Logistic Regression by C.Looney

What is Logistic Regression?

Logistic Regression is a “Supervised machine learning” algorithm that can be used to model the probability of a certain class or event. It is used when the data is linearly separable and the outcome is binary or dichotomous in nature.

That means Logistic regression is usually used for Binary classification problems.

Binary Classification refers to predicting the output variable that is discrete in **two** classes.

This article was published as a part of the Data Science Blogathon

Overview:

In this article, we will learn the in-depth working and implementation of **Logistic Regression** in Python using the Scikit-learn library.

Topics covered:

1. What is Logistic Regression?
2. Types of Logistic Regression
3. Extensions of Logistic Regression
4. Use Linear Regression for classification
5. How does Logistic Regression work?
6. Implementation in Python using Scikit-learn library

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A few examples of Binary classification are Yes/No, Pass/Fail, Win/Lose, Cancerous/Non-cancerous, etc.

Types of Logistic Regression

- **Simple Logistic Regression:** a single independent is used to predict the output
- **Multiple logistic regression:** multiple independent variables are used to predict the output

Extensions of Logistic Regression

Although it is said Logistic regression is used for Binary Classification, it can be extended to solve multiclass classification problems.

Multinomial Logistic Regression: The output variable is discrete in three or more classes with no natural ordering.

Food texture: Crunchy, Mushy, Crispy

Hair colour: Blonde, Brown, Brunette, Red

Ordered Logistic Regression: Aka Ordinal regression model. The output variable is discrete in three or more classes with the ordering of the levels.

Customer Rating: extremely dislike, dislike, neutral, like, extremely like

Income level: low income, middle income, high income

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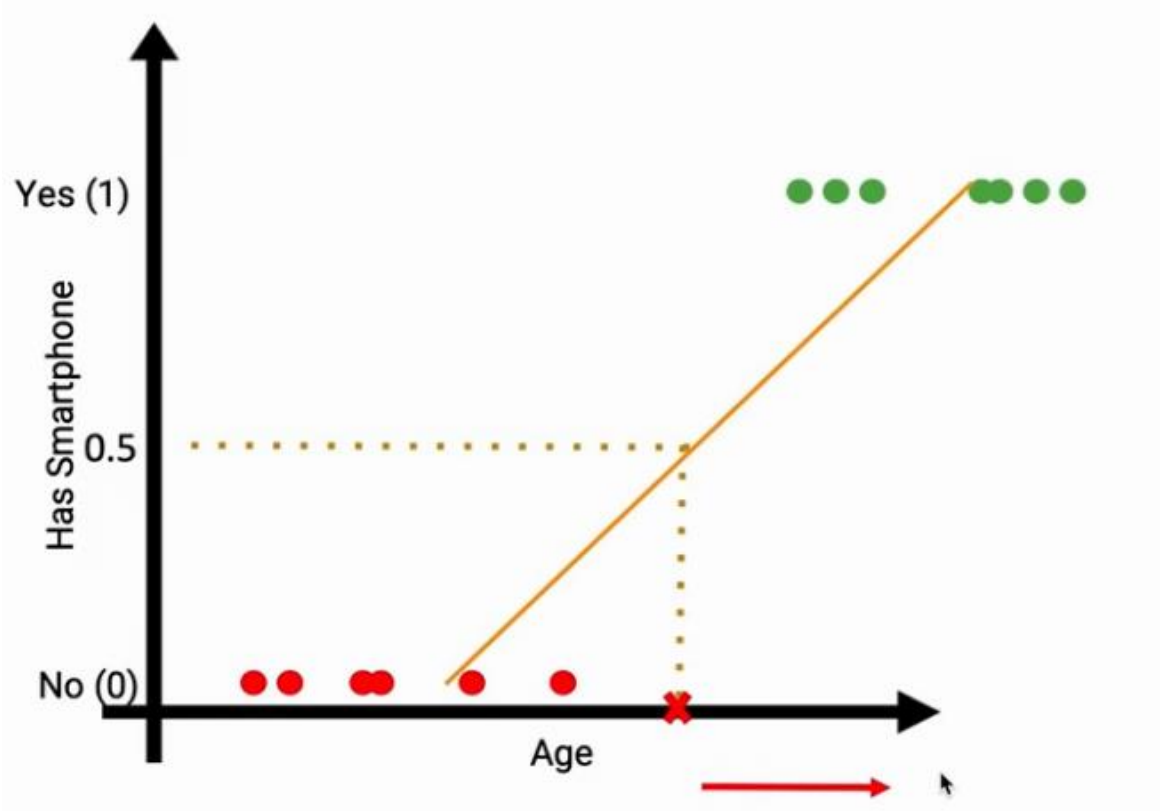

