Modelling, Control System Design and Simulation of an Autonomous Bicycle

Conference Paper · February 2014

DOI: 10.2316/P.2014.809-007

Zeki Y Bayraktaroglu

Istanbul Technical University

Proceedings of the IASTED International Conference
Modelling, Identification and Control (MIC 2014)
February 17 - 19, 2014 Innsbruck, Austria

MODELLING, CONTROL SYSTEM DESIGN AND SIMULATION OF AN AUTONOMOUS BICYCLE

Omer Faruk Argin, Zeki Yagiz Bayraktaroglu
Istanbul Technical University, Faculty of Mechanical Engineering
MA1 Gumussuyu 34437 Istanbul/Turkey
oargin@itu.edu.tr, zeki.bayraktaroglu@itu.edu.tr

ABSTRACT
This paper presents the mathematical modelling and control system design of an autonomous bicycle. The nonlinear equations of motion have been derived and the proposed control system has been used in the simulation of the dynamical behaviour of the bicycle. With the assumption of rolling without slipping condition for the wheels-ground interaction, the system is constrained by nonholonomic equations, and the equations of motion are highly nonlinear. Unlike many other approaches present in related literature, the dynamical model is preserved in simulations in its original nonlinear form without any simplifying assumptions and linearization. Numerical results of the simulations show that the proposed closed-loop control system is achievable. Design of the experimental system has been based on a commercially available bicycle. The mechanical modifications and control system hardware have been designed according to the simulation results.

KEY WORDS
Dynamical modelling, Bicycle control.

1. Introduction

The equations of motion of bicycle are highly nonlinear and rolling of wheels without slipping can only be expressed by nonholonomic constraint equations. In previously published works on the subject, many assumptions are made in order to simplify the equations of motion of the system. Simplified models lead to the deviation of the analysis results from the physical reality. Motion analysis based on simplified dynamics is valid only around specific nominal states such as the equilibrium of the bicycle. Since the system is nonlinear, controllers developed around such special states will perform poorly in the large.

The first mathematically rigorous studies of bicycle dynamics were at end of the 19th century, with the Sharp's manual [1], Rankine [2] and Bourlett [3]. Carvalho presented the equation of motion of bicycle linearized around vertical equilibrium [4]. In the second half of the 20th century, various mathematical models for two-wheel vehicles have been proposed with the increasing popularity of bicycle and motorcycle races. Neimark and Fufaev revealed linear bicycle models for different wheel structures such as rigid disc, torus and pneumatic tire [5].

Roland's work was the first computer simulation of the bicycle behaviour [6]. In 1990s, nonlinear bicycle models with certain assumptions have been proposed [7],[8]. The head tube angle (λ) is one of the most important parameters that effect stability of motion and maneuverability. The bicycle dynamics can be expressed in a simpler manner if the head tube angle (λ) is 0 degree, i.e. perpendicular to the ground [9]. Guo et al. have proposed a nonlinear model of a two-body bicycle, consisting of a frame and a combined mass of the wheels [10]. In another two-body model, the bicycle mass is distributed between the frame and gyroscopic stabilizer, with λ=0 [11]. Nonholonomic constraints are not taken into account in [10] and [11]. Huang et al. have used the Kane method in order to obtain the dynamics with nonholonomic constraints of a front wheel driven bicycle [12].

Especially in the last ten years, studies have focused on tracking control over reference trajectories and improved the bicycle balance control. The first study on balance control of a bicycle was proposed with nonlinear model and nonzero speed in 1994 by Getz [8]. Once certain speed conditions are met, the bicycle balance can be controlled by the position of the handlebar [10],[12],[13]. External force to ensure the bicycle balance can be achieved with the aid of an inverted pendulum [14]. Gyroscopic stabilization is another method used in closed-loop control of self-balancing bicycle [11], [15].

In this study, the head tube angle has been taken into account in dynamical modelling. The highly nonlinear equations of motion have also been preserved without any simplification through assumptions or linearization. The external force required for balance control is supplied by the help of the rotating disc mounted on the frame.

In the second section, the dynamical model of a two-wheeled bicycle is obtained through the Euler-Lagrange equations. In the third section, the design of the closed-loop control system is presented. The fourth section presents the results of dynamical analysis with the proposed model and controller. Finally in the last section, the simulation results are discussed and the future work is described.

2. Modelling

The bicycle dynamics is obtained through the Euler-Lagrange equations. The vehicle consists of the main frame, handlebars, rear and front wheel and balance disc (Figure 1).

The scheme of the bicycle with the ground fixed reference frame is shown in Figure 2. The body fixed reference frames are located at the mass centres. In order to observe the position of the bicycle in the ground fixed frame, an additional frame is located at the contact point of the rear-wheel with the ground (Table 1).

