Worksheet L16: Grey atmospheres

Goal of this worksheet: find the wavelength dependent flux (the spectrum!) predicted by our analytical estimate for the temperature structure of a star (with a given effective temperature.

$$F_\lambda(\tau) = -2\pi \int_{\tau'=0}^{\tau} S_\lambda(\tau') E_2(\tau - \tau') d\tau' + 2\pi \int_{\tau'=\tau}^{\infty} S_\lambda(\tau') E_2(\tau' - \tau) d\tau'$$

We know that that $S_\lambda(\tau) = B_\lambda(T(\tau))$, that $B_\nu(T) = \frac{2h}{c^2}\nu^3 \frac{1}{e^{h\nu/kT} - 1}$, and that $T^4(\tau_z) = \frac{3T_{\rm eff}^4}{4}(\tau_z + q(\tau_z))$.

Step1: We will do a change of variable where $\alpha = \frac{h\nu}{kT_{\rm eff}} = \frac{hc}{\lambda k T_{\rm eff}}$. Find an expression for $B_\alpha(T)$ (remember that $B_\alpha d\alpha = B_\nu d\nu$).

Step2: Replace the source function in the equation for the flux by your expression for B_α in Step 1. This means that the resulting quantity will be $F_\alpha(\tau)$. Make an additional substitution so that $p(\tau) = T_{\rm eff}/T(\tau)$. Pull all of the constant quantities outside of the integrals.

Step3: Check your answer with the solution, and replace the term in [ ] with $C(\alpha, \tau)$.

Step4: In the notebook, we will be interested to make a graph of $F_\lambda/\tilde{F}$ (at a given optical depth). First, let's find $\tilde{F}$. We know what the wavelength-integrated flux will be $\sigma T_{\rm eff}^4$ at all layers. Find the definition of σ in terms of fundamental constants (see the BB lecture — there will be some cs, hs, πs, etc in there).

Step 5: Now find $F_\alpha/\tilde{F}$ by dividing your expression for F_α from Step 3 by the expression for $\sigma T_{\rm eff}^4$ from Step 4 (a whole bunch of stuff should cancel out!)

Step 6: Now, we need to convert $F_\alpha/\tilde{F}$ into $F_\lambda/\tilde{F}$. Find the factor you need to multiply $F_\alpha/\tilde{F}$ by to do this.

Step7: Check your answer on the solution.

Step 8: I lied a bit — in fact we will actually graph F_λ, and not $F_\lambda/\tilde{F}$. If you have already calculated $F_\lambda/\tilde{F}$ in your code (after all the expression you found in Step7 is very neat to type in python!), how can you quickly transform an array containing $F_\lambda/\tilde{F}$ into an array containing F_λ? (Hint: in the astropy python package, σ is const.sigma_sb; 'SB' stands for Stefan-Boltzmann).