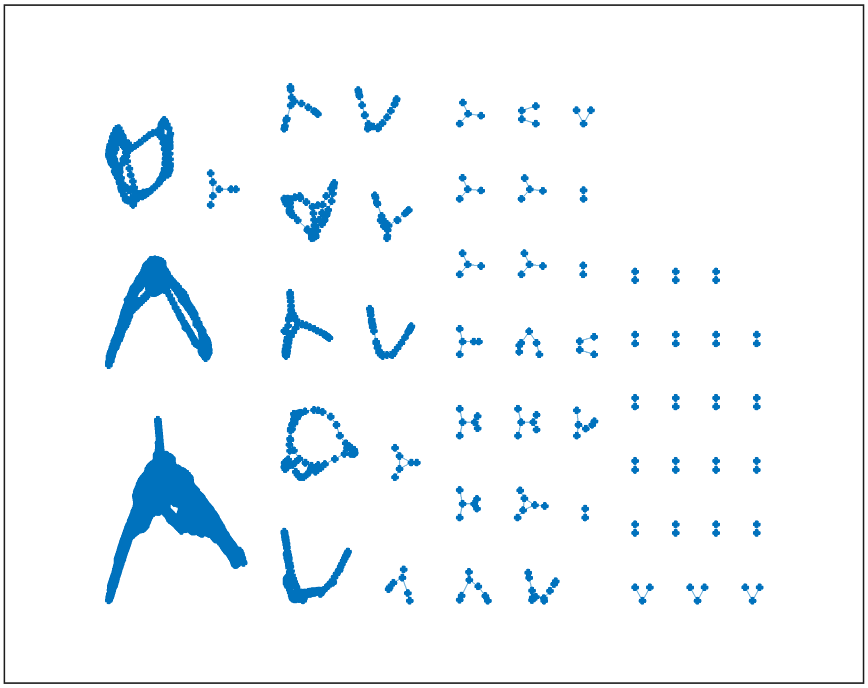
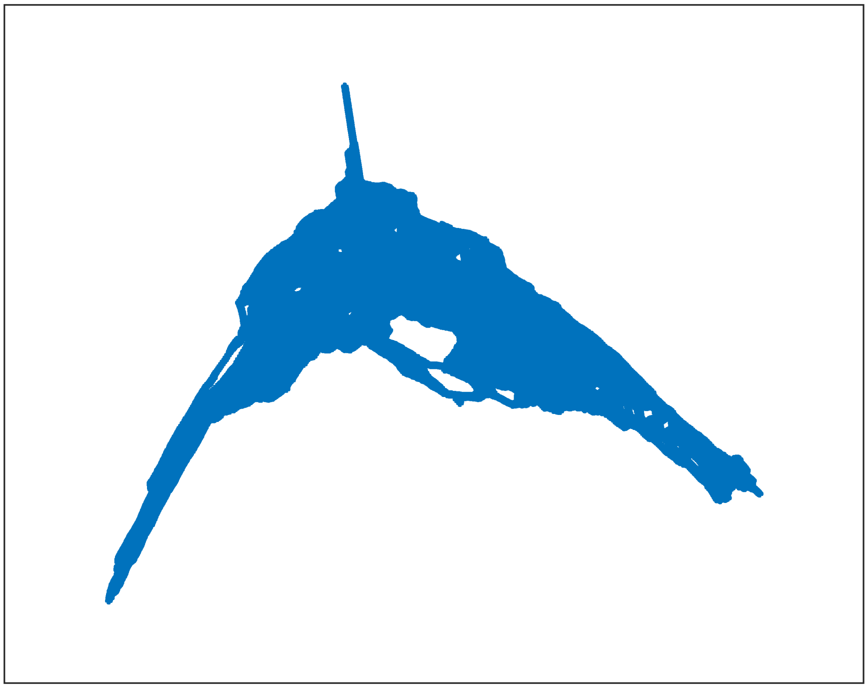
Problem 1

```
% Loading Data

data = load("usroads.mat");

G = graph(data.Problem.A);
plot(G)
```



```
comps = conncomp(G);

SG = subgraph(G, comps==28);

plot(SG)
```

