Duane-Hunt relation and the determination of Planck's constant.

Experimental Aim

1. To investigate how the spectrum of an X-ray tube varies with accelerating voltage.
2. To collect measurements via computer software to verify the Duane-Hunt relation and estimate Planck's constant.

Skills under development

Through this experiment, we want you to develop your experimental skills further. This is what your lab demonstrator will be looking for when they mark your work.

By the end of the experiment;

- ☐ You will have shown an understanding of the physics of this experiment and the recommended parameters for recording the measurement series.
- ☐ You will be confident and competent performing data collection via computer software.
- ☐ You will have estimated the uncertainty in your measurements.
- ☐ You will have graphed your experimental results and determined an estimate of Planck's constant, h, from them.
- ☐ You will clearly present information (including title, date, partner) in your lab book report and communicate your efforts and achievements ethically.

Background

An X-Ray tube generates x-ray photons by colliding accelerated electrons into and anode. Some of the electron energy is transferred to a photon.

The Bremsstrahlung continuum in the emission spectrum of an X-ray tube is characterized by the limit wavelength, λ_{min}, as shown in Figure 1.

In 1915, the American physicists, William Duane and Franklin L. Hunt discovered an inverse proportionality between the limit wavelength, λ_{min}, and the tube high voltage, U:

$$\lambda_{min} \sim 1/U. \tag{1}$$

This Duane-Hunt relationship can be sufficiently explained by examining some basic quantum mechanical considerations. As the wavelength, λ, and the frequency, ν, for any electromagnetic radiation are related in the manner,

$$\lambda = c/\nu, \tag{2}$$

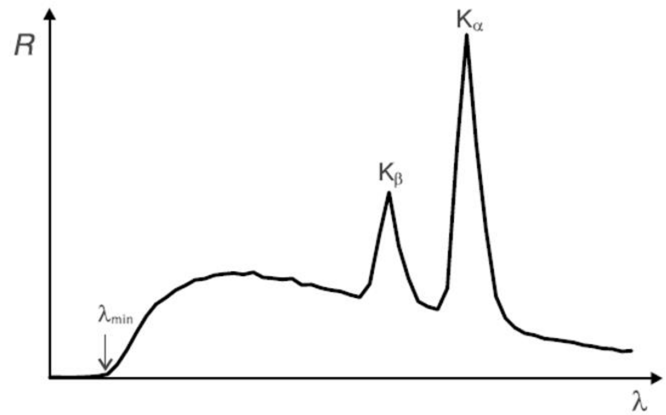

Figure 1: Emission spectrum of an X-ray tube with the limit wavelength λ_{min}, of the Bremsstrahlung continuum and the characteristic κ_α and κ_β lines.

where $c = 299792458\, m/s$ (the velocity of light in a vacuum), the minimum wavelength λ_{min} corresponds to the emitted x-ray quanta with maximum frequency ν_{max} and maximum energy,

$$E_{max} = h\nu_{max}, \qquad (3)$$

where h is Planck's constant.

An x-ray quantum attains maximum energy if it acquires the total kinetic energy of the accelerated electron:

$$E = eU, \qquad (4)$$

(Here $e = 1.6022 \times 10^{-19} As$, the charge on the electron.) Putting Equations (2) - (4) together gives us,

$$\lambda_{min} = \left(\frac{hc}{e}\right) \cdot \frac{1}{U} \qquad (5)$$

Equation (5) corresponds to the Duane-Hunt relation (Equation (1)) with the constant of proportionality included. Assuming the constants c and e are known, it is possible to determine Planck's constant h experimentally from a plot of λ_{min} as a function of $1/U$; the relation should appear as a straight line with gradient given by hc/e.

The detector in the X-ray apparatus you will using for this experiment does not measure photon energy directly. Instead the apparatus effectively contains a spectrometer comprised of a goniometer with NaCl crystal and a Geiger-Muller counter tube. The crystal and counter tube are pivoted with respect to the incident x-ray beam in 2θ coupling as shown in Figure 2.

According to Bragg's law for reflection off crystalline lattices with lattice plane spacing d, the scattering angle θ in the first order diffraction corresponds to the wavelength,

$$\lambda = 2d \sin\theta, \qquad (6)$$

$d = 282.01pm$ for the NaCl mono-crystal.

Thus, with different λ reflected at different θ, we can use the counter tube signal as a function of angle to measure the x-ray spectrum.

