Week 12 Thursday

L-22

Radiation pressure

$$\frac{dM_r(r)}{dr} = 4\pi r^2 \rho(r)$$

$$\frac{dP(r)}{dr} = -\rho(r)\frac{GM_r(r)}{r^2}$$

$$\frac{dT(r)}{dr} = -\frac{3\kappa_R(r)\rho(r)}{64\pi\sigma}\frac{L_r(r)}{r^2 T(r)^3}$$

$$\frac{dL_r(r)}{dr} = 4\pi r^2 \rho(r)\epsilon(r)$$

$$M_r(r) \quad P(r) \quad L_r(r) \quad T(r)$$

$$\rho(r) \quad \mu(r) \quad \epsilon_{\text{nuc}}(r) \quad \kappa_R(r)$$

$$P(r) = \frac{\rho(r)}{\mu(r)} kT(r)$$

$$\mu(r) = f(\text{comp}, T(r), P(r))$$

$$\kappa_R(r) = f(\text{comp}, T(r), P(r))$$

$$\epsilon_{\text{nuc}}(r) = f(\text{comp}, T(r), P(r))$$

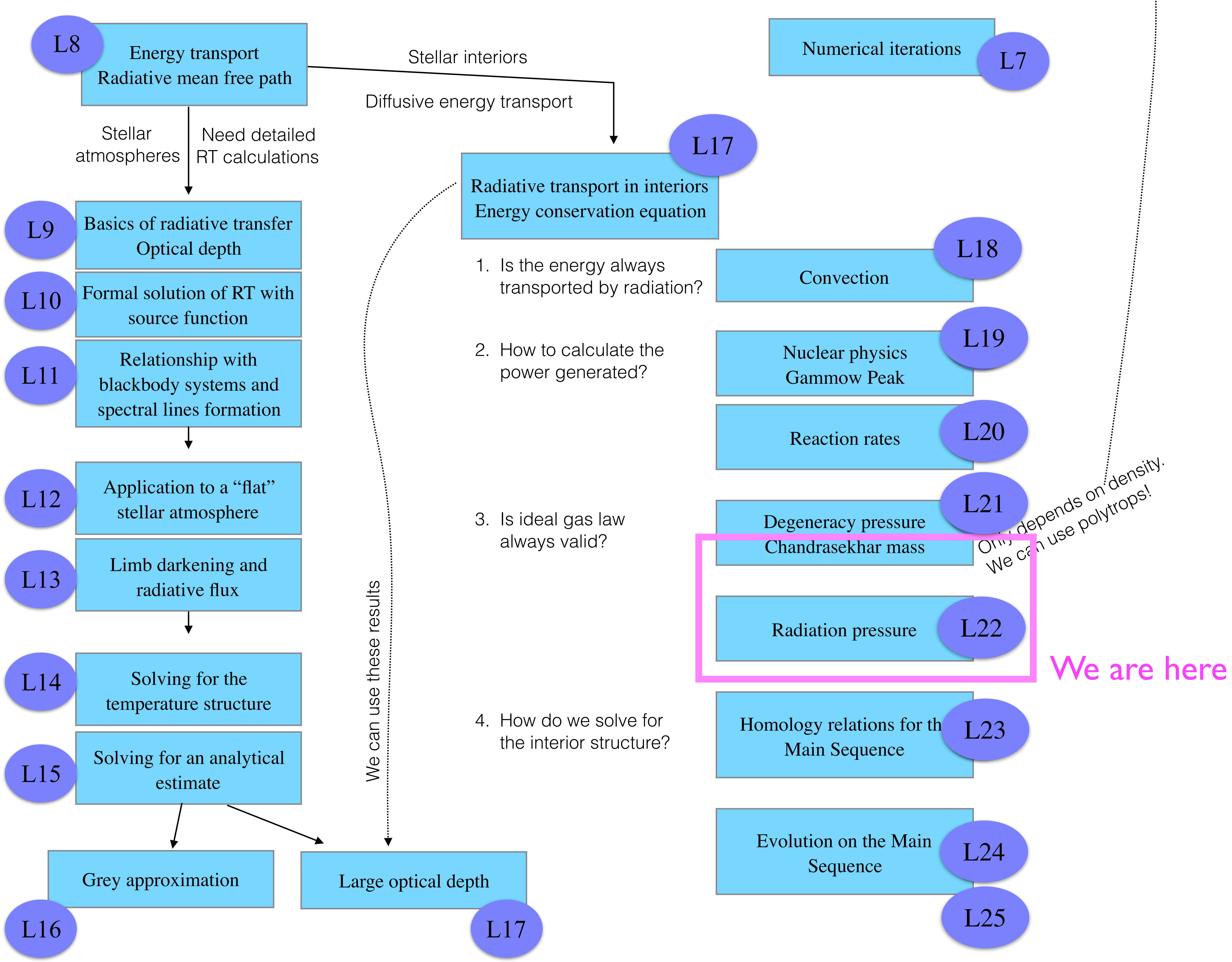
Other energy transport?

Ideal gas always valid?

Nuclear mechanism?

How to solve?

What changes with time?



