Fluid Mechanical Models in Physical and Computational Contexts

Cameron Beebe

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Abstract

Related fluid mechanical models are discussed in two important contexts: (i) in conceptualizations and table-top models of the quantum world, and (ii) in conceptualizations and models of electronics and computational research in micro-fluidics. In physics, recent table-top models have been constructed which recreate behavior analogous to several important quantum physical phenomena. Vibrating fluid baths are used to produce quantum-like behavior—for example 'walking' droplets on the fluid surface giving similar wavelike statistics characteristic of the quantum "wave particle duality". In electronics, idealized fluid flow has been used as a pedagogical and conceptual tool to teach elements of electric current. A fluid circuit model can even be constructed in analogy with classical electronic circuits, and modern micro-fluidic computational devices have been utilized in a variety of research areas. Various arguments are discussed in these contexts, including arguments against the many worlds approach in quantum mechanics. At the intersection of these contexts we find the debate over explaining speed up in quantum computation. This debate is addressed with the fluid analogy in mind. Considerations of these various fluid mechanical models, in particular the flexibility of these models, helps support the central thesis: that fluid mechanical models are uniquely useful aspects of scientific reasoning and conceptualizations.

Contents

1 Introduction

In the philosophical analysis of reasoning in argumentation, or what is known as argumentation theory, there is a particular type of argument known as an argument from analogy. Such an argument involves the use of perceived similarities between two concepts, premise domains, physical objects, or abstract structures and formalisms. Similarity in such an argument is known as the positive analogical relationship between two (or more) of these domains, objects, or structures (I will use domain henceforth since it is arguably the more general notion). One can have positive and negative analogies between two domains.

In this thesis we will discuss fluid mechanics and the behaviors of various idealized fluid systems on the one hand, and the analogical relationships with aspects from the quantum world (the physical context) and electronics, circuitry, and logic gates (the computational context) on the other. Using these as background, we will see in the later sections that at the intersection of these two contexts—the field of quantum computation—fluid analog models may also be of heuristic value. The potential benefits of viewing quantum computations in light of the lessons learned from fluid analog models in the previous sections are evaluated in an in-depth discussion of quantum speed-up.

This is used to conclude in the final sections that fluid models seem to have a particularly elevated place among reasoning with models in the sciences, particularly in the physics-related fields discussed. This is due primarily to the ability to form analogical models and construct arguments from analogy using fluid mechanics, but also related abilities such as that fluid mechanical models are liable to idealization and are applicable to a variety of specialized fields (such as quantum computation). Fluid mechanical models do not need to be analogous—for example in scale models used in aerodynamic testing, which rely on what are called scaling laws (that phenomena observed in the scale model will apply to the larger system). (see e.g. [Batterman, 2008]) However, for the majority of the thesis we will be concerned with the scientific use of fluid analog models—fluid mechanical models that are analogous in some manner to other systems.

Fluid analog models have been used in a variety of contexts, from characterizing economic forces (think liquidity), to black holes (see e.g. [Dardashti et al., 2014], [Unruh, 2008]), to those discussed here in electronics and quantum physics. To begin with, consider an analogy between the idealized notion of a classical physical field and an ideal fluid.[1] Positively analogous aspects could be if both the field and fluid

[1]In these idealized physical notions, it is perhaps not so simple to separate the 'mathematical' and 'physical' aspects of what it is to be an "idealized physical concept". The mathematics could

were continuous, and could propagate waves which behaved roughly linearly. That is, if both of the respective wave systems obeyed the general superposition principle this would be one obvious positive analogical aspect.

On the other hand, perhaps our knowledge of atomic and molecular physics seems to make the continuous mathematical (and physical) aspect of the idealized fluid unrepresentative of actual fluids—whereas for a classical field we may still be okay with continuity. If one agreed with this latter statement, continuity would be an aspect between a classical field and an ideal fluid which is dis-analogous. One can see that there are arguments over determining exactly the degree of similarity between two domains. This particular example, and the ways in which one might feel regarding the analogy between these two domains, will prove to be at the heart of how one will view the discussion in the next sections. I will argue that, in the case of fluid analog models of the quantum world, the similarities outweigh the dissimilarities.

1.1 Arguments from Analogy

In this section I provide a minimal working basis with which to utilize in subsequent discussions concerning fluid systems, their similarities with various target phenomena, and arguments based on considerations of these fluid models in both the physical and computational contexts. There are many different approaches to characterizing arguments from analogy, and it is not my intention to resolve controversies in the literature concerning the proper characterization of all types of analogical reasoning or arguments from analogy. The reader may wish to familiarize themselves with a survey, for example in [Bartha, 2013], of various approaches in philosophy towards analyzing analogies—and some of the problems these approaches encounter.

I am mostly concerned in this present work with describing certain aspects concerning fluid mechanical analogies in science, and not in achieving a normative framework for how analogical reasoning in science should function. It is apparent that analogical reasoning and arguments from analogy *do* perform a function in science—and it is my job to convince the reader that this is the case (i.e. that I am accurately describing some aspects of scientific methodology). Then, we will see that we are able to draw some philosophical conclusions with this shared knowledge of fluid models.

However, it is still useful to have some working background on what is meant by arguments from analogy and basic analogical reasoning before pressing onward.

be used to precisely explicate an experienced mental representation of something 'physical', but are also important in their own right as mathematical objects first and foremost. This is an interesting discussion, but for the purposes here the formal representations of the field and fluid are presumed to be in some sense secondary to the more primary physical concepts they are associated with.

To begin a general characterization of arguments from analogy, it is perhaps best to discuss examples. There are many available in the literature, illustrating different ways in which similarities have been drawn between a variety of domains. I have chosen ones that are relevant—both historically and topically—to the subject matter of this thesis. That is, analogies concerning fluid mechanical systems in the physical and computational contexts.[2] Furthermore, I have tried to relay some lesser known examples than can commonly be found in the literature.

The first example comes to us from Nikola Tesla, an electrical engineer perhaps best known for his work on alternating current (AC), although his scientific achievements are numerous. In an attempt at reproducing and expanding upon Hertz's famous experiments taken to confirm Maxwell's theory of electromagnetism, Tesla apparently came up with an alternative theory (along with a plethora of inventions, devices, and demonstrations) to produce wireless transmission. In an article for the February, 1919 issue of *The Electrical Experimenter*, Tesla argues against Hertz's experimental approach by using a fluid analog model to support his earth-conduction theory of the transmission of wireless power.[3] According to Tesla, this argument was relayed to none other than Lord Kelvin:

> "In my exposition to [Lord Kelvin] I resorted to the following mechanical analogues of my own and the Hertz wave system. Imagine the earth to be a bag of rubber filled with water, a small quantity of which is periodically forced in and out of the same [bag] by means of a reciprocating pump, as illustrated. If the strokes of the latter are effected in intervals of more than one hour and forty-eight minutes, sufficient for the transmission of the impulse [through] the whole mass, the entire bag will expand and contract and corresponding movements will be imparted to pressure gauges or movable pistons with the same intensity, irrespective of distance. By working the pump faster, shorter waves will be produced

[2]The knowledgeable reader may be familiar with a number of hydrodynamical analogies. I will refrain from using the term hydrodynamical, since the more general field of fluid mechanics encompasses hydrodynamics. While I will be primarily discussing *liquids*, I do not want to rule out gas behavior and hydraulics as contributing to potential fluid mechanical models. My general thesis does not particularly depend on the inclusion of gases under the umbrella, however. I think it is more agreeable that fluid mechanical models are uniquely useful in science than just hydrodynamics, even though by encompassing more we dilute this 'uniqueness'. That said, the systems discussed are mainly hydrodynamical.

[3]Tesla contended that the earth was a near-perfect conductor, and that the ground connection of circuits involved in early radio transmission technology was actually essential for wireless transmissions, in opposition to the theory that radio waves reflect off of the atmosphere and travel above ground.[Tesla, 1919]

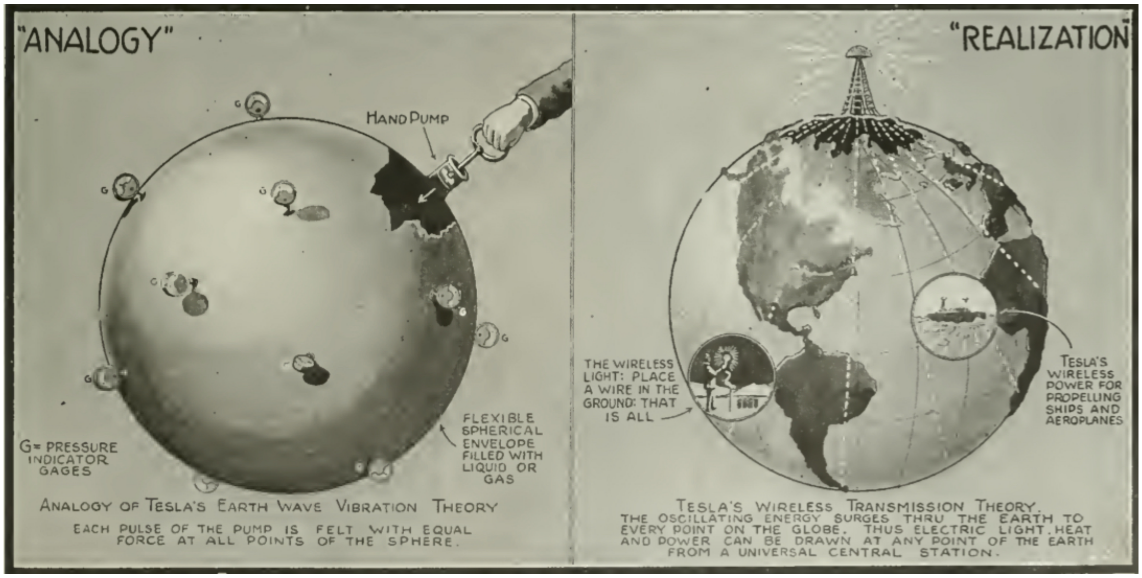

Figure 1: The diagram accompanying Tesla's article in the Electrical Experimenter.

> which, on reaching the opposite end of the bag, may be reflected and give rise to stationary nodes and loops. But in any case, the fluid being incompressible, its inclosure perfectly elastic, and the frequency of oscillations not very high, the energy will be economically transmitted and very little power consumed so long as no work is done in the receivers. [...] The Hertz wave system is in many respects the very opposite of this. To explain it by analogy, the piston of the pump is assumed to vibrate to and fro at a terrific rate and the orifice [through] which the fluid passes in and out of the cylinder is reduced to a small hole. There is scarcely any movement of the fluid and almost the whole work performed results in the production of radiant heat, of which an infinitesimal part is recovered in a remote locality."[Tesla, 1919]

I am not concerned here with whether Tesla was correct. However, I argue this is a paradigm example of analogical reasoning in science—particularly through a fluid analog model. There are several aspects present in this model, also to be seen throughout the thesis, which I argue helps support the conclusion that fluid mechanical models are unique as scientific devices. First, there are idealizations in the model such as the characterization of the fluid as incompressible, and the boundary conditions as perfectly elastic. Another aspect is that this model has been constructed for a specific purpose, relevant in a highly specialized case concerning wireless transmission. Finally, it enabled Tesla to construct an easily visualizable conception, and through analogy enabled the possibility for an argument to be constructed. In fact, it is used by Tesla to make both a *positive* and *negative* argument. The model admits a positive argument in favor of his own theory of wireless transmission, and Tesla then provides a (rather unfavorable) translation of the Hertzian account into the model to make a negative argument.

Another relevant example of analogical reasoning in science, which will help introduce quantum theory to the reader (more to come in the next section) comes to us from Erwin Schrödinger. He formulates the following 'symbolic proportion' when discussing how we might arrive in quantum theory at a wave mechanical description:

$$\text{Ordinary mechanics} : \text{Wave mechanics} = \text{Geometrical optics} : \text{Undulatory optics}$$

This reads something like "the relationship between ordinary mechanics and wave mechanics is equivalent to the relationship between geometrical optics and undulatory optics". The analogy comes via a historical consideration of optics, in that geometrical optics are an approximation (that light rays are like lines) for what is really an undulatory or wave-like entity. (For more on the history of this analogy see e.g. [Joas and Lehner, 2009].)

> "[The] description of a wave-motion in terms of *rays* is merely an approximation (called "geometrical optics" in the case of light-waves). It only holds if the structure of the wave phenomenon that we happen to be dealing with is coarse compared with the wave-length, and as long as we are only interested in its "coarse structure". The detailed fine structure of a wave phenomenon can never be revealed by a treatment in terms of rays ("geometrical optics"), and there always exist wave-phenomena which are altogether so minute that the ray-method is of no use and furnishes no information whatever. Hence in replacing ordinary mechanics by wave mechanics we may hope on the one hand to retain ordinary mechanics as an approximation which is valid for the coarse "macro-mechanical" phenomena, and on the other hand to get an explanation of those minute "micro-mechanical" phenomena (motion of the electrons in the atom), about which ordinary mechanics was quite unable to give any information. [...] The step which leads from ordinary mechanics to wave mechanics is an advance similar in kind to Huygens' theory of light, which replaced Newton's theory. [...] Typical quantum phenomena are analogous to typical wave phenomena like diffraction and interference."[Schrödinger, 1982, p. 160-162]

In this example, Schrödinger is not constructing an analog model so much as using an argument from analogy to characterize a legitimate theory about the world. It is an example of analogical reasoning which is perhaps used in a different sense compared to what we saw in Tesla's fluid analog model. This can be seen as one of the difficulties of pinning down the character of analogies used in science—there is more than one way in which analogies are used. Throughout this thesis, we will see a number of further examples which do not alleviate this worry. Rather, there is a very rich diversity in the ways in which analogies are utilized in science—and the ability of fluid mechanical models to match this diversity is, I argue, one of the supporting reasons for why fluid mechanical models have a unique status in science.

