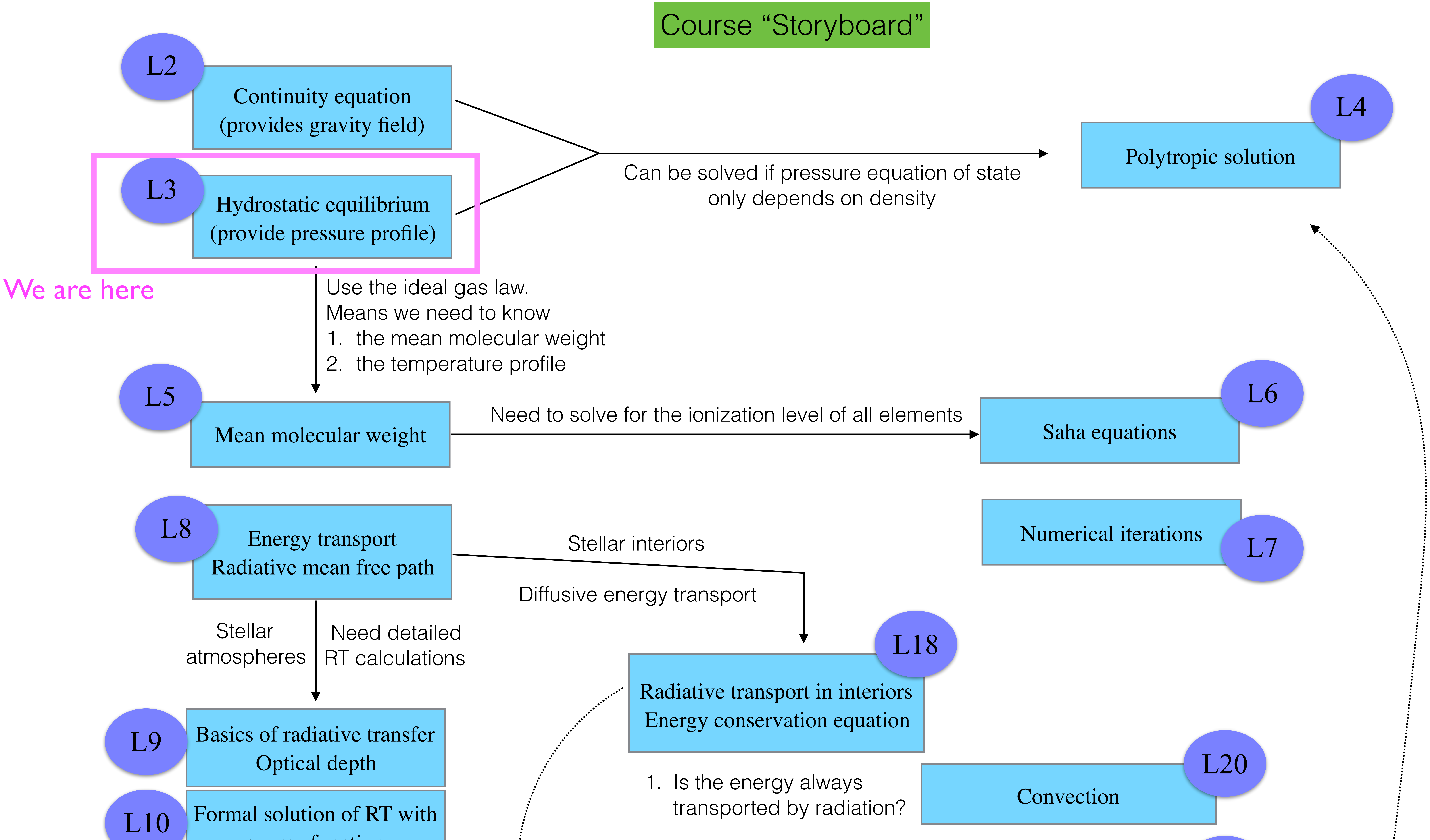
Week 2 Tuesday

L-3

Hydro EQ


Course “Storyboard”
L2
Continuity equation
(provides gravity field)
L3
Hydrostatic equilibrium
(provide pressure profile)
We are here
Can be solved if pressure equation of state
only depends on density
L4
Polytropic solution
Use the ideal gas law.
Means we need to know
1. the mean molecular weight
2. the temperature profile
L5
Mean molecular weight
Need to solve for the ionization level of all elements
L6
Saha equations
L7
Numerical iterations
L8
Energy transport
Radiative mean free path
Stellar interiors
Diffusive energy transport
Stellar
atmospheres
Need detailed
RT calculations
L18
Radiative transport in interiors
Energy conservation equation
L9
Basics of radiative transfer
Optical depth
L10
Formal solution of RT with
1. Is the energy always
transported by radiation?
L20
Convection

