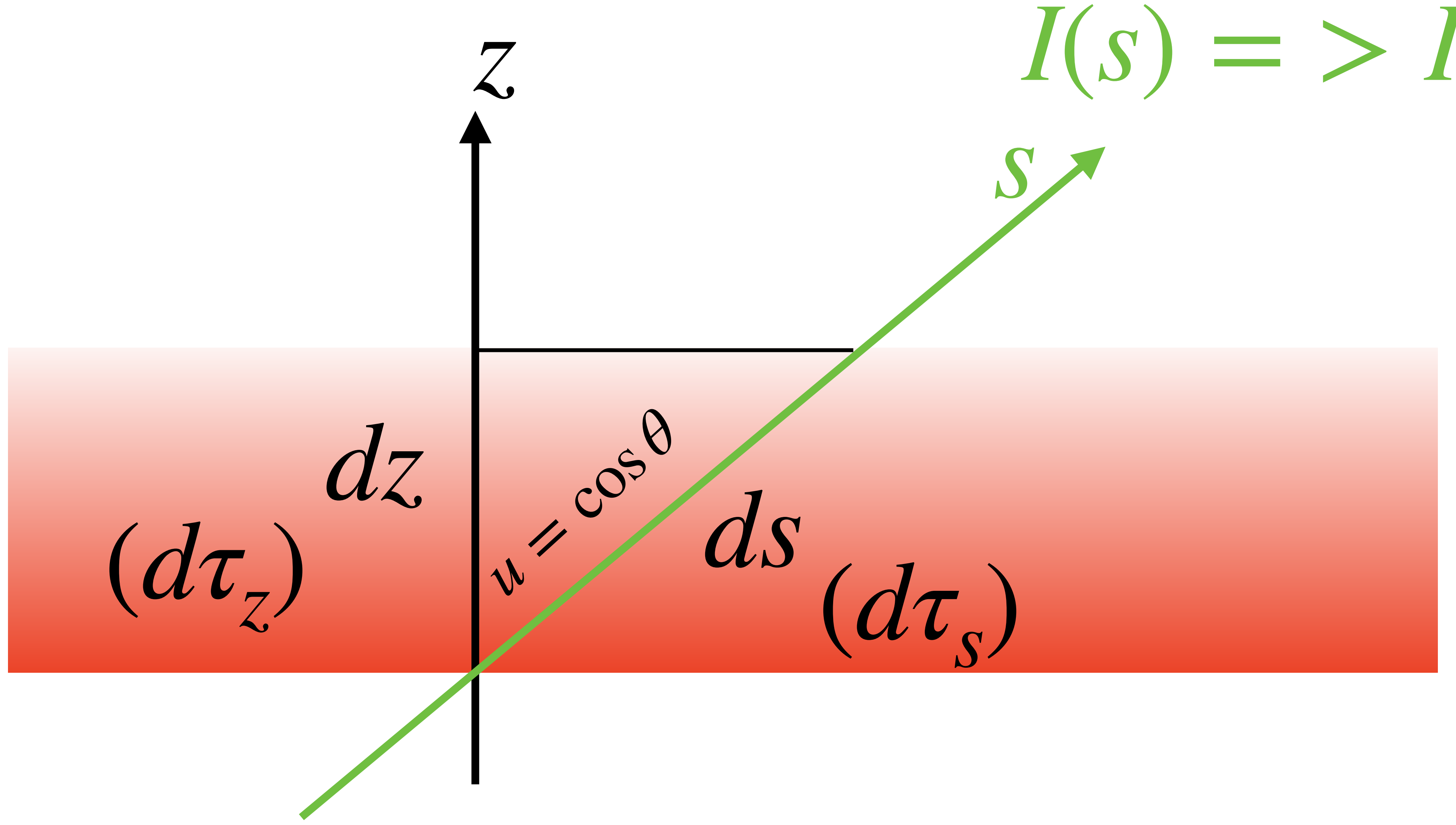
Week 7 Tuesday
L-13
Limb darkening

Re-writing the formal solution in terms of z and u

$$I(s) => I(z, u)$$



$$\frac{d\tau_z}{u} = d\tau_s$$

$$\frac{dI(s)}{d\tau_s} = I(s) - S(s)$$

Change of variable

$$u\frac{dI(z, u)}{d\tau_z} = I(z, u) - S(z)$$

Q: why no u dependence here?

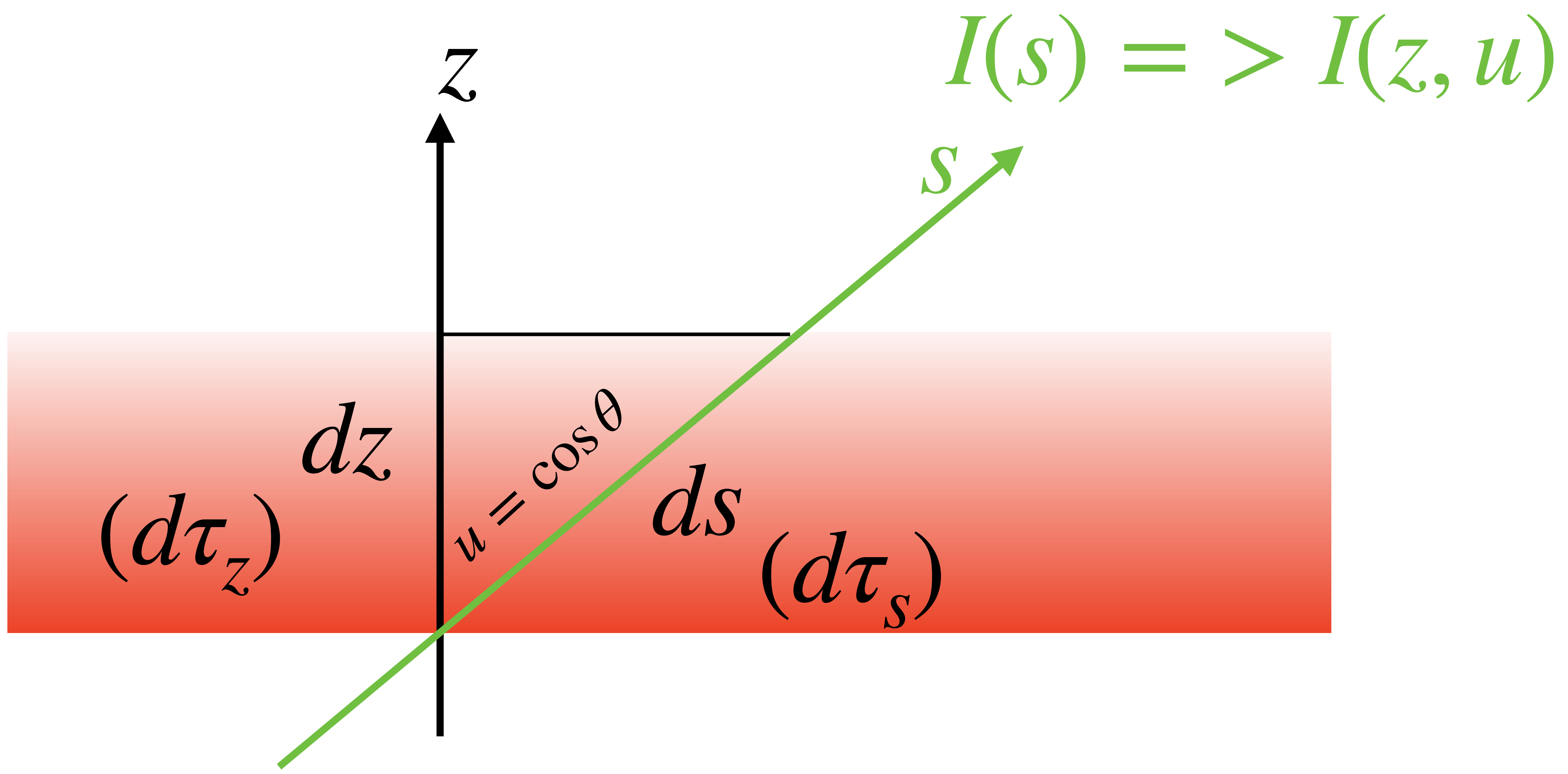
Solving the same way as the general formal solution
from a place we know the intensity $I(\tau_o, u) = I_o(u)$

$$I(\tau_z, u) = I_o(u)\ \mathrm{e}^{\frac{\tau_z - \tau_o}{u}} + \int_{\tau_z'=\tau_z}^{\tau_z'=\tau_o} S(\tau_z')\ \mathrm{e}^{\frac{\tau_z - \tau_z'}{u}}\ d\tau_z$$

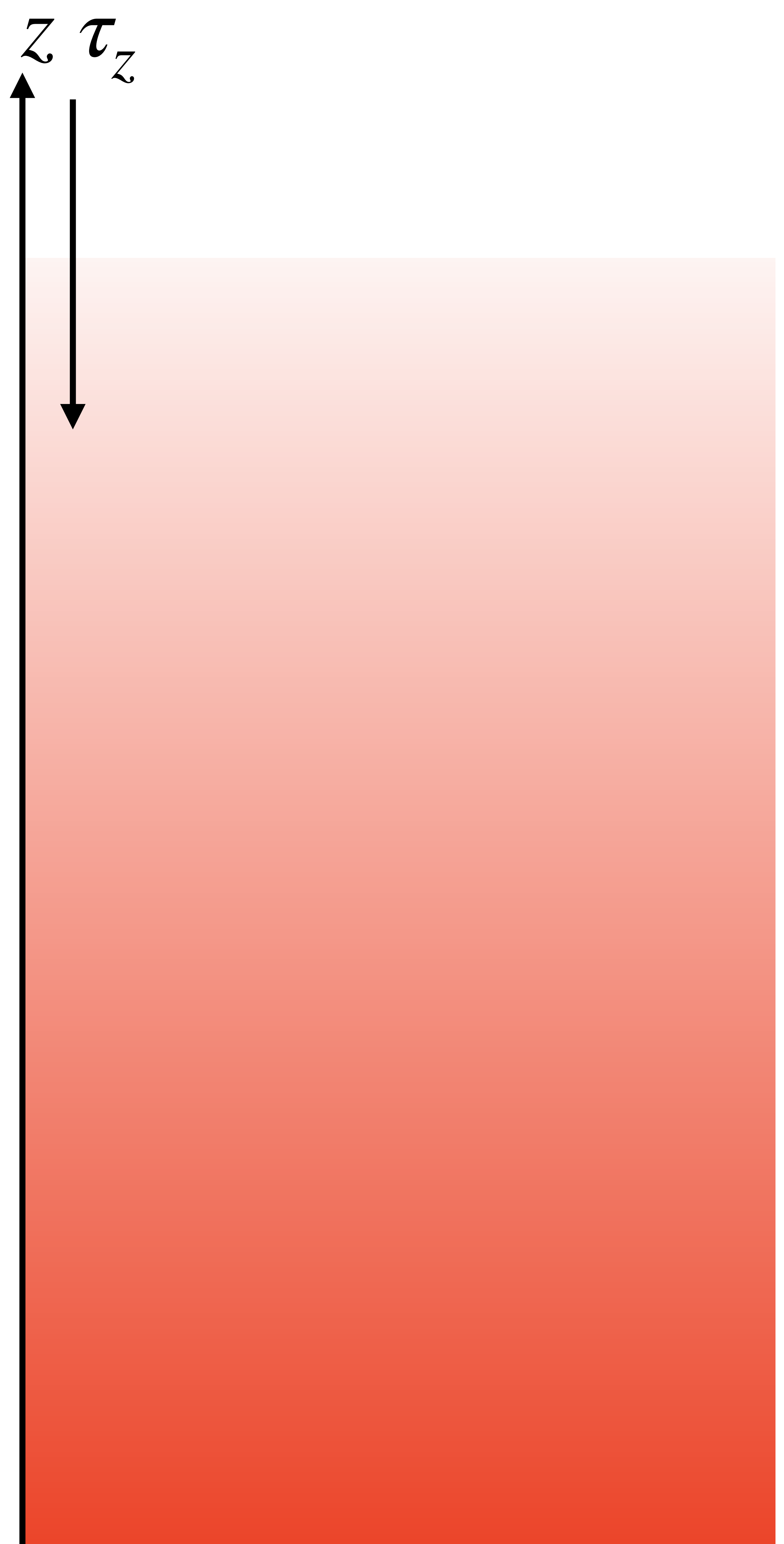
Re-writing the formal solution in terms of z and u

Solving the same way as the general formal solution
from a place we know the intensity $I(\tau_o, u) = I_o(u)$

Okay…… but where in hell is that gonna be?