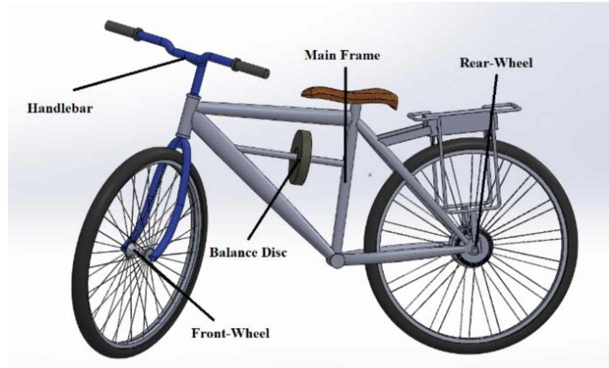

Figure 1: Bicycle solid model.

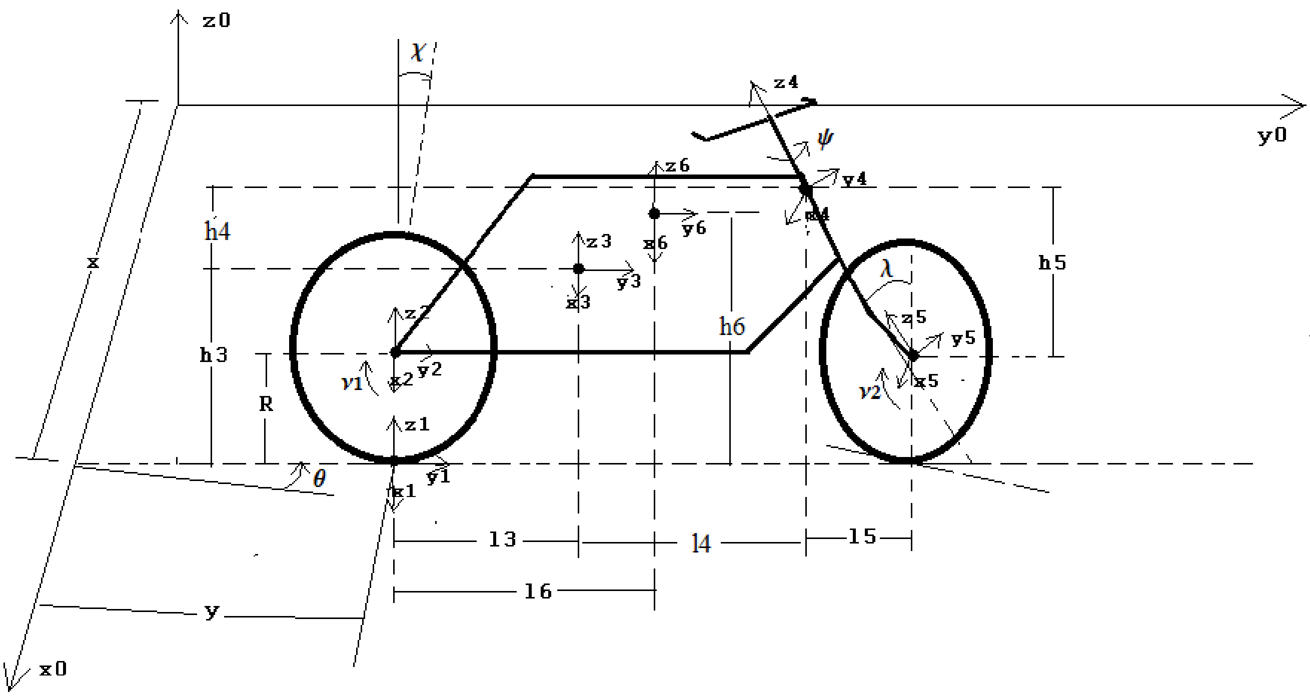

Figure 2: Coordinate axes, variables and constant parameters of the system.

Table 1: Coordinate axes.

Coordinate Axes	Location
R_0 : x_0, y_0, z_0	Axes fixed to the ground
R_1 : x_1, y_1, z_1	Axes fixed to the rear-wheel ground contact point
R_2 : x_2, y_2, z_2	Axes fixed to the centre of mass of rear-wheel
R_3 : x_3, y_3, z_3	Axes fixed to the centre of mass of frame
R_4 : x_4, y_4, z_4	Axes fixed to the centre of mass of handlebar
R_5 : x_5, y_5, z_5	Axes fixed to the centre of mass of front-wheel
R_6 : x_6, y_6, z_6	Axes fixed to the centre of mass of balance disc

The constant parameters and generalized coordinates used in the model are respectively given in Tables 2 and 3.