L8
Energy transport
Radiative mean free path
Stellar interiors
Diffusive energy transport
Stellar atmospheres
Need detailed RT calculations
L7
Numerical iterations
L17
Radiative transport in interiors
Energy conservation equation
L9
Basics of radiative transfer
Optical depth
L10
Formal solution of RT with source function
L11
Relationship with blackbody systems and spectral lines formation
L12
Application to a "flat" stellar atmosphere
L13
Limb darkening and radiative flux
L14
Solving for the temperature structure
L15
Solving for an analytical estimate
Grey approximation
L16
Large optical depth
L17
We can use these results
1. Is the energy always transported by radiation?
L18
Convection
2. How to calculate the power generated?
L19
Nuclear physics
Gammow Peak
L20
Reaction rates
3. Is ideal gas law always valid?
L21
Degeneracy pressure
Chandrasekhar mass
depends on density.
use polytrops!
L22
Radiation pressure
We are here
4. How do we solve for the interior structure?
L23
Homology relations for th
Main Sequence
L24
Evolution on the Main Sequence
L25

Pressure is a momentum flux:
intermediate step from last lecture

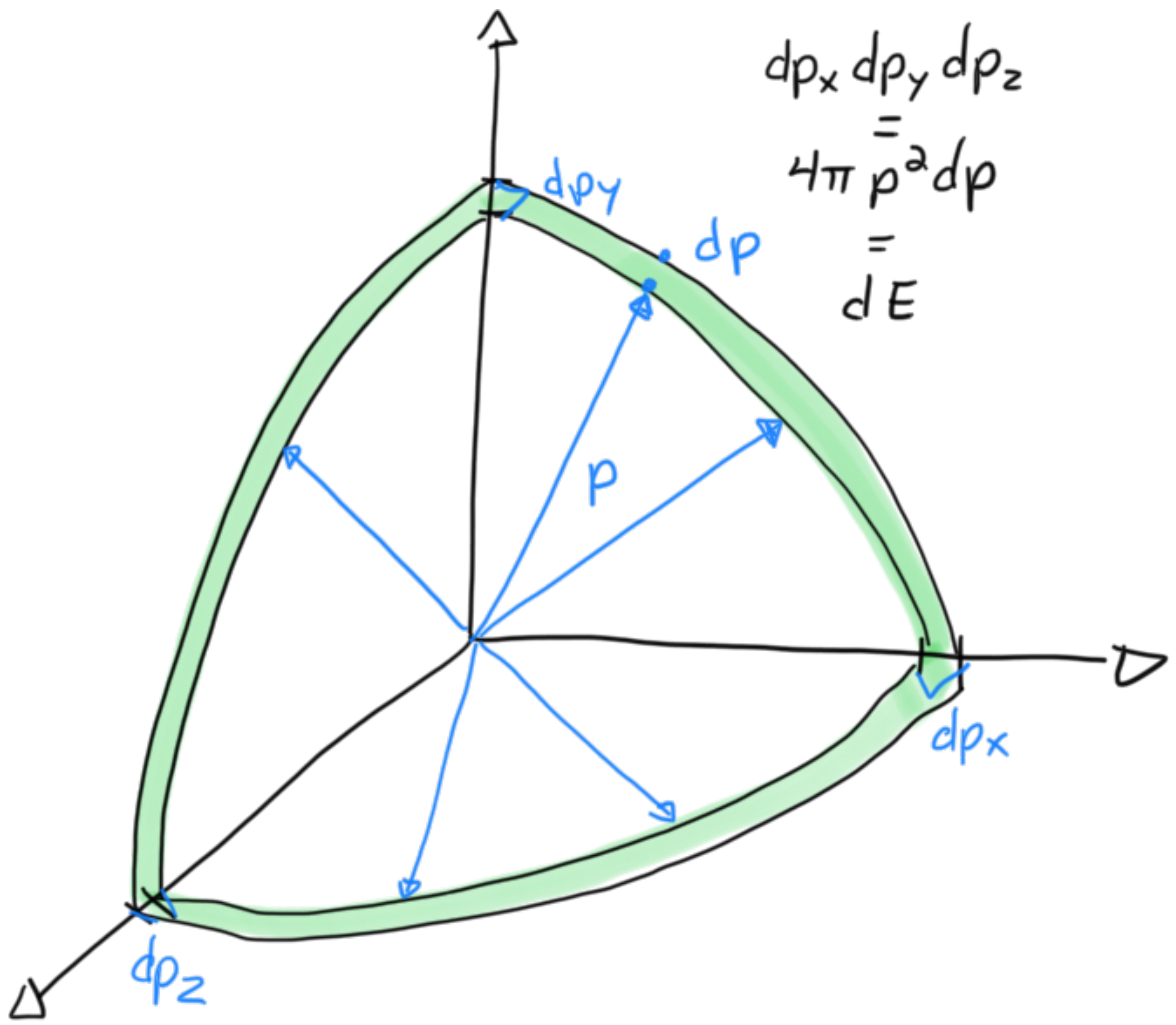
$$P = \frac{1}{3}\int_0^{p_F} p\ v(p)\ n(p)\ 4\pi p^2\ dp$$

$v(p) = c$

Adapt this for photons:

Not degenerate, no limit on momentum

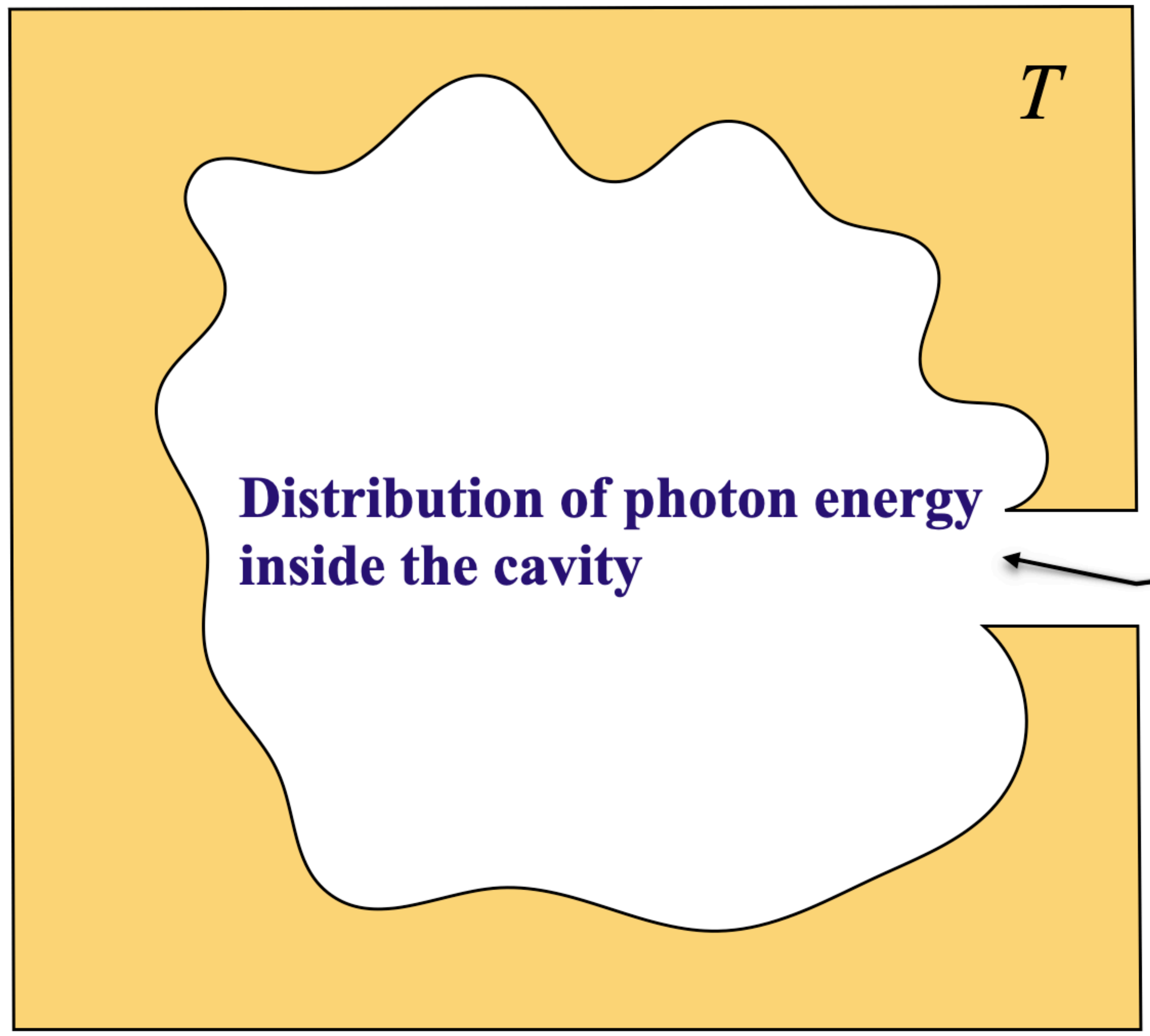
$$P = \frac{1}{3}\int_0^{\infty} p\ c\ n(p)\ 4\pi p^2\ dp$$



pc is also the energy of a photon.
We can relate this to the energy density of a “gas” of photons

$$P = \frac{1}{3}\int_0^{\infty} E\ n(E)\ dE$$

Remember that $n(p)$ is the number of particle per volume with momentum between p and $p + dp$
So $n(E)$ is the number of particle per volume with energy between E and $E + dE$