A semi-infinite flat atmosphere



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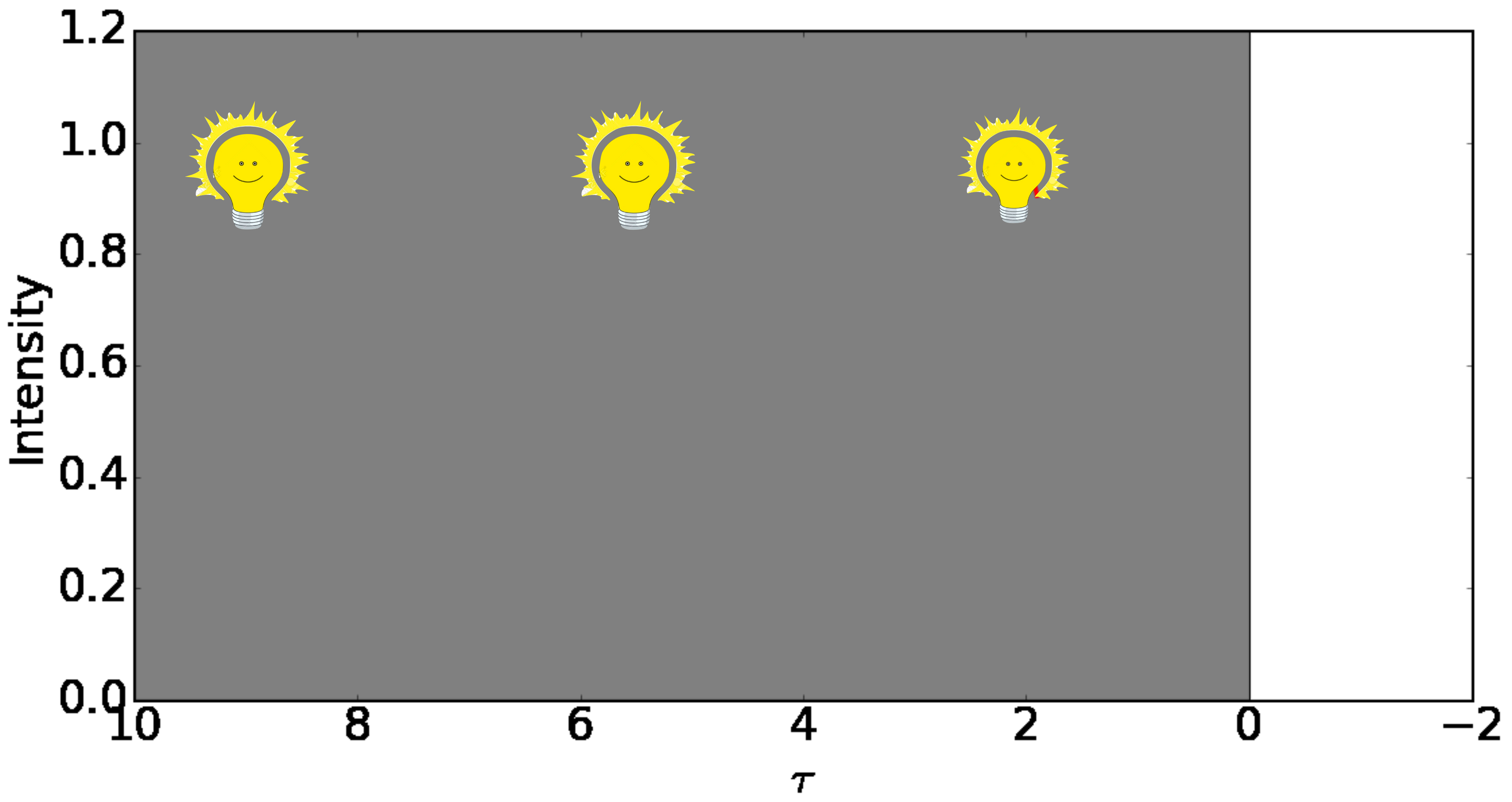
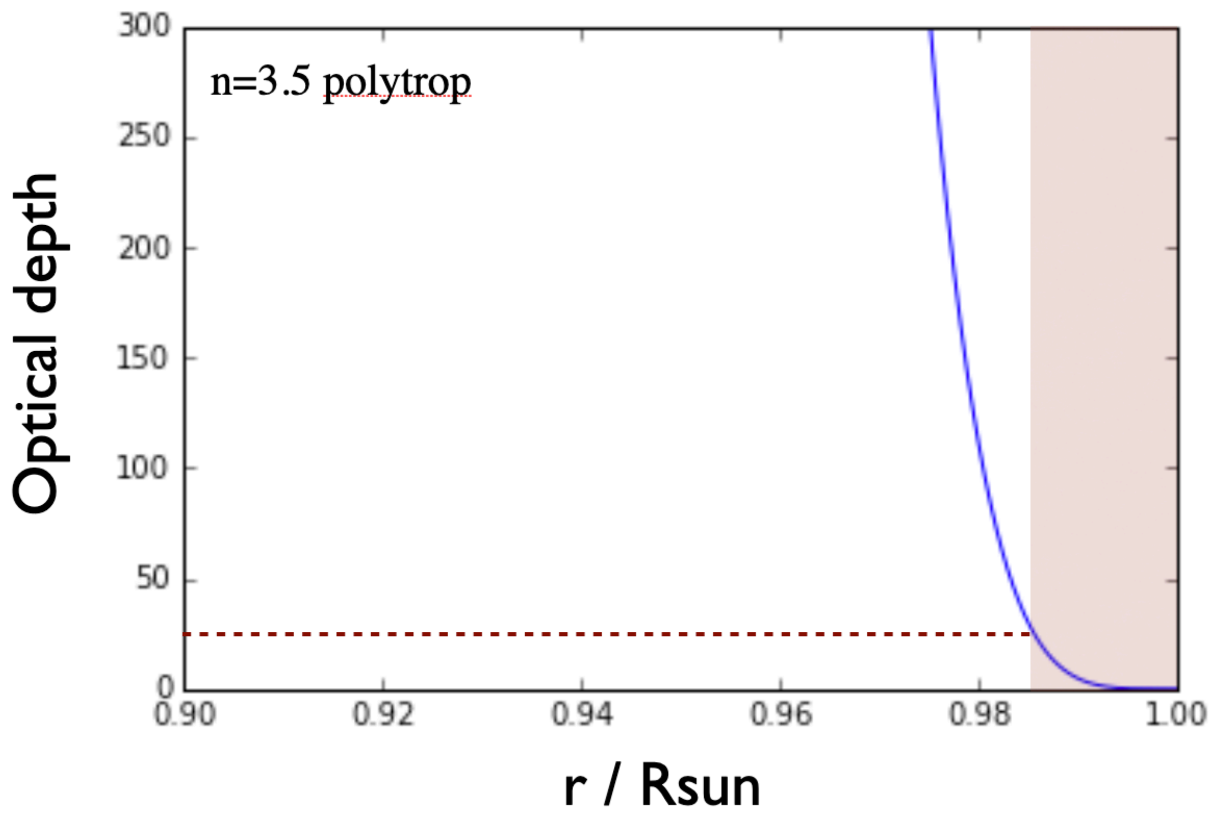
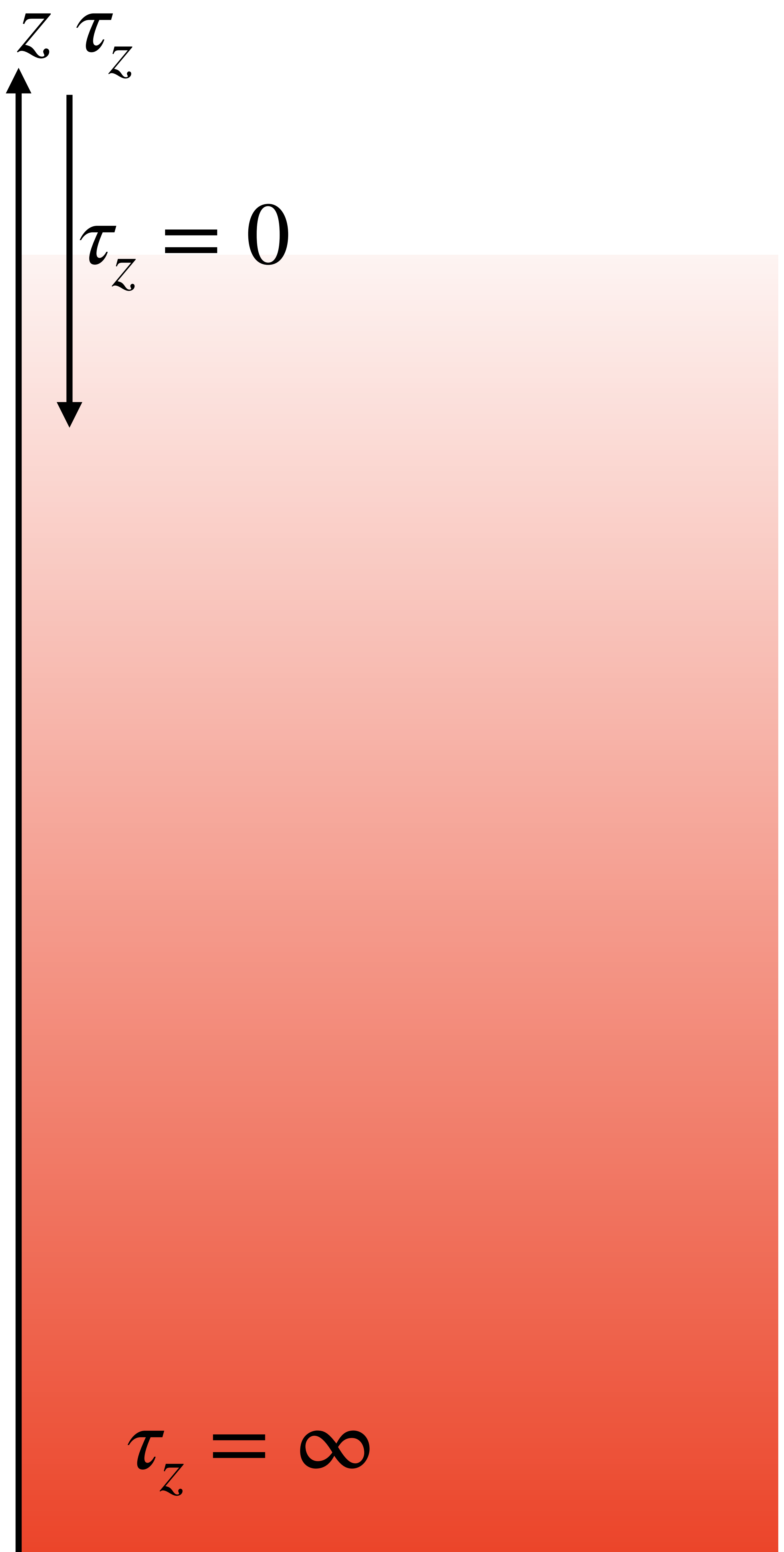