Differential equations:

Continuity equation

$$\frac{dM_r(r)}{dr} = 4\pi r^2 \rho(r)$$

One equations, two unknowns

Variables

$M_r(r)$
$\rho(r)$

Boundary conditions

$M_r(r = 0) = 0$

Differential equations:

Continuity equation

$$\frac{dM_r(r)}{dr} = 4\pi r^2 \rho(r)$$

Let's say we guess the density

Variables

$M_r(r)$
$\rho(r)$

Boundary conditions

$M_r(r = 0) = 0$

Constant density case

Variables

Boundary conditions

$M_r(r = 0) = 0$

$M_r(r)$

$\boxed{\begin{array}{l}\rho_o \\ R_\star\end{array}}$ Known

$$M_r(r) - M_r(r = 0) = \int_0^r 4\pi r^2 \rho_o dr \qquad \text{if } r \leq R_\star$$

$$M_r(r) = \frac{4\pi r^3 \rho_0}{3}$$

$$M_\star = M_r(r = R_\star)$$

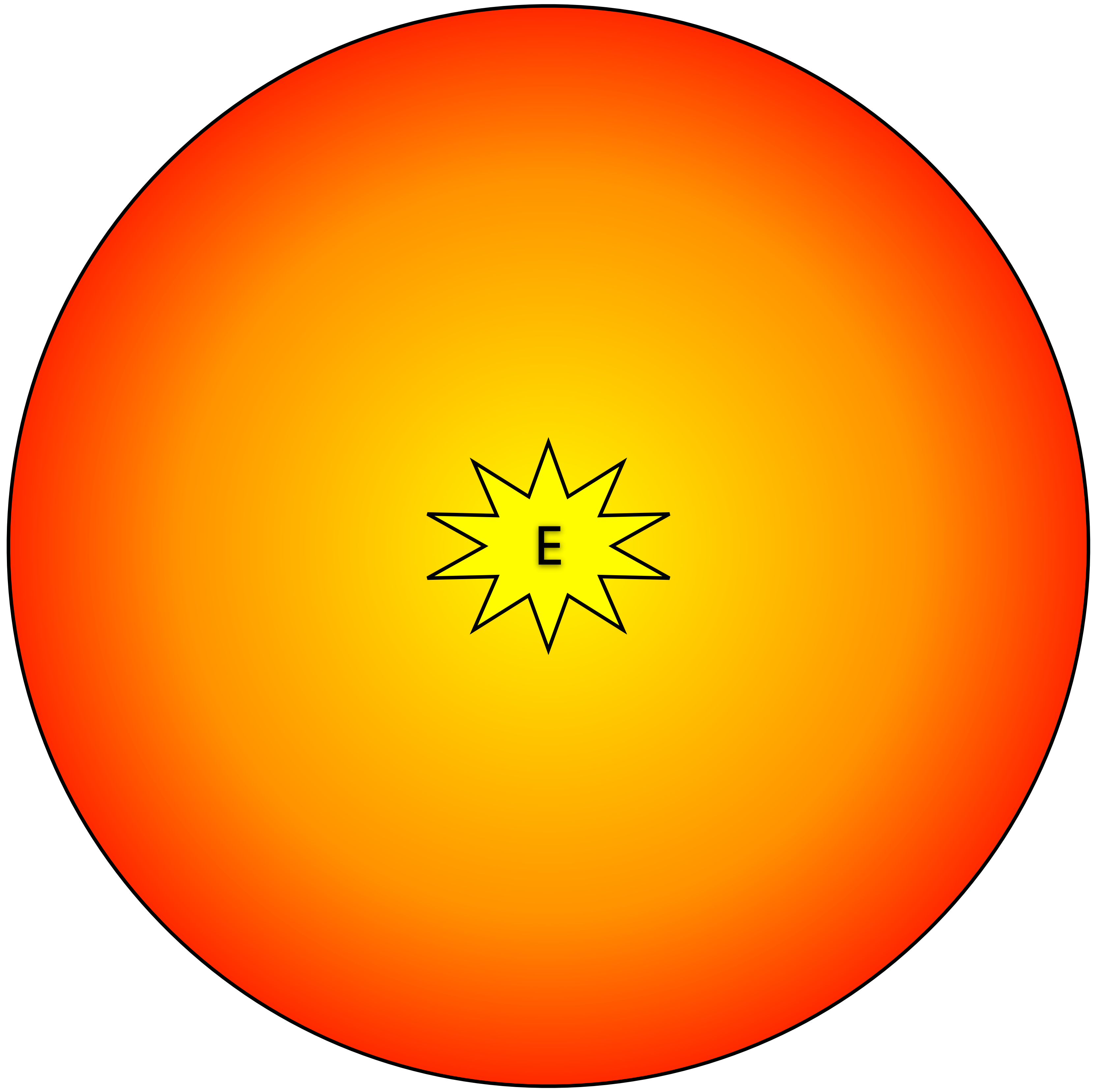
$M_\star$

$$M_\star = \frac{4\pi R_\star^3 \rho_0}{3}$$

A star is:

Self-gravitating celestial object, in which there is, or once was, sustained thermonuclear fusion of hydrogen in their core.

It is not (quickly) contracting nor expanding



A jellyfish has nearly the same density as water. If you know the density, pressure, and temperature everywhere in this ocean, under which condition will the jellyfish remain still?