While the above mentioned disagreements over characterizing analogical reasoning and arguments from analogy no doubt extend to any analogical formalization, it is useful to briefly present some background on what I take to be a general formalism for analogical reasoning. I will be following in spirit the treatment found in [Hesse, 1966].[4] (See also [Bartha, 2013]) A useful formalism is found in which arguments from analogy are presented like proportion problems in mathematics, similar to the notation used by Schrödinger. We are given or have assumed a relationship between two terms, and one from another relation of two terms. From these three terms we can construct a fourth term. Using fractional notation such problems for x are of the form

$$\frac{a}{b} :: \frac{c}{x} \tag{1}$$

For simple mathematical ratios, we could have for example

$$\frac{2}{3} :: \frac{x}{9} \tag{2}$$

where we say "two is to three, as x is to nine". Solving for $x = 6$ is very basic and straightforward, treating the :: as an equality we simply multiply the diagonals getting $3x = 18$.[5] The double colon notation '::' can denote an analog or similarity

[4]This discussion uses Hesse's work as a jumping-off point, however I do not claim my treatment is ultimately consistent with hers nor without similar issues she has noted regarding analog reasoning in science.

[5]Although without some further clarification three possibilities might seem equally likely, that $x = 4, 6$ or 8. The latter option interprets the relationship between 3 and 9 as the addition of 6, and applies the same to the numerator. The first option interprets the relationship as squaring 3, and thus squaring 2 is 4. By convention, and use in analogical reasoning, we say that in this case $x = 6$ since it is the *proportion* that must remain the same—that is, it is not the relationship between 3 and 9 that counts but the relationship between 2 and 3 that we wish to preserve on the right hand side.

relation between observed phenomena or theoretical terms. While in strict problems of mathematical proportion an equality may be appropriate, for theoretical or inter-theoretical relations of analogy an equality is clearly not appropriate. Thus, I prefer this latter notation over Schrödinger's.

When we use the form of (1) to talk about scientific reasoning from an analog model, however, we might not be aware of certain negative or neutral analogical aspects that would allow us to easily solve for some property x. We may not be able to enumerate the conditions under which an analogical argument could be made—i.e. there is no clear 'fourth term' or there may be many of them.

Call the relationships on the left side of (1) in this simple representation the *analog model relations*, while those on the right side are the *target system relations*. Ideally we will have greater empirical access to or knowledge of the analog model, using it to gain information about the target system. The vertical relation (i.e. the fraction bar) is presumed to represent some kind of causal connection between aspects.[6] When a and c are sufficiently similar properties (i.e. $c = a'$), and since we know that 'a is to b', analogical reasoning suggests that $x = b'$.[7] For the philosopher of science, it should be clear that this mode of analogical inference that restricts the possible values of x has great potential as a heuristic device in a theoretical research program. A sufficient formal approach regarding analogies also explicitly represents the reasoning used in the program. We are not left wondering *why* certain experiments might be suggested (or invested in). As long as we can explain the analogy sufficiently, the suggested set of what should constitute the 'fourth term' is relatively clear. At the very least, this set of terms will be a subset of the available terms.

I will characterize arguments from analogy by using a *list* of pairs generated from positive or negative analogical aspects between domains. To make an argument from analogy we construct an extension of this list. This extension is the output of the argument. For elements in the domains X, Y being compared, a typical analogy is constructed by enumerating n similarities $x :: y$ such that $x_i \in X$ and $y_j \in Y$. The relation can thus be defined as pairs $\langle x, y \rangle$ which are unordered since $::$ is symmetric (but not necessarily transitive). When we say $X ::_n Y$, we mean that there are n pairs of elements x, y that are $::$ related. This is the ideal case, and the subscript will be omitted in practice when the number of pairs is unknown or under question.

A list of n such pairs plus a further term from one of the domains (forming half

[6] This is simply a preliminary presentation, Hesse also notes there are some very important and nuanced different ways to depict and analyze different argument forms that all might be considered 'analogical'.

[7] The equality here should be read as "c is some a' ", and given this we can say $a :: c$.

of the $n+1$ pair), can be considered the basis on which the existence of a further pair is asserted in an argument from analogy. In other words, the argument extends the list to include $n+1$ similarity relations between elements in the domains. In another direction, one can spell out a symmetrical argument for how two domains are *dissimilar* or dis-analogous. For now we will stick with the positively analogous lists between two domains. The argument from analogy is suggested when we have a list of positively analogous elements from two domains, and use this as a justification to extend the list to another pair.

In considering particular lists of similarities and dissimilarities between two domains, it may be helpful to layout a table outlining the known similarities, known dissimilarities, unknown similarities and unknown dissimilarities. Lowercase variables are here used for aspects (primed in the target system), but later it will be helpful to use capital letters when referring to specific aspects as they will be discussed in the text. The exposition here follows somewhat the expositions both in the work of Hesse as well as the expansions found for example in [Bartha, 2013].

	Analog Domain	Target Domain
Positive	$a, b, \ldots$	$a', b', \ldots$
Negative	$g, h, \ldots \neg l, \neg m, \ldots$	$\neg g', \neg h', \ldots l', m', \ldots$
Unknown	$p, q, \ldots$	$x, y, \ldots$

We use capital letters here X, Y to refer to domains (sets of aspects), and domains are constructed by the terms in vertical relationship.

$$\mathrm{X} :: \mathrm{Y} = \begin{array}{c} a \\ \hline \vdots \\ \hline n \end{array} :: \begin{array}{c} a' \\ \hline \vdots \\ \hline n' \end{array} \tag{3}$$

That is, what we mean by asserting a similarity relation between two domains is that there are n pairs, with each pair being constructed by an analogous aspect from each domain. I consider the above to represent a minimal working formalism, to be used in subsequent sections when discussing further fluid analog models.

2 Wave Mechanics and Many Worlds in the Physical Context

There will be several arguments throughout the course of this present work. These will each support the central thesis in some way. This thesis, as mentioned earlier, claims that fluid mechanical models are not only useful in scientific conceptualizations and arguments—they are in many ways *uniquely* useful in their ability to be applied to a wide range of problems. One of the most important ideas to be analyzed using fluid analog models in this work, is the notion of many worlds in the quantum world—as it will be important to consider in both the physical and computational contexts. Some background on quantum theory and the many worlds approach will be helpful.

We will have to revisit some of the fundamental concepts that Einstein, de Broglie,[8] and Schrödinger in particular were engaged in debating in the foundations of quantum theory. Cushing notes that they *"... shared a commitment to a continuous wave as a basic physical entity subject to a causal description. [...] One could well deem it better to have an understandable, if imperfect, theory with a definite, classical type of wave ontology than an abstract theoretical framework with a conceptually opaque ontology based on some concept of a wave-particle duality."* [Cushing, 1998, p. 286] Additionally, Cushing quotes Schrödinger on the same page (I substitute the translation from [Joas and Lehner, 2009]): *"[This means nothing else] but to get serious about the de Broglie-Einstein undulatory theory of the moving particle, according to which the latter is nothing but a kind of 'crest' on a wave radiation forming the substratum of the world."*

A similar field-theoretic notion can be found later in the works of Bohm:

> "What is implied by [the 'zero-point' energy] is that what we call empty space contains an immense background of energy, and that matter as we know it is a small, 'quantized' wavelike excitation on top of this background, rather like a tiny ripple on a vast sea. [...] An interesting image is obtained by considering that in the middle of the actual ocean (i.e., on the surface of the Earth) myriads of small waves occasionally come together fortuitously with such phase relationships that they end up in a certain small region of space, suddenly to produce a very high wave

[8]Both de Broglie brothers had a big part to play in discovering matter waves (see e.g. [Wheaton, 2007]), and this is the foundation for Schrödinger's subsequent work. A growing position questions the notion of quantum 'particles', and for a growing number of phenomena we are realizing that the particle picture is not necessary nor accurate. [Bitbol, 2007] I agree that we should relinquish, or push off as far as possible, the particle picture.

which just appears as if from nowhere and out of nothing. [Bohm, 1980, p. 242-244]

I wish to take these quotes at face value as succinct formulations of one of the central ideas to be discussed in this paper: that the quantum world should be visualized or conceptualized in a fluid-like manner—or through *analogy* with fluid behavior. For these reasons, and others to be discussed throughout, it should be clear that there is a deep conceptual and historical connection between systems and concepts of fluids and the quantum world. The connection of idealized fluid models in physics, particularly in addressing action at a distance and field theories, goes back at least to Maxwell, Kelvin, and others. The curious reader may wish to take a look at [Hesse, 1962], [Hesse, 1955] among other references for more on this issue.

2.1 A Brief Introduction to Wave Mechanics

Erwin Schrödinger's wave equations are famous developments in the history of quantum theory, and their relevance and use today is widespread. A brief introduction will be helpful before entering into subsequent sections. Intuitively, in a wave equation we want to represent the change of a medium as a wave propagates through it. A classical wave equation for one dimension takes the following form:

$$\frac{\partial^2 y(x,t)}{\partial x^2} = c^2 \frac{\partial^2 y(x,t)}{\partial t^2} \tag{4}$$

This is a linear second order partial differential equation, representing the time evolution t of a wave with amplitude extending in the y dimension of an $x - y$ plane, traveling with velocity c. Solutions of this wave equation will take the general form

$$\psi(x,t) = p(x - ct) + q(x + ct) \tag{5}$$

We refer to ψ as a wave function. A combination of functions p and q, when themselves solutions, will combine to give us a solution to the wave equation. Schrödinger's time dependent quantum wave equation, in one dimension, is the following:

$$\frac{-\hbar^2}{2m} \frac{\partial^2 \psi(x,t)}{\partial x^2} + U(x)\psi(x,t) = i\hbar \frac{\partial \psi(x,t)}{\partial t} \tag{6}$$

The potential $U(x)$ denotes boundary conditions, while $\hbar = \frac{h}{2\pi}$ is the reduced Planck constant and m a mass term. The boundary conditions are what restrict the solutions of this wave equation to discrete values—and this is the hallmark of quantum mechanics. This equation is also linear, and two superposed solutions will likewise be another solution. By superposition, we mean the combination (denoted by '+') of two functions.

However, it is interesting to note that although Schrödinger is considered the best-known proponent (and also perhaps the originator, after de Broglie) of the wave mechanical approach, his own interpretations of the formalism are *not* typically used today. His goal was to visualize matter in terms of standing wave packets, where the modulus-square of the amplitude was related to an electrodynamical electric charge density $\rho = -e\psi\psi^*$. [Bitbol, 1996, §2.3-2.4] (Here ψ^* denotes the complex conjugate of the wave function ψ.) Max Born interpreted values calculated by $|\psi|^2$ (where $|\cdot|$ denotes the complex modulus) in terms of probability densities of observing *particles* or discrete quanta in those regions. This became incorporated into the standard Copenhagen Interpretation. I take Schrödinger's attempt to be a more fruitful approach for someone like myself with realist intuitions and a sympathetic taste for visualizable models.[9]

A contemporary of Schrödinger, Erwin Madelung, even provided an interpretation of Schrödinger's equations likening the terms to an ideal fluid:

> "In two papers written in the fall of 1926, [Madelung] developed a hydrodynamical-like formulation of Schrödinger's theory. By expressing the wave function as $\psi = R(\exp(2\pi i \frac{S}{h}))$, Madelung tranformed the Schrödinger equation into two equations, one corresponding to the imaginary part and the other to the real part; he suggested that the equations represented a charged but non-viscous ideal fluid of density $\rho = \psi\psi^*$ and velocity $v = \nabla\frac{S}{m}$." [Kragh and Carazza, 2000, p. 51]

Schrödinger referred to the characteristic solutions of boundary conditions in quantum wave mechanics as the 'proper values' of atoms, in analogy with traditional vibration problems (such as in determining harmonics of musical instruments). (See e.g. various sections of [Schrödinger, 1982].) These would be certain harmonic solutions of a wave equation in given boundary conditions. The result is standing waves, particularly in spherical harmonics. One objection to this approach which came primarily from Lorentz, but also Heisenberg, was that such wave packets generally spread out in space relatively quickly when unbounded, and that the individual atoms would quickly become delocalized.

In a famous article translated as "The continuous transition from micro- to macromechanics" Schrödinger had shown a case of a wave packet that did not spread

[9] I think something akin to the approach of [Hobson, 2012, p. 7] as interpreting the Born rule in terms of *interactions* between real fields is more satisfactory, but this should be investigated further.

out, and stayed localized during time evolution. [Schrödinger, 1926], [Bitbol, 1996, p. 45-46] Under clearly defined (finite) boundary conditions (such as a container or box), standing wave packets are apparently stable and function in the discrete way Schrödinger intended. Michel Bitbol notes that Schrödinger was influenced by Eddington's idea that considered "[...] *'particles' as proper modes of vibration of the closed universe as a whole* [...]" with a finite universe analogously corresponding to the finite walls of a box.

