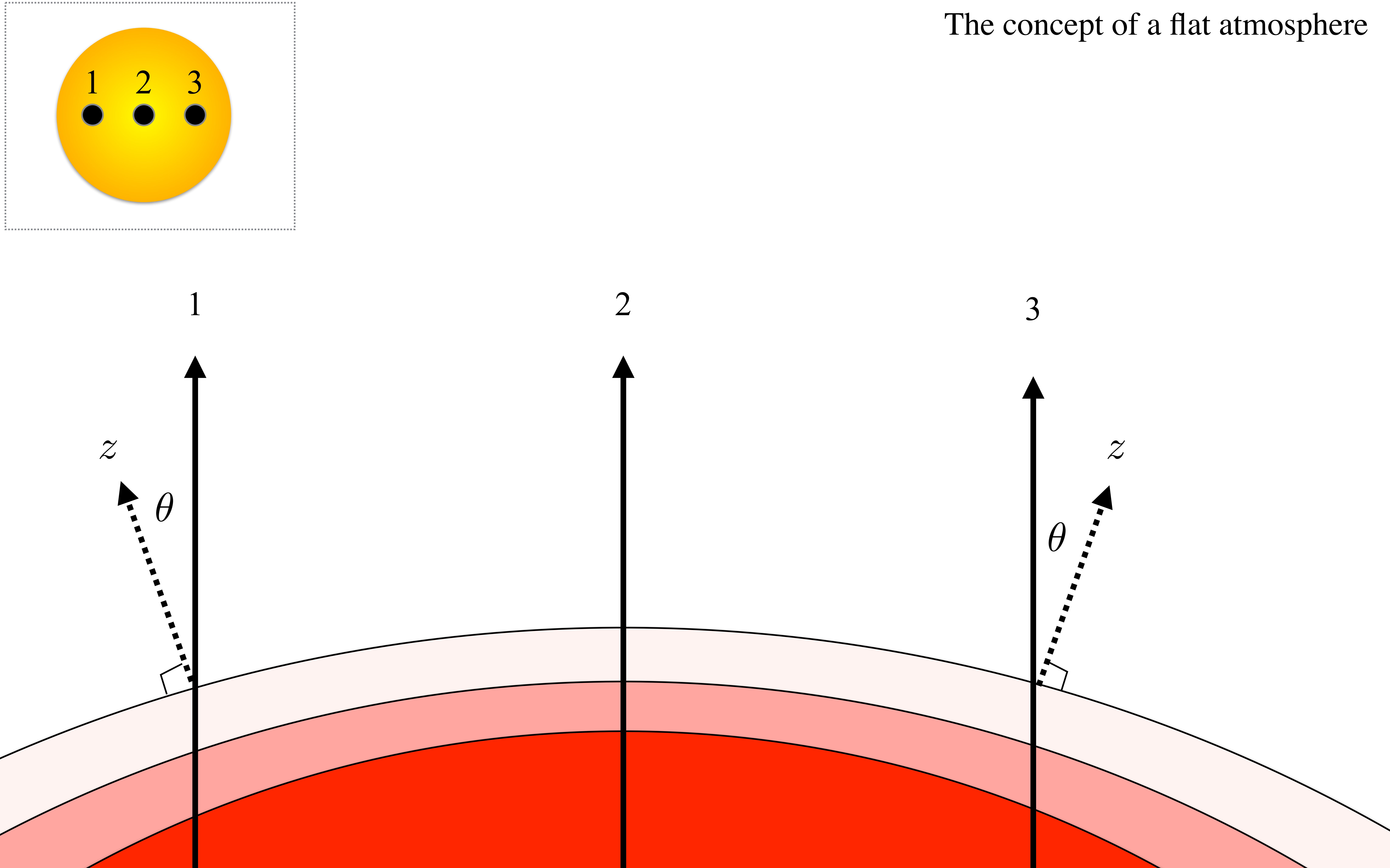
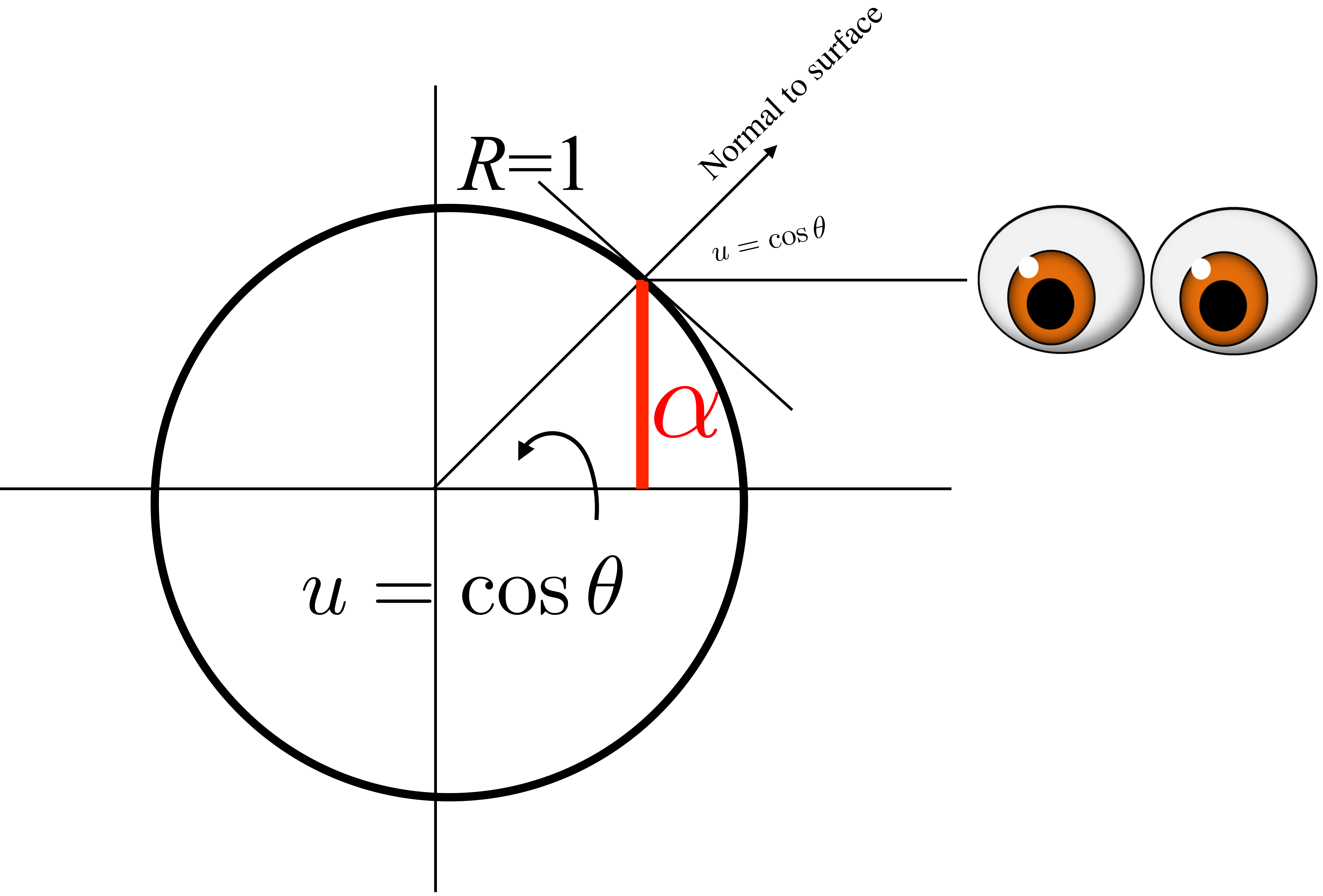
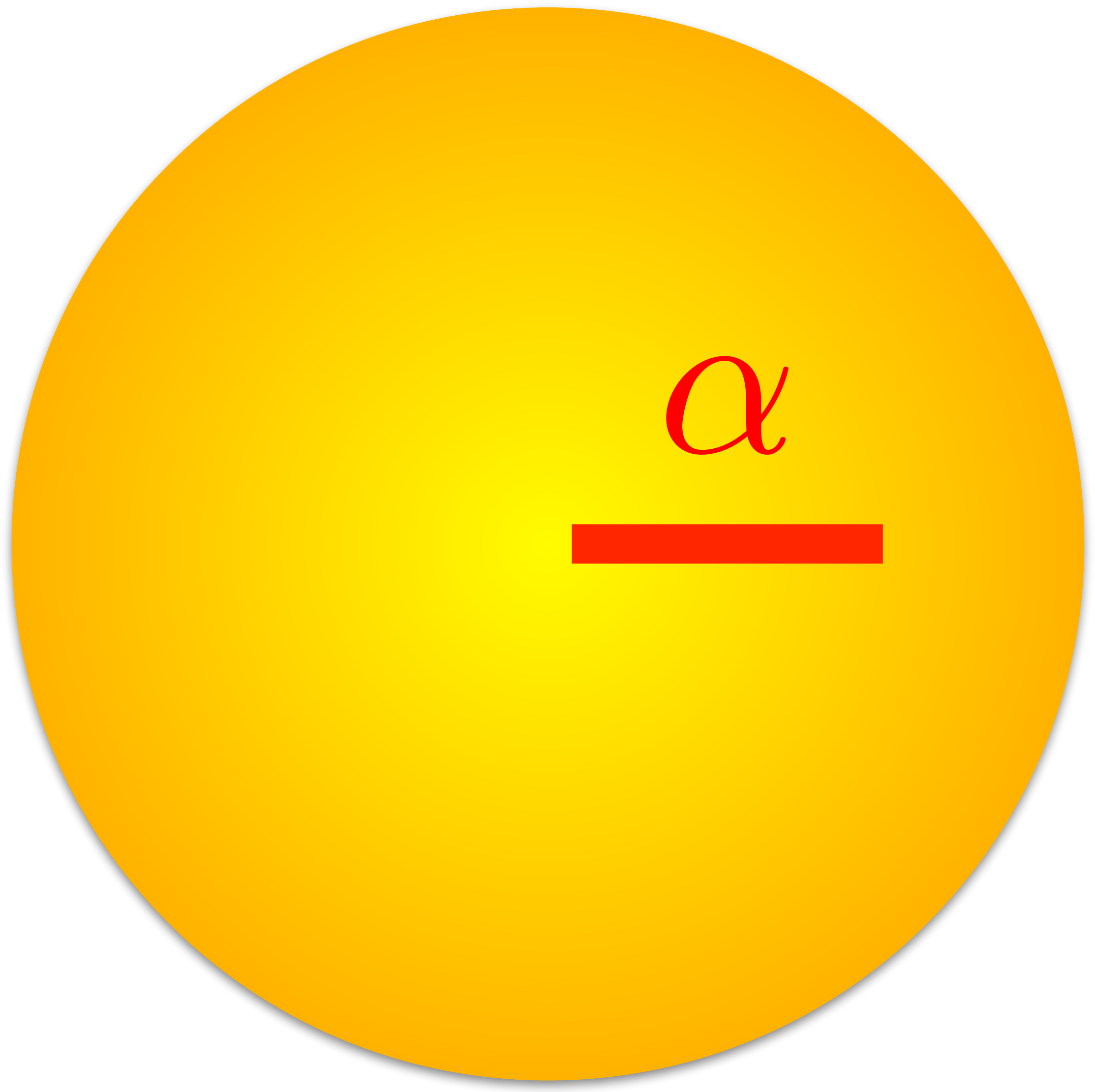
Week 6 Thursday
L-12

L8
Energy transport
Radiative mean free path
Stellar interiors
Diffusive energy transport
Stellar atmospheres
Need detailed RT calculations
L9
Basics of radiative transfer
Optical depth
L10
Formal solution of RT with source function
L11
Relationship with blackbody systems and spectral lines formation
We are here
L12
Application to a "flat" stellar atmosphere
L13
Limb darkening and radiative flux
L14
Solving for the temperature structure
Solving for an analytical
We can use these results
L7
Numerical iterations
L18
Radiative transport in interiors
Energy conservation equation
1. Is the energy always transported by radiation?
2. How to calculate the power generated?
3. Is ideal gas law always valid?
4. How do we solve for the interior structure?
L20
Convection
L21
Nuclear physics
Gammow Peak
L22
Reaction rates
L23
Degeneracy pressure
Chandrasekhar mass
Only depends on density.
We can use polytrops!
L24
Radiation pressure
L25
Homology relations for the Main Sequence

