Week 4 Thursday
L-8


WHAT GREEK LETTERS MEAN IN EQUATIONS
π THIS MATH IS EITHER VERY SIMPLE OR IMPOSSIBLE.
Δ SOMETHING HAS CHANGED.
δ SOMETHING HAS CHANGED AND IT'S A MATHEMATICIAN'S FAULT.
θ CIRCLES!
ϕ ORBS
ϵ NOT IMPORTANT, DON'T WORRY ABOUT IT.
υ,ν IS THAT A V OR A U? OR...OH NO, IT'S ONE OF THOSE.
μ THIS MATH IS COOL BUT IT'S NOT ABOUT ANYTHING THAT YOU WILL EVER SEE OR TOUCH, SO WHATEVER.
Σ THANK YOU FOR PURCHASING ADDITION PRO®!
Π ...AND THE MULTIPLICATION® EXPANSION PACK!
ζ THIS MATH WILL ONLY LEAD TO MORE MATH.
β THERE ARE JUST TOO MANY COEFFICIENTS.
α OH BOY, NOW THIS IS MATH ABOUT SOMETHING REAL. THIS IS MATH THAT COULD KILL SOMEONE.
Ω OOOH, SOME MATHEMATICIAN THINKS THEIR FUNCTION IS COOL AND IMPORTANT.
ω A LOT OF WORK WENT INTO THESE EQUATIONS AND YOU ARE GOING TO DIE HERE AMONG THEM.
σ SOME POOR SOUL IS TRYING TO APPLY THIS MATH TO REAL LIFE AND IT'S NOT WORKING.
ξ EITHER THIS IS TERRIFYING MATHEMATICS OR THERE WAS A HAIR ON THE SCANNED PAGE.
γ ZOOM PEW PEW PEW [SPACE NOISES] ZOOOOM!
ρ UNFORTUNATELY, THE TEST VEHICLE SUFFERED AN UNEXPECTED WING SEPARATION EVENT.
Ξ GREETINGS! WE HOPE TO LEARN A GREAT DEAL BY EXCHANGING KNOWLEDGE WITH YOUR EARTH MATHEMATICIANS.
ψ YOU HAVE ENTERED THE DOMAIN OF KING TRITON, RULER OF THE WAVES.


Use the ideal gas law.
Means we need to know
1. the mean molecular weight
2. the temperature profile
L5
Mean molecular weight
Need to solve for the ionization level of all elements
Saha equations
L6
Numerical iterations
L7
We are here
L8
Energy transport
Radiative mean free path
Stellar interiors
Diffusive energy transport
Stellar atmospheres
Need detailed RT calculations
L18
Radiative transport in interiors
Energy conservation equation
L9
Basics of radiative transfer
Optical depth
L10
Formal solution of RT with source function
L11
Relationship with blackbody systems and spectral lines formation
L12
Application to a “flat” stellar atmosphere
L13
Limb darkening and radiative flux
1. Is the energy always transported by radiation?
Convection
L20
2. How to calculate the power generated?
Nuclear physics
Gammow Peak
L21
Reaction rates
L22
3. Is ideal gas law always valid?
Degeneracy pressure
Chandrasekhar mass
L23
Only depends on density.
We can use polytrops!

Continuity equation

$$\frac{dM_r(r)}{dr} = 4\pi r^2 \rho(r)$$

$M_r(r)$ $P(r)$ $T(r)$

$\rho(r)$ $\mu(r)$

We assume:
Initial comp: X_i

Hydrostatic equilibrium

$$\frac{dP(r)}{dr} = -\rho(r)\frac{GM_r(r)}{r^2}$$

Ideal gas law

$$P(r) = \frac{\rho(r)}{\mu(r) m_H} kT(r)$$

Mean molecular weight

$$\mu(r) = f(\mathrm{comp}, T(r), P(r))$$

Need more physics:

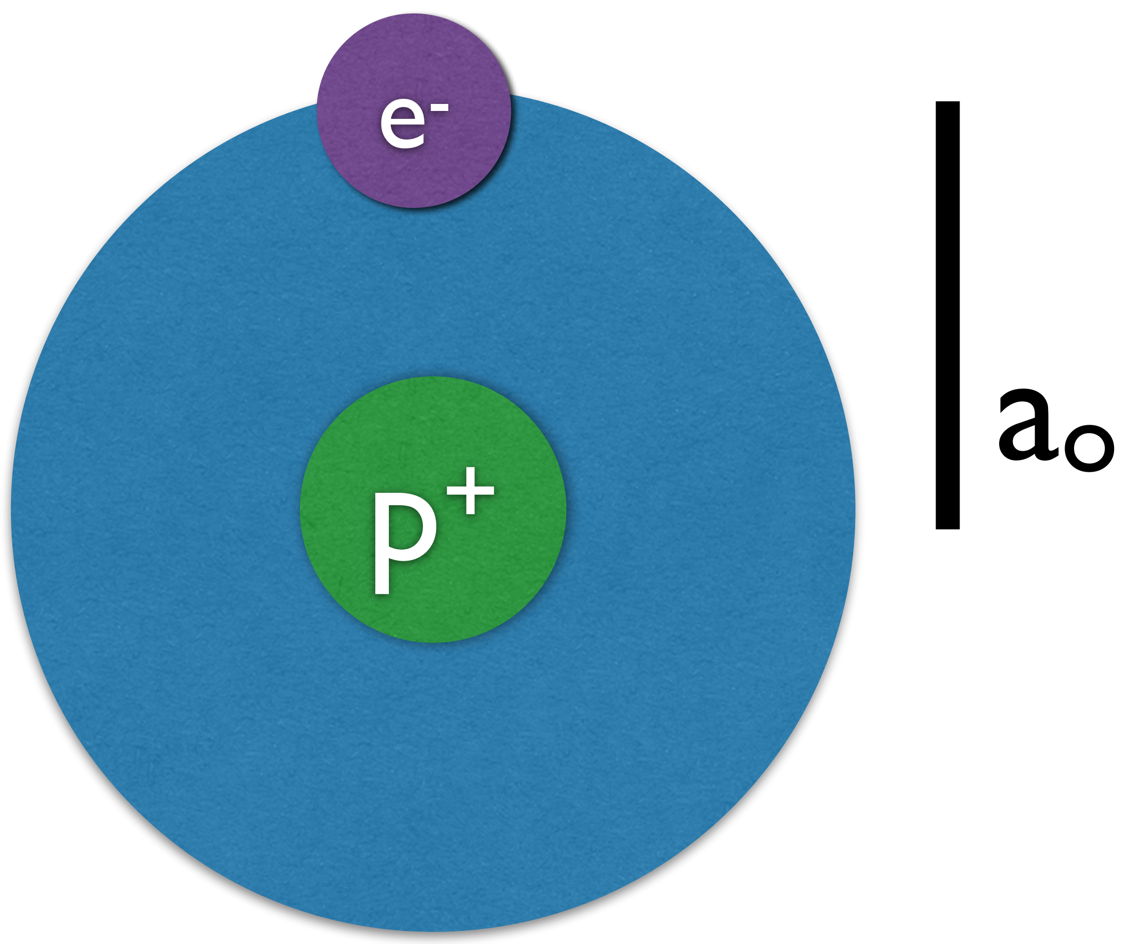
<u>Transport of energy</u> by photon will be related to the gas temperature.