In this study, the bicycle is assumed to move over perfectly horizontal ground with the wheels satisfying the rolling without slipping condition. The type of ground-wheel contact is modelled as a point contact.

Table 2: Constant parameters.

Parameter	Description
m_2	mass of rear-wheel
m_3	mass of frame
m_4	mass of handlebar
m_5	mass of front-wheel
m_6	mass of balance disc
g	gravitational acceleration
R	Rear and front wheel radius
h_3	R_3 height from the ground
h_4	R_4 height from R_3

h_5	R_5 height from R_4
h_6	R_6 height from the ground
l_3	Distance between frame and contact point of the rear-wheel with the ground
l_4	Distance between handlebar and frame
l_5	Distance between handlebar and front wheel
l_6	Distance between balance disc and contact point of the rear-wheel with the ground
λ	Bicycle head angle
I_2	Mass moment of inertia matrix of rear-wheel
I_3	Mass moment of inertia matrix of frame
I_4	Mass moment of inertia matrix of handlebar
I_5	Mass moment of inertia matrix of front-wheel
I_6	Mass moment of inertia matrix of balance disc

Table 3: Generalized coordinates.

Variable	Description
x	Position of R_1 in the direction of x_0 axis
y	Position of R_1 in the direction of y_0 axis
θ	R_1 angular position relative to z_1 axis (Orientation angle)
χ	R_1 angular position relative to y_1 (Tilt angle)
ψ	R_4 angular position relative to z_4 axis (Handlebar angle)
υ_1	R_2 angular position relative to x_2 axis (Rear wheel angle)
υ_2	R_5 angular position relative to x_5 axis (Front wheel angle)
α	R_6 angular position relative to y_6 axis (Balance disc angle)

Transformation matrices between successive reference frames are written as follows;

$$
{}^0_1T = \begin{pmatrix} c\theta c\chi & -s\theta & -c\theta s\chi & x \\ s\theta c\chi & c\theta & -s\theta s\chi & y \\ s\chi & 0 & c\chi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad {}^1_2T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c\upsilon_1 & -s\upsilon_1 & 0 \\ 0 & s\upsilon_1 & c\upsilon_1 & R \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

$$
{}^2_3T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_3 \\ 0 & 0 & 1 & h_3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad {}^3_4T = \begin{pmatrix} c\psi & -s\psi c\lambda & -s\psi s\lambda & 0 \\ s\psi & s\psi c\lambda & -c\psi s\lambda & l_4 \\ 0 & s\lambda & c\lambda & h_4 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{1}
$$

$$
{}^4_5T = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c\upsilon_2 & -s\upsilon_2 & l_5 c\lambda - h_5 s\lambda \\ 0 & s\upsilon_2 & c\upsilon_2 & h_5 c\lambda + l_5 s\lambda \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad {}^5_6T = \begin{pmatrix} c\alpha & 0 & -s\alpha & 0 \\ 0 & 1 & 0 & l_6 \\ s\alpha & 0 & c\alpha & h_6 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

By using these transformation matrices (1), the dynamics of the bicycle motion can be expressed with respect to the ground fixed reference frame. The total kinetic energy due to translational and rotational motions and the potential energy of the system are written as follows;

$$T = \frac{1}{2} m_i {v_i}^T v_i + \frac{1}{2} {\omega_i}^T I_i \omega_i \tag{2}$$

$$V = m_i g h_i \tag{3}$$

The Lagrangian function L of the system is obtained as follows;

$$L = T - V \tag{4}$$

In cases where the number of degrees of freedom is greater than the number of generalized coordinates, additionally defined coordinates are not independent of the present generalized coordinates. Equations with terms in the time derivatives of the generalized coordinates and which cannot be integrated are called nonholonomic constraint equations [16]. Although the autonomous bicycle model has eight generalized coordinates, four of them are not independent from the remaining four. The following equation is the general form of nonholonomic constraint equations encountered in mechanics;

$$\dot{q}_{m+l} - \sum_{j=1}^{m} a_{lj}\, \dot{q}_j = 0 \quad , \quad l = 1, \dots, k \tag{5}$$

where k is the number of equations and m is the number of independent generalized coordinates. The equations of nonholonomic constaints for the bicycle model are written as follows;

$$\dot{x} = -R\dot{\upsilon}_1 \sin\theta \tag{6}$$

$$\dot{y} = R\dot{\upsilon}_1 \cos\theta \tag{7}$$

$$\dot{\theta} = \frac{R}{l_4+l_5} \dot{\upsilon}_1 (\cos\theta \tan(\theta + \psi\cos\lambda) - \sin\theta) + \frac{h_4 \sin\lambda - l_5 \cos\lambda}{l_4+l_5} \dot{\psi}_1 \tag{8}$$

$$\dot{\upsilon}_2 = \frac{\cos\theta}{\cos(\theta+\psi\cos\lambda)} \dot{\upsilon}_1 \tag{9}$$