The concept of a flat atmosphere

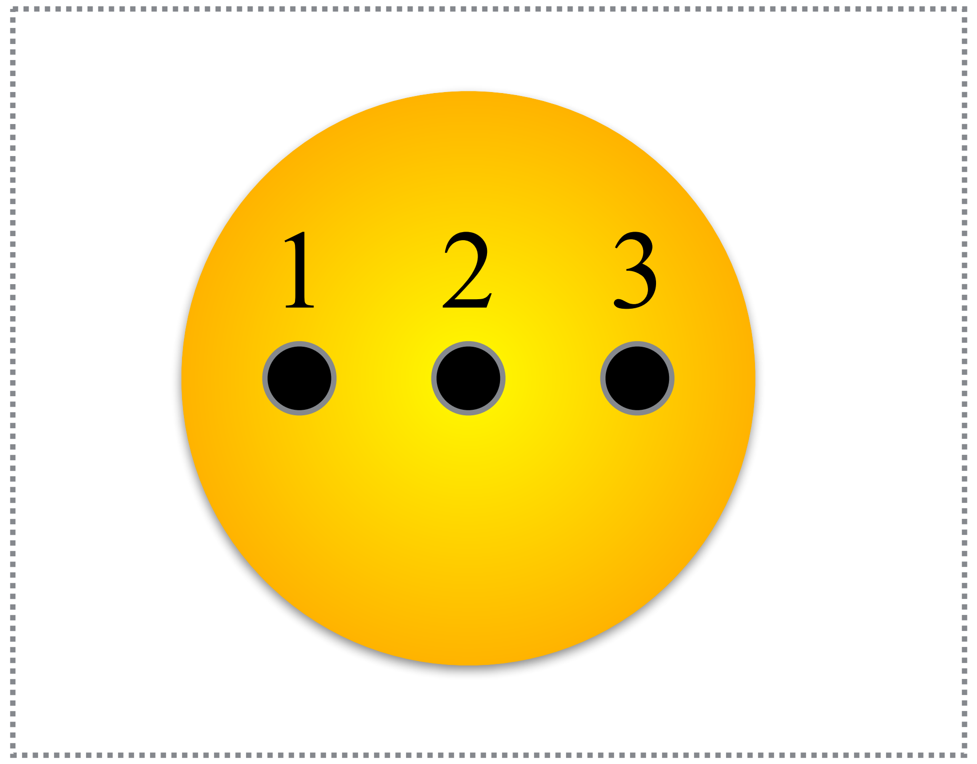






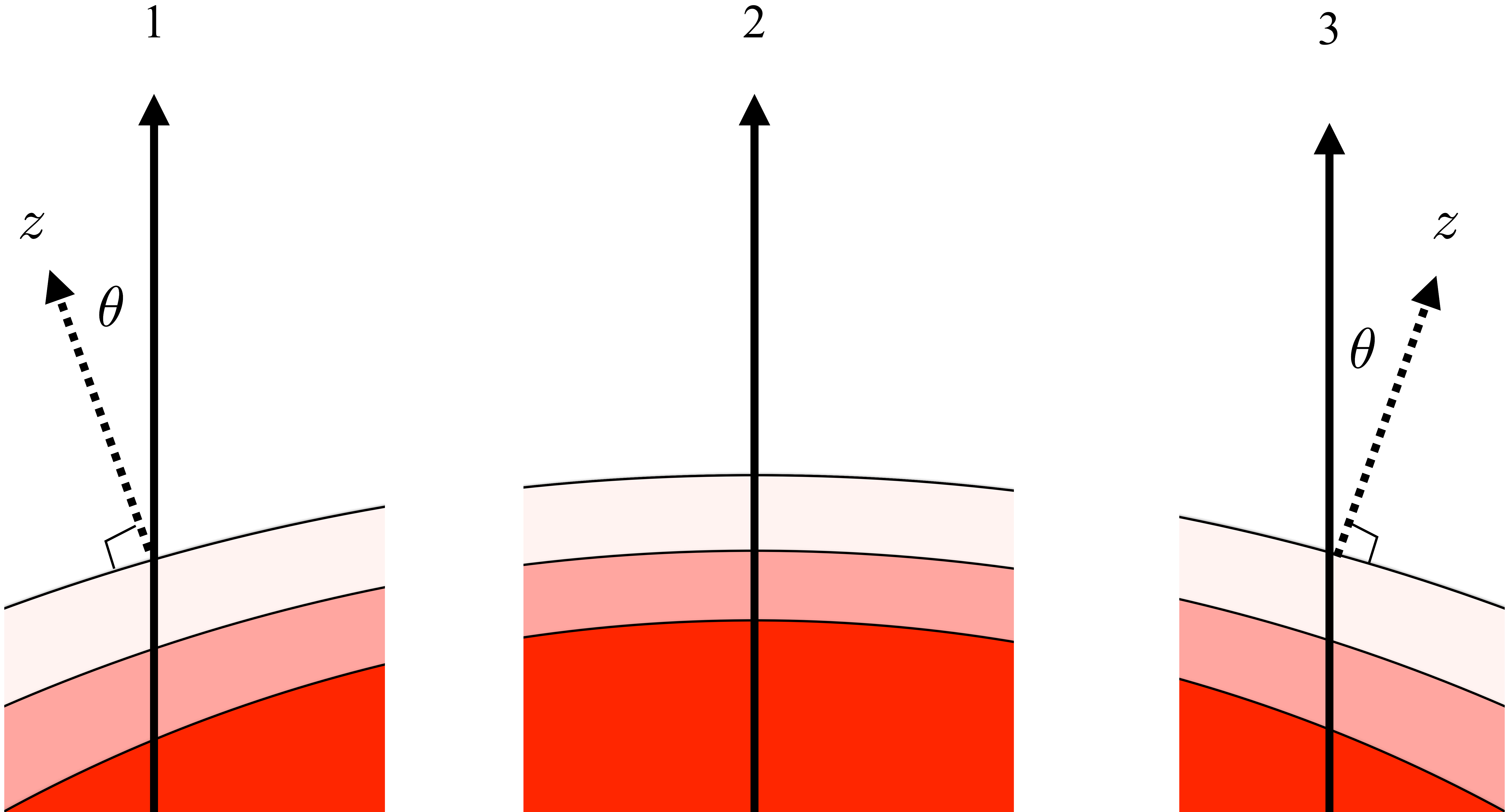
$$u^2 = 1 - \alpha^2$$

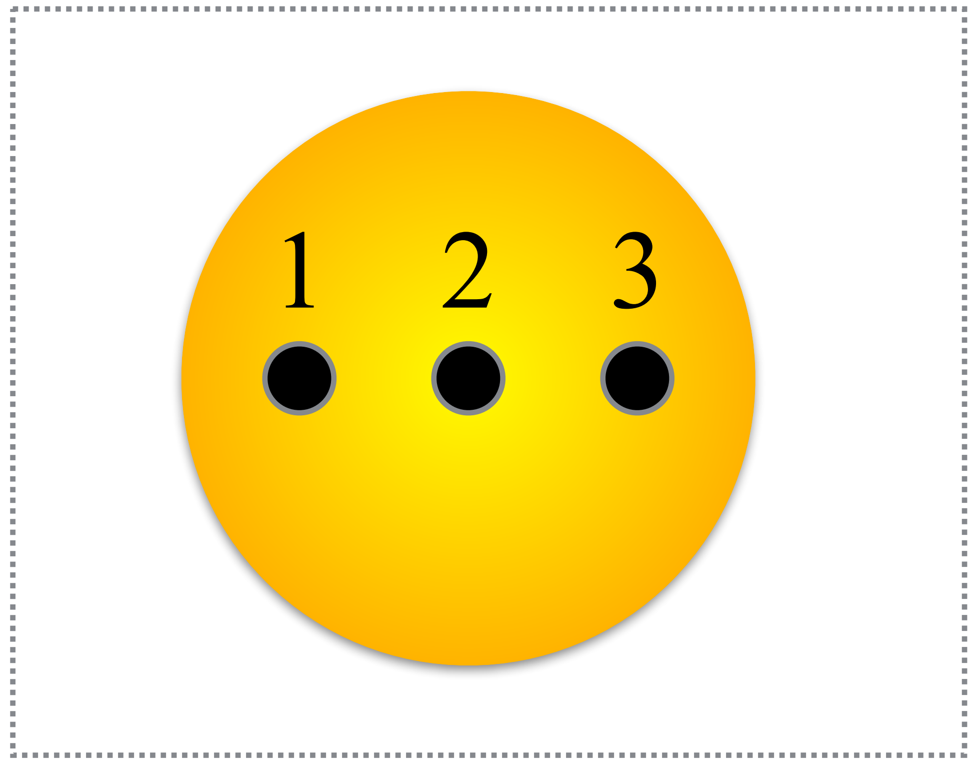
The concept of a flat atmosphere



Q: where is the $u = +1$ ray?

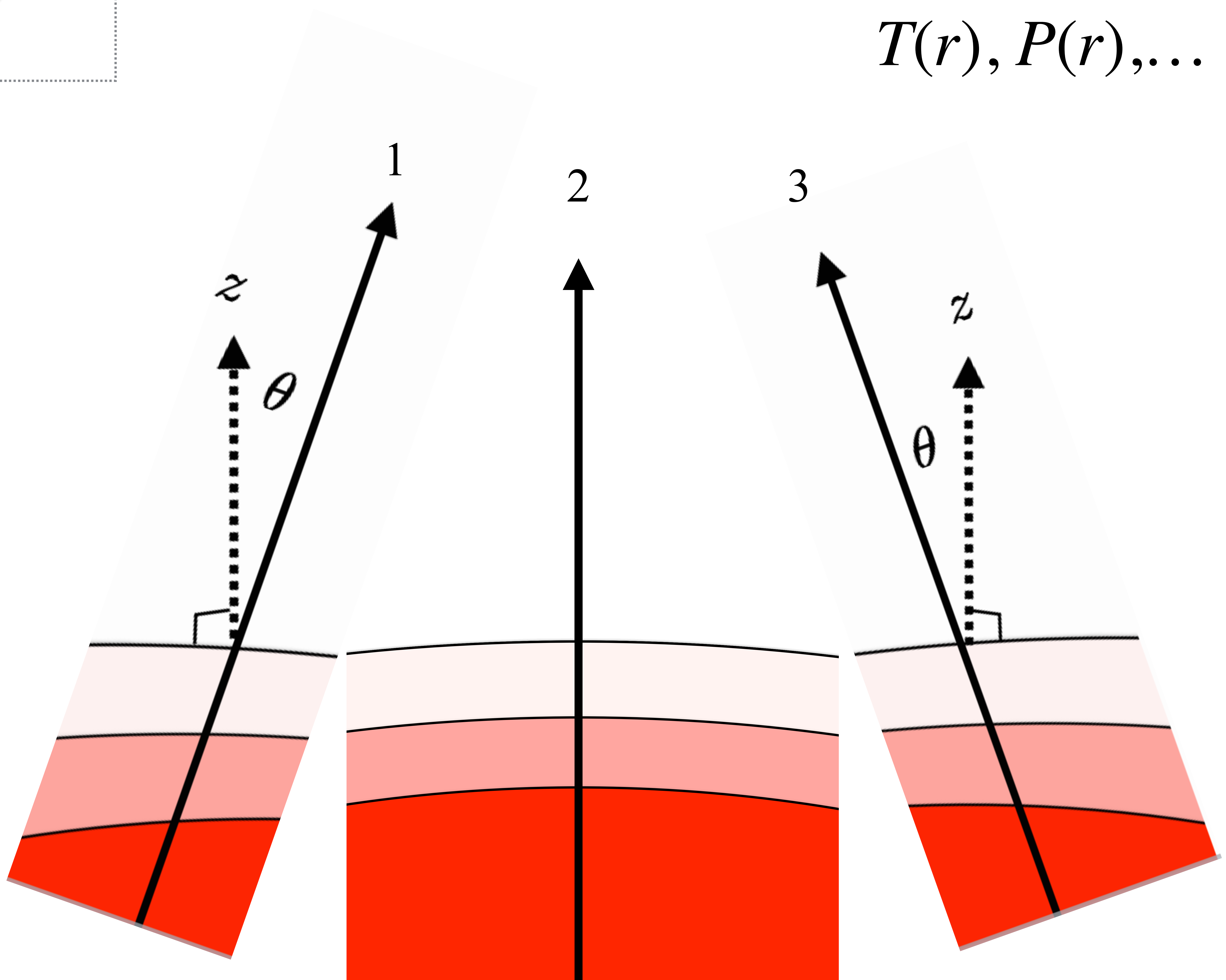
Q: where is the $u = 0$ ray?