Energy transport by radiation

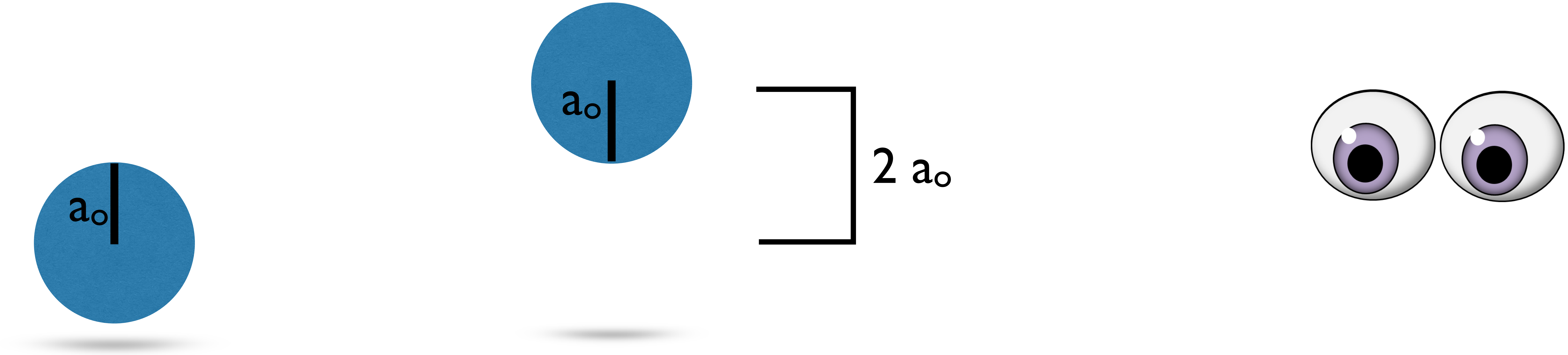
Let's start with the concept of <u>mean free path</u>: average distance traveled by an object between 'interactions'

$$l = \frac{\text{Distance traveled in a time } \Delta t}{\text{\# of interactions in a time } \Delta t}$$

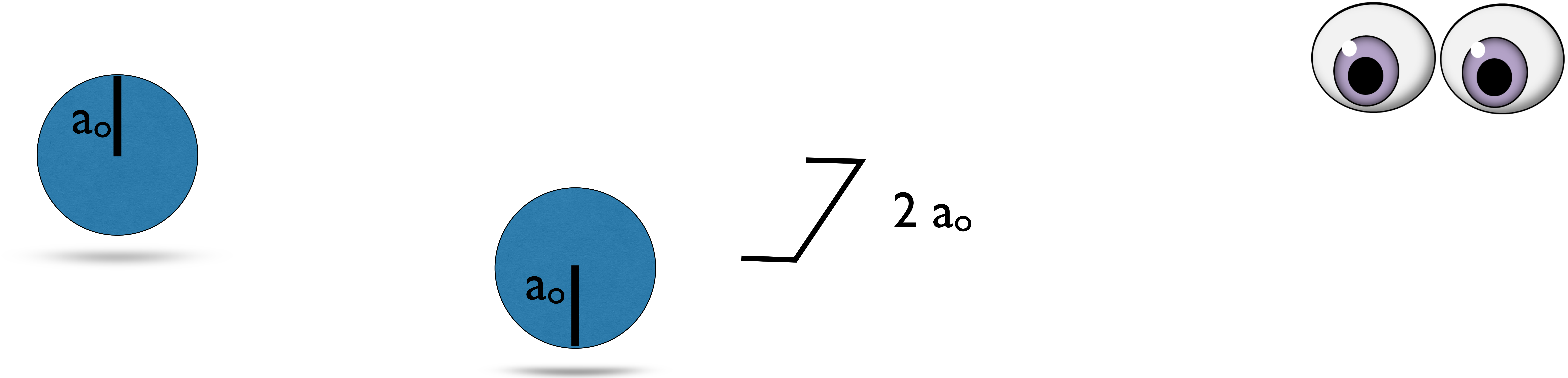
Starting with <u>particle</u> mean free path



How close do they have to pass each other in order to interact?