> "Schrödinger's initial picture of quantized matter waves was not only retained and extended beyond the domain of a gas sample, but the boundary conditions it needed (the "box") was made completely "natural" by invoking the finiteness of the universe." [Bitbol, 1996, p. 48-49]

The quantized standing wave packets of Schrödinger were based on a conception that they were extensions or excitations (or 'crests') of a background (fluid-like) medium. Following Cushing's analogical explanation, we start with a simple standing wave on a string that is bounded on either side (Fig. 1).[Cushing, 1998, p. 274; 292] For wavelengths λ, we get solutions $\frac{\lambda n}{2} = l$, where n is an integer and l is the length between the boundaries.

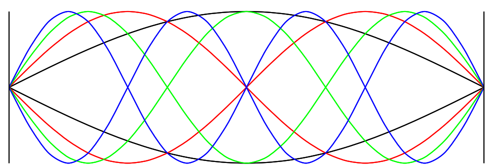

Figure 1. *The appearance of the first four harmonics of a standing wave, where* $n = 1, 2, 3, 4$.

Each standing wave is made from two superposed oppositely traveling waves of the same frequency. These standing wave 'solutions' of the boundary conditions are analogous to the discrete energy levels, or allowed orbits, in quantum mechanics. [Cushing, 1998, p. 292] Wave packets are depicted in a similar manner, traveling along a one-dimensional line. (e.g. see [Schrödinger, 1926])

We have clearly seen in the previous sections that historically a fluid analogy or conceptualization has been integral to particular approaches to quantum theory. Conceptualizing the quantum world in a fluid-like manner is justified both conceptually and historically, and we will build on this background in the next sections.

2.2 Many Worlds

Many Worlds as an approach to quantum mechanics is also referred to as the Everettian interpretation, named after the physicist Hugh Everett. In this approach, all superposed components of a quantum wave function continue to evolve. This includes the components which we do not apparently 'observe' through a measurement, since a measurement will only show us one of the possible outcomes in a superposition. However, a proponent of a many worlds approach may say that there is another 'world' where the other components are measured. There are many variations of this approach which attempt to reconcile the appearance of one (classically appearing) world with the 'actual' time evolution of all components in a superposition of waves.

There are two senses in which I argue the many worlds proponent characterizes her worlds. The first I will call *naïve* many worlds (NMW), and the second is called decohered many worlds (DMW). The NMW considers worlds to simply be components in a linear superposition, whereas the DMW utilizes what is called decoherence to pick out worlds. Coherence is the consistent phase relationship between multiple wave functions, and decoherence is the breakdown of this consistent phase relationship—perhaps through interactions with the 'environment'.

The difference is important: the NMW position takes all components to be functioning worlds, without addressing why we seem to only see *one* of these worlds. The DMW fixes this drawback by taking decoherence processes to be critical in the many worlds account, allowing us to explain why we observe some worlds and not others. The worlds we observe are the ones that are picked out (or 'survive') decoherence.

However, the benefit of the NMW position is that it utilizes a mathematically rigorous idea rather than an abstract notion relative to the human experience. This is opposed to the exposition in e.g. [Vaidman, 2014], where the author claims that the notion of worlds is in an attempt to provide a "correspondence between the quantum state of the Universe and our experiences." Even though decoherence is a rigorous mathematical and physical concept, we are here concerned with a coherent quantum world—and presume that the universe is on the whole coherent. The decohered worlds are still a subset of a quantum universe which (I assume) is coherent. That NMW does not rely on decoherence to pick out worlds proves to have an additional benefit: it can be discussed in the quantum computational context, where there is ideally no decoherence.[10]

NMW does have to account for interference terms, so the definition needs to be

[10] Or, to put it another way, under the assumption that the universe is coherent we may question what exactly decoherence *is*. Is it a theory of perception, or metaphysics? Should we see worlds via 'decoherence' as patterns such as advocated by [Wallace, 2005]?

clarified slightly to say that worlds are *orthogonal* terms in a decomposition. Representing classically observed states by these orthogonal terms, we can still construct a superposition that has interference terms in which there is a non-zero amplitude for states that are not classically orthogonal. This begs the question: do quantum worlds actually interfere? If they do, then they must be present in the same space and overlapping. If worlds are only vaguely representing some aspect of our experience or interaction with the world, and are not actually representing waves, then we have no reason to suppose that they are overlapping or capable of interference.

We *do* have interference terms in quantum mechanics, so the many worlds proponent must admit that the worlds (i.e. orthogonal state representations) not only overlap but interfere. These interference terms are furthermore not related with ignorance or probability, but with coherent amplitudes. Unfortunately, this does not bode well for the DMW approach, since the universe (and the 'universal wave function' $|\Psi\rangle$) is arguably coherent. We can justify this claim by considering the Eddington-Schrödinger notion of the finite universe corresponding to boundary conditions determining proper modes. There are even macroscopic quantum effects which rely on coherence, and we can observe these. How do we define the world in which I observe macroscopic quantum coherence? The object, such as a superconducting wire, superfluid, or Bose-Einstein Condensate (BEC), should be decoherent because it is a macroscopic object (i.e. appears in a definite classical state)—but it *isn't*. Could this be because quantum waves are not just epistemic?

The discussion in this thesis, utilizing fluid analog models, provides an affirmative response to this question. Through analog models and analog reasoning, we will see a number of critical issues discussed—particularly the notion of many worlds in both quantum mechanics and in the context of quantum computation.

2.3 The Many Worlds Argument

For the negative analogical argument against many worlds, we could simply refuse to extend the list of similarities between the fluid analog models discussed and the quantum world to include the aspect of 'many worlds'. This is not particularly interesting, and it seems that perhaps we can grant that worlds may be useful to characterize surface waves in a fluid. However, I will argue that even *if* such a similarity relation is granted between many worlds in both domains, this is still detrimental to the MW account of quantum mechanics since it means there are worlds of the same *kind* present in the fluid models. A list of proposed analogical aspects between the fluid and quantum domains is presented in the following table, where the similarity relation :: is read horizontally, and the (somewhat) causal relation is read vertically. The last three entries are the crucial pairs to be discussed in the following sections.

Fluid Analog Models	**Quantum World/Theory**
Droplets	Quantum 'Particles'
Faraday Surface Waves	Pilot Waves
Surface Wave Interference	Quantum Interference
'Wavelike' Statistics	Probability Amplitudes
'Tunneling'	Quantum Tunneling
Analog Casimir Effect	Casimir Effect
?	Entanglement
?	Many Worlds (?)
?	Quantum Speed Up

As we proceed through the dissertation the various analogical aspects I have asserted in the above list will become more apparent. It will be seen that there are many formal and conceptual similarities (that is, *positive* analogical aspects) between fluid models and the quantum world. Nonetheless, I will note dissimilarities when they arise. Regarding many worlds, I will argue that even *if* we grant the similarity relation holds regarding many worlds, then a triviality results from the existence of many worlds in surface waves and vibrating fluid baths (VFB). In fact, the triviality result extends to any linear system. Additionally, there are important cases where it is more practical to refer to the superposed state rather than the 'worlds' composing it.

That the notion of many worlds in a VFB does not seem necessary is, I argue, an important physical reason *not* to take the many worlds account of quantum mechanics—and, by extension, quantum computation—seriously. What I mean by 'physical reason' is that we literally have material, observable, physical systems whose behavior can be used as justification for reasons in an argument. This is an argument made by analogy—against the notion of many worlds. That the behavior of the discussed fluid systems are similar to the quantum world—formally, conceptually, and physically—means they *are* relevant for our discussions about the quantum world, and the explanations we try to construct using interpretations of quantum mechanics.

I argue that the fluid analog models are important in a fundamental explanatory way, and that the quantum and fluid worlds are more similar than dissimilar. That they are similar explains why we see the analogous phenomena produced. Those who reject this will have to make an argument as to why the fluid systems are more dissimilar to the quantum world. Furthermore, they will have to explain why similar phenomena are produced in the fluid systems—not to mention explaining away the formal similarities between the domains which have been fruitfully exploited by field theorists and fluid analog users in various domains.

This is, I think, a difficult position to defend given the enormous historical and conceptual links between field theory, the development of hydrodynamics and wave mechanics, and the mathematical developments of Erwin Schrödinger among others. In other words, there is a sort of 'built in' similarity in quantum mechanics with the fluid systems exhibiting the quantum-like behavior, and this not only bolsters the approach taken in this dissertation but makes the argument against many worlds even more devastating. It seems no one ever thought to introduce the existence of many worlds into our explanation and description of waves on the surface of fluids, and the notion is not necessary in this context—even for all practical purposes. In fact, as mentioned, I argue that for all practical purposes (FAPP) there are cases in which the notion of worlds is misleading and *un*practical. For example, in the important case of standing waves, we practically deal with the standing wave and not the two oppositely phased components that construct it. It may be a superposition of *two* surface-wave 'worlds', but FAPP it is *one* wave—one *world*.

2.4 My Job

My job in this thesis is to first of all convince the reader that fluid analog models are relevant and, on the whole, more similar than dissimilar to the quantum world. Once this is achieved, the reader should agree with me that the notion of many worlds loses much of its explanatory appeal. The main dis-analogies between fluid analog models and the quantum world come in the difference of dimensions and the substance or medium involved in producing the phenomena, while the positively analogous aspects are mainly found in highly controlled and idealized conditions. For example, similar kinds of boundary conditions and the linearity of wave mechanics are found in the respective domains.

The next sections will serve as a detailed exposition of the relations mentioned in the table above. Those similarities under question such as entanglement, many worlds, and speed up will each be addressed in turn. After hopefully convincing the reader that there is indeed a relevant similarity between the fluid analog systems involved and the quantum world, we will transition into a discussion of fluid analog models in the computational context. Once this context is also established for the reader, we will turn towards the intersection of these two contexts: fluid analog models for quantum computation.

What we will see throughout the thesis, is a consistent usefulness of not only scientific reasoning through arguments by analogy, but that the analogies considered are also useful as conceptual aids independently of their use in reasoning. While this latter sense may many times be part of the inspiration for constructing an analogical argument, it is independently interesting in its own right. Thus, the lesson to be learned is that—particularly in the fluid mechanical analogies discussed—analogies are an important part of scientific reasoning and scientific conceptualizations. Additionally, we will continue to see the presence of idealization, and the continued ability of fluid mechanical models to specialize in a number of areas.[11]

[11] An objection to the aspect of specialization of fluid mechanical models might be that actually it is the opposite of specialization—fluid mechanical models have a wide range because of general properties. I would counter this point by noting that most of the models are independently constructed for their relevant targets. In other words, it seems to be rare that the same table-top fluid model is used for two separate targets. Rather, the experimental set-ups for the analog model need to reflect the target system's experimental set-ups.

3 Fluid :: Quantum

3.1 Faraday Surface Modes :: Quantum 'Waves'

Following the work of Robert Hooke and Ernst Chladni on the investigation of patterns arising in powdered materials on vibrating plates of glass or metal, Michael Faraday undertook a series of experiments on the behavior of different fluids on similar plates. [Faraday, 1831, appendix] The interference patterns and curiously beautiful regularities—what Faraday calls 'crispations'—that occur in the fluids under such circumstances are often referred to as "Faraday waves". It is important for the reader to understand what these waves are, as they will prove to be central to the analysis of our fluid analog models in both the physical and computational contexts. Faraday waves are standing waves in the higher-dimensional fluid system (compared to the one-dimensional 'string' example above). Faraday recognized the key characteristic of the 'flickering' up and down of antinodes.

> "[...] notwithstanding the apparent permanency of the crispated surface, [...] the heaps were not constant, but were raised and destroyed with each vibration of the plate; and also [the] heaps did not all exist at once, but (referring to locality) formed two sets of equal number and arrangement, fig. 23, never existing together, but alternating with, and being resolved into each other, and by their rapidity of recurrence giving the appearance of simultaneous and even permanent existence."[12]
> [Faraday, 1831, p. 328-329]

The vibrations or *modes* of the plate distributes energy non-locally, in the sense that the 'heaps' of powder or amplitudes of the fluid appear localized—*but are actually formed from the harmonic solutions of the boundary conditions.* In other words, the crests and troughs are going up and down simultaneously even though each crest or trough is localized with respect to points on the surface. Of course this is only a model, and this point must be stressed. Many times the string-in-a-box analogy is given, but I have not seen an exploration into how the analogy might change or influence our conception of an atom once we depict the model in more dimensions.

As noted by scientists working on vibrating fluid bath analog systems, Faraday waves—the standing surface waves created by vertical oscillations of the bath—bear a striking resemblance to de Broglie's notion of guiding 'pilot' waves in quantum theory. Faraday waves are standing surface waves formed in a fluid, non-linear in practice due

[12] His figure 23 is a checkerboard pattern, presumably representing crests and troughs (or 'heaps' and anti-'heaps').

to dampening and non-ideal fluid terms, but in the ideal fluid case it is reasonable to assume linearity in an ideally tuned vibrating fluid bath that is vertically oscillated. We assume a system of ideal Faraday waves obey the superposition principle. See e.g. [Eddi et al., 2011].

