Assignment 2

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Exercise 1

Every mathematical statement, such as $\sum_{n=1}^{\infty} 1/n^2 = \pi^2/6$, should be part of a sentence. Even equations need punctuation!

The notation $\lim_{x\to a} f(x) = L$ means that for every $\varepsilon > 0$ there is a $\delta > 0$ such that $|x-a| < \delta$ implies $|f(x) - L| < \varepsilon$. An incorrect way to typeset this definition is

$$\forall\,(\varepsilon > 0)\,\exists\,(\delta > 0) \ni (|x-a| < \delta \Longrightarrow |f(x) - L| < \varepsilon)\,.$$

The symbols $\forall$, $\exists$, $\ni$, and $\Longrightarrow$ should be used only in the context of the mathematical subject of formal logic and should not replace the words "for all", "exists", and "such that", and "implies".

One of your instructor's favorite mathematical statements is

$$n! \sim \sqrt{2\pi n}\left(\frac{n}{e}\right)^n,$$

otherwise known as Stirling's formula. As an example, 100! is approximately equal to $\sqrt{200\pi}\,(100/e)^{100} \approx 9.32 * 10^{157}$.

The following is true:

$$\begin{aligned} \left|\int_1^a \frac{\sin x}{x}\,dx\right| &\le \int_1^a \left|\frac{\sin x}{x}\right|\,dx \\ &\le \int_1^a \frac{1}{x}\,dx \\ &= \ln a. \end{aligned}$$

After first simplfying using the exponential and the natural log functions, L'Hôpital's rule can be used to evaluate $\lim_{x\to 2^-}(4-x)^{1/(2-x)}$

Take $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$. The inner product of $\mathbf{x}$ and $\mathbf{y}$ is defined by $\langle \mathbf{x}, \mathbf{y}\rangle = \mathbf{x}^\mathsf{T}\mathbf{y}$. It follows that

$$\langle \mathbf{x}, \mathbf{x}\rangle = \mathbf{x}^\mathsf{T}\mathbf{x} = \|\mathbf{x}\|^2,$$

which is a non-negative real number.

Exercise 2

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1. Navier Stokes Equations

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\,\mathbf{v} = -\nabla p + v\Delta\mathbf{v} + \mathbf{f}\,(\mathbf{x}, t)$$

2. Additive Identity or $\mathbf{O(x)}$

3. Green's Theorem

A method of surface integration, Green's Theorem allows you to integrate over a region D enclosed by a closed curve C in a counter-clockwise direction. (What are P and Q?)

$$\oint_C (P\,dx + Q\,dy) = \iint_D \left(\frac{\partial P}{\partial x} - \frac{\partial Q}{\partial y}\,dx\,dy\right)$$