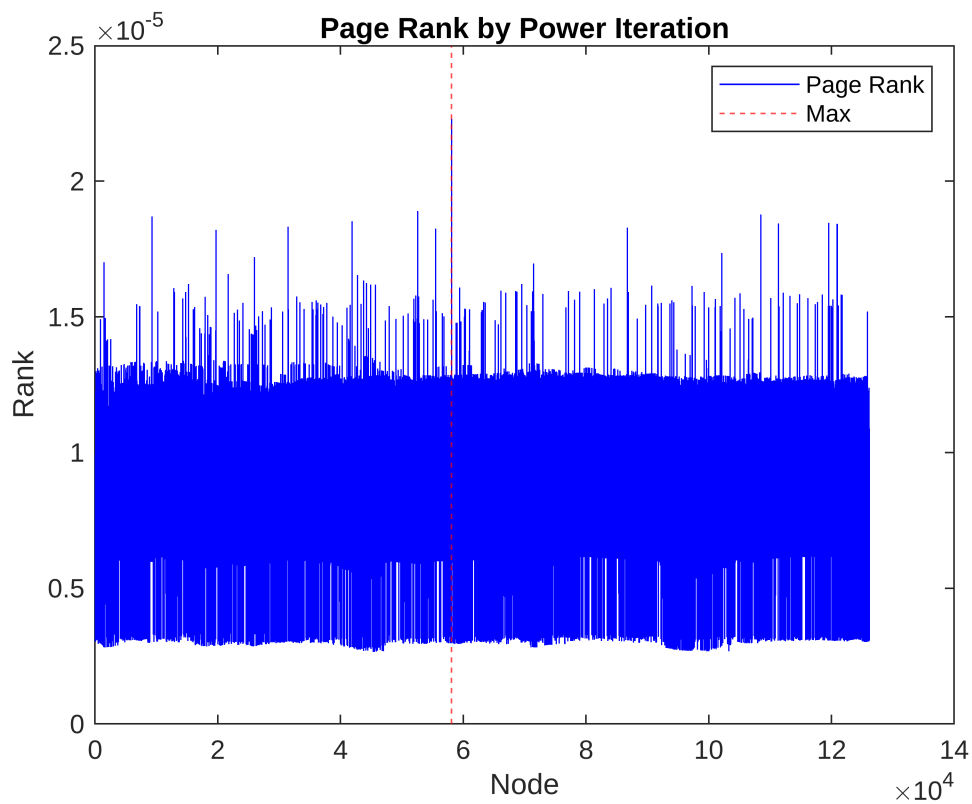


Problem 1.1

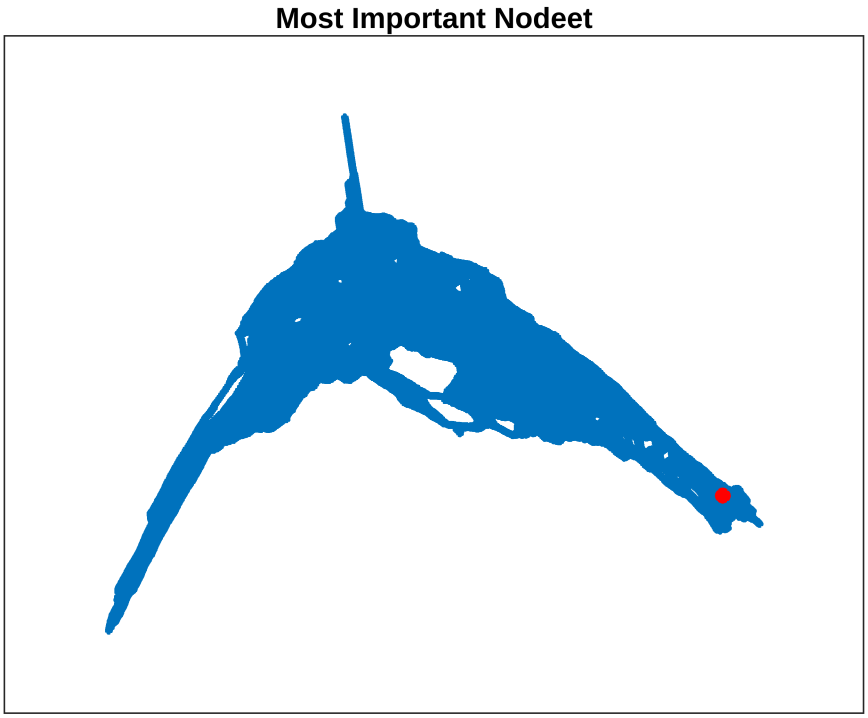
```
A_SG = adjacency(SG);

%Adjacency matrix for Floritine families
A_SG2= [0 0 0 0 0 0 0 0 1 0 0 0 0 0 0;
0 0 0 0 0 1 1 0 1 0 0 0 0 0 0;
0 0 0 0 1 0 0 0 1 0 0 0 0 0 0;
0 0 0 0 0 0 1 0 0 0 1 0 0 1 0;
0 0 1 0 0 0 0 0 0 0 1 0 0 1 0;
0 1 0 0 0 0 0 0 0 0 0 0 0 0 0;
0 1 0 1 0 0 0 1 0 0 0 0 0 0 1;
0 0 0 0 0 0 1 0 0 0 0 0 0 0 0;
1 1 1 0 0 0 0 0 0 0 0 1 1 0 1;
0 0 0 0 0 0 0 0 0 0 0 0 1 0 0;
0 0 0 1 1 0 0 0 0 0 0 0 0 1 0;
0 0 0 0 0 0 0 0 1 0 0 0 0 1 1;
0 0 0 0 0 0 0 0 1 1 0 0 0 0 0;
0 0 0 1 1 0 0 0 0 1 1 0 0 0;
0 0 0 0 0 0 1 0 1 0 0 1 0 0 0];

A_SG = A_SG./sum(A_SG);
```

Problem 1.2

```
% Power Iteration
% Find the page rank of this network using power iteration.

n = length(A_SG);
iterations = 1000;

%Begin power iteration
x0=ones(n,1)/n;

for i = 1:iterations
    y0=A_SG*x0;
    x0=y0./sum(y0);
end

pr_pi = x0./sum(x0);

[v, I] = max(pr_pi);

plot(pr_pi, 'b')
hold on
xline(I, '--r')
hold off
title("Page Rank by Power Iteration");
xlabel("Node")
ylabel("Rank")
legend("Page Rank", "Max")
```



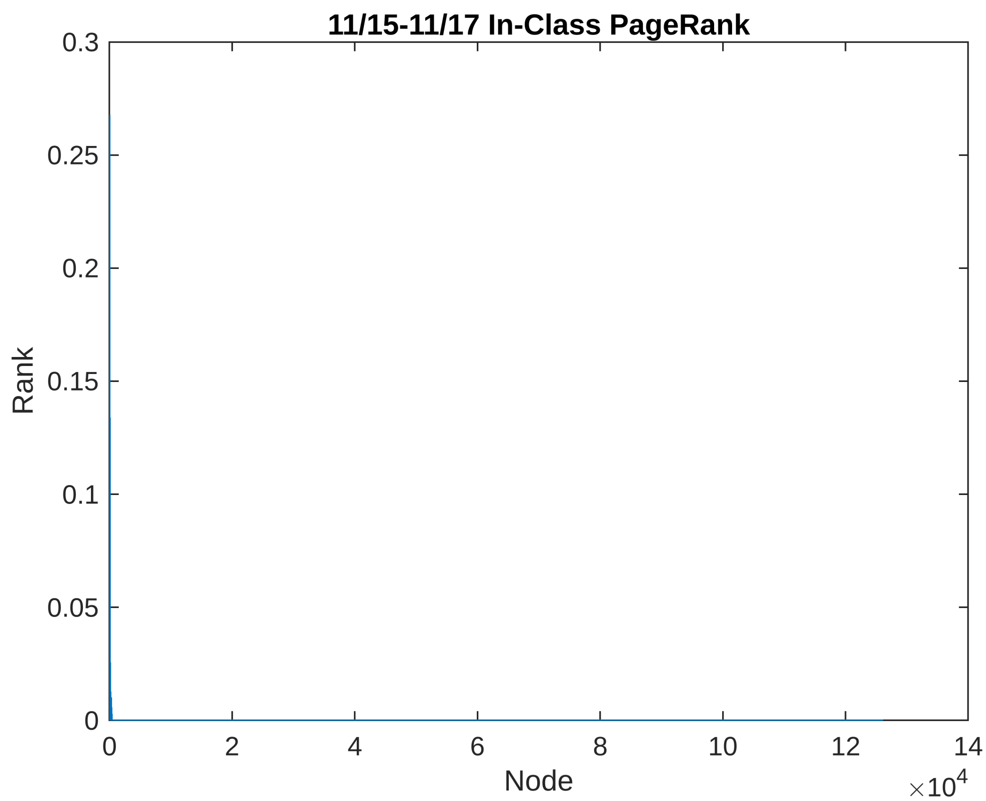
```
Og_g = plot(SG);
highlight(Og_g, I, 'NodeColor','red', 'MarkerSize',5)
title("Most Important Nodeet")
```



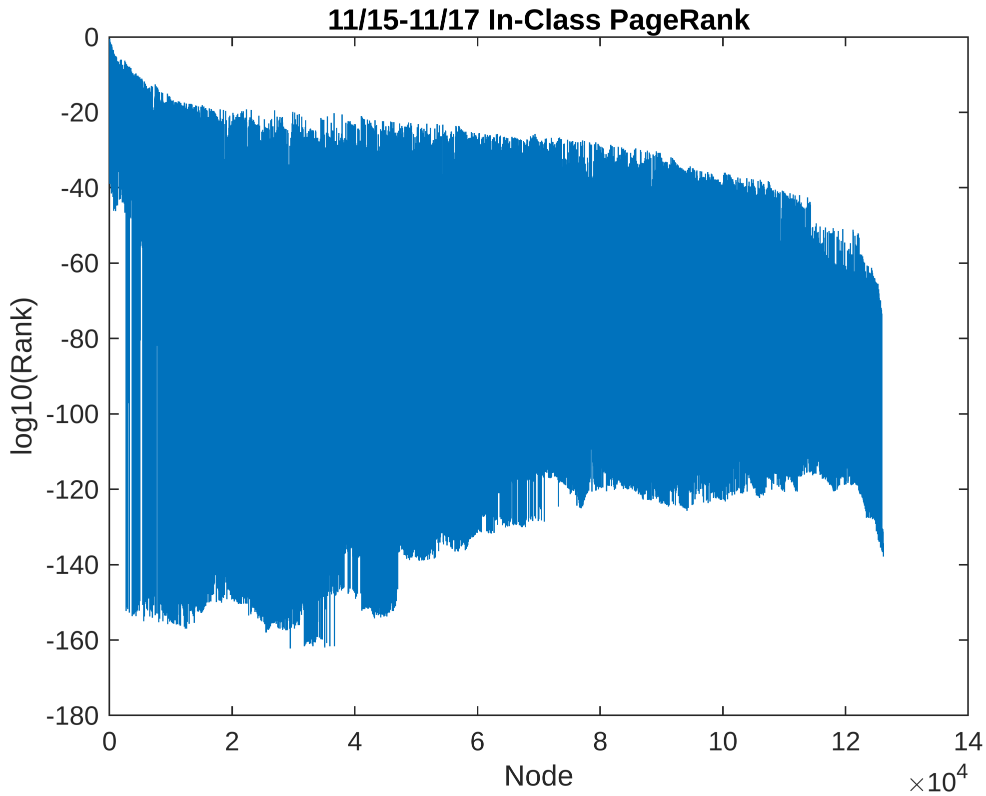
Problem 1.3

```
% Solve using the method discussed in class 11/15 11/17

alpha = 0.9;
k = 1000;
n = length(A_SG);

%Make a column vector of 1's normalized to n
v = eye(n, 1) / n;

%Do the zeroth iteration
partial = v;
v_sum = partial;

for i = 1:k
    partial = alpha*A_SG*partial;
    v_sum = v_sum + partial;
end

pr_iter = (1 - alpha)*v_sum;
pr_iter = pr_iter./sum(pr_iter);
```

```
plot(pr_iter)

title("11/15-11/17 In-Class PageRank");
xlabel("Node")
ylabel("Rank")
```

