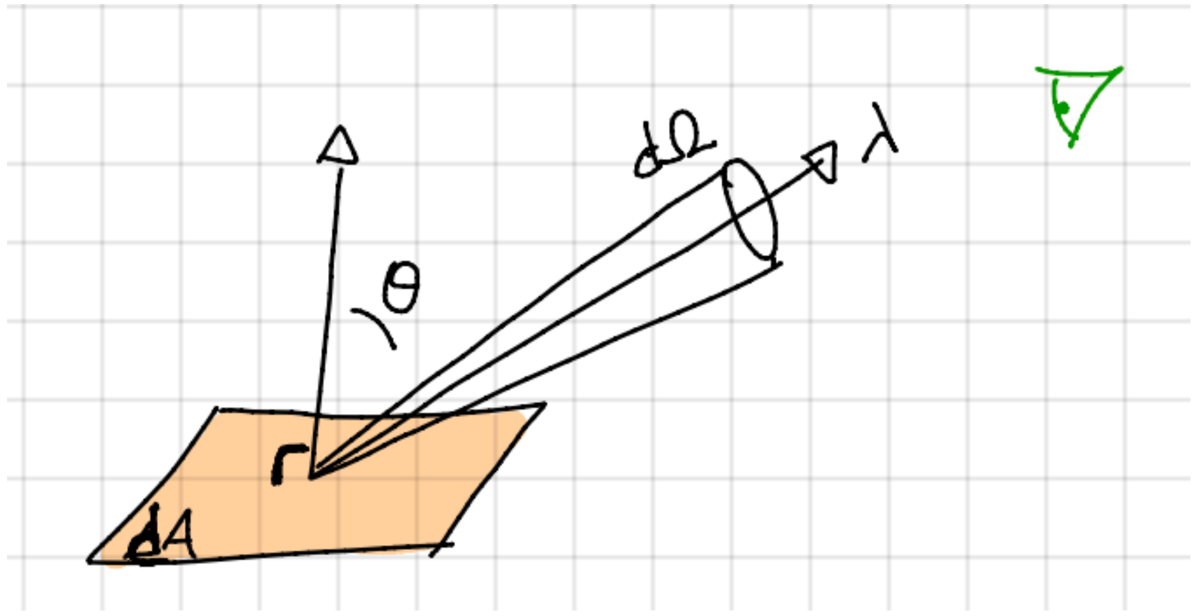
Week 5 Tuesday
L-9


Use the ideal gas law.
Means we need to know
1. the mean molecular weight
2. the temperature profile
L5
Mean molecular weight
Need to solve for the ionization level of all elements
L6
Saha equations
Numerical iterations
L7
L8
Energy transport
Radiative mean free path
Stellar interiors
Diffusive energy transport
Stellar atmospheres
Need detailed RT calculations
We are here
L18
Radiative transport in interiors
Energy conservation equation
L9
Basics of radiative transfer
Optical depth
1. Is the energy always transported by radiation?
L20
Convection
L10
Formal solution of RT with source function
L11
Relationship with blackbody systems and spectral lines formation
2. How to calculate the power generated?
L21
Nuclear physics
Gammow Peak
L22
Reaction rates
L12
Application to a “flat” stellar atmosphere
3. Is ideal gas law always valid?
L23
Degeneracy pressure
Chandrasekhar mass
Only depends on density.
We can use polytrops!
L13
Limb darkening and
radiative flux

Specific flux F_λ:

$$d\Omega = \sin\theta d\theta d\phi$$



$F_\lambda =$ Quantity of energy:

- per unit of time dt
- per unit of wavelength $d\lambda$
- going though a surface dA

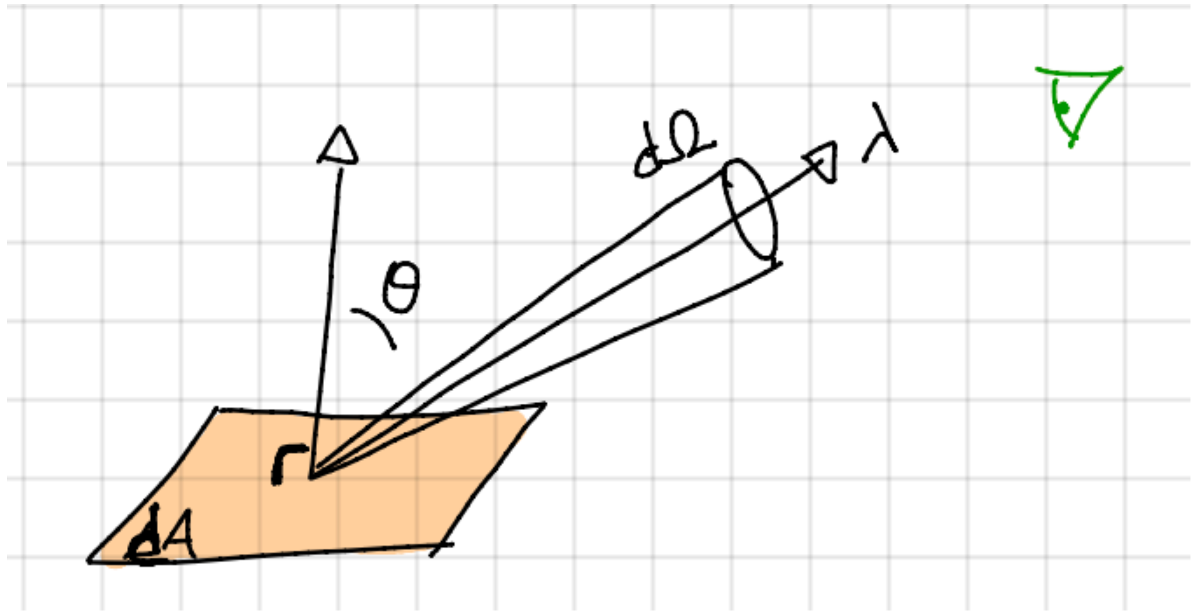
Total energy transported by a ray:

$$\frac{E}{d\lambda\ dt\ dA} = \int \frac{I_\lambda\ d\lambda\ dt\ dA}{d\lambda\ dt\ dA} \cos\theta\ \boxed{d\Omega}$$

$$F_\lambda = \int I_\lambda(\Omega)\ \cos\theta\ d\Omega$$

Specific flux F_λ:

$$d\Omega = \sin\theta d\theta d\phi$$



$$F_\lambda = \int I_\lambda(\Omega)\ \cos\theta\ d\Omega$$

$$= \int_0^{2\pi}\int_0^{\pi} I_\lambda(\theta,\phi)\ \cos\theta\ \ \sin\theta d\theta d\phi$$

$$u = \cos\theta$$

$$du = -\sin\theta d\theta$$

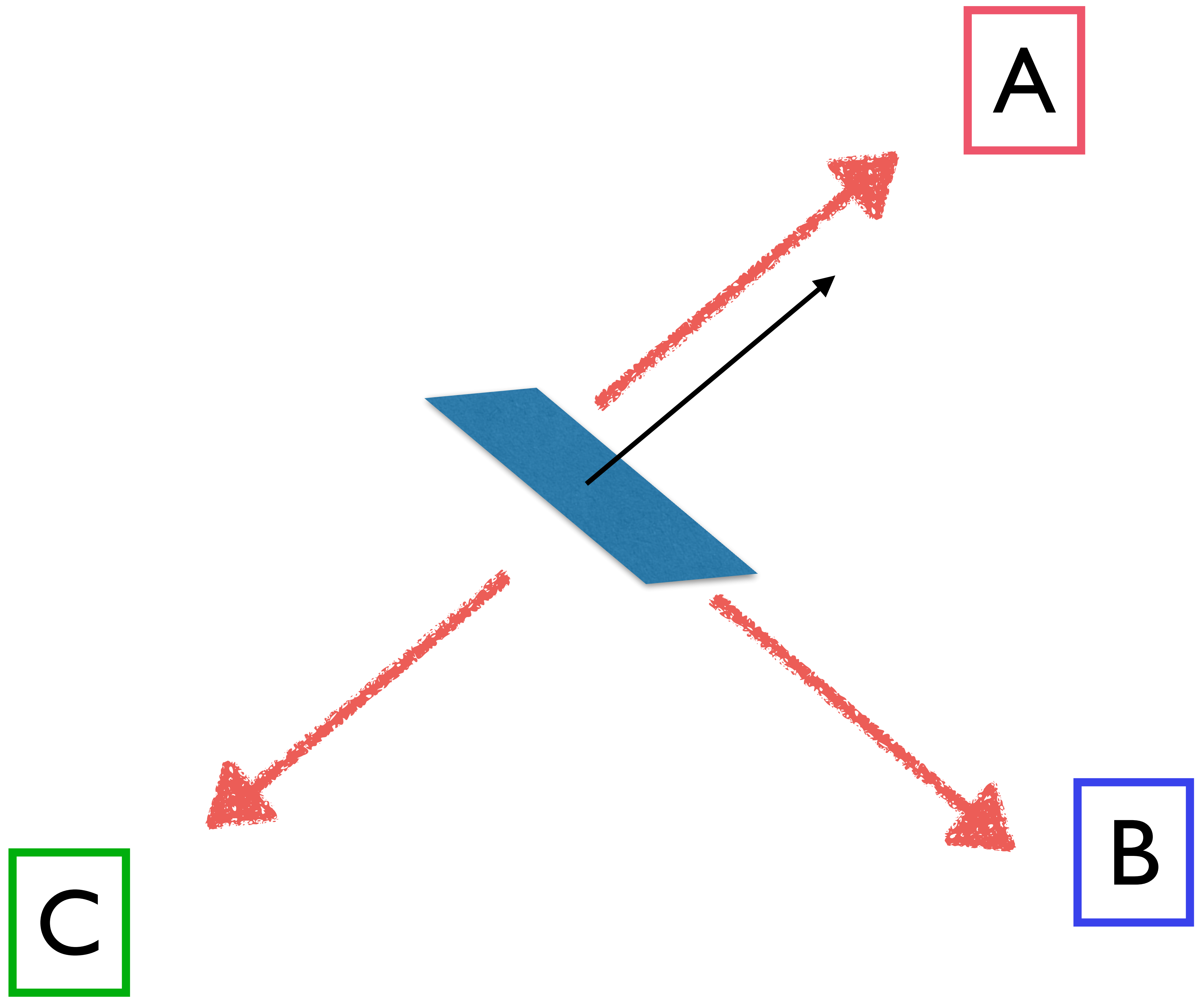
When $\theta = 0, u = +1$

When $\theta = \pi, u = -1$

$$= \int_0^{2\pi}\int_{-1}^{+1} I_\lambda(u,\phi)\ u du\ \ d\phi$$

(Q: If the radiation is isotropic, what is the specific flux?)