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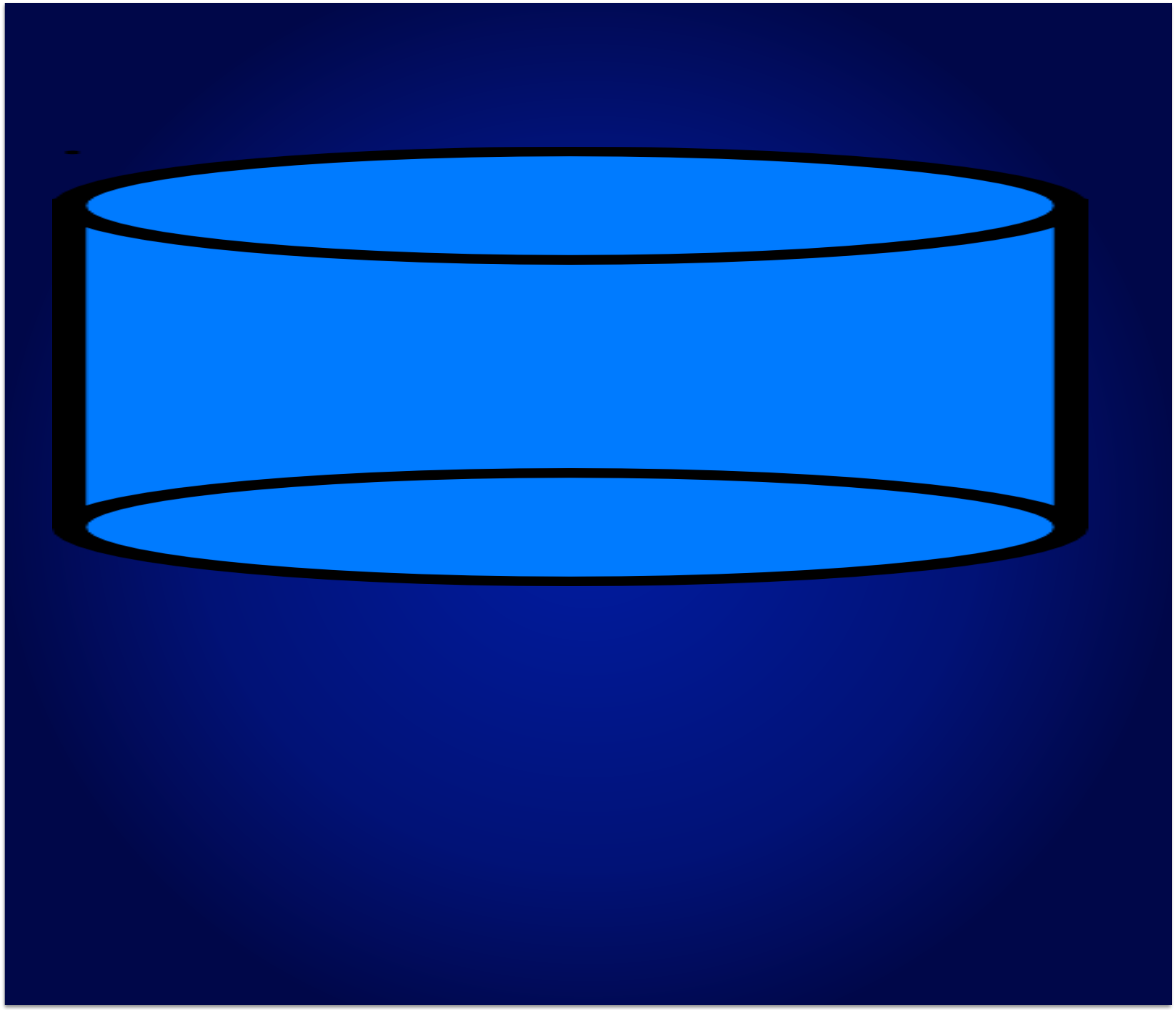
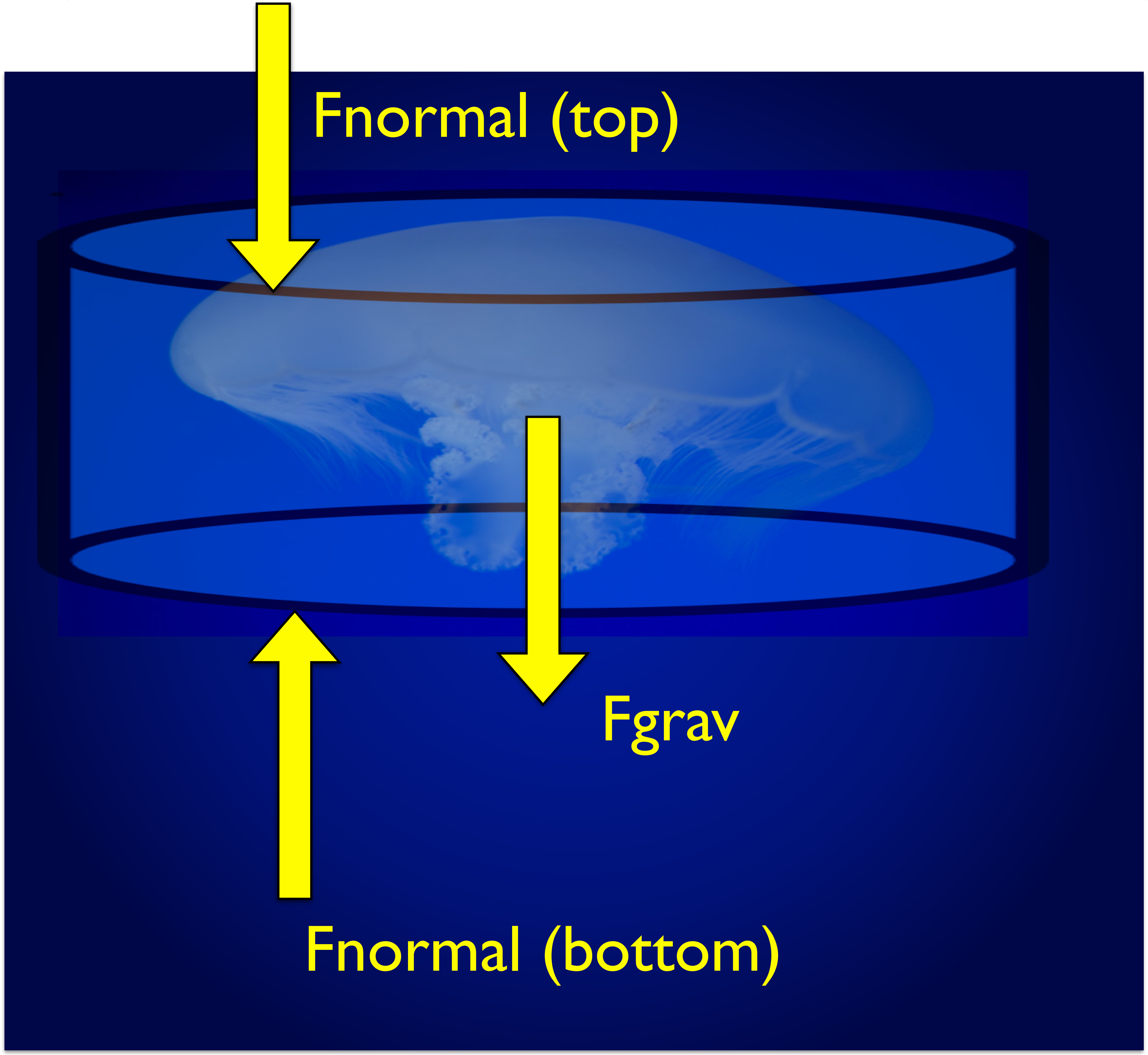