3.1.1 Surface Wave Interference :: Quantum Interference

In the traditional exposition of quantum mechanics, the double-slit experiment (DSE) is used to convey the sense in which both particle and wave pictures are necessary to adequately explain quantum phenomena. A typical set-up involves aiming a laser, or a coherent light source, at a barrier in which two holes (or slits) are cut. A screen behind the barrier records the distribution of light, in which an interference pattern is apparent. We attribute this to the wave nature of light, allowing for interference between the two modes emerging from each respective slit just like that seen in surface waves of a fluid. When one slit is open, we do not observe the interference pattern.

Double slit experiments have even been performed with other atomic-scale objects, which we would typically think of as discrete particle-like entities more than photons of light. The Copenhagen adherent, utilizing Bohr's concept of complementarity, would say that ascribing either particle-like properties or wave-like properties to quantum entities is thus misleading—and we should just embrace a fundamental duality between the particle and wave pictures.

It is remarkable that the VFB systems and fluid analog models can produce similar statistics to those observed in the DSE. Fluid droplets on the surface of VFBs can have prolonged bouncing behavior, associated with the surface waves which they produce upon each bounce. The droplets and their associated surface waves which serve to determine their trajectories have the very same (or very *similar*) dualistic quality explicitly represented at the macroscopic scale. We will discuss these droplets more in the next sections, but first we will talk about surface wave interference as an analogical aspect to quantum interference.

In fluid analog reproductions of the DSE, one sees that the surface wave modes associated with walking droplets pass through both slits—thus creating two centers of propagation capable of interference. This is obviously similar to the Bohmian account, where our guiding quantum wave for the quantum particle (or point particle, for Bohmians) goes through both slits in the experiment. There is a rather direct analogy between surface wave interference and quantum interference. Taking this a step further, I propose the following reconstruction using the similarity relation between the waves which must *cause* this interference. Placing the effect of interference on the top, the wave causes from the fluid and quantum domains are below each of the respective interferences.

$$\frac{I_Q}{W_Q} :: \frac{I_F}{W_F} \tag{7}$$

This reads something like "quantum interference from (or caused by) quantum waves is analogous to surface wave interference from surface waves". The usual qualifications, such as that the waves are both assumed to be linear, are still in effect.

3.1.2 'Wavelike' Statistics :: Probability Amplitudes

Yet another crucial analogical aspect between the fluid and quantum domains is that we can characterize statistics of walking droplets according to the Faraday modes of the boundary conditions in the fluid bath. In other words, the interference of surface waves and underlying Faraday modes of the vibrating bath determine probabilities of droplet location. The previous analogy between surface waves and quantum waves (including the aspect of interference) can be extended to include the statistical behavior associated with the droplets and observed probabilities in quantum mechanics (and calculated via the Born Rule).

The notion of probability amplitude in quantum mechanics, that the amplitudes somehow represent probabilities, is somewhat clarified when the analogy is granted: amplitudes are amplitudes, and probabilities are probabilities. This allows the formulation of a relation such as the following:

$$\frac{S_Q}{M_Q} :: \frac{S_F}{M_F} \tag{8}$$

This one reads like "statistics in quantum mechanics from (or caused by) quantum modes is analogous to statistics in fluid systems from fluid modes (Faraday modes)". In the fluid there are real waves (with real amplitudes), and we associate a certain statistical behavior of the walking droplets as a function of the surface waves. In the quantum case, it seems that probabilities and amplitudes are conflated in the notion of a probability amplitude. However, we can take a hint from the analog model and keep our analysis tidy by keeping the quantum amplitudes and probabilities separate.

3.2 Droplets :: Quantum 'Particles'

At first glance, perhaps the most obvious aspect of similarity which is present between the fluid analog systems and the quantum world, is that droplets which are sustained on the surface of a vibrating fluid bath (VFB) behave as more or less inelastic, discrete, localized, identifiable bodies. They are, in other words, like *particles* as long as they do not coalesce back into the fluid bath. While a similar definition of particles has traditionally encountered severe difficulties in the quantum world, it is nonetheless popular to refer to quantum entities as ‘particles’. I have no issue with this characterization so long as one remains conscious of the limitations of this terminology, and of the essential wave nature of matter.[13]

The groundbreaking work of, for example [Fort et al., 2010], involves vibrating a fluid bath just below what is called the Faraday threshold. Informally, the ‘Faraday threshold’ is the critical acceleration applied by a bow on the edge of a metal plate (or the critical vertical vibrations of the fluid achieved in modern experiments) needed to produce surface waves. Specifically tuning a VFB just below this threshold allows the surface to remain stable and flat. Faraday instability is the term for when the surface of the fluid becomes unstable at (or above) the threshold, allowing the vibrations in the fluid to become sufficiently violent to overcome the surface tension, gravitation, and resistance of air on the free surface.

Adding a droplet of the same fluid to the surface (perhaps by quickly dipping a needle and withdrawing it as mentioned by [Terwagne and Bush, 2011]) generates interesting behavior. These droplets are sufficiently robust and stable from the fluid’s surface tension, viscosity and surrounding air pressure to survive multiple bounces on the vibrating bath. The interaction between the bouncing droplet and the ‘readied’ vibrating bath creates sustained surface-wave-walking-droplet systems. Stable fine-tuned conditions can prolong this ‘walking’ behavior indefinitely (at least in our ideal discussion of such a bath).

> “[. . .] the points of the surface disturbed by the bouncing droplet keep emitting waves. The motion of the droplet is thus driven by its interaction with a superposition of waves emitted by the points it has visited in the recent past. This phenomenon, easily observed in the wave pattern of a linearly moving walker [. . .], generates a path memory, a hitherto unexplored type of spatial and temporal nonlocality.” [. . .] “At all times [the droplet’s motion] is driven by its interaction with a superposition of

[13] I do not subscribe to the complementarity of Bohr, and indeed it fits the context here to remain attached to Schrödinger’s (among others) wavelike conception as more primary or more fundamental than the particle picture. Quantum ‘particles’ are *defined* by de Broglie’s relations, not the other way around. In other words, one can talk of particles as an approximation for the crest-like excitations of a background field, but one cannot grant these particles existence since they are merely appearances. This wave-only position, similar to a field theoretic approach, has been argued for elsewhere and will not be directly argued for in this dissertation.

> waves emitted at the points it visited in the past. Through the mediation of these waves, the present motion is influenced by the past trajectory. This is what we call the 'wave-mediated path memory' of the system." [Fort et al., 2010, p. 17515, 17516]

The motion of the droplet is driven by its interaction with the superposition of waves emitted from its past locations in a bath tuned near the Faraday threshold. The droplet would not continue to bounce, and would coalesce if the bath did not have the appropriately tuned amount of energy continuously being input. Furthermore, as [Fort et al., 2010, p. 17518] even note, the damping of surface waves increases (decreases) further from (closer to) the instability threshold. This means that the temporal persistence of the interfering 'memory waves' from past locations of the droplet are primarily due to the fact that the vibrating bath is in such a state (near the threshold) as to allow bouncing and sustained surface waves in the first place. The bath is near an *idealized* state, much in the same way we idealize surface waves on water as behaving linearly.

At energy levels sufficiently beyond the Faraday threshold, the surface breaks from high-amplitude waves and droplets can be ejected from the fluid. This is examined in [Terwagne and Bush, 2011, §3.4] in an interesting example with Tibetan 'singing' bowls. A critical 'droplet ejection acceleration threshold' Γ_d (the same kind of parameter as the Faraday threshold) is characterized, dependent on the forcing frequency f. This non-linear aspect observed in VFBs is apparently dis-analogous with the quantum world, but it is not present in systems producing analogous quantum behavior.

The analogy with the quantum world does break down in another important way, however. The amplitudes of surface waves on the *free surface* of the vibrating bath extend vertically into the layer of air above the bath. A corresponding quantum 'fluid' (or fluid-*like* field), which would fill all space three-dimensionally, does not have a free surface.

An ideal gas, for example, fills a spherical container whereas a liquid forms a free surface in the presence of gravity. Without a free surface, there is no means for the easily visualized transverse amplitudes to extend. Thus, there seems to be a conceptual limit to the classical notion of transverse waves in a medium purely from an analysis of dimensions and free surfaces. The concept of an 'amplitude' as representing the difference in pressure or density from equilibrium is then a natural proposal, since the intensity must still be accounted for.

One can imagine that certain gases, or entirely enclosed fluids with no free surface or gravitational influence, could be investigated in spherical cavities, which are vibrated by spherical incoming waves. Amplitudes of spherical harmonics, however, do not extend transversely in a conceptually straightforward way into a *fourth* dimension. Thus, amplitudes of quantum waves in three dimensional space also provide a problem of interpretation. This is well known, but it is useful to see the problem arise from the boundary between classical transverse and longitudinal waves in a higher-dimensional fluid.

3.3 'Tunneling' :: Quantum Tunneling

Quantum tunneling is an effect where the amplitude of a state is distributed beyond certain boundary conditions (i.e. a potential or barrier). This results, through the Born Rule calculation, in a non-zero probability of observing a quanta outside of this boundary. Classically, such as in the case of a ball or rigid particle in a valley, such a body is confined to the valley—it cannot escape under 'natural' conditions. Unless we impart a force upon the body sufficient for it to roll up and beyond the side of the valley or crash directly through, it will remain in the valley.

Vibrating fluid bath systems can also exhibit similar phenomena to tunneling. Droplets as 'walkers' on these baths can occasionally jump over barriers constructed in the bath. [Eddi et al., 2009] In the vibrating fluid bath analog models, one actually has surface waves associated with walking droplets—and the impinging of these surface waves on a barrier (such as a circular corral) can interfere so as to allow, with certain probability, the droplet to jump over the barrier.

The analogy between 'tunneling' in the fluid bath and quantum tunneling is, then, quite nice. It involves both the particular and wave mechanical accounts in the quantum world, since in the bath there are surface waves associated with the walking droplets.

3.4 Analog Casimir Effect :: Casimir Effect

In this particular analogical relation, we will see that a good example of analogical reasoning is found by scientists working on a macroscopic fluid model of a well-known quantum behavior. In a similar set up to the VFBs discussed earlier, there is a table-top fluid model of the so-called Casimir Effect. (see [Denardo et al., 2009]) The (quantum) Casimir effect is produced between two very small (and very thin) uncharged parallel metal plates that are placed close together in a vacuum. At certain distances d the plates are pushed together by a force, while at others they are pushed apart.

In quantum theory we can explain this from a non-zero energy associated with the ground state of each mode in the quantum vacuum $hf/2$ (where f is the frequency of the harmonic oscillator associated with the mode and h is Planck's constant). Following Planck's approach (see [Hobson, 2012, p. 8-9], [Kragh, 2012, §3], also [Mehra and Rechenberg, 1999]) the energy levels of a quantum harmonic oscillator are given by

$$E = hf(n + \frac{1}{2}), \tag{9}$$

where h is Planck's constant and f the frequency according to integers n. Even in the ground state where $n = 0$ the energy of the oscillator is $\frac{hf}{2}$, which is called the *zero point energy* or, historically in German *nullpunktsenergie*.[14] Thus, the ground state of an associated quantum field has a non-zero energy density.[15]

We can account for such behavior in quantum electrodynamics by calculating the relative difference in pressure between the force of electromagnetic radiation outside the plates and inside the plates. The closeness of the plates excludes certain wavelengths of the background spectrum from the interior of the cavity. The spectrum of zero point frequencies is infinite both on the outside and inside of the plates, but after renormalization [Denardo et al., 2009, p. 1095] note that the result of the calculation gives a force of $\pi^2 \hbar c/240d^4$ per unit area.

The water wave analog model that the authors construct consists of a table-top bath which is vertically vibrated according to a range of frequencies (10-20 Hz), exciting various surface waves in the bath. Two acrylic or PVC plates are hung in parallel above the bath and dipped deeply but partially into the VFB. The surface waves in the VFB are analogous to zero point fluctuations, giving us the opportunity for a similar explanation to above for the quantum Casimir effect. Again, this explanation stems from the difference in pressure between outer and inner surfaces

[14] This quantity can also be derived as a consequence of Heisenberg's uncertainty relations according to [Kragh, 2012, p. 212], and is an important aspect of the theoretical *interpretation* of zero point energy. However, it is unclear whether this is supposed to be an *explanation*, since if one maintains an epistemological interpretation of Heisenberg's relations, it is not clear exactly what we are uncertain *about*. Subsequently, it is unclear what relationship the epistemological limitation from the uncertainty relations should have with the apparently *ontological* aspect of zero point energy. In other words, isn't the zero point energy *there* in the world regardless of the accuracy limitations of measuring conjugate variables?