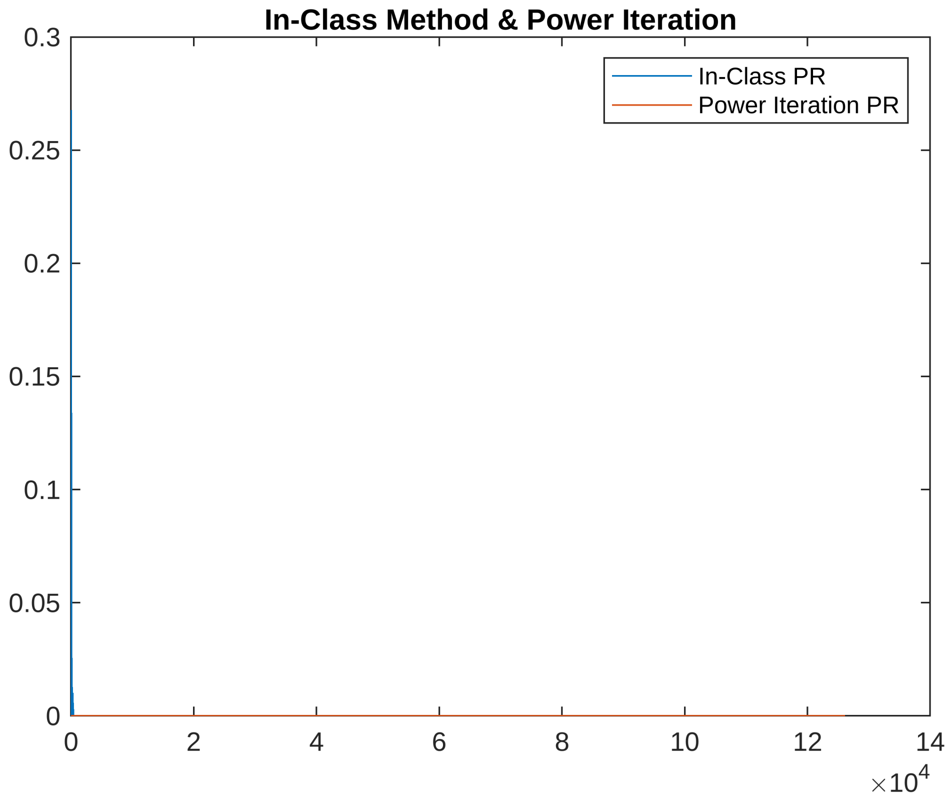



```
plot(log10(pr_iter))

title("11/15-11/17 In-Class PageRank");
xlabel("Node")
ylabel("log10(Rank)")
```



Plot Comparisons with Power Iteration

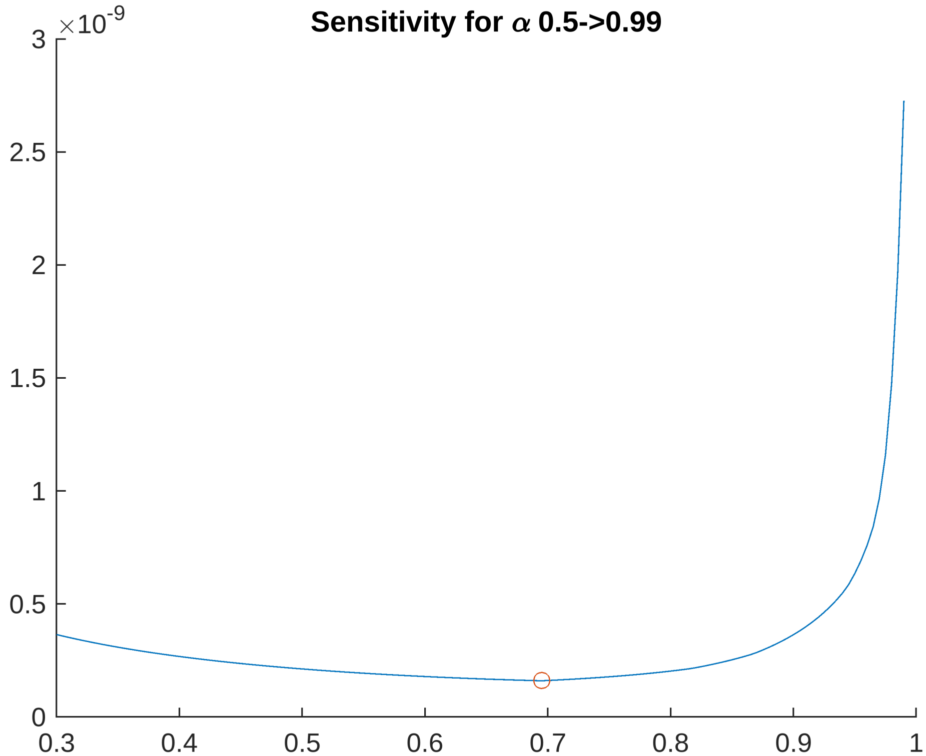
```
plot(pr_iter);
hold on
plot(pr_pi)
%plot(pr_eq)
%plot(pr_eig)
legend("In-Class PR", "Power Iteration PR");
title("In-Class Method & Power Iteration")
hold off
```



Calculate Sensitivity

```
alphas = 0.3:0.005:0.99;
k = 150;
n = length(A_SG);

%Make a column vector of 1's normalized to n
v = eye(n, 1) / n;
count = 1;
s = zeros(n, count);

for alpha = alphas
    partial = A_SG * v;
    v_sum = alpha^(-1) * v + 2 * partial;

    for j = 2:k
        partial = alpha * A_SG * partial;

        v_sum = v_sum + (j + 1) * partial;
    end

    s(:, count) = v_sum - A_SG * v_sum;
    count = count + 1;
end

sense = mean(abs(s));
```

```
[lowest, idx] = min(sense);

figure()
hold on
plot(alphas, sense)
scatter(alphas(idx), lowest, 'o')
title("Sensitivity for \alpha 0.5->0.99")
hold off
```