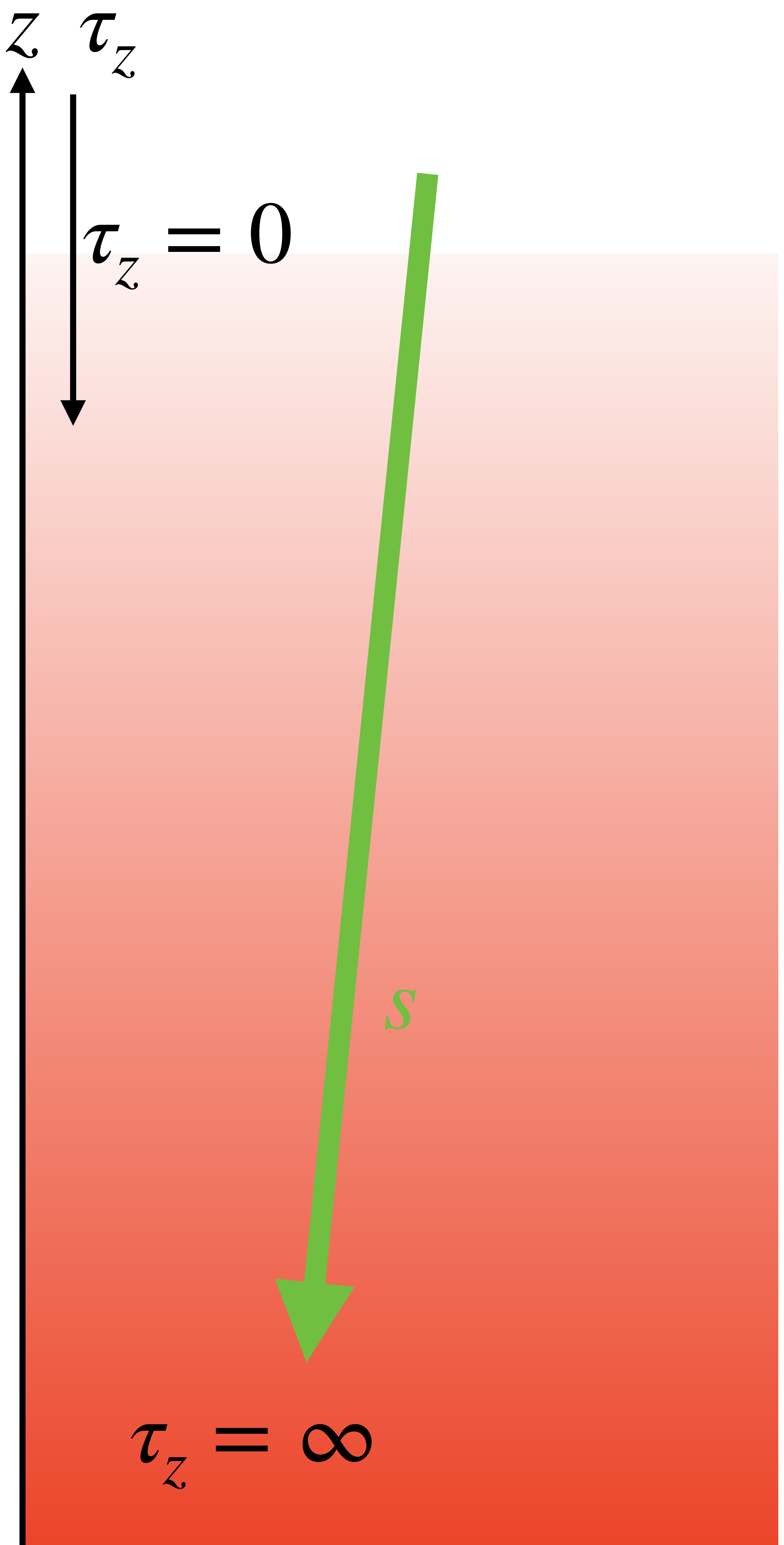


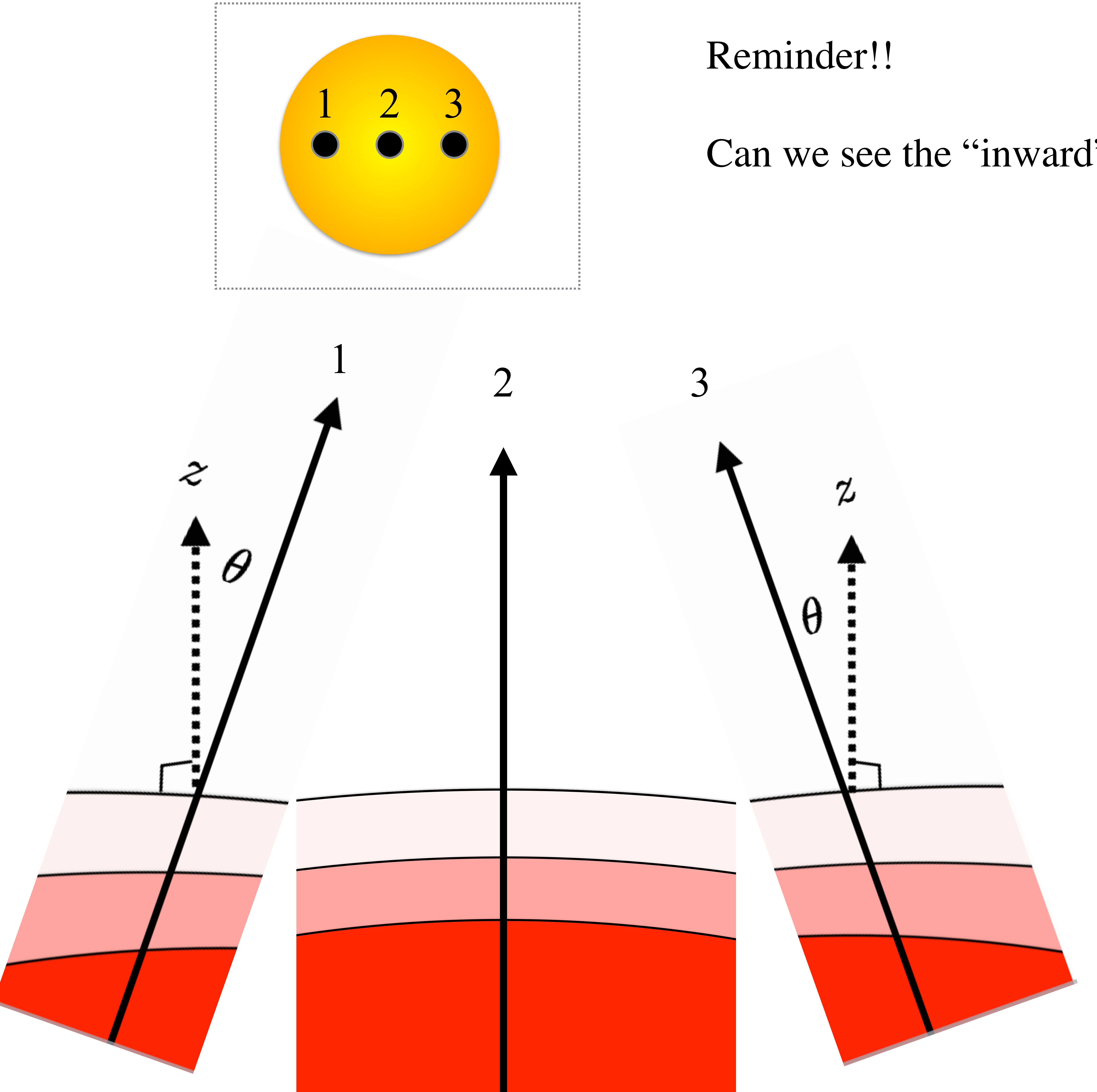
A semi-infinite flat atmosphere

$$I(\tau_z, u) = I_o(u)\ e^{\frac{\tau_z - \tau_o}{u}} + \int_{\tau_z'=\tau_z}^{\tau_z'=\tau_o} S(\tau_z')\ e^{\frac{\tau_z - \tau_z'}{u}}\ d\tau_z$$

The inward ("in") rays ($u < 0$)

Q: Is there a place where we know intensity and/or optical depth?



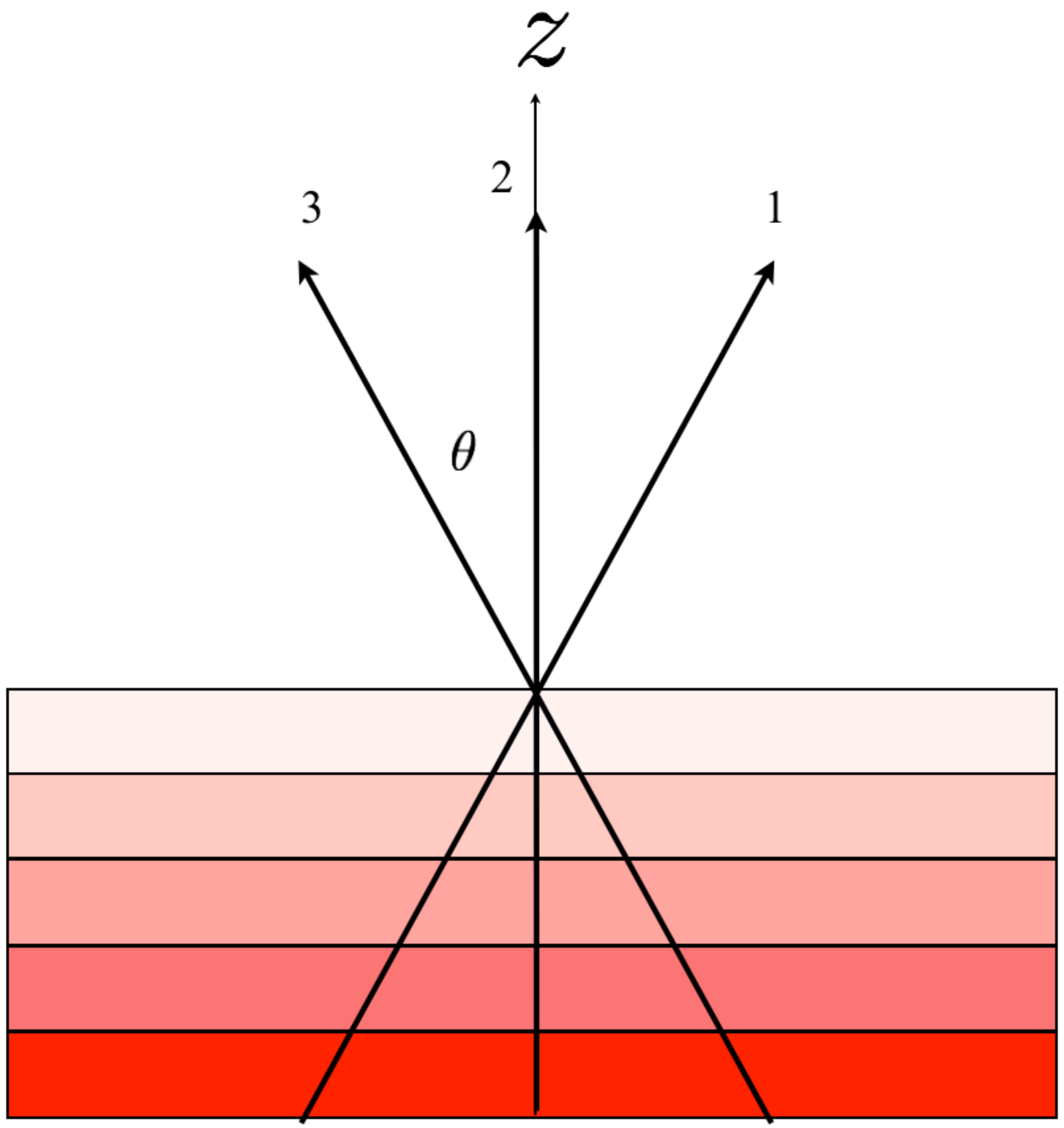


Reminder!!

Can we see the “inward” rays?

Spherical symmetry becomes a symmetry in the z direction:

$$T(r), P(r), \ldots \rightarrow T(z), P(z), \ldots$$



A semi-infinite flat atmosphere

$$I(\tau_z, u) = I_o(u)\, e^{\frac{\tau_z - \tau_o}{u}} + \int_{\tau_z'=\tau_z}^{\tau_z'=\tau_o} S(\tau_z')\, e^{\frac{\tau_z - \tau_z'}{u}}\, d\tau_z$$

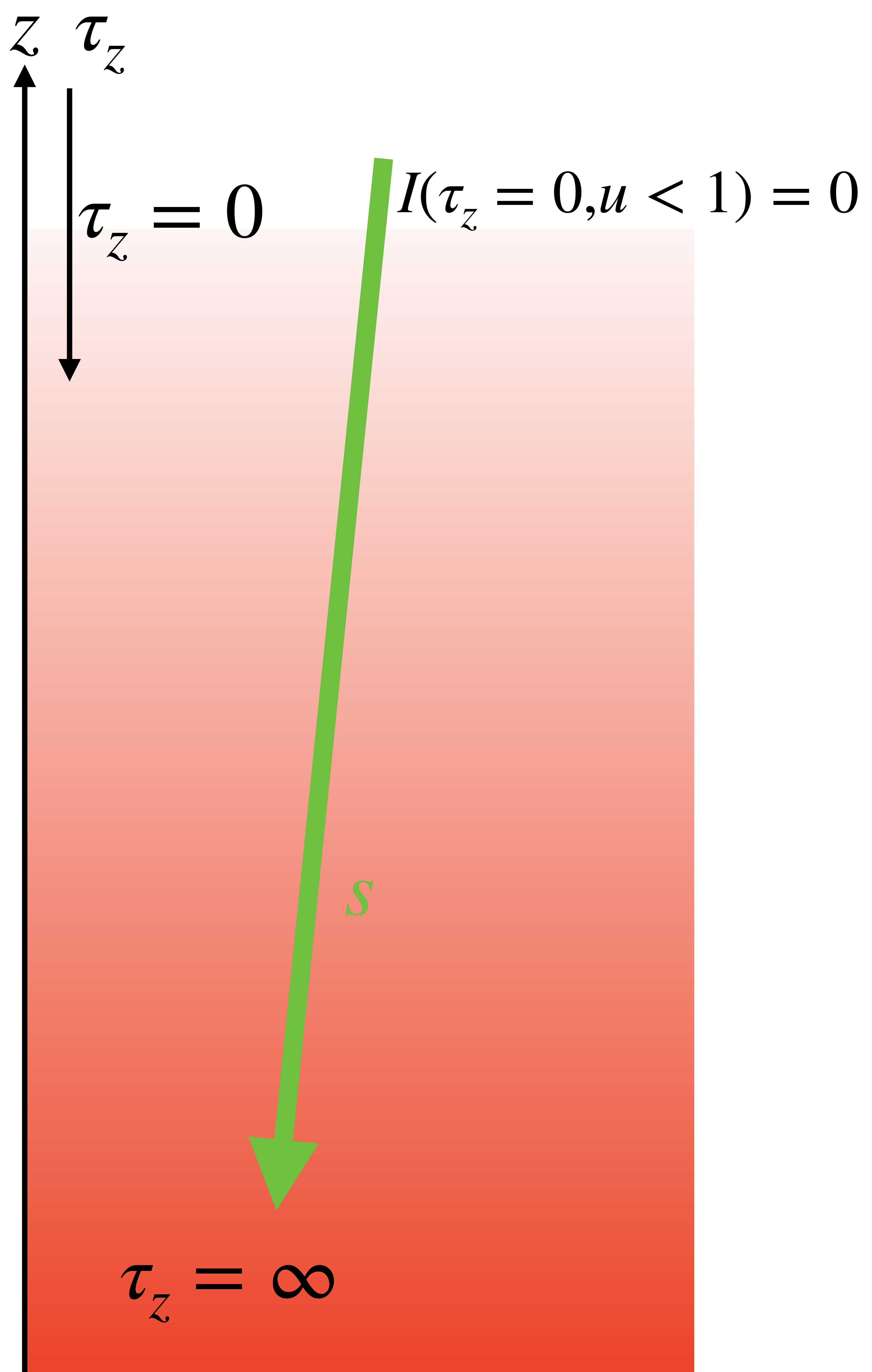
The inward ("in") rays ($u < 0$)

Q: Is there a place where we know intensity and/or optical depth?