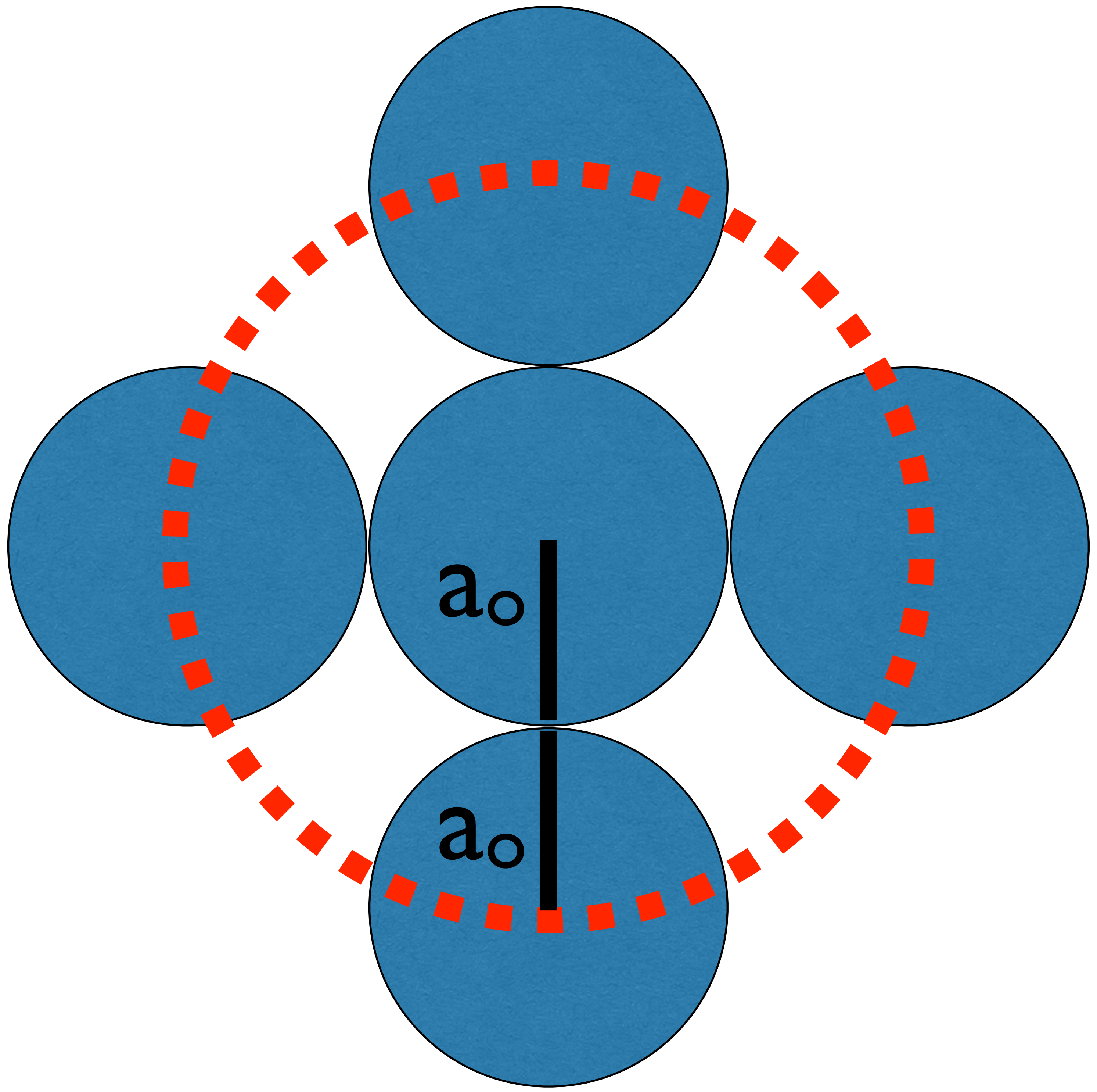
a_o
a_o
$2\ a_o$

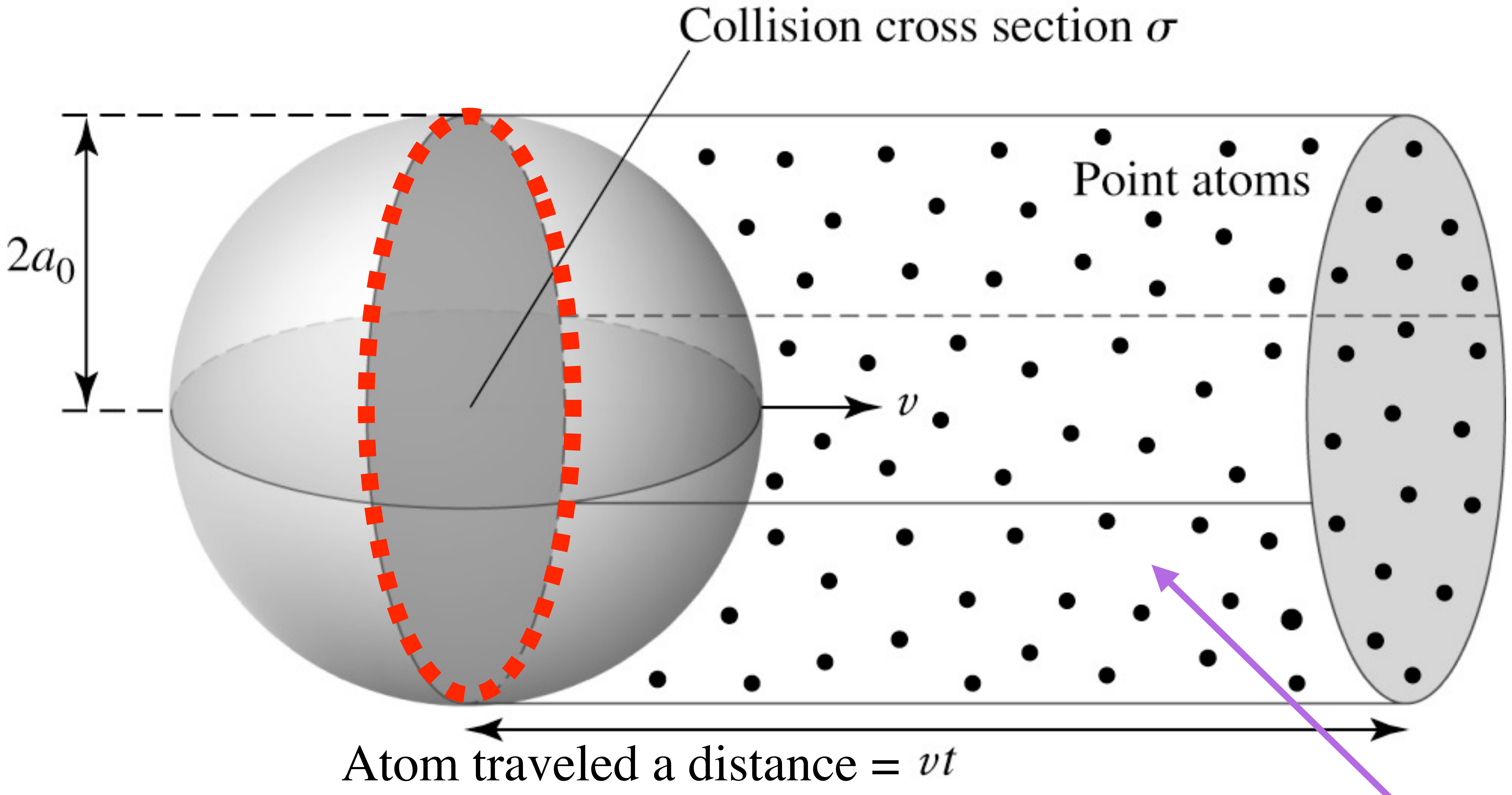


a_o
a_o
$2\ a_o$

The interaction 'cross-section' σ of an atom:
How far does the middle of another atom
has to pass from the middle of our atom to interact

In a time Δt:





$$\sigma = \pi(2a_o)^2 = 3.5 \times 10^{-16}\ \mathrm{cm}^2$$

How many atom will it interact with?