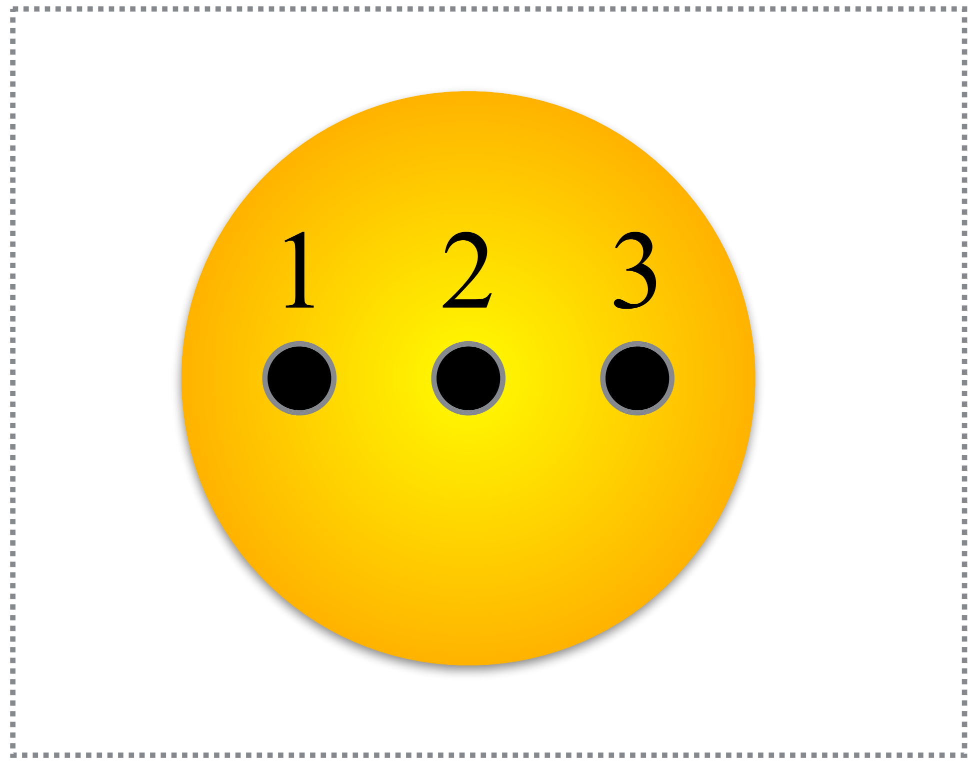




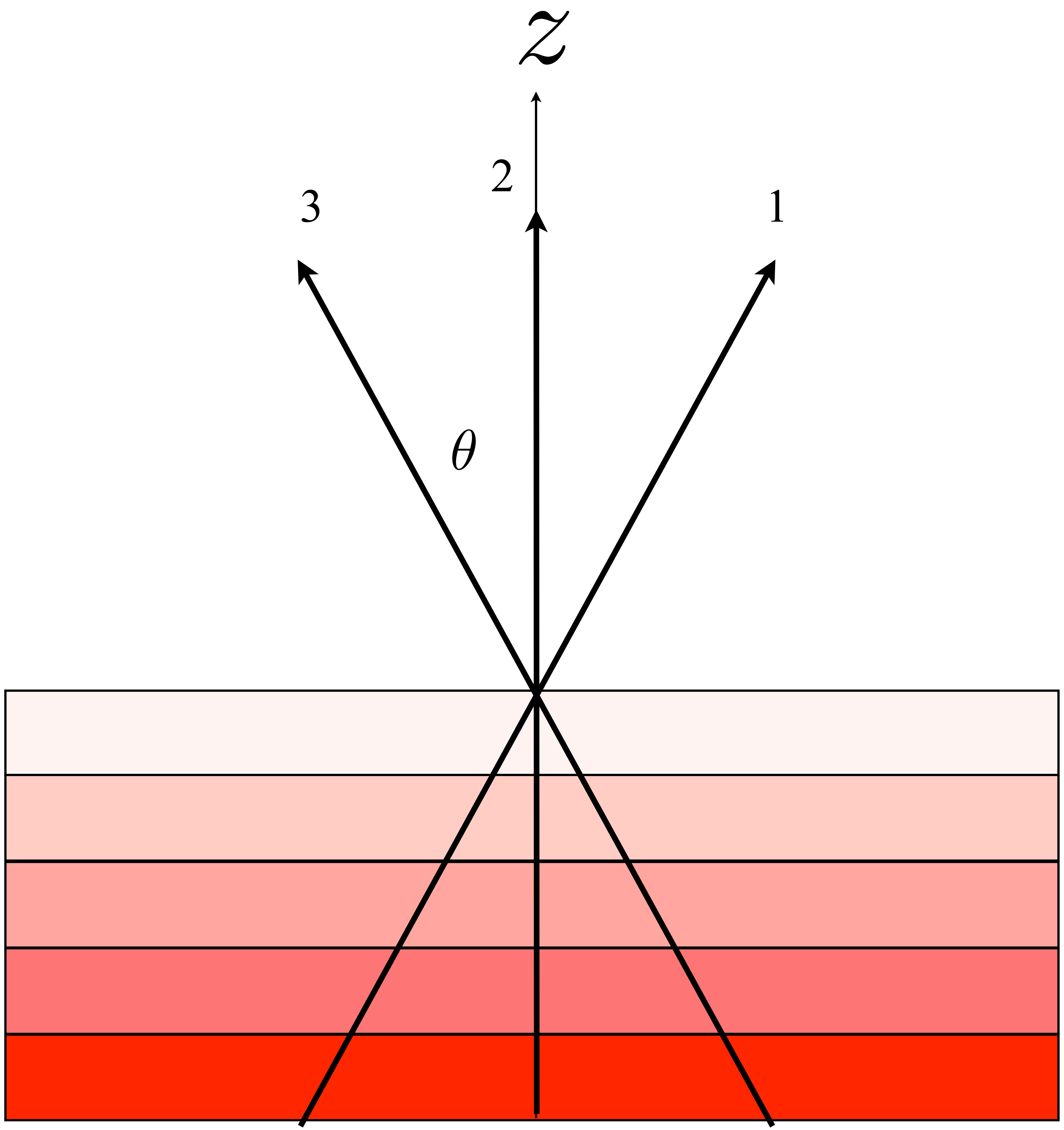
Spherical symmetry becomes a symmetry in the z direction:

$T(r), P(r), \ldots \rightarrow T(z), P(z), \ldots$





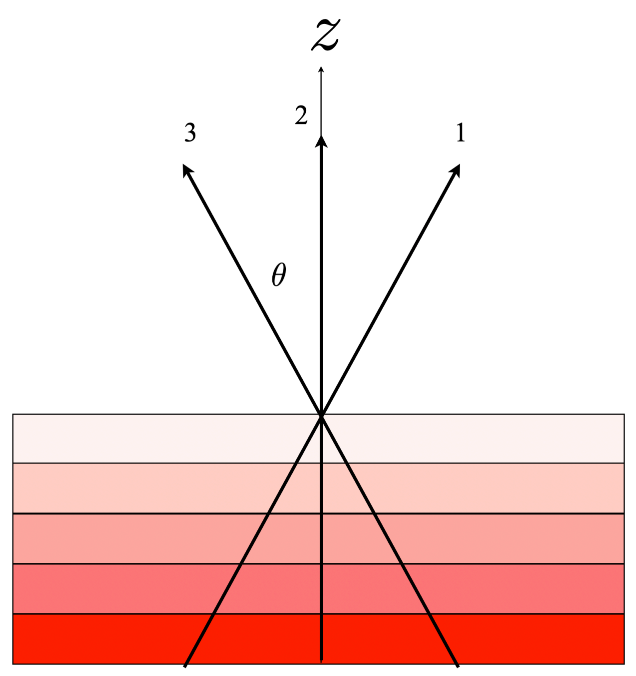
1 2 3

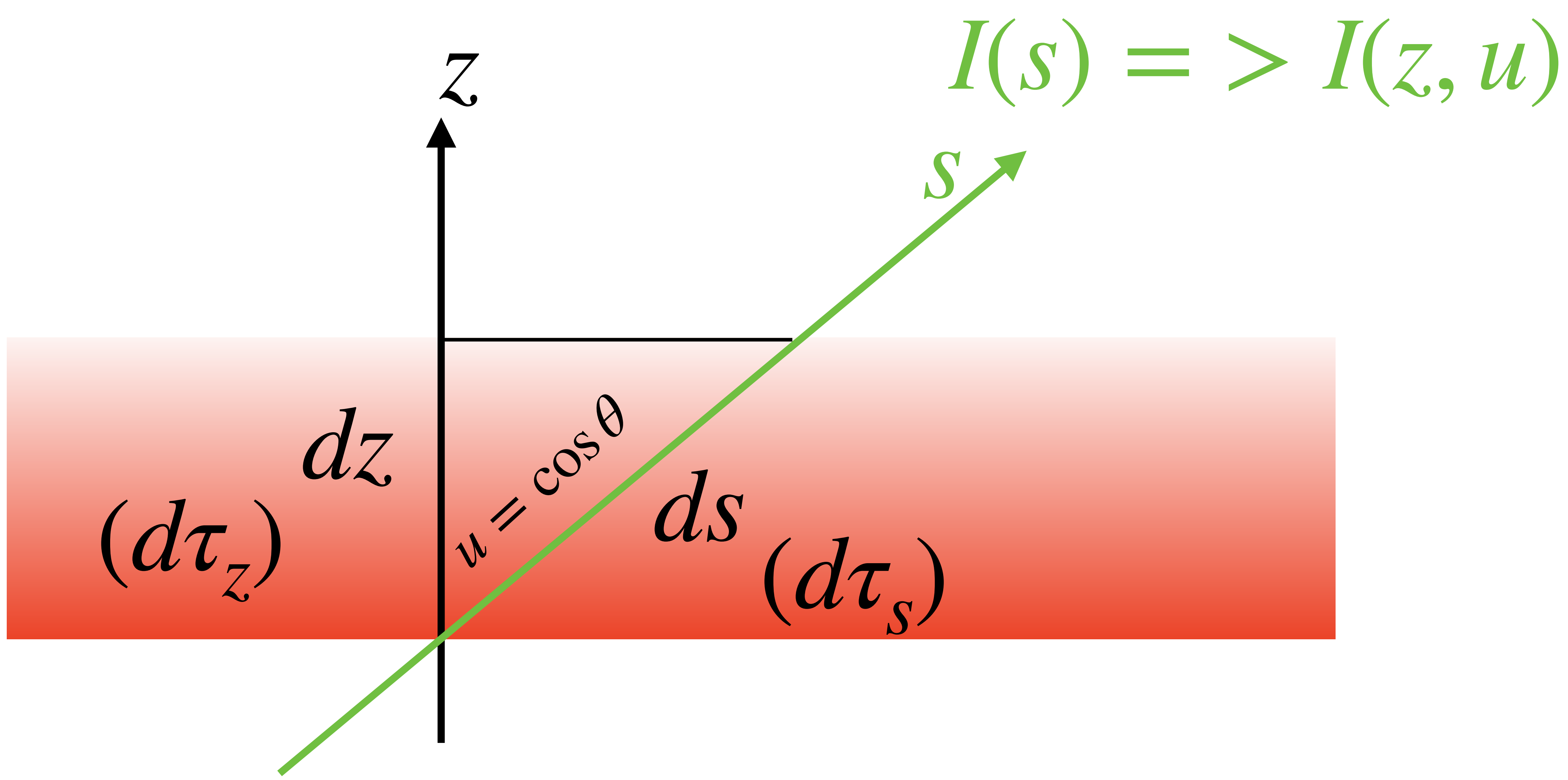


z
3
2
1
θ

The (multiple!) coordinate systems:

- z: the spatial coordinate (up is towards the outside of the star)
- s: the coordinate along a specific ray (direction of the photons)



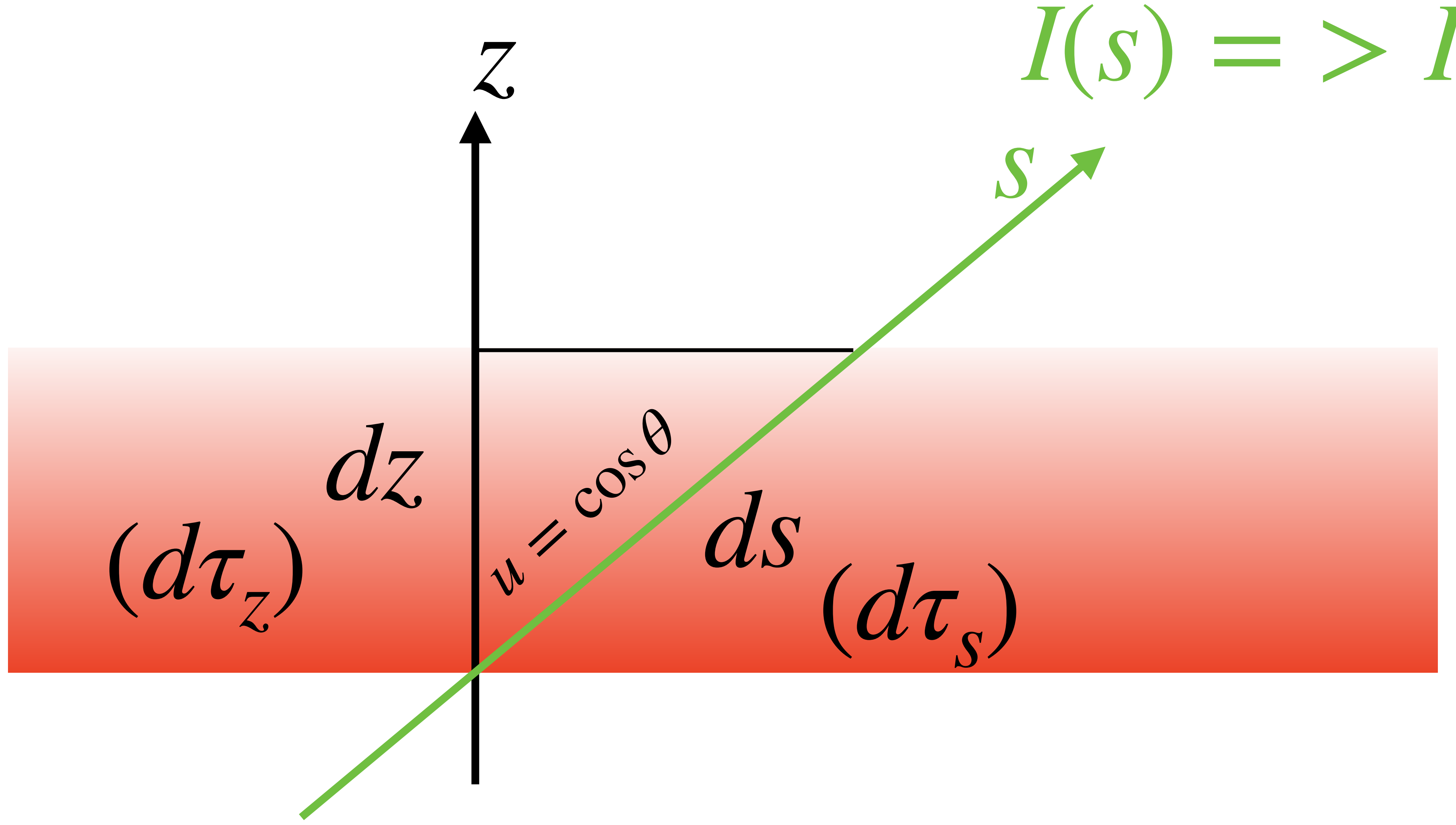


- $d\tau_s$: the change in optical depth along the ray (which is the important one for RT)
- $d\tau_z$: the optical depth measured from the vertical ray — we will use that as your independent coordinate

$$\frac{d\tau_z}{u} = d\tau_s$$

Re-writing the formal solution in terms of z and u

$$I(s) => I(z, u)$$



$$\frac{d\tau_z}{u} = d\tau_s$$

$$\frac{dI(s)}{d\tau_s} = I(s) - S(s)$$

Change of variable

$$u\frac{dI(z, u)}{d\tau_z} = I(z, u) - S(z)$$

Q: why no u dependence here?

