Week 13 Thursday
L-24

Homology for composition

Let's consider stars with the same mass, but different composition.

$$\mu \sim \frac{2}{3X + Y/2 + 1} \qquad \text{using } X + Y \approx 1, \text{ we get } \frac{1}{\mu} \sim \frac{5X+3}{4}$$

1. $P = \dfrac{\rho}{\mu m_H} kT = \dfrac{5X+3}{4} \dfrac{k}{m_H} T$

2. pp-chain $\epsilon \sim \epsilon_0 X^2 \rho T^4$

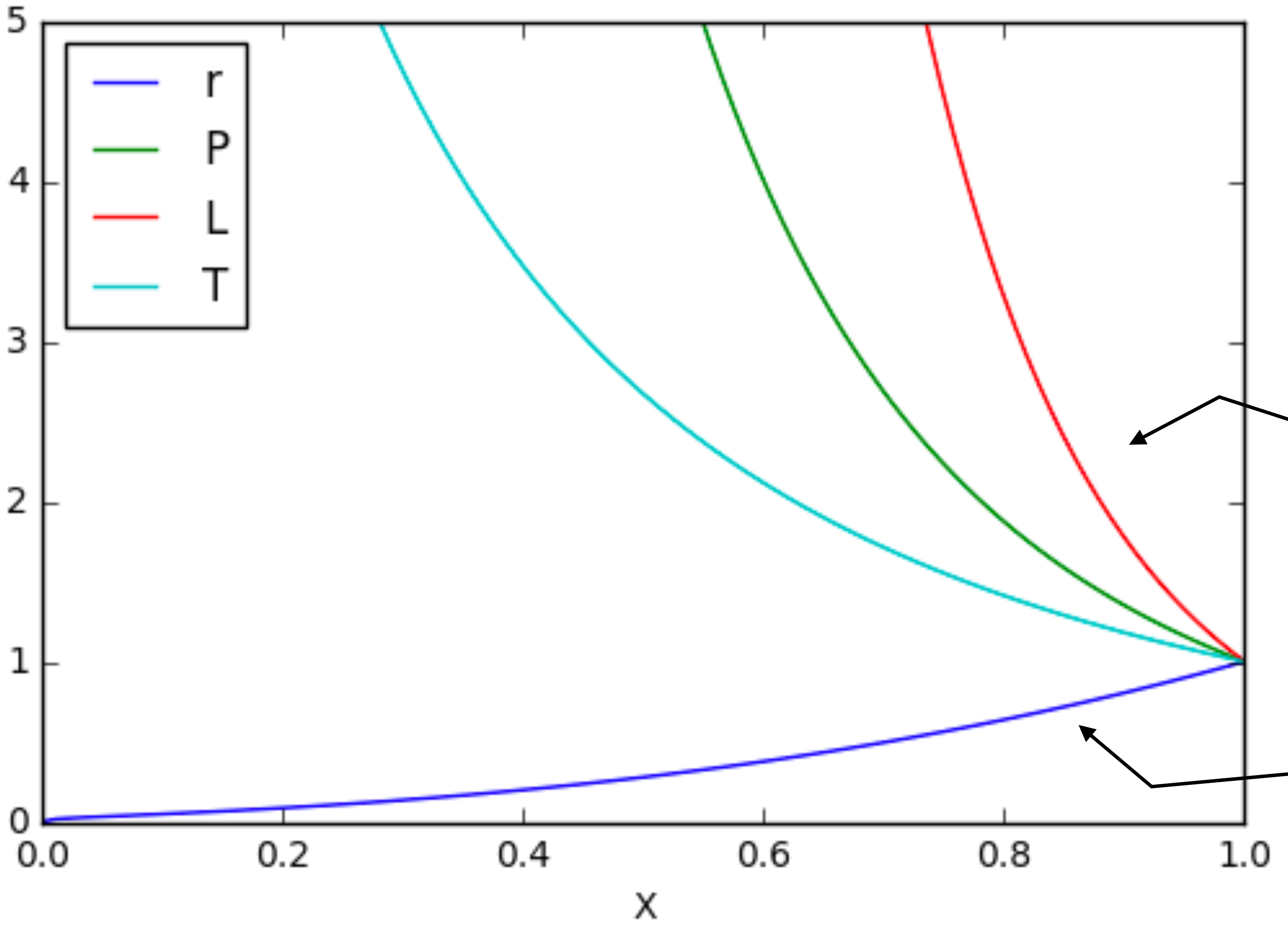
3. opacity $\kappa \sim \kappa_0 Z (1+X) \rho T^{-7/2}$