All of those currently located in this volume

Volume = $\sigma v \Delta t$

# of atoms = Volume * concentration = $\sigma v \Delta t \ n$

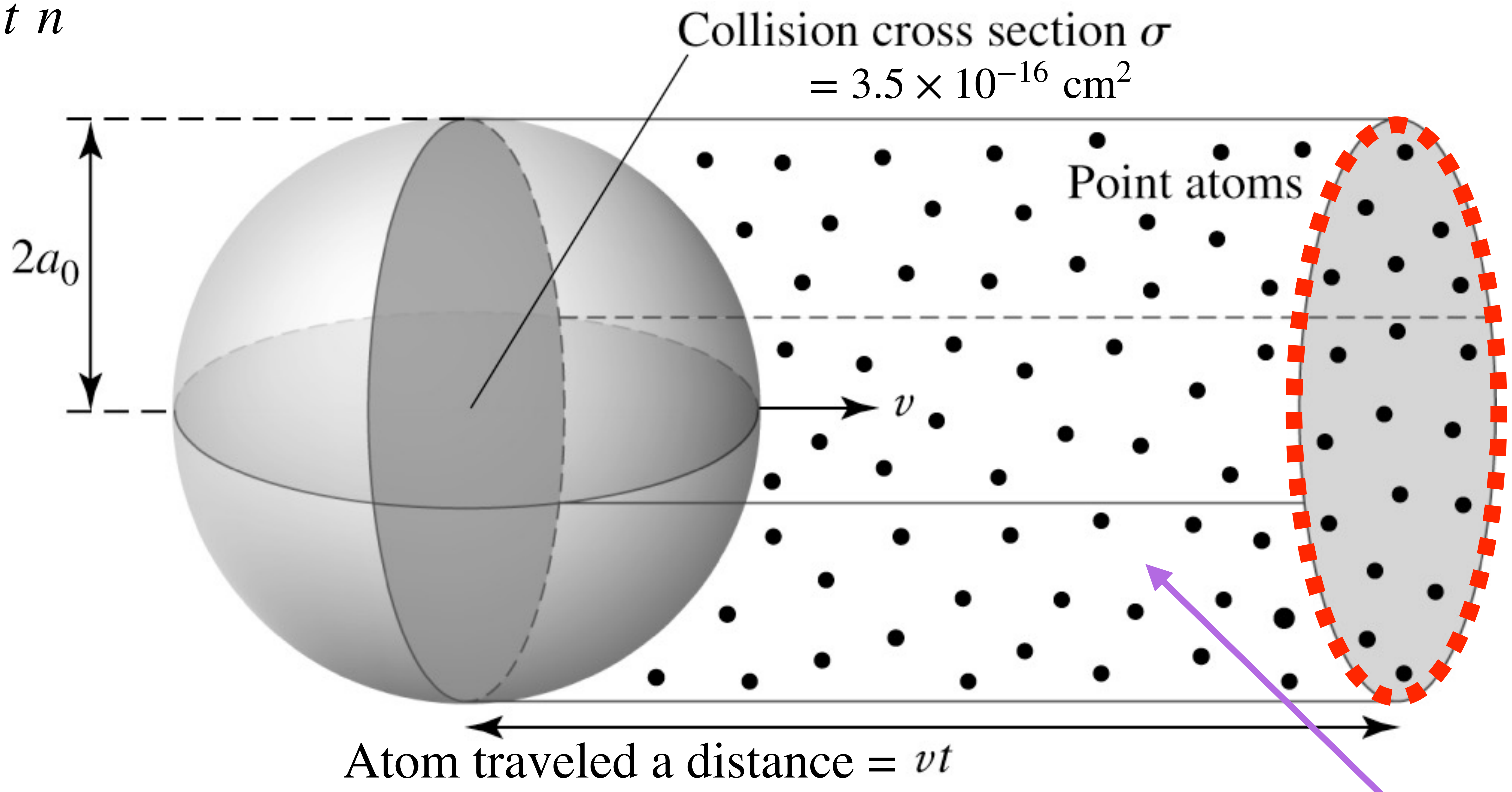
$$l = \frac{\text{Distance traveled in a time } \Delta t}{\text{\# of interactions in a time } \Delta t}$$

$$l = \frac{v\Delta t}{v\Delta t \sigma n}$$

$$= \frac{1}{\sigma n}$$

$$= \frac{\mu m_H}{\sigma \rho}$$

In a time Δt:



How many atom will it interact with?

All of those currently located in this volume

$$l = \frac{\mu m_H}{\sigma \rho}$$

At the surface (=upper layers) of the sun, the density is $\sim 10^{-7}$ g/cm^3
Let's assume pure neutral hydrogen (as an estimate)

```
rho = 1e-7 * u.g /u.cm**3
sigma = np.pi*4*cds.a0**2
mu = 1 # pure neutral hydrogen

l = (mu * const.u / sigma / rho).decompose()

print( 'The mean free path of atoms at the surface of the sun is {0:0.2g}'.format(l.cgs) )
```

```
The mean free path of atoms at the surface of the sun is 0.047 cm
```

How small is that, really?

Temperature gradient in the whole sun:

$T_\star \sim 10^4$ K

$T_o \sim 10^7$ K

$R_\star \sim 10^{11}$ cm

$$\frac{\Delta T}{\Delta r} \sim 10^{-4} \text{ K / cm}$$

Atoms interact with other atoms that have similar characteristics
('Local Thermodynamic Equilibrium' or LTE)

Energy transport by radiation

Now, what about the photons' mean free path?

$$l = \frac{1}{\kappa\rho}$$

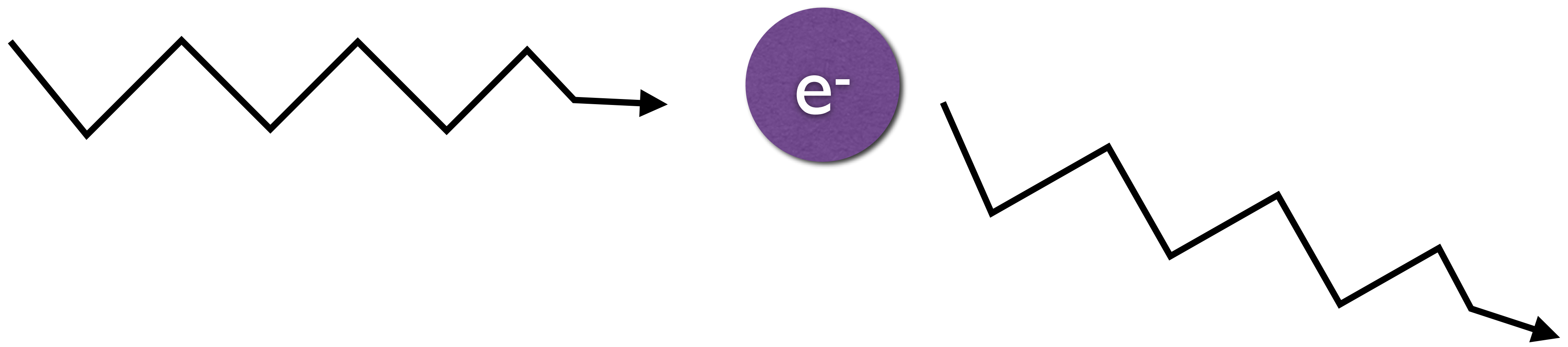
Opacity: Capacity of a material to absorb/scatter photons.