[15] In [Mehra and Rechenberg, 1999, p. 120-121] the authors conclude that zero-point energy *cannot* be linked with the ground state of a quantum field. Rather, it should be considered a heuristic concept used to explain certain physical and chemical consequences. However, I think the zero point energy itself is in need of an explanation, and is a physical consequence of *something else*. Especially given its potential importance to the cosmological constant, we should not think of the half-integer zero-point energy as a heuristic invention any longer—although that is certainly a way it was *historically* handled by some physicists. See [Kragh, 2012].

due to the exclusion of certain waves between the plates.[16]

We can model the relationship between these two systems in the following way according to the similarity relation discussed above. Represent the effect of two parallel plates being pushed together when dipped in a VFB as E. To differentiate, let us say that E' represents the *similar* effect on Casimir plates. The VFB for the analog model is B, and it has a causal relationship to E in that it causes the relative pressure due to wave motion to be greater on the surface area outside the plates than on the surface of the interior. Thus far we can say:

$$\frac{E}{B} :: \frac{E'}{x} \tag{10}$$

I denote the target system (the system we have constructed the model for) with primed variables since it is actually the table-top fluid analog model that we have more concrete knowledge of. We can observe the spectrum of surface waves in the bath, whereas we cannot directly observe the zero point fluctuations.[17] In fact, the Casimir effect is taken as evidence for the existence of zero point modes in the first place. We cannot observe directly the *cause* of the quantum Casimir effect—the Casimir phenomena is the effect of a presumed zero point energy cause.[18] We see that observing the behavior of the macroscopic fluid system can be used to reason about the explanation of the quantum Casimir effect. This will serve as a preview of the type of reasoning used later in an argument against the Many Worlds account of quantum mechanics. Naturally, the type of reasoning is through analogy.

Our choice for x, if we had no other knowledge of the target system, would arguably be such an x that fulfills similar conditions as B (that is, x should be some causal explanation B'). Intuitively, it seems overwhelmingly more likely that the chances for $x = B'$ are *much greater* than other options—given that we have

[16]Also mentioned in both [Denardo et al., 2009] and [Barrow, 2002, §7] is the even larger practical application of macroscopic 'Casimir' effects to parallel marine vessels whose rolling on a swell will result in the destructive interference of waves roughly 180° out of phase in between the ships, thus allowing the relative pressure difference from the waves on the outer surfaces to push the ships dangerously close together. The destructive interference aspect of the explanation is, however, dis-analogous to the Casimir effect since the ships are producing surface waves between themselves that cancel, whereas the difference in pressure in the Casimir effect is explained by the exclusion of certain wavelengths of waves. This aspect is more clearly analogous in the table-top model considered here.

[17]That is, certainly not in the same direct sense that we can view surface waves at the macroscopic scale.

[18]Assuming that the Casimir effect is not the result of Van der Waals force. The treatment here is sympathetic to a field-theoretic approach which, some might argue, is more 'fundamental' than the molecular level.

established some justification for applying the analog model in the first place (i.e. we have $E :: E'$). In this case, we have a theory of vacuum fluctuations from quantum electrodynamics that can supply such a B'.[19]

However, we know from earlier that the analogical relationship at least breaks down with respect to the *dimensions* of the analog model and the target system. The authors consciously note other deficiencies in the analogy:

> "The analogy of our water wave system to the Casimir effect is not exact. Because the water waves are driven, the energy density of the spectrum is not infinite, so a regularization procedure is not needed. Furthermore, we are primarily concerned with the case of closely spaced plates, which yields a force that is independent of the separation distance d. This behavior is in contrast to the Casimir force, which has a $1/d^4$ dependence due to the divergence of the ω^3 spectrum at high frequencies." [Denardo et al., 2009, p. 1095]

Furthermore, there are terms in our formal representation of the fluid such as viscosity and surface tension which have unclear analogical relationships. However, in my view these are not particularly troublesome. The analogy concerns the relative difference in pressure between the exterior and interior of two parallel surfaces in an oscillating medium composed of a range of frequencies. An ideal fluid system qualifies as such a medium, and similarly the zero point spectrum in the quantum electrodynamic vacuum.

In this sense, a particular interpretation of the Casimir effect seems to be implied by the conceptual aspect of the explanation: the B' presupposes a field-theoretic explanation in terms of a spectrum of modes rather than that the effect is due to the attraction of charged particles, as in the Van der Waals force.[20] Furthermore, there are many other systems which exhibit similar phenomena:

> "Casimir-type effects occur, in general, for two bodies in a homogeneous and isotropic spectrum of any kind of random waves that carry momentum. A net attractive force occurs between two parallel plates in

[19] Although, one should agree that any talk of 'virtual particles' does not seem to count as a B'. This conception is fraught with the same difficulties that has plagued quantum theory from the beginning: quantum particles can only be defined through de Broglie's relations—or some other mathematical relation that serves to associate the wave aspects of matter with the appearance of discrete, localized particle-like behavior.

[20] Also, an explanation in terms of 'virtual particles' flitting in and out of existence seems inferior, given these results, to an explanation in terms of crests and troughs of waves in a fluctuating medium.

> the typical case where the radiation force is reduced between them." [Denardo et al., 2009, p. 1100]

The various analog models of Casimir-type effects seem to provide a unifying explanation of the phenomena, whereas an alternative explanation of the quantum Casimir effect in terms of van der Waals force, for example, places the quantum effect in stark contrast.[21]

To sum up this section: an analogical argument is made between with the relative pressures of waves on parallel partitions in the (practically) random 'bath'—either surface waves in a macroscopic fluid B or electromagnetic radiation in a quantum vacuum B'. We extend the list of positive similarities between the systems to include the respective bath-representing terms in each domain (the fluid and quantum). These waves are assumed to be linear, since non-linear mechanics begin to appear upon sufficiently high amplitude vertical oscillations of the fluid bath when droplets are ejected. [Denardo et al., 2009] Thus, idealized linearity is a relevant aspect of the analogy that we can call an *auxiliary* or *boundary* condition that qualifies the argument to a restricted form. Auxiliary conditions from the experimental set up that produces both the Casimir and analog effects, plus the respective relevant fluid and quantum theories, gives us B and B' which are related through the analogical argument given earlier.

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[21] Unless van der Waals and Casimir forces were shown to be the same, but this seems unlikely. The van der Waals force sums molecular forces, whereas the quantum field theoretic explanation of the Casimir effect sums quantum field modes.

4 Summing Up the Fluid Analogy in the Physical Context

In the previous sections, we have seen that a number of fundamental properties and behaviors typically associated with the quantum world[22] have been convincingly shown to have formal, conceptual, and physical similarities with various properties and behaviors of fluid systems at the macroscopic level. These do not represent all of the possible, or even known, analogical behaviors similar to quantum phenomena. Analog systems involving fluids have shown behavior similar to the Zeeman effect, similarities with quantized orbits, in addition to behavior analogous with the Aharonov-Bohm effect. (See e.g. [Eddi et al., 2012], [Oza et al., 2014], also [Berry et al., 1980])

This is important to keep in mind: the analogy between the fluid models and the quantum world is constructed under various experimental conditions—in other words multiple table-top set-ups are required to exhibit all of the analogous behavior. This is not necessarily a drawback when one realizes that the analogous quantum phenomena are also produced under various experimental conditions. Additionally, it has been discussed that idealizations such as those involved in characterizing an ideal fluid or assuming linearity are critical to granting the arguments from analogy involved.

Before moving on to the second half of this thesis (the computational context), we must note that there are two critical aspects of the fluid models that have more questionable correspondences with certain elements of quantum mechanics and quantum theory: Many Worlds and Entanglement. These will also return in the discussion over explanations in quantum computation, by extension (i.e. the discussion of quantum speed up depends on the foundational considerations of the analogy with the physical world). That is, the extent to which we grant the similarities between the fluid and quantum domains should also hold in the quantum computational context.

While the practical set-up of an implemented quantum computer may differ from the previously discussed experiments exhibiting quantum behavior, the same general properties of quantum mechanics are involved in the computational process. Thus, if we grant certain aspects of similarity in the general physical context discussed above, we should also grant them in the computational context. This leads to questions such as: could fluid analogies help us understand quantum computation? Are there table top models that could perform computations which are analogous to quantum

[22]Noting that the Born Rule and de Broglie pilot waves are not always recognized by philosophers as clearly included in what is called quantum mechanics, but surely they are part of quantum theory as a whole.

computational procedures? First, though, we take a closer look at many worlds and entanglement in the physical context.

4.1 Many Worlds in the Physical Context

Finally, we are ready to make the argument from analogy against a Many Worlds approach to quantum mechanics. We achieve this by noticing that if worlds are present in VFBs then they are similar in kind to the quantum worlds of the Many Worlds approach. Earlier, we took quantum worlds to be defined as orthogonal components in a superposition—called the naïve definition—and these represent the 'classical' states which we are potentially able to observe (i.e. when we make a measurement we will only measure one of these states and not an interference of multiple).

However, there is a problem with the interpretation of worlds in VFBs—and it stems from a practicality condition. Some proponents of many (quantum) worlds suggest that these many worlds exist only *for all practical purposes* (FAPP). The neo-Everettian interpretation, as explicated e.g. in [Hewitt-Horsman, 2009], says that

> "The totality of physical reality is represented by the state $|\Psi\rangle$: the 'universal state'. There is no larger system than this. Within this main structure we can identify substructures that behave like what we would intuitively call a single universe (the sort of world we see around us). [...] The identification of these substructures is not fundamental or precise—rather, they are defined for all practical purposes (FAPP), using criteria such as distinguishableness and stability over time, with decoherence playing an important role [in] such an identification. [...] In each of these universes in turn we can find smaller substructures which are in general more localized than an entire universe. These are known as *worlds*." [Hewitt-Horsman, 2009, p. 5]

It seems that in the case of standing waves, the more practical interpretation consists in the usefulness of the composed wave function (i.e. the superposed world), and not in the picture of separate worlds in the decomposition. In other words, I argue that what Hewitt-Horsman calls 'substructures' are of less practical importance than the whole 'structure' for standing waves in the fluid case.

Worlds, as defined as components of a superposition, are directly related to the linear waves that are superposed. In the fluid case, these are surface waves as opposed to quantum waves. We would have a similarity relation of the form:

$$\frac{MW_Q}{W_Q} :: \frac{MW_F}{W_F} \tag{11}$$

This reads "many worlds from quantum waves are analogous to many worlds from surface waves". This relation is when the analogy is *granted*, otherwise there should be a big question mark over the :: symbol. In fact, I suggest that the situation is similar to the Casimir effect discussed earlier. I argue that we have similarly indirect evidence for MW_Q as we did for B'. We do not directly observe the zero point spectrum in the same way that we do not directly observe many worlds in the quantum world.

Unfortunately, the case for MW_F is more controversial than it was for the analog Casimir effect. In fact, if we grant MW_F then, having three of the four terms in our similarity relation, we would be inclined to accept the argument for MW_Q. However, for the reason given above concerning the characterization of standing waves in a fluid, I could deny that MW_F is a proper characterization of superposed surface waves.

In a standing wave, the individual components in the superposition are less important than the resulting function for characterizing the behavior of the surface. It is more practical in general to discuss standing waves as entities-in-themselves than to characterize them as a superposition of oppositely phased waves of equal frequency in overlapping coordinates. It should be recalled that standing waves are also integral to the wave mechanical approach to quantum mechanics—and that this reason for denying the practicality of the MW approach (and for asserting the practicality of the 'one world' approach) is not confined to the fluid analog model. Thus, there are crucial cases where it is at least unclear why we would wish to use the many worlds characterization over the characterization of the global states (i.e. proper modes) themselves.

An objection from the Many Worlds proponent may counter this point by arguing that *deep down* a standing wave *really is* the two components. This objection may work for this example, however for more complex wave functions—especially ones with non-unique decompositions—the MW proponent cannot use this counter-point since we are talking about a token wave function. Exchanging one decomposition for another is only trivial at the level of the superposed function and not at the level of the components, which is where the worlds are and where the objection of the MW proponent stemmed from initially.

Without MW_F we have two missing terms X and Y:

$$\frac{X}{W_Q} :: \frac{Y}{W_F} \tag{12}$$

Concerning the analogical argument across the top of the relation, one should not just fill these both in willy nilly with the respective MW terms—doing so would trivialize the extended pair in the list of similarities between the fluid and quantum domains. To have a term in the domain requires some justification (i.e. theoretical or empirical). One of two things is needed from the MW proponent: (i) evidence of many quantum worlds; or (ii) a reason why we should characterize superpositions of surface modes in terms of many worlds, particularly in the case of standing waves. An extension of the list based on neither of these is, I argue, trivial. Then, macroscopic many worlds have always been with us in every linear wave system—and always will be.

There are then two senses in which we can see the argument against many worlds in quantum mechanics. One reflects the position that there are important cases (standing waves) in which such a characterization does not seem practical—in either the fluid or quantum domains. That is, neither MW_F nor MW_Q are granted. This argument does not extend the list of similarities between the fluid models and the quantum world to include a similarity between many worlds in both domains, based on the practical grounds of characterizing these states. Call this the *practicality* argument against many worlds.

Without MW_F, we are left with blank numerators in our similarity relation. Since it is the analog system which we have more direct access to, the third term in the relation should be a fluid term. However, to move further, a many worlds proponent would have to object to the practical characterization of standing waves in both the fluid and quantum domains, or would have to provide more direct evidence of many quantum worlds. Otherwise, if the terms are filled in without achieving either of these, there is a trivial sense in which all linear wave mechanical systems have many worlds. Call this follow-up argument the *triviality* argument against MW.

It should also be noted now, and this will be discussed in a little more detail later in the computational context, that there should be a careful distinction between many worlds and many universes. Only a single universe is required for a many worlds approach. As we have seen in previous sections, there can be multiple superposed modes in the same bath—and I take this to be true also of the quantum world. It simply depends on the linearity of the systems, and the overlapping spatial coordinates of the mathematical functions. The entire bath, then, is a 'universe' just the same as the quantum universe has multiple wave functions in superposition.

