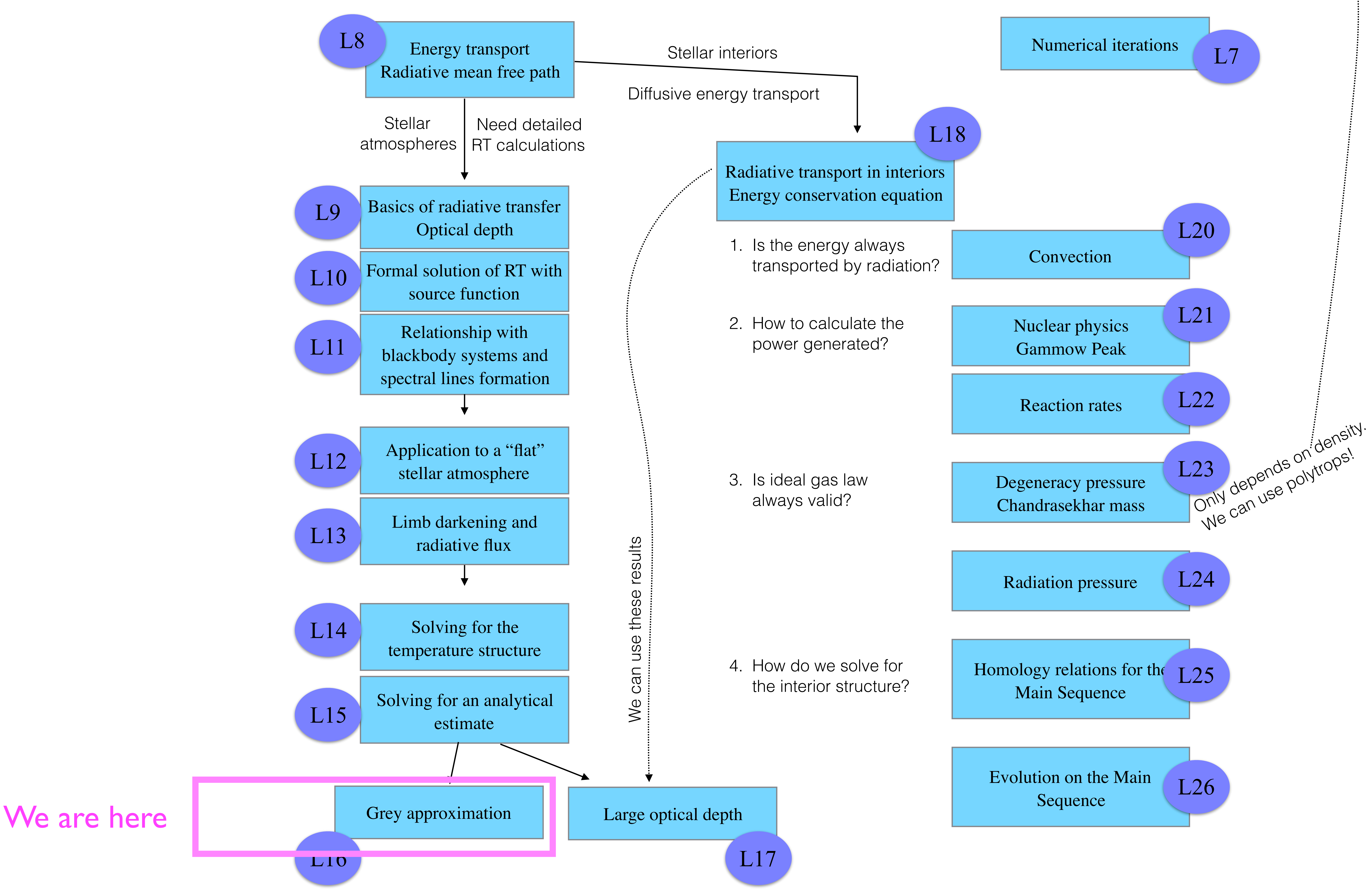
Week 9 Thursday
L-16
Grey atmospheres


L8
Energy transport
Radiative mean free path
Stellar interiors
Diffusive energy transport
Numerical iterations
L7
Stellar atmospheres
Need detailed RT calculations
L18
Radiative transport in interiors
Energy conservation equation
L9
Basics of radiative transfer
Optical depth
L10
Formal solution of RT with source function
L11
Relationship with blackbody systems and spectral lines formation
L12
Application to a “flat” stellar atmosphere
L13
Limb darkening and radiative flux
L14
Solving for the temperature structure
L15
Solving for an analytical estimate
We are here
Grey approximation
L16
Large optical depth
L17
We can use these results
1. Is the energy always transported by radiation?
2. How to calculate the power generated?
3. Is ideal gas law always valid?
4. How do we solve for the interior structure?
Convection
L20
Nuclear physics
Gammow Peak
L21
Reaction rates
L22
Degeneracy pressure
Chandrasekhar mass
L23
Only depends on density.
We can use polytrops!
Radiation pressure
L24
Homology relations for the Main Sequence
L25
Evolution on the Main Sequence
L26