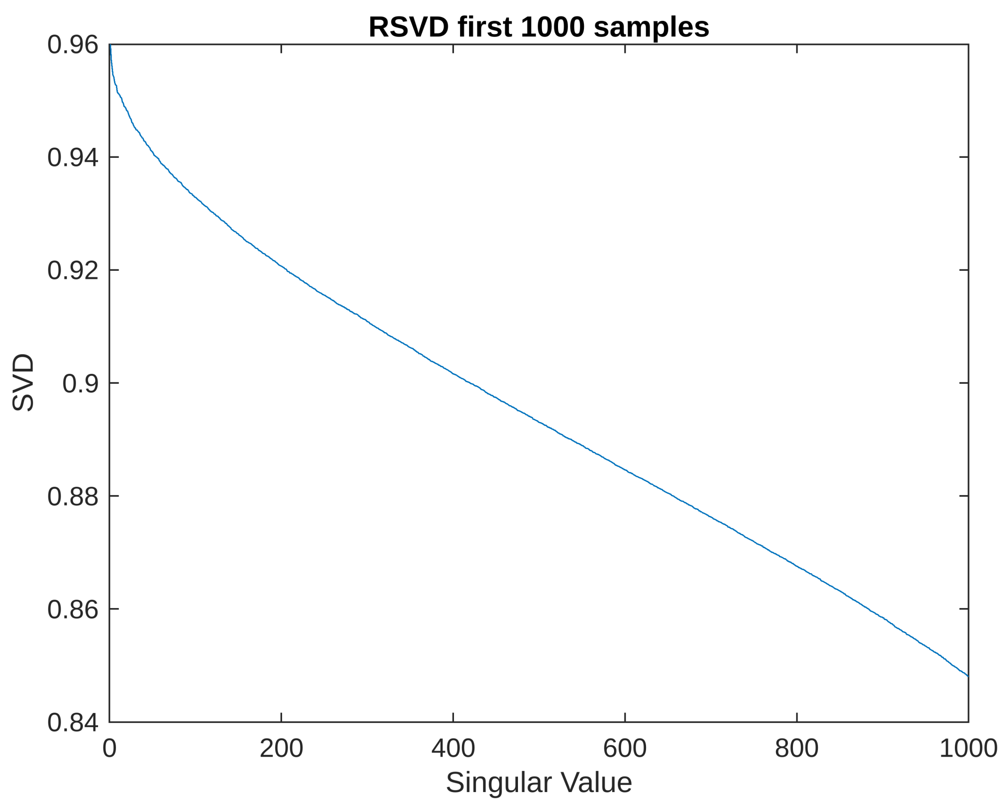
First 1000 Singular Values By RSVD

```
%Step 2
%Pull the size of the matrix
Asize = size(A_SG);
%m is x axis
m = Asize(1);
%n is y axis
n = Asize(2);
%Problem says find the first 1000 singular values using RSVD so r is 1100
oversample a bit
r = 1100;
%Create a random matrix that is n x r
X = randn(n,r);
%Multiply the matrices together to get AX
q = orth(A_SG*X);
%Do the QR factorization of AX to AX = QR
```

```
QTA = q'*A_SG
```

```
QTA = 1100×126146
    0.0013    0.0013    0.0070    0.0016   -0.0019    0.0068   -0.0033   -0.0090 ...
   -0.0003   -0.0003    0.0032   -0.0134    0.0012    0.0175    0.0023   -0.0014
   -0.0065   -0.0065   -0.0006   -0.0001    0.0000   -0.0014    0.0036   -0.0073
    0.0139    0.0139   -0.0004    0.0004   -0.0060    0.0057    0.0025   -0.0055
   -0.0064   -0.0064   -0.0093    0.0049    0.0067   -0.0172    0.0032    0.0029
    0.0014    0.0014    0.0031   -0.0012    0.0012    0.0061   -0.0008   -0.0030
    0.0082    0.0082    0.0005    0.0015    0.0033   -0.0001    0.0001   -0.0026
    0.0114    0.0114   -0.0017   -0.0039    0.0036    0.0059    0.0045    0.0059
   -0.0030   -0.0030   -0.0033   -0.0113    0.0074   -0.0155    0.0004   -0.0081
    0.0091    0.0091    0.0040    0.0020    0.0051   -0.0003    0.0051    0.0088
       ⋮
```

```
%Take the SVD of QTA
[uo,so,vot] = svd(QTA,"econ");
%Take the diagonal of the
singvals = diag(so);
%figure(10)
plot(singvals(1:1000))
xlabel('Singular Value')
ylabel('SVD')
title('RSVD first 1000 samples')
```



Minimal Route From 1000 to 1100:

```
src = 1000;
dest = 1100;

n = length(A_SG);

%Make a sparse vector
S = sparse(n,1);

%Index the sparse matrix using the endpoint
S(dest,1) = 1;

%Multiply the matrix by the sparse matrix
power = A_SG * S;

%Initialize distance
dist = 1;

while power(src, 1) == 0
    power = A_SG * power;
    dist=dist+1;
end

dist
```

```
dist = 140
```

Split Graph with Fielder Method

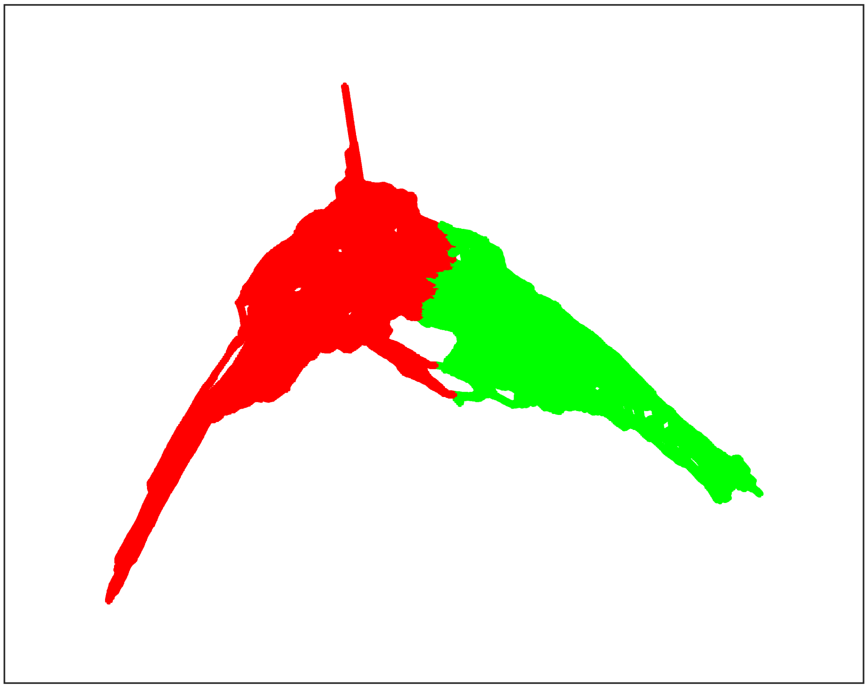
```
L_SG = laplacian(SG);

[V_LSG, L_LSG, f_LSG] = eigs(L_SG,100,0);

Fiedler = V_LSG(:,2);

sum(Fiedler)
```

```
ans = 9.4670e-11
```

```
Positions = 1:height(SG.Nodes);

Cut = Fiedler > 0;

SG_1_nodes = Positions(Cut);
SG_2_nodes = Positions(~Cut);
```

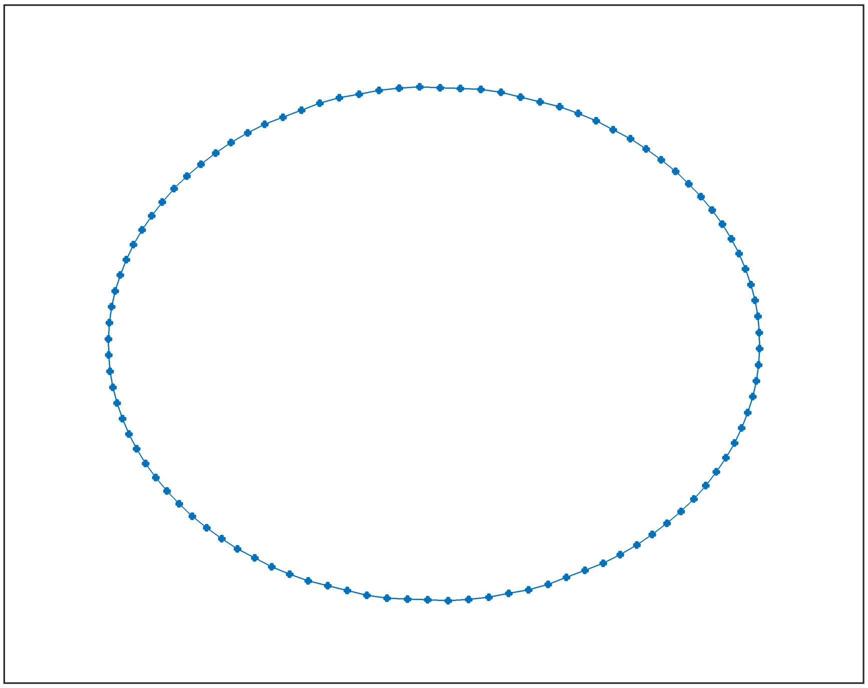
Plot the Cuts

```
SG_plot = plot(SG);

%plot(SG)
highlight(SG_plot,SG_1_nodes,'NodeColor','r');
highlight(SG_plot,SG_2_nodes,'NodeColor','g');
```