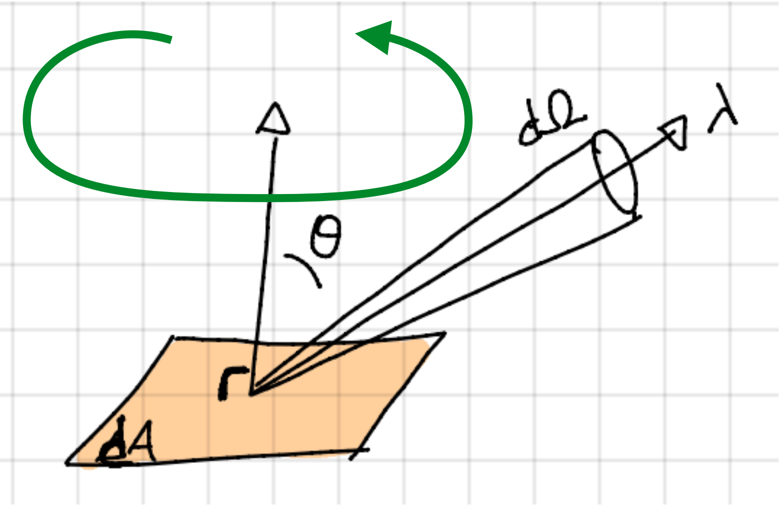
Which of these rays of light has $u = 0$?



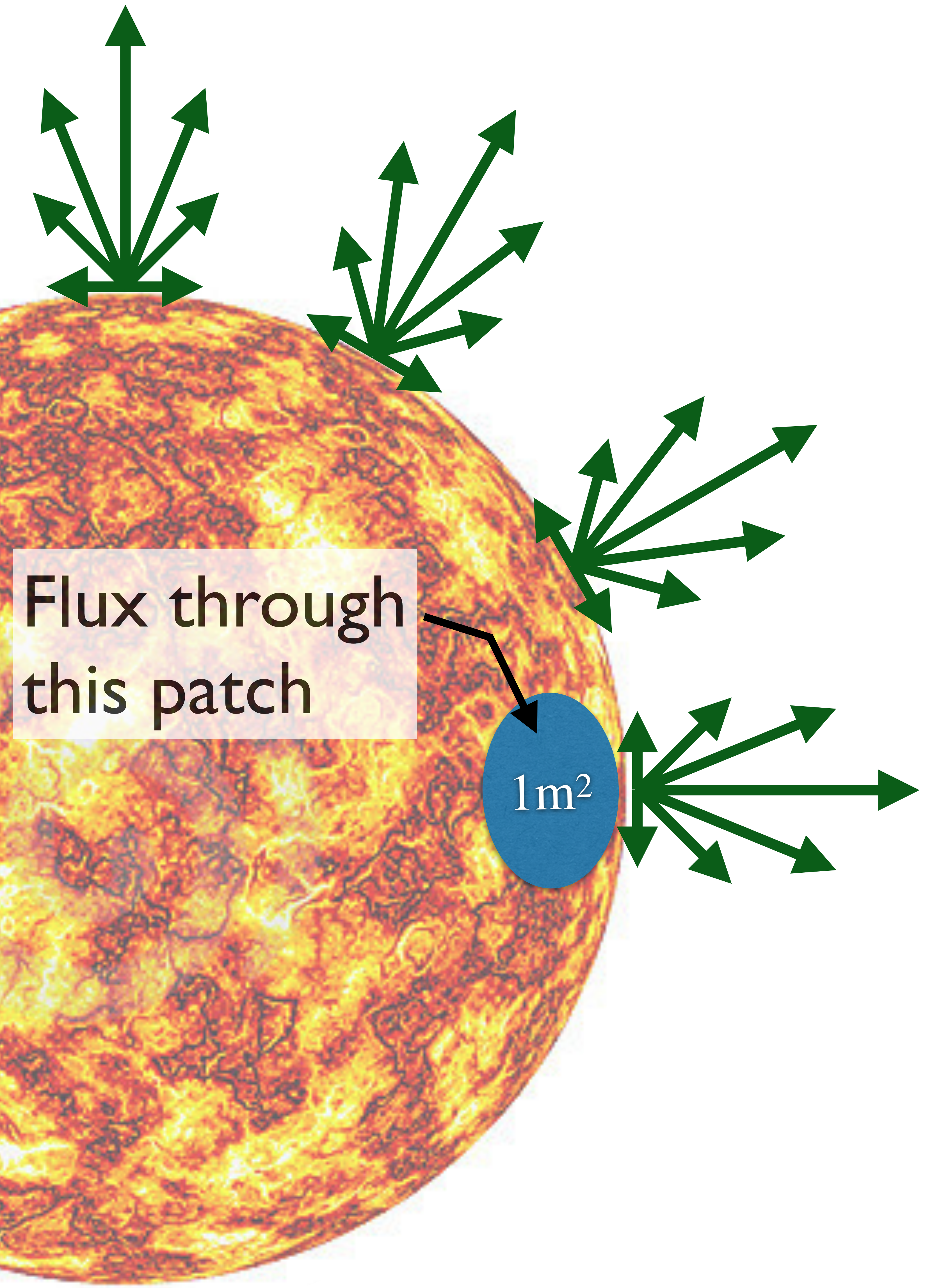
<u>Surface flux</u> by integrating intensity at one point at the surface

$$F_\lambda = \int_0^{2\pi}\int_{-1}^{+1} I_\lambda(u,\phi)\ u du \ \ d\phi$$

If there is azimuthal (ϕ) symmetry:



$$= \ 2\pi \int_{-1}^{+1} I_\lambda(u)\ u du$$



$$\frac{F_\lambda}{4\pi} = \frac{1}{2}\int_{\substack{\cancel{-1}\\ 0}}^{+1} I_\lambda(u)\ u\ du$$

$$I_\lambda(u<0) = 0 \qquad \text{(Q:Why?)}$$

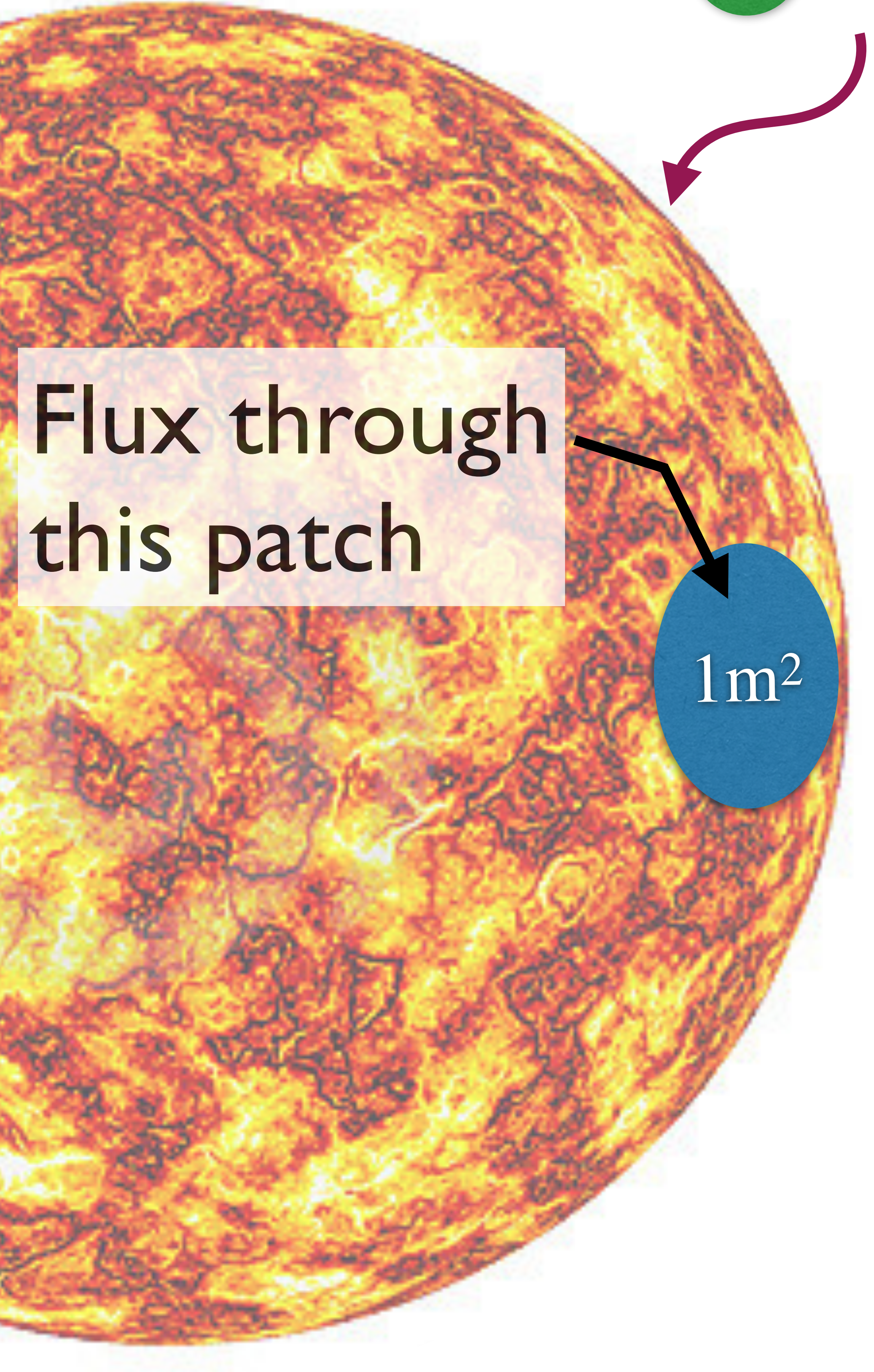
(For the Sun: 63 MegaWatt / m²)