The grey case (opacity is not a function of wavelength) + large τ

Reminder

1 $$\tilde{J}(\tau_z) = \frac{\sigma \boxed{T^4(\tau_z)}}{\pi}$$

2 $$\frac{d\tilde{K}(\tau_z)}{d\tau_z} = \frac{\sigma T_{\text{eff}}^4}{4\pi}$$

$\rightarrow$ Step 1: Integrate this $\tilde{K}(\tau_z) = \frac{\sigma T_{\text{eff}}^4}{4\pi}(\tau_z + \text{const})$

3 $$\tilde{J}(\tau_z) = 3\tilde{K}(\tau_z)$$

$\rightarrow$ Step 2: replace $\tilde{K}$ in here $\tilde{J}(\tau_z) = \frac{3\sigma T_{\text{eff}}^4}{4\pi}(\tau_z + \text{const})$

Step 3: replace $\tilde{J}$ in here $\frac{\sigma \boxed{T^4(\tau_z)}}{\pi} = \frac{3\sigma T_{\text{eff}}^4}{4\pi}(\tau_z + \text{const})$

$$T^4(\tau_z) = \frac{3T_{\text{eff}}^4}{4}(\tau_z + q(\tau_z))$$

The grey case (opacity is not a function of wavelength) + large τ

Reminder

1 $$\tilde{J}(\tau_z) = \frac{\sigma T^4(\tau_z)}{\pi}$$

→Remember, this came for out result that $\tilde{J} = \tilde{S}$

If the opacity is "grey" $\kappa \neq \kappa_\lambda$:

$$0 = \kappa \int_0^\infty (J_\lambda - S_\lambda) d\lambda$$

$$\int_0^\infty J_\lambda \, d\lambda = \int_0^\infty S_\lambda \, d\lambda$$

$$\tilde{J}(\tau_z) = \tilde{S}(\tau_z)$$

But if $S_\lambda \simeq B_\lambda$, we can also write:

$$\tilde{J}(\tau_z) = \tilde{B}(T(\tau_z)) = \frac{\sigma T^4(\tau_z)}{\pi}$$

2 $$\frac{d\tilde{K}(\tau_z)}{d\tau_z} = \frac{\sigma T_{\rm eff}^4}{4\pi}$$

3 $$\tilde{J}(\tau_z) = 3\tilde{K}(\tau_z)$$

→ Step 2: replace $\tilde{K}$ in here $\tilde{J}(\tau_z) = \frac{3\sigma T_{\rm eff}^4}{4\pi}(\tau_z + \text{const})$

$$J_\lambda(\tau) = +\frac{1}{2}\int_{\tau'=0}^{\tau} S_\lambda(\tau')E_1(\tau - \tau')d\tau' + \frac{1}{2}\int_{\tau'=\tau}^{\infty} S_\lambda(\tau')E_1(\tau' - \tau)d\tau'$$

$$\tilde{J}(\tau) = +\frac{1}{2}\int_{\tau'=0}^{\tau} \tilde{S}(\tau')E_1(\tau - \tau')d\tau' + \frac{1}{2}\int_{\tau'=\tau}^{\infty} \tilde{S}(\tau')E_1(\tau' - \tau)d\tau'$$

$$\frac{3\sigma T_{\rm eff}^4}{4\pi}(\tau_z + \boxed{q(\tau_z)}) \qquad \frac{3\sigma T_{\rm eff}^4}{4\pi}(\tau_z + \boxed{q(\tau_z)}) \qquad \frac{3\sigma T_{\rm eff}^4}{4\pi}(\tau_z + \boxed{q(\tau_z)})$$

$$\boxed{q(\tau) = 0.7104 - 0.133\text{e}^{-3.4488\tau_z}}$$

We can find another approximation for q,
using again the expressions for the large optical depth case.

