OCG - Owner vs. Collector

1. Model Overview

This case study recreates the Owner–Collector Game (OCG) as described in Wu et al. (2020) using a Vickery-Clarke-Groves style auction, represented as a taxi hailing game. In the OCG setting, a data owner wants the benefit of a service (a taxi), and the data collector wants to extract truthful location reports while protecting user privacy. The model we implement follows the representative approach used in ride-matching systems:

- A VCG mechanism incentivizes honest reporting of user valuations.
- A differentially private exponential mechanism randomizes which user is selected, protecting location privacy.
- The system studies the tradeoff between efficiency (accurate allocation) and privacy (strength of noise).

This game fits the Owner vs. Collector category because the collector designs the mechanism (allocation + payment rule), and the owner chooses what information to reveal based on incentives and privacy concerns.

2. Players

- Owner (Passenger):
  - Reports a valuation for service and a location. Wants high utility while preserving privacy.
- Collector (Platform):
  - Wants accurate service allocation but must incorporate privacy protections that reduce utility.

3. Strategies

- Owner (Passenger):
  - Truthfully report valuation and location
  - Misreport valuation and/or location
- Collector (Platform):
  - Choose privacy level ε
  - Choose allocation rule (exponential mechanism)
  - Compute VCG-style payments

4. Payoff Functions

BLUF:

If s_i is the best score and ε is high, that person is almost always chosen. If ε is low, exponentials compress and all choices look similar, having stronger privacy. Let:

- v_i = valuation of user i
- c_i = distance/cost to serve user i
- $s_i = v_i - c_i$ (score)
- ε = DP privacy parameter

4.1 Allocation Probability (Exponential Mechanism)

The probability of any one person being chosen is $\exp(\varepsilon s_i)/\mathrm{sum}_j(\exp(\varepsilon s_j))$. This formula ensures two things:

1. probabilities are always positive (since exponentials are never negative).
2. Amplifies the difference in scores when ε is large.

4.2 Owner Utility

If the user is chosen their utility is:

- Utility of chosen owner = (valuation of user i) - (payment to user i)
- $U_i = v_i - \mathrm{payment}_i$

If the user is not chosen, their utility is 0.

4.3 Collector Payoff

Welfare of the collector is calculated by:

- $\mathrm{sum}_i(\Pr(i\ \mathrm{chosen})(v_i-c_i))$
- Pr = probability
- Higher ε, more accurate welfare. Lower ε, more privacy

5. Equilibrium Concept

Mechanism Design Optimum (Dominant-Strategy Truthfulness)

- The collector implements a VCG-style payment rule, ensuring that the owner's best response is truth-telling, regardless of other players' reports.
- Differential privacy introduces noise but does not break truthfulness; expected utilities preserve incentive compatibility.

Why this fits:

- OCG models are typically evaluated using truthfulness as the equilibrium. The owner optimally reveals truthful valuations, and the collector optimally chooses ε to balance efficiency vs. privacy.

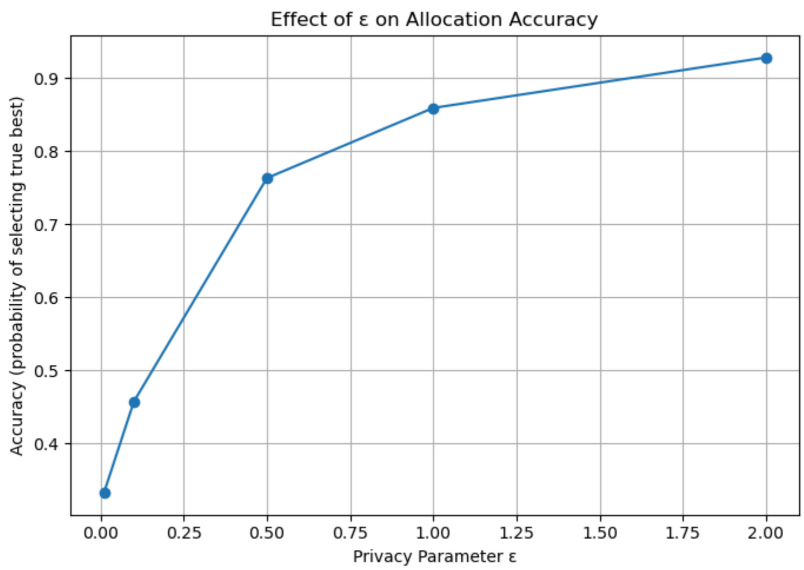
6. Implementation Details

This game is performed through a set of computations. The following steps is the processes of playing the game.