4.2 Entanglement and Non-Locality in the Fluid Models

At this time, it is unclear how (or if) there could be an analogical correspondence of entanglement in the fluid analog model. One could chalk this up to the seeming locality of surface waves in the classical fluid bath—it is unclear whether the distributed surface waves could constitute some kind of analogous non-locality. Since standing Faraday waves are produced in the fluid by the simultaneous vertical oscillation of the entire bath, one could consider this as an analogous aspect of entanglement. In other words, consider the bath as sitting on top of a rigid, bounded plane. In the ideal case (i.e. the plane is non-elastic), each point in this plane oscillates simultaneously. It is this forcing input from an orthogonal dimension (i.e. the vertical axis) which results in the distributed standing wave behavior of the fluid surface.

For now this is all I can say regarding whether or not there seems to be an analogy to entanglement in fluid behavior. We do *not*, as far as I know, have a demonstration of some sort of EPR analogy. However, just because we don't know how to characterize entanglement in the fluid analogy doesn't mean there isn't an analogy. Additionally, for the argument from analogy against many worlds, this lack of similarity in this particular aspect is not problematic.

For example, the many worlds proponent may want to say that the lack of an entanglement-like aspect in the fluid models means that the fluid models really are more dis-analogous than analogous to the quantum world. The dis-analogy, they may say, is enough that we should not take seriously the notion that the fluid systems are actually relevant for discussions concerning the quantum world (e.g. discussions over interpretations of quantum mechanics).

However, it should be noted that this dis-analogy is irrelevant for the arguments made above. Lets assume, for the sake of argument, that there is not a convincing analogy to entanglement in the relevant fluid systems. This has no bearing on the interpretation of components in a superposition as 'worlds', and thus both the argument from practicality and the triviality argument just discussed above are unscathed. The relevant aspects of the analogy, I argue, are intact for the arguments against many worlds. These arguments will be extended to the quantum computational context since, as mentioned earlier, the function of a quantum computer is dependent on quantum mechanics. If we deny the many worlds account in quantum mechanics generally, then we should also deny the account in the computational context.

5 Fluid Analog Models in the Computational Context

Having hopefully convinced the reader in the first half that the analogy between fluid and quantum domains is more similar than dissimilar, we now proceed to the second argument in this dissertation. In the following sections, we will see that by keeping in mind the above conceptual, physical, and formal similarity some theoretical results can be obtained. As a concrete example, we will consider the foundational debate found in quantum information science over explanations of speed up in quantum computation for certain tasks compared to classical computation.

To match the presentation from the first half of this thesis, I will first give some background on the use of conceptual fluid analogies in simple electronics and circuitry, as well as discuss modern research on highly controlled experimental fluid systems which can perform computational operations. This will also highlight the important notion of analog computing, which uses potentially continuous valued variables to perform computations (as opposed to digital computing). Fluid analog computers have even been made to solve differential equations. [Roffman and Katz, 1967] The reader may wish to keep the following question in mind during the discussion: is quantum computing just another form of analog computing, or is it different somehow from both analog and digital computing?[23]

We do not *measure* continuous variables in quantum computation, but we do manipulate them.[24] I do not venture into an answer here, but this dissertation will end with the suggestion that this is a useful question to pursue in future research. For example, if we can reproduce enough quantum-like behavior in a cheaper micro-fluidic device to produce "analog speed-up" then perhaps research in quantum information theory will take on a new era of macroscopic experiments (perhaps with higher amounts of analog qubits). Perhaps not.

5.1 Hydraulic Analogy in Electronics

The Hydraulic Analogy in electronics is a common way to illustrate and teach some of the fundamental principles involved in classical circuits. Analogous parallel and series circuits are associated with hydraulic flow in pipes and reservoirs. (see e.g.

[23] I will try to make it clear by context, where I am referring to an analogy in terms discussed previously, and where I am referring to analog as in the type of computing. In some cases, such as when discussing a potential analog quantum computer, these two senses may overlap (i.e. the analog computer has analogous aspects to the quantum world).

[24] Although, there seems to be a growing interest in continuous variable quantum computation.

[Schönfeld, 1954]) Historically, electric current has been conceptualized as a sort of electric fluid. [Stocklmayer and Treagust, 1994] The historical development of the notion of fields was likewise associated with developments in formalizing idealized continuous fluids in fluid mechanics. [Hesse, 1962, §8] Earlier, we saw how fluid conceptions were involved in the development of quantum theory. In this section, a few relevant analog models are considered to further establish the use of a fluid analog model in quantum computing.

In [Schönfeld, 1954] we are shown some examples of analog systems in hydraulics and electronics. Using idealized incompressible fluids as an analog model for electronic current has strong historical precedence in many of the foundational contributors in the field.

> "The fluid theories have occupied a prominent place in the teaching of electricity from the time they were first propounded and their influence has been dominant to the present day. All the early researchers in electricity worked from a fluid perspective: Coulomb, in particular, held strongly to the two-fluid theory and his ideas were followed by Ampère and other French researchers. He also believed that there was a 'magnetic fluid' which accounted for the phenomenon of magnetism." [Stocklmayer and Treagust, 1994]

A fluid analogy for voltage is another great example.(see e.g. [Bartha, 2013, §3.3] For the mass flow rate of an idealized incompressible fluid in a pipe we have

$$\Delta p = V_f k \tag{13}$$

where V_f is the flow volume and k is some constant (which would be computed for non-ideal fluids, taking into account viscosity, length of pipe section, and the radius of the pipe for example), giving us the difference in pressure Δp in the system. This is the so-called Hagen-Poiseuille equation. Analogously, Ohms law is

$$V = IR \tag{14}$$

where V is the difference in volts across the conductor, I is current in amps, and R is ohms resistance.[25] We can use the proportional notation to describe the analogical relationship between these two equations in the following way:

[25] This holds under certain idealized conditions, for so-called *ohmic* materials. There are materials and conditions where R, for example, is not a constant.

$$\frac{\Delta p}{\frac{V_f}{k}} :: \frac{V}{\frac{I}{R}} \tag{15}$$

The vertical relationship again represents something like the causal connection between theoretical terms in a given theory, whereas the horizontal relation is the inter-theoretical similarity relation. [Schönfeld, 1954, p. 425] suggests the following list of analogical correspondences for electronics:

	HYDRAULIC	::	ELECTRIC
	Volume		Charge
	Discharge		Current
	Potential Difference		Potential Difference
	Resistance		Resistance
	Deliverance		Conductance
Causal $\updownarrow$	Inertance		Inductance
	Capacitance		Capacitance
	Kinetic Energy		Magnetic Energy
	Gravitational Energy		Electric Energy
	Sea		Earth
	Generator (pump)		Generator (element)

While this list of analogical properties may be relatively complete, it might be worried that making concrete analogical arguments seems to rely on the complete enumeration of analogical properties between systems. Since analogies drawn between two domains are many times controversial, complete enumerations in the above manner may never occur. Some readers may no doubt object to a few of the above relations asserted in the table above. However, we can say that while such a complete enumeration of properties is sufficient for an argument from analogy, it is not necessary.

In the next brief section, we will discuss some modern computational attempts involving controlled fluid systems—which many times can be considered to function in a continuous manner in accordance with the physical fluids used. These will show that the hydraulic analogy being made to electronics above is not just conceptual, but that practical fluid systems can be implemented which perform computations. Thus, we see that in both the physical and computational context, more or less macroscopic (with respect to the quantum world) fluid systems can and are used both conceptually and in a practical manner to obtain scientific results—whether these be theoretical, pedagogical, or practical.

5.2 Analog Computing and Microfluidics

After the previous discussion of a fluid analogy in electronics, we now begin to transition towards a discussion of computing.[26] Analog computing is a field of practical and theoretical computer science which is concerned with building, implementing, and studying computational procedures using logical operations whose data types are not digital or discrete. These operations are based on the physical systems used to implement them, and can take on a continuous range of values.

One type of analog computer can be constructed using carefully controlled fluid systems. These systems can be used in a highly controlled manner to perform certain functions. As an example, an analog computer of the fluid sort could be used to solve differential equations as in [Monadjemi, 2001]. The continuous level of fluid in a transparent container could be read out, for example, as a solution to an equation. This could be implemented in similar orders of magnitude as the VFBs discussed earlier.

Microfluidics is also an area of active research concerned with the control over very small fluid systems involving cappillaries (small tubes), circuits, pumps, bubbles, and micro-droplets, which has recently become relevant in particular to biomedical research. [Whitesides, 2006] Other areas of microfluidics involve carefully controlled fluid systems such as droplets on the surface of a liquid, or even suspension of multiple particles in lattice formations in the fluid. These latter systems can use standing waves in the fluid (similar to the kind of waves discussed earlier) to 'force' the particles or bubbles into the stationary nodes determined by the standing wave solutions of a container. Carefully adjusting the phase of these waves can program or control the motion of particles in these fluids. See for example [Wood et al., 2008], also Figure 2.

We can see that using fluids in these ways begins to blur the distinction between analog and digital computing. One can imagine hybrid set-ups utilizing many different fluid behaviors. A hybrid fluid computer could be discrete in using droplets or particles but continuous also in a read-out of a volume level in a container, or in the densities of longitudinal waves (or amplitudes of surface waves). Furthermore, the use of standing waves to control particulate matter in the fluid provides a unique cross-section with the VFBs discussed in analogical relation to the quantum world.

It is this aspect which serves to connect the discussion of the use of fluids in analogy with both the physical and computational contexts. Whereas earlier we were concerned with quantum-like behavior being produced in the fluid models, now we

[26] Be wary of the usage of analog as a type of computing, rather than analog as referring to an analogy!

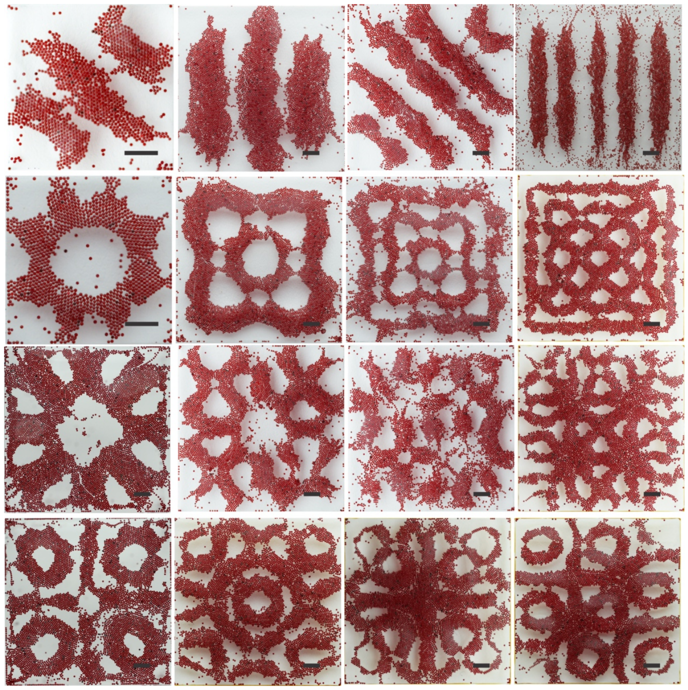

Figure 2: Micro-particles arranged according to nodal lines of standing waves in a fluid. Photo source: Wikimedia Commons.

are interested in discussing computational-like behavior—whether analog or digital or somewhere in-between. The following discussion concerning quantum computation is then directly located in the cross-section between these two contexts. In the next section, we will use the previous discussion over fluid analog models in the physical context and extend it further into the computational context. Particularly, in theoretical quantum computer science there is the question of how we explain quantum speed up—i.e. what physical resources are exploited, and how are they used.

The reader should by now have a good grasp on why I have chosen to discuss fluid analog models in *both* the physical and computational contexts. There is not only significant overlap between the kinds of fluid systems discussed, but also there is the critical contextual overlap in a discussion of quantum computation that helps support the overall thesis: that fluid mechanical models uniquely enable scientific argumentation and conceptualizations.

6 Fluids and Quantum Computation

In this section, I will briefly introduce the basics of quantum computation. Afterwards, we will use the earlier discussions on fluid systems to consider whether we can learn anything about quantum computation. That is, this thesis will conclude with a discussion on whether the intersection of the physical and computational contexts offered by the conceptual and practical aspects of fluids has something to offer the field of quantum computation. I will suggest that, at least on the conceptual side, there is something to gain in the form of a visualization benefit—the fluid analogy makes it easier to visualize what exactly quantum computation is all about. That we can apply fluid analog models to such a unique and specialized modern research area supports my argument that fluid models are not only unique among analog models in science, but also among models in science generally.