$$r \propto X^{4/13} (1+X)^{2/13} (5X+3)^{7/3} Z^{2/13}$$

$$P \propto X^{-16/13} (1+X)^{-8/13} (5X+3)^{-28/13} Z^{-8/13}$$

$$L \propto X^{-2/13} (1+X)^{-14/13} (5X+3)^{-101/13} Z^{-14/13}$$

$$T \propto X^{-4/13} (1+X)^{-2/13} (5X+3)^{-20/13} Z^{-2/13}$$



More luminous

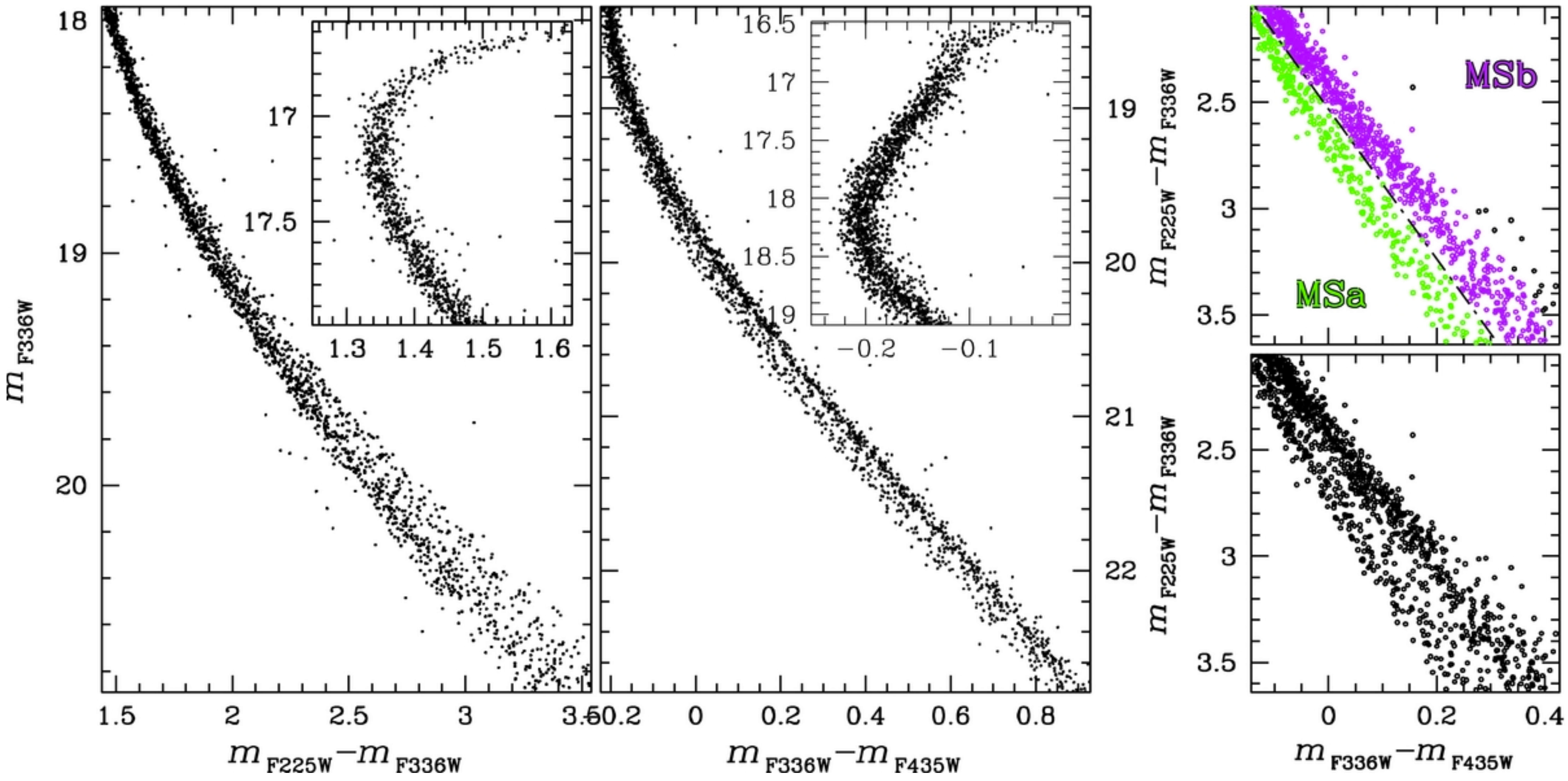
Smaller star

Less H (e.g., more He)

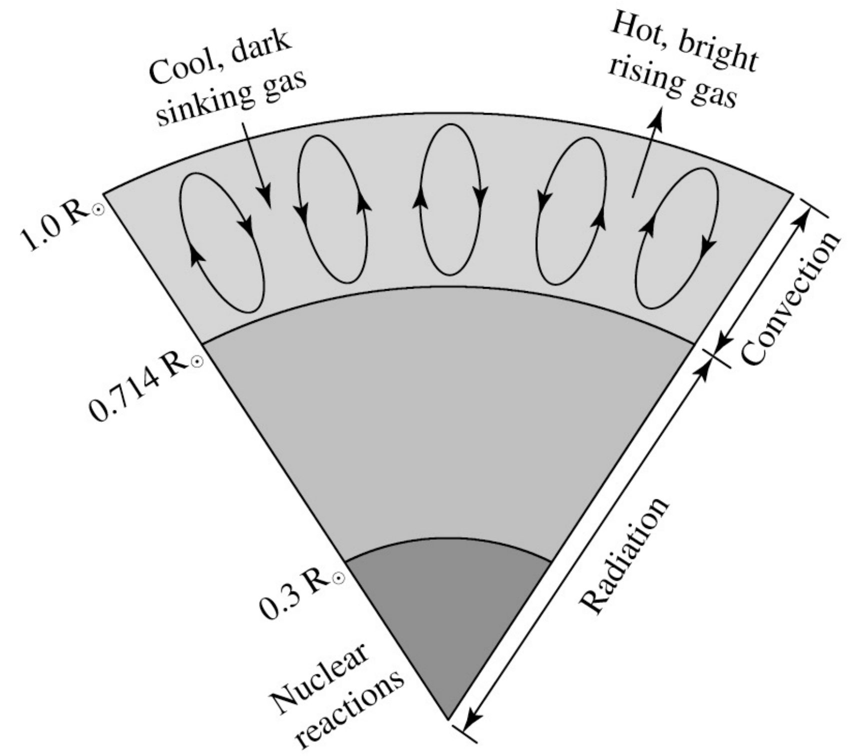
$$\left(\frac{T_{\rm eff}}{T_\odot}\right)^4 = \frac{L_\star}{L_\odot}\left(\frac{R_\odot}{R_\star}\right)^2$$