In many cases, the nonholonomic constraints can be used in the construction of the Lagrangian function of the system and the resulting equations lead to the so-called Voronec equations [17]. The reduced Euler-Lagrange function $\bar{L}$ is obtained by replacing the expressions of the constraint equations in the original Lagrangian function L. The Voronec equations of motion are finally written as follows [5];

$$\frac{d}{dt}\left(\frac{\partial \bar{L}}{\partial \dot{q}_i}\right) - \frac{\partial \bar{L}}{\partial q_i} = \tau_i + \sum_{v=1}^{k} \frac{\partial \bar{L}}{\partial q_{m+v}} a_{vi} + \sum_{v=1}^{k} \sum_{j=1}^{m} \frac{\partial \bar{L}}{\partial \dot{q}_{m+v}} b_{ij}^{v} \dot{q}_j \quad , \quad i = 1, \dots, m \tag{10}$$

where,

$$b_{ij}^{\nu} = \frac{\partial a_{\nu i}}{\partial q_j} - \frac{\partial a_{\nu j}}{\partial q_i} + \sum_{\mu=1}^{k} \left(\frac{\partial a_{\nu i}}{\partial q_{m+\mu}} a_{\mu j} - \frac{\partial a_{\nu j}}{\partial q_{m+\mu}} a_{\mu i} \right)$$
(11)

The nonlinear dynamical model of the bicycle is written based on equations (10). The model consists of four second-order differential equation and four nonholonomic constraint equation. The equation of χ consists of 1162 terms, equation of ψ 6375 terms, equation of υ_1 7061 terms and equation of α 4 terms. The equations of motion have been derived using Mathematica®.

3. Control System Design

The required reference trajectories must be generated for the autonomous motion of the bicycle. Generation of the reference trajectories is presented in two parts: Generation of reference trajectories for the rear-wheel and computation of the reference angle for the handlebar. Design and implementation of the closed-loop controller are presented at the end of this section.

3.1 Trajectory Generation

The path planning is based on ensuring continuity of the bicycle speed. The position of the contact point of the rear-wheel with the ground and the rear-wheel angular velocity references are generated by the trajectory generator. Between the start and end positions of a desired path, intermediate points are selected to design the entire path. Third-order polynomials are generated between successive intermediate position references. Boundary conditions are determined by the continuity of velocity at intermediate points.

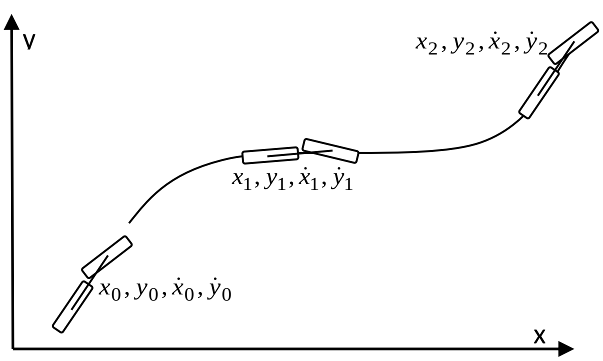

Figure 3: Trajectory planning and boundary conditions.

The third-order polynomial trajectories are computed by using cubic splines [18]. The bicycle reference position, velocity and acceleration profiles are written as follows;

$$x = a_{x0} + a_{x1}s + a_{x2}s^2 + a_{x3}s^3$$
$$y = a_{y0} + a_{y1}s + a_{y2}s^2 + a_{y3}s^3$$

$$\begin{aligned} \dot{x} &= a_{x1} + 2a_{x2}s + 3a_{x3}s^2 \\ \dot{y} &= a_{y1} + 2a_{y2}s + 3a_{y3}s^2 \\ \ddot{x} &= 2a_{x2} + 6a_{x3}s \\ \ddot{y} &= 2a_{y2} + 6a_{y3}s \end{aligned} \tag{12}$$

The reference angle θ can be calculated as follows;

$$\theta = \tan^{-1}\left(\frac{dy/dt}{dx/dt}\right) = \tan^{-1}\left(\frac{a_{y1}+2a_{y2}s+3a_{y3}s^2}{a_{x1}+2a_{x2}s+3a_{x3}s^2}\right)$$
(13)

The rear-wheel speed reference is obtained as follows;

$$\dot{\upsilon}_1 = \frac{1}{R}\sqrt{\dot{x}^2 + \dot{y}^2} \tag{14}$$

3.2 Tracking Algorithm

In this study, the pure pursuit algorithm, which is among the widely used geometric trajectory tracking algorithm [19], has been implemented for tracking control purpose.