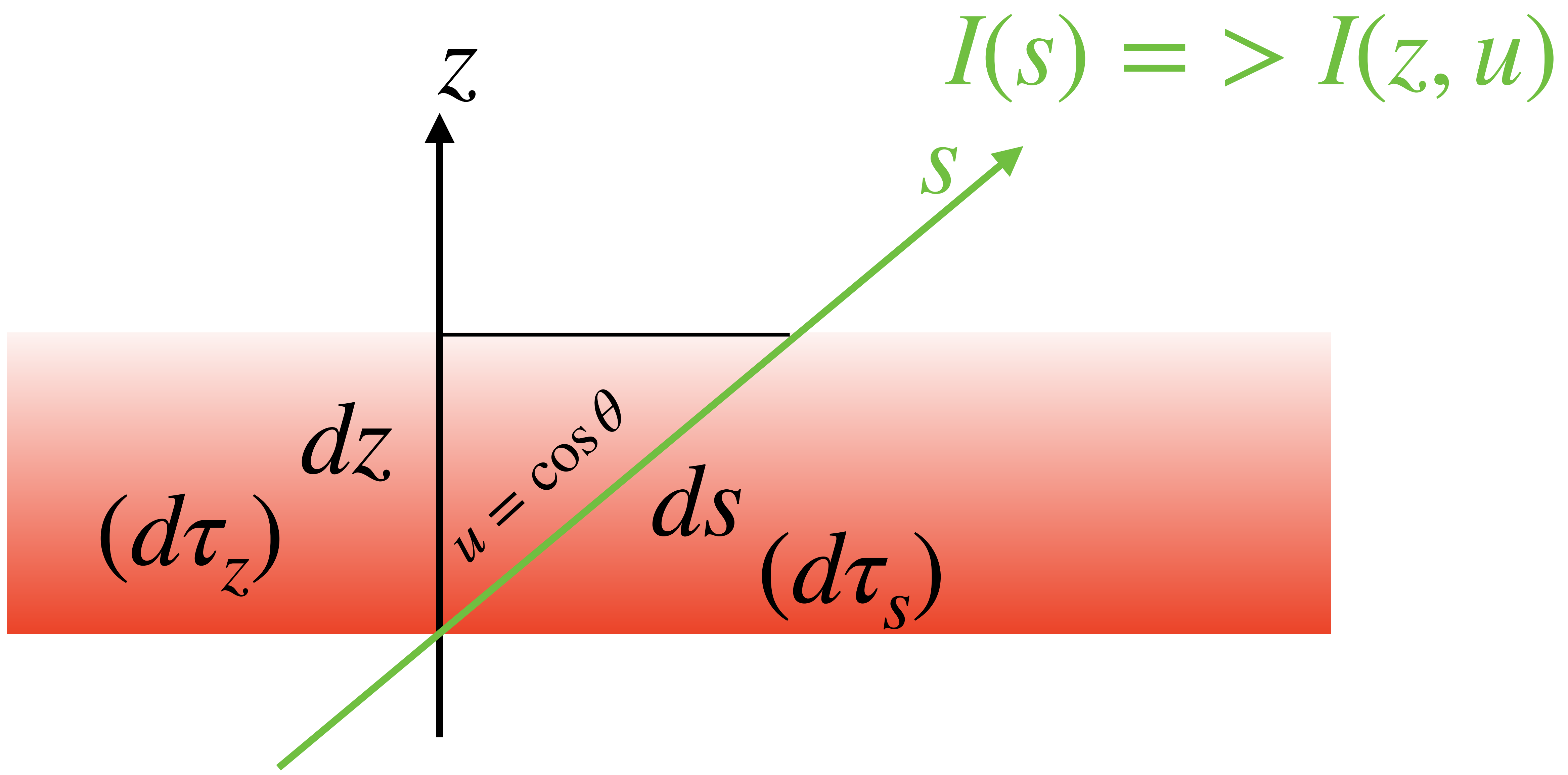
Solving the same way as the general formal solution
from a place we know the intensity $I(\tau_o, u) = I_o(u)$

$$I(\tau_z, u) = I_o(u)\ \mathrm{e}^{\frac{\tau_z - \tau_o}{u}} + \int_{\tau_z'=\tau_z}^{\tau_z'=\tau_o} S(\tau_z')\ \mathrm{e}^{\frac{\tau_z - \tau_z'}{u}}\ d\tau_z$$

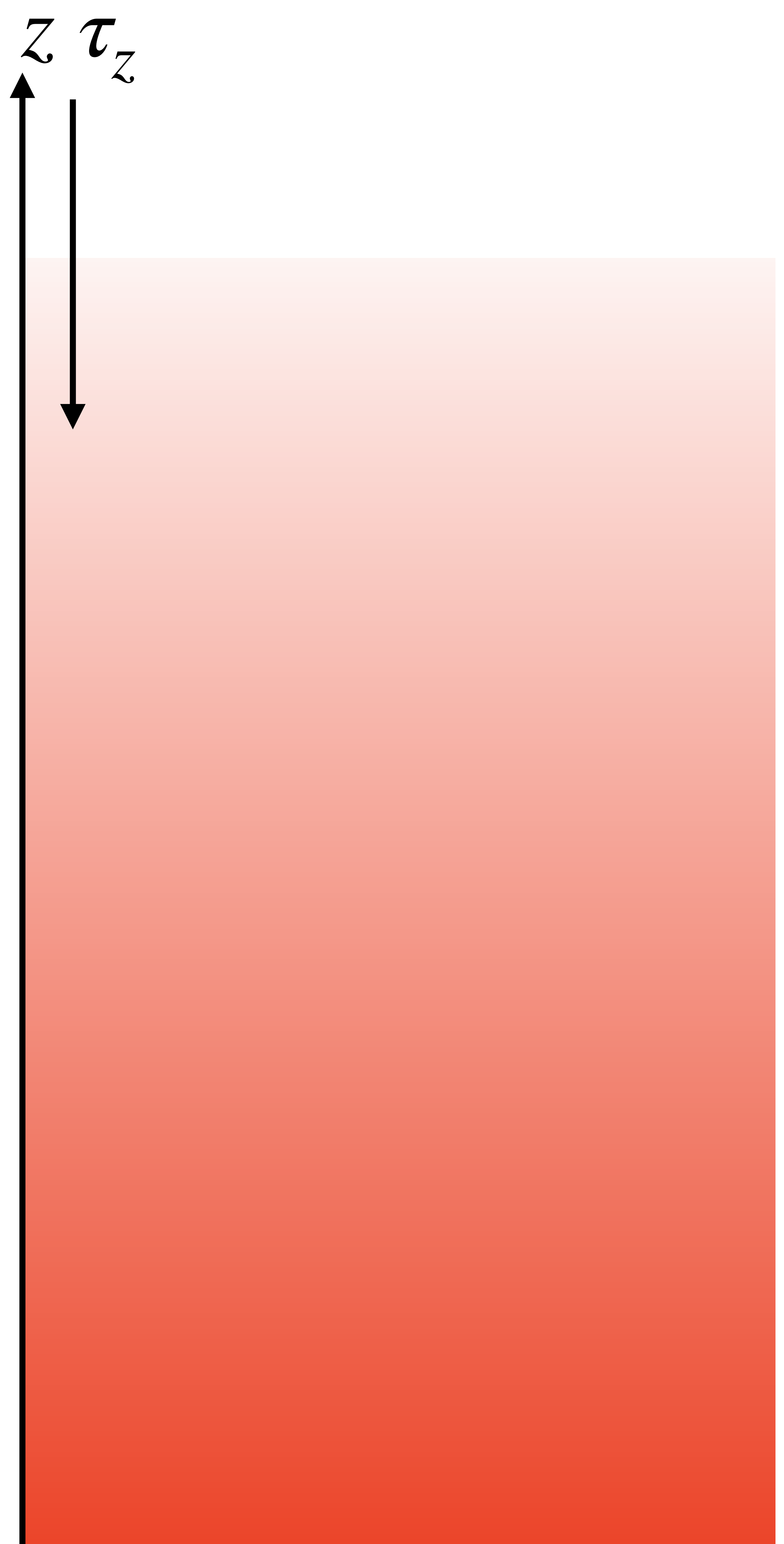
Re-writing the formal solution in terms of z and u

Solving the same way as the general formal solution from a place we know the intensity $I(\tau_o, u) = I_o(u)$

Okay…… but where in hell is that gonna be?



A semi-infinite flat atmosphere



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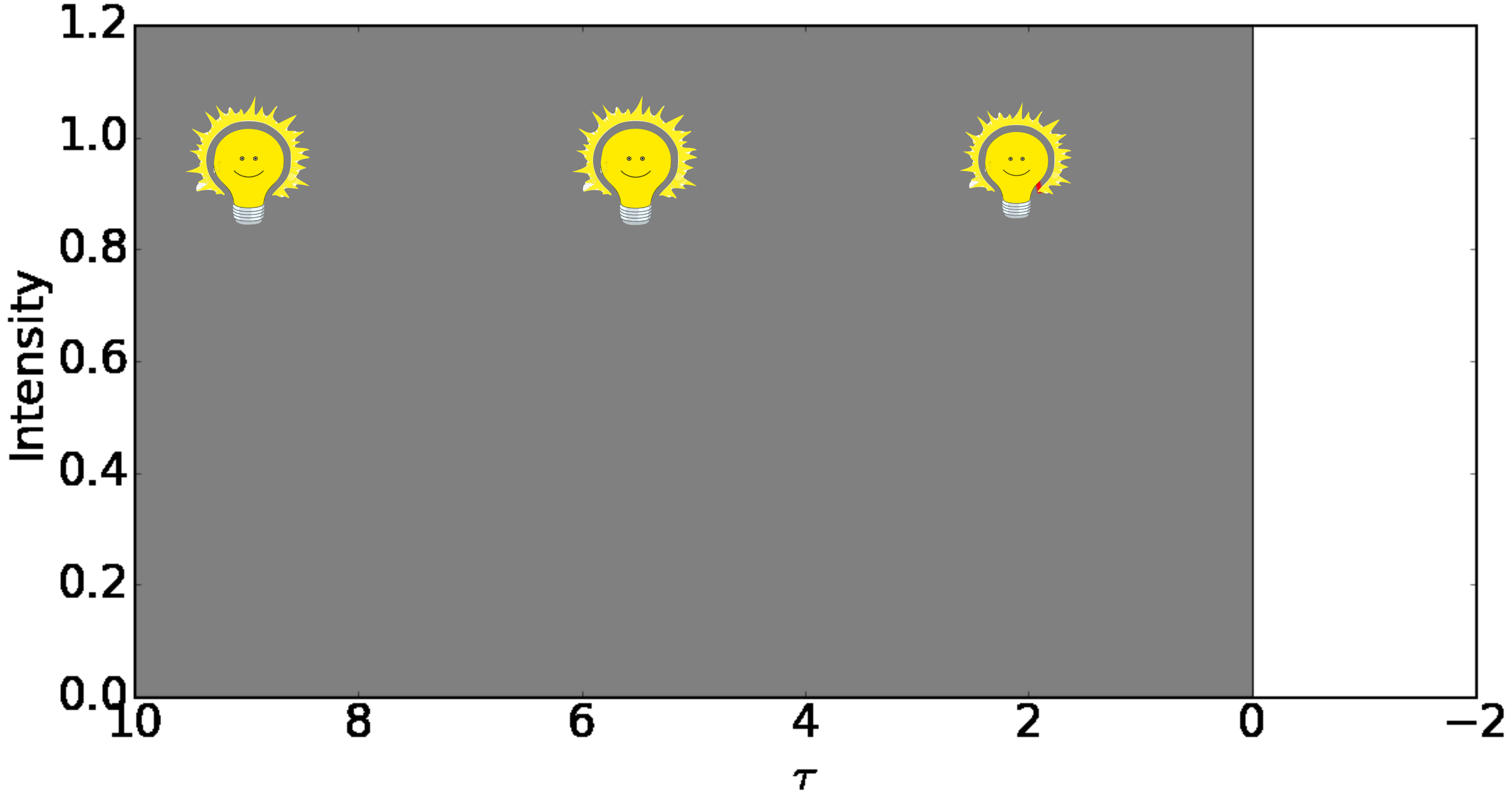
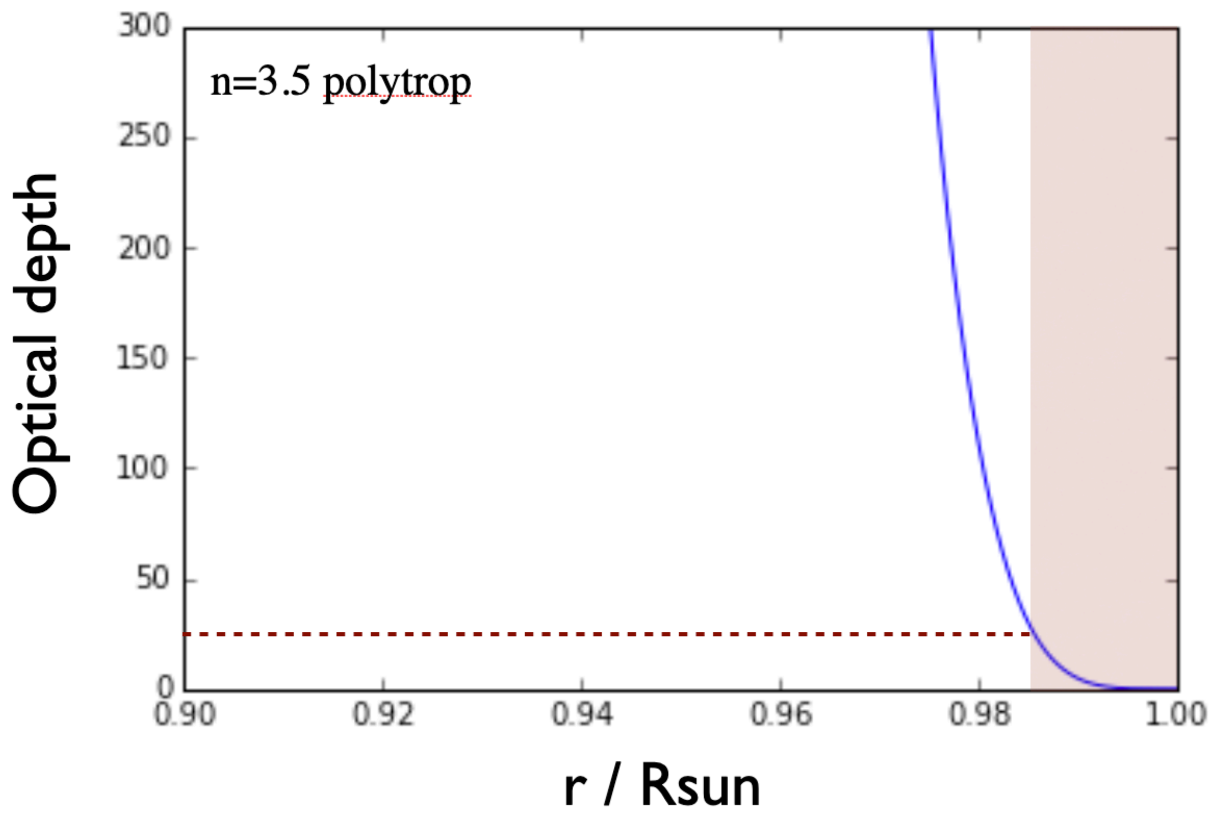