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Case 1

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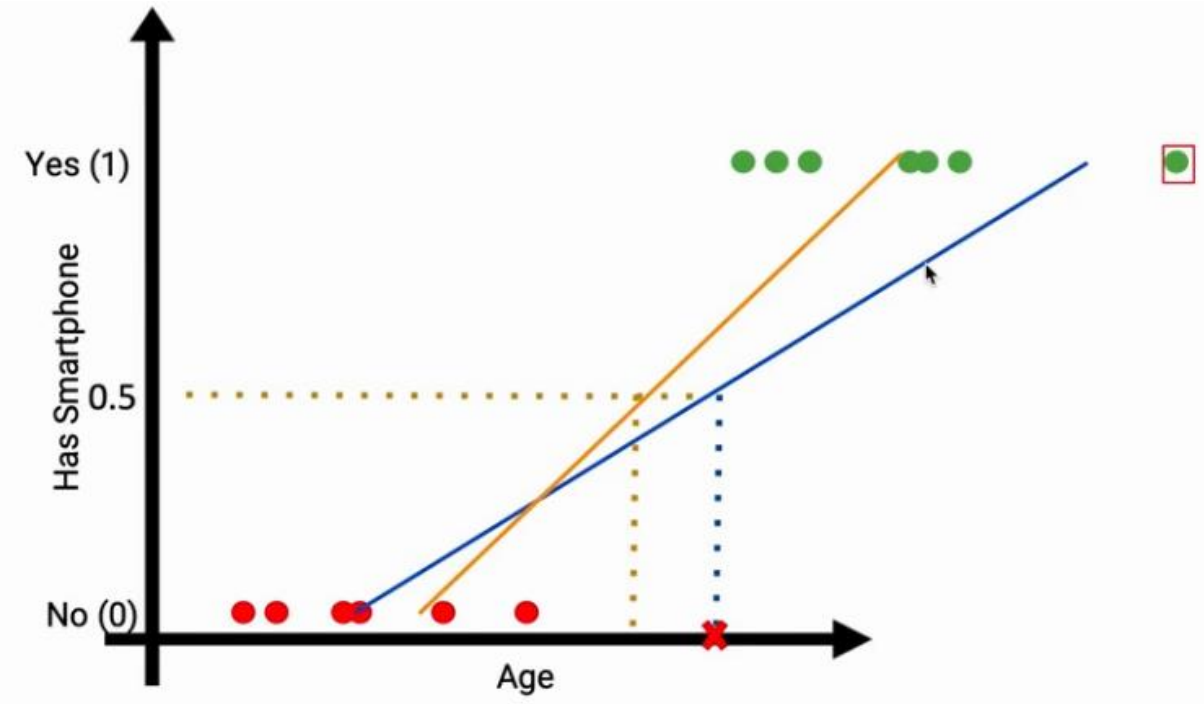

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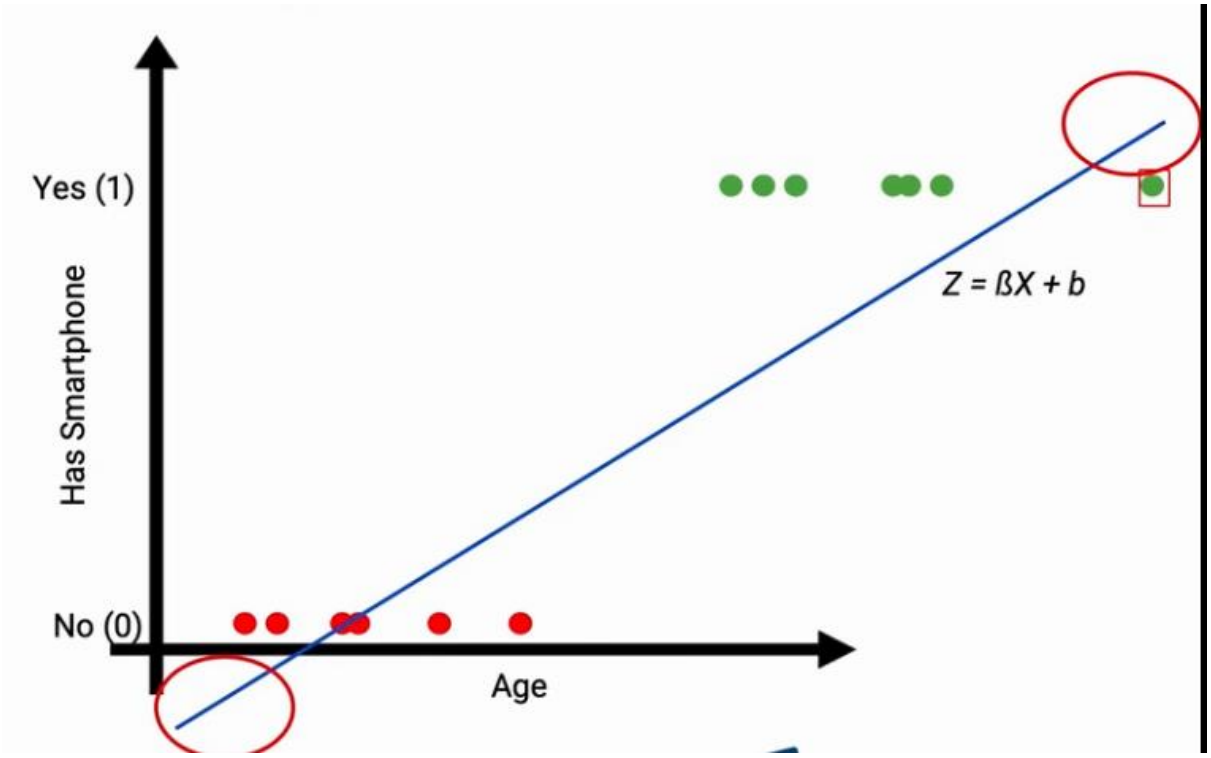
The other issue with Linear regression is when you extend this line it will give you values above 1 and below 0. In our classification problem, we do not know what the values greater than one and below 0 represents. so it is not the natural extension of the linear model. Further, it makes the model interpretation at extremes a challenge.