κ = Cross section (cm^2 / g)

Depends on:

- Properties of the material (T, μ, P, X_i, etc)
- Wavelength λ of the photons

Example for now: electron scattering process (no dependence on λ)



$$\kappa = 0.34 \text{ cm}^2/\text{g}$$

$$l = \frac{1}{\kappa\rho}$$

$$\frac{\Delta T}{\Delta r} \sim 10^{-4}\ \mathrm{K / cm}$$

a. In the interior of the Sun

The mean density inside the Sun is $1\,\mathrm{g/cm^3}$. Let's use the opacity (0.34 $\mathrm{cm^2/g}$) for electron scattering for our estimate.

```
rho = 1 * u.g /u.cm**3 # mean density in the Sun
kappa = 0.34 * u.cm**2 / u.g # We use the electron scattering for an estimate

print 'The mean free path of photons in the interior of the Sun is {0:0.2g}'.format(MeanFreePhoton(rho, kappa).cgs)

## At home, format your answer like illustrated above.
```

```
The mean free path of photons in the interior of the Sun is 2.9 cm
```

In the interior, photons mostly interact with particles that have the same properties.

$$l = \frac{1}{\kappa \rho}$$

$$\frac{\Delta T}{\Delta r} \sim 10^{-4}\ \mathrm{K\,/\,cm}$$

a. In the interior of the Sun

The mean density inside the Sun is 1g/cm^3. Let's use the opacity (0.34 cm^2/g) for electron scattering for our estimate.

```
rho = 1 * u.g /u.cm**3 # mean density in the Sun
kappa = 0.34 * u.cm**2 / u.g # We use the electron scattering for an estimate

print 'The mean free path of photons in the interior of the Sun is {0:0.2g}'.format(MeanFreePhoton(rho, kappa).cgs)

## At home, format your answer like illustrated above.
```

```
The mean free path of photons in the interior of the Sun is 2.9 cm
```

In the interior, photons mostly interact with particles that have the same properties.

b. At the surface of the Sun

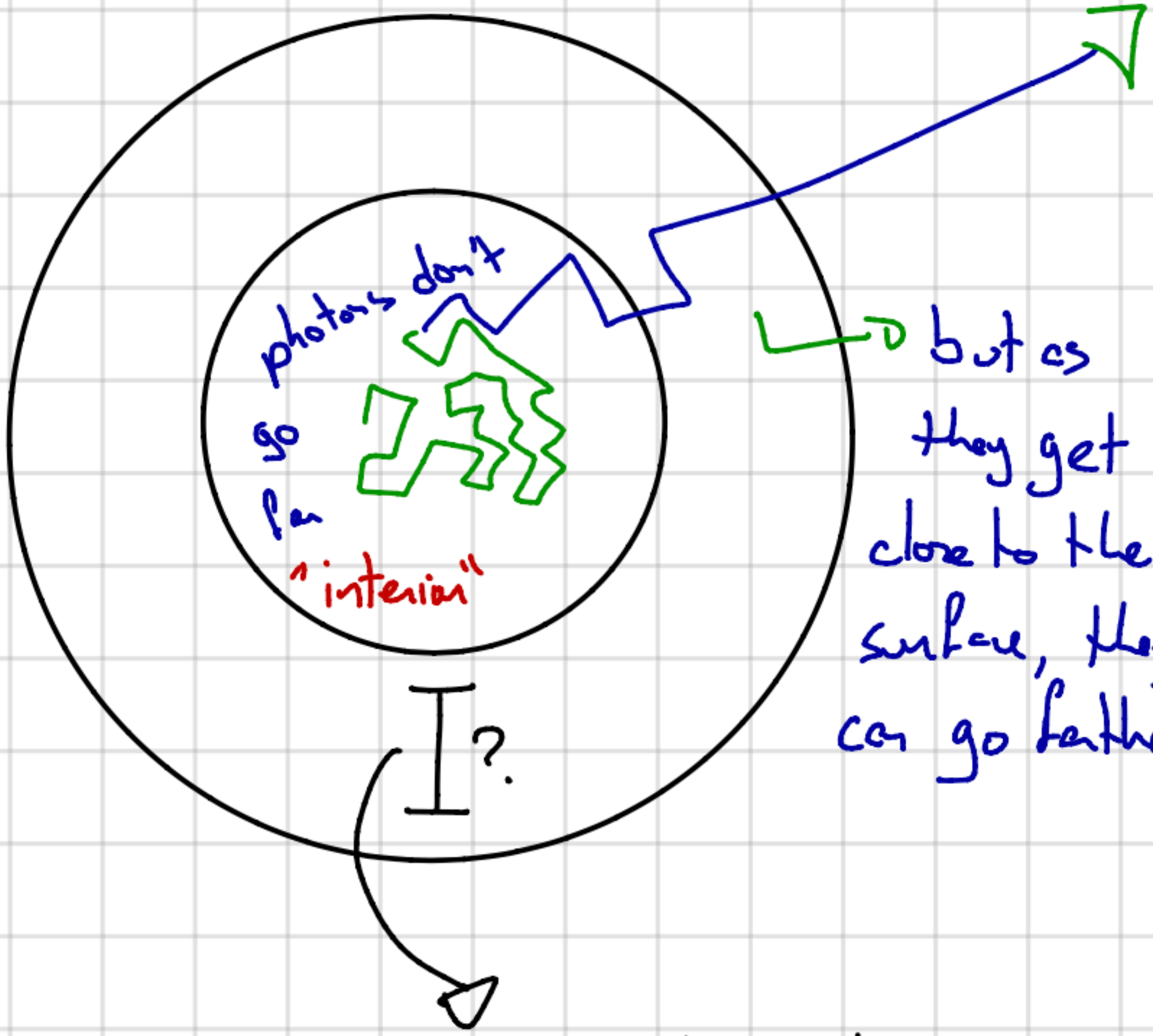
The mean density at the surface the Sun is 10^{-7}g/cm^3. Let's use the opacity (0.34 cm^2/g) for electron scattering for our estimate.

```
rho = 1e-7 * u.g /u.cm**3
kappa = 0.34 * u.cm**2 / u.g # We use the electron scattering for an estimate

print 'The mean free path of photons at the surface of the sun is {0:0.2f}'.format(MeanFreePhoton(rho, kappa).to(u.km))
```

```
The mean free path of photons at the surface of the sun is 294.12 km
```

In the upper layers, photons will interact with particles that have different properties