Flux at a distance by scaling the surface flux

1 Total power (energy/s) = L (for luminosity)

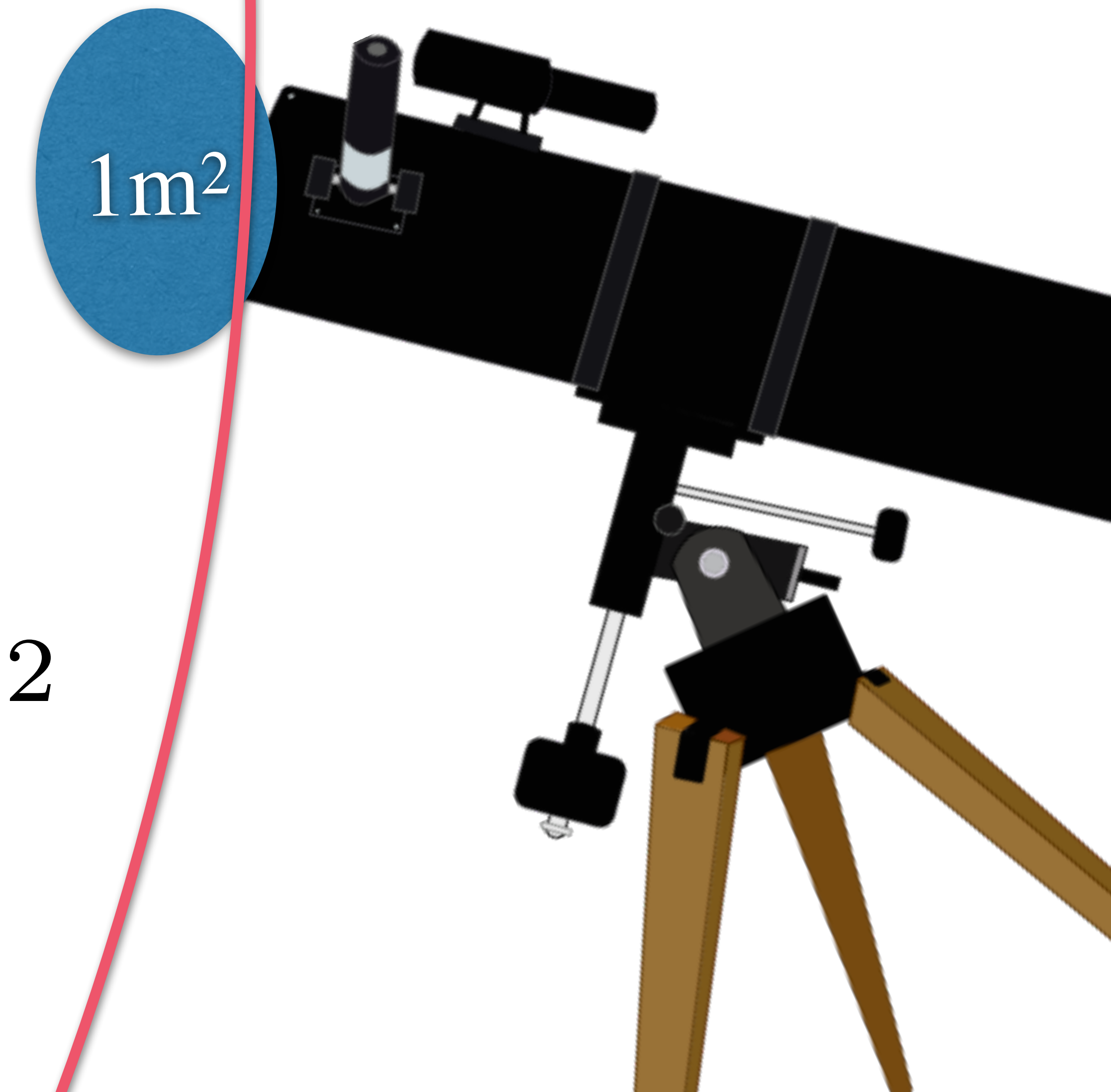
4 $\text{Area} = 4\pi d^2$

2 $\text{Area} = 4\pi R_\star^2$



3 $$F_{\text{surface}} = \frac{L}{4\pi R_\star^2}$$

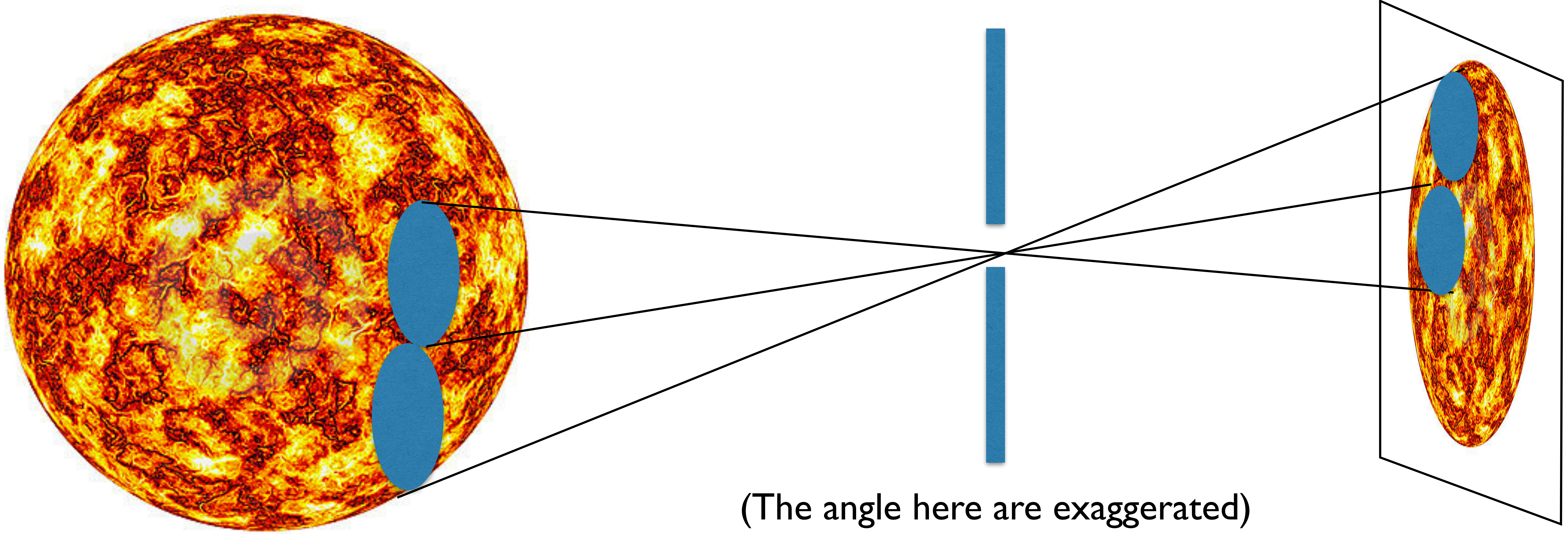
(For the Sun: 63 MegaWatt / m²)



5 $$F(d) = \frac{L}{4\pi d^2} = F_{\text{surface}} \left(\frac{R_\star}{d}\right)^2$$

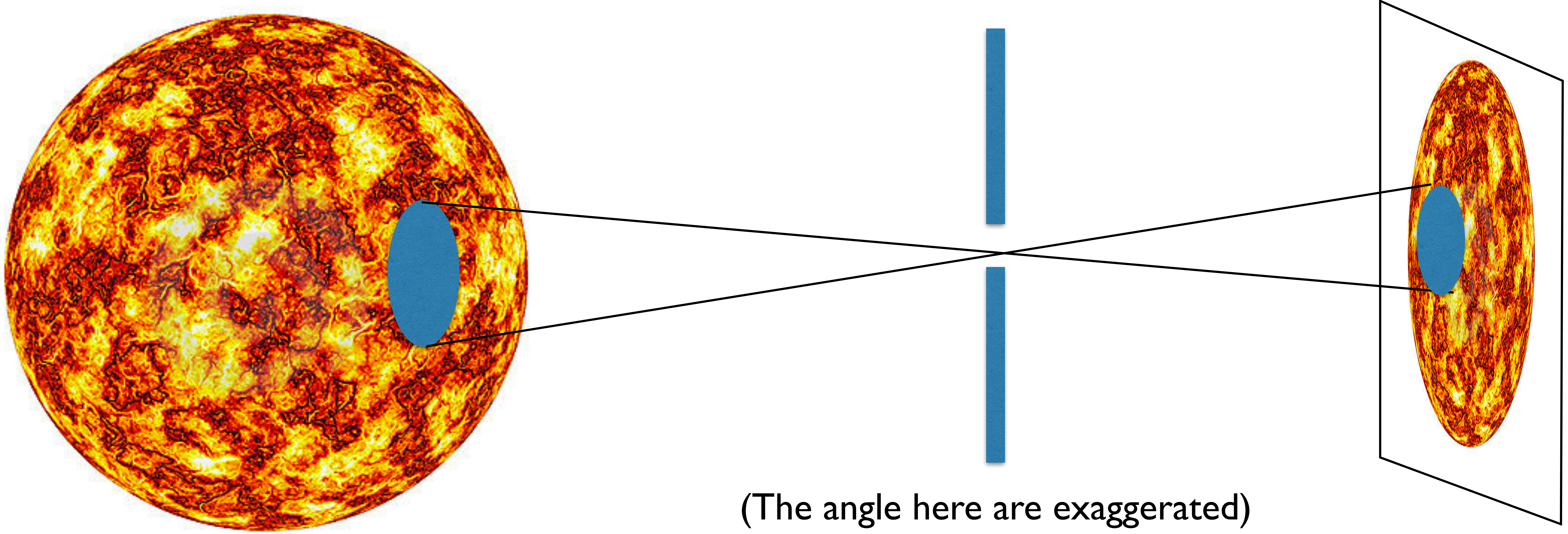
(At the distance of the Earth: 1300 Watt / m²)

Intensity versus Flux in observations



(The angle here are exaggerated)