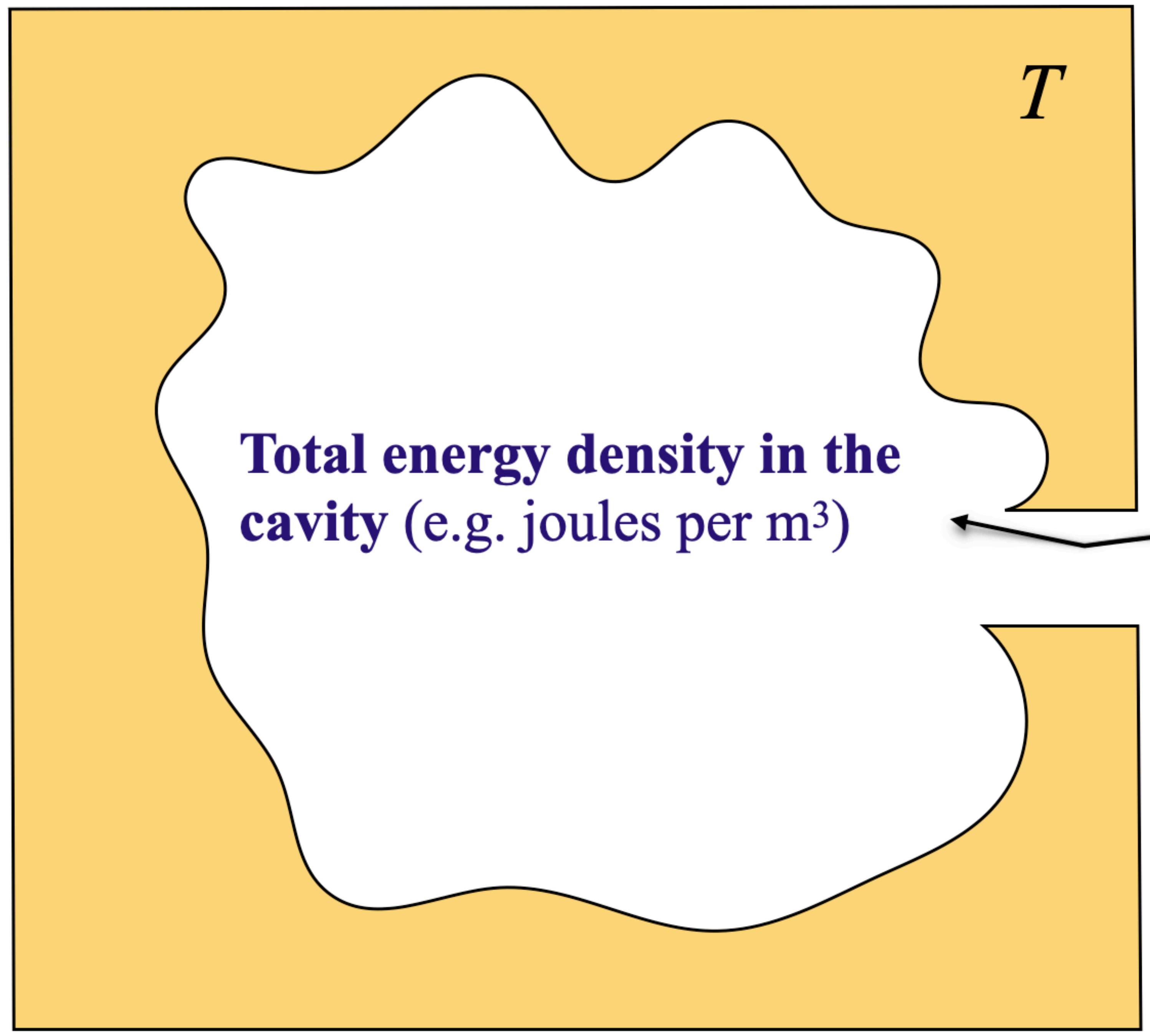


Energy density (u) per unit of photon energy (E)

e.g. joules per m³ per keV

$$u(E) = \frac{8\pi}{(hc)^3} \frac{E^3}{e^{E/kT} - 1}$$

(Undergrad Thermal Physics textbook by Schroeder, Sec 7.4)



$$U = \frac{8\pi}{(hc)^3} \int_0^\infty \frac{E^3}{e^{E/kT} - 1}$$

$$= \frac{4}{c} \boxed{\frac{2\pi^4 k^4}{15 h^3 c^2}} T^4$$

Stefan-Boltzmann constant σ

$$U = \frac{4\sigma}{c} T^4$$

But if we were to "add up" the energy of all the photons in the cavity, we could also express the energy density as:

$$U = \int_0^\infty E\ n(E)\ dE$$

Let's compare this with our expression for pressure from the previous slide:

$$P = \frac{1}{3} \int_0^\infty E\ n(E)\ dE$$

So

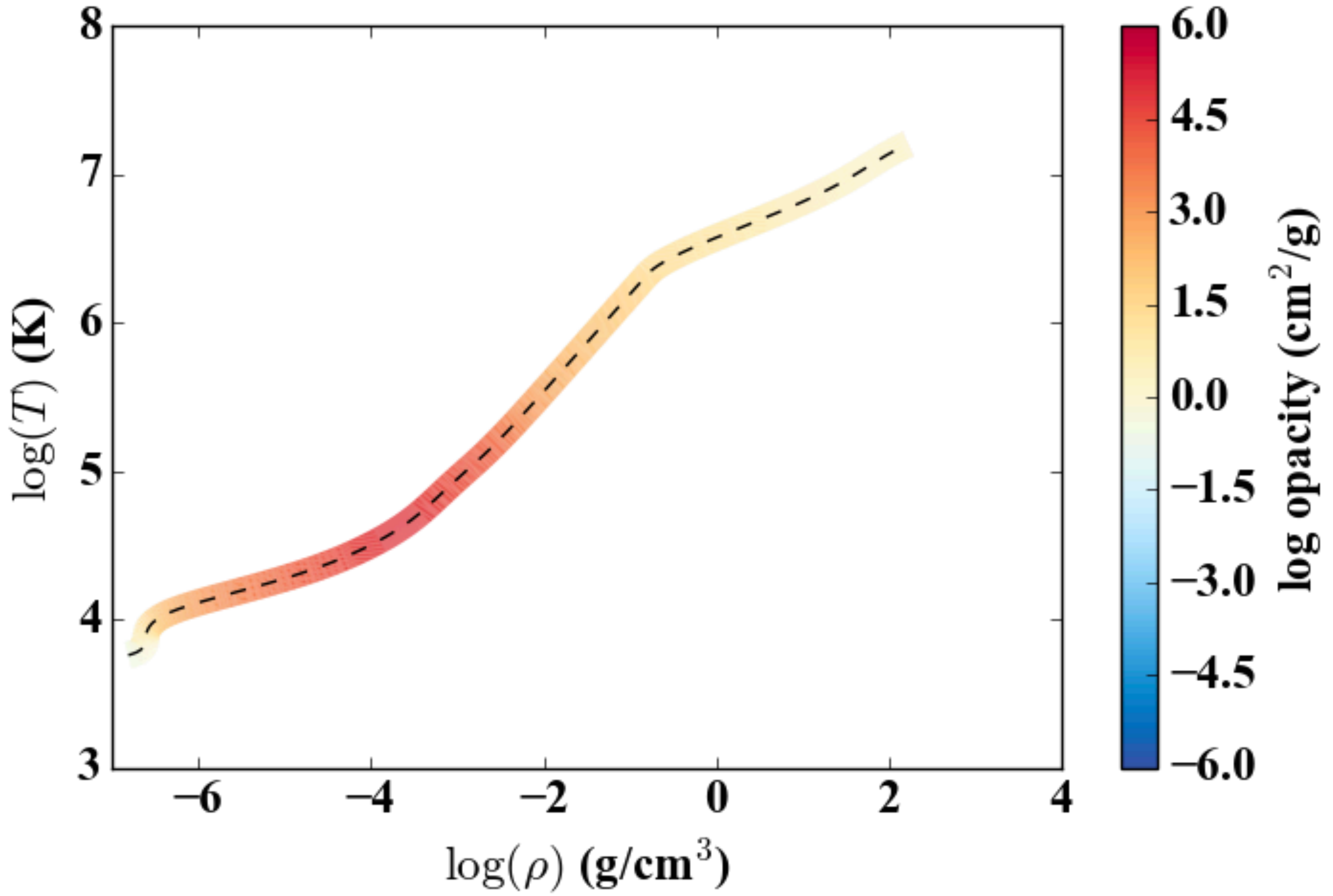
$$P = \frac{1}{3} U$$

$$P_{\rm rad} = \frac{4\sigma}{3c}\ T^4$$

At which ρ and T does the radiation pressure becomes important?

1. Transition from ideal gas law to radiation pressure (on the board)

$$\left(\frac{\rho}{\mu}\right)_{\mathrm{trans}} = 3.05 \times 10^{-23} \quad T^3 \quad [\mathrm{dyn/cm}]$$



Let's add this to our notebook

If there is a mixture of P_{ideal} and P_{rad}: The Eddington model

$$P_{\text{tot}} = \frac{\rho k}{\mu m_H} T + \frac{4\sigma}{3c} T^4$$

Let's set $\quad \beta = \dfrac{P_{\text{ideal}}}{P_{\text{tot}}}$

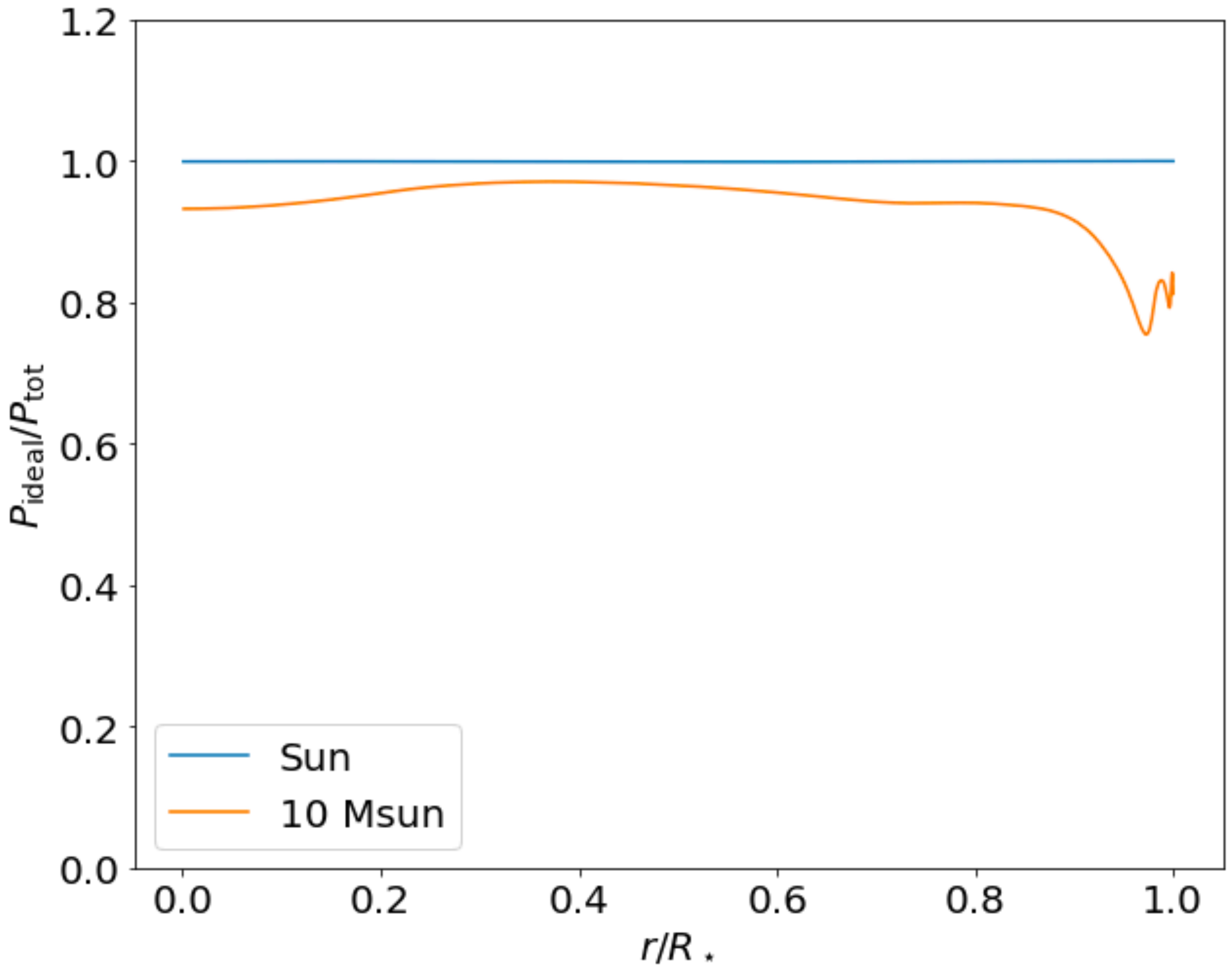
Some algebra later….

$$P_{\text{tot}} = \left[\frac{(1-\beta)}{\beta^4} \frac{3c}{4\sigma} \left(\frac{k}{\mu m_H} \right)^4 \right]^{1/3} \rho^{4/3}$$

Polytrops

$$\boxed{P = k\rho^{1+1/n}}$$

So if β is constant as a function of radius,
this would be a $n = 3$ polytrope….
So, is $P_{\text{gas}}/P_{\text{tot}}$ roughly constant?