6.1 A Brief Introduction to Quantum Computation

In the field of theoretical quantum computation, the following expression represents the quantum analogy of a binary digit or *bit*, the *qubit*:

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle \tag{16}$$

Here, $|\psi\rangle$ (read as "ket psi" in what is known as Dirac notation) is a superposition, i.e. a linear combination represented by the addition sign, of the two basis states $|0\rangle$ and $|1\rangle$. Kets are taken to be column vectors whose entries are complex numbers, whereas 'bras' $\langle\cdot|$ are row vectors whose entries are the complex conjugates of the corresponding ket. For example, our basis states are the following column vectors:

$$|0\rangle = \begin{bmatrix}1\\0\end{bmatrix} \qquad\qquad |1\rangle = \begin{bmatrix}0\\1\end{bmatrix} \tag{17}$$

The coefficients α, β in (16) are complex numbers called *amplitudes*, and this title is best understood through the wave mechanical formalism of quantum mechanics. We require of these amplitudes that $|\alpha|^2 + |\beta|^2 = 1$, where $|\cdot|$ denotes the modulus of the complex number. From earlier we remember such a calculation is known as the Born Rule.

From combinations of these building blocks, including unitary linear operators on state vectors and vector spaces containing them, linear algebra (along with Dirac notation) forms a formal language which is capable of representing procedures useful in theoretical quantum computer science.

6.2 Quantum Speed-Up

One of the primary benefits we are supposed to obtain by utilizing quantum computation is that certain computational tasks can be achieved faster. From a complexity theorists' view, less resources (such as time) would be needed to perform some procedures. Whereas on a classical computer factoring the product of two large prime numbers is hard (i.e. takes exponential time with respect to the input)[27], a quantum computer implementing what is called Shor's algorithm could efficiently achieve factorization—that is, it achieves this in polynomial time with respect to the input.

This ability to factor composites efficiently is a well-known example of quantum speed up. The popular RSA scheme of cryptography relies on the fact that, while computing the product of two large integers (primes) is easy, the reverse operation of factoring when the factors are unknown is hard. Shor's algorithm then shows not only the potential of formal machinery in quantum computing, but practical implications of a quantum computer once (or if) reliable and scalable hardware is available.

There are several candidates, perhaps not all of them exclusive, which can be considered to provide an explanation for the particular behaviors of quantum algorithms and features of the quantum world which enable less resources to be used in quantum computations for certain tasks, such as optimization problems. The primary four explanations in the literature seem to be the following:

1. Parallelism
2. Quantum Logical
3. Entanglement
4. Interference

In the next subsections I will summarize the ways in which these explanations are typically used in the literature, and discuss the impact that our fluid analog models may have on them. Unfortunately, no space will be spent in the main text on the quantum logical explanation here, since this can be considered a different *kind* of explanation compared to the other more physical options. A lengthy appendix is attached concerning a new approach to quantum logic inspired by considerations from the fluid analog models, in which the resource of interference is highlighted.

[27]The fastest approach, the generalized number field sieve, is apparently sub-exponential.

6.2.1 Parallelism and Many Worlds Explanation

Parallelism, in terms of computation, is the idea that a computational architecture performs processes simultaneously. The more tasks which can be performed 'in parallel' means the computation will take less time. This can be thought of as a trade of using more space to reduce the time of a computation. In a coherent quantum computation, the evolution of all components in superposition is liable to be interpreted in a parallel manner. Proponents of a many worlds approach can embrace this appearance and account for speed-up by saying that parallel computations could be achieved by each of the many worlds simultaneously.

Thus, as an explanation of where the computational power of quantum algorithms comes from, some proponents of many worlds may suggest that multiple evaluations of a function are computed simultaneously in a quantum computer. Armond Duwell calls this the *quantum parallelism thesis* (QPT) in his argument against many worlds as being the only interpretation of quantum mechanics capable of explaining quantum speed up. [Duwell, 2007] He then goes on to characterize a typical unitary function which could be taken to support the QPT, called the *quantum parallelism process* (QPP):

$$\sum_{x=0}^{2^n-1} |x\rangle |y\rangle \rightarrow_{U_f} \sum_{x=0}^{2^n-1} |x\rangle |y \oplus f(x)\rangle \qquad (18)$$

Here n is the number of qubits, where x is an integer represented in binary basis components ($|0\rangle$, $|1\rangle$). A typical reading of this in the quantum computational literature is that the tensor product $|x\rangle |y\rangle$ represents a pair of input and output registers, $|x\rangle$ being the input and $|y\rangle$ the output. The notation $|x\rangle |y \oplus f(x)\rangle$ denotes that the output register depends on the evaluation of the unitary function U_f for x, where $\oplus$ denotes addition modulo 2 (i.e. $1 \oplus 1 = 0$).

This expression *looks like* the function has been evaluated (in parallel) for every value of x by the quantum computer, since the input register is a superposition (summation) of all x values. A proponent of many-worlds-parallelism would say that the function *has* been evaluated by many worlds evolving in parallel. The aforementioned 'neo-Everettian' position of [Hewitt-Horsman, 2009] may say that for all practical purposes (FAPP) we talk of the situation as if there were many worlds performing in parallel. This might be taken as a 'weaker' position, since worlds are arguably less ontologically rigid than in other many worlds accounts.

However, as [Steane, 2003] points out, such a notion of computation which would admit the QPP for the example above is not particularly useful, since no measurement has yet been made. Once we have done a measurement, we will only find out one value of the function. The notation is therefore misleading, and the concept of parallel computations performing multiple evaluations of the function becomes dubious—i.e. QPT does not follow from the QPP. One reason the notation is misleading is that we cannot actually find out the result of all of these supposed parallel computations, in contrast to a classical parallel process.

Duwell continues on to argue that other ontological approaches which differ from the MWI to explaining the computational processes in QPP (which justify the QPT) are acceptable, and that the proponents of many world parallelism need to show why their approach is uniquely better.

> "Any interpretation in which ['benign realism'] is true and subscribes to the universal validity of the Schrödinger evolution will at least be on equal footing with the MWI regarding an account of quantum computational efficiency." [Duwell, 2007, p. 15]

There are other problems facing the proponents of the many world account of quantum computation and parallelism as construed above. For one, as argued in [Cuffaro, 2012], the lack of decoherence in quantum computations (quantum algorithms manipulate *coherent* states[28]) removes one of the critical columns supporting the many worlds account in general:

> "[...] unlike its close cousin, the neo-Everettian many worlds interpretation of quantum *mechanics*, where the *decoherence* criterion is able to fulfill the role assigned to it, of determining the preferred basis for world decomposition with respect to macro-experience, the corresponding criterion for world decomposition in the context of quantum computing cannot fulfill this role except in an ad hoc way." [Cuffaro, 2012, p. 36]

That is, the problem of the preferred basis returns for the many worlds account of quantum computation. This problem arises out of the fact that a wave function has more than one decomposition, i.e. there is no unique decomposition. Another rather serious problem for the many worlds account, which the fluid analog models discussed earlier help illustrate, is that phase is important if one grants some ontological weight or computational significance to components in a superposition. (See e.g. [Duwell, 2007]) In the case of standing waves, the oppositely phased reflected components are crucial—without the opposite phase there would be no formation of a standing wave. Additionally, as mentioned in the argument earlier, it is the

[28] Reminder: coherence is a property of a system of waves in which the phase relationships between components remain constant.

'global state' (i.e. standing wave) that is useful for practical purposes—and not the individual worlds composing it. This further supports the reasoning from Duwell against many world approaches to quantum computation.

6.2.2 Entanglement as Explanation

Another potential explanation of quantum speed up is the explanation from *entanglement*. An entangled quantum state is one for which we cannot re-write it as a (tensor) product of two factors. Typical entangled states are also called EPR or Bell states, for historical reasons. Such a state is:

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle \tag{19}$$

Where here we could with probability one-half measure either the state $|00\rangle$ or the state $|11\rangle$ from what is typically called the Bell state $|\Phi^+\rangle$. In this formalism by convention $|00\rangle = |0\rangle|0\rangle = |0\rangle \otimes |0\rangle$, and the first state vector is considered the input register while the second is considered the output. Or, the first is Alice's qubit and the second is Bob's. One can see then that even though there is a one-half probability of getting either $|0\rangle_A |0\rangle_B$ or $|1\rangle_A |1\rangle_B$ by measurement, every time Alice and Bob will observe the same outcome of either 0 or 1. This kind of perfect correlation (or perfect anti-correlation for other Bell states) are entangled correlations since they derive from an entangled state.

Exploiting these correlations is thought to be a primary way in which certain algorithms in quantum computation are able to achieve speed-up over classical algorithms. Considering that the VFB systems do not seem to display an analogous behavior to that of quantum entanglement, the resource of entanglement does not seem to get any boost from the fluid models. However, as mentioned earlier, whether or not the fluid analog systems can display an analogy of entanglement is still under question.

6.3 Interference as Explanation

There are several ways in which the term interference can be thought to refer to some quantum property, effect, or computational resource, and it will be helpful to distinguish between these since not all of them will be suitable for an *explanation* of speed up. In this section, we will go through these in depth in order that the reader may have a better sense afterwards of the nuanced way in which interference may affect quantum computations. I will argue that coherent interference is the most likely candidate for explaining quantum speed up that is supported by the fluid analog models of the quantum world.

6.3.1 Double-Slit Interference

The statistical distributions of the double slit experiment in quantum mechanics are known for their characteristic interference patterns. That this distribution displays interference warrants an explanation, and such explanation is usually given in terms of the interference of waves with the acknowledgment of a fundamental wave-particle duality we cannot avoid in our description. The sort of explanation supported by the fluid analog models relevant here is that there really are quantum waves, whatever one wants to think of the word real—they are FAPP real. Guiding or pilot waves interfere since they distribute through both slits—and this is true even when the associated 'particle' (or droplet) goes through definitely one or the other slit.

Interference apparent in the *analogous* statistics is due directly to the interference of surface wave modes which go through *both* slits. So, there is a crucial distinction between interference terms in an observed probability distribution—and the interference of an ontological wave field. An explanation of quantum speed up cannot come from interference in posterior probability distributions—since the distribution characterizes what happens upon measurement. Speed up happens during the coherent quantum computation, before measurement. We will get to coherent interference in a moment, but first there is a second notion of interference—interference between computational paths.

6.3.2 Interference of 'Computational Paths'

In [Fortnow, 2003] it is suggested that the primary component in an explanation of quantum speed up is *interference of computational paths*. A similar notion is found in [Deutsch et al., 2000]:

> "The basic idea of quantum computation is to use quantum interference to amplify the correct outcomes and to suppress the incorrect outcomes of computations."[Deutsch et al., 2000, p. 10]

To explain these computational paths, consider the $\sqrt{NOT^*}$-gate from [Deutsch et al., 2000]:[29]

[29] I call this gate the $\sqrt{NOT^*}$ as opposed to $\sqrt{NOT}$ from the original article, since when concatenated twice in a row the operation is not quite like NOT but still exhibits the notion of computational paths interfering. For some reason, the authors give amplitudes of $\frac{i}{\sqrt{2}}$ for paths in their $\sqrt{NOT}$ machine, but it is unclear why.

$$\sqrt{NOT^*} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \tag{20}$$

Applied to $|0\rangle$ we get the superposition $\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$. When two $\sqrt{NOT^*}$-gates are concatenated, we get something similar to the normal NOT operation:

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \tag{21}$$

We can call the concatenation NOT*. It takes $|0\rangle \rightarrow |1\rangle$, and $|1\rangle \rightarrow -|0\rangle$. Thus, it switches the phase of the second operation, where a normal NOT would not change the phase. In any case, the top left and bottom right entries of NOT* have been *cancelled* due to 'destructive interference' $(\frac{1}{2} + \frac{-1}{2})$ along their respective paths. Similarly, the top right and bottom left paths have 'constructively interfered'. Saying that *paths* have interfered, however, seems to obscure rather than clarify. Actually, it is amplitudes which are interfering—the 'paths' are just representations of the transformations applied to these amplitudes.

One might be sympathetic to the notion of 'probability amplitudes', and talk of the probabilities as interfering. However, until we apply the Born rule and calculate the modulus-squared of the complex numbers, the amplitudes are not probabilities. In any case, amplitudes of *waves* are capable of constructive and destructive interference, and this will be mathematically expressed as positive and negative values of the same components which cancel or add to each other.

Interference of computational paths is definitely a candidate for an explanation of quantum speed up, but only because it can be interpreted as a different formal representation of the interference of coherent quantum waves (whatever these may be). However, if the interference is taken as interference of something like probabilities, probability amplitudes, or probability waves—as opposed to *actual* waves—then we run into the same problem of needing an explanation for these and would have to justify the deviation of 'normal' probability theory. The amplitudes interfere *before* we calculate probabilities through the Born Rule. The fluid models clearly support maintaining the distinction between amplitudes and probabilities.

6.3.3 Coherent Interference

The third kind of interference, and what I take to be the correct notion of interference to be a candidate for an explanation of speed up, is what can be called *coherent* interference. For classical waves, coherence is the consistent phase relationship between superposed modes. Coherence is an interference phenomena that qualifies in general as a quantum feature which can be used in an explanation of speed up. From the coherent interference in the fluid analog model, we can argue that the other notions of interference actually boil down to coherent interference. If we take the amplitudes of wave functions to correspond to actual waves, then we can justify the interference in double-slit statistics through *correspondence principles* with these wave entities. Additionally, the negative matrix entries in the computational path notion of interference can be seen as just representing these same amplitudes.