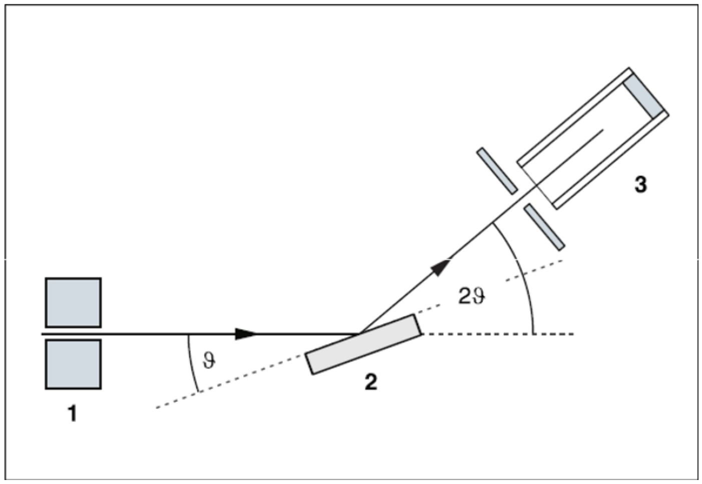

*Figure 2: Schematic diagram of diffraction of x-rays at a mono-crystal and 2θ coupling between counter-tube and scattering angle. **1**: collimator, **2**: mono-crystal, **3**: counter tube.*

Theoretical Estimates

The accepted value of Planck's constant is $h = 6.626070040 \times 10^{-34} Js$ [1]. To experimentally determine your own estimate of Planck's constant in this experiment, you will vary the accelerating voltage, U, of the X-ray tube and measure the corresponding minimum wavelength of the Bremmstrahlung x-ray photons. Before doing so, however, we'd like you to gain an understanding of the results you'd expect to get assuming the established theory is correct.

- Complete the python notebook DuaneHunt-TheoreticalEstimate-SV.ipynb

Now consider Table 1; the recommended parameters for recording the measurement series for this experiment.

U (kV)	I (mA)	Δt (s)	β_{min} (°)	β_{max} (°)	$\Delta\beta$ (°)
35	1.00	10	2.5	6.0	0.1
34	1.00	10	2.5	6.0	0.1
32	1.00	10	2.5	6.0	0.1
30	1.00	10	3.2	6.0	0.1
28	1.00	20	3.8	6.0	0.1
26	1.00	20	4.5	6.2	0.1
24	1.00	30	5.0	6.2	0.1
22	1.00	30	5.2	6.2	0.1

Table 1: Recommended parameters for recording the measurement series.

- ❖ Given the results of your above calculations, explain the recommended values of β_{min}, β_{max} and $\Delta\beta$ in Table 1.
- ❖ Discuss why the recommended acquisition time, Δt, increases as the accelerating voltage, U, decreases.
- ❖ Why isn't the recommended current varied? (You may like to discuss this with your demonstrator.)

Experimental procedure

1. Ask the demonstrator to install the goniometer with the NaCl crystal on the target holder and remove the Zr filter.
2. Check that the spacing between the collimator and the NaCl crystal is approximately 5cm and that the spacing between the NaCl crystal and the sensor is approximately 6cm.

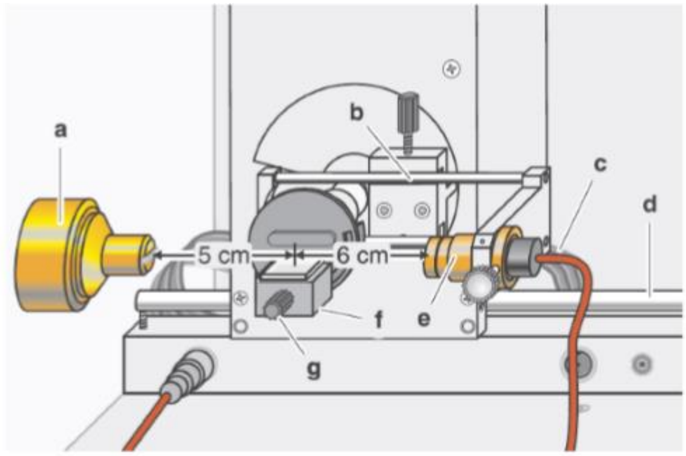

Figure 3: The required separation between collimator and the target holder (5cm), and the target holder and sensor (6cm).