Intensity versus Flux in observations

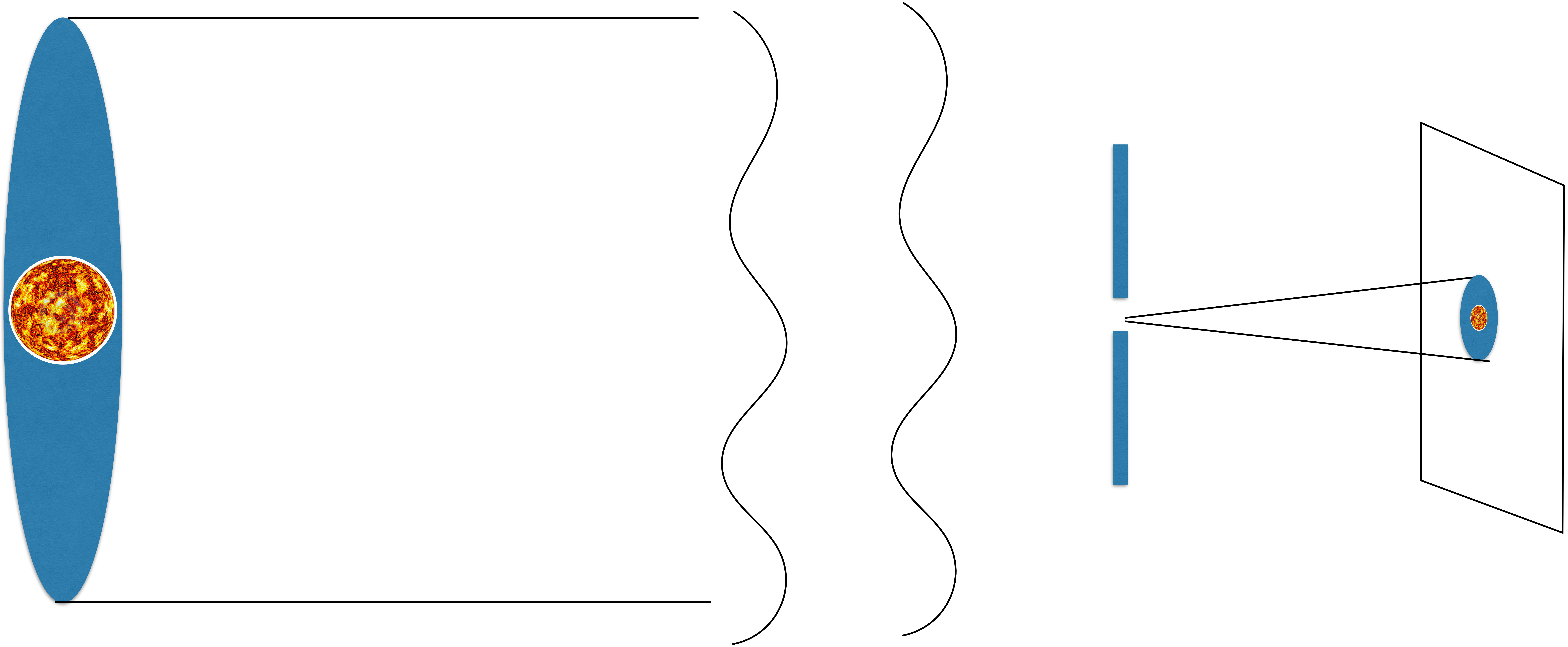


(The angle here are exaggerated)

Intensity is conserved along a ray (when propagating in vacuum)

Intensity versus Flux in observations

If the angular size of the object is smaller than one resolution element, the object is "unresolved".
We are now measuring the flux.

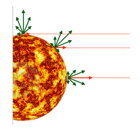


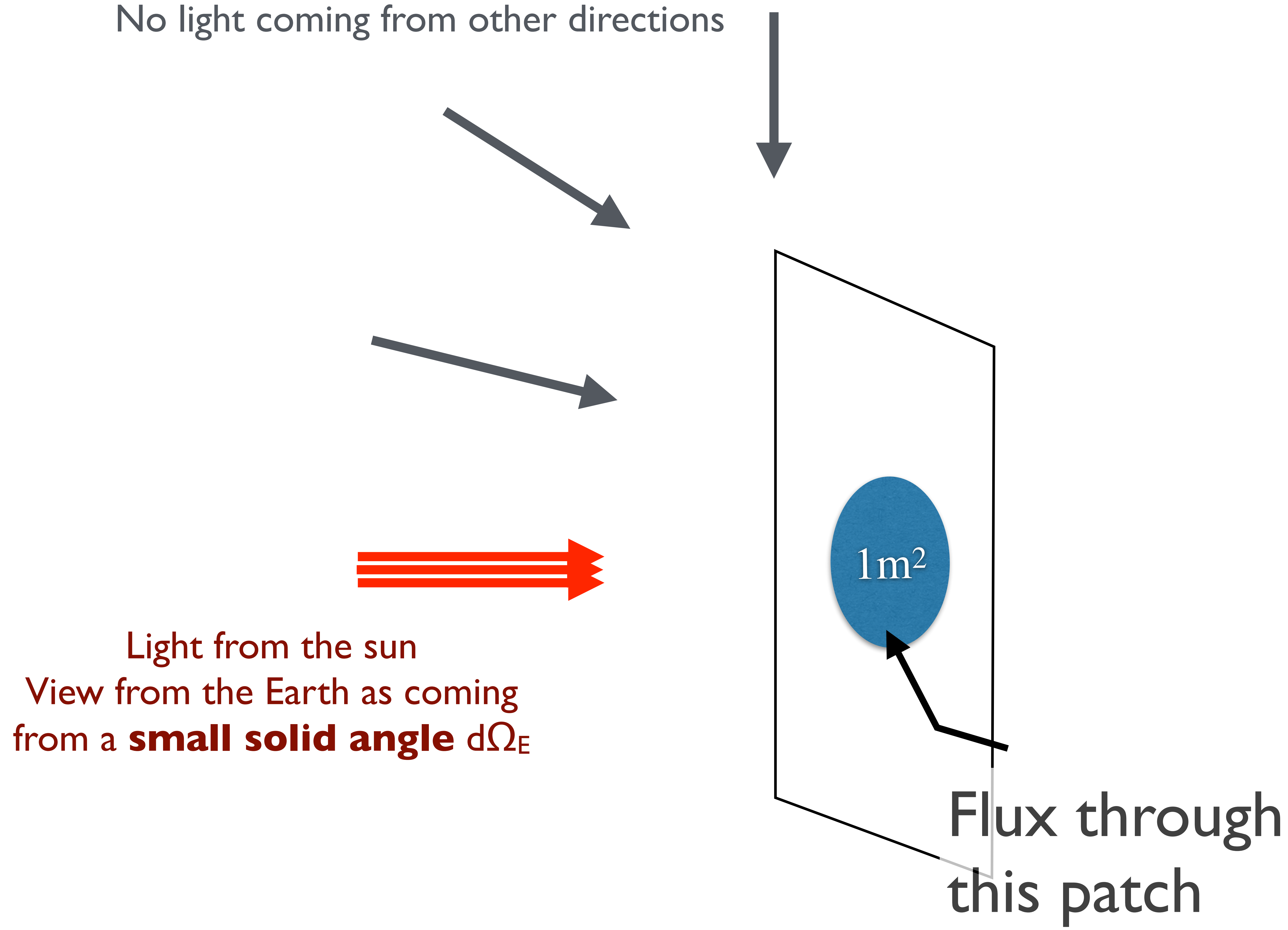
Flux at a distance by integrating at the detector

Flux at a distance by integrating at the detector

A bit more to scale

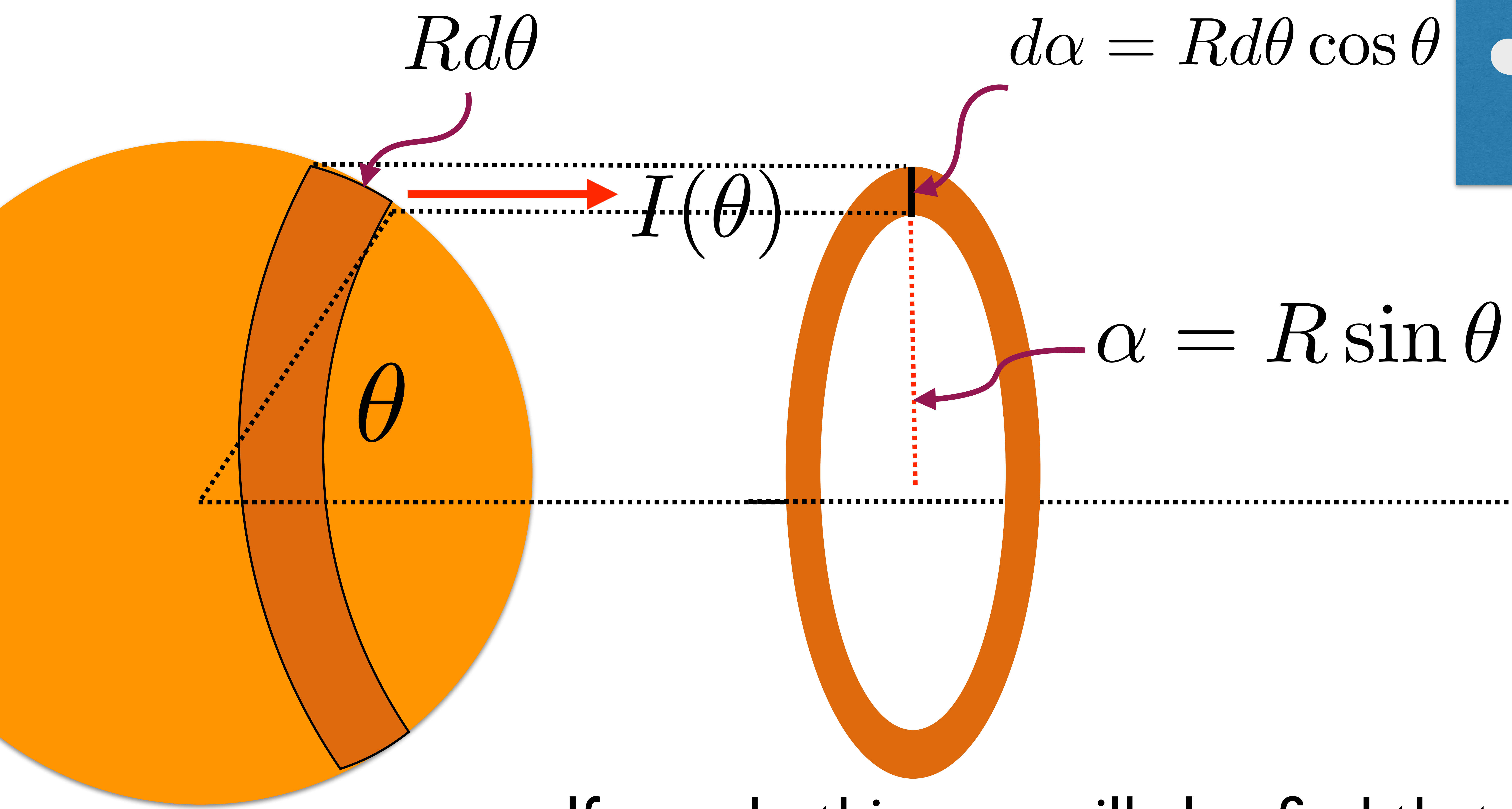
(That's the sun here)