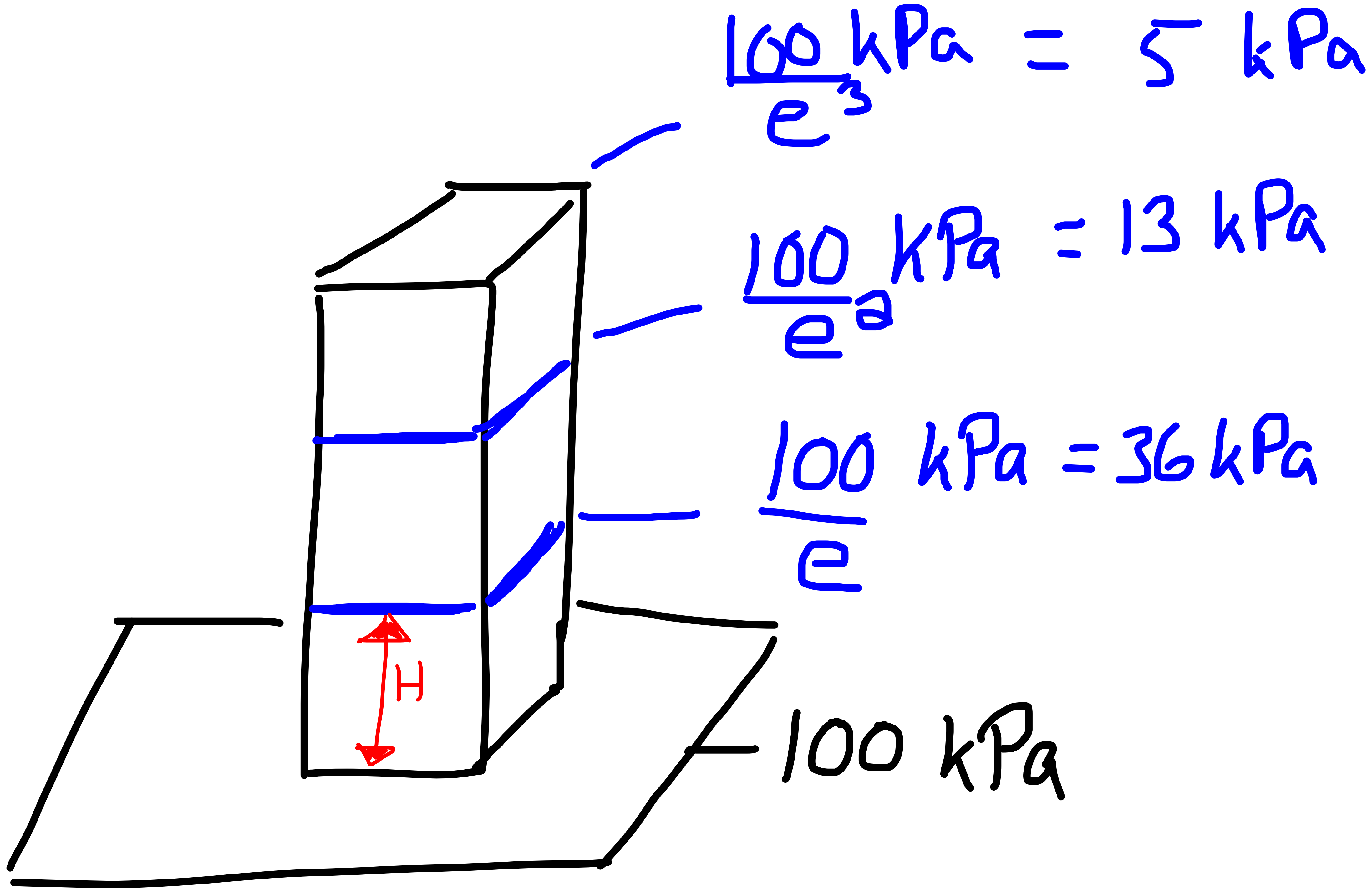
but as they get close to the surface, they can go farther.

and eventually escape (which is handy, otherwise we wouldn't be able to see the Sun, right?)

(atmosphere)

Now the question is, how big is, how big is the "atmosphere"

The pressure scale height: distance over which the pressure change by a factor of “e”



$$P(r) = \frac{\rho(r)}{\mu(r) m_H} kT(r)$$

$$H = \frac{P}{dP/dr}$$

$$\frac{dP(r)}{dr} = -\rho(r) g(r)$$

$$= \frac{kT/\mu m_H}{g}$$

For the Sun near the surface:
$T_\star = 5800$ K
$g_\star = 10^4$ cm/s^2
$\mu \sim 1$

$$H_\odot \sim 500 \text{ km}$$

$$H_\odot \sim 0.0007 \text{ R}_\odot$$

The ‘atmosphere’ is very thin compared to the radius of the star

Interiors

Photons don't go far =
energy transport is easier
(diffusion approximation)

We will have to include the
energy created by nuclear
reactions

Atmospheres

Transport of energy more tricky
(<u>radiative transfer</u>)

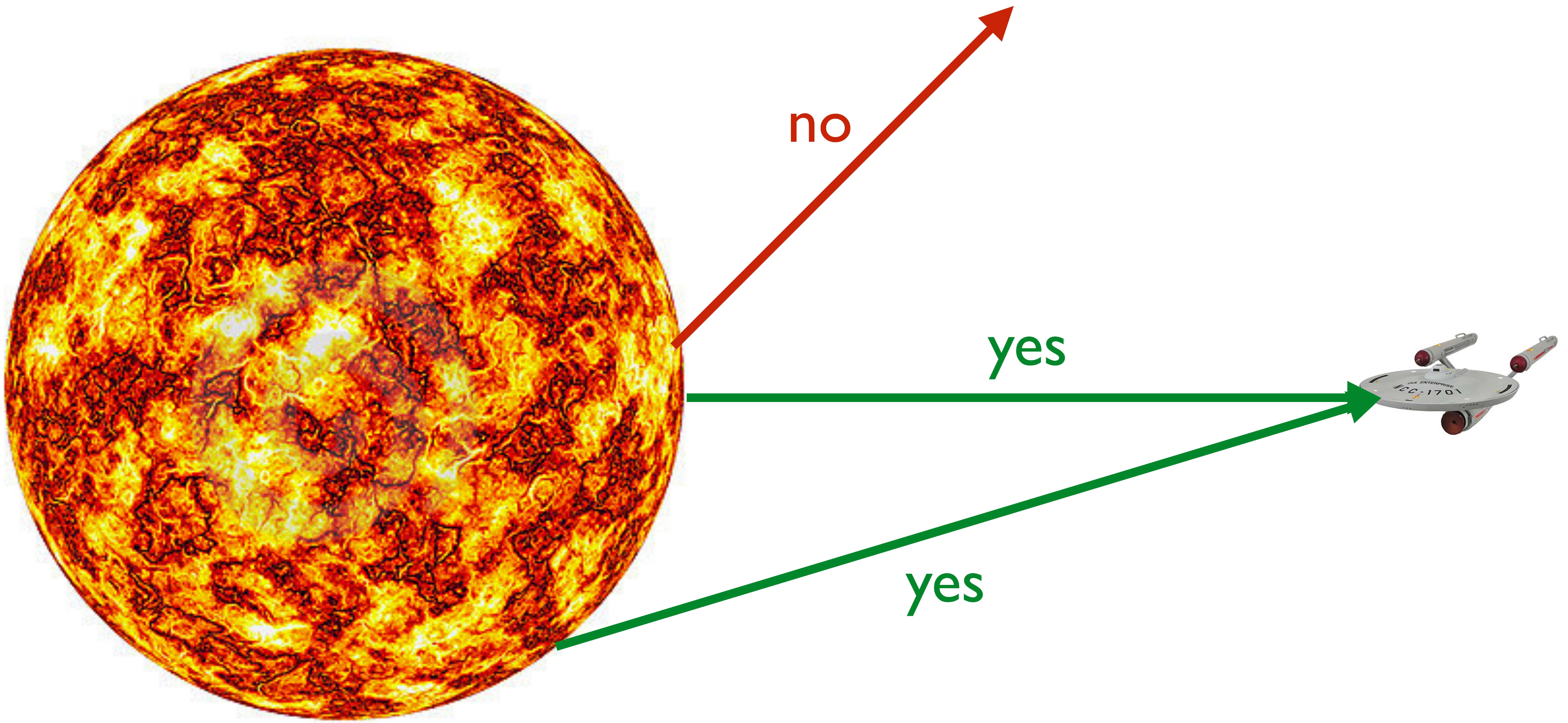
No energy production

Scale height is small:
Flat geometry approximation

The basics of Radiative Transfer

Specific intensity: fancy name of “a ray of light”