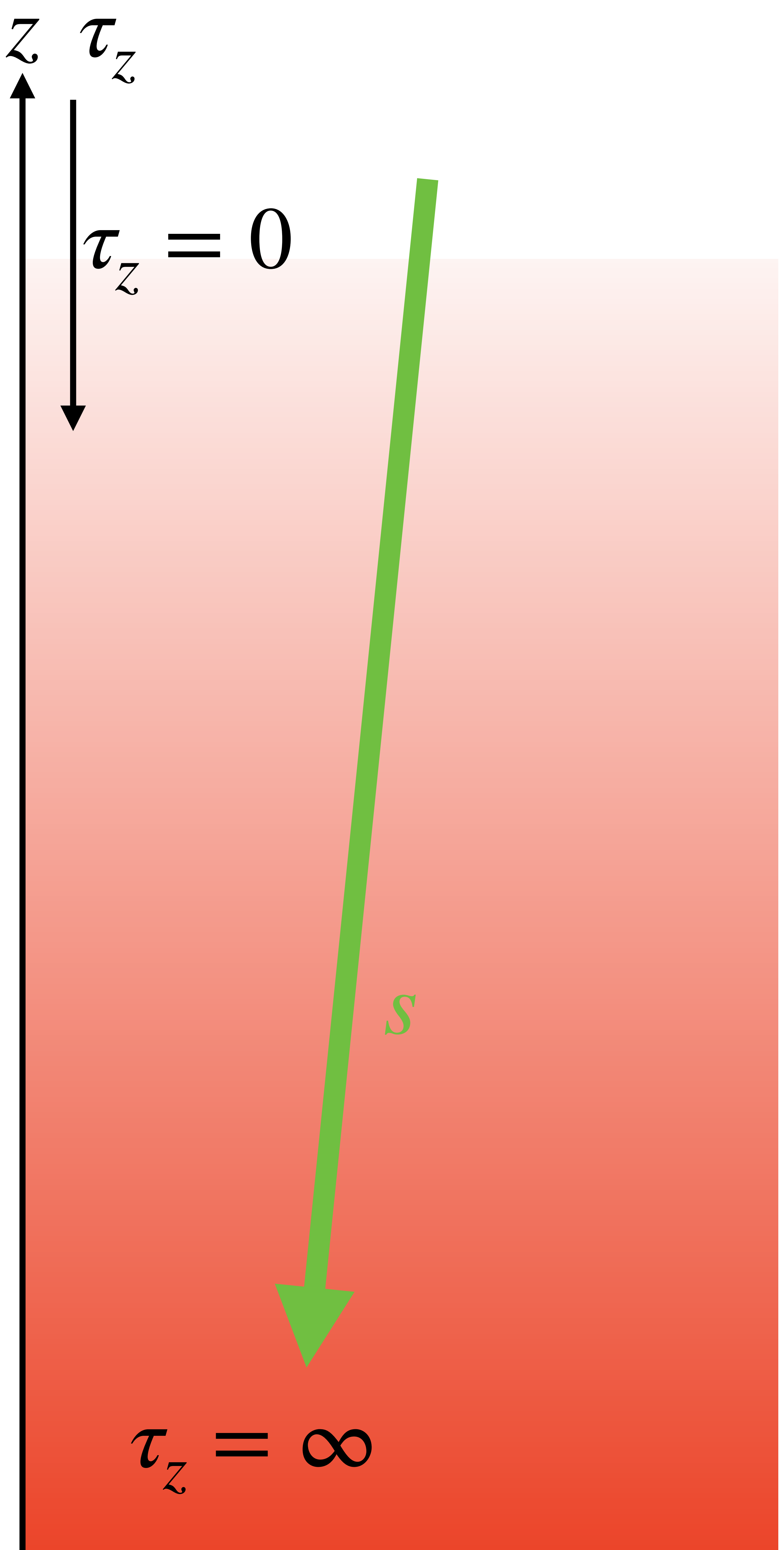


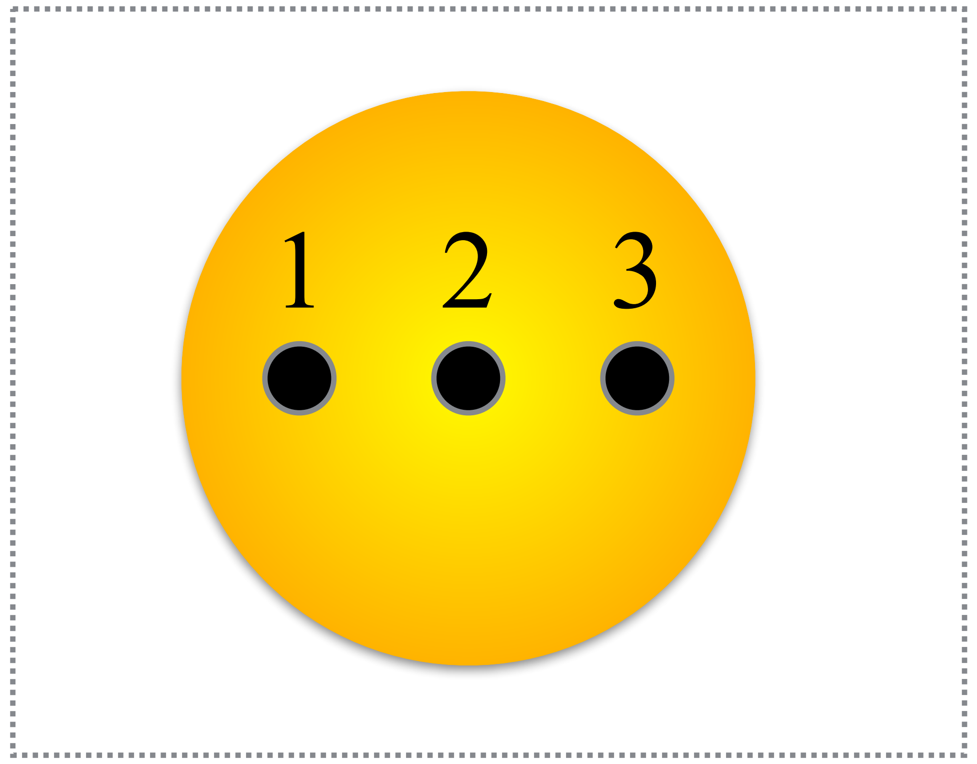
A semi-infinite flat atmosphere

$$I(\tau_z, u) = I_o(u)\ \mathrm{e}^{\frac{\tau_z - \tau_o}{u}} + \int_{\tau'_z=\tau_z}^{\tau'_z=\tau_o} S(\tau'_z)\ \mathrm{e}^{\frac{\tau_z - \tau'_z}{u}}\ d\tau_z$$

The inward ("in") rays ($u < 0$)

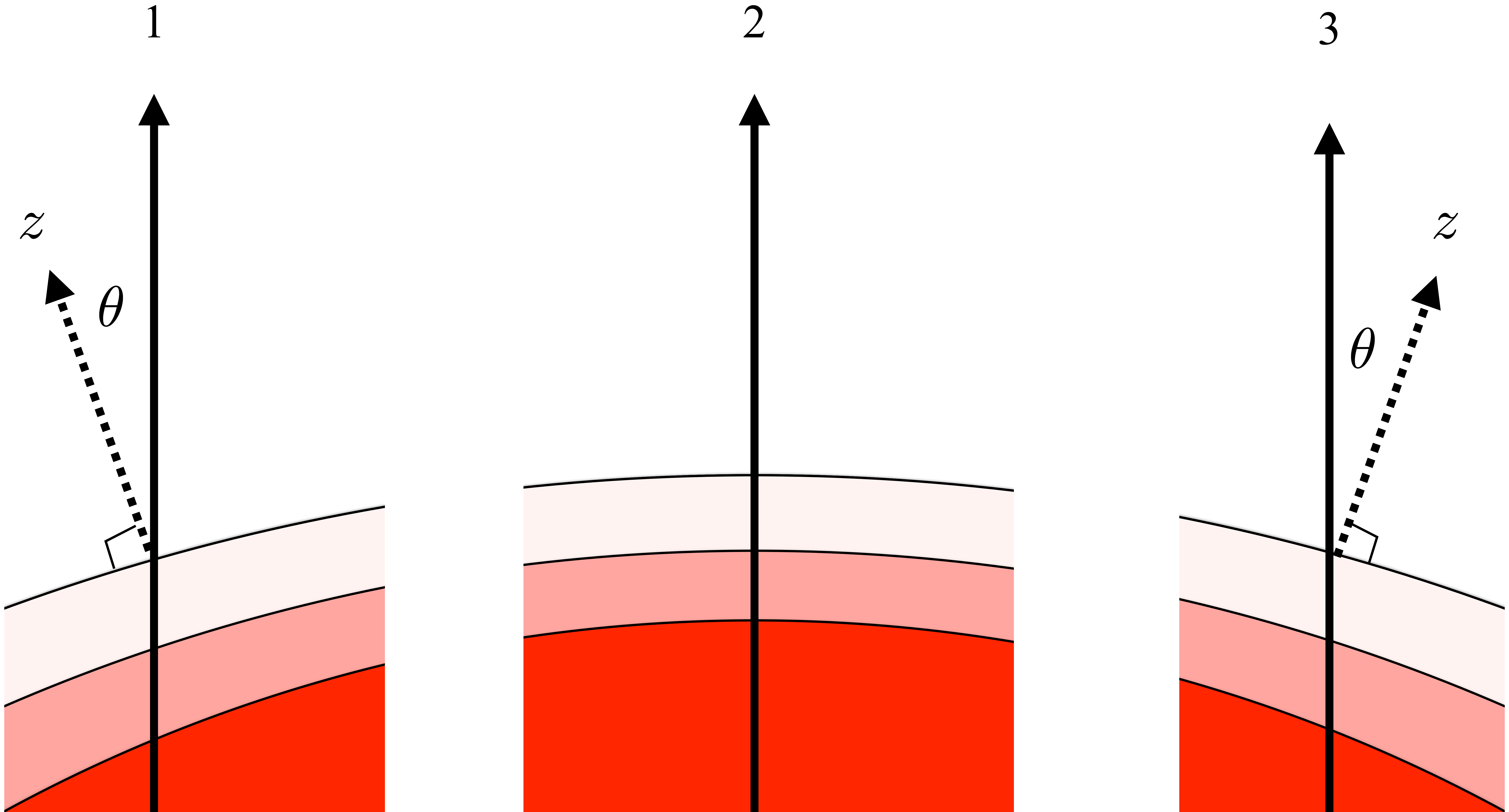
Q: Is there a place where we know intensity and/or optical depth?

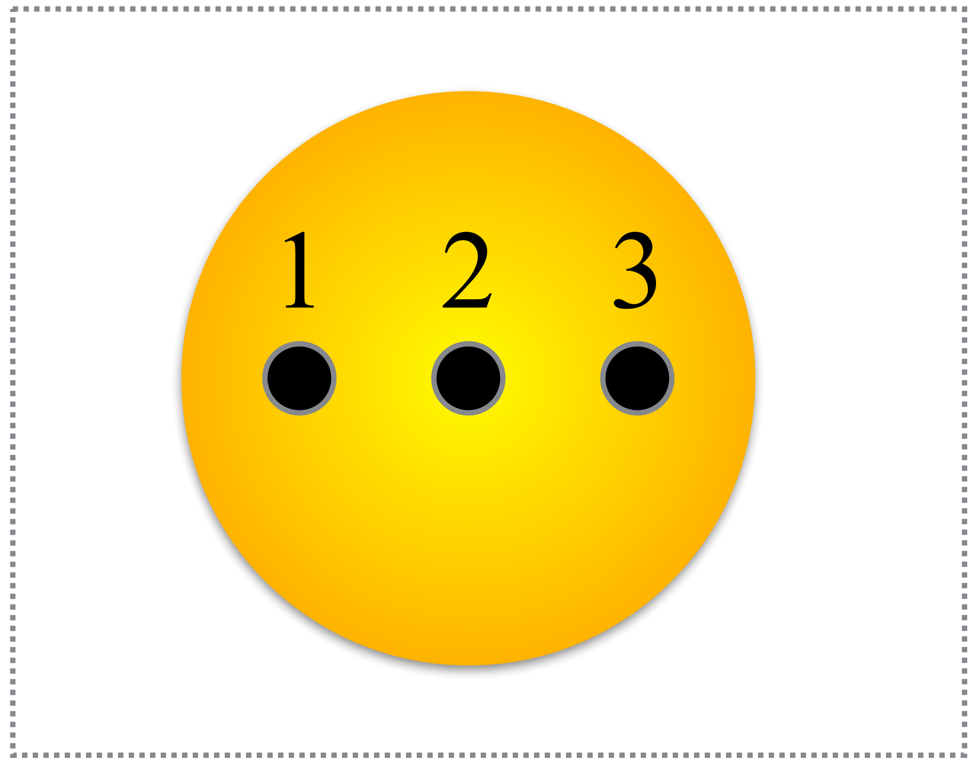




Reminder!!

