Linear Algebra II Midterm 1 Name:

1. State the definitions of:

a. The span of the vectors $v_1, \dots, v_k$.

b. Linearly independent vectors $v_1, \dots, v_k$.

c. The dimension of a subspace U of a vector space V.

d. A linear map $T \in \mathscr{L}(V, W)$.

2. Let $v_1, \ldots, v_n$ be a basis for V. Prove that each $v \in V$ is a *unique* linear combination of $v_1, \ldots, v_n$.

3. The range of a linear map $T : V \to W$ is $\operatorname{range} T = \{Tv : v \in V\}$. Prove that $\operatorname{range} T$ is a subspace of W.

4. Let $v_1, \ldots, v_m$ and $w_1, \ldots, w_n$ be linearly independent vectors in V. Let $U = \text{span}(v_1, \ldots, v_m)$ and $W = \text{span}(w_1, \ldots, w_n)$. Prove that $U + W$ is a direct sum.

5. Give an example of a linear map $T : M_{2,2}(\mathbb{C}) \to P_2(\mathbb{C})$ such that $\dim(\operatorname{null} T) = 2$.

6. Give an example of a nontrivial subspace of $\mathbb{R}^{\mathbb{R}}$ that does not have finite dimension.