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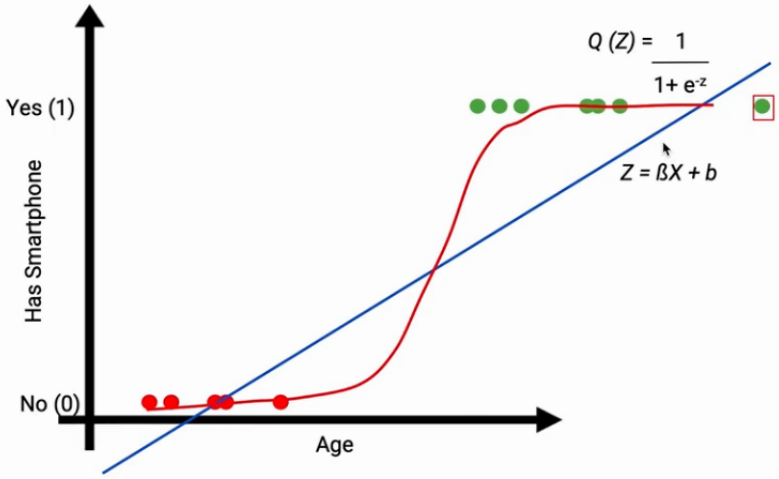

This sigmoid function transforms the linear line into a curve. This will constraint the values between 0 and 1. Now it doesn't matter how many new points I add to each extreme it will not affect my model.

The other important aspect is, for each observation model will give a continuous value between 0 and 1. This continuous value is the prediction probability of that data point. If the prediction probability is near 1 then the data point will be classified as 1 else 0.

The cost function for Logistic regression

For linear regression, the cost function is mostly we use Mean squared error represented as the difference y_predicted and y_actual iterated overall data points, and then you do a square and take the average. It is a convex function as shown below. This cost function can be optimized easily using gradient descent.

Linear Regression
Cost Function

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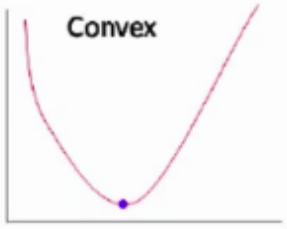

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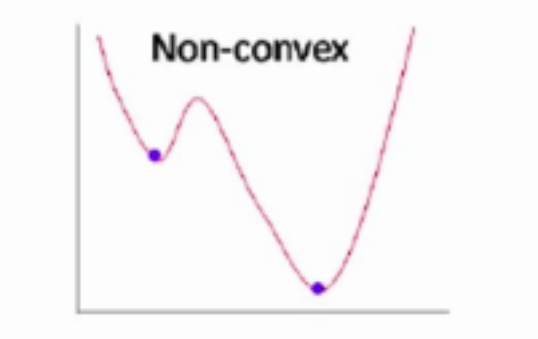

Hence, we need a different cost function for our new model. Here comes the log loss in the picture. As you can see, we have replaced the probability in the log loss equation with y_hat

$$\text{Log loss} = \frac{1}{N} \sum_{i=1}^{N} -(y_i * \log(p_i) + (1-y_i) * \log(1-p_i))$$