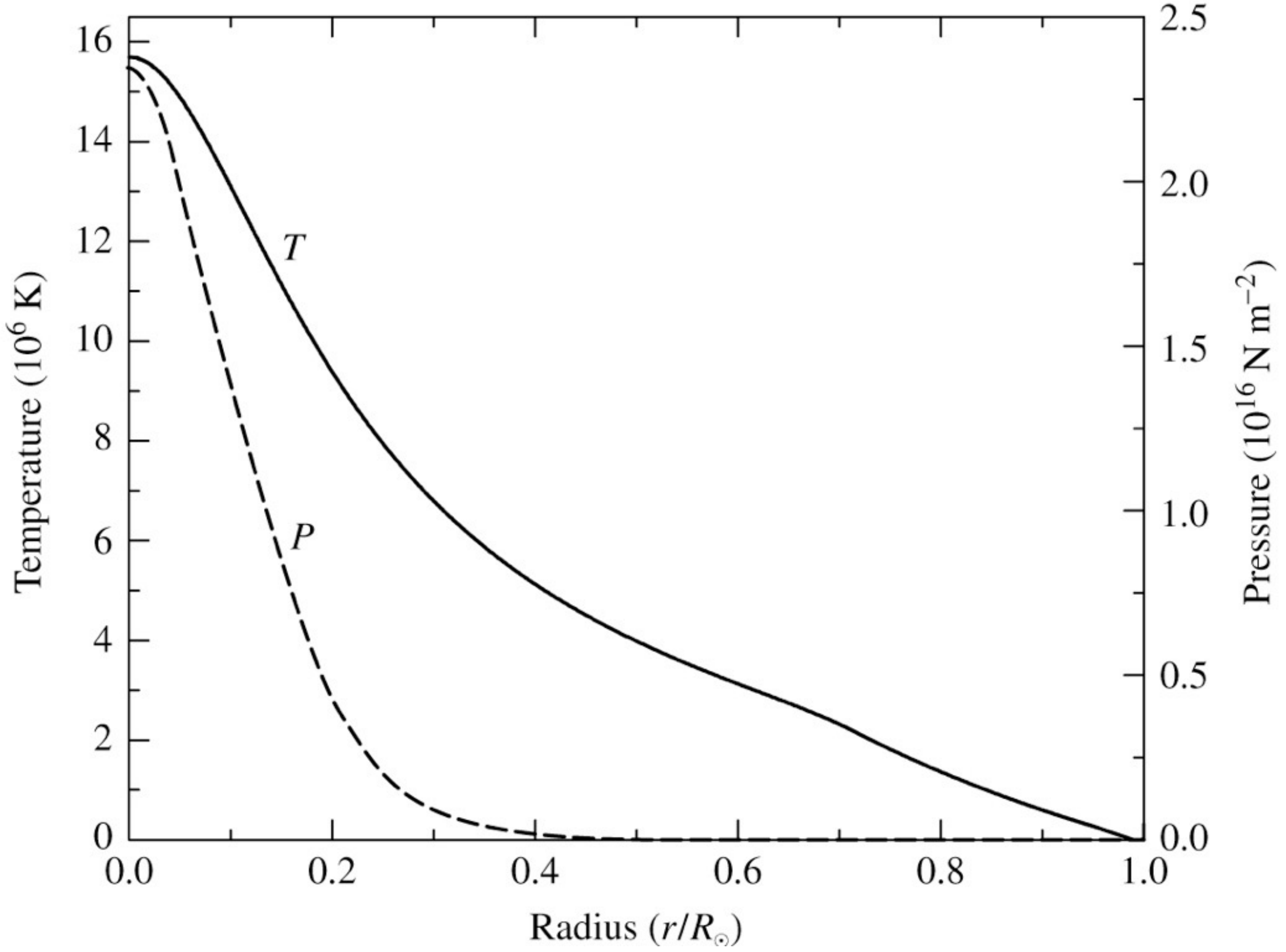
Bluer star

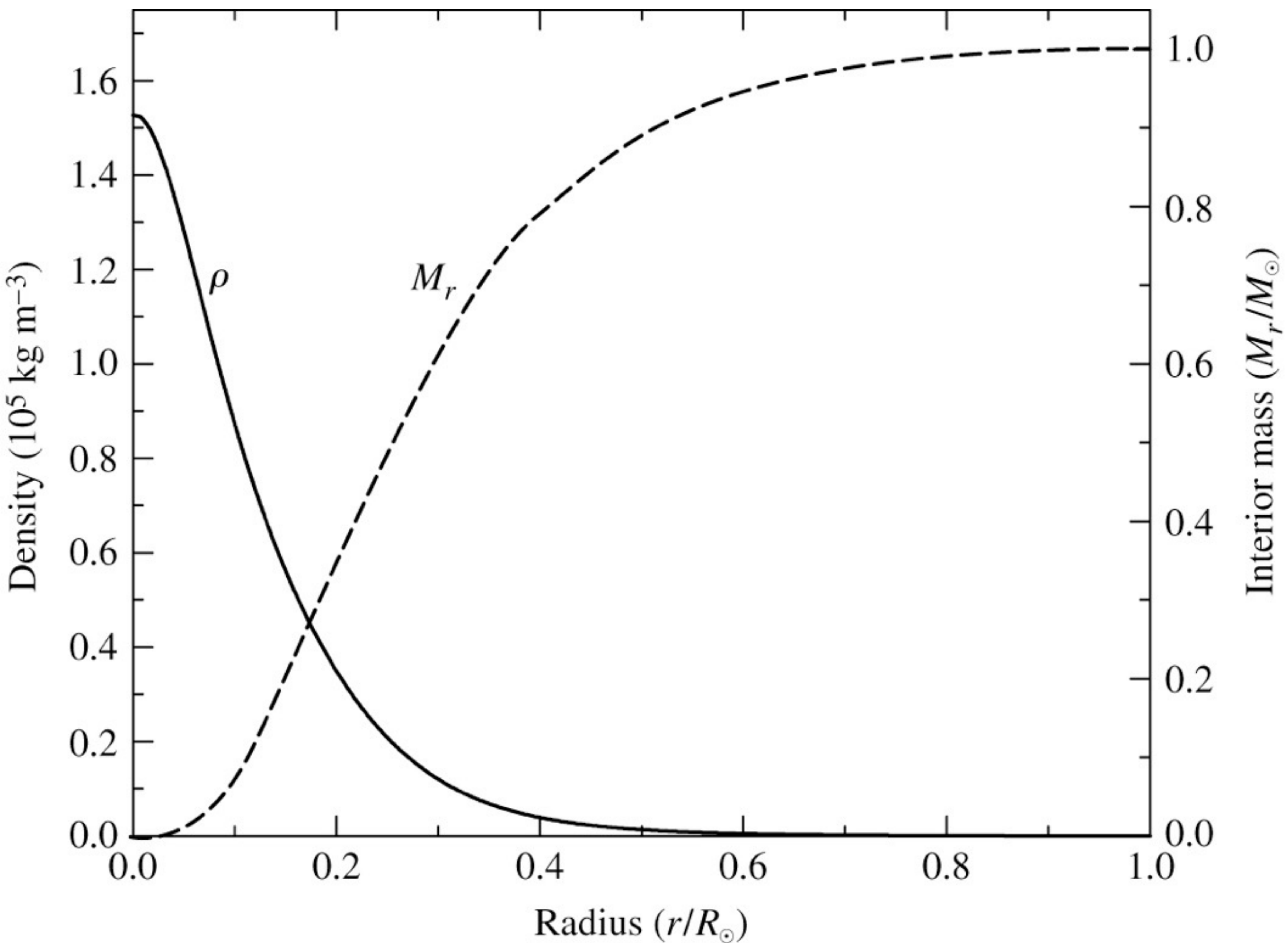
Globular cluster NGC 6397, with two generations of stars with different composition