$$I(\tau_z, u < 0) = \int_{\tau_z'=\tau_z}^{\tau_z'=0} S(\tau_z')\, e^{\frac{\tau_z - \tau_z'}{u}}\, d\tau_z$$

In the textbook, they then flip the bounds:

$$I(\tau_z, u < 0) = -\int_{\tau_z'=0}^{\tau_z'=\tau_z} S(\tau_z')\, e^{\frac{\tau_z - \tau_z'}{u}}\, d\tau_z$$



A semi-infinite flat atmosphere

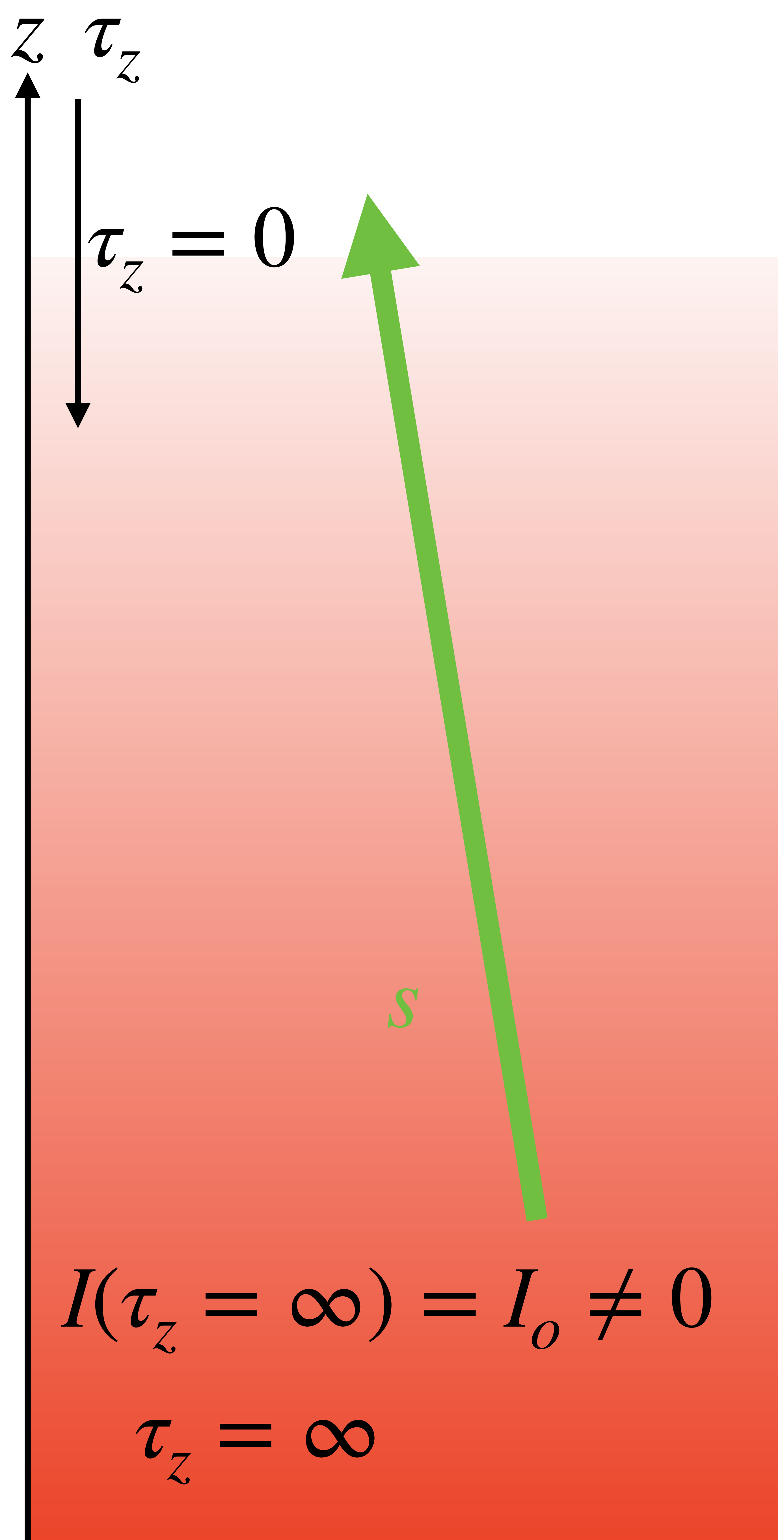
$$I(\tau_z, u) = I_o(u)\ \mathrm{e}^{\frac{\tau_z - \tau_o}{u}} + \int_{\tau_z'=\tau_z}^{\tau_z'=\tau_o} S(\tau_z')\ \mathrm{e}^{\frac{\tau_z - \tau_z'}{u}}\ d\tau_z$$

The ourward (“out”) rays ($u > 0$)

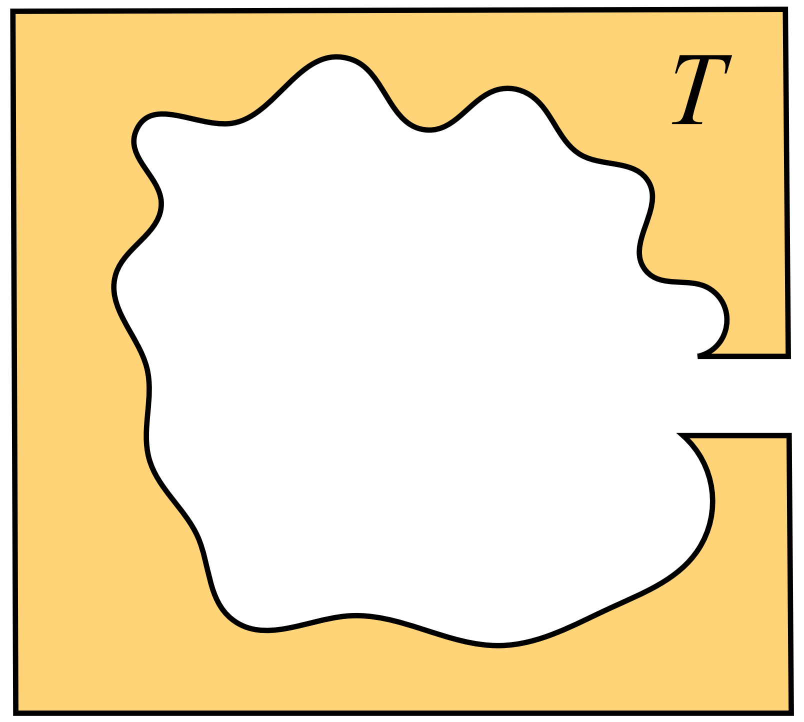
Q: Is there a place where we know intensity and/or optical depth?

$$I(\tau_z, u > 0) = \int_{\tau_z'=\tau_z}^{\tau_z'=\infty} S(\tau_z')\ \mathrm{e}^{\frac{\tau_z - \tau_z'}{u}}\ d\tau_z$$

But!



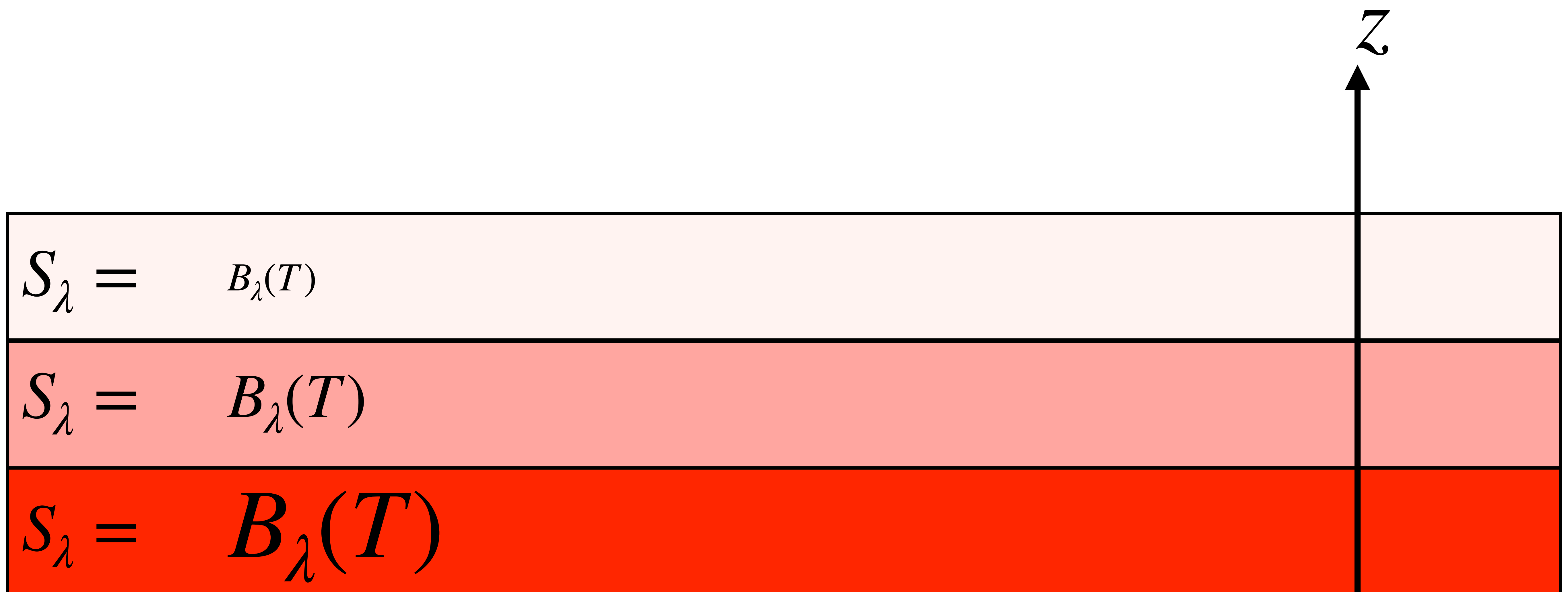
The concept of “Limb-darkening”



Intensity of a black body = Planck function B_λ or B_ν

A stellar atmosphere is not a perfect Blackbody
(not at a single temperature!)

In Local Thermodynamical Equilibrium (LTE),
(where the particles interact only with their peers)
if pure absorption is more important than scattering:
$S(z)$ = Planck function $B_\lambda(\ T(z)\ )$



Remember this?

5. At home: Formal solution with source function that increases linearly with optical depth

Let's assume that the density in the slab is constant, such that $\kappa\rho = 2.0$ per unit length.

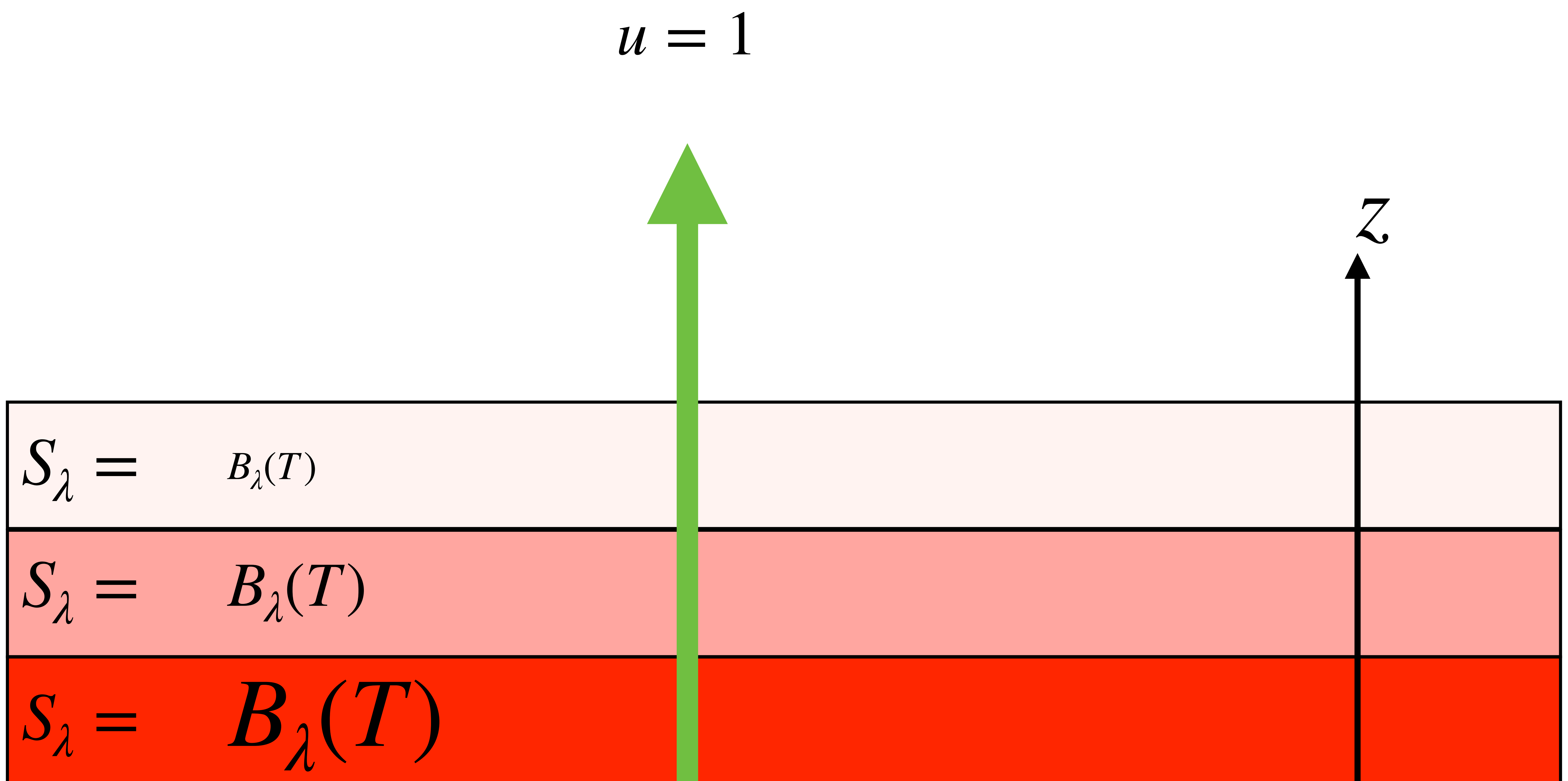
The source function is a function of τ such that:

$$S(\tau) = S_0 + S_1\tau$$

where $S_0 = 0.5$ intensity unit, and $S_1 = 1.3$ intensity units per optical depth unit.