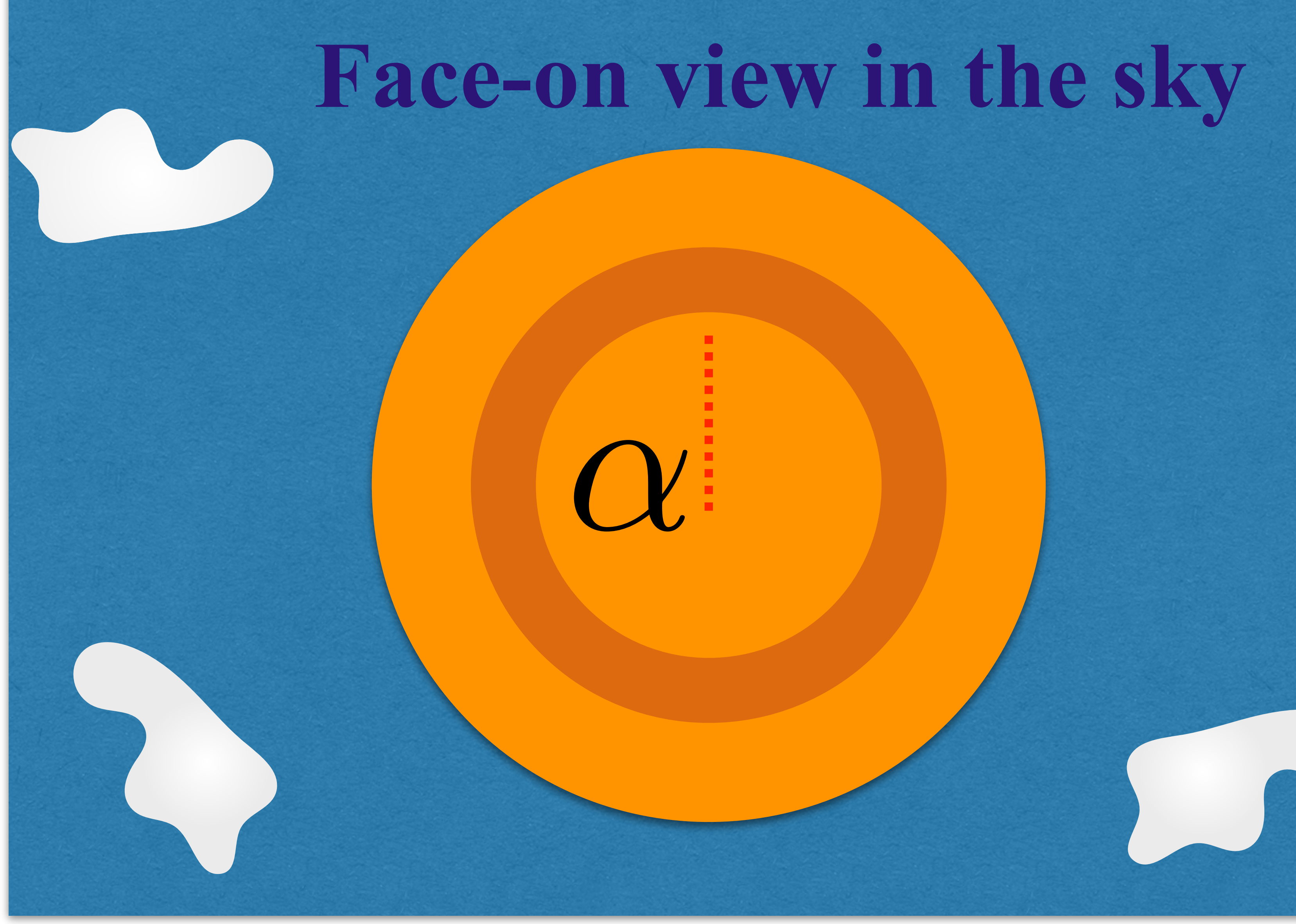




If you do this, you will also find that: $F(d) = F_{\mathrm{surface}} \left(\frac{R_\star}{d} \right)^2$

Flux at a distance by integrating over the sky-projected surface

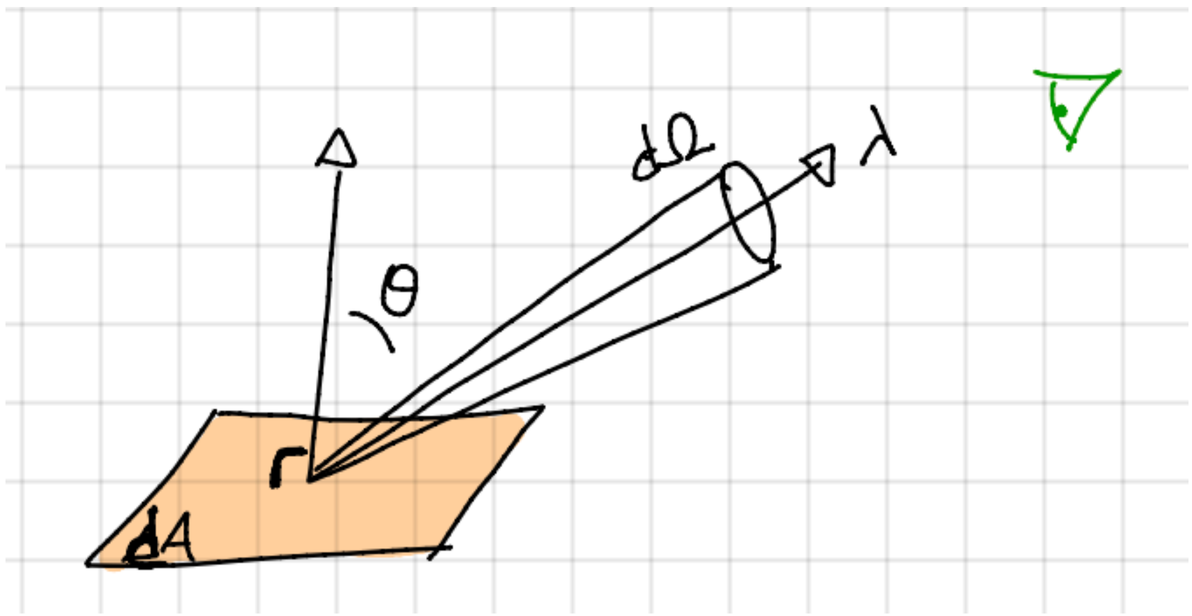




If you do this, you will also find that: $F(d) = F_{\text{surface}}\left(\frac{R_\star}{d}\right)^2$

Radiative Transfer:

What happens to intensity when a ray encounters matter



Absorption: how can we loose intensity?

1. Pure absorption: destroy a photon

2. Scattering: deflect a photon

Absorption: how can we loose intensity?

1. Pure absorption: destroy a photon

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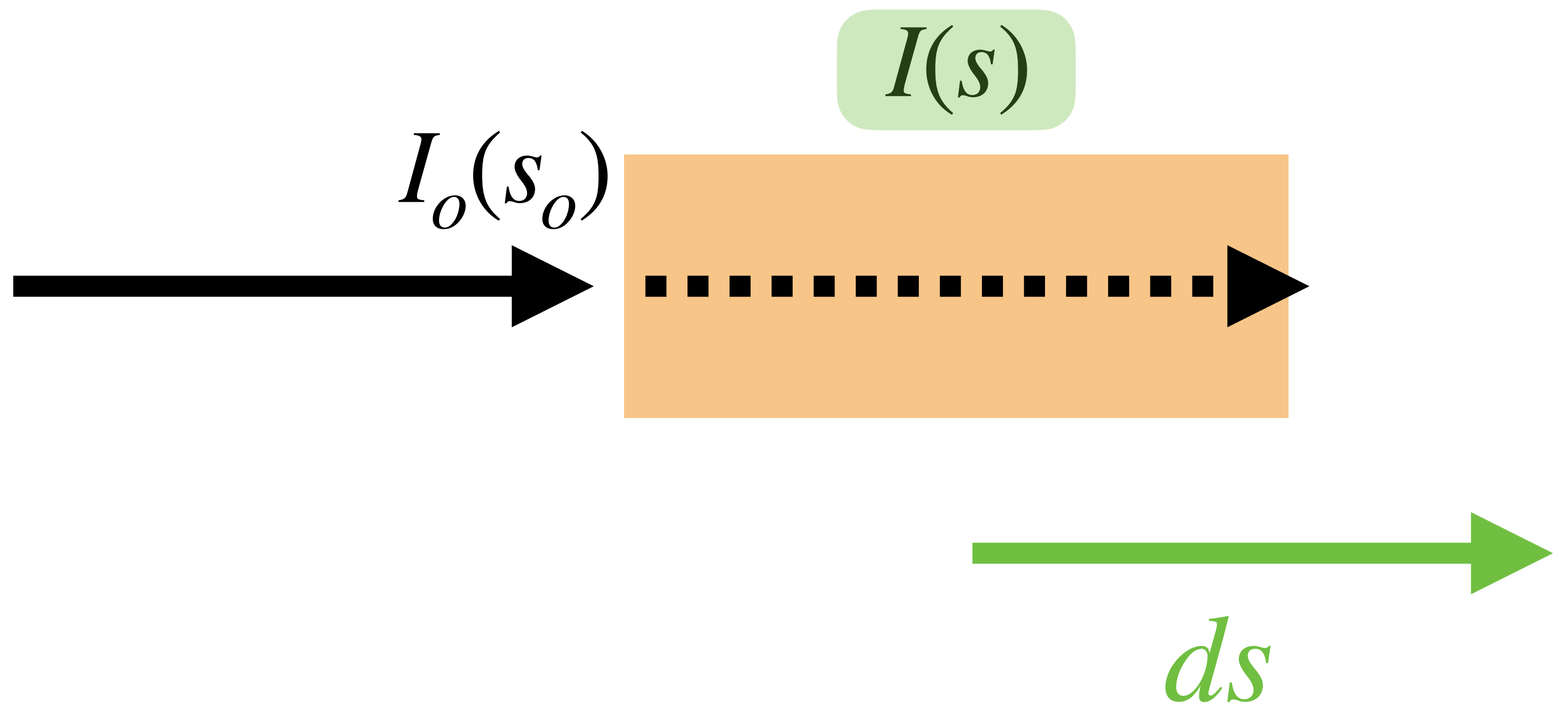
$$dI_\lambda \quad = \quad - \quad \rho \quad ds \quad \kappa_\lambda \quad I_\lambda$$