You only “see” a ray if it is directed at you



Specific intensity

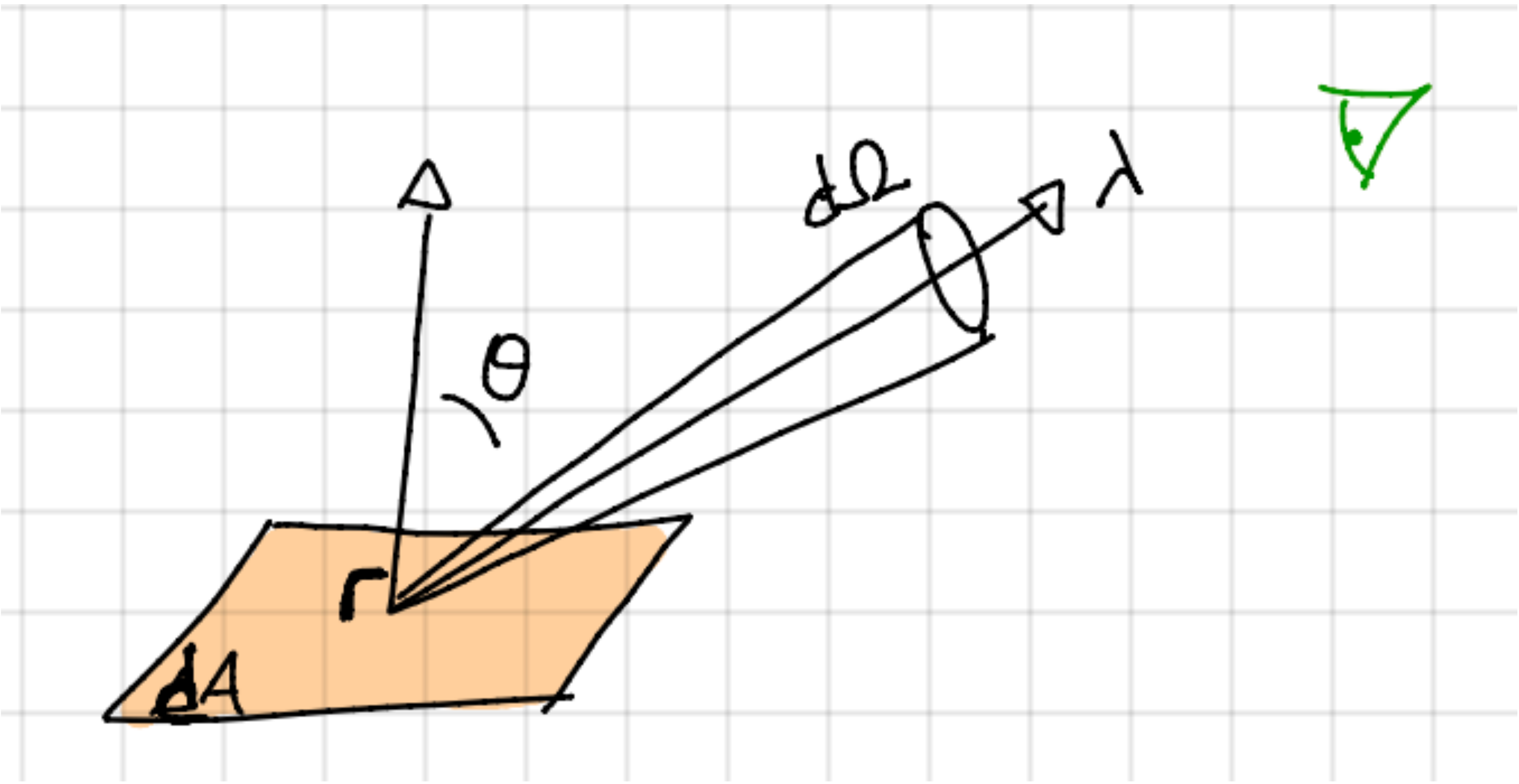
Even for the Sun, we can consider that the ray we observed at all parallel to each other.

Specific intensity

Rays don’t diverge

Specific intensity: the physics definition

$I_\lambda =$ Quantity of energy:



- per frequency/wavelength unit $d\nu$ or $d\lambda$
- per unit of time dt
- that passes through a surface element dA $\cos\theta$
- inside a solid angle $d\Omega$
- oriented in the direction θ with respect to the normal to the surface

Making it an effective surface

Total energy transported by a ray:

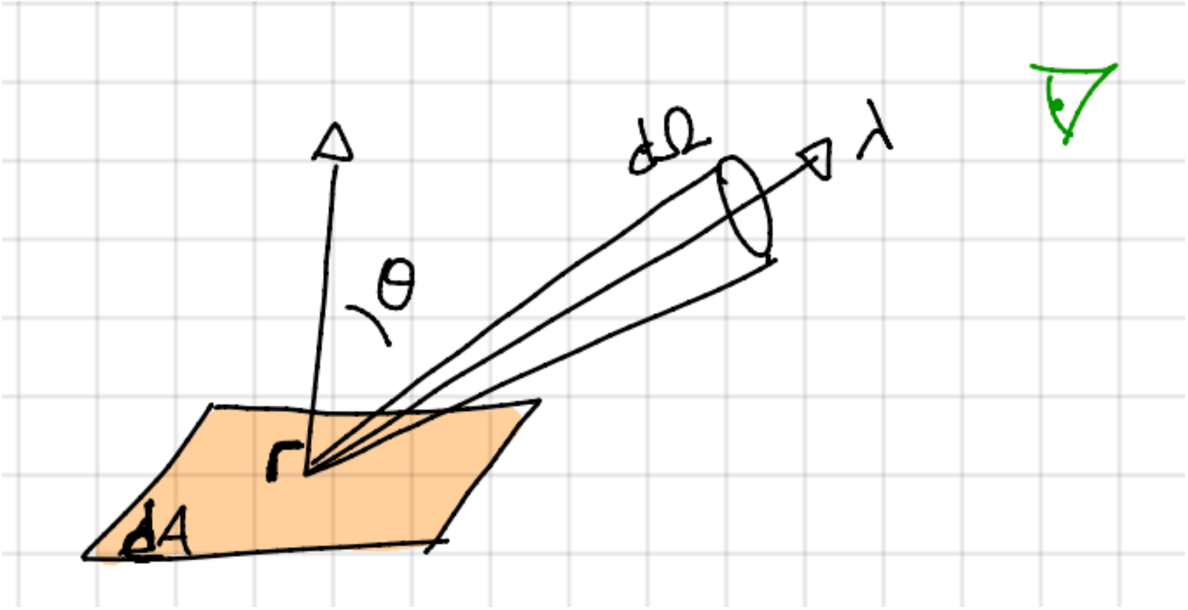
$$E = I_\lambda \, d\lambda \; dt \; dA \cos\theta \; d\Omega$$

(So, what are the units of intensity?)

