# Logic Basics

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- Classical logic sometimes taken to be crucial aspect of foundations of math.
- Philosophers use to formalize arguments and sharpen critical thinking skills.
- BUT Philosophers and Logicians also argue about Logic itself. Pure, Applied, Pluralism, Non-Classical, etc.

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    - $\neg R$

# Classical Logic

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- Classical Logic is typically defined over a set of "sentences" $\mathcal{L}$ via the set of connectives $\{\neg, \wedge, \vee, \rightarrow\}$ with a binary valuation function $v(i) \mapsto \{0, 1\}$ or $\{F, T\}$ for $i \in \mathcal{L}$. Proofs are constructed according to classical logical rules of inference.

# Truth Table Examples

DeMorgan's Law: $\neg(A \wedge B) = \neg A \vee \neg B$

| $A$ | $B$ | $A \wedge B$ | $\neg(A \wedge B)$ | $\neg A \vee \neg B$ |
|---|---|---|---|---|
| T | T | T | F | F |
| T | F | F | T | T |
| F | T | F | T | T |
| F | F | F | T | T |

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Material Implication: $A \rightarrow B = \neg A \vee B$

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|---|---|---|---|
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| T | F | F | F |
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- Thus, it is possible for an argument to be Valid, yet not Sound. (Most philosophizing concerns soundness.)

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- Important link between logic and computer science. Algorithms.

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- (Plug in raining example again)

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- Venn diagrams!