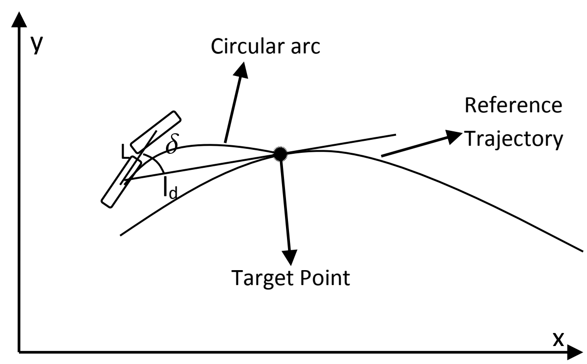

Figure 4: Trajectory tracking.

The pure tracking algorithm as seen in Figure 3, is based on calculating a circular arc between the rear-wheel ground contact and the target point. Bicycle handlebar angle can be calculated from the distance between the rear-wheel contact and the target position l_d, orientation error δ and the bicycle base length $L = l_4 + l_5$ [14]. The pure tracking control algorithm is given as follows;

$$\psi = \frac{2L}{l_4}\sin\delta \tag{15}$$

3.3 Closed-loop Control

The reference inputs for the position of the rear-wheel contact with the ground (x_{ref}, y_{ref}), the orientation of the bicycle (θ_{ref}), and the angular velocity of the rear wheel ($\dot{\upsilon}_{1ref}$) were calculated as presented in section 3.1. The reference for the handlebar angular position was calculated as a function of position in section 3.2.

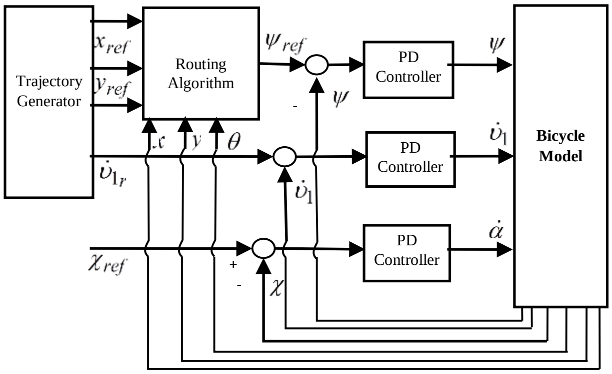

Figure 5: Closed loop control scheme.

The tilt angle χ, which is an important variable affecting on the stability of motion, is fixed to zero degree during the motion. The angular momentum of the balance disc is used to control the tilt angle. The handlebar position is used to control bicycle orientation and the rear-wheel angular velocity gives bicycle velocity. All of those variables constitute the applied control inputs. Coefficients of the PD controllers used in simulations can be determined by trial and error in successive simulations.

4. Dynamical Simulations

The bicycle to be used in experimental implementation is shown in Figure 6. Parameters of the vehicle have been measured and identified. The list of constant parameters used in dynamical simulations is given in Table 4.

Figure 6: Electric bicycle.

Simulations have been performed in Matlab® environment. The Runge-Kutta solver (ode4) with 0.01 seconds step size is used for the solution of differential equations.

Table 4: Parameters

Parameters
m_2=5.75kg, m_3=11.5kg, m_4=1.95kg, m_5=2.3kg,
m_6=10.07kg, R= 0.33m, h_3=0.54m, h_4=0.75m,
h_5=-0.42m, h_6=0.55m, l_3=0.32m, l_4=0.89m,
l_5=0.20m, l_6=0.51m, λ=21°, g=9.81 m/s²
$I_2=\begin{bmatrix}0.44 & 0 & 0\\ 0 & 0.22 & 0\\ 0 & 0 & 0.22\end{bmatrix}$, $I_3=\begin{bmatrix}1.29 & 0 & 0\\ 0 & 0.32 & 0.04\\ 0 & 0.04 & 0.99\end{bmatrix}$
$I_4=\begin{bmatrix}0.17 & 0 & 0\\ 0 & 0.19 & 0\\ 0 & 0 & 0.03\end{bmatrix}$, $I_5=\begin{bmatrix}0.21 & 0 & 0\\ 0 & 0.10 & 0\\ 0 & 0 & 0.10\end{bmatrix}$
$I_6=\begin{bmatrix}0.034 & 0 & 0\\ 0 & 0.065 & 0\\ 0 & 0 & 0.034\end{bmatrix}$ kg.m²

Figures 7-12 present the results of dynamical simulations of the bicycle motion. The tracking performances of the proposed control system with and without limitation on the handlebar angular velocity are given in Figure 7. In the limited case, the maximum velocity $\dot{\psi}_{ref}$ is $0,873\ rad/s$. The initial and final velocities of the bicycle are zero at the start and end positions. A total displacement of $20m$ is achieved in $30sec$. The position errors at the end of tracking simulations are respectively $0,56m$ and $0,07m$ for limited and unlimited handlebar angular velocities.