There is no intial intensity entering the slab so $I_o = 0$.

Prepare your code such that you can vary the values of the paramters.



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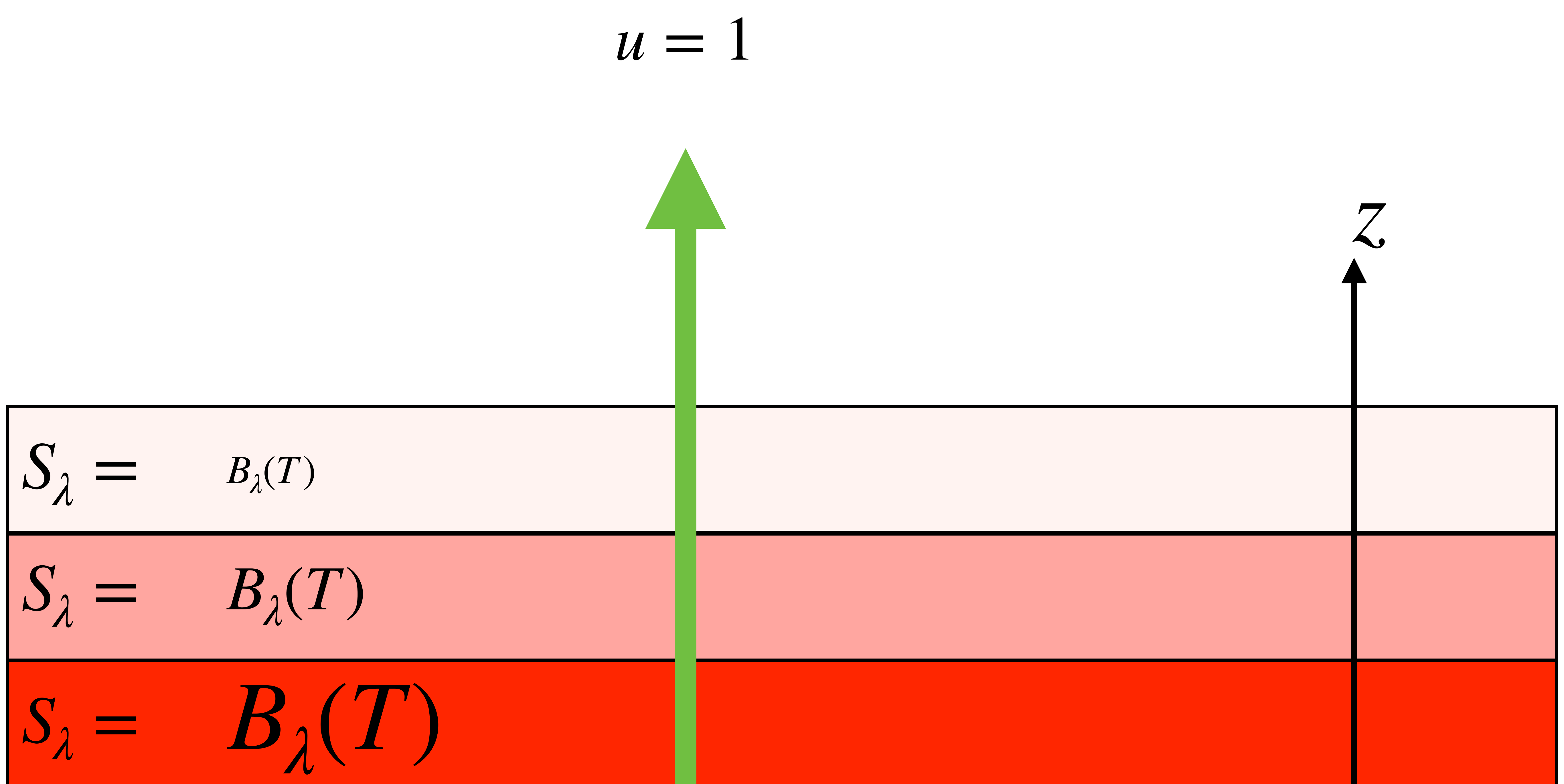
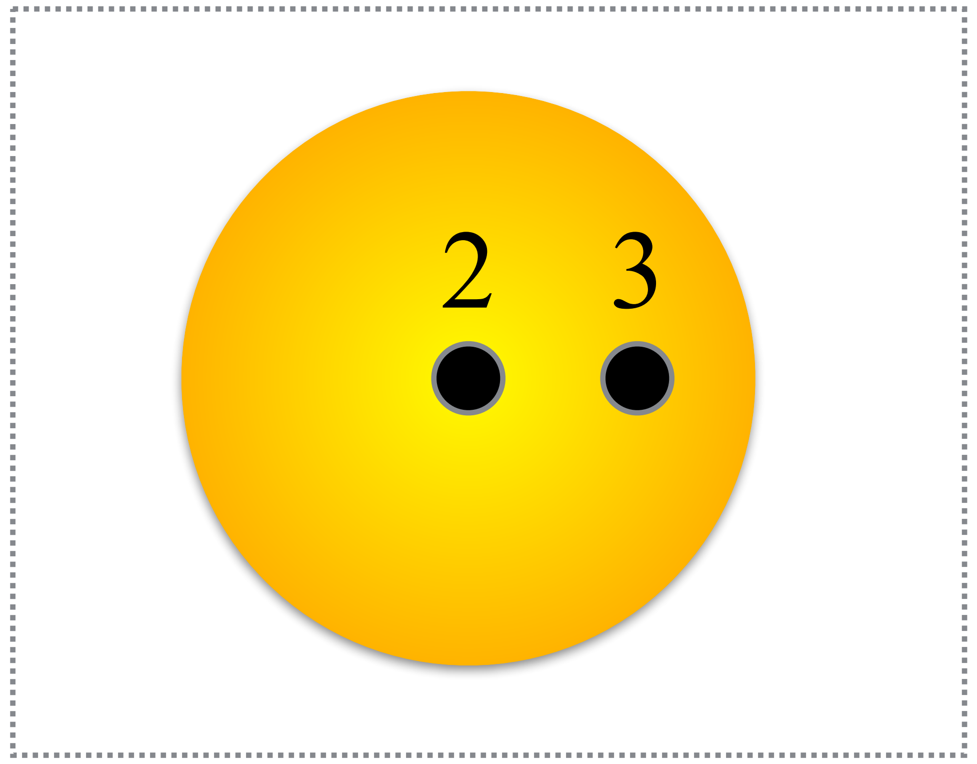
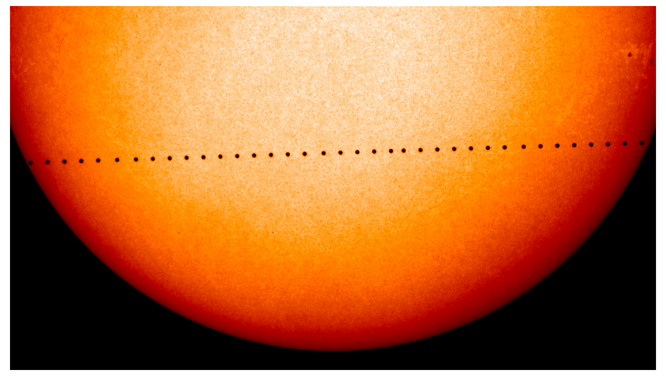
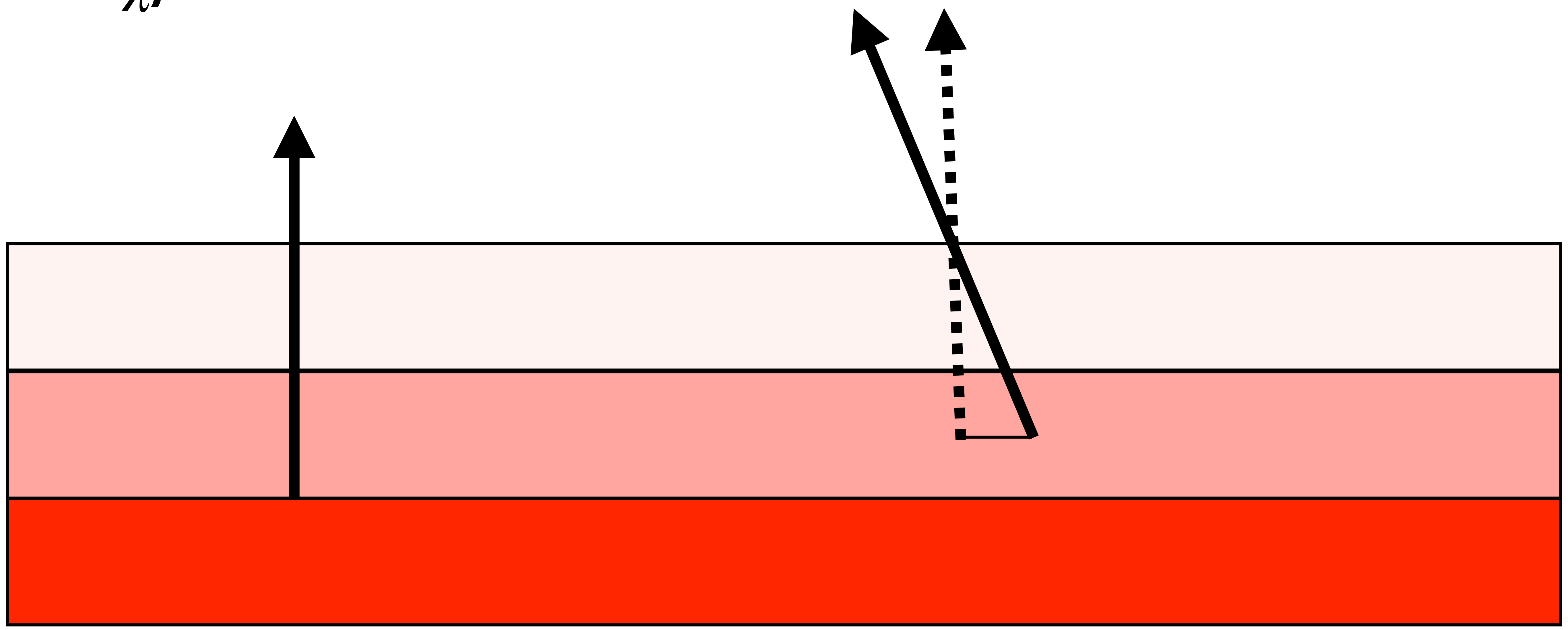