A jellyfish has nearly the same density as water. If you **know the density, pressure, and temperature** everywhere in this ocean, under which condition will the jellyfish remain still?



On the board:

* Using this free-body diagram to derive the hydrostatic equilibrium equation.

$$\frac{dP(r)}{dr} = -\rho(r)\overbrace{\frac{GM_r(r)}{r^2}}^{g(r)}$$

Differential equations:

Continuity equation

$$\frac{dM_r(r)}{dr} = 4\pi r^2 \rho(r)$$

Hydrostatic equilibrium

$$\frac{dP(r)}{dr} = -\rho(r)\overbrace{\frac{GM_r(r)}{r^2}}^{g(r)}$$

Two equations, three unknowns (plus R)

Variables

$M_r(r)$
$\rho(r)$
$P(r)$

Boundary conditions

$M_r(r = 0) = 0$
$P(r = R) = 0$

Integral form equations:

$$M_r(r) - 0 = \int_0^r 4\pi r^2 \rho(r) dr$$

$$P(r) - 0 = \int_{R_\star}^r - \rho(r) \overbrace{\frac{GM_r(r)}{r^2}}^{g(r)} dr$$

Variables

$M_r(r)$
$\rho(r)$
$P(r)$

Boundary conditions

$M_r(r = 0) = 0$
$P(r = \mathbf{R}) = 0$

Two equations, three unknowns (plus R)

Integral form equations:

Let's say we know the density (and thus radius)
From last week, with constant density:

$$\rho(r) \equiv \rho_o$$

or $\rho(r) \equiv \rho_o \left(1 - \frac{R_\star}{r}\right)$

$$M_r(r) - 0 = \int_0^r 4\pi r^2 \rho(r) dr$$

$$\frac{M_r(r)}{M_\star} = \left(\frac{r}{R_\star}\right)^3$$

$$\frac{g(r)}{g_\star} = \frac{r}{R_\star}$$

$$P(r) - 0 = \int_{R_\star}^r -\rho(r) \underbrace{\frac{GM_r(r)}{r^2}}_{g(r)} dr$$

$$P(r) - 0 = \int_{R_\star}^r -\rho_o g(r) dr$$

Let's practice transforming our integrals into a unit-less form

$$P(r) - 0 = \int_{R_\star}^{r} -\rho_o g(r) dr$$

$$\frac{g(r)}{g_\star} = \frac{r}{R_\star}$$

$$g_\star = \frac{GM_\star}{R_\star^2}$$

On the board:

- Step 1: change of variable for "*dr*"
- Step 2: change the bounds
- Step 3: pull all constants to the front.

Should have units of pressure!

No unit

$$P(r) = \boxed{-\rho_o g_\star R_\star} \boxed{\int_1^x x dx}$$

On the board:

- Check the units
- Do the integral
- Calculate the central pressure (at r=0) and scale to $P(r)/P_o$.

In the notebook: calculating the central density using the awesome Astropy “units” and “constant” packages