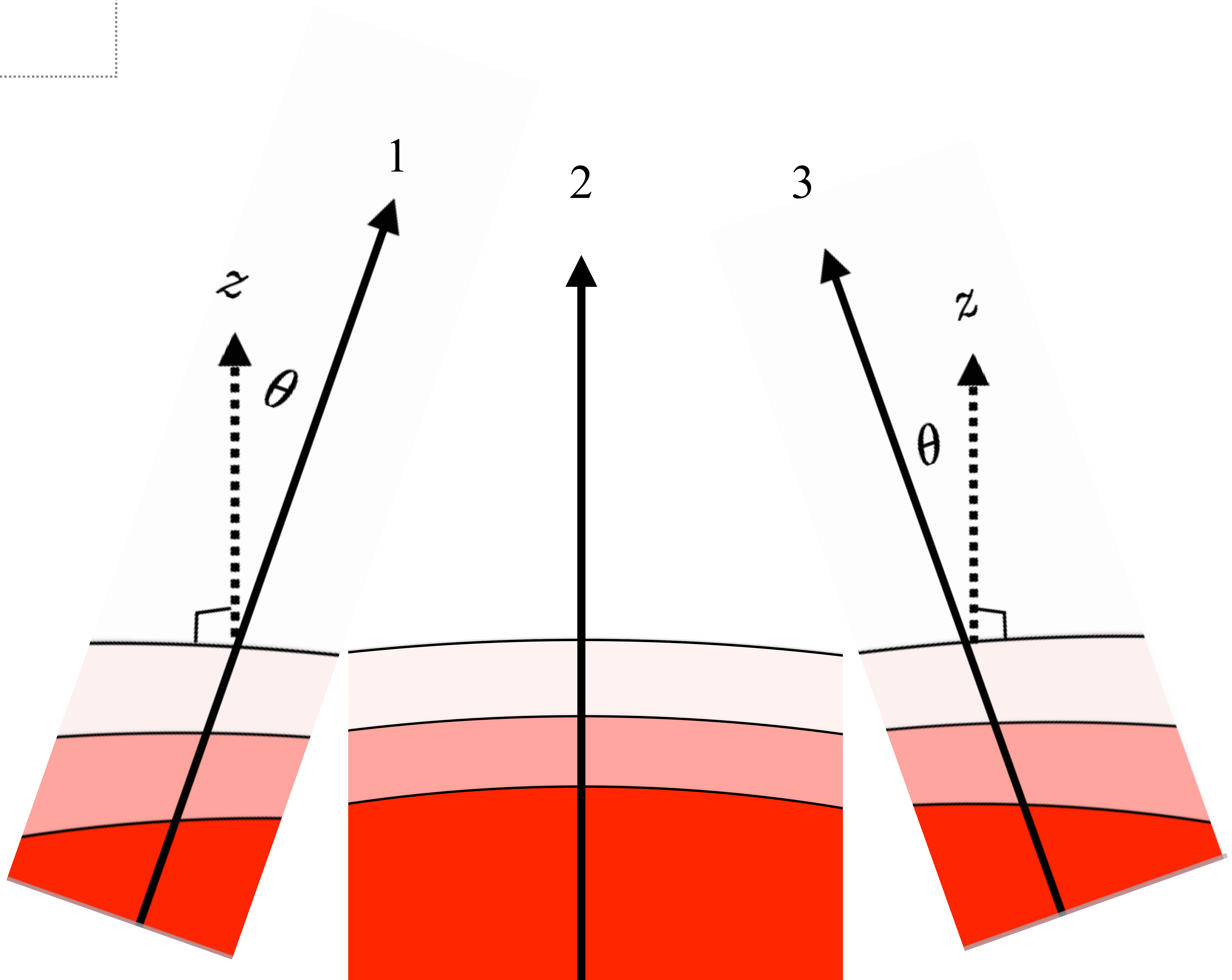
Can we see the "inward" rays?





Reminder!!

Can we see the “inward” rays?



A semi-infinite flat atmosphere

$$I(\tau_z, u) = I_o(u)\ e^{\frac{\tau_z - \tau_o}{u}} + \int_{\tau_z'=\tau_z}^{\tau_z'=\tau_o} S(\tau_z')\ e^{\frac{\tau_z - \tau_z'}{u}}\ d\tau_z$$

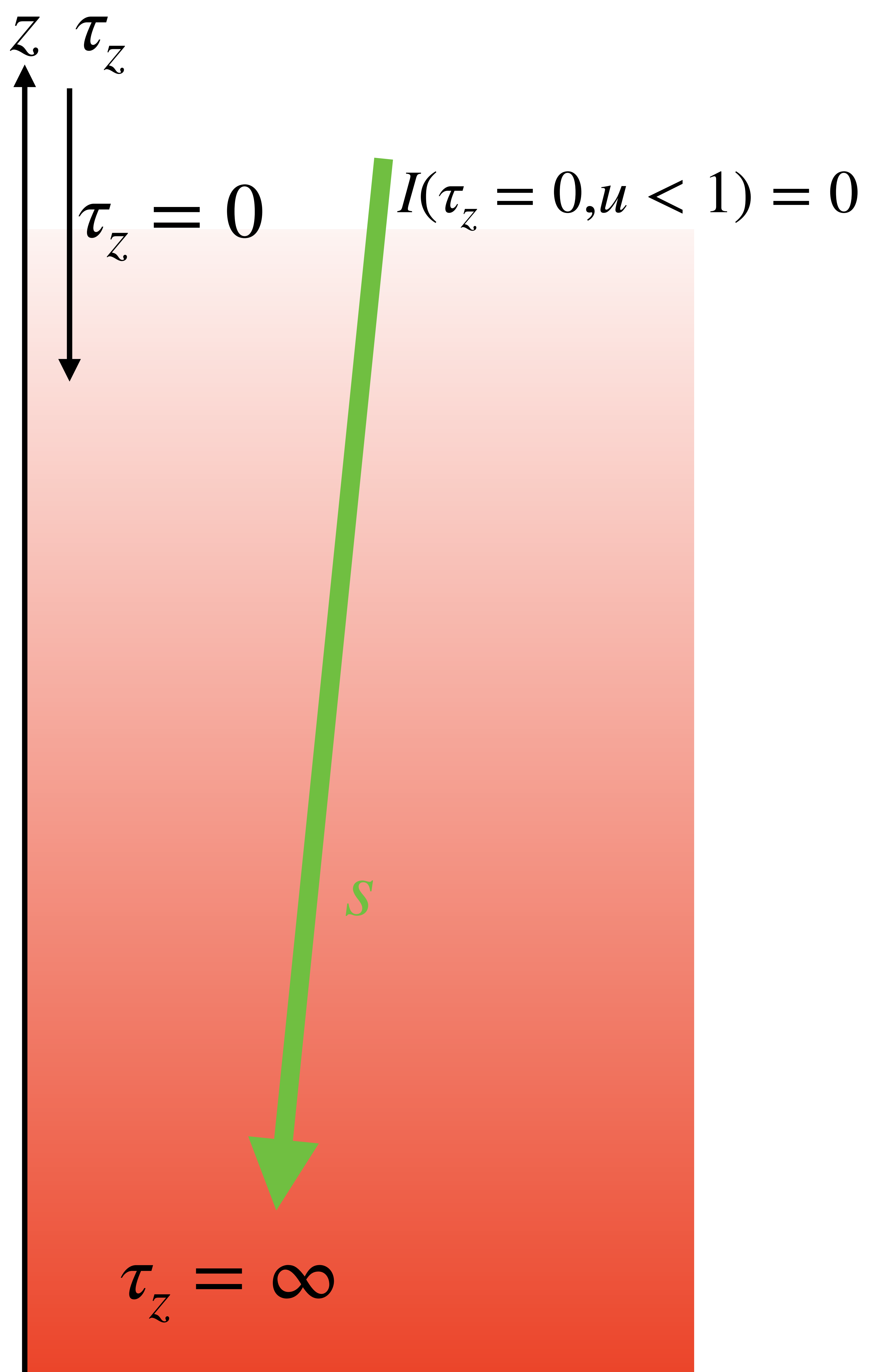
The inward ("in") rays ($u < 0$)

Q: Is there a place where we know intensity and/or optical depth?

$$I(\tau_z, u < 1) = \int_{\tau_z'=\tau_z}^{\tau_z'=0} S(\tau_z')\ e^{\frac{\tau_z - \tau_z'}{u}}\ d\tau_z$$

In the textbook, they then flip the bounds:

$$I(\tau_z, u < 1) = -\int_{\tau_z'=0}^{\tau_z'=\tau_z} S(\tau_z')\ e^{\frac{\tau_z - \tau_z'}{u}}\ d\tau_z$$



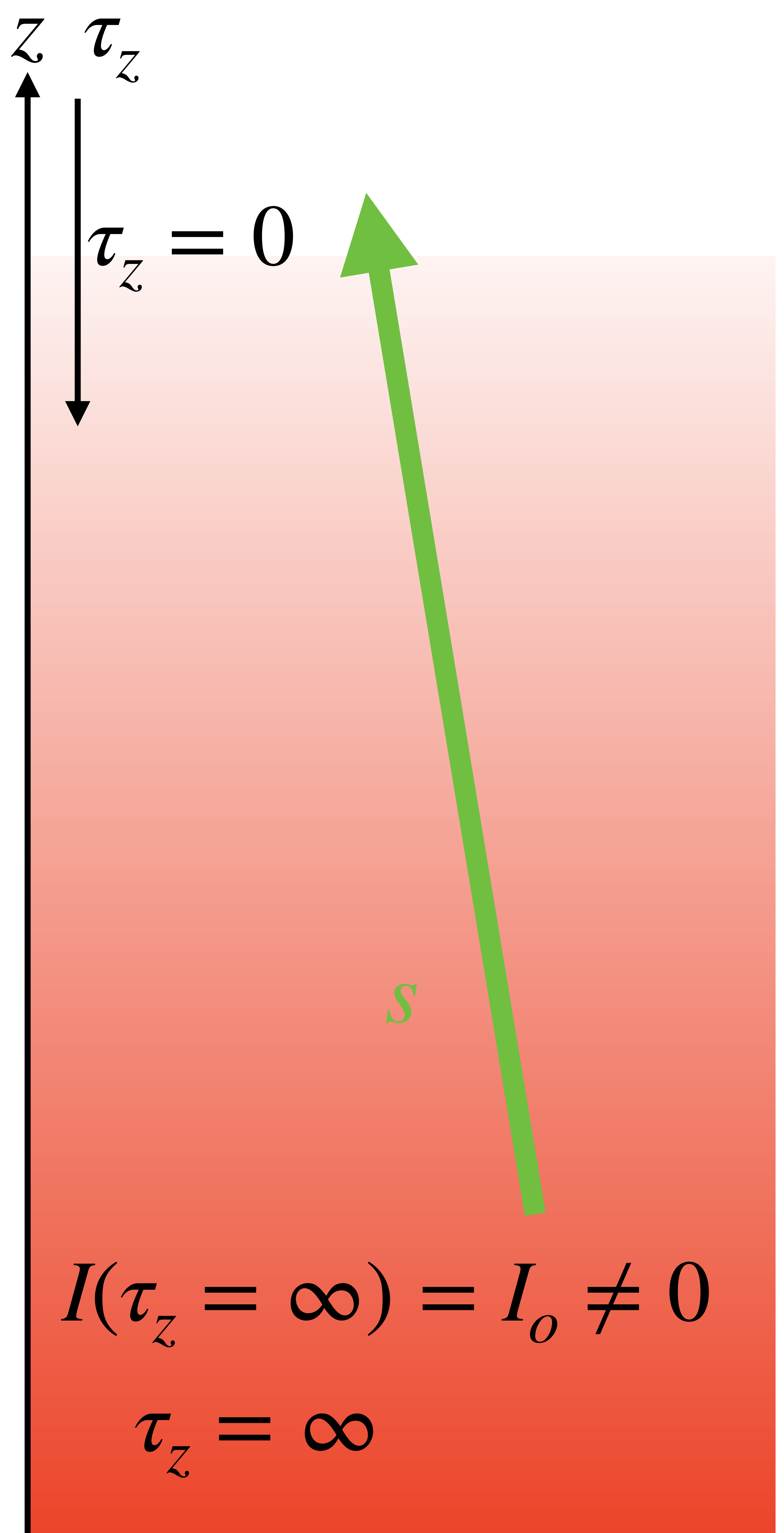
A semi-infinite flat atmosphere

$$I(\tau_z, u) = I_o(u)\ \mathrm{e}^{\frac{\tau_z - \tau_o}{u}} + \int_{\tau_z'=\tau_z}^{\tau_z'=\tau_o} S(\tau_z')\ \mathrm{e}^{\frac{\tau_z - \tau_z'}{u}}\ d\tau_z$$

The ourward ("out") rays ($u > 0$)

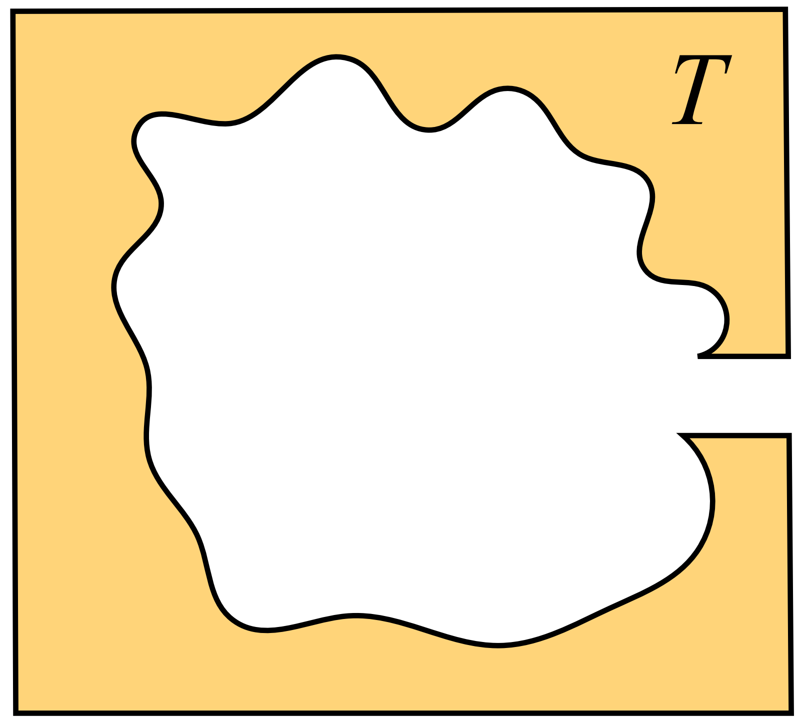
Q: Is there a place where we know intensity and/or optical depth?

$$I(\tau_z, u > 1) = \int_{\tau_z'=\tau_z}^{\tau_z'=\infty} S(\tau_z')\ \mathrm{e}^{\frac{\tau_z - \tau_z'}{u}}\ d\tau_z$$



But!