3. Check that the computer is connected to the x-ray apparatus.
4. Turn on the x-ray apparatus.
5. Turn on the computer.
6. Start the X-ray program by clicking on the X-ray icon on the Desktop.
7. Clear any previous data using the F4 key on the keyboard.
8. If a COM port error is raised, reset the COM port to COM4.
9. Now on the X-ray apparatus, press the **U** button and use the "Adjust" control to set the operating voltage to 35kV.
10. Press the **I** button and set the current to 1.0mA.
11. Press the $\Delta \boldsymbol{t}$ button and set the counting time to 10.0 seconds.
12. Press the $\Delta \boldsymbol{\beta}$ button and set the angular step size to $0.1°$.
13. Press the 'Coupled' mode, and press the $\boldsymbol{\beta}$ **Limits** button and set the lower limit of the target angle to $2.5°$.
14. Press the $\boldsymbol{\beta}$ **Limits** button again and set the upper limit of the target angle to $6.0°$.
15. To display a graph of count rate against wavelength on the PC, ensure the Bragg tab is selected and press the F5 key on the keyboard. The settings dialogue box will open. Click on "**NaCl**" and select "***Angular representation***" and then **OK**. The computer is converting readings taken as a function of angle to become a function of wavelength using Equation 6.
16. Start a measurement by pressing the **SCAN** button on the x-ray apparatus.
17. Once the computer has built up the graph of count rate against wavelength and the scan is complete, save the result by pressing the F2 key and enter a suitable name to identify it (i.e. 35kV scan).
18. Repeat steps 9-17 using each row of recommended parameters given in Table 1. Do not clear any data between scans. You are aiming to build up a graph containing 8 measurement curves – one for each of the different operating voltages used.

19. Once your graph is complete (containing 8 measurement curves), click on the printer icon and then select ‘Diagramm Drucken’.
20. Stick the printed graph in your lab book and comment on your results with respect to the first experimental aim.

For each of the recorded diffraction spectrum, associated with each of the operating voltages, you now need to estimate limit wavelength λ_{min} and its uncertainty.

21. In the diagram, click the right mouse button to access the evaluation functions of the software “X-ray apparatus” and select the comment “Best-fit straight line”.
22. Mark the curve range to which you want to fit a straight line to determine λ_{min} using the left mouse. There is some “flexibility” or uncertainty in how you do this, so use your best judgement.
    a. The best fitting line to the range you specified should now appear on the curve, as shown in Figure 4.

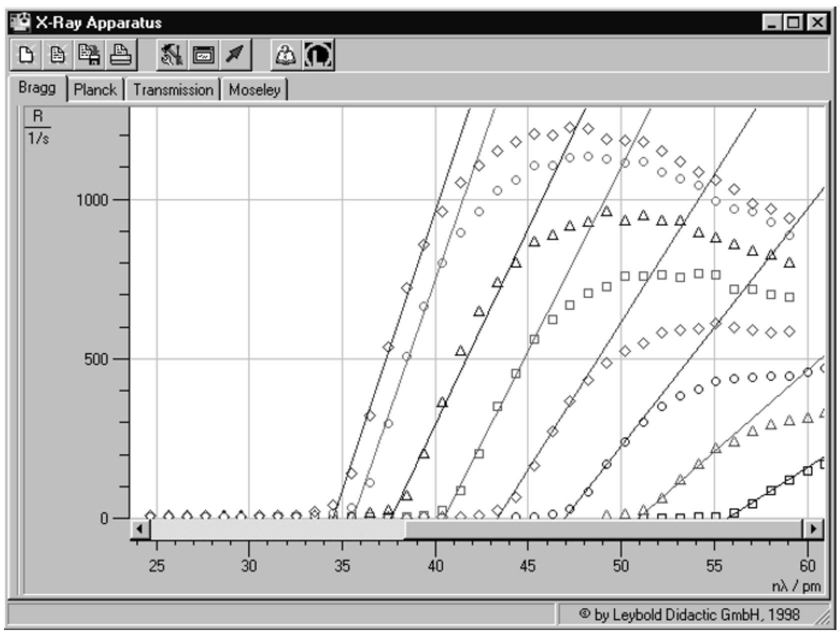

Figure 4: Sections from the diffraction spectra of x-radiation for the tube high voltages U=22-35kV with best-fit straight lines added to determine minimum wavelength in each case.

23. Record the λ_{min} value that then appears in the bottom left of the screen.
24. To estimate the uncertainty in the λ_{min} value, fit another straight line to the same curve that is different from your previous but also a reasonable fit to your curve.
25. Record the λ_{min} value that then appears in the bottom left of the screen.
26. Use the two estimates of λ_{min} to obtain an average estimate and an estimate of uncertainty.
27. Repeat steps 21-26 to obtain an average estimate of λ_{min} and its uncertainty for each of the operating voltages.
28. Determine an experimental estimate of h, and a measure of its uncertainty, by completing the python notebook DuaneHuntRelation_DeterminingPlancksConstant-SV.ipynb.
29. Compare your experimental estimate of h with the accepted value of h.