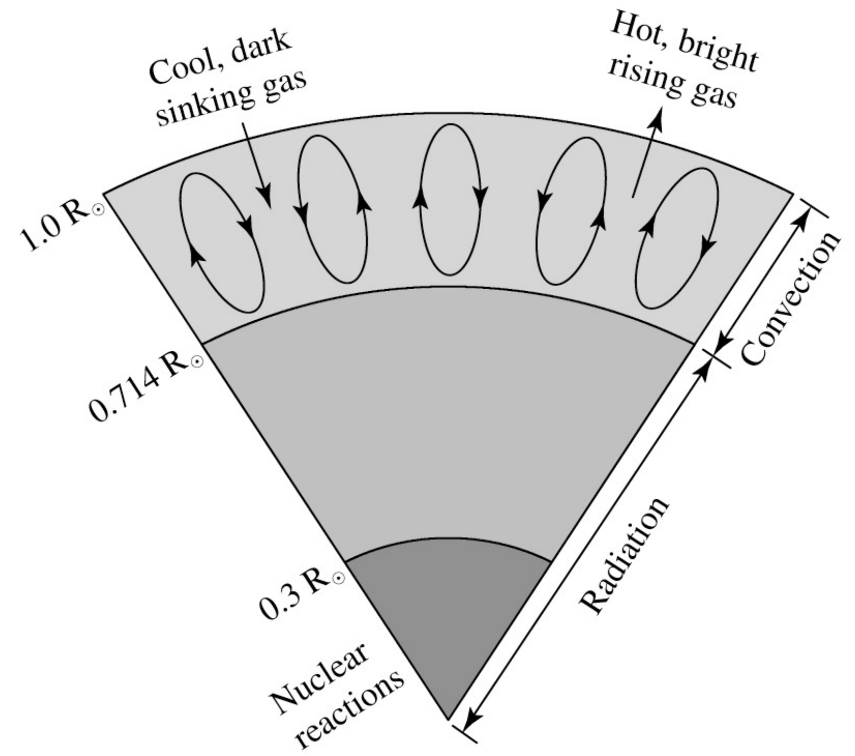
The Sun's structure today

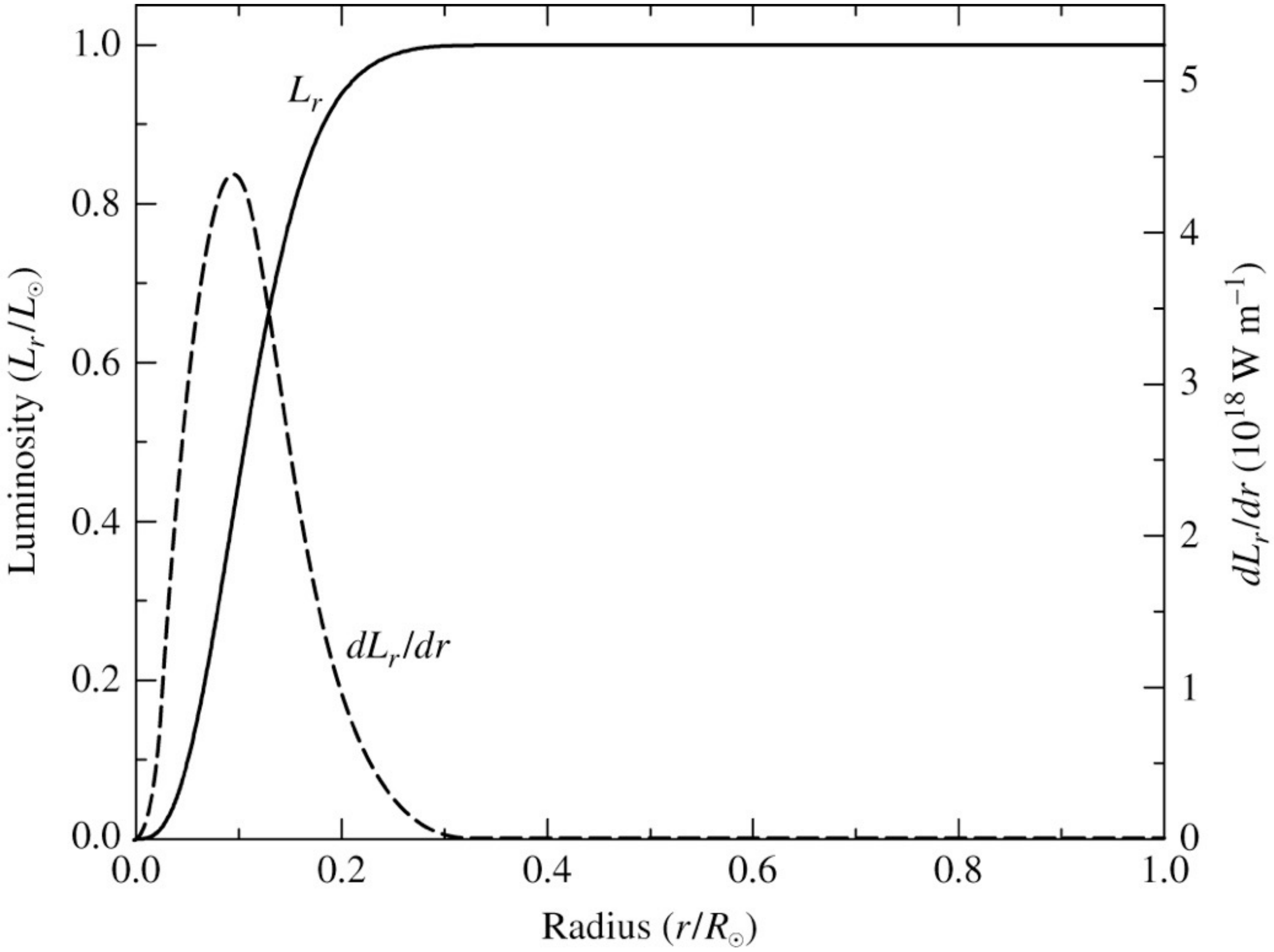




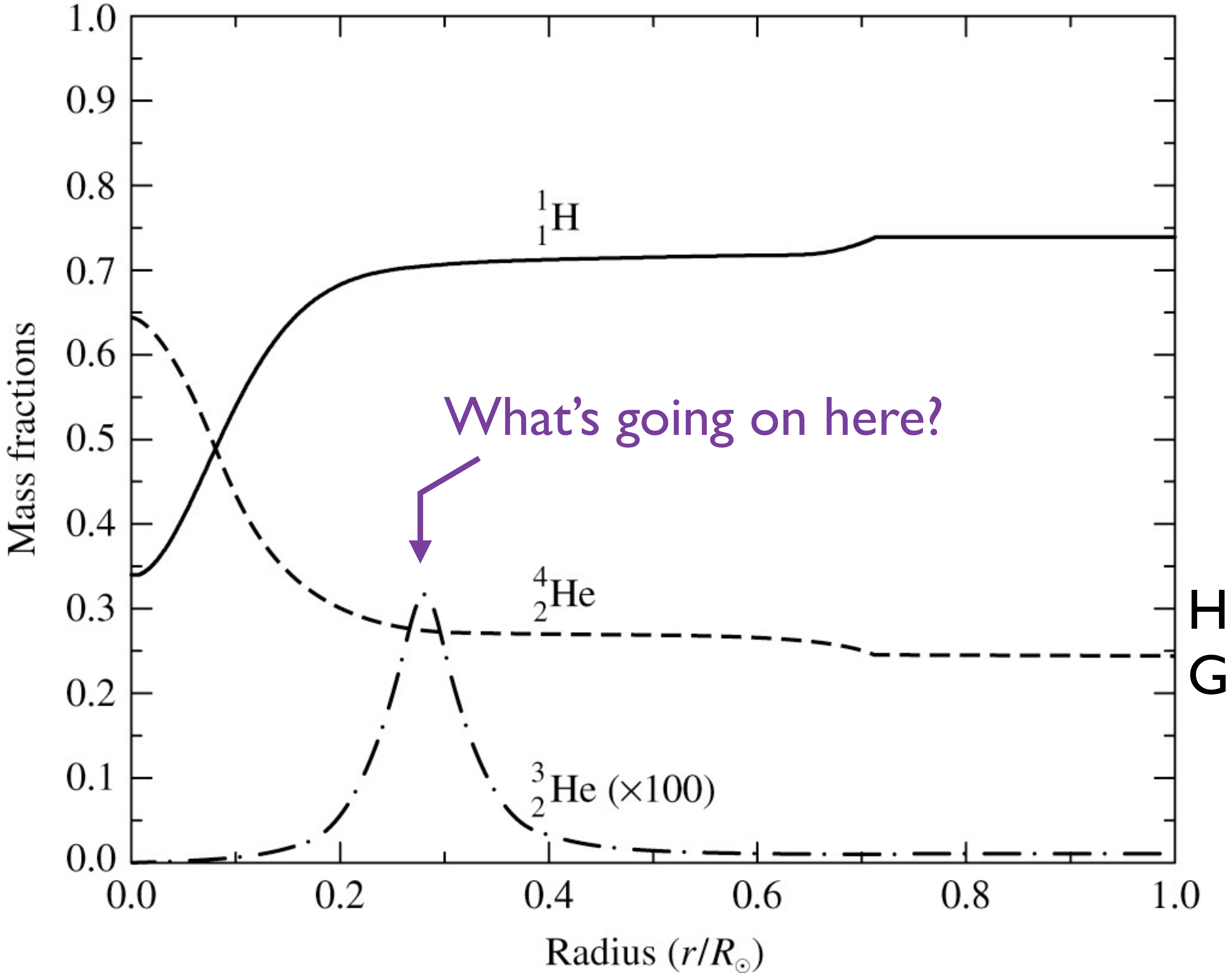


The Sun’s structure today



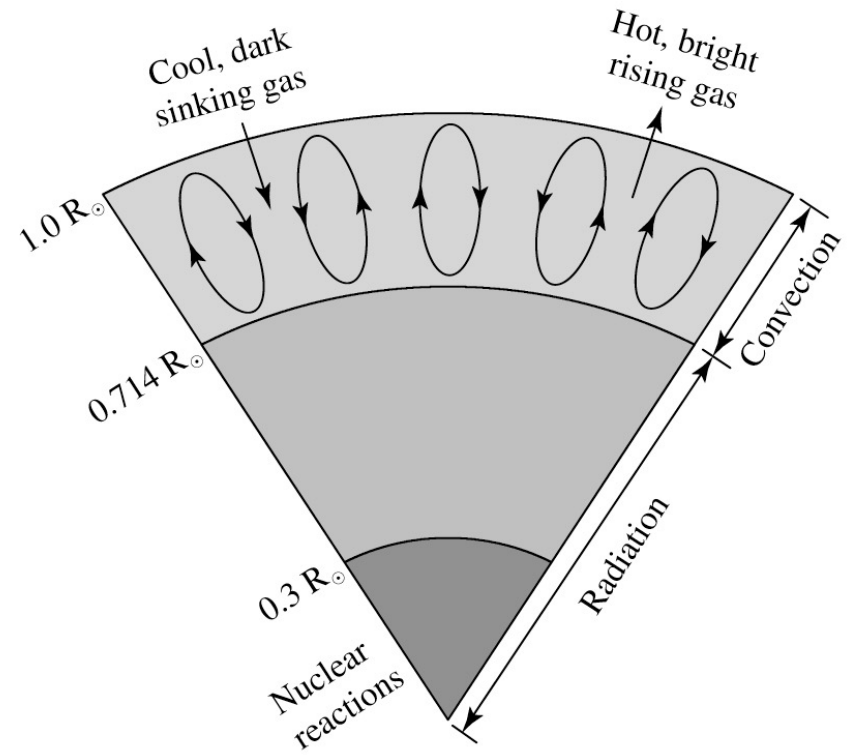


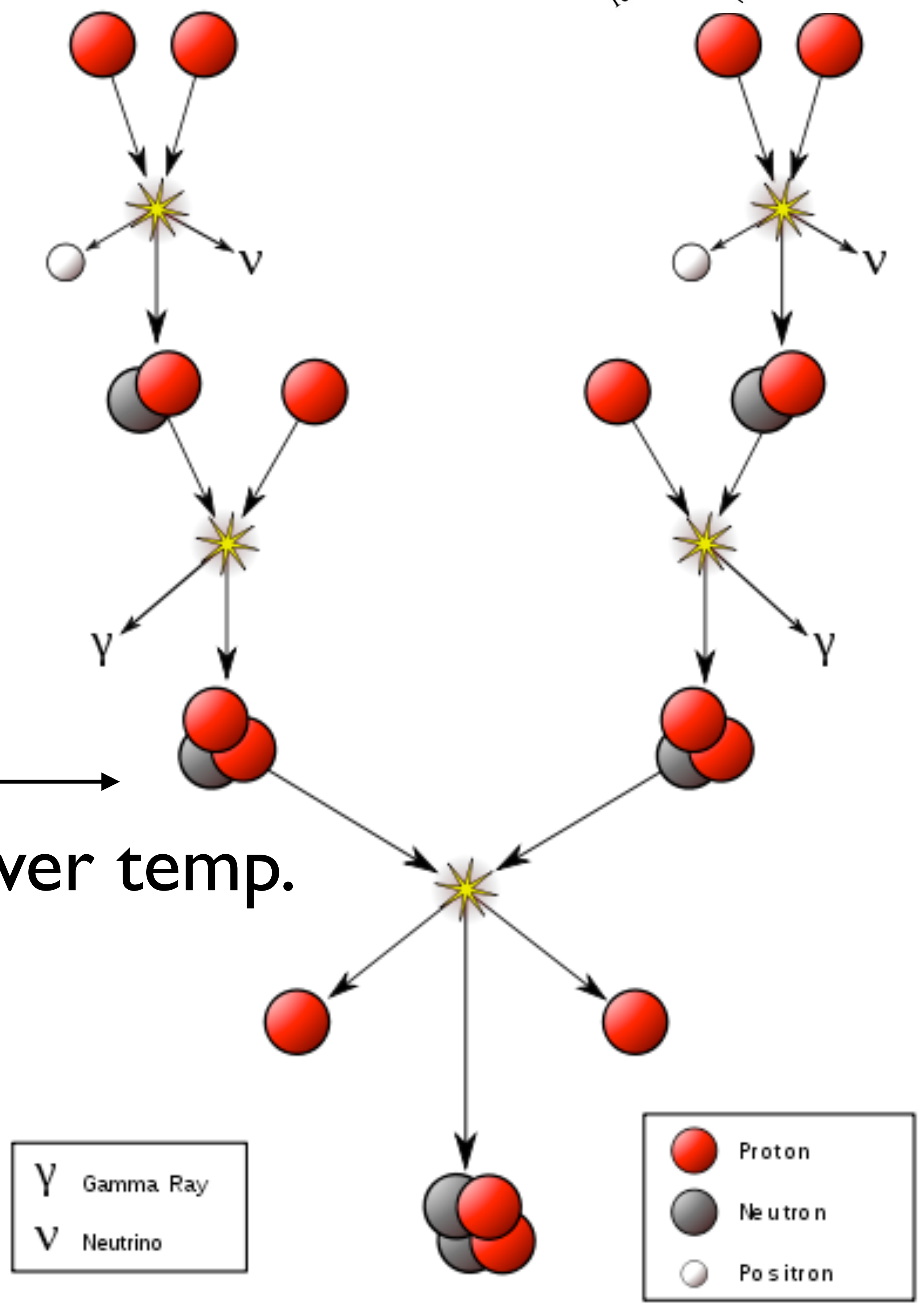
The Sun's structure today



Higher barrier

Goes slower at lower temp.





$$\frac{dM_r(r)}{dr} = 4\pi r^2 \rho(r)$$

$$\frac{dP(r)}{dr} = -\rho(r)\frac{GM_r(r)}{r^2}$$

$$\frac{dT(r)}{dr} = -\frac{3\kappa_R(r)\rho(r)}{64\pi\sigma}\frac{L_r(r)}{r^2T(r)^3}$$

$$\frac{dL_r(r)}{dr} = 4\pi r^2 \rho(r)\epsilon(r)$$

$$M_r(r) \quad P(r) \quad L_r(r) \quad T(r)$$

$$\rho(r) \quad \mu(r) \quad \epsilon_{\text{nuc}}(r) \quad \kappa_R(r)$$

$$P(r) = \frac{\rho(r)}{\mu(r)}kT(r)$$

$$\mu(r) = f(\text{comp}, T(r), P(r))$$

$$\kappa_R(r) = f(\text{comp}, T(r), P(r))$$

$$\epsilon_{\text{nuc}}(r) = f(\text{comp}, T(r), P(r))$$

Other energy transport?