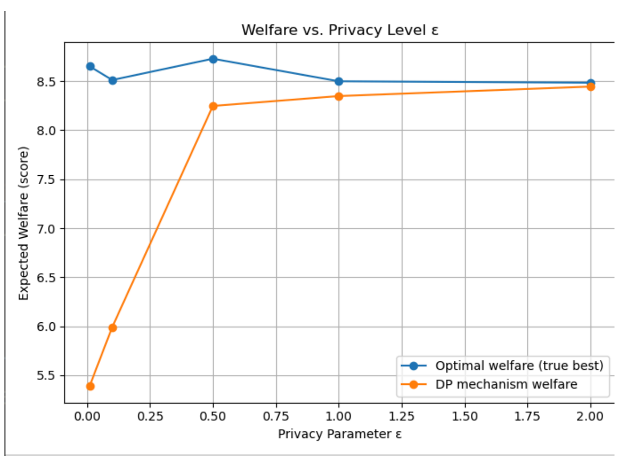
- Generate a set of passengers with valuations and costs.
- Compute each passenger's score s_i.

- For a chosen ε, compute exponential-mechanism allocation probabilities.
- Randomly draw a selected passenger based on these probabilities.
- Compute VCG-style payments.
- Repeat over many runs to estimate expected utilities and welfare under different ε values.

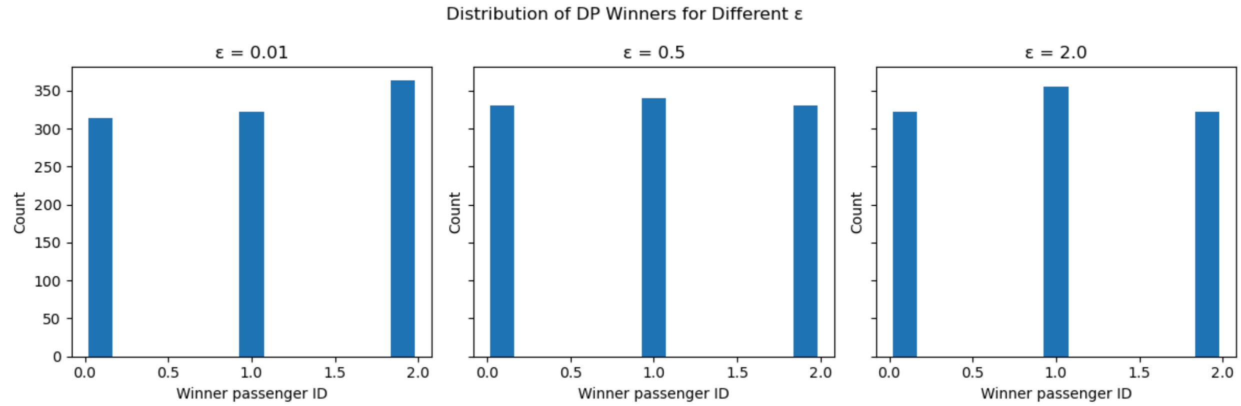
7. Results



The probability of selecting the highest-score passenger increases sharply with ε. For ε = 0, allocation becomes almost uniform.



Stronger privacy (small ε) hurts system performance, because choices become nearly random. Increasing ε improves welfare, because the mechanism increasingly favors high-score passengers. At sufficiently large ε, DP noise becomes negligible, so welfare converges to the optimal.



Differential privacy randomizes outcomes, and low ε makes the mechanism nearly uniform. As ε increases, the distribution becomes more peaked toward the true best passenger. The figure visually captures the privacy accuracy tradeoff, showing how noise drops and preference emerges as ε grows.

7.1 Observations

- Higher ε = weaker privacy, higher welfare, more accuracy.
- Lower ε = stronger privacy, lower welfare.
- The tradeoff is smooth and matches the pattern described in Wu et al. (2020): owners benefit from privacy protection but face increased randomness.

8. Interpretation

This OCG simulation shows how a platform can use differential privacy to protect user location data while still extracting truthful inputs through a VCG-based mechanism. Owners are incentivized to report truthfully, but their probability of being chosen depends on the privacy parameter ε.

As Wu et al. emphasizes, the key insight is the efficiency–privacy tradeoff:

- Stronger privacy requires more randomization (low ε),
- But randomization reduces allocation accuracy and welfare.

The collector must therefore choose ε to balance user trust and system performance. Our results match the general trend found in the paper.