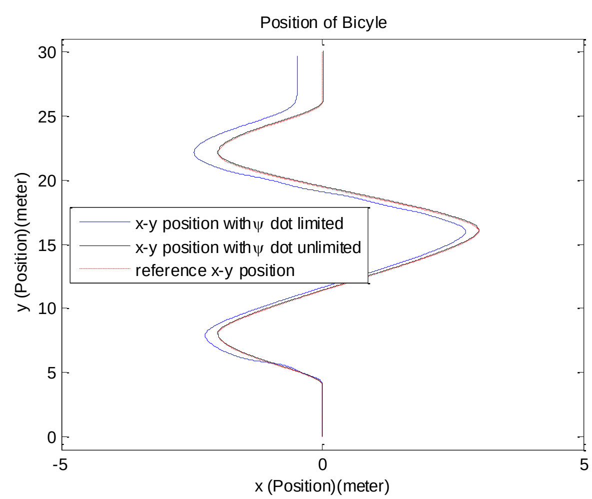

Figure 7: Trajectory tracking.

The angular position reference ψ_{ref} of the handlebar which is generated by the pure tracking algorithm and the responses of the model are given in Figures 8 and 9 for limited and unlimited handlebar angular velocities. The range of motion for the handlebar is limited to $\pm 45°$.

The bicycle orientation reference θ_{ref} and the corresponding simulation responses are given in Figure 10.

The rear-wheel speed reference $\dot{\upsilon}_{1_{ref}}$ computed by the trajectory generator and corresponding model responses are given in Figure 11. The tracking error is seen to increase in the portions of trajectory where the curvature is relatively smaller.

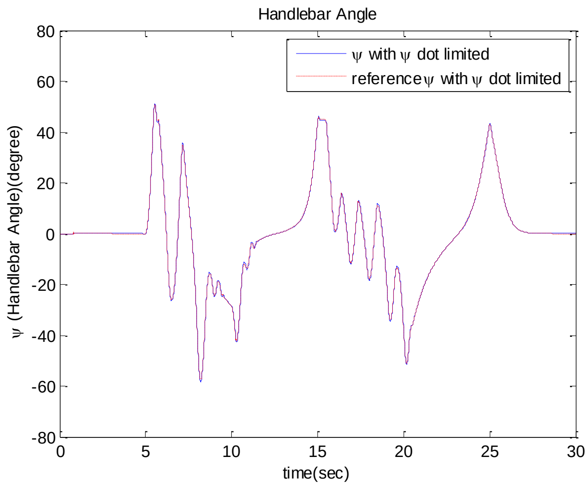

Figure 8: Handlebar angle with $\dot{\psi}_{ref}$ is limited.

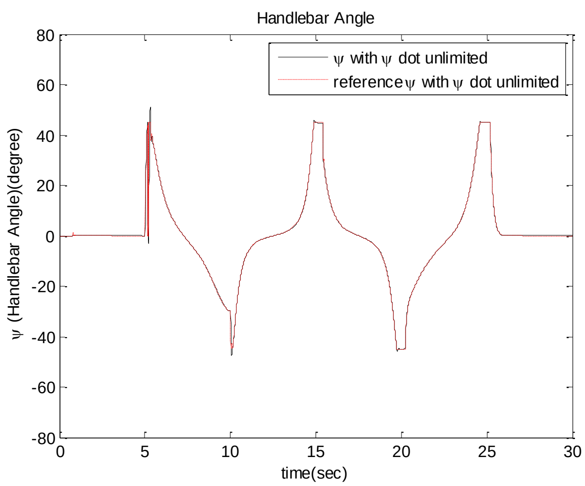

Figure 9: Handlebar angle with $\dot{\psi}_{ref}$ is unlimited.

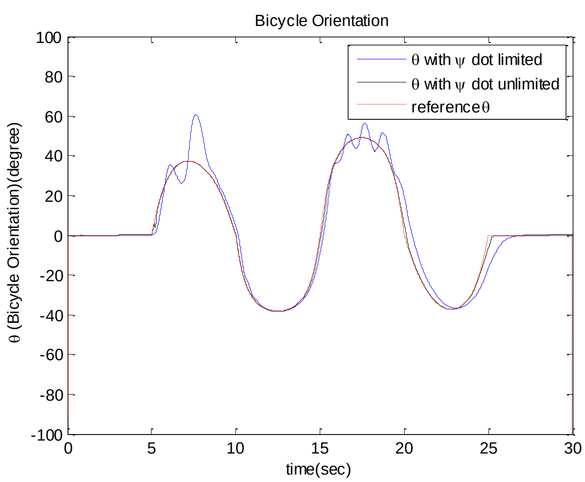

Figure 10: Bicycle orientation.