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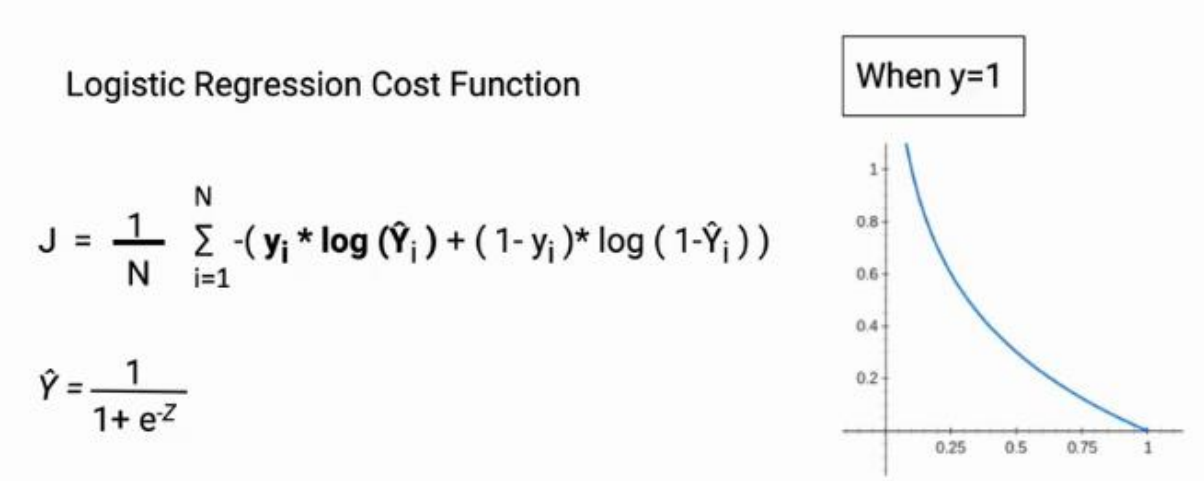

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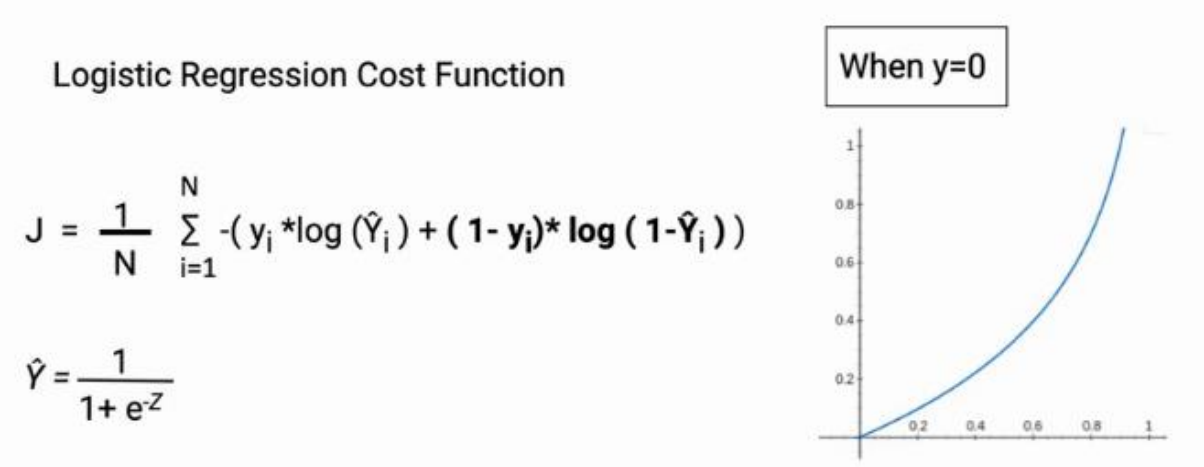


Log Loss

Introduction to Loss function

Before jumping into Log Loss let's first understand what is Loss function. Imagine the scenario, Once you developed your machine learning model that you believe, successfully identifying the cats and dogs but how do you know this is the best result?

Here, we are looking for the metrics or a function that we can use to optimize our model performance. The loss function tells how good your model is in predictions. If the model predictions are closer to the actual values the Loss will be minimum and if the predictions are totally away from the original values the loss value will be the maximum.

In mathematical connotations

Loss= abs(Y_pred – Y_actual)

On the basis of the Loss value, you can update your model until you get the best result.

In this article, we will specifically focus on Binary Cross Entropy also known as Log loss, it is the most common loss function used for binary classification problems.

What is Binary Cross Entropy Or Logs Loss?

Binary cross entropy compares each of the predicted probabilities to actual class output which can be either 0 or 1. It then calculates the score that penalizes the probabilities based on the distance from the expected value. That means how close or far from the actual value.

Let's first get a formal definition of binary cross-entropy

Binary Cross Entropy is the negative average of the log of corrected predicted probabilities.

Objective

- Why the Linear regression model will not work for classification problems.
- How Logistic regression model is derived from a simple linear model.

Introduction

While working with the machine learning models, one question that generally comes into our mind for a given problem whether I should use the regression model or the classification model.

Regression and Classification both are supervised learning algorithms. In regression, the predicted values are of continuous nature and in classification predicted values are of a categorical type.