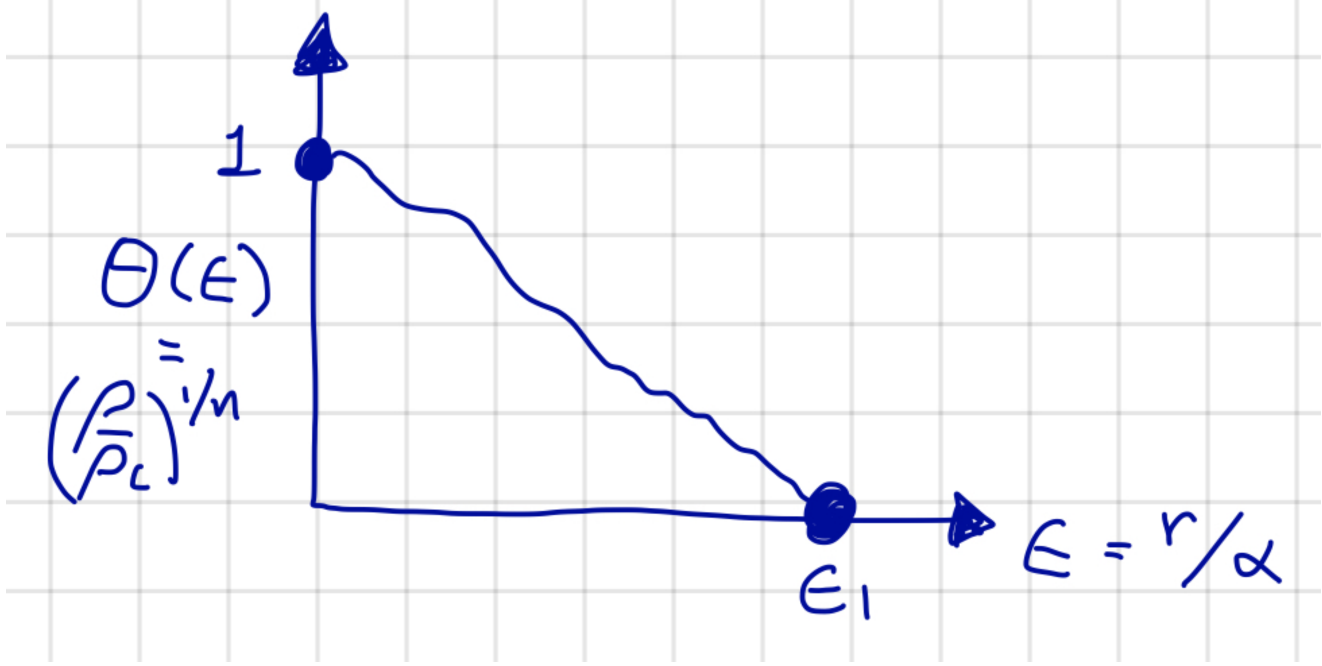


1.2
1.0
0.8
0.6
0.4
0.2
0.0
$P_{\mathrm{ideal}}/P_{\mathrm{tot}}$
0.0
0.2
0.4
0.6
0.8
1.0
$r/R_\star$
Sun
10 Msun

$$P_{tot} = \underbrace{\left[\frac{(1-\beta)}{\beta^4}\frac{3c}{4\sigma}\left(\frac{k}{\mu m_H}\right)^4\right]^{1/3}}_{K}\rho^{4/3}$$

Polytrops

$$\boxed{P = K\rho^{1+1/n}}$$



n

1. $$R_\star = \alpha\epsilon_1 = \left(\frac{(n+1)P_c}{4\pi G\rho_c^2}\right)^{1/2}\epsilon_1$$

2. $$P_c = K\rho_c^{(n+1)/n}$$

3. $$M_\star = -\frac{1}{(4\pi)^{1/2}}\left(\frac{n+1}{G}\right)^{3/2}\frac{P_c^{3/2}}{\rho_c^2}\,\epsilon_1^2\,\theta'(\epsilon_1)$$

P_c ρ_c $\boxed{R_\star}$

then known K
(so does β)