Mean specific intensity J_λ: average over all directions

direction = solid angle

$$d\Omega = \sin\theta d\theta d\phi$$



Anisotropic radiation: $I = I(\theta, \phi)$

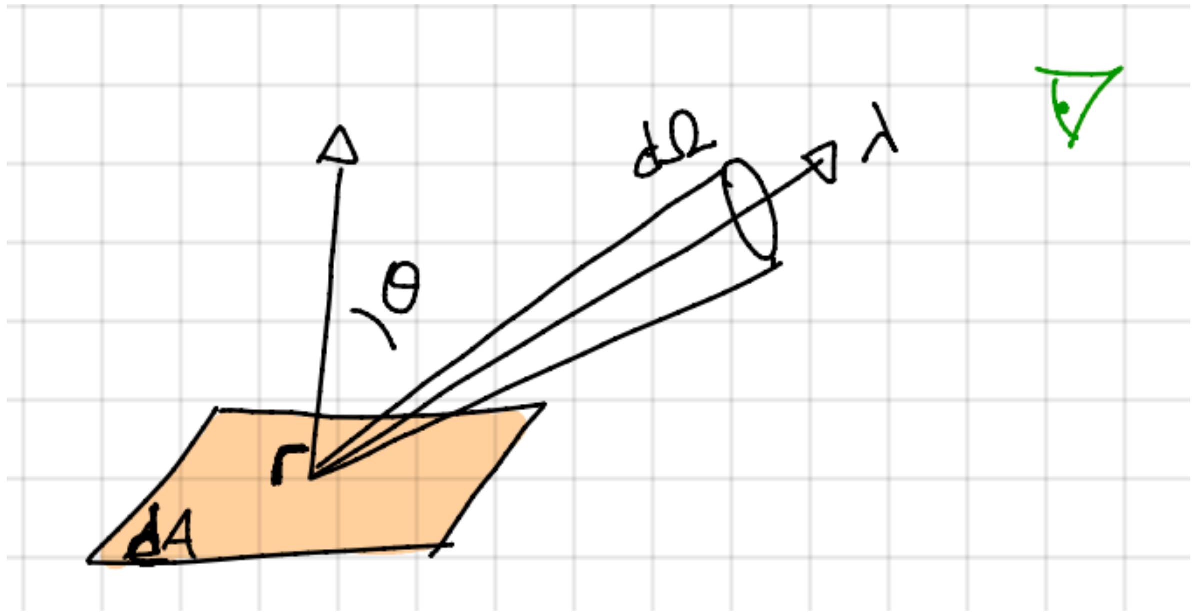
$= I(\Omega)$

Isotropic radiation: $I = J$

$$J_\lambda = \frac{\int I_\lambda(\Omega) d\Omega}{\int d\Omega} = \frac{1}{4\pi} \int I_\lambda(\Omega) d\Omega$$

Specific flux F_λ:

$d\Omega = \sin\theta d\theta d\phi$



$F_\lambda =$ Quantity of energy:

- per unit of time dt
- per unit of wavelength $d\lambda$
- going though a surface dA

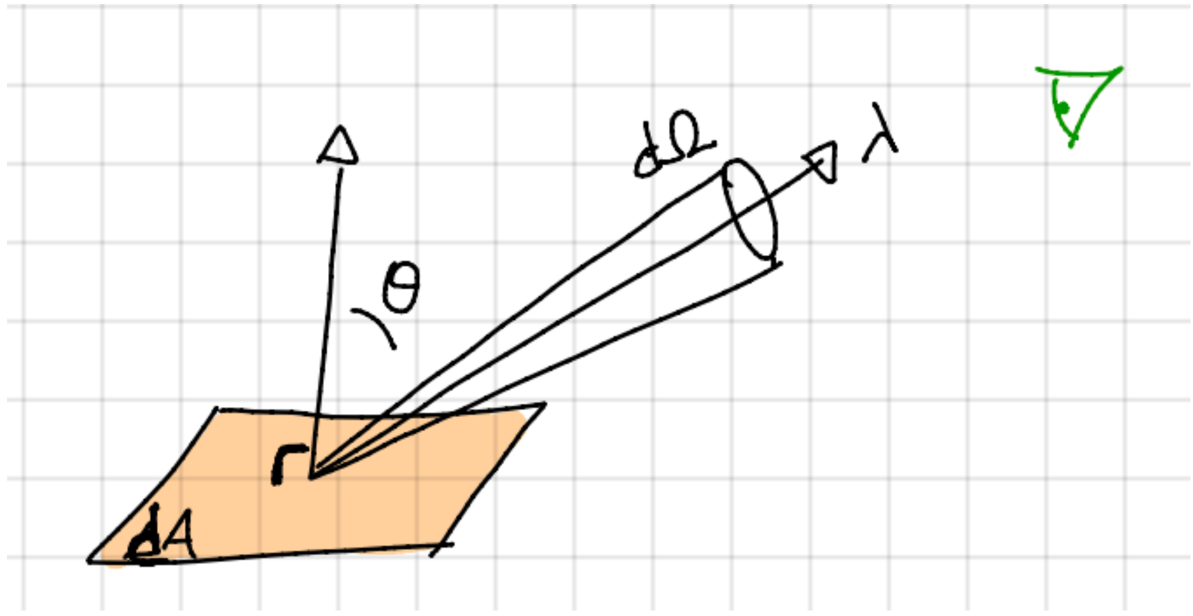
Total energy transported by a ray:

$$\frac{E}{d\lambda\ dt\ dA} = \int I_\lambda \frac{d\lambda\ dt\ dA}{d\lambda\ dt\ dA} \cos\theta\ d\Omega$$

$$F_\lambda = \int I_\lambda(\Omega)\ \cos\theta\ d\Omega$$

Specific flux F_λ:

$$d\Omega = \sin\theta d\theta d\phi$$



$$F_\lambda = \int I_\lambda(\Omega) \; \cos\theta \; d\Omega$$

$$= \int_0^{2\pi} \int_0^{\pi} I_\lambda(\theta, \phi) \; \cos\theta \quad \sin\theta d\theta d\phi$$

$$= \int_0^{2\pi} \int_{-1}^{+1} I_\lambda(u, \phi) \; u du$$

$$u = \cos\theta$$

$$du = -\sin\theta d\theta$$

When $\theta = 0, u = +1$

When $\theta = \pi, u = -1$

(Q: If the radiation is isotropic, what is the specific flux?)

Which of these rays of light has $u = 0$?