Assignment 2

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Exercise 1

Every mathematical statement, such as $\sum_{n=1}^{\infty} 1/n^2 = \pi^2/6$, should be part of a sentence. Even equations need punctuation!

The notation $\lim\limits_{x\to a} f(x) = L$ means that for every $\varepsilon > 0$ there is a $\delta > 0$ such that $|x-a| < \delta$ implies $|f(x) - L| < \varepsilon$. An incorrect way to typeset this definition is

$$\forall\,(\varepsilon > 0)\,\exists\,(\delta > 0) \ni (|x-a| < \delta \Longrightarrow |f(x) - L| < \varepsilon)\,.$$

The symbols $\forall$, $\exists$, $\ni$, and $\Longrightarrow$ should be used only in the context of the mathematical subject of formal logic and should not replace the words "for all", "exists", and "such that", and "implies".

One of your instructor's favorite mathematical statements is

$$n! \sim \sqrt{2\pi n}\left(\frac{n}{e}\right)^n,$$

otherwise known as Stirling's formula. As an example, 100! is approximately equal to $\sqrt{200\pi}\,(100/e)^{100} \approx 9.32 \times 10^{157}$.

The following is true:

$$\begin{aligned}\left|\int_1^a \frac{\sin x}{x}\,dx\right| &\leq \int_1^a \left|\frac{\sin x}{x}\right|\,dx \\ &\leq \int_1^a \frac{1}{x}\,dx \\ &= \ln a.\end{aligned}$$

After first simplfying using the exponential and the natural log functions, L'Hôpital's rule can be used to evaluate $\lim\limits_{x\to 2^-} (4-x)^{1/(2-x)}$.

Take $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$. The inner product of $\mathbf{x}$ and $\mathbf{y}$ is defined by $\langle \mathbf{x}, \mathbf{y}\rangle = \mathbf{x}^\mathsf{T}\mathbf{y}$. It follows that

$$\langle \mathbf{x}, \mathbf{x}\rangle = \mathbf{x}^\mathsf{T}\mathbf{x} = \|\mathbf{x}\|^2,$$

which is a non-negative real number.

Exercise 2

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1. Navier Stokes Equations

I have found this equation to be interesting since I first learned of it. I don't even understand what the laplacian operator is. I just have read these equations (one them is displayed below) describe the motion of a fluid (liquid or gas) in space. I find it fascinating that although technologies in several inindustries from videogames to areospace rely on these equations today, new math must be invented to show that: "In three space dimensions and time, given an initial velocity field, there exists a vector velocity and a scalar pressure field, which are both smooth and globally defined, that solve the NavierStokes equations.". This the problem statement is given my the Clay Mathematics Institute which is offerring a \$1,000,000 to anyone who can prove or provide a counter example to the statement above.

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\,\mathbf{v} = -\nabla p + v\Delta \mathbf{v} + \mathbf{f}\,(\mathbf{x}, t)$$

2. Additive Identity

For a vector space V the property of additive identity holds. The additive identity states there exists an element $0 \in V$ such that $v + 0 = v$ for all $v \in V$. 0 is pretty special.

3. Green's Theorem

A method of surface integration, Green's Theorem allows you to take a line integral over a region D enclosed by a curve C in a counter-clockwise direction. Where P and Q are functions of x and y that have continuous partial derivatives over D.

$$\oint_C (P\,dx + Q\,dy) = \iint_D \left(\frac{\partial P}{\partial x} - \frac{\partial Q}{\partial y}\,dx\,dy\right)$$