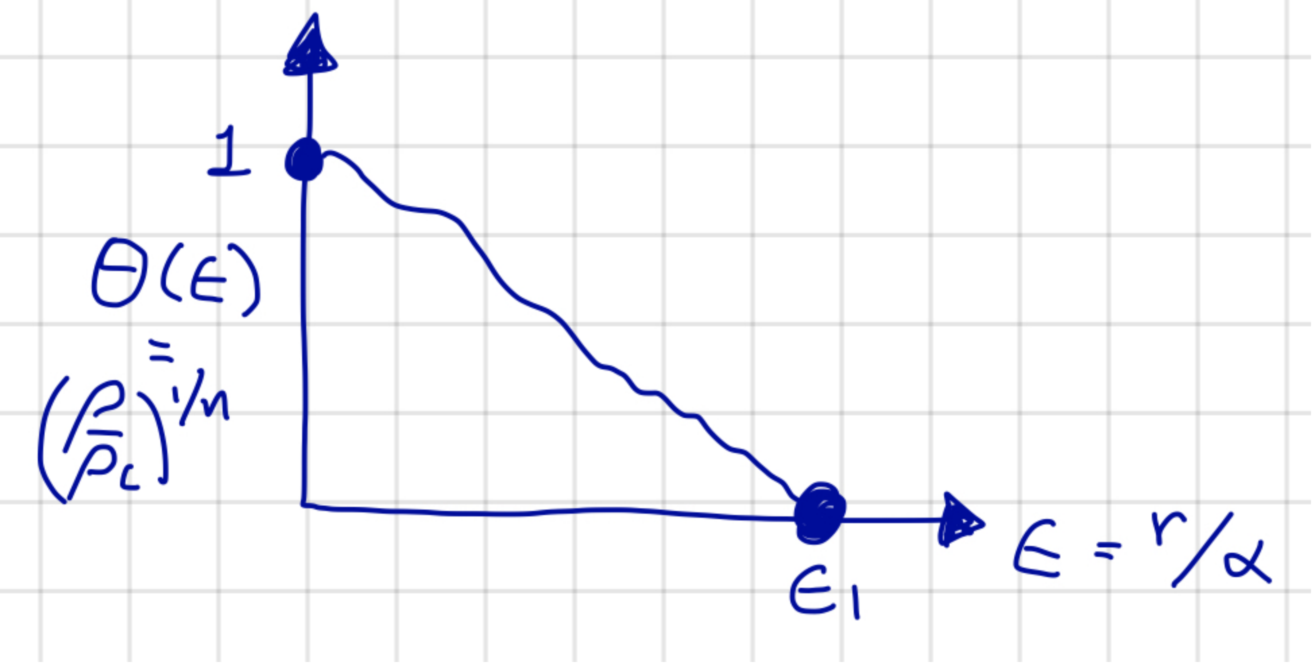
if we pick

$\boxed{M_\star}$

$$P_{tot} = \underbrace{\left[\frac{(1-\beta)}{\beta^4}\,\frac{3c}{4\sigma}\left(\frac{k}{\mu m_H}\right)^4\right]^{1/3}}_{K} \rho^{4/3}$$

Polytrops

$$\boxed{P = K\rho^{1+1/n}}$$



1 $$R_\star = \alpha\epsilon_1 = \left(\frac{(n+1)P_c}{4\pi G\rho_c^2}\right)^{1/2}\epsilon_1$$

2 $$P_c = K\rho_c^{(n+1)/n}$$

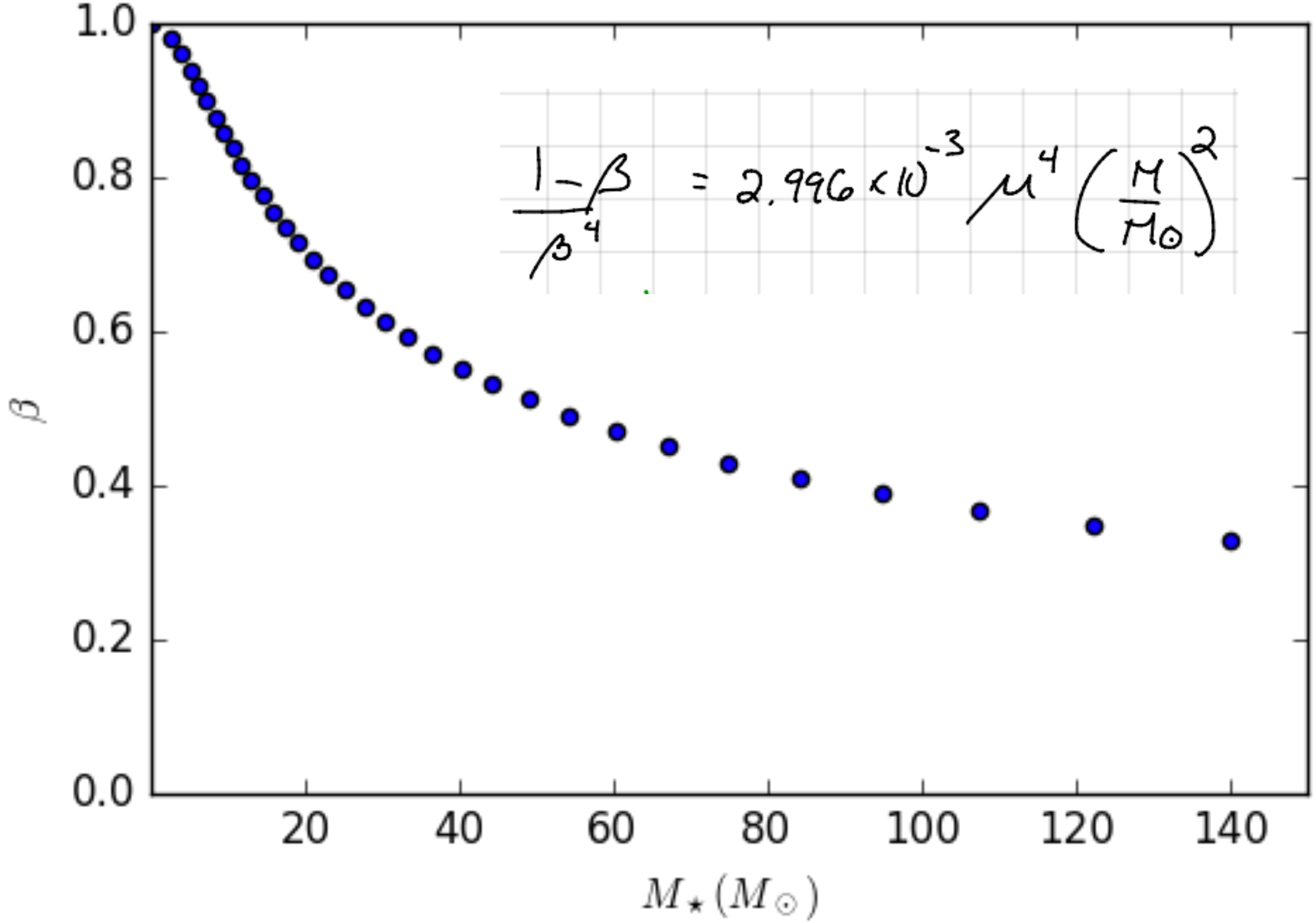
If $n = 3$, $P_c = K\rho_c^{4/3}$, so $K = \frac{P_c}{\rho_c^{4/3}}$ and $K^{3/2} = \frac{P_c^{3/2}}{\rho_c^2}$