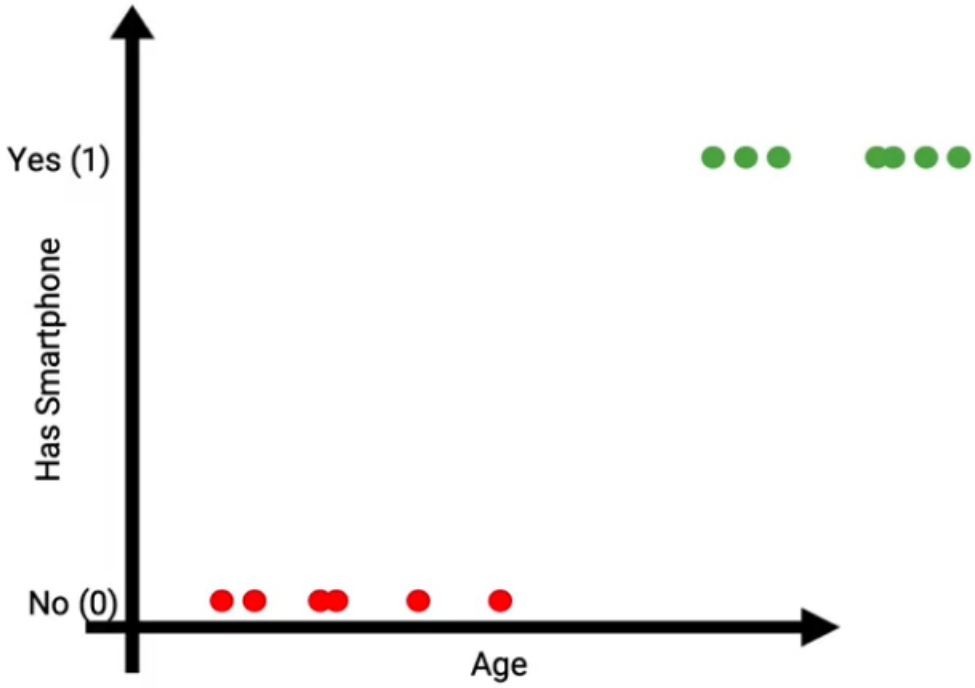
Note: If you are more interested in learning concepts in an Audio-Visual format, We have this entire article explained in the video below. If not, you may continue reading.

In simple terms, If you have a dataset with marks of a student in five subjects and you have to predict the marks in another subject it will be a regression problem. On the other hand, if I ask you to predict whether a student is pass or fail based on the marks it will be a classification problem.

Now let's talk about Logistic regression. What do you think of what kind of algorithm is Logistic regression? A classification or a regression one.

To our surprise, Logistic regression is actually a classification algorithm. Now you must be wondering if it is a classification algorithm why it is called regression.

Let's consider a small example, here is a plot on the x-axis Age of the persons, and the y-axis shows they have a smartphone. It is a classification problem where given the age of a person and we have to predict if he posses a smartphone or not.



In such a classification problem, can we use linear regression?

Issues with Linear regression

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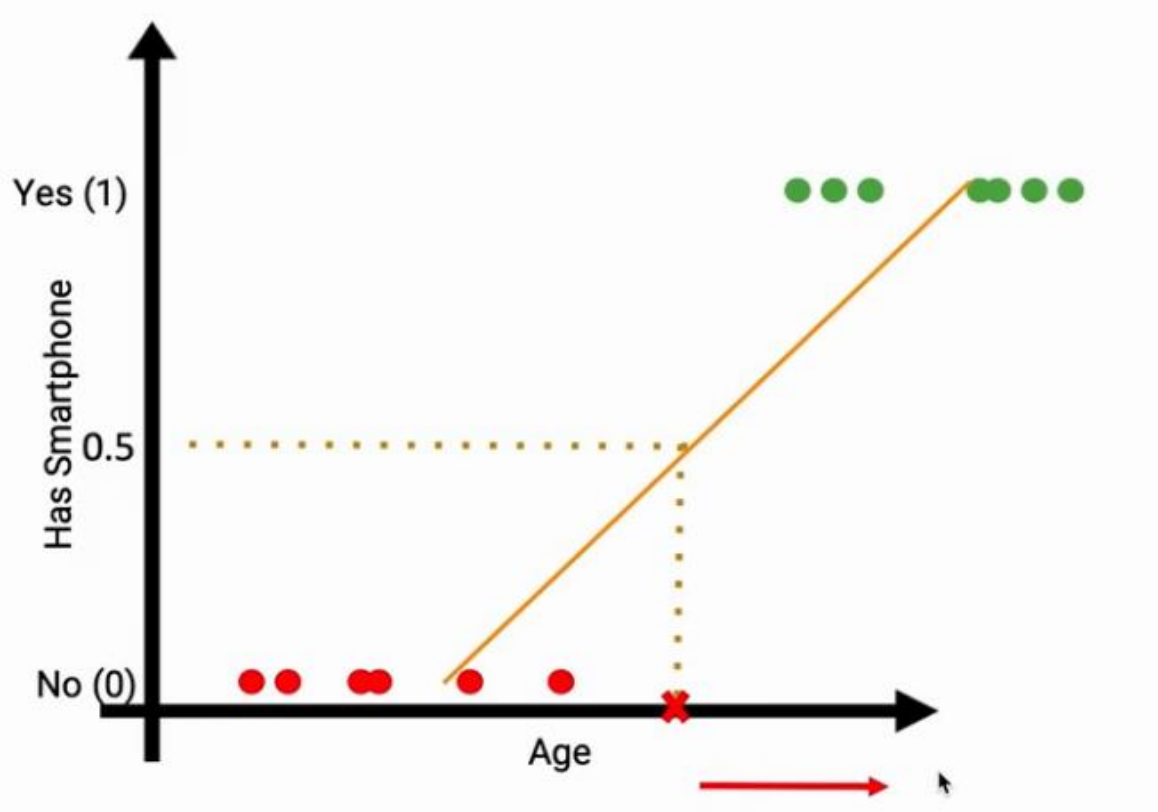