Let’s solve this equation, if we know $I(s_o) = I_o$



In the direction of the ray’s propagation

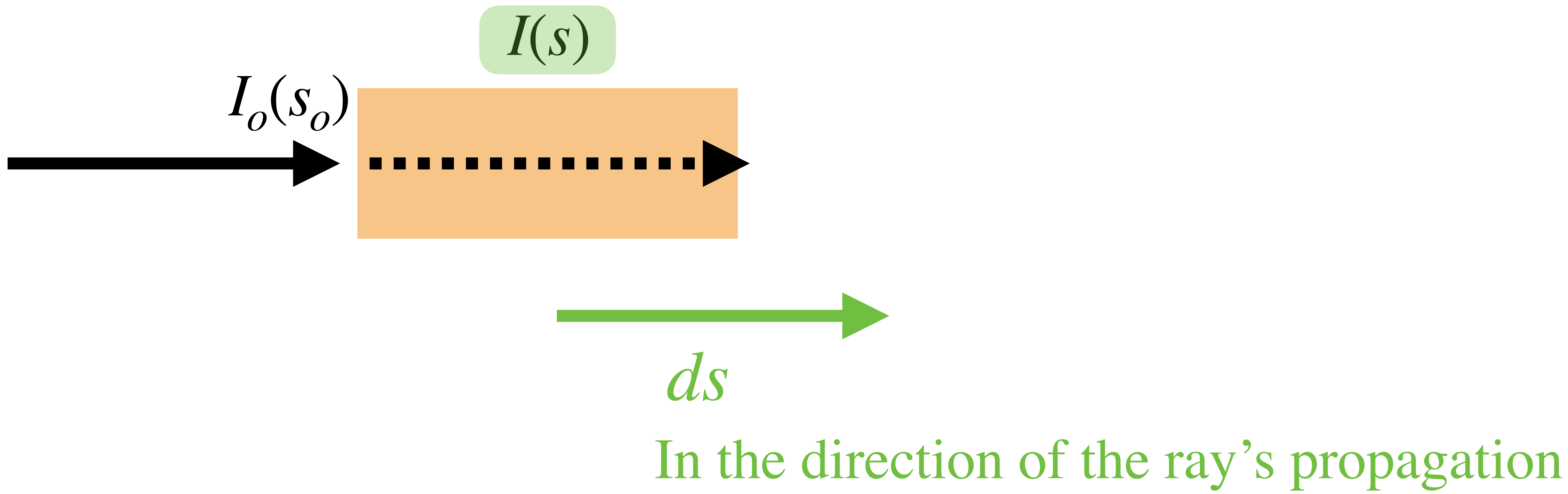
$$dI_\lambda = -\,\kappa_\lambda(s)\ \rho(s)\ I_\lambda(s)\ ds$$

Together on the board…

$$I(s) = I_o e^{-\int_{s_o}^{s} \kappa(s)\rho(s)ds}$$

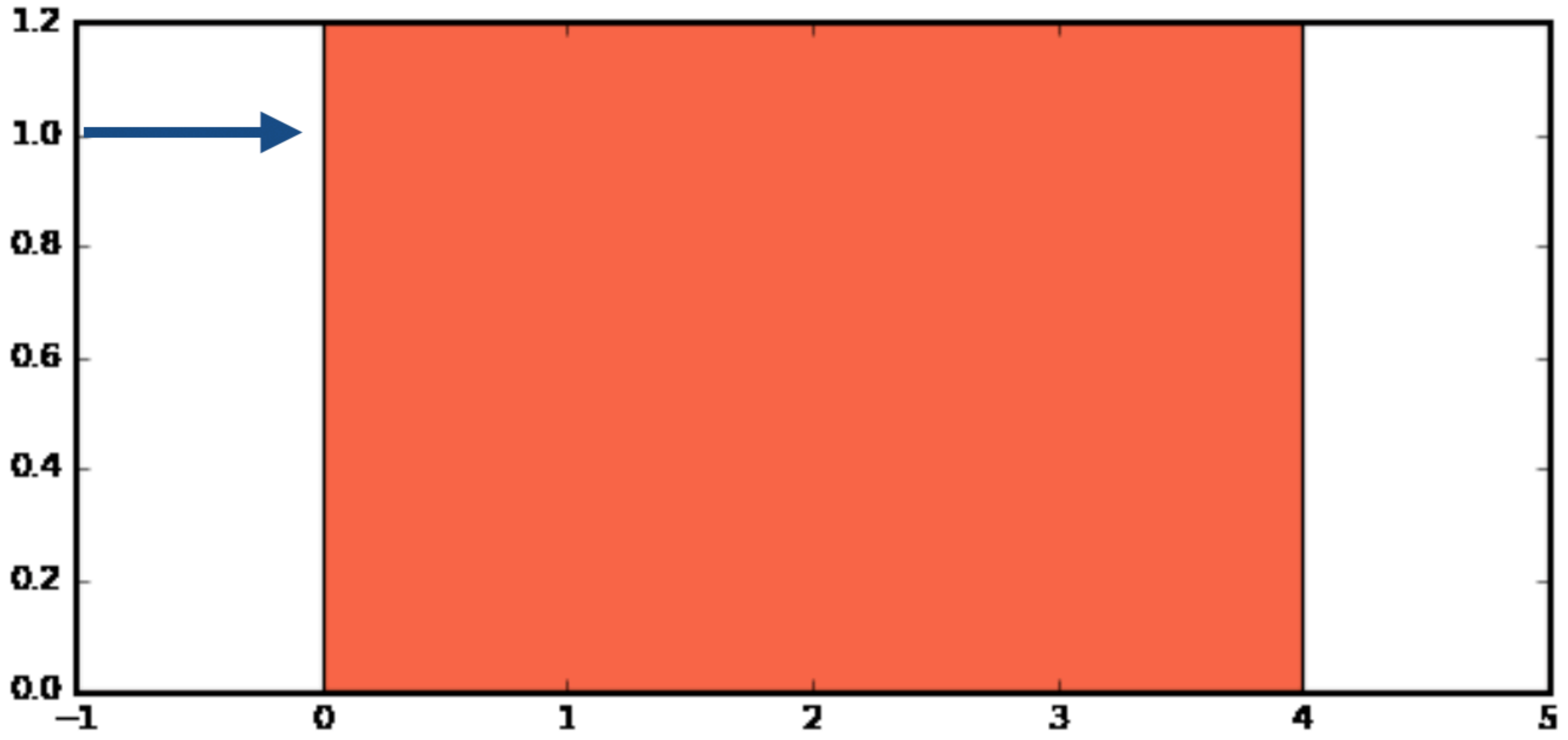
What if the properties of the material are constant?

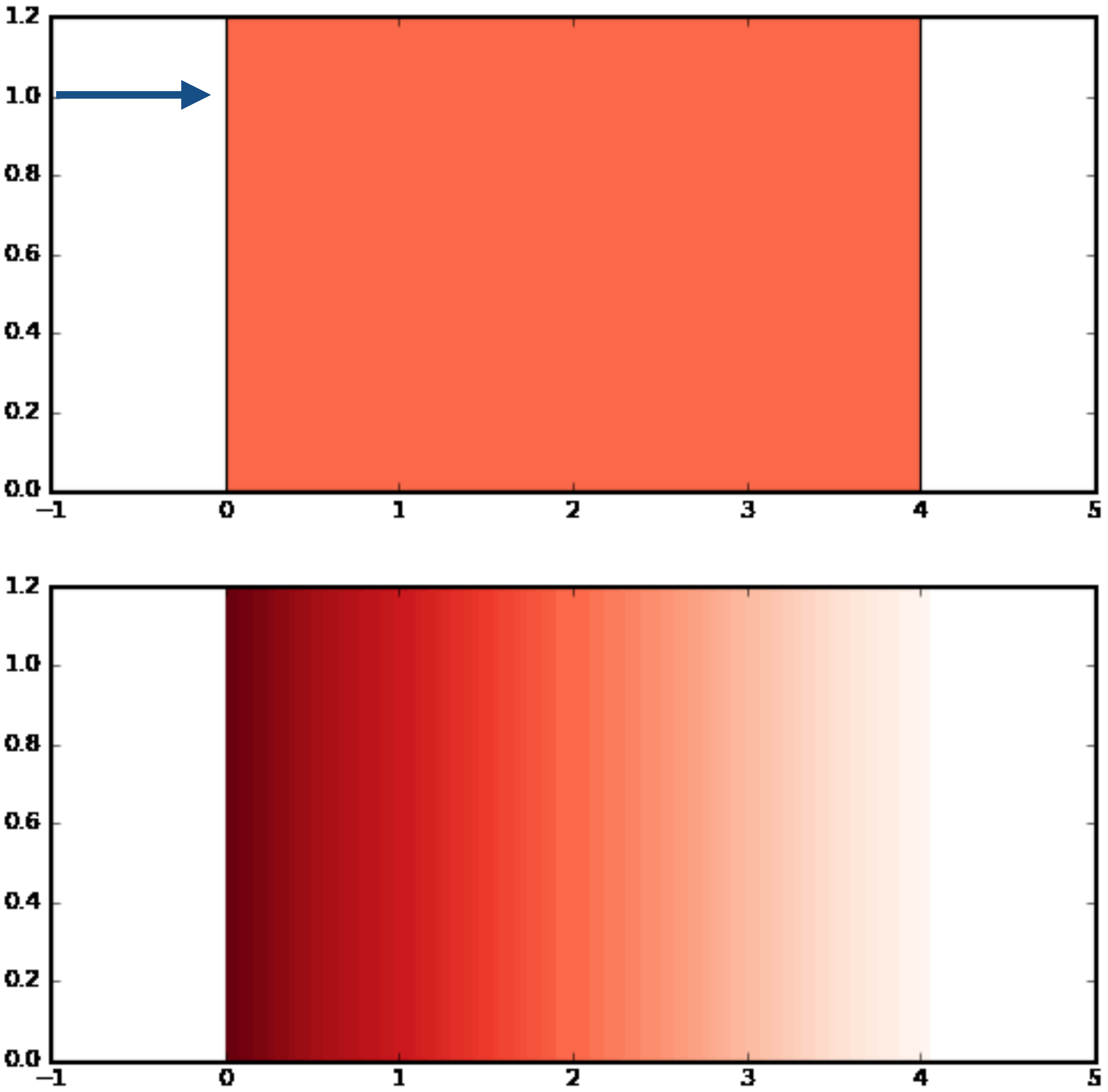
$$I(s) = I_o e^{-\int_{s_o}^{s} \kappa(s)\rho(s)ds}$$



In the notebook #1

- Find an equation for $I(s)$
- Find $I(s = 4 \text{ length units})$ if $\kappa\rho = 0.25 \text{ length}^{-1}$
- Graph $I(s)$