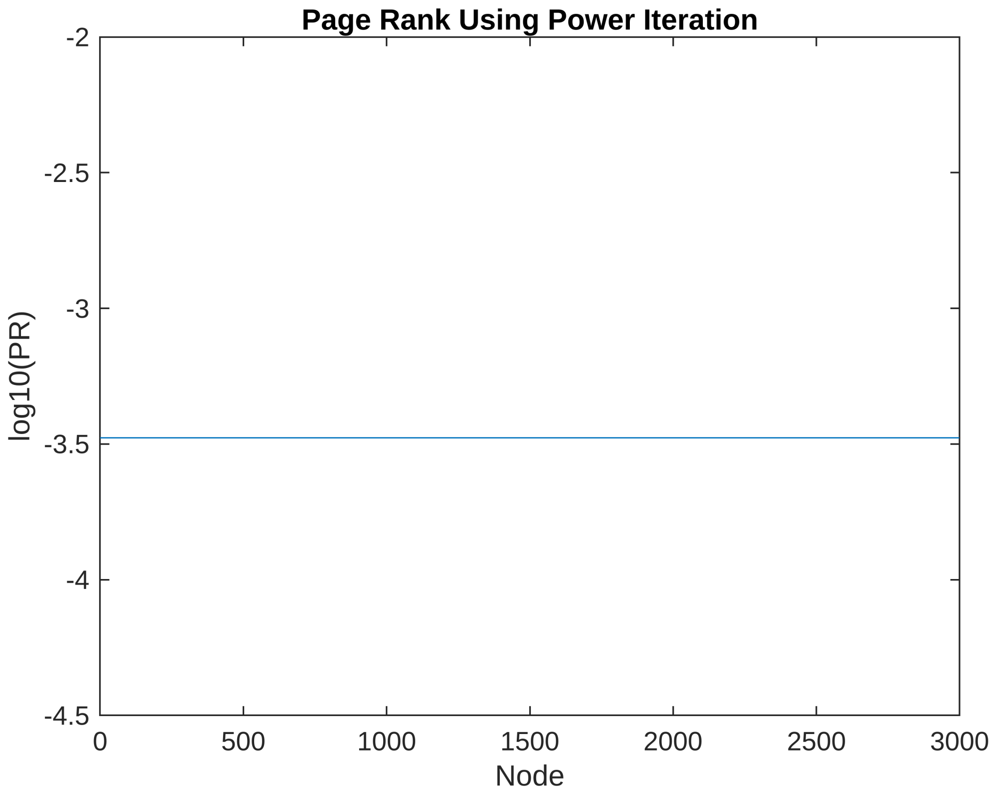
G49.mat

```
load('G49.mat')

Graph_G49 = graph(Problem.A);
plot(Graph_G49)
```



Calculate the Page Rank with Power Iteration

```
% Power Iteration
% Find the page rank of this network using power iteration.

A = Problem.A./sum(Problem.A);

n = length(A);
iterations = 1000;

%Begin power iteration
x0=ones(n,1)/n;

for i = 1:iterations
    y0=A*x0;
    x0=y0./sum(y0);
end

pr_pi = x0./sum(x0);
plot(log10(pr_pi));
ylabel('log10(PR)')
xlabel('Node')
title('Page Rank Using Power Iteration')
```



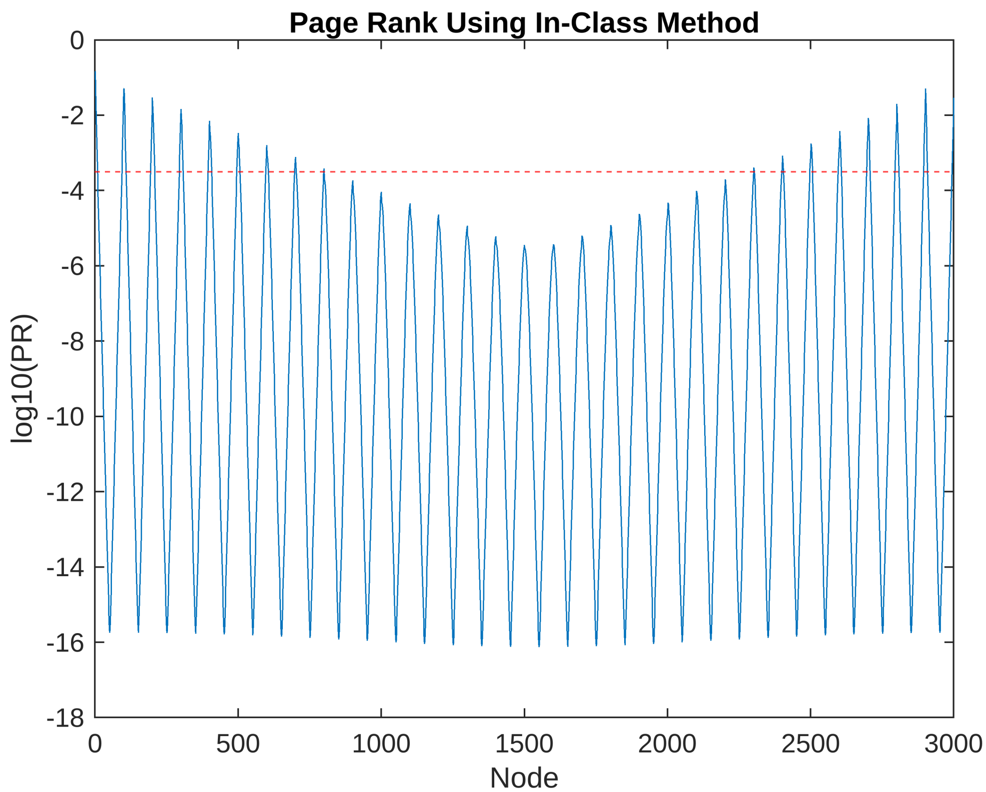

```
9
```

```
ans = 9
```

Calculate PageRank

```
% Solve using the method discussed in class 11/15 11/17

alpha = 0.9;
k = 1000;
n = length(A);

%Make a column vector of 1's normalized to n
v = eye(n, 1) / n;

%Do the zeroth iteration
partial = v;
v_sum = partial;

for i = 1:k
    partial = alpha*A*partial;
    v_sum = v_sum + partial;
end

pr_iter = (1 - alpha)*v_sum;
pr_iter = pr_iter./sum(pr_iter);
```