The bicycle's tilt angle reference of $0°$ and model responses can be seen in Figure 12. Deviations from the tilt angle reference occur at the direction changes of bicycle, with greater magnitude for unlimited handlebar angular velocity. The stability of motion is shown to be maintained during the motion. The angular velocity and acceleration of the balance disk during the entire motion are given in Figures 13 and 14.

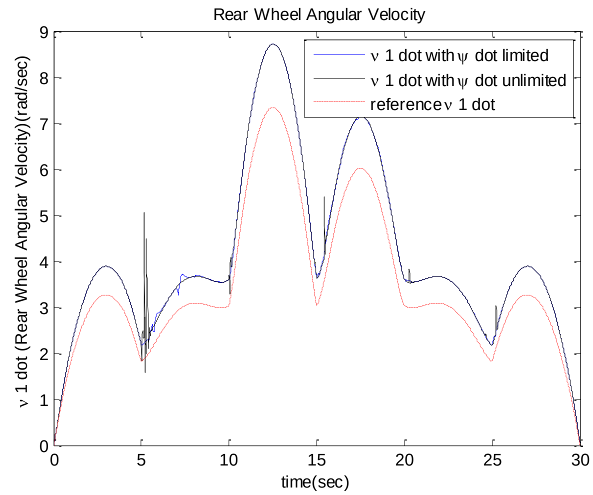

Figure 11: Rear-wheel angular velocity.

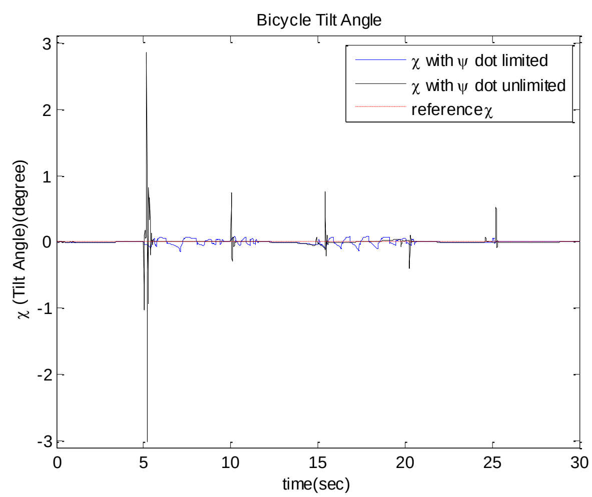

Figure 12: χ tilt angle.

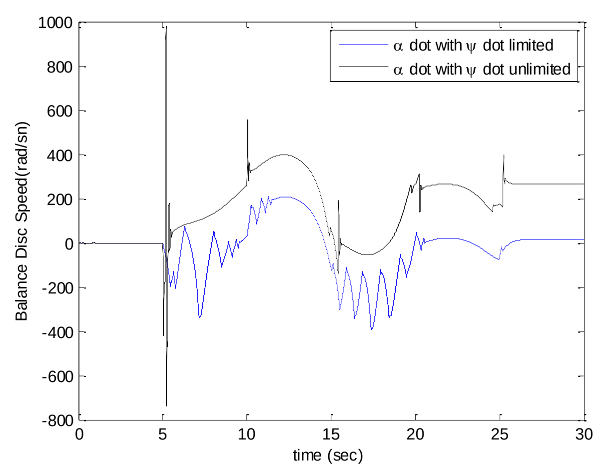

Figure 13: Balance disc speed.

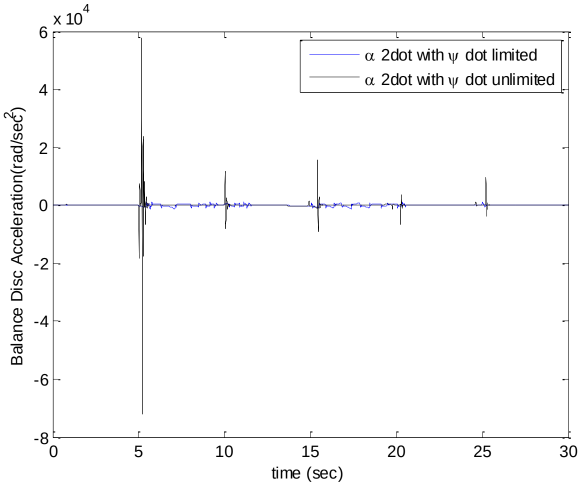

Figure 14: Balance disc acceleration.