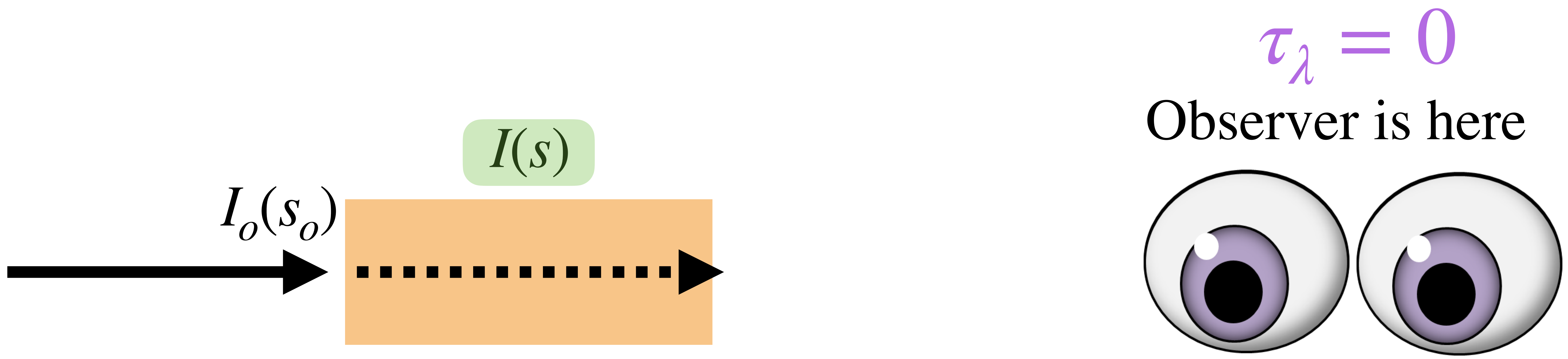




1.2
1.0
0.8
0.6
0.4
0.2
0.0
−1
0
1
2
3
4
5

The concept of optical depth

$$dI_\lambda = \boxed{-\,\kappa_\lambda(s)\ \rho(s)\ ds}\ I_\lambda(s)$$



$$d\tau_\lambda = -\,\kappa_\lambda(s)\ \rho(s)\ ds$$

ds

In the direction of the ray's propagation

$d\tau_\lambda$

In the direction opposite to the ray's propagation

The usual definition is to set the optical depth to <u>zero at the observer</u>

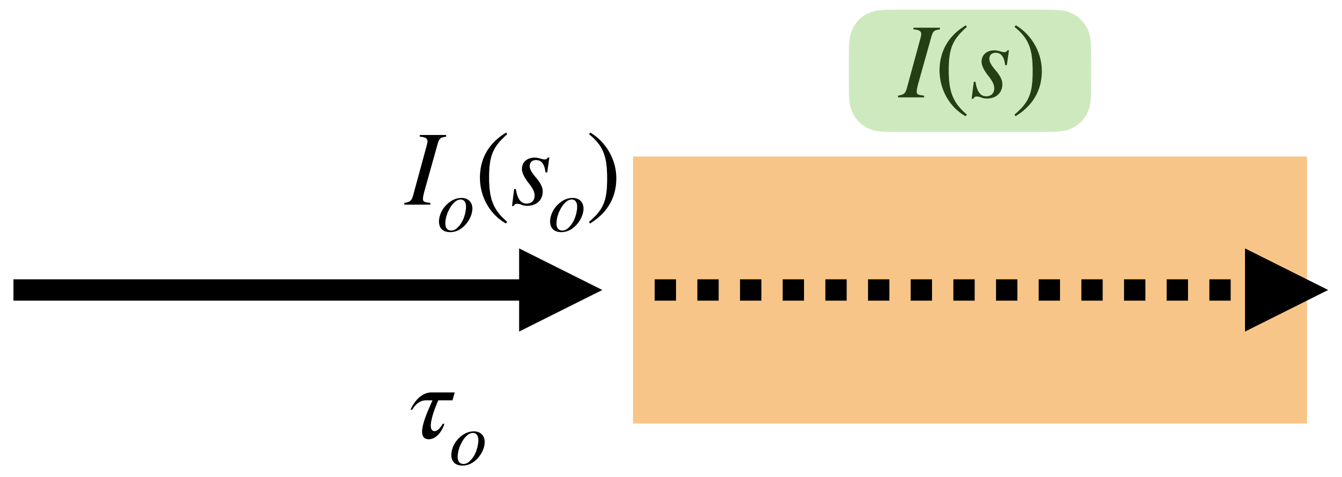
$$\int_{\text{observer}}^{\tau} d\tau_\lambda = -\int_{\text{observer}}^{s} \kappa_\lambda(s)\ \rho(s)\ ds$$

$$\tau_\lambda(s) - 0 = -\int_{s_{\text{obs}}}^{s} \kappa_\lambda(s)\ \rho(s)\ ds$$

∞

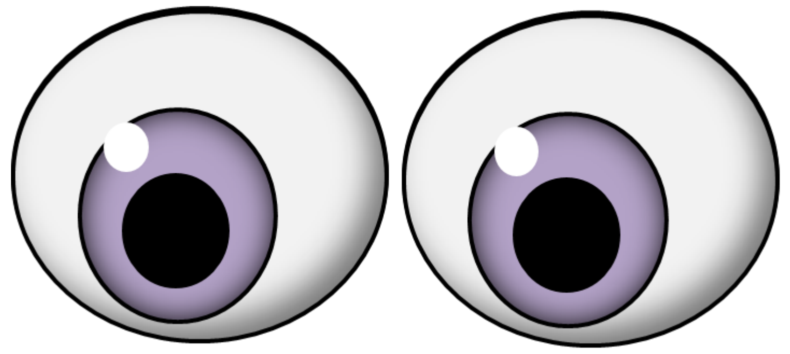
Often just denoted by ∞, because empty space between the end of the slab and the observer has zero density.

Let's solve this equation, if we know $I(s_o) = I_o$



$\tau_\lambda = 0$

Observer is here



$$dI_\lambda = \boxed{-\,\kappa_\lambda(s)\ \rho(s)\ ds}\ I_\lambda(s)$$

$$dI_\lambda = d\tau_\lambda\ \ I_\lambda(s)$$

ds

In the direction of the ray's propagation

$d\tau_\lambda$

$$\tau_\lambda(s) - 0 = -\int_{s_{\mathrm{obs}}}^{s} \kappa_\lambda(s)\ \rho(s)\ ds$$

In the direction opposite to the ray's propagation

Solution:

$$I(s) = I_o\ \mathrm{e}^{\tau(s)-\tau_o}$$

This is what we had before:

$$I(s) = I_o \mathrm{e}^{-\int_{s_o}^{s} \kappa(s)\rho(s)ds}$$

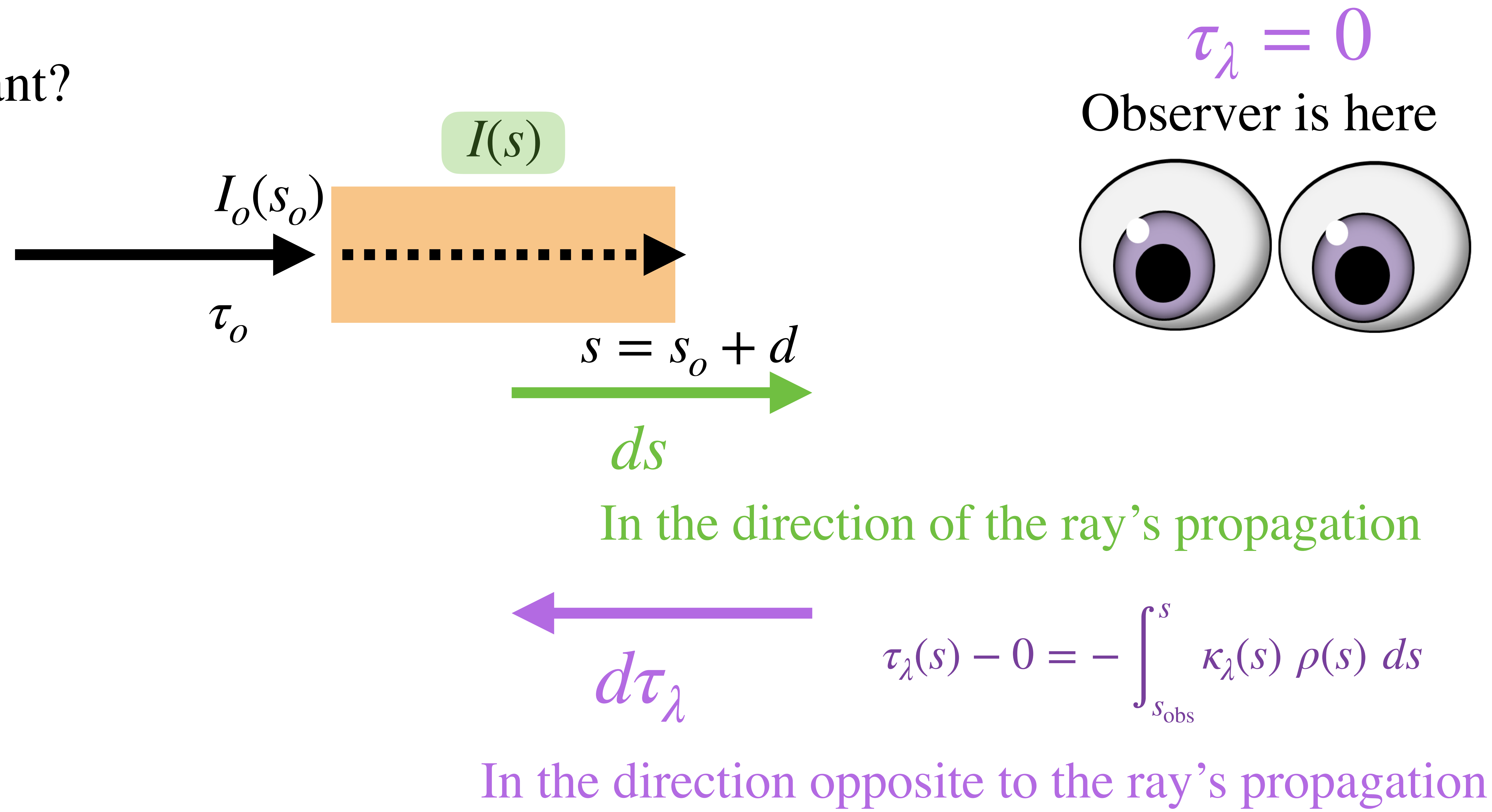
We can also use τ as a kind of coordinate to measure 'depth' in the material
(Q: what are the units of $d\tau_\lambda$?)

$$I(\tau) = I_o\ \mathrm{e}^{\tau-\tau_o}$$

Q: The intensity at the observer is?

What if the properties of the material are constant?