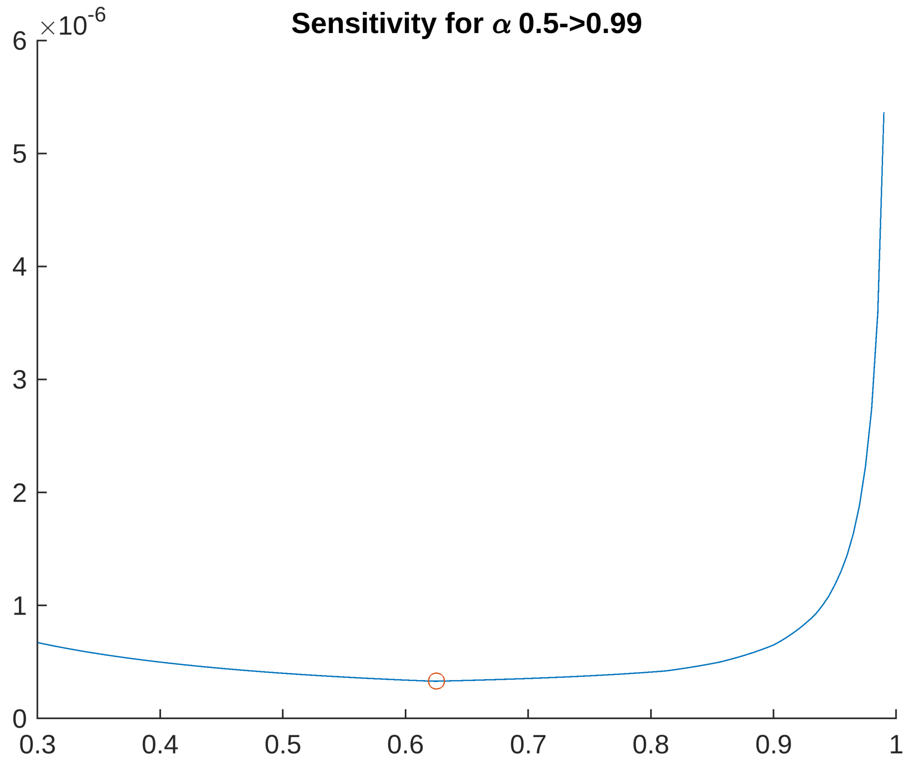
```
plot(log10(pr_iter))
hold on
yline(-3.5, '--r')
hold off
ylabel('log10(PR)')
xlabel('Node')
title('Page Rank Using In-Class Method')
```



Calculate Sensitivity

```
alphas = 0.3:0.005:0.99;
k = 150;
n = length(A);

%Make a column vector of 1's normalized to n
v = eye(n, 1) / n;
count = 1;
s = zeros(n, count);

for alpha = alphas
    partial = A * v;
    v_sum = alpha^(-1) * v + 2 * partial;

    for j = 2:k
        partial = alpha * A * partial;
```

```
        v_sum = v_sum + (j + 1) * partial;
    end

    s(:, count) = v_sum - A * v_sum;
    count = count + 1;
end

sense = mean(abs(s));
[lowest, idx] = min(sense);

figure()
hold on
plot(alphas, sense)
scatter(alphas(idx), lowest, 'o')
title("Sensitivity for \alpha 0.5->0.99")
hold off
```



First 1000 Singular Values

```
%Step 2distendpoint
%Pull the size of the %Find the minimum of S that isn't ~zero
```

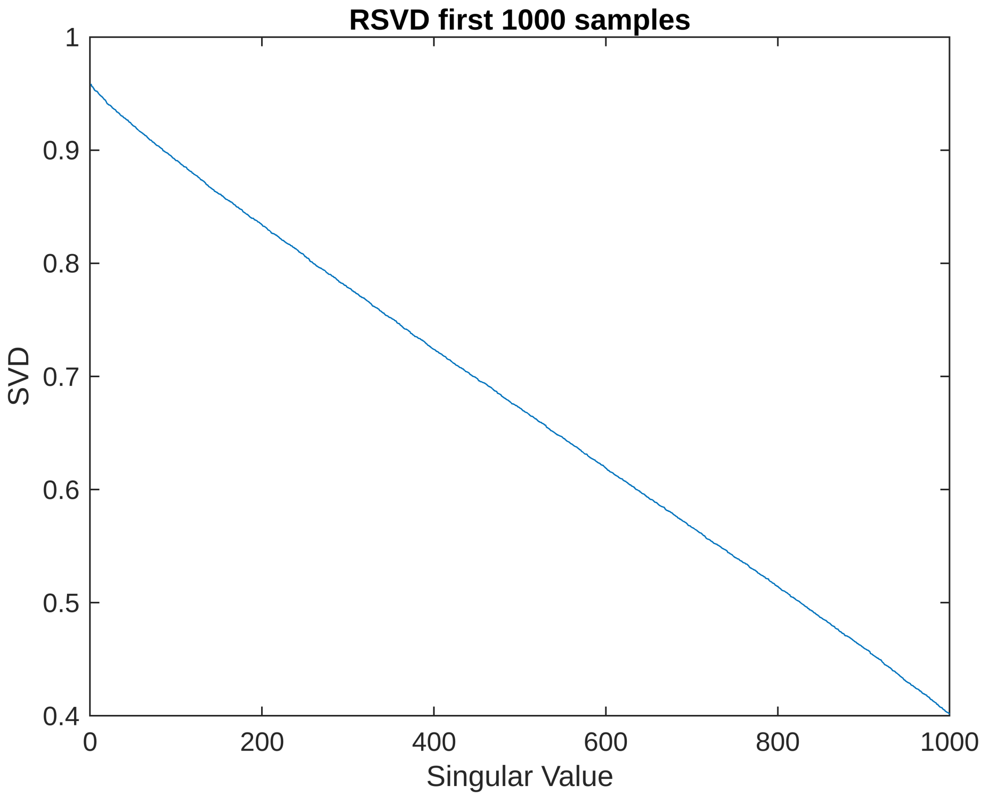
[M, I] = min(s(abs(s) > 1e-15));

```
Asize = size(A);
%m is x axis
```

```
m = Asize(1);
%n is y axis
n = Asize(2);
%Problem says find the first 1000 singular values using RSVD so r is 1100
oversample a bit
r = 1100;
%Create a random matrix that is n x r
X = randn(n,r);
%Multiply the matrices together to get AX
q = orth(A*X);
%Do the QR factorization of AX to AX = QR
QTA = q'*A
```

```
QTA = 1100×3000
   -0.0130    0.0020    0.0063    0.0171   -0.0048    0.0134   -0.0033    0.0092 ···
    0.0048   -0.0086   -0.0046    0.0005   -0.0197    0.0049   -0.0364    0.0106
    0.0083   -0.0072   -0.0071   -0.0072   -0.0116   -0.0046   -0.0019    0.0015
   -0.0073    0.0067   -0.0061   -0.0081    0.0205   -0.0128    0.0156   -0.0200
   -0.0151    0.0155   -0.0023    0.0219    0.0075    0.0033   -0.0063    0.0009
    0.0152   -0.0131    0.0348   -0.0038    0.0205    0.0015    0.0076    0.0008
   -0.0106    0.0089    0.0088   -0.0014   -0.0019    0.0051   -0.0011   -0.0136
    0.0036    0.0315   -0.0037    0.0245   -0.0087    0.0171   -0.0156    0.0148
    0.0430   -0.0274    0.0273   -0.0070    0.0078    0.0137    0.0039    0.0079
   -0.0191    0.0127    0.0038   -0.0048   -0.0105   -0.0145   -0.0231    0.0134
      ⋮
```

```
%Take the SVD of QTA
[uo,so,vot] = svd(QTA,"econ");
%Take the diagonal of the
singvals = diag(so);
%figure(10)
plot(singvals(1:1000))
xlabel('Singular Value')
ylabel('SVD')
title('RSVD first 1000 samples')
```