3 $$M_\star = -\frac{1}{(4\pi)^{1/2}}\left(\frac{n+1}{G}\right)^{3/2}\boxed{\frac{P_c^{3/2}}{\rho_c^2}}\,\epsilon_1^2\,\theta'(\epsilon_1)$$

$$K = \frac{(4\pi)^{1/3}}{4}\,\frac{GM_*^{2/3}}{\left[\epsilon_1^2(-\theta'_{\epsilon_1})\right]^{2/3}}$$

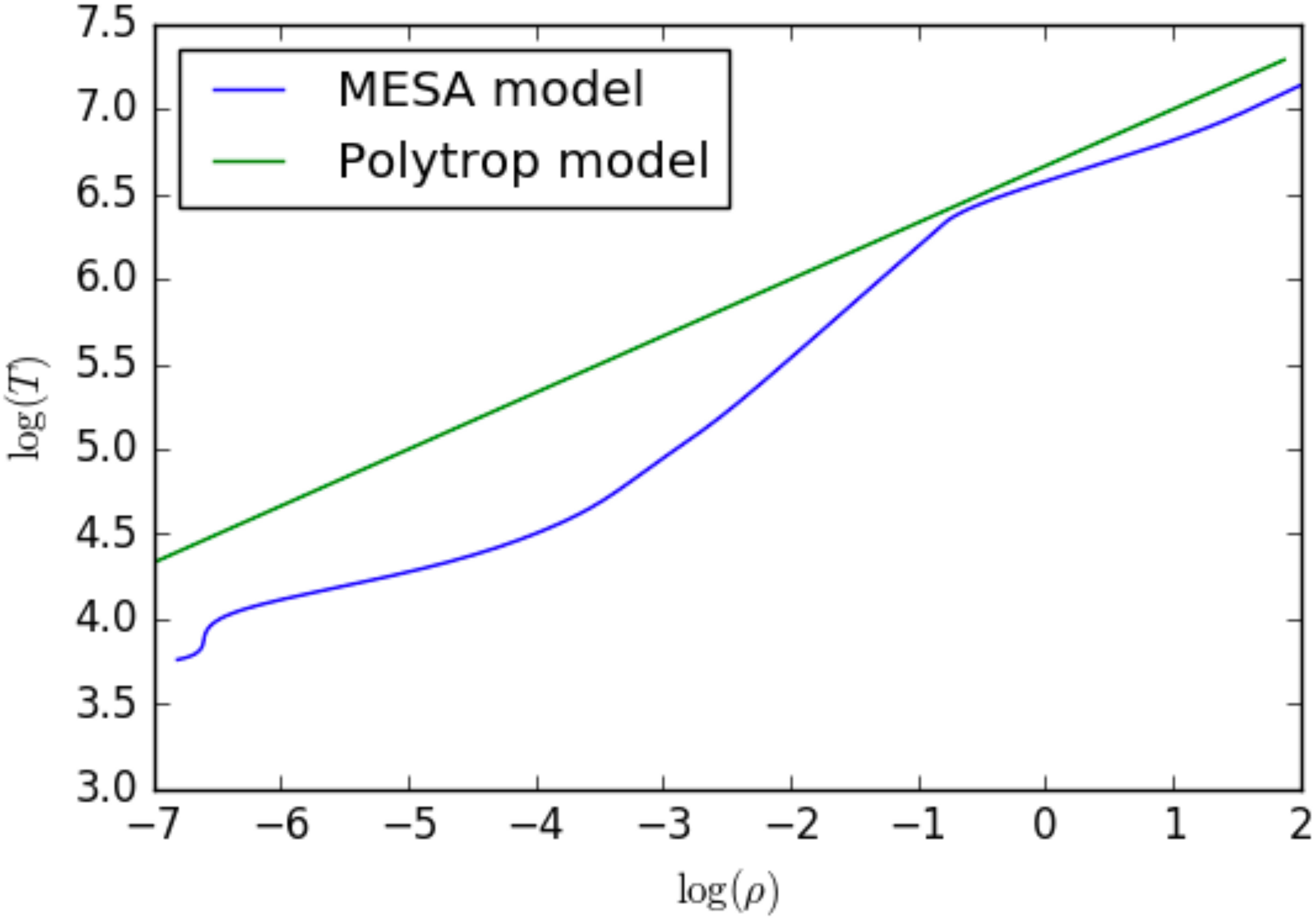
$$P_{tot} = \underbrace{\left[\frac{(1-\beta)}{\beta^4} \frac{3c}{4\sigma} \left(\frac{k}{\mu m_H} \right)^4 \right]^{1/3}}_{K} \rho^{4/3}$$

$$K = \frac{(4\pi)^{1/3}}{4} \frac{G M_*^{2/3}}{\left[\epsilon_1^2 (-\theta'_{\epsilon_1}) \right]^{2/3}}$$



$$\frac{1-\beta}{\beta^4} = 2.996 \times 10^{-3} \mu^4 \left(\frac{M}{M_\odot} \right)^2$$

$$T(r) = \left[\frac{(1-\beta)}{\beta} \frac{3c}{4\sigma} \frac{k_B}{\mu m_H} \right]^{1/3} \rho^{1/3}$$



We already know how to calculate the density for a polytrop