We can do the same procedure (approximation for large τ) for all of the 'moment' equations, and also for the intensity solution.

$$\boxed{I(\tau,u) = S(\tau) + u\frac{dS(\tau')}{d\tau'}\bigg|_{\tau}} + 2!\ u^2\frac{d^2S(\tau')}{d\tau'^2}\bigg|_{\tau} + \ldots$$

Reminder

$$\boxed{J(\tau) = S(\tau)} + \frac{1}{3}\frac{d^2S(\tau')}{d\tau'^2}\bigg|_{\tau} + \ldots$$

$$\boxed{\frac{F}{4\pi} = \frac{1}{3}\frac{dS(\tau')}{d\tau'}\bigg|_{\tau}} + \frac{1}{5}\frac{d^3S(\tau')}{d\tau'^3}\bigg|_{\tau} + \ldots$$

$$\boxed{K(\tau) = \frac{1}{3}S(\tau)} + \frac{1}{5}\frac{d^2S(\tau')}{d\tau'^2}\bigg|_{\tau} + \ldots$$

$$\frac{F_\lambda(\tau=0)}{4\pi} = \frac{1}{2}\int_{-1}^{+1} I_\lambda \, u du$$

Surface flux, no “in” rays

$$\boxed{I(\tau,u) = S(\tau) + u\left.\frac{dS(\tau')}{d\tau'}\right|_\tau}$$

$$F_\lambda(\tau=0) = 2\pi\int_0^{+1}\left[S_\lambda(\tau=0) + u\left.\frac{dS_\lambda}{d\tau}\right|_{\tau=0}\right] udu$$

Source function does not depend on direction

$$= 2\pi S_\lambda(\tau=0)\int_0^{+1} udu + 2\pi\left.\frac{dS_\lambda}{d\tau}\right|_{\tau=0}\int_0^{+1} u^2 du$$

Integrate

$$= 2\pi S_\lambda(\tau=0)\left[\frac{1}{2}\right] + 2\pi\boxed{\left.\frac{dS_\lambda}{d\tau}\right|_{\tau=0}\left[\frac{1}{3}\right]}$$

$$\boxed{\frac{F}{4\pi} = \frac{1}{3}\left.\frac{dS(\tau')}{d\tau'}\right|_\tau}$$

$$= \pi S_\lambda(\tau=0) + \frac{1}{2}F_\lambda(\tau=0)$$

$$F_\lambda(\tau=0) = 2\pi S_\lambda(\tau=0)$$

$$F_\lambda(\tau = 0) = 2\pi S_\lambda(\tau = 0)$$

Source function is the plank function

$$= 2\pi B_\lambda(\tau = 0)$$

Integrate over wavelengths

$$\int_\lambda F_\lambda(\tau = 0)d\lambda = 2\pi \int_\lambda B_\lambda(\tau = 0)d\lambda$$