5. Conclusion

In this study, a nonlinear autonomous bicycle model is firstly written. We presented a proposed control for a bicycle with an balancer and also presented a closed-loop stability analysis of the bicycle using a nonlinear model.
Parameters of the bicycle to be used in experimental setup have also been used in the dynamical model. A closed-loop tracking controller has been designed and tested in dynamical simulations. The simulation results confirm a satisfactory closed-loop performance with the proposed model and controller. Tracking errors are partly due to the imperfect controller coefficients, obtained by trial and error in simulations. Simulation results show that for limited handlebar angular velocity, the trajectory tracking error increases while better stability and velocity tracking are achieved. The resulted ranges of numerical values of the control variables are compatible with physically achievable behaviour. Therefore, the simulation results confirm also the feasibility of the physical autonomous bicycle with commercially available hardware.

Hardware modifications are designed for the bicycle shown in Figure 6. The control system tested in simulations will be implemented on the experimental autonomous bicycle. The angular velocity of the balance disc and the angular position of the handlebars will be provided by DC actuators. The position of the handlebar will be measured with an absolute encoder and the tilt angle will be measured by an IMU to be fixed to the frame. Closed loop control system will be implemented in an embedded computer.

References

[1] A. Sharp, *Bicycles & Tricycles: A Classic Treatise on Their Design and Construction* (New York: Dover, 2003).
[2] W.J.M. Rankine, On the dynamical principles of the motion of velocipedes, *Engineer*, 28, 1869, pp.79, 129, 153, 175.
[3] C. Bourlett, *Traité des Bicycles et Bicyclettes* (Paris: Gauthier-Villars,1898).
[4] E. Carvallo, Théorie du mouvement du monocycle et de la bicyclette, *J. de L'Ecole Polytechnique*, 5, 1900, 119-188.
[5] I.U.I. Neimark & N.A. Fufaev, *Dynamics of Nonholonomic Systems* (Providence, Rhode Island: American Mathematical Society, 1972).
[6] R.D. Roland, *Computer Simulation of Bicycle Dynamics*, (New York: ASME, 1973).
[7] G. Franke, W. Suhr, & F. Rie*β*, An advanced model of bicycle dynamics, *Eur. J. Phys.*, 11(2), 1990, 116-121.
[8] N.H. Getz, Control of balance for a nonlinear nonholonomic nonminimum phase model of a bicycle, *Proc. Amer. Control Conf.*, Baltimore, MD, 1994, 148-151.
[9] N.H. Getz & J.E. Marsden, Control for an autonomous bicycle, *Proc. IEEE Conf. Robotics Control*, Nagoya, Japan, 1995, 1397–1402.
[10] L.Guo, Q. Liao,S. Wei, Y. Huang, A kind of bicycle robot dynamic modelling and nonlinear control, *Proc. of Int. IEEE Conf. Information and Automation*, Harbin,China,2010, 1613-1617.
[11] S. Suntharasantic, Manop Wongsaisuwan, Piecewise affine model and control of bicycle by gyroscopic stabilization, *Proc. 8th ECTI Conference*, Khon Kaen,Thailand,2011,549-552.
[12] Y. Huang, Q. Liao, S. Wei, L. Guo, Dynamic Modelling of a bicycle robot with front wheel drive based on Kane's method., *Proc. of Int. IEEE Conf. Information and Automation*, Harbin,China,2010, 758-764.
[13] M.Yamakita & A.Utano, Automatic Control of Bicycles with a Balancer, *Proc. of Int. IEEE Conf. Advanced Intelligent Mechatronics*, Monterey, CA, 2005, 1245-1250.
[14] G.Frosali, F.Ricci, A nonlinear mathematical model for a bicycle, *Proc. XIX Congresso AIMETA*, Ancona, 2009, 1-10.
[15] P.Y. Lam, T.K. Sin, Gyroscopic stabilization of a self-balancing robot bicycle, Int. J. of Automation Technology, 5(6), 2011, 916-923.
[16] F. Pasin, *Mekanik Sistemler Dinamiği,* (Istanbul: I.T.U, 1994).
[17] N. A. Lemos, Nonholonomic constraints and Voronec's equations, *Rev. Bras. Ens. Fis*. 25, 2003,25-28
[18] Waheed, Imran, Trajectory and temporal planning of a wheeled mobile robot, *MSc thesis in Saskatchewan University*, Canada, 2006.
[19] J. M. Snider, Automatic steering methods for autonomous automobile path tracking, *Technical report, CMU–RI–09–08, Robotics Institute, Carnegie Mellon University*, 2009, 9-13.