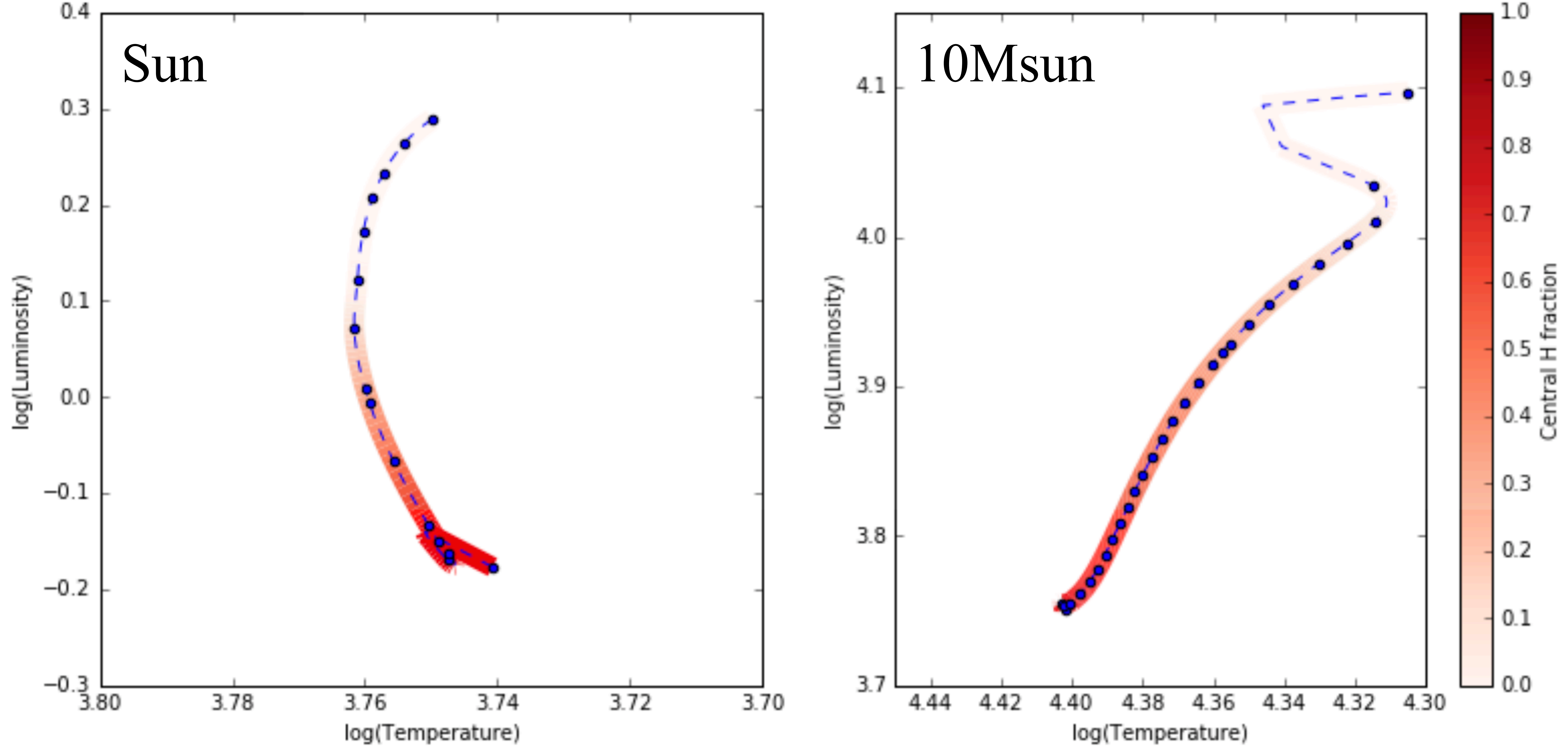
Ideal gas always valid?

Nuclear mechanism?

How to solve?

What changes with time?

When X goes down, L goes up: OK
But what is happening to the temperature?



1. $$\frac{dM_r(r)}{dr} = 4\pi r^2 \rho(r)$$

2. $$\frac{dP(r)}{dr} = -\rho(r)\frac{GM_r(r)}{r^2}$$

3. $$\frac{dT(r)}{dr} = -\frac{3\kappa_R(r)\rho(r)}{64\pi\sigma}\frac{L_r(r)}{r^2 T(r)^3}$$

4. $$\frac{dL_r(r)}{dr} = 4\pi r^2 \rho(r)\epsilon(r)$$

$$P = \frac{\rho k T}{\mu}$$

μ: higher when more He,
higher when neutral

$\varkappa_R$: higher when neutral

ε_{PP}: $\propto T^4$

ε_{CNO}: $\propto T^{20}$

When X_{core} goes down, L goes up

Easy to do, nuclear rates go up

Tcore goes up a bit for pp,
and nearly not at all for CNO

1. $$\frac{dM_r(r)}{dr} = 4\pi r^2 \rho(r)$$

2. $$\frac{dP(r)}{dr} = -\rho(r)\frac{GM_r(r)}{r^2}$$

3. $$\frac{dT(r)}{dr} = -\frac{3\kappa_R(r)\rho(r)}{64\pi\sigma}\frac{L_r(r)}{r^2T(r)^3}$$

4. $$\frac{dL_r(r)}{dr} = 4\pi r^2 \rho(r)\epsilon(r)$$

$$P = \frac{\rho k T}{\mu}$$

μ: higher when more He,
higher when neutral

κ_R: higher when neutral

ε_{PP}: $\propto T^4$

ε_{CNO}: $\propto T^{20}$

For pp:

$$\frac{P_c \downarrow}{\rho_c} = \frac{kT_c \uparrow}{\mu m_H} \uparrow$$

ρ_c goes up

For cno:

$$\frac{P_c}{\rho_c} = \frac{kT_c}{\mu m_H} \uparrow$$

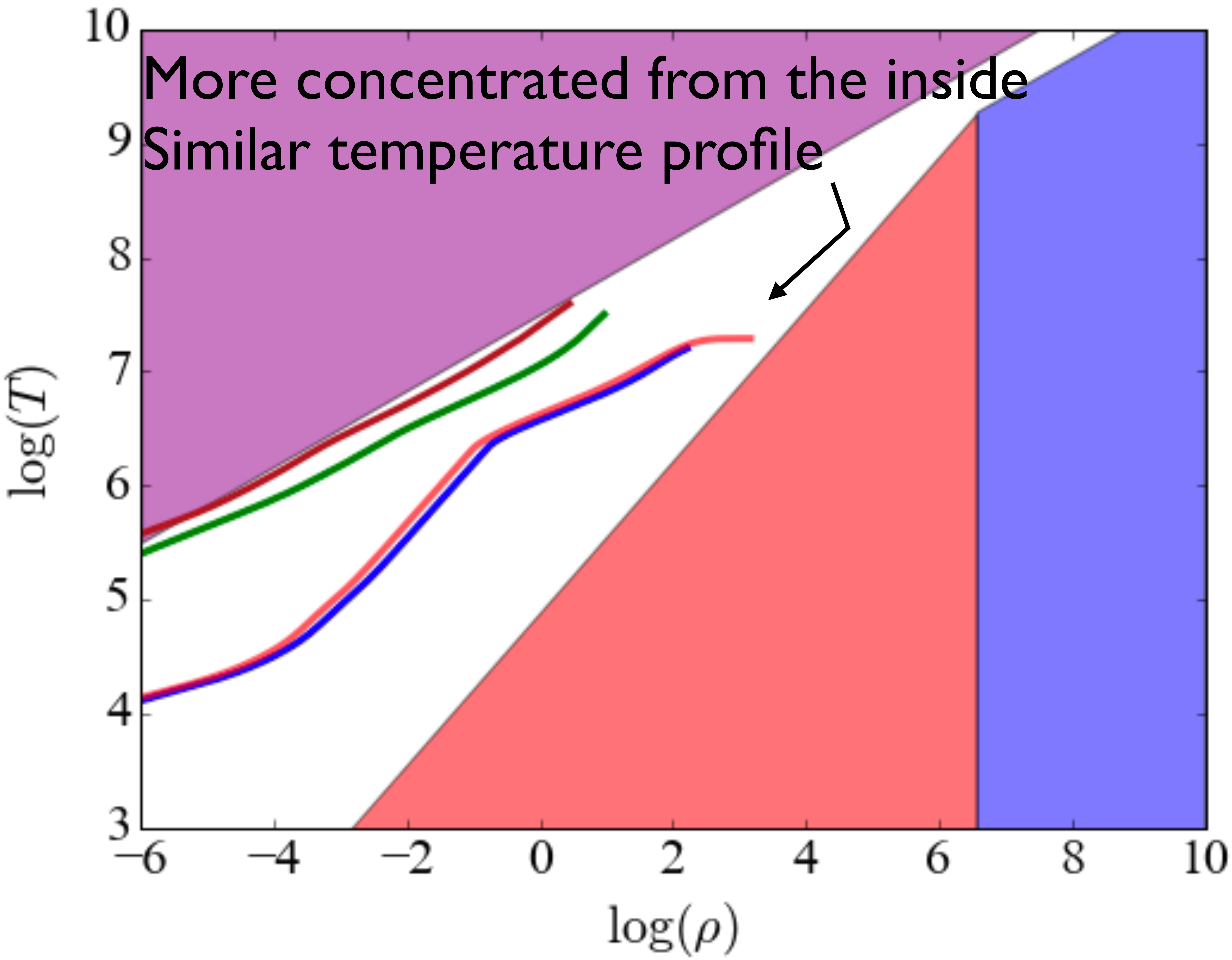
ρ_c goes up and P_c goes down

For pp:

$$\frac{P_c}{\rho_c} = \frac{kT_c}{\mu m_H}$$

ρ_c goes up

$$L_\star = 4\pi R_\star^2 \sigma T_{\text{eff}}^4$$



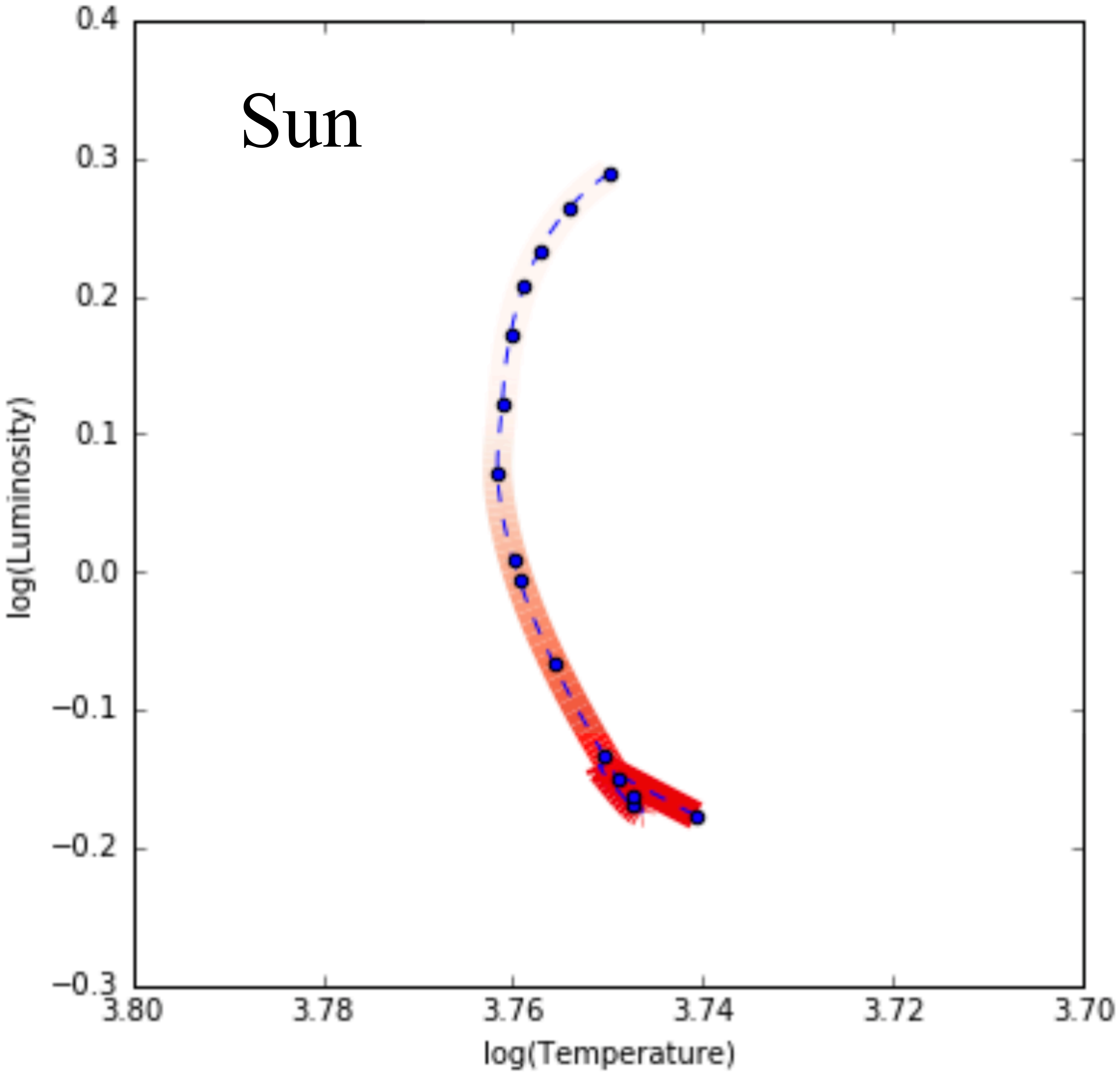
For pp:

$$\frac{P_c}{\rho_c} = \frac{kT_c}{\mu m_H}$$

ρ_c goes up

$$L_\star = 4\pi R_\star^2 \sigma T_{\rm eff}^4$$

Evolve at nearly constant effective temperature



For cno:

$$\frac{P_c}{\rho_c} = \frac{kT_c}{\mu m_H}$$

