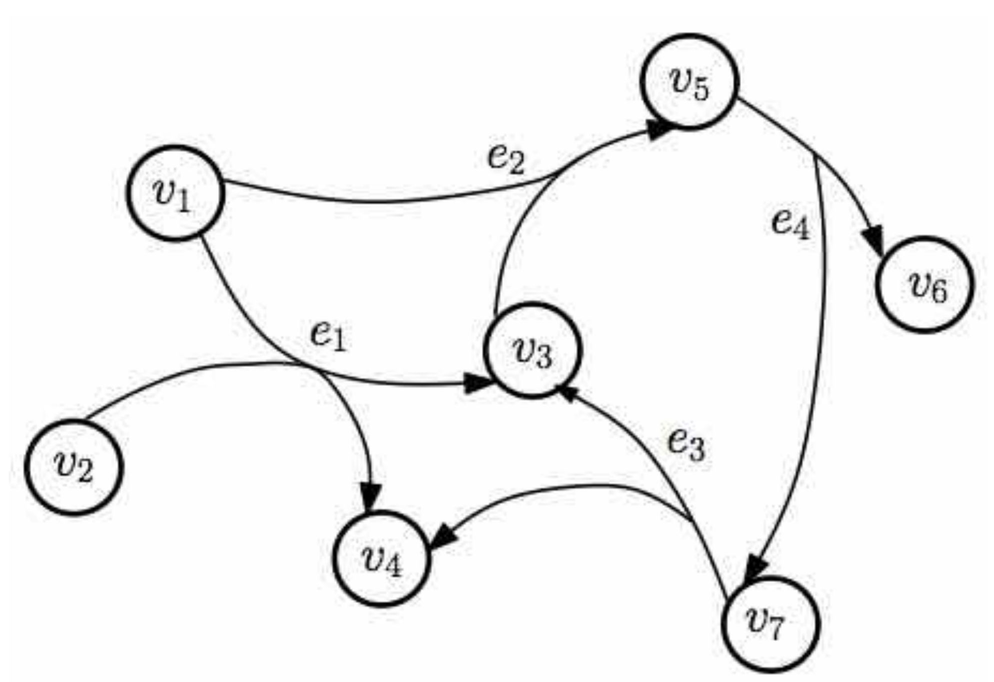
## What Are Hypergraphs?

### Undirected Hypergraphs




Undirected hyperedges connect groups of nodes.

### Directed Hypergraphs




Directed hyperedges connect "tail" groups of nodes to "head" groups of nodes.

## *halp*: Hypergraph Algorithms Package

### Features

**Open Source:** Thoroughly tested Python package publicly-available on GitHub [1]

**Data Structures:** Directed and undirected hypergraph data structures to easily model complex networks

**Usable Algorithms:** Implementations of important and canonical hypergraph algorithms

**Utilities:** Quick extraction of hypergraph properties and statistics + conversion to other formats/structures

### Algorithms

The algorithms currently implemented in *halp* span:

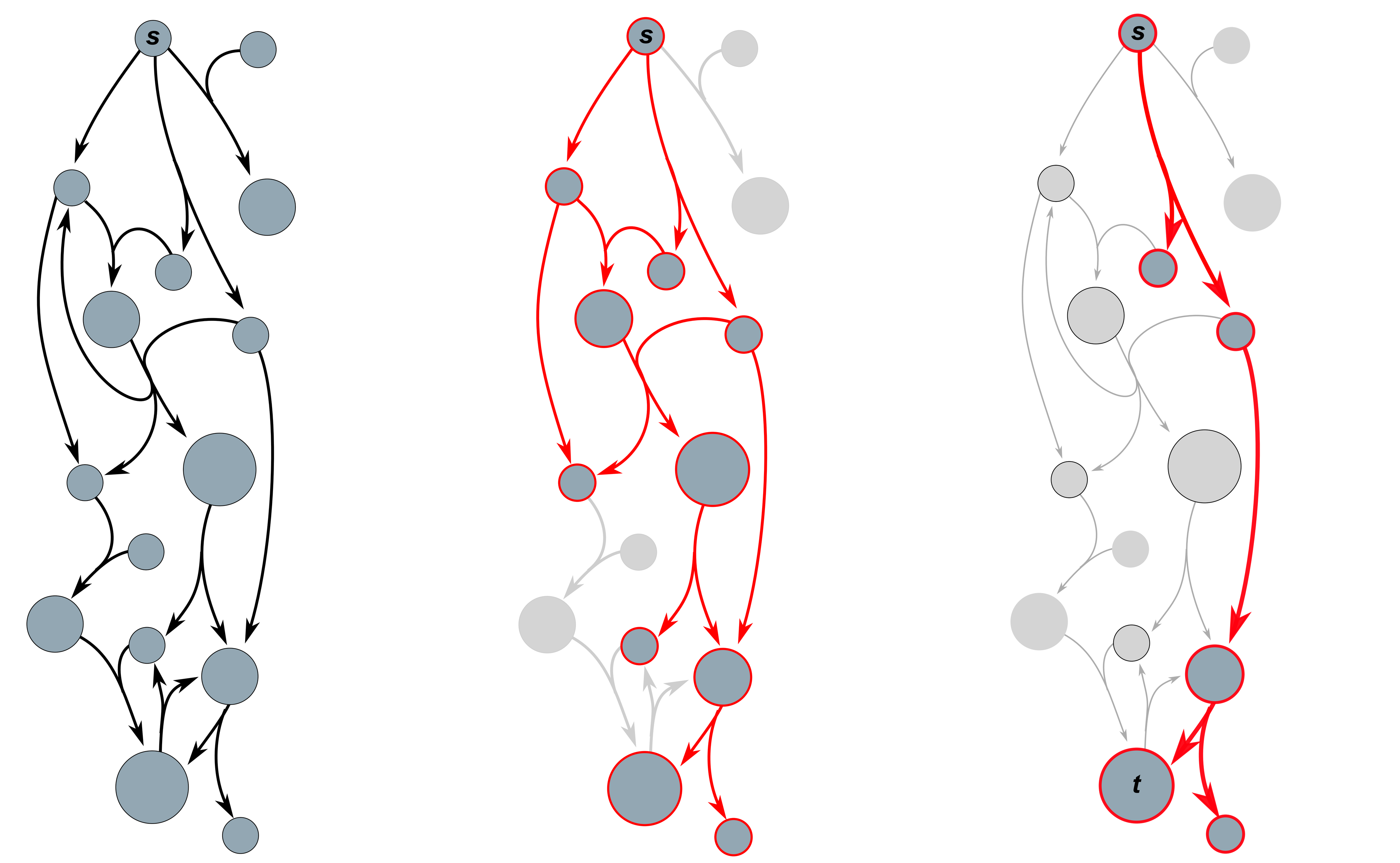
- **Connectivity** [2]
- **Hyperpaths** [2] [3] [6]
- **Hypertrees** [2]
- **Random Walks and Partitioning** [4] [5]

These algorithms are illustrated to the right:

- *B-Visit* algorithm, for computing *B-connectivity*
- *s-t B-hyperpath* algorithm, for computing a minimal B-connected hyperpath