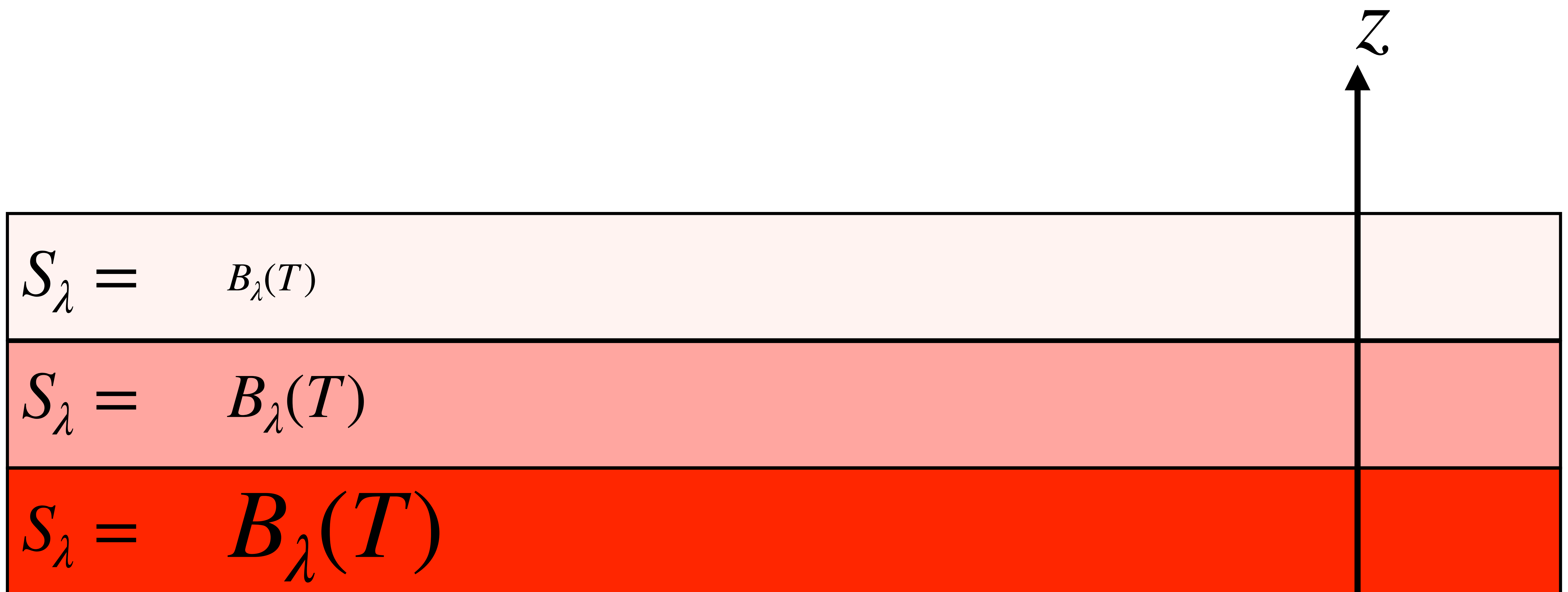
The concept of “Limb-darkening”



Intensity of a black body = Planck function B_λ or B_ν

A stellar atmosphere is not a perfect Blackbody
(not at a single temperature!)

In Local Thermodynamical Equilibrium (LTE),
(where the particles interact only with their peers)
if pure absorption is more important than scattering:
$S(z)$ = Planck function $B_\lambda(T(z))$



Remember this?

5. At home: Formal solution with source function that increases linearly with optical depth

Let's assume that the density in the slab is constant, such that $\kappa\rho = 2.0$ per unit length.

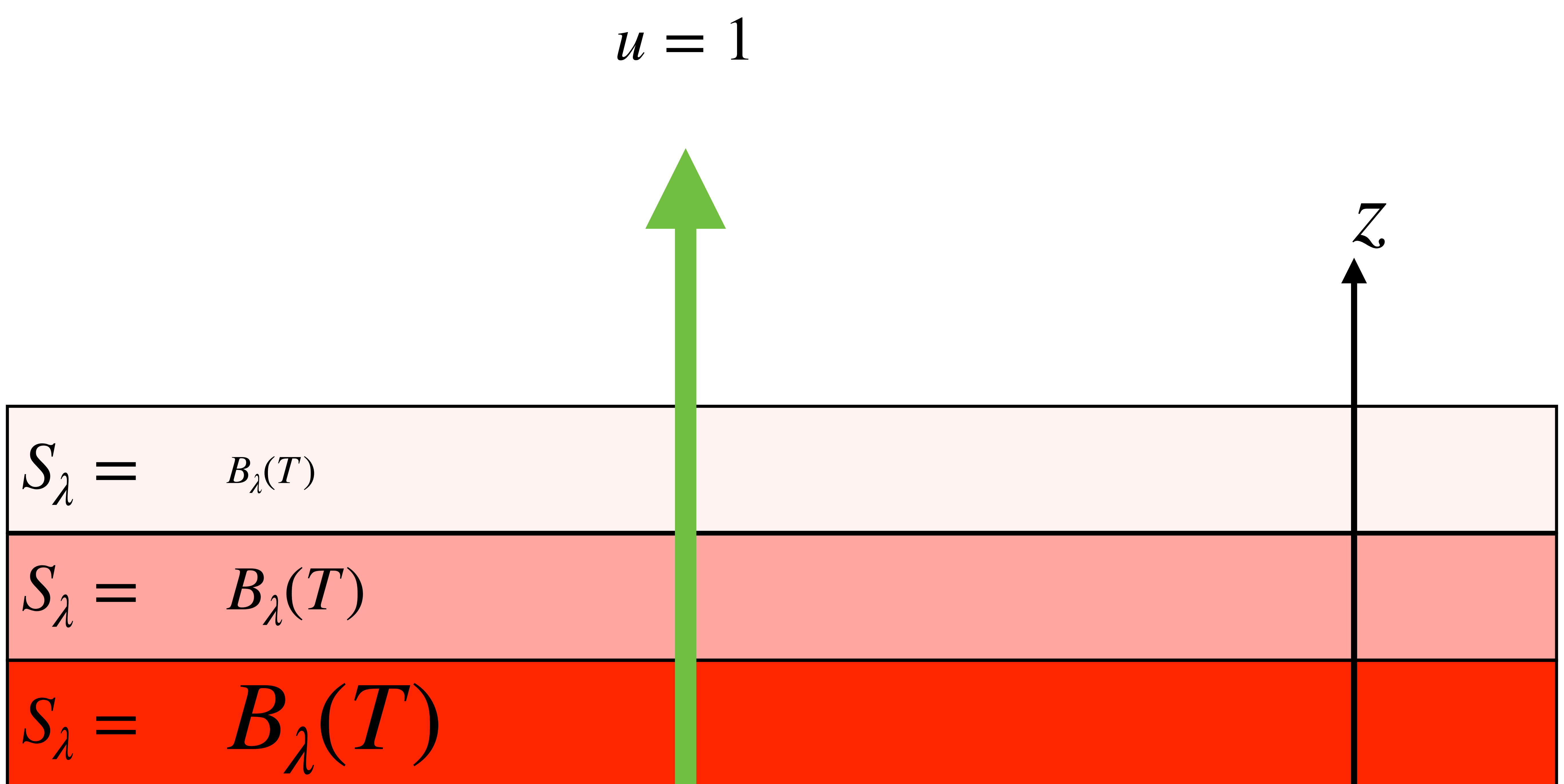
The source function is a function of τ such that:

$$S(\tau) = S_0 + S_1\tau$$

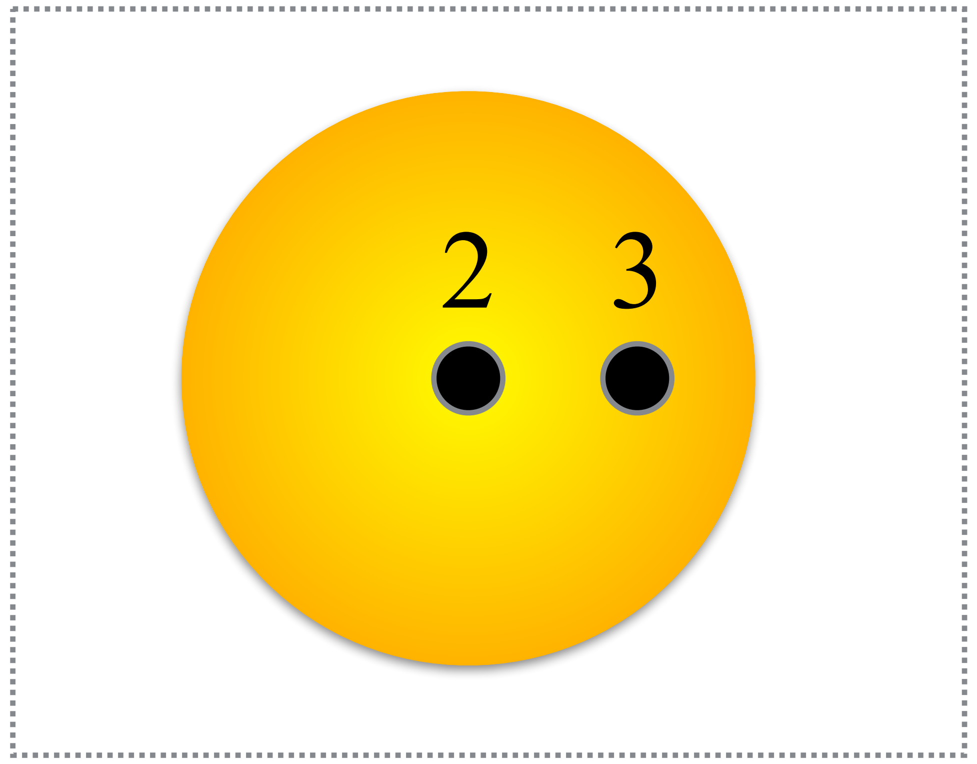
where $S_0 = 0.5$ intensity unit, and $S_1 = 1.3$ intensity units per optical depth unit.

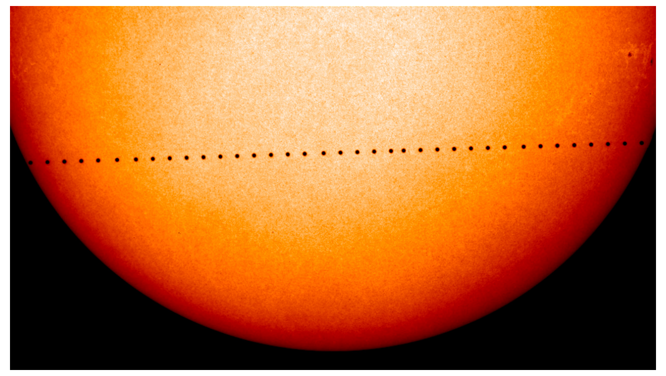
There is no intial intensity entering the slab so $I_o = 0$.