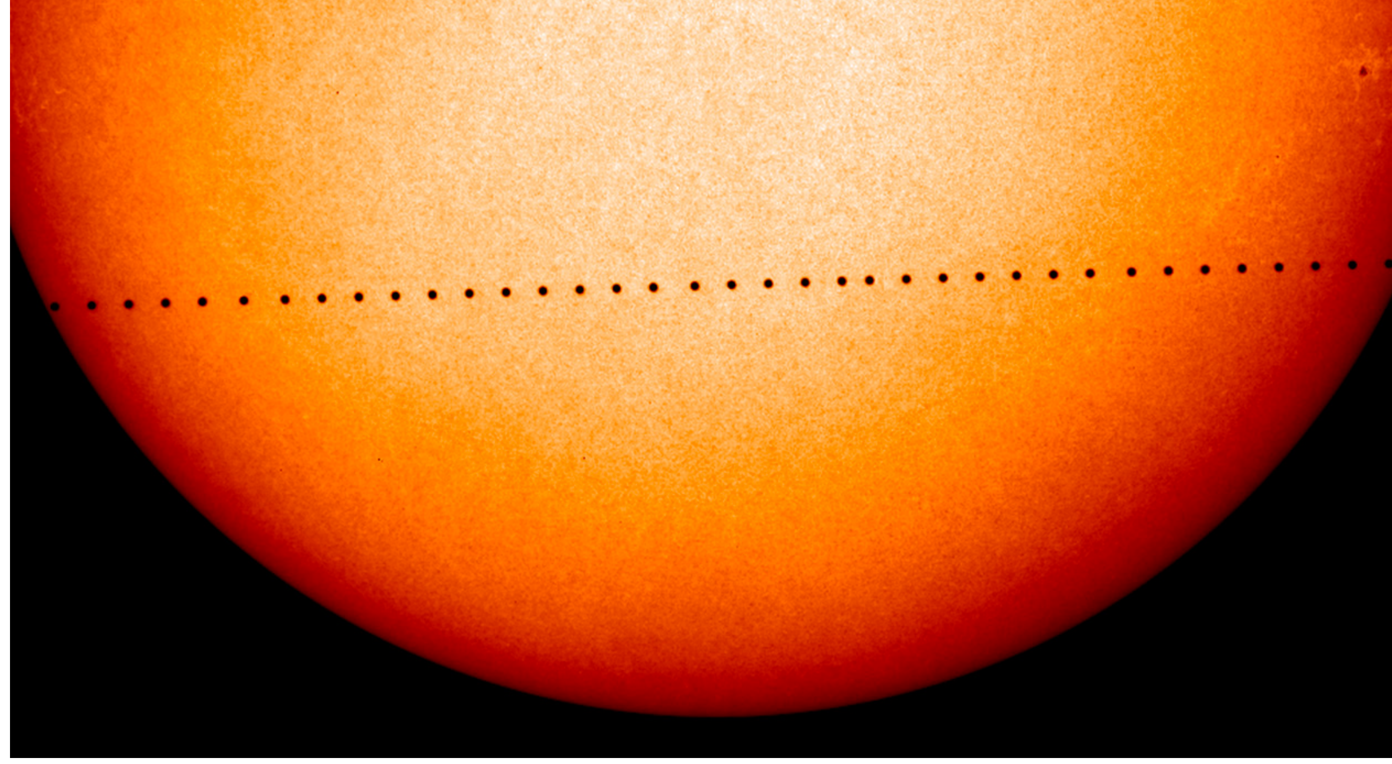


$$d\tau_\lambda = \kappa_\lambda \rho ds$$





Intensity for a flat, semi-infinite atmosphere



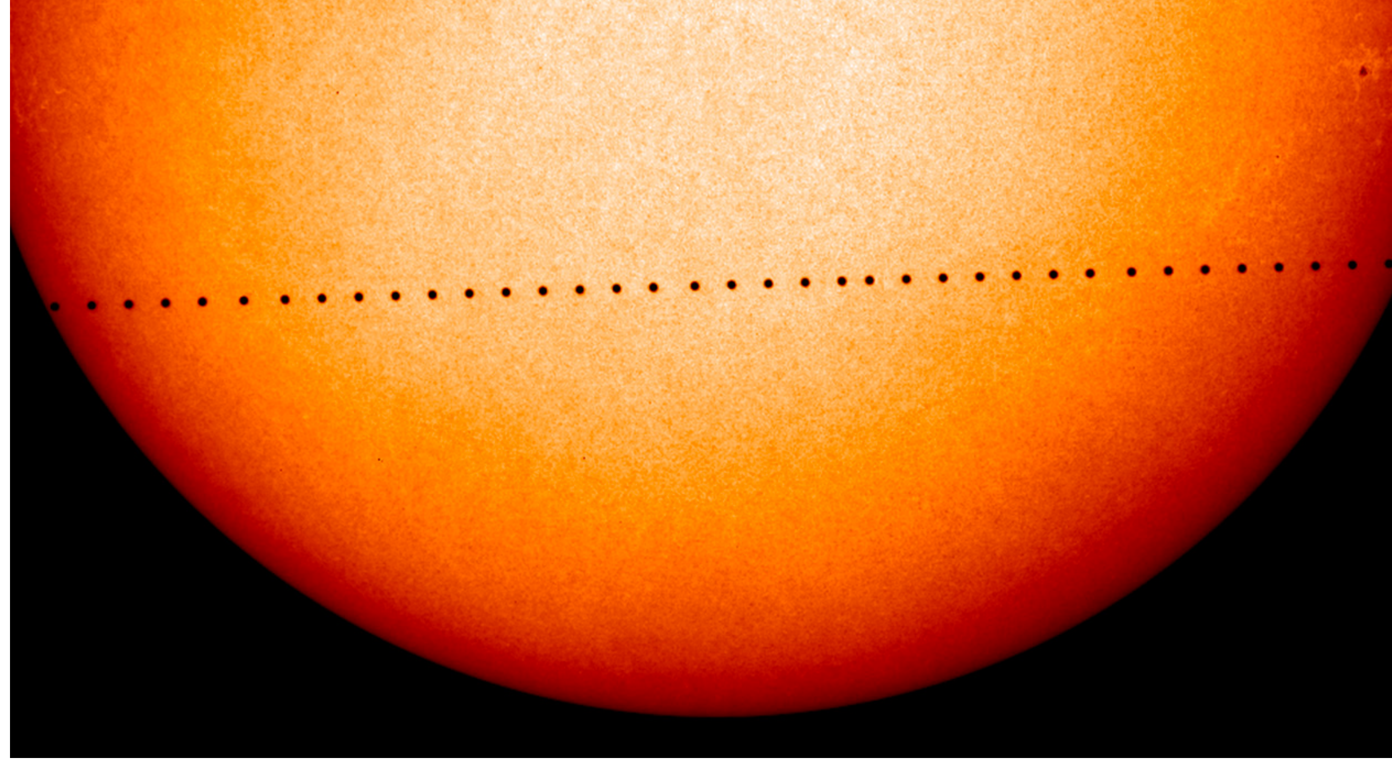
out

$$I(\tau_z, u > 0) = \int_{\tau_z'=\tau_z}^{\tau_z'=\infty} S(\tau_z')\ \mathrm{e}^{\frac{\tau_z - \tau_z'}{u}}\ d\tau_z$$

in

$$I(\tau_z, u < 0) = \int_{\tau_z'=\tau_z}^{\tau_z'=0} S(\tau_z')\ \mathrm{e}^{\frac{\tau_z - \tau_z'}{u}}\ d\tau_z$$

Intensity for a flat, semi-infinite atmosphere



out

$$I(\tau_z = 0, u > 0) = \int_{\tau_z'=\tau_z}^{\tau_z'=\infty} \boxed{S(\tau_z')}\, e^{\frac{\tau_z - \tau_z'}{u}}\, d\tau_z$$

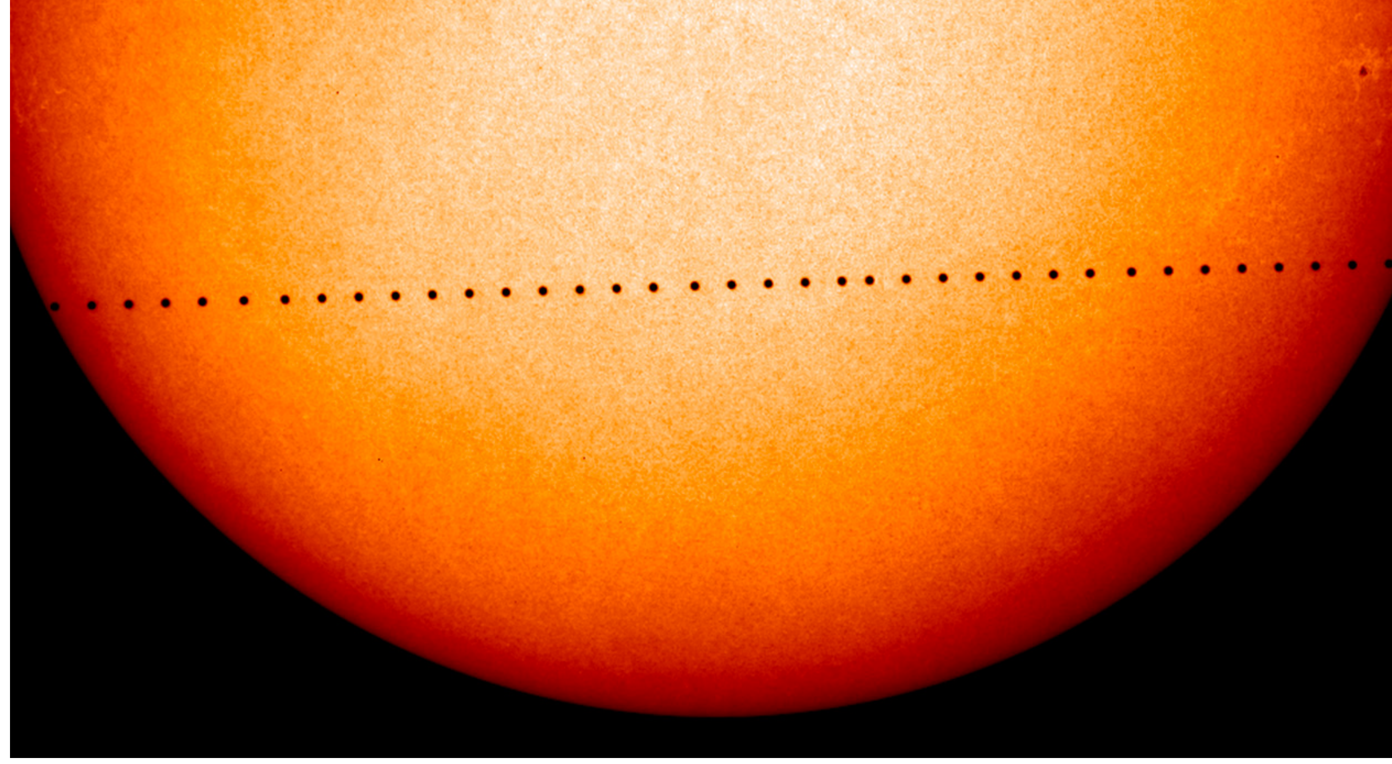
in

$$I(\tau_z = 0, u < 0) = \int_{\tau_z'=\tau_z}^{\tau_z'=0} \boxed{S(\tau_z')}\, e^{\frac{\tau_z - \tau_z'}{u}}\, d\tau_z$$

Means we can find $S(\tau)$ (and $T(z)$)

For the Sun, we can measure!

Intensity for a flat, semi-infinite atmosphere



$$S(\tau_z') = S_0 + S_1\ \tau_z'$$

$$I_{\mathrm{Obs}}(u > 0) \quad = \quad I(\tau_z = 0, u > 0) = \int_{\tau_z'=\tau_z}^{\tau_z'=\infty} S(\tau_z')\ e^{\frac{\tau_z - \tau_z'}{u}}\ d\tau_z$$

… math here :)

$$I(\tau_z = 0,\ u > 0) = S_0 + S_1\ u$$

$$= S(\tau_z = u)$$

5. At home: Formal solution with source functio increases linearly with optical depth

Let's assume that the density in the slab is constant, such that $\kappa\rho = 2.0$ per un

The source function is a function of τ such that:

$$S(\tau) = S_0 + S_1\tau$$

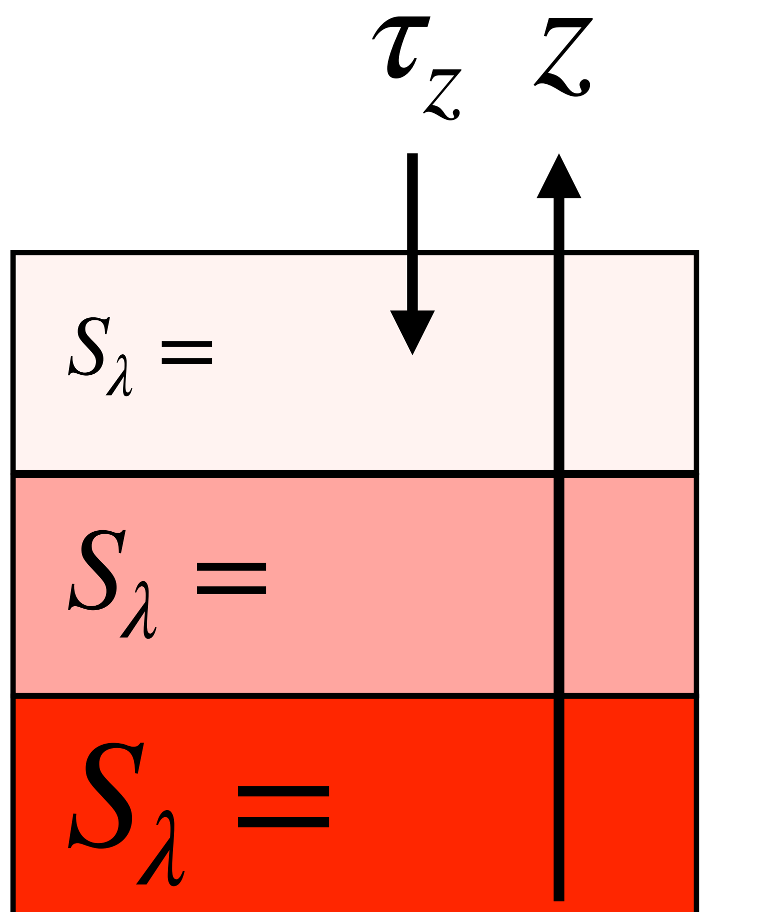
where $S_0 = 0.5$ intensity unit, and $S_1 = 1.3$ intensity units per optical depth un

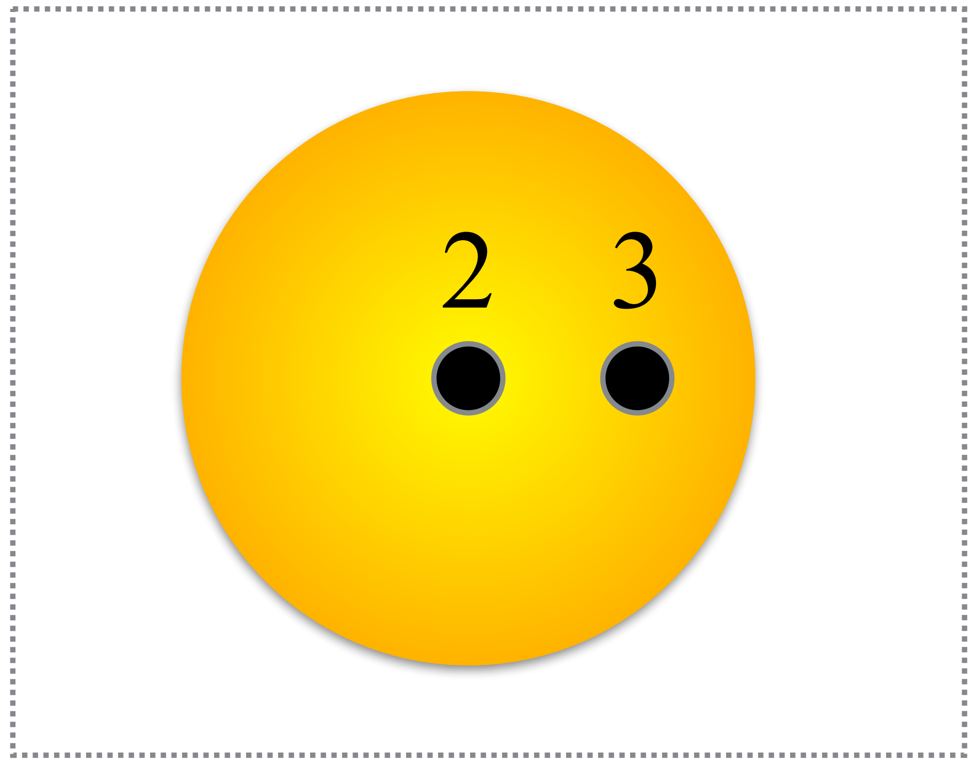
There is no intial intensity entering the slab so $I_o = 0$.

