

# Assignment 3

Math 351

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## 1 Instructions

There are two exercises (which were typeset using the theorem environment).

**Exercise 1.** *Recreate this entire document.*<sup>1</sup>

**Exercise 2.** *Create a new document containing a short description of three of your favorite books, papers, or other publications. Be sure to include a bibliography, created using `BIBTEX`.*

An assignment which completes Exercise 2 in an interesting way or makes amusing use of mathematical typesetting will earn the coveted `LATEX`er of the week distinction.

### 1.1 When to turn it in

Please upload the `.tex` and `.bib` source files and the `.pdf` output files to your solutions to Assignment 3 on or before Sunday.

## 2 Euler was excellent

Euler proved many statements, such as

$$\prod_{m=1}^{\infty} (1 - q^m) = \sum_{n=-\infty}^{\infty} (-1)^n q^{(3n^2-n)/2}. \quad (1)$$

where  $q$  is an indeterminate. Equation (1) is known as Euler's pentagonal number theorem. Euler also proved theorem 1 below.

**Theorem 1** (The Basel Problem.). *We have  $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ .*

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<sup>1</sup>How meta.

Euler's original proof of Theorem 1 makes unjustified assumptions that infinite products and sums behave like finite products and sums, but interesting nonetheless and worth displaying.

*Proof.* Using the power series for  $\sin x$ , we have

$$\begin{aligned}\frac{\sin x}{x} &= \frac{1}{x} \left( x - \frac{x^3}{3!} - \frac{x^5}{5!} - \dots \right) \\ &= 1 - \frac{x^2}{3!} - \frac{x^4}{5!} - \dots \\ &= \left( 1 - \frac{x}{\pi} \right) \left( 1 + \frac{x}{\pi} \right) \left( 1 - \frac{x}{\pi} \right) \left( 1 + \frac{x}{\pi} \right) \dots\end{aligned}\tag{2}$$

where the reasoning<sup>2</sup>

□

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<sup>2</sup>This reasoning is actually true, just needs further justification.