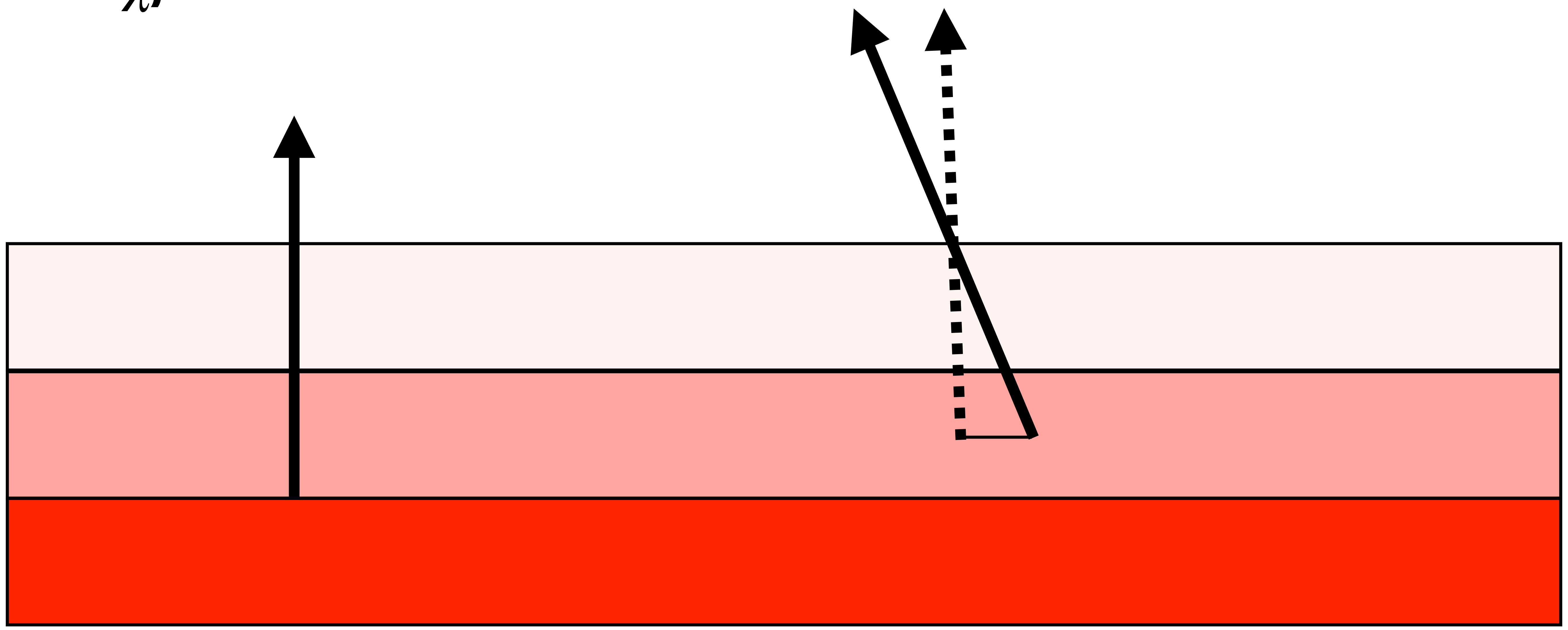
Prepare your code such that you can vary the values of the paramters.

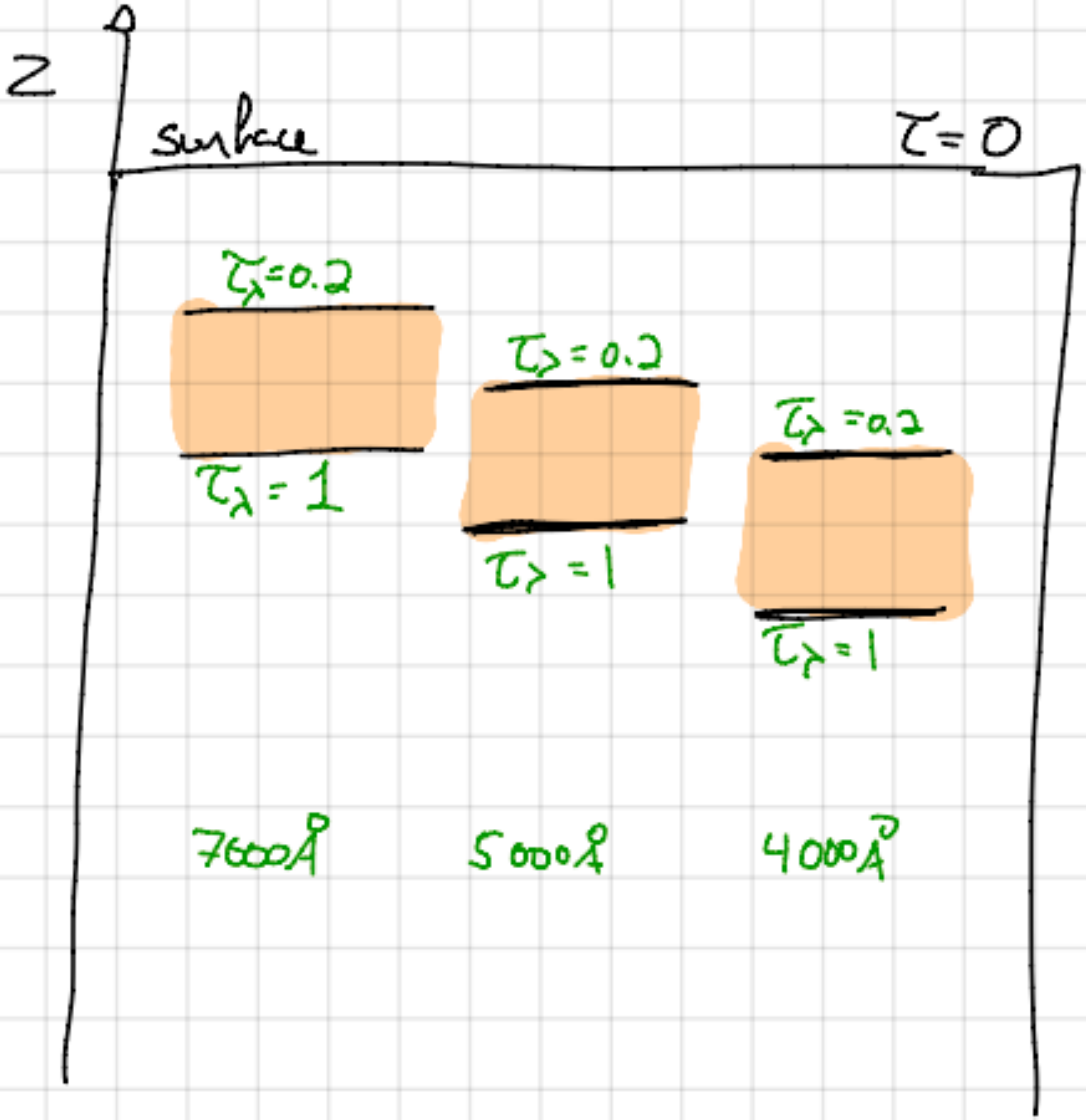


$$d\tau_\lambda = \kappa_\lambda \rho ds$$

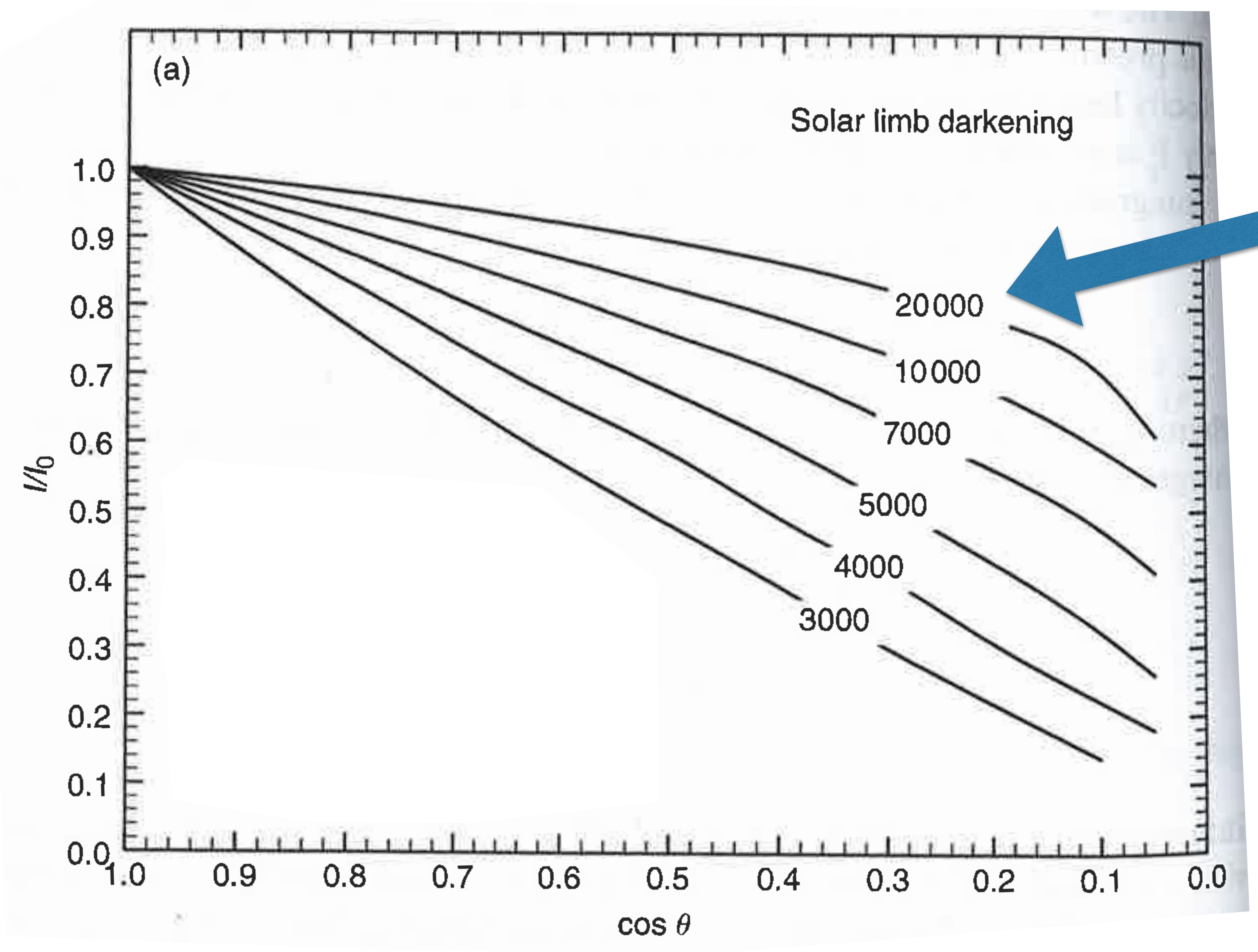






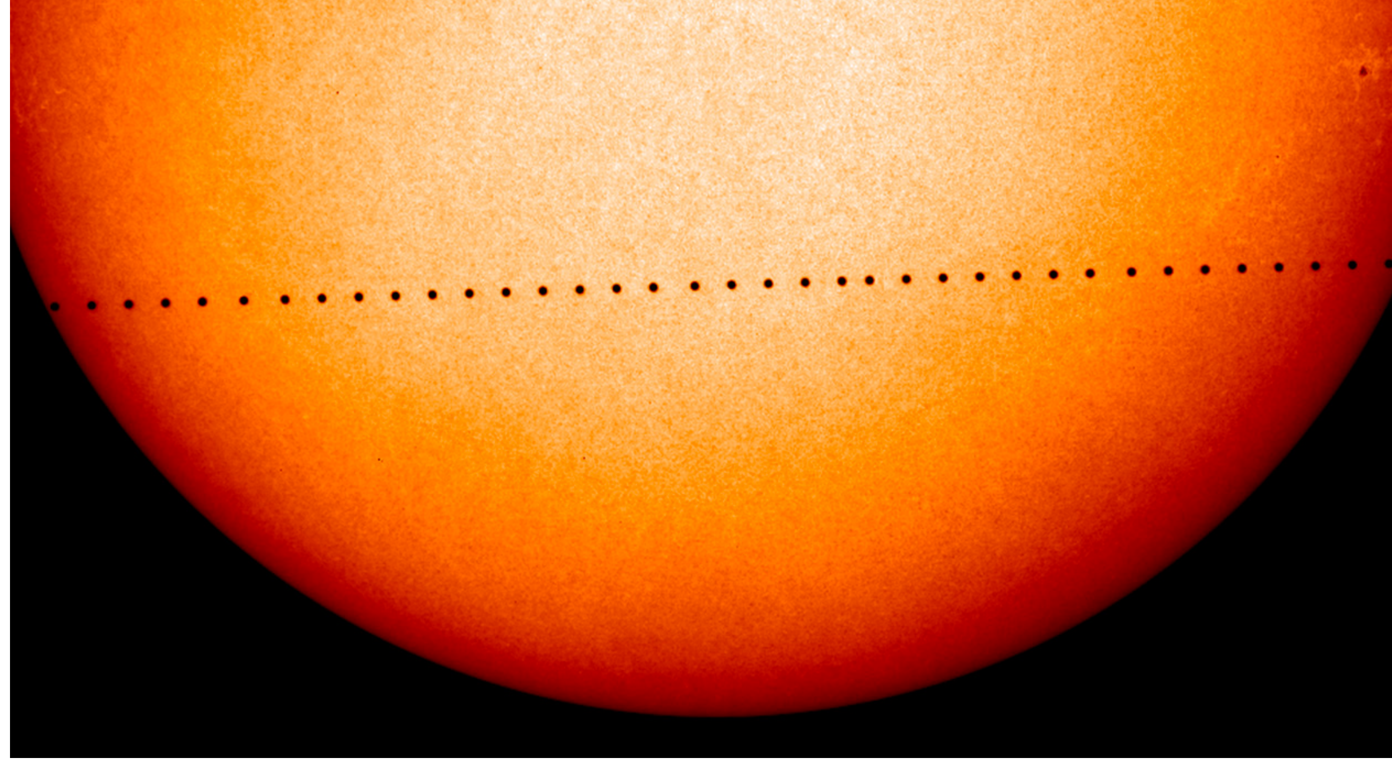


z
surface
$\tau = 0$
$\tau_\lambda = 0.2$
$\tau_\lambda = 1$
$\tau_\lambda = 0.2$
$\tau_\lambda = 1$
$\tau_\lambda = 0.2$
$\tau_\lambda = 1$
7600Å
5000Å
4000Å



Lambda in A
(10 A/nm)

Intensity for a flat, semi-infinite atmosphere



out

$$I(\tau, u > 0) = \int_{\tau'=\tau}^{\infty} \boxed{S(\tau')} \mathrm{e}^{\frac{\tau - \tau'}{u}} \frac{d\tau'}{u}$$

Means we can find S(τ) (and T(z))

in

$$I(\tau, u < 0) = -\int_{\tau'=0}^{\tau} \boxed{S(\tau')} \mathrm{e}^{\frac{\tau - \tau'}{u}} \frac{d\tau'}{u}$$

For the Sun, we can measure!