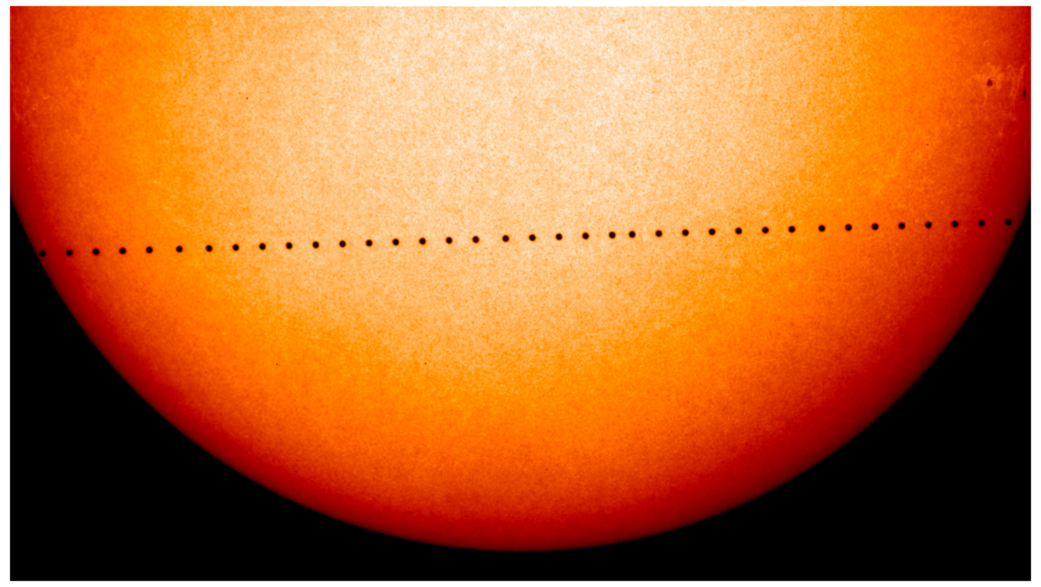
Prepare your code such that you can vary the values of the paramters.

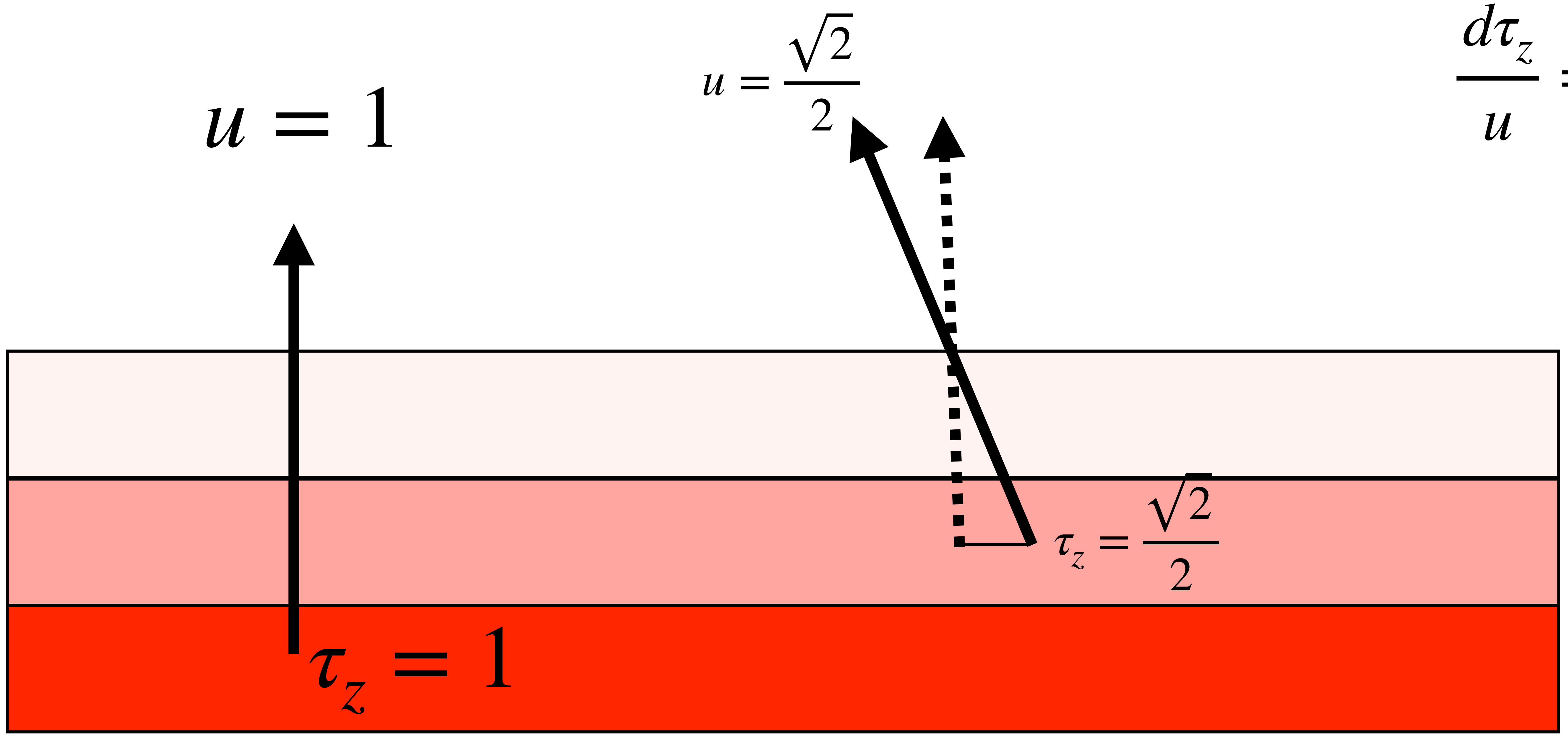
Let's make the approximation that S increases linearly with τ_z

So the intensity for a given u ray is equal to the value of the source function S at the layer where the vertical optional depth τ_z is equal to u