Take for example a beam-splitter implementing a Hadamard gate $\mathbf{H}$ twice on the state $|0\rangle$. First, the Hadamard will create a superposition of $|0\rangle$ and $|1\rangle$. Applying the gate a second time to this superposition means that, since it is a linear operator, it will distribute onto each of these state vectors creating a total of four equally weighted basis terms.

$$|0\rangle \rightarrow_{\mathbf{H}} \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \rightarrow_{\mathbf{H}} \frac{1}{2}|0\rangle + \frac{1}{2}|1\rangle + \frac{1}{2}|0\rangle - \frac{1}{2}|1\rangle \tag{22}$$

However, the phase relations shows that there will be interference—destructively eliminating, for example, the $|1\rangle$ state. This simply leaves us in the same state we began with, namely $|0\rangle$. No Born rule calculation is involved, and thus the interference is of coherent quantum amplitudes and not of probability of any kind. This is interference at the most general level, in that there is an operation which can remove states that have been present (i.e. $|1\rangle$).

Coherent interference is present in a quantum computation *before* measurements are made, allowing this notion of interference to qualify as an explanation of speed up. It is important to realize that the coherent interference in the linear combination of components in, say, a standing wave, *need* to be at the same place, in the same universe, and of the same medium. Furthermore, coherence is a relationship between the *amplitudes* and is importantly *not* a property of 'probabilities'. Thus, coherent interference is the prime candidate for an explanation of the various senses of interference—and coherence can also be maintained in interfering surface waves in a vibrating bath. The interference between 'in' and 'out' components which form standing waves in VFBs is also coherent interference, since a shift in relative phase will make the resulting wave travel (i.e. it will 'decohere' and cease to be a standing wave).

Whether coherent interference is the primary (or only) explanation of speed-up is still up for debate. Perhaps the competing explanations are not all mutually exclusive, although the fluid systems discussed here do not seem to support an explanation from entanglement. If it is any interest to the reader, there is at least an attempt from information theory to characterize an entanglement measure explicitly in terms of interference. [Luo and Zhang, 2003]

6.4 Fluid Analog for Quantum Computation?

In the previous sections one can see that an intimate connection can be made between the use of standing waves (and finely tuned and controlled fluid systems) in analog models for physics and electronics. Also, in terms of computation, we can consider similar controlled functions of a fluid in analog computers and microfluidic applications.

A natural course of inquiry has suggested itself: *Can a fluid analog model help us conceptualize some operations in quantum computing?* Specifically, can we get a clearer grasp of what is going on in the so-called 'quantum speed up', or use the fluid analog in a heuristic manner for engineering quantum circuits? Could we construct a fluid analog quantum computer? These are big questions whose final answers surely lie beyond the scope of this paper, but I argue that the previous discussions allow an affirmative response. The analogical relationships are sufficient to warrant further research in this direction. It seems that, at the very least, we can apply the concepts involved in the relevant fluid models to quantum information theory, since quantum information theory is based on quantum mechanics. In other words, an analog model for the quantum world should also be of interest to quantum information theory.

To be able to better visualize what is 'happening' in the quantum world during a quantum computational process would be immensely beneficial for several reasons. Practical reasons such as helping engineer quantum computers, or helping discover new algorithms are clearly areas that would benefit from this visualization. Of course, issues in quantum computing are hard to separate from interpretational debates of quantum theory in general—and so there is strong philosophical motivation for seeking such *anschauliche* models regardless of these practical considerations. Thus, there are strong philosophical reasons for also seeking a model, particularly the macroscopic and analogous kinds of models which have been discussed here.

I would like to leave the reader with a few open questions. Can we use an analog (as in continuous) fluid system to represent analogous (as in similar) quantum information processes? If we could perform quantum-*like* computations using such a fluid system, we might have a better grasp on what needs to be done to build and scale actual quantum computers. Additionally: is there really a clear cut boundary between classical, analog, and quantum computational models? Is quantum computing really different from analog or hybrid computing? I would answer that, based on the fluid systems considered here, it seems unlikely that the theoretical computer scientist can keep sharp distinctions between these different computational models.

7 Fluid Models as Unique Scientific Devices

In closing this present work, it is useful to recount the general results in support of the thesis and suggest future research sympathetic with the approach advocated for here. I argue that fluid mechanical models are unique scientific devices on the basis three primary reasons. First, we have seen the plethora of uses that fluid models have in an analogical capacity. Fluid analog models and systems have been used to form scientific arguments from analogy, as well in conceptualizing difficult-to-access phenomena.

The ability for fluid analog models to support analog reasoning, visualizations and conceptualizations in science stem from a variety of attributes. There is the ability to observe macroscopic systems, the existence of a variety of formal similarities involved in idealized fluid systems, and also the conceptual link with field theories and continuums. These attributes apply to a range of considerations from both applied and theoretical fields in physics and computer science. I have illustrated a number of examples of analog reasoning used in both of these contexts.

We have seen that, while there are dis-analogous aspects between the domains of fluid analog models and the quantum world, there are arguably plenty of conceptual, formal, and physical similarities to be considered between them. Overall, I argue, the fluid mechanical systems and the quantum world are more similar than dissimilar. It seems that the aspects which are controversial, such as the existence of many worlds and the presence of entangled states, are not deal breakers so much as they are conversation starters. The analogical framework provides a useful and explicit context with which we can address foundational questions in both the physical and computational contexts. In particular, at the intersection of these contexts is the field of quantum computation. We can use the fluid models to help visualize and think about quantum computational procedures.

We saw that the quantum-like behavior present in VFBs can be attributed, at least in part, to the fact that the bath is near an *ideal* state. In a discussion of models we assume that even though a truly ideal system may not exist, idealizations may be helpful in explaining and characterizing the observed phenomena. These idealizations in fluid analog systems include that the waves in the bath are roughly linear (i.e. that they obey the superposition principle) and that the bath is tuned as close to the (ideal) Faraday instability threshold as practically possible. Other characteristics of an ideal fluid may be that it is inviscid (or approaches an inviscid state), is incompressible, or continuous.

Other interesting quantum phenomena, including those under the category of "macroscopic" quantum phenomena, are also present at near idealized conditions.

These conditions most importantly include the experimental set-up of the system at *near* absolute zero temperature. Superfluid helium, perhaps the most 'ideal' fluid observed in actual experimental situations, obtains a near ideal state due to the fact that it is cooled to almost absolute zero. At this temperature, the superfluid helium becomes inviscid as the coherent wave aspect of matter takes over.

Thus the second reason fluid analog models obtain their unique status is, I argue, because of these available idealizations. In other words, we can use fluid models because we can utilize (in theory and in practice) a number of fluid systems combining a variety of idealized (or *near* idealized) aspects. For example, I support the contention of Robert Batterman (e.g. in [Batterman, 2006] and [Batterman, 2008]) that for certain phenomena (particularly relevant fluid phenomena) the continuous idealization of a fluid can be part of the explanatory appeal of a model.

The quantum hall effect is also produced in quantum fluids at very cold temperatures. This is an important phenomena for so-called topological quantum computing (TQC) in which specific topological features of particular quantum states are manipulated to perform computations. [Nayak et al., 2008] Could a fluid analog model be constructed for TQC, in which topological features of vortices in a non-superfluid are manipulated? More work, both theoretical and experimental, should be done with regard to the carefully controlled behavior of fluid systems. As briefly noted earlier, we already have serious research being done with regard to micro-fluidics and fluid circuits. It seems that we are also justified in pursuing computational aspects of fluid systems which reproduce behavior that is analogous to the quantum world. At the very least, they may help clarify quantum computational models, and may suggest certain niche uses as previous models of analog computing have done (i.e. solving differential equations).

Further research could explore the extent to which fluid models are used in physics to a greater ontological purpose—i.e. the extent to which the line between model and theoretical entity is blurred. For example, some theoretical physicists consider that if spacetime were like a fluid, a possible restriction may be that it must behave like a superfluid with zero viscosity. This approach has been connected with attempts at formulating a theory of quantum gravity. [Liberati and Maccione, 2014] In this approach, is the superfluid a model of spacetime or does it represent more of an ontological commitment to the actual physical world? A further philosophical question one might ask is whether the number of similarities between various fluid behaviors and the quantum world actually confirms (or supports in any way) hydrodynamic approaches to quantum mechanics, or a superfluid approach to quantum gravity.

That we can ask these highly specialized questions in the first place is the third reason in support of the unique strength that fluid analog models have in science.

Future research questions into such specialized areas such as quantum computation and cutting edge theoretical physics may benefit from a conscious consideration of fluid analog models. In total, the ability to provide idealized models and heuristic analogical devices, combined with the ability to specialize, provides sufficient justification for the thesis that fluid mechanical models are uniquely useful scientific devices. This has been shown to particularly apply to physics and alternative approaches to computing. Uses vary from scientific arguments and conceptualizations, to potential applications in highly specialized fields. Taking into account the variety of other contexts (and cases) in which these models are utilized, which have not been discussed in depth here, we begin to see a larger picture in which fluid mechanical systems generally appear to have a unique modeling status in science.

In conclusion, the previous philosophical analysis seems to warrant the formulation of three general statements in support of this thesis: fluid models and systems have an ample ability to form analogies, the flexibility to specialize in a number of areas, and are liable to many idealizations (some of which are approached in physical systems). Each of these has been present throughout the discussion, and are by no means exclusive reasons in support of the thesis. In fact throughout the dissertation they are rarely present alone. Different specializations may require or rely on different idealizations, and the extent to which a fluid model is analogous or is used in an argument from analogy varies. Nonetheless, I argue that these three characterizations justify the view that fluid mechanical models have a unique usefulness in the methodology of science.

8 Appendix A: Sequent Representation and Circuit Proofs

8.1 Structural and Operational Rules for Quantum Programs

It is of interest, in the light of the discussion in the preceding thesis, to briefly discuss the quantum logic approach to explaining quantum speed-up. There are a few aspects which we have seen are incredibly important when viewed from the perspective of fluid analog models. These aspects should be represented in a quantum logic, in order to fully capture potential resources in quantum computation. We should in particular be able to represent phase, and the difference between the interference of coherent amplitudes and the measurement probabilities calculated by the Born Rule.

Unitary transformations are the types of rule applications that are relevant for quantum computation, and instead of abstracting away the phase relations and probabilities (as I noted some previous attempts in this field do) we abstract away from the linear algebra calculations (which should be computed in the background). In addition to considering unitary transformations (such as a universal set of such operators) as inference rules in the proof system, we add structural rules that can be gleaned from discussions on quantum computation in the circuit model—particularly *single* quantum circuits. The rules listed here are not exhaustive. Note: the blank succedents should be read as "we have not yet calculated the Born Rule (BR) yet, but will do so just before a measurement M".

8.2 Definition of QMC

Here, subscripts n, m are added to denote the general rules for finite numbers of qubits of generalized superpositions denoted Σ. The sequent arrow '$\Rightarrow$' is read as a counterfactual 'measurement' implication: "if we were to measure Σ then the outcomes are determined by probability distribution P". The turnstile '$\vdash$' is used to denote post-measurement what was measured from this distribution, with probability subscript p. The pseudo-predicate P denotes the calculation of a probability distribution by the Born Rule (BR), and the left conjunction rule is denoted by $\otimes$ to mimic tensor product. First there are what can be considered more the 'identity' and 'structural' rules:

$$|0\rangle \Rightarrow (Axiom) \qquad \frac{\Sigma_n \vdash_p |x\rangle_n}{|x\rangle_n \Rightarrow} (Prep)$$

Then perhaps the 'operational' rules:[30]

$$\frac{\Sigma_n \Rightarrow}{\Sigma_n \Rightarrow P(\Sigma_n)}\,(BR)$$

$$(\otimes)\,\frac{\Sigma_n \Rightarrow \qquad \Sigma_m \Rightarrow}{\Sigma_n \otimes \Sigma_m \Rightarrow}$$

$$(M_n)\,\frac{\Sigma_n \Rightarrow P(\Sigma_n)}{\Sigma_n \vdash_p |x\rangle_n}$$

This last rule is called measurement. Measurement on n qubit systems can be simulated by n singular qubit measurement gates. There are many unitary transformations that perform logical operations on antecedents. That is, there is a group of left rules U, where U is ideally some universal set of unitary operators. These rules are more difficult to represent in a general form, and in multi-partite operations their applications to specific qubits requires care. They have inductive definitions on the number of different ways they can be applied to any number of states present in the antecedent. The form of the Hadamard gate for example, for x in the basis states $\{0, 1\}$, is:

$$(\mathbf{H})\,\frac{|x\rangle \Rightarrow}{\frac{(-1)^x}{\sqrt{2}}\,|x\rangle + \frac{1}{\sqrt{2}}\,|1-x\rangle \Rightarrow}$$

A few brief remarks should be made. The system as discussed above, for single quantum circuits, is intuitionistic in that only ever one term will be derived in the succedent. There is no left or right weakening or contraction. That there is no left weakening means the system is *non-monotonic*, and this can be explained in the system by noticing that adding another linear wave component to a superposition will not result in the same probability distribution as calculated through Born's rule. The physical and mathematical reason for this is that the coherent amplitudes *interfere* with each other.

This kind of quantum logic, emphasizing interference and non-monotonicity, has not to my knowledge been considered in the literature concerning quantum speed up from a logical perspective. I have written extensively about this calculus elsewhere, but it is of interest that it initially stemmed from the considerations of fluid analog models in the development of this thesis.

[30] The traditional distinction between these types of rules, however, seems a bit unwarranted (or at least unclear) for M and *Prep*.

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