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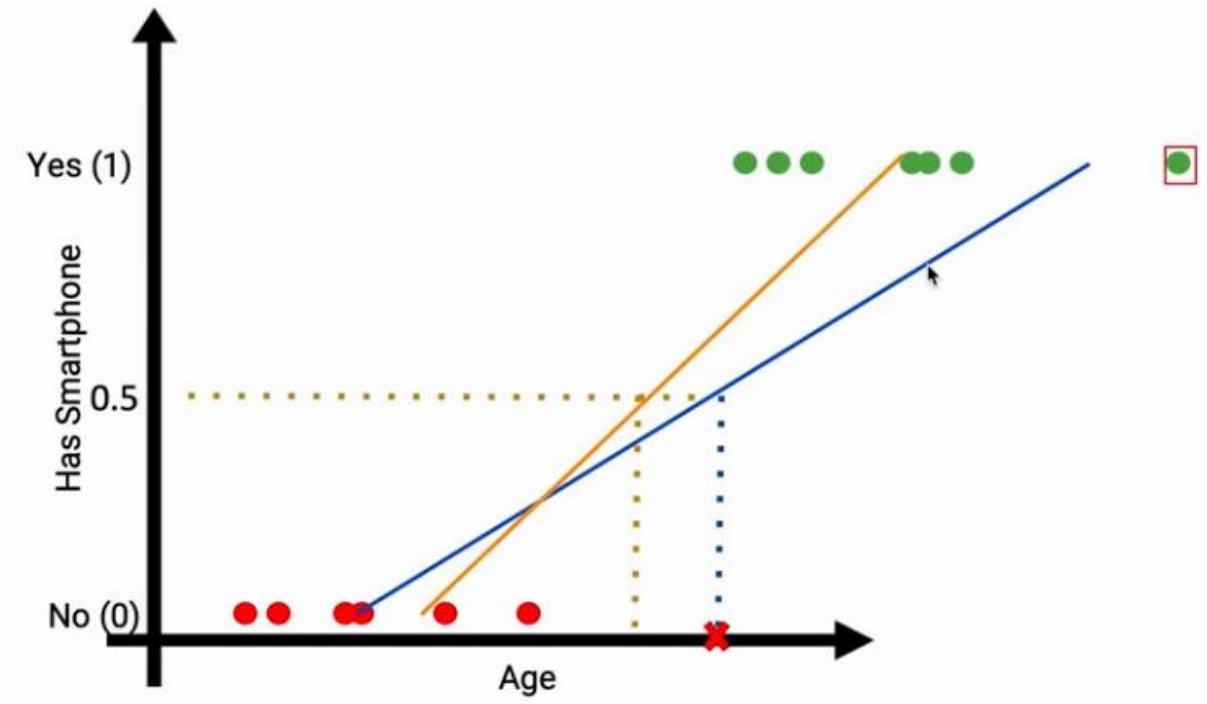

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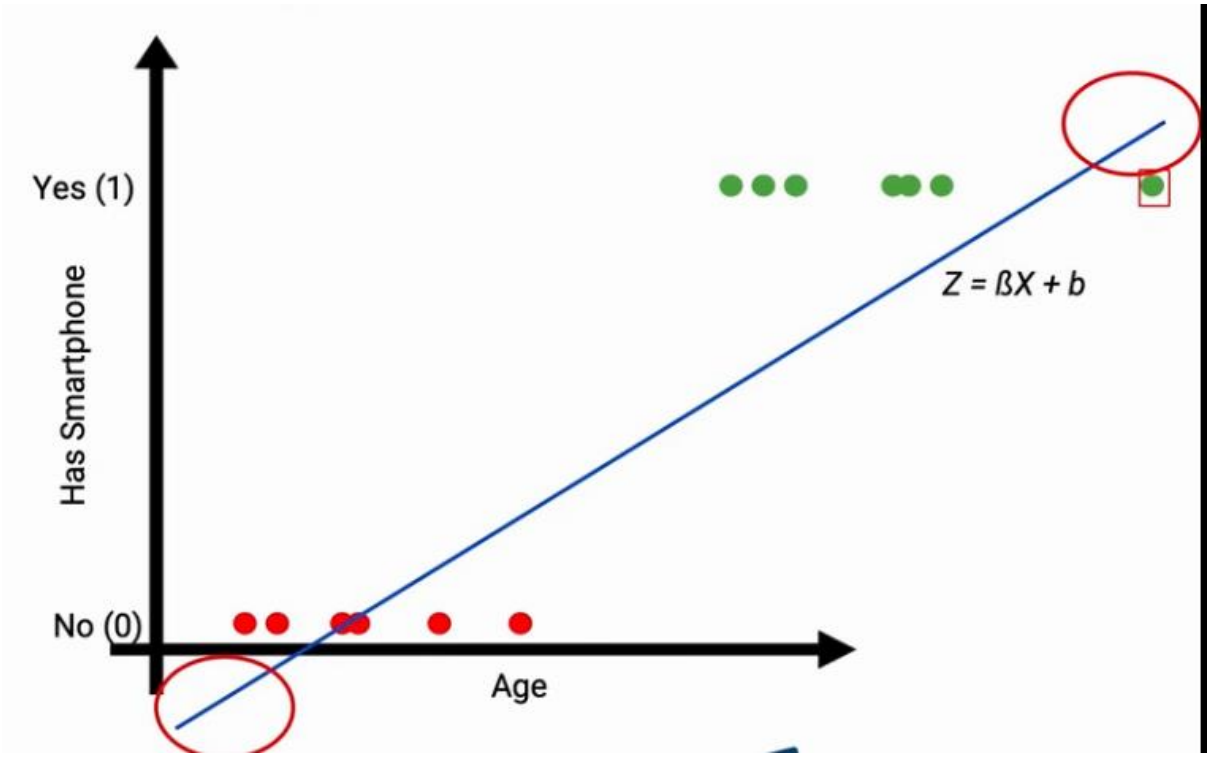