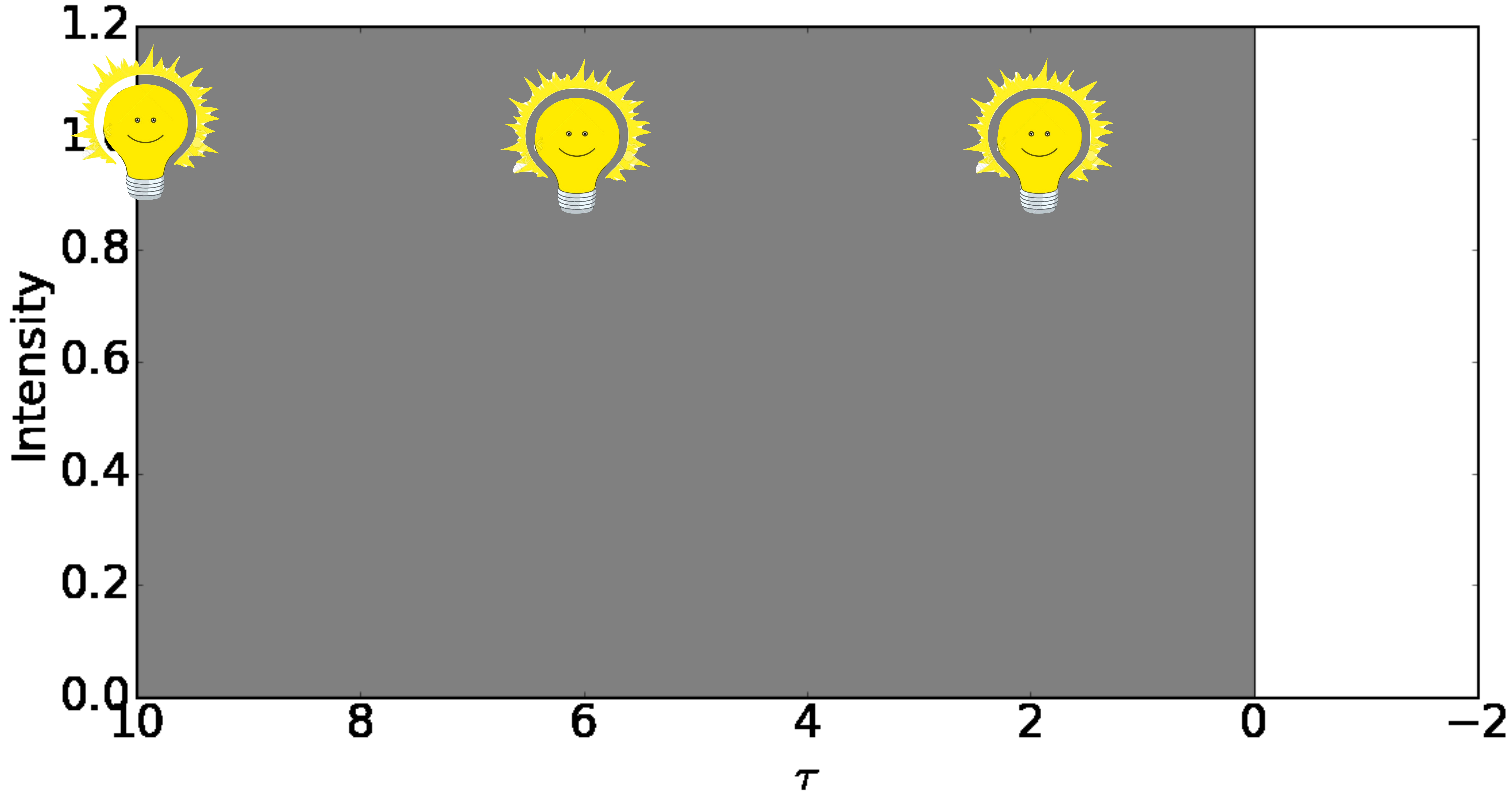
$$I(s) = I_o \, e^{\tau(s)-\tau_o}$$



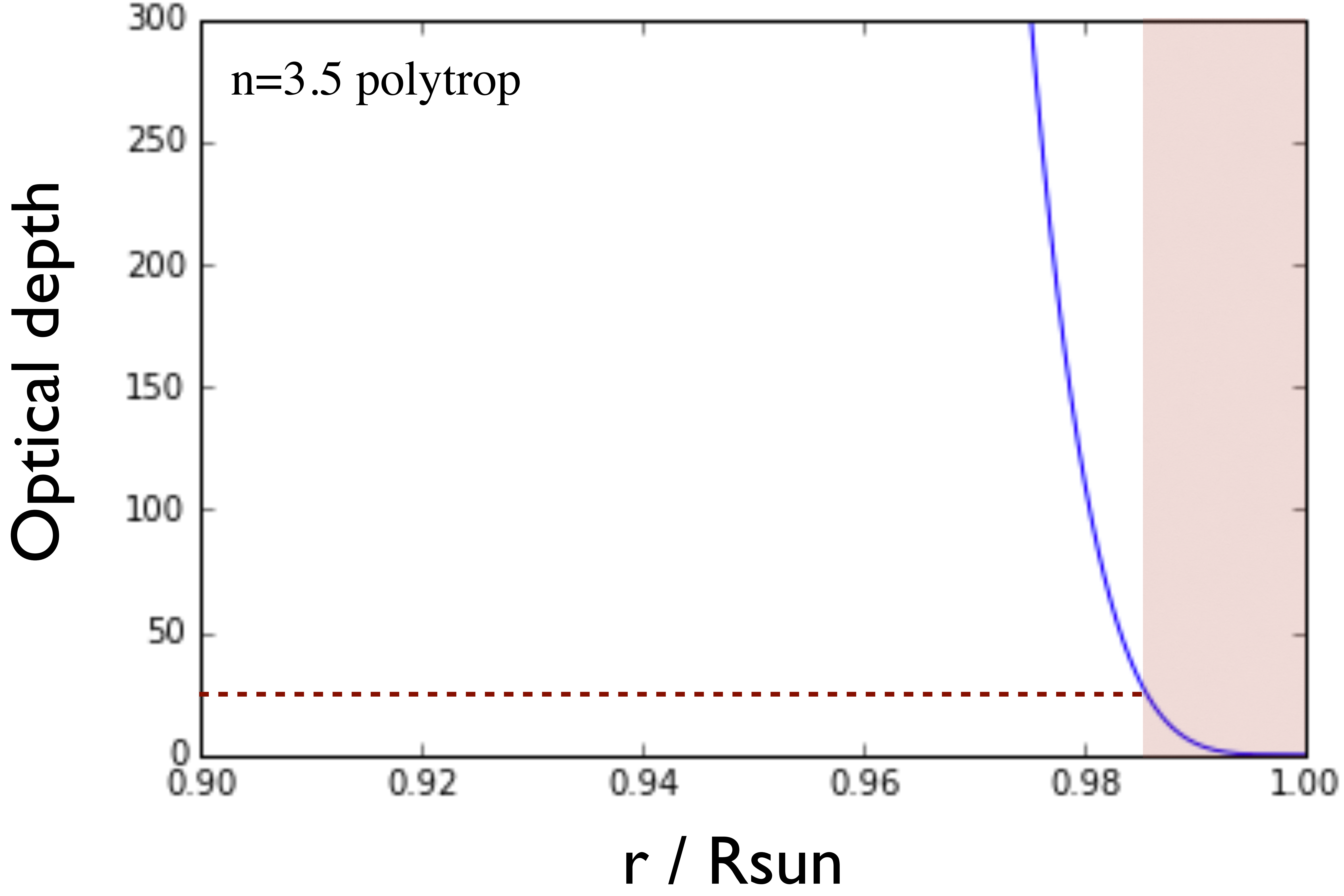
In the notebook #1

- Find an equation for $\tau(s)$
- Graph $\tau(s)$

The utility of optical depth: we only case about what happens within a few optical depths….

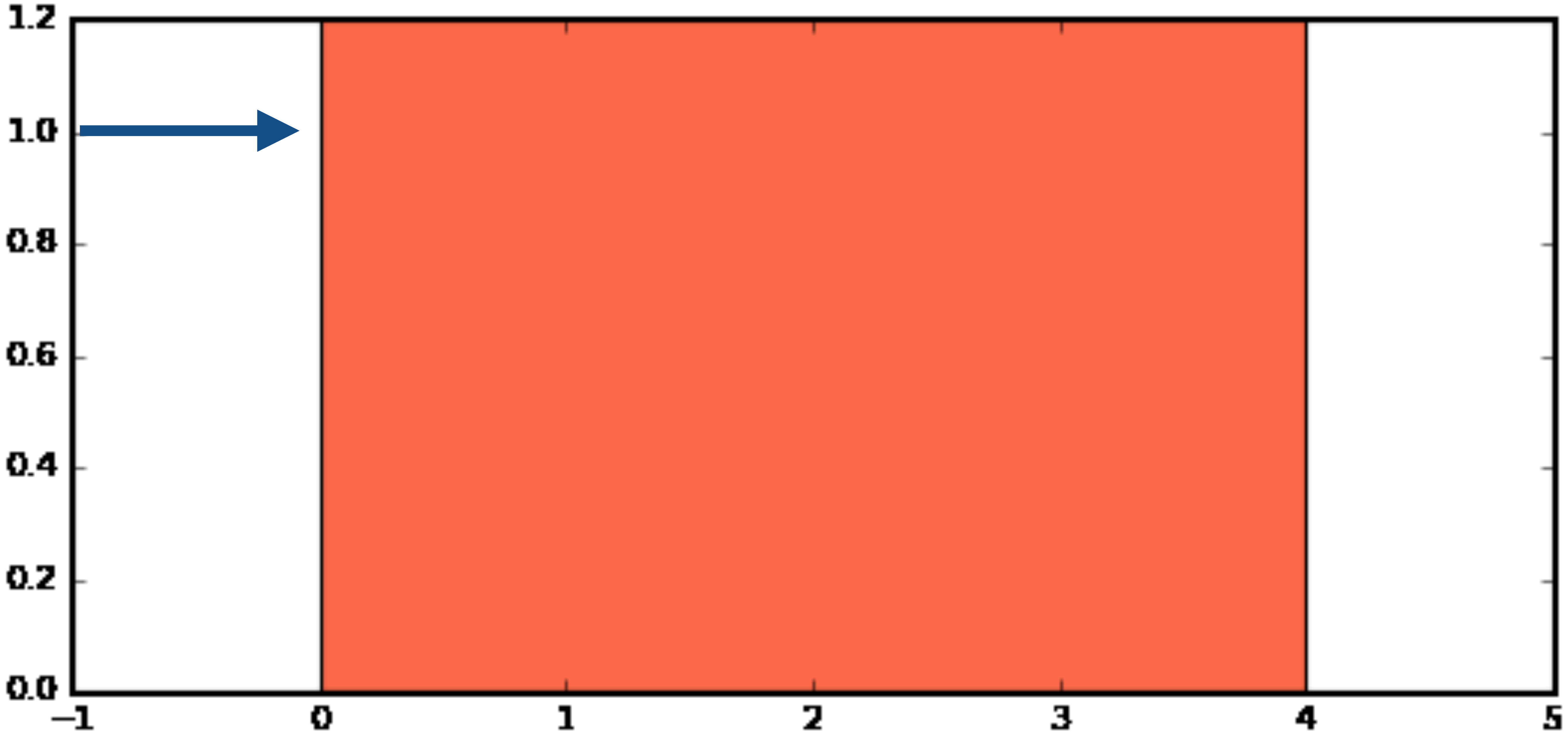


$$I_{\text{out}}\,(u,\ \tau = 0)\ ?$$



The emergent intensity only depends on what's going on within few optical depths from the surface.

We can use the optical depth as our independent variable instead of the radius



In the notebook #2

What is ρ is linearly decreasing?