$$\sigma T_{\text{eff}}^4 = 2\pi \frac{\sigma T^4(\tau = 0)}{\pi}$$

$$T^4(\tau_z) = \frac{3T_{\text{eff}}^4}{4}(\tau_z + q(\tau_z))$$

$$T^4(\tau = 0) = \frac{3T_{\text{eff}}^4}{4}(0 + q(0))$$

$$T_{\text{eff}}^4 = \frac{3}{2}T_{\text{eff}}^4\, q(0)$$

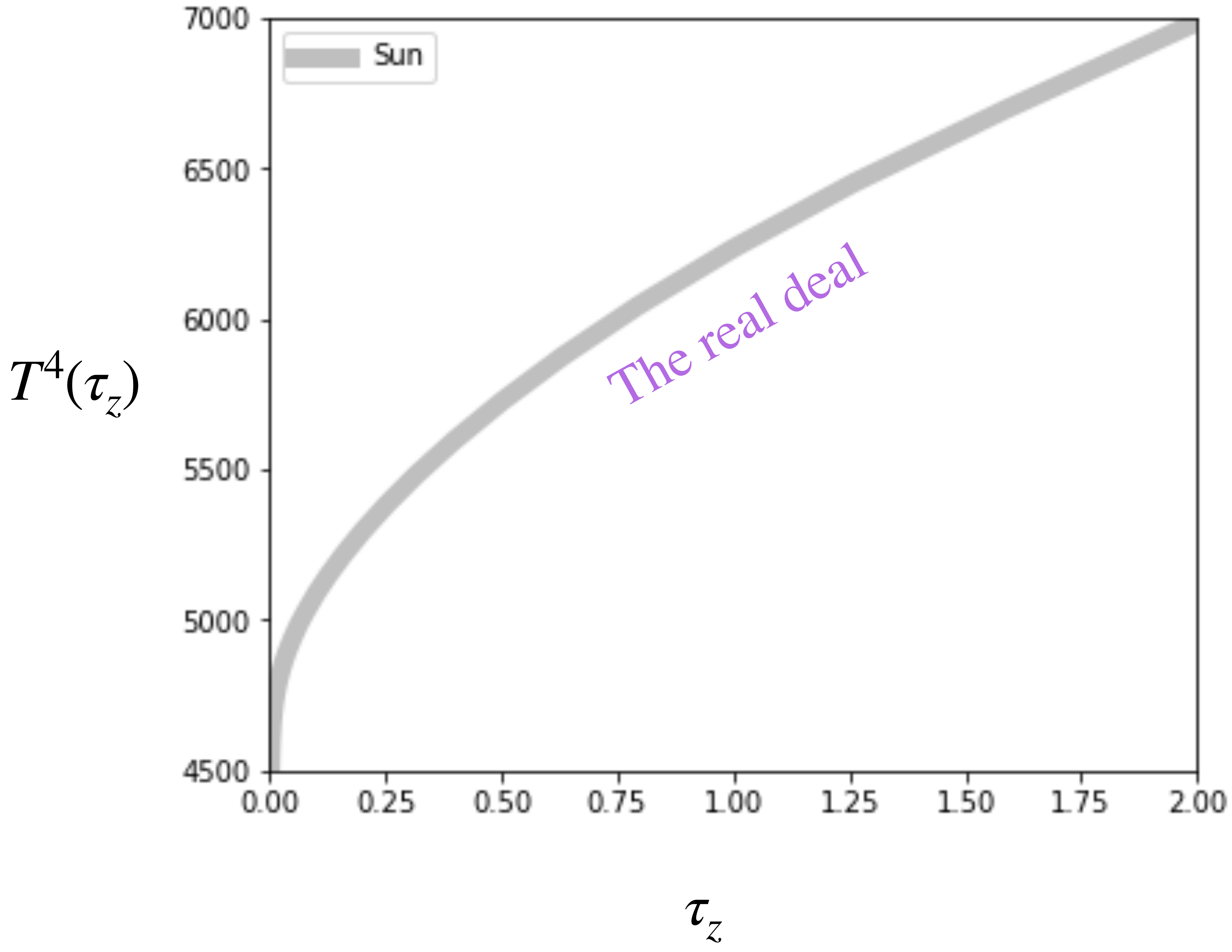
$$q(0) = \frac{2}{3}$$

$$q(\tau_z) \sim \frac{2}{3}$$

The grey case (opacity is not a function of wavelength) + large τ

$$T^4(\tau_z) = \frac{3T_{\text{eff}}^4}{4}(\tau_z + q(\tau_z))$$

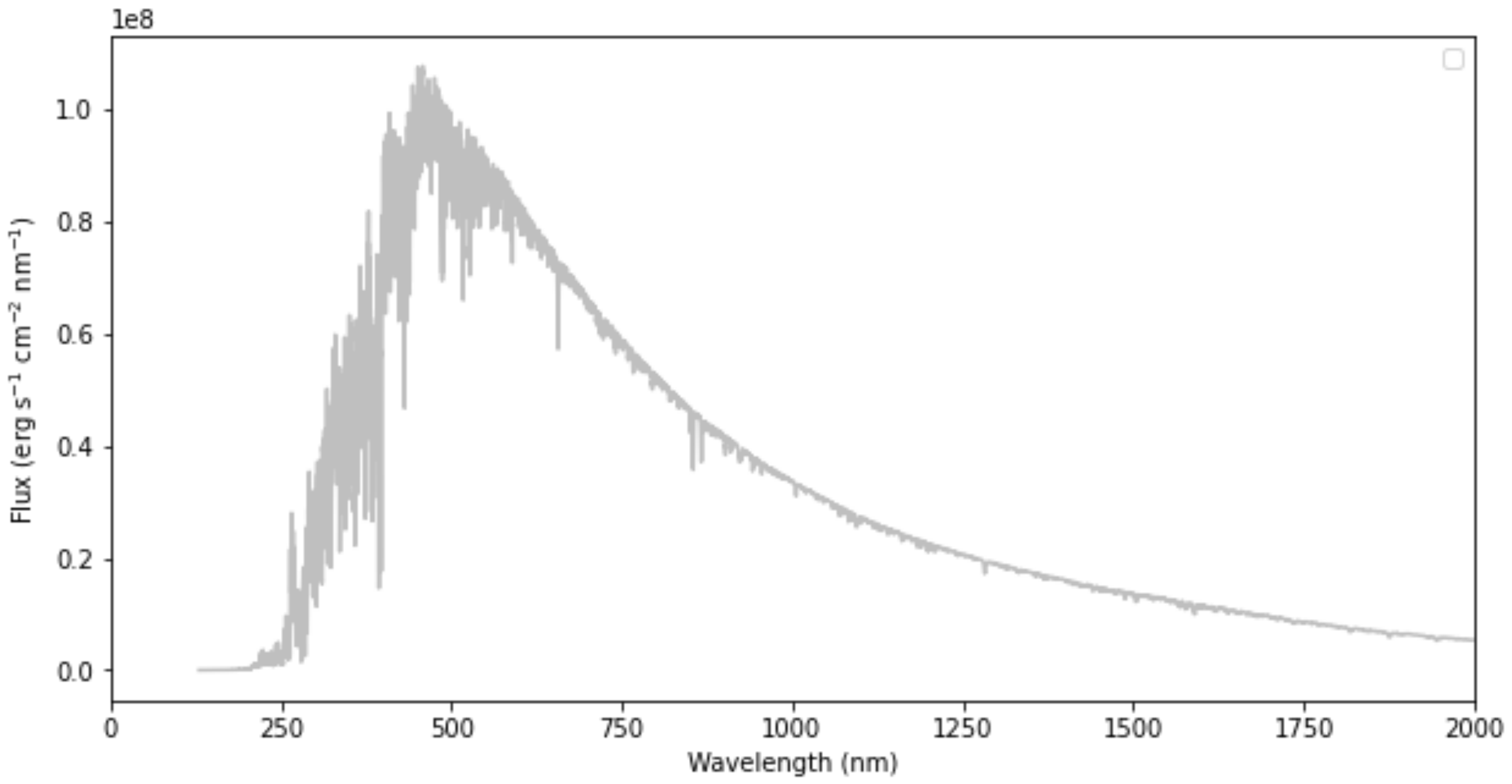
So how good is it?