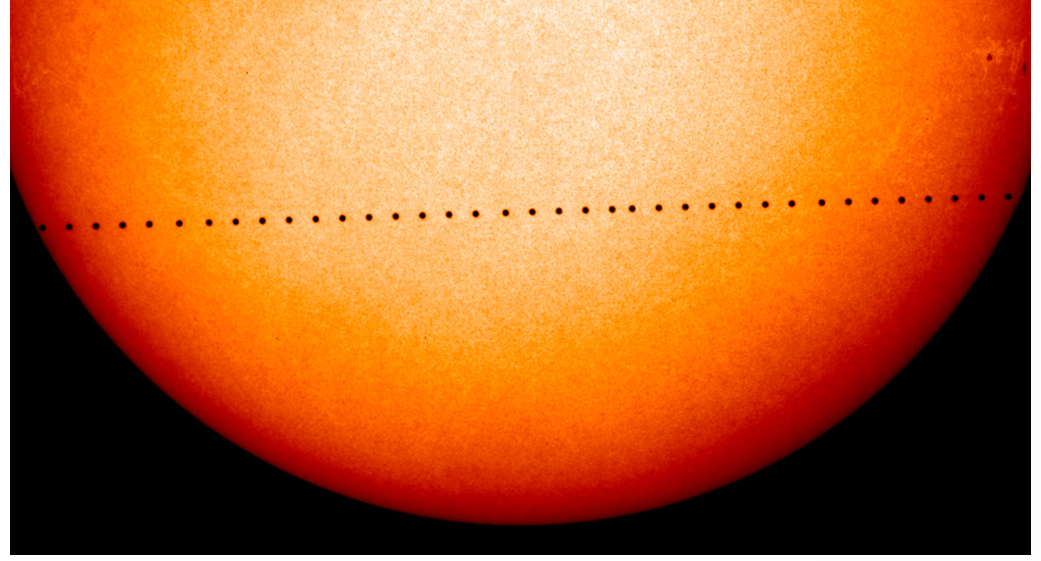








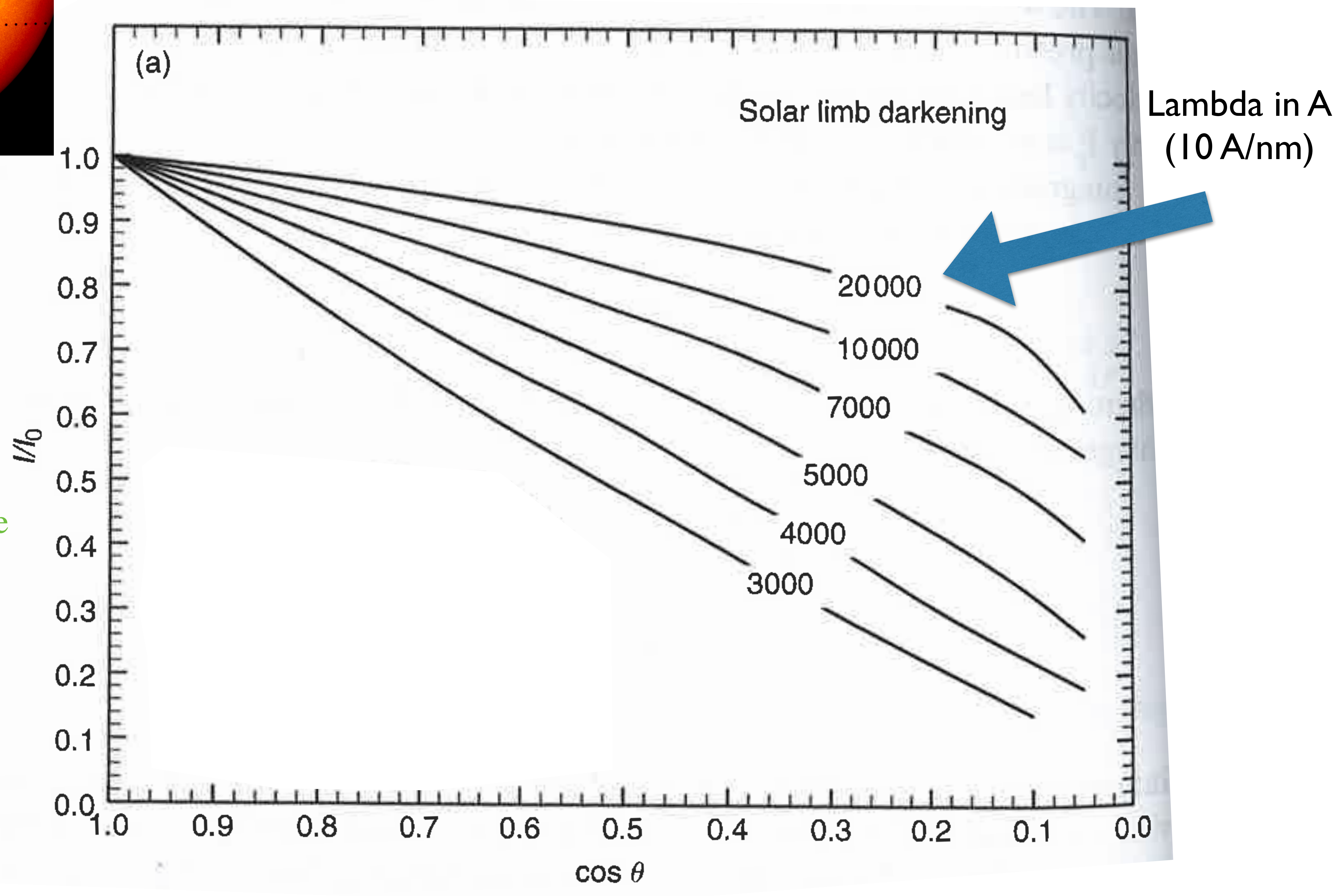
$$\frac{d\tau_z}{u} = d\tau_s$$

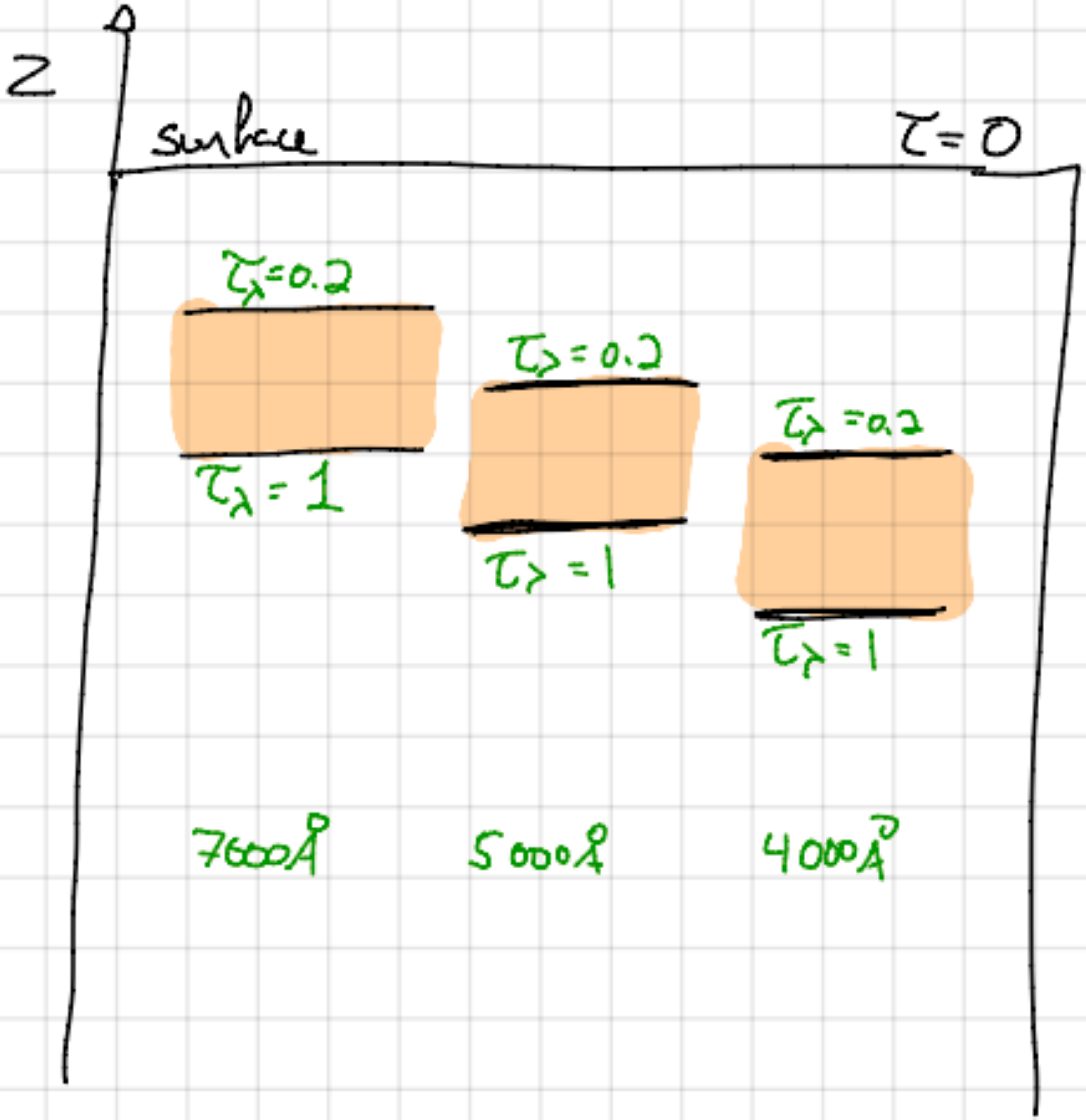


$$\frac{I_{obs}(u)}{I_{obs}(u=1)} = \frac{S_0 + S_1\ u}{S_0 + S_1}$$

In notebook: slope

$$= \frac{1 + \boxed{\frac{S_1}{S_0}} u}{1 + \frac{S_1}{S_0}}$$

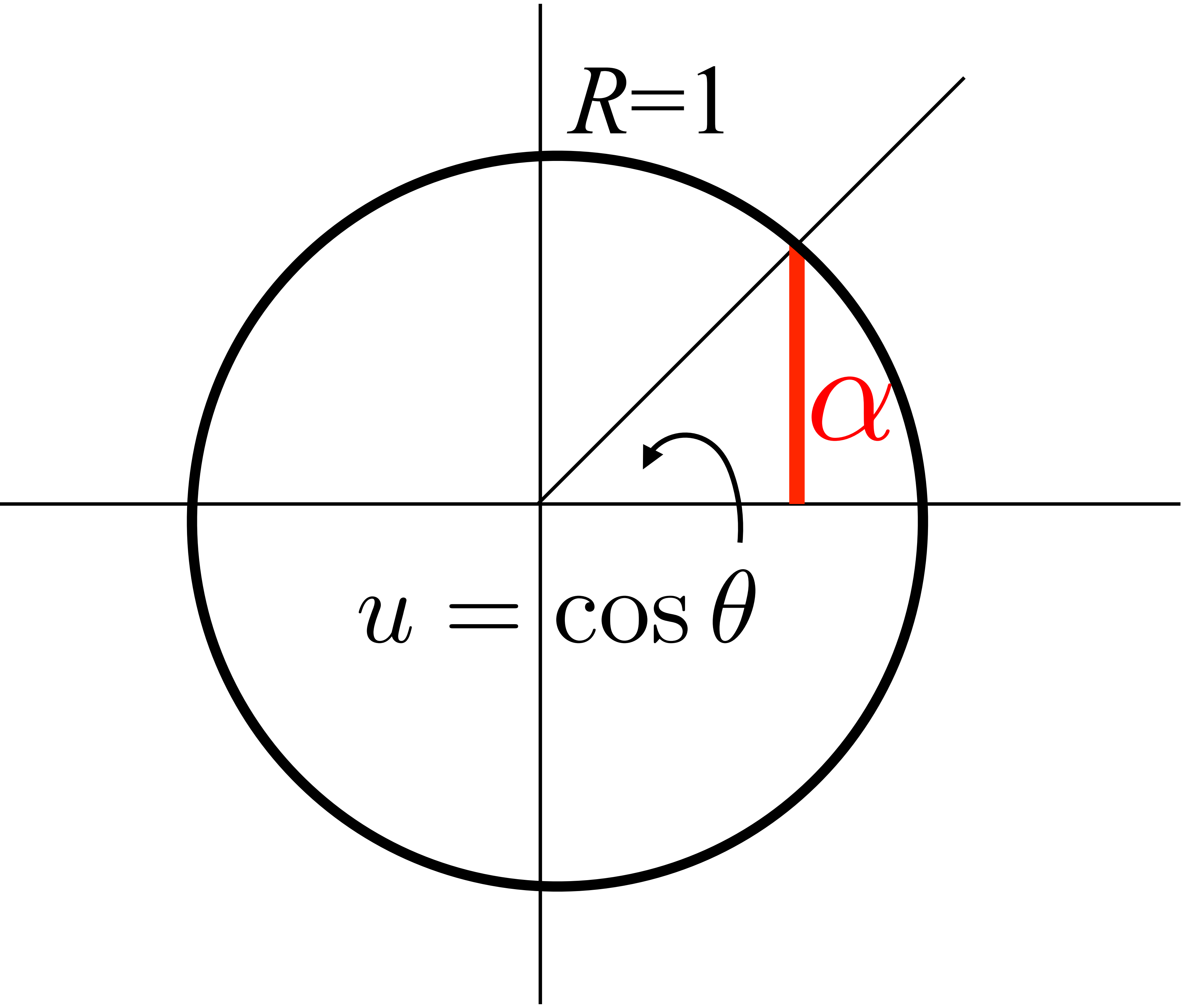


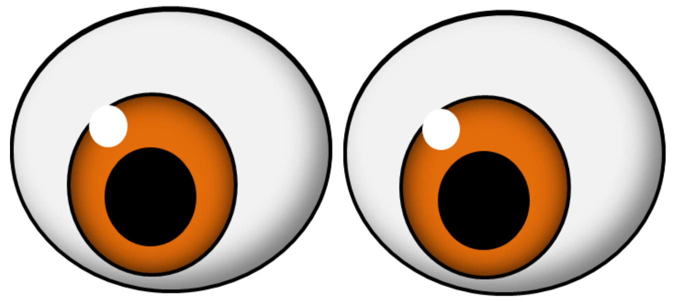


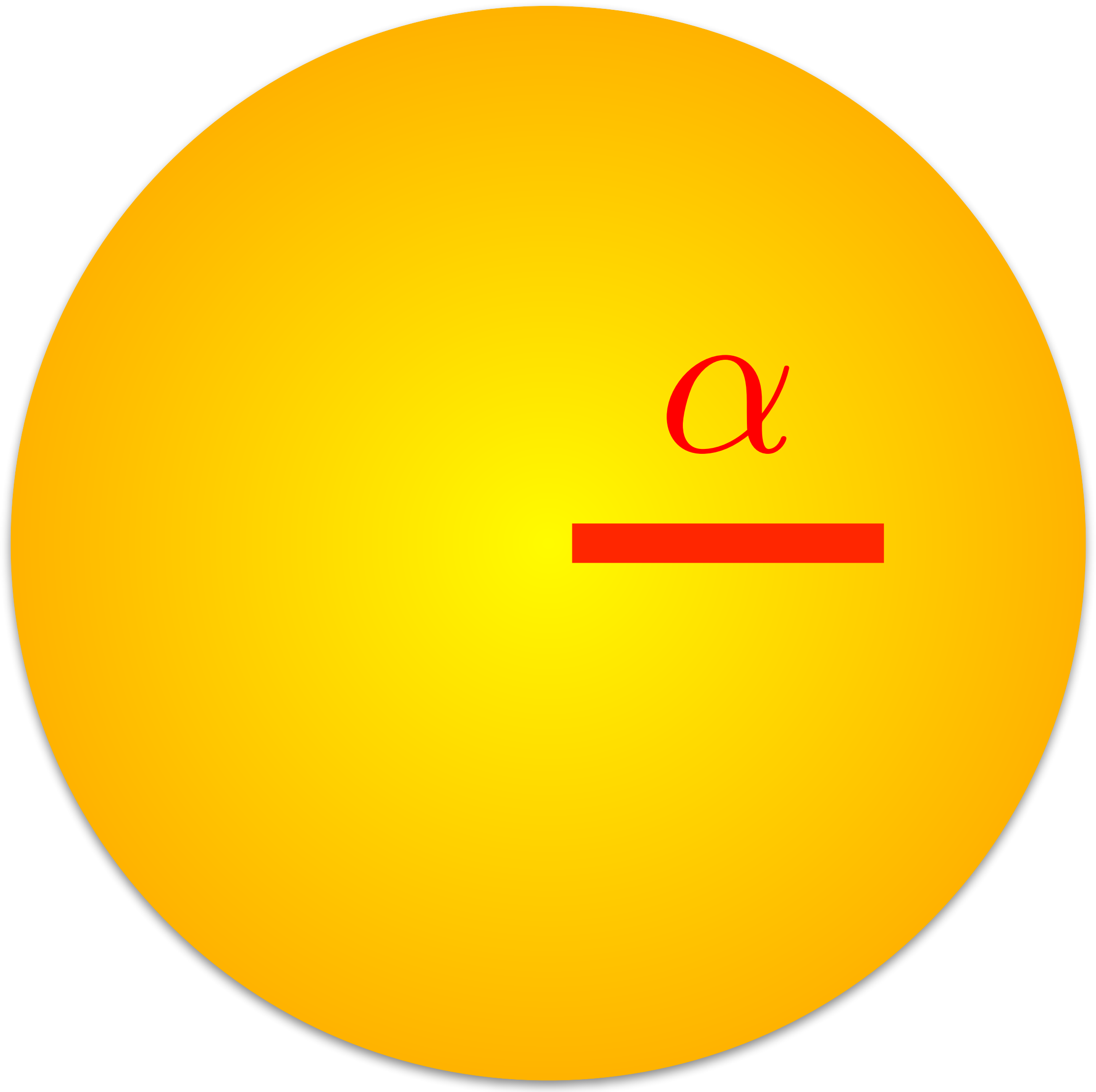
z
surface
$\tau = 0$
$\tau_\lambda = 0.2$
$\tau_\lambda = 1$
$\tau_\lambda = 0.2$
$\tau_\lambda = 1$
$\tau_\lambda = 0.2$
$\tau_\lambda = 1$
7600Å
5000Å
4000Å

Notebook: Let's see what the limb-darkening looks like for various slopes of the source function

1. How to create polar grids
2. How to make a color map







$$u^2 = 1 - \alpha^2$$

Notebook: Now let's make a simple model for planet transits, and see how limb-darkening can change the transit shape!