Path from 1000 to 1100

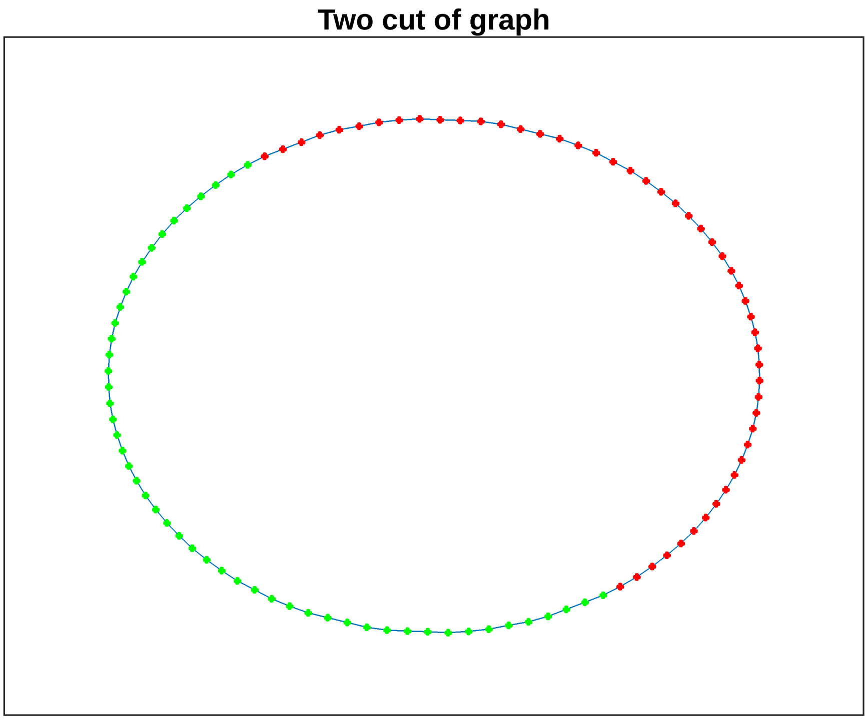
```
src = 1000;
dest = 1100;
n = length(A);

%Make a sparse vector
S = sparse(n,1);

%Index the sparse matrix using the endpoint
S(dest,1) = 1;

%Multiply the matrix by the sparse matrix
PA = A * S;

%Initialize distance
dist = 1;

while PA(src, 1) == 0
    PA = A * PA;
    dist=dist+1;
end
```

```
dist
```

```
dist = 1
```

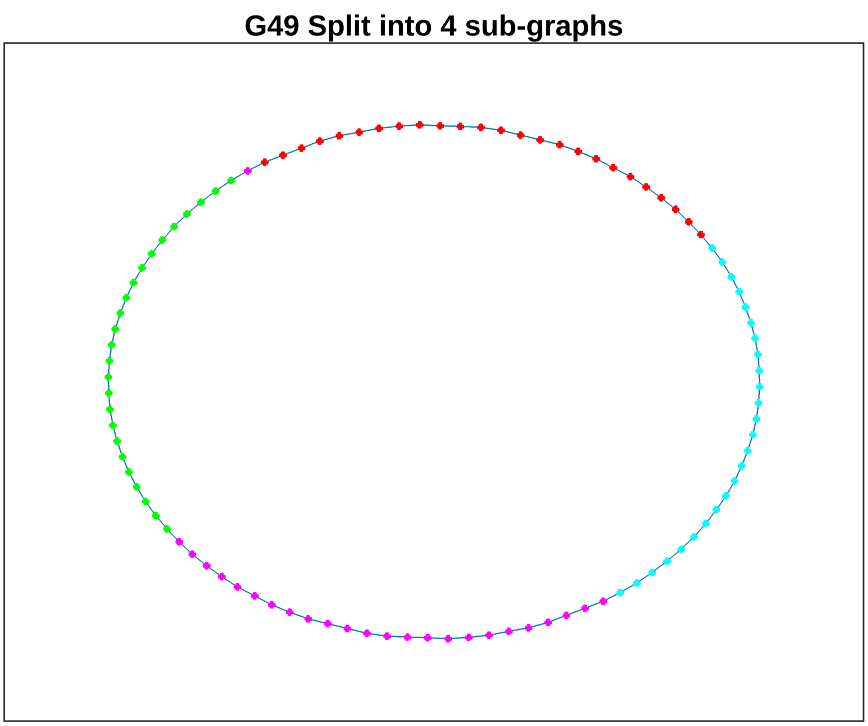
Cut Graph

```

L = laplacian(Graph_G49);
[D, S] = eigs(L, 3000);

Positions = 1:height(Graph_G49.Nodes);

s = diag(S);

%Find the minimum of S that isn't ~zero
[M, I] = min(s(abs(s) > 1e-15));

Cut = D(:,I) > 0;

H1_nodes = Positions(Cut);
H2_nodes = Positions(~Cut);

H1 = rmnode(Graph_G49, H1_nodes);
H2 = rmnode(Graph_G49, H2_nodes);

G_2plot = plot(Graph_G49);
highlight(G_2plot,H1_nodes,'NodeColor','g');
highlight(G_2plot,H2_nodes,'NodeColor','r')
title("Two cut of graph")
```




```
G_plot = plot(Graph_G49);
highlight(G_plot,H1_nodes,'NodeColor','g');
highlight(G_plot,H2_nodes,'NodeColor','r')

% Split the first half in two
L1 = laplacian(H1);
n1 = height(H1.Nodes);
[D1, S1] = eigs(L1, n1);

Pn1 = 1:n1;

s1 = diag(S1);

%Find the minimum of S that isn't ~zero
[M1, I1] = min(s1(abs(s1) > 1e-10));

Cut1 = D1(:,I1) > 0;

H11 = rmnode(H1, Pn1(~Cut1));
H12 = rmnode(H1, Pn1(Cut1));

H11_nodes = H1_nodes(Cut1);
H12_nodes = H1_nodes(~Cut1);

highlight(G_plot,H11_nodes,'NodeColor','magenta');
```

```
% Split the second half in two
L2 = laplacian(H2);
n2 = height(H2.Nodes);
[D2, S2] = eigs(L2, n2);

Pn2 = 1:n2;

s2 = diag(S2);

%Find the minimum of S that isn't ~zero
[M2, I2] = min(s2(abs(s2) > 1e-10));

Cut2 = D1(:,I2) > 0;

H21 = rmnode(H2, Pn2(~Cut2));
H22 = rmnode(H2, Pn2(Cut2));

H21_nodes = H2_nodes(Cut2);
H22_nodes = H2_nodes(~Cut2);

highlight(G_plot,H21_nodes,'NodeColor','cyan');
title("G49 Split into 4 sub-graphs")
```



Find the Dirichlet PageRank

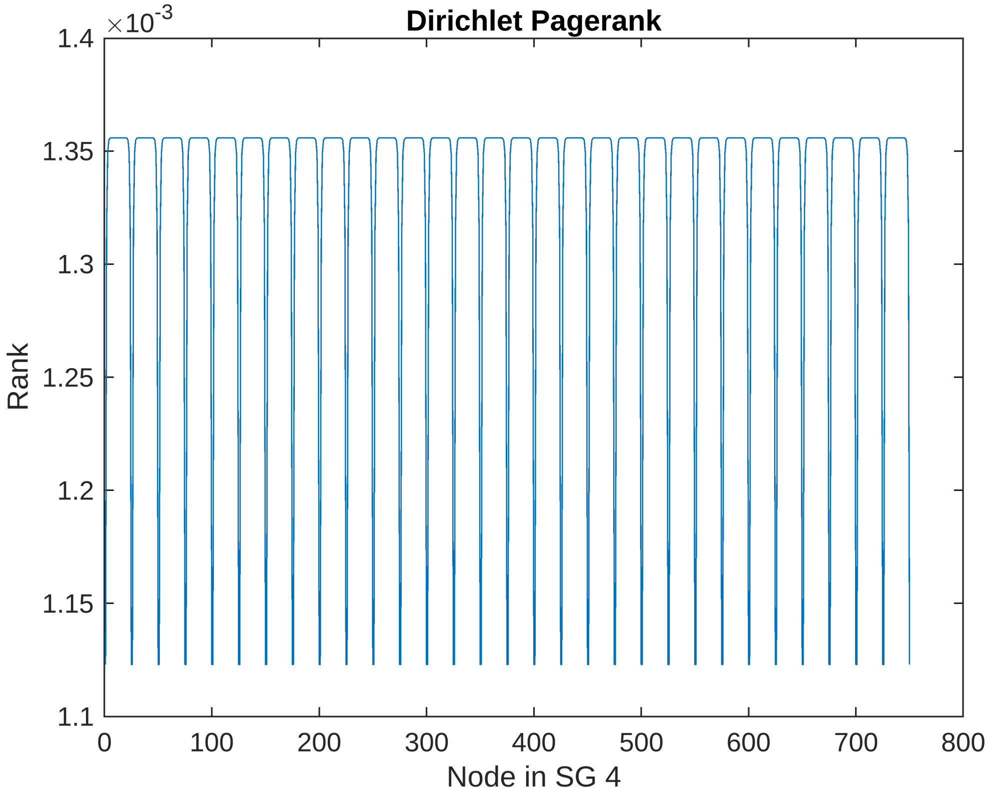
```
An_G49 = full(Problem.A ./ sum(Problem.A))
```

```
An_G49 = 3000×3000
         0    0.2500         0         0         0         0         0         0 ···
    0.2500         0    0.2500         0         0         0         0         0
         0    0.2500         0    0.2500         0         0         0         0
         0         0    0.2500         0    0.2500         0         0         0
         0         0         0    0.2500         0    0.2500         0         0
         0         0         0         0    0.2500         0    0.2500         0
         0         0         0         0         0    0.2500         0    0.2500
         0         0         0         0         0         0    0.2500         0
         0         0         0         0         0         0         0    0.2500
         0         0         0         0         0         0         0         0
        ⋮
```

```
n = length(H22_nodes);
alpha = 0.5;

A21 = An_G49(H22_nodes,H22_nodes);
A22 = An_G49(H21_nodes,H21_nodes);

Adir = [eye(750,750),zeros(750,750);
        A22,A21];

Adir = Adir./sum(Adir);

xdir=inv(eye(n) - alpha*A22) * ((1-alpha)*ones(n,1)+alpha*A21*ones(n,1)/n)
```

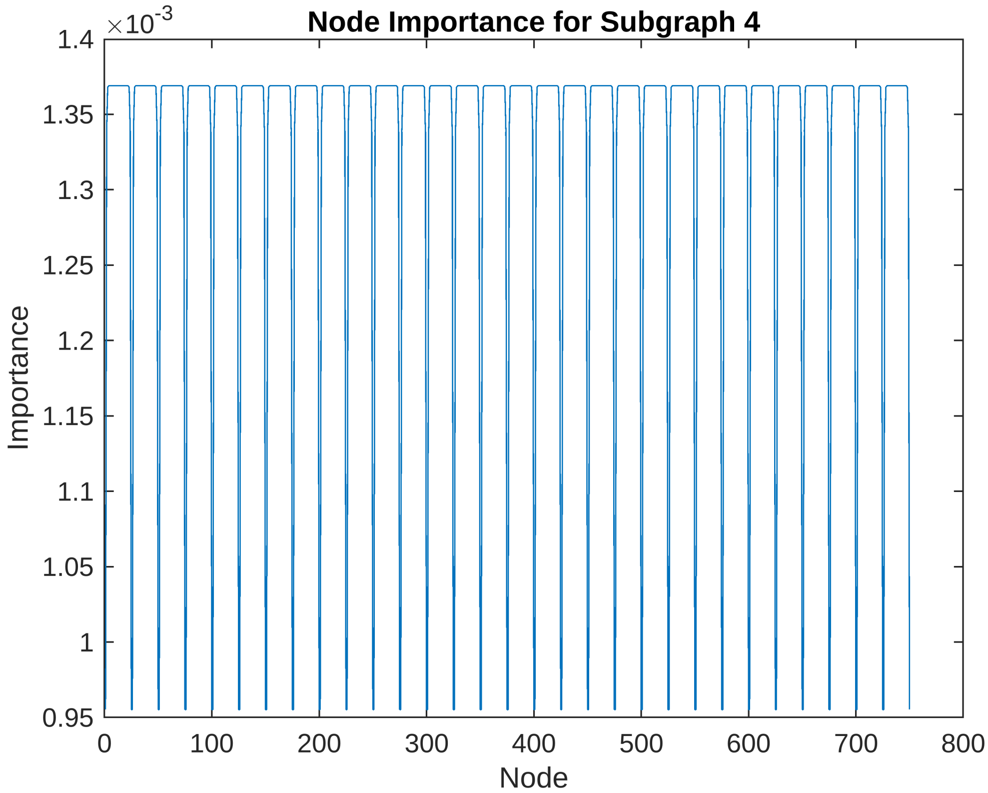
```
xdir = 750×1
    0.8293
    0.9718
    0.9963
    1.0005
    1.0012
    1.0013
    1.0013
    1.0013
    1.0013
    1.0013
        ⋮
```

```
xdir = xdir ./ sum(xdir);
plot(xdir)
title("Dirichlet Pagerank")
xlabel("Node in SG 4")
ylabel("Rank")
```



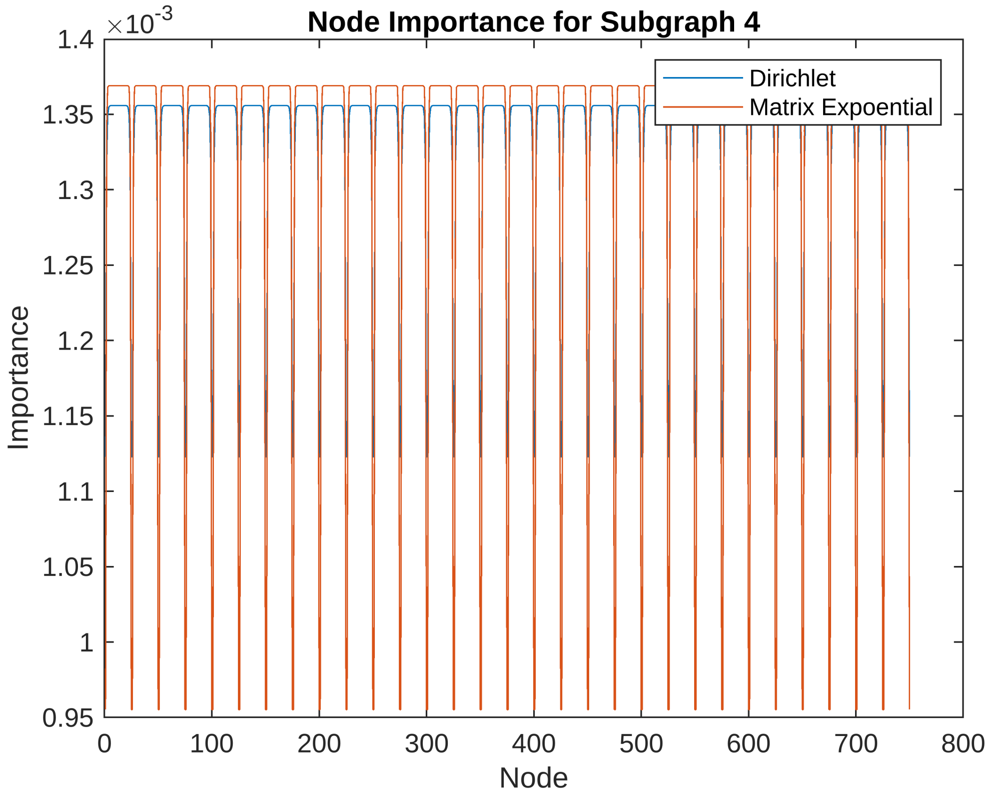
Centrality of Nodes

```
A_small = adjacency(H22);
Central = expm(A_small);
central_nodes = diag(Central);
central_nodes = central_nodes ./ sum(central_nodes);

plot(central_nodes)
xlabel("Node")
ylabel("Importance")
title("Node Importance for Subgraph 4")
```



Comparing Centralities

```
plot(xdir)
hold on
plot(central_nodes)
hold off
xlabel("Node")
ylabel("Importance")
title("Node Importance for Subgraph 4")
legend(["Dirichlet", "Matrix Expoential"])
```


Node Importance for Subgraph 4
×10^-3
1.4
1.35
1.3
1.25
1.2
1.15
1.1
1.05
1
0.95
Importance
0
100
200
300
400
500
600
700
800
Node
Dirichlet
Matrix Expoential