Notebook part 1

The grey case (opacity is not a function of wavelength) + large τ

We can also use the temperature structure to find the wavelength-depend flux (i.e. the spectrum)



Worksheet for the math part

Worksheet L16: Grey atmospheres

Goal of this worksheet: find the wavelength dependent flux (the spectrum!) predicted by our analytical estimate for the temperature structure of a star (with a given effective temperature.

$$F_\lambda(\tau) = -2\pi \int_{\tau'=0}^{\tau} S_\lambda(\tau') E_2(\tau - \tau') d\tau' + 2\pi \int_{\tau'=\tau}^{\infty} S_\lambda(\tau') E_2(\tau' - \tau) d\tau'$$

We know that that $S_\lambda(\tau) = B_\lambda(T(\tau))$, that $B_\nu(T) = \frac{2h}{c^2}\nu^3 \frac{1}{e^{h\nu/kT} - 1}$, and that $T^4(\tau_z) = \frac{3T_{\rm eff}^4}{4}(\tau_z + q(\tau_z))$.

Step1: We will do a change of variable where $\alpha = \frac{h\nu}{kT_{\rm eff}} = \frac{hc}{\lambda k T_{\rm eff}}$. Find an expression for $B_\alpha(T)$ (remember that $B_\alpha d\alpha = B_\nu d\nu$).

Step2: Replace the source function in the equation for the flux by your expression for B_α in Step 1. This means that the resulting quantity will be $F_\alpha(\tau)$. Make an additional substitution so that $p(\tau) = T_{\rm eff}/T(\tau)$. Pull all of the constant quantities outside of the integrals.

Step3: Check your answer with the solution, and replace the term in [ ] with $C(\alpha, \tau)$.

Worksheet L16: Grey atmospheres

Step4: In the notebook, we will be interested to make a graph of $F_\lambda/\tilde{F}$ (at a given optical depth). First, let's find $\tilde{F}$. We know what the wavelength-integrated flux will be $\sigma T_{\rm eff}^4$ at all layers. Find the definition of σ in terms of fundamental constants (see the BB lecture — there will be some cs, hs, πs, etc in there).

Step 5: Now find $F_\alpha/\tilde{F}$ by dividing your expression for F_α from Step 3 by the expression for $\sigma T_{\rm eff}^4$ from Step 4 (a whole bunch of stuff should cancel out!)

Step 6: Now, we need to convert $F_\alpha/\tilde{F}$ into $F_\lambda/\tilde{F}$. Find the factor you need to multiply $F_\alpha/\tilde{F}$ by to do this.

Step7: Check your answer on the solution.

The grey case (opacity is not a function of wavelength) + large τ

How good is it?

Notebook part 2