### Example Algorithms

***Initial Hypergraph*** ***B-Visit from s*** ***s-t B-Hyperpath***




## New Algorithm with Application to Biological Networks

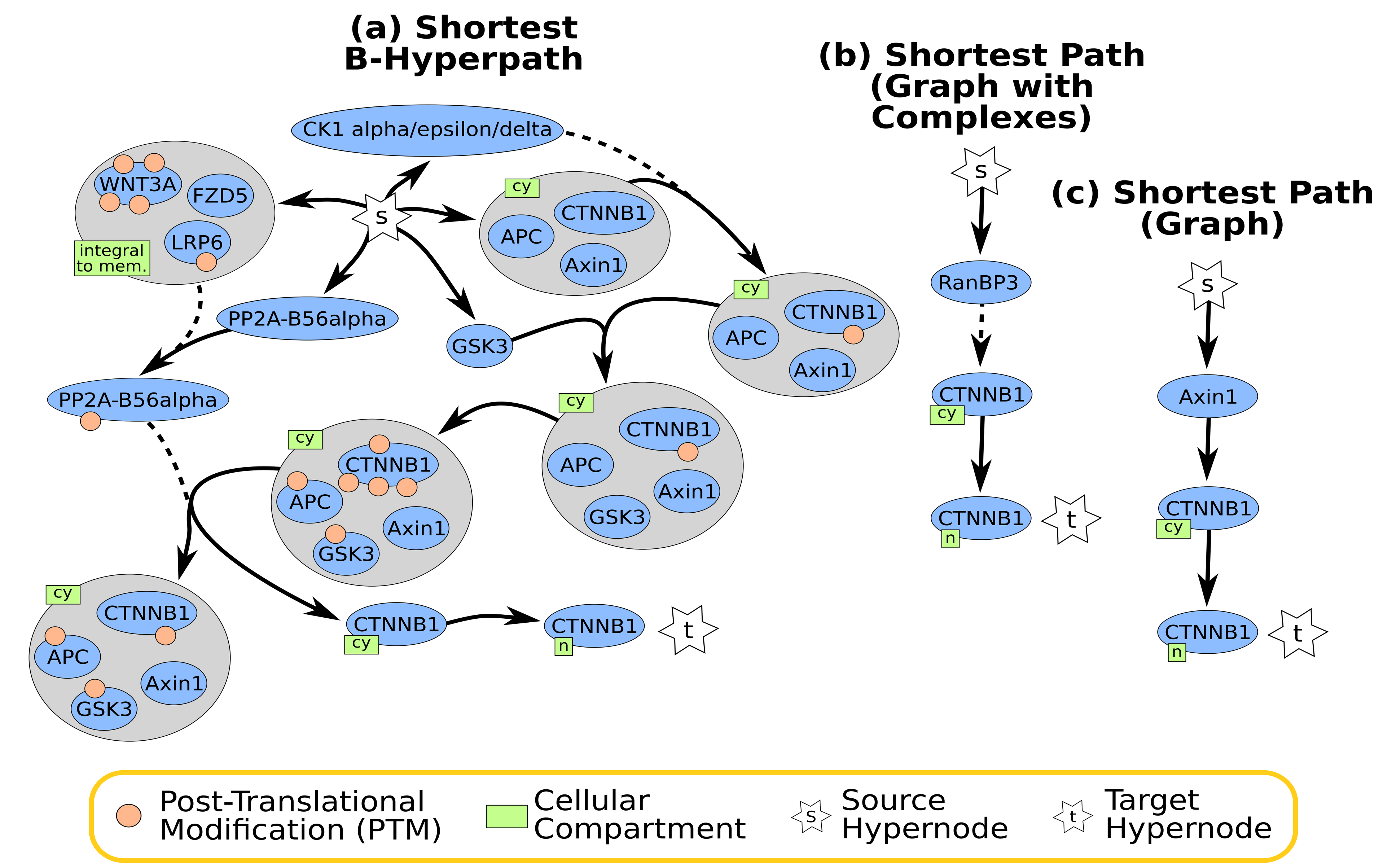
**Cellular Signaling Pathways**

- Cells respond to environmental signals through "signaling pathways"
- Many types of reactions can occur along these paths
- Graphs cannot model these interactions adequately...but hypergraphs can!

**Shortest B-Hyperpath Algorithm**

- The biological interpretation of a B-hyperpath is a path from node s to node t that contains all intermediate reactants and products needed to reach t from s
- We developed an algorithm using mixed integer linear programming to find the *shortest* acyclic B-hyperpath of all possible B-hyperpaths in a directed hypergraph [6]

### Shortest B-Hyperpaths in Signaling Pathways




(a) Shortest B-hyperpath in the Wnt signaling pathway when represented as a directed hypergraph. Nodes represent complexes (in grey) and standalone proteins (in blue outside of any complexes). (b, c) Shortest paths in the Wnt signaling pathway when represented as a graph with complexes (b) or as a graph (c). We see that the hyperpath is much more informative than the path in the graphs.

## References

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