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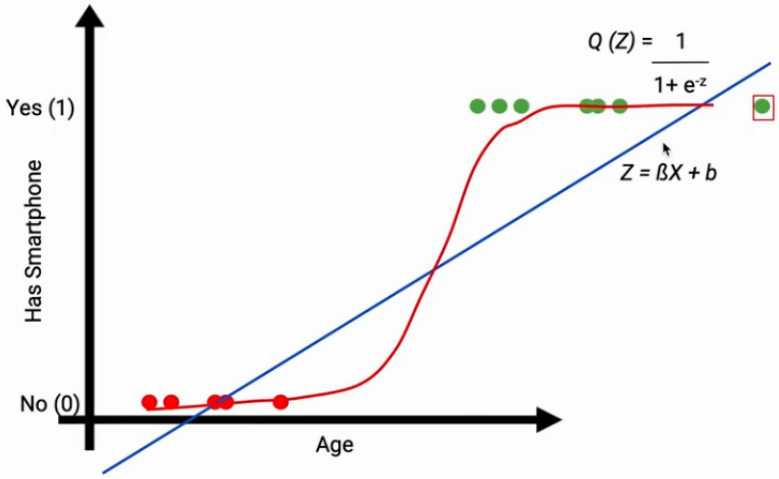

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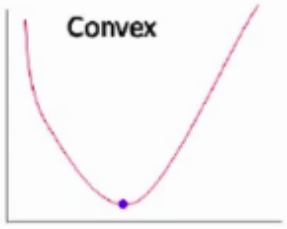

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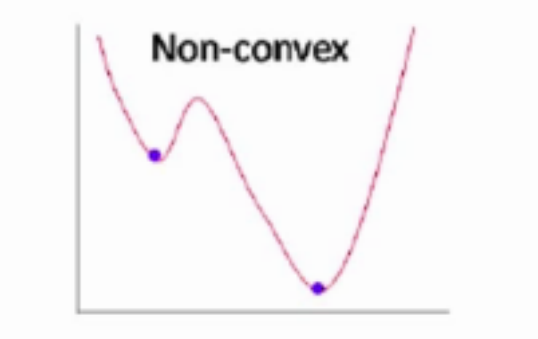

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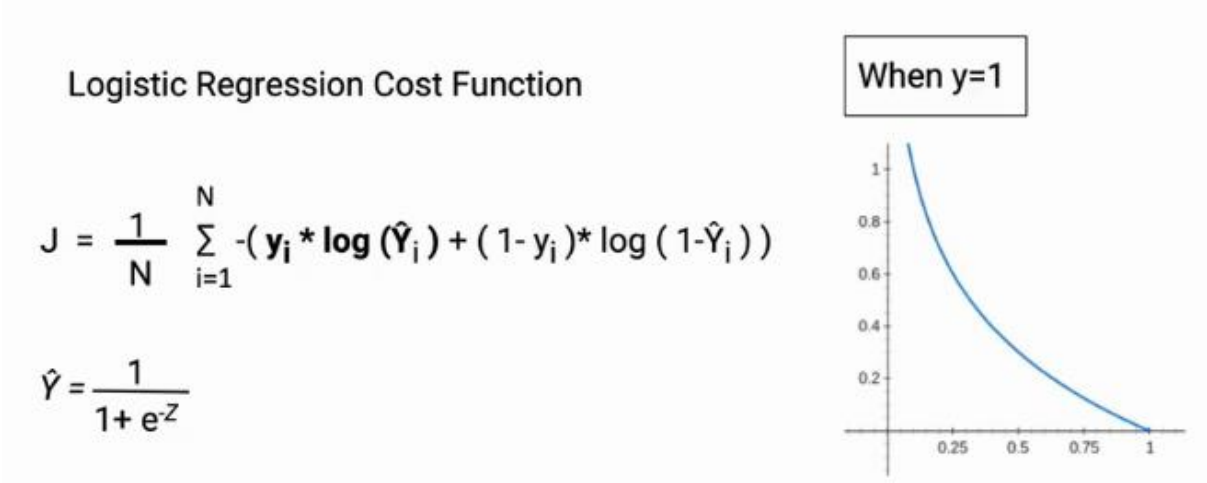

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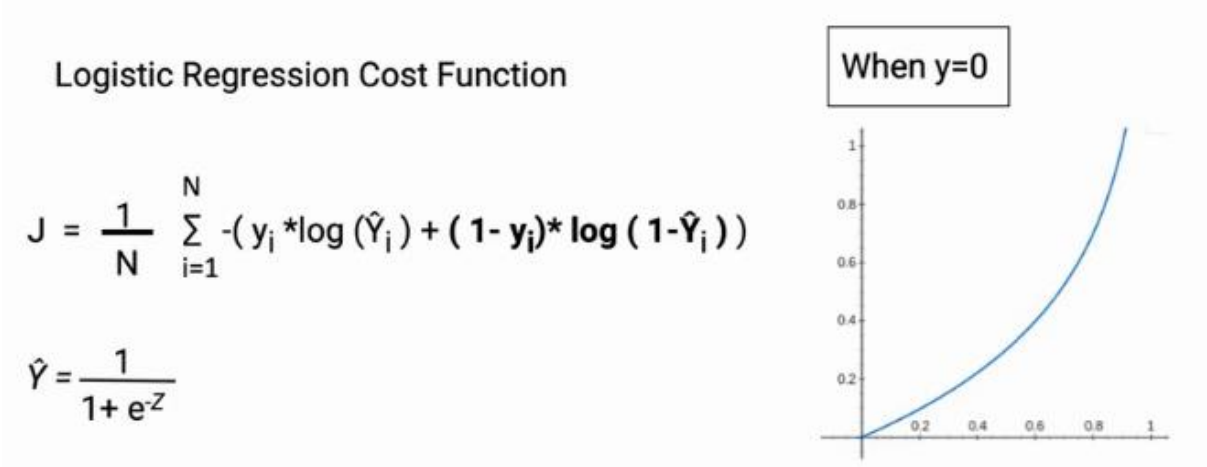

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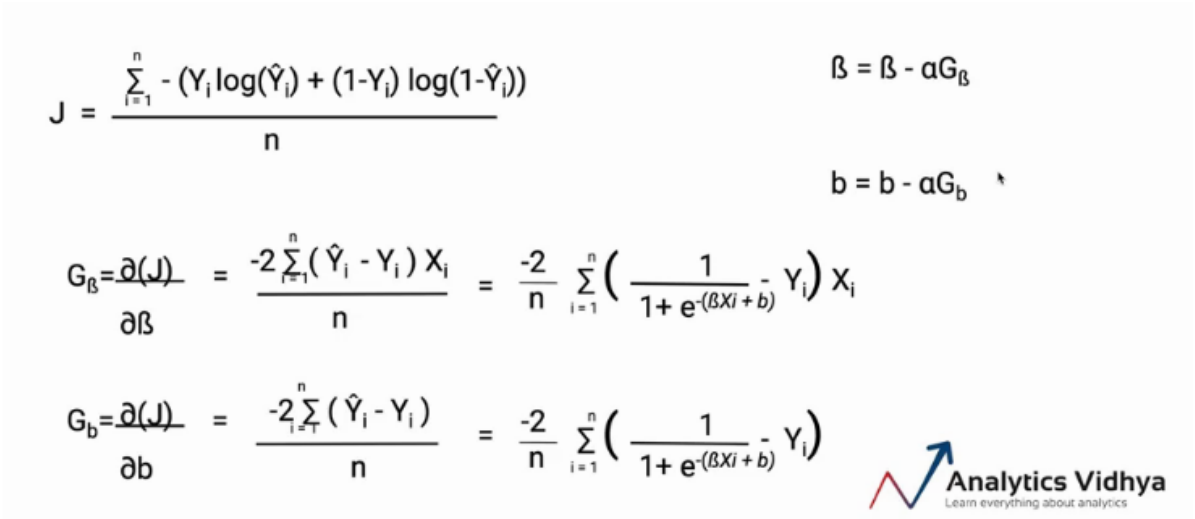
Optimize the model

Once we have our model and the appropriate cost function handy, we can use **"The Gradient Descent Algorithm"** to optimize our model parameters. As we do in the case of linear regression.

Although gradient descent is a separate topic still I will quickly explain it as shown in the following image. We have the cost function J and the parameters B and b. Now we can differentiate the cost function J with parameters B and b. Once we have our values of GB and Gb, we can update our parameters.



The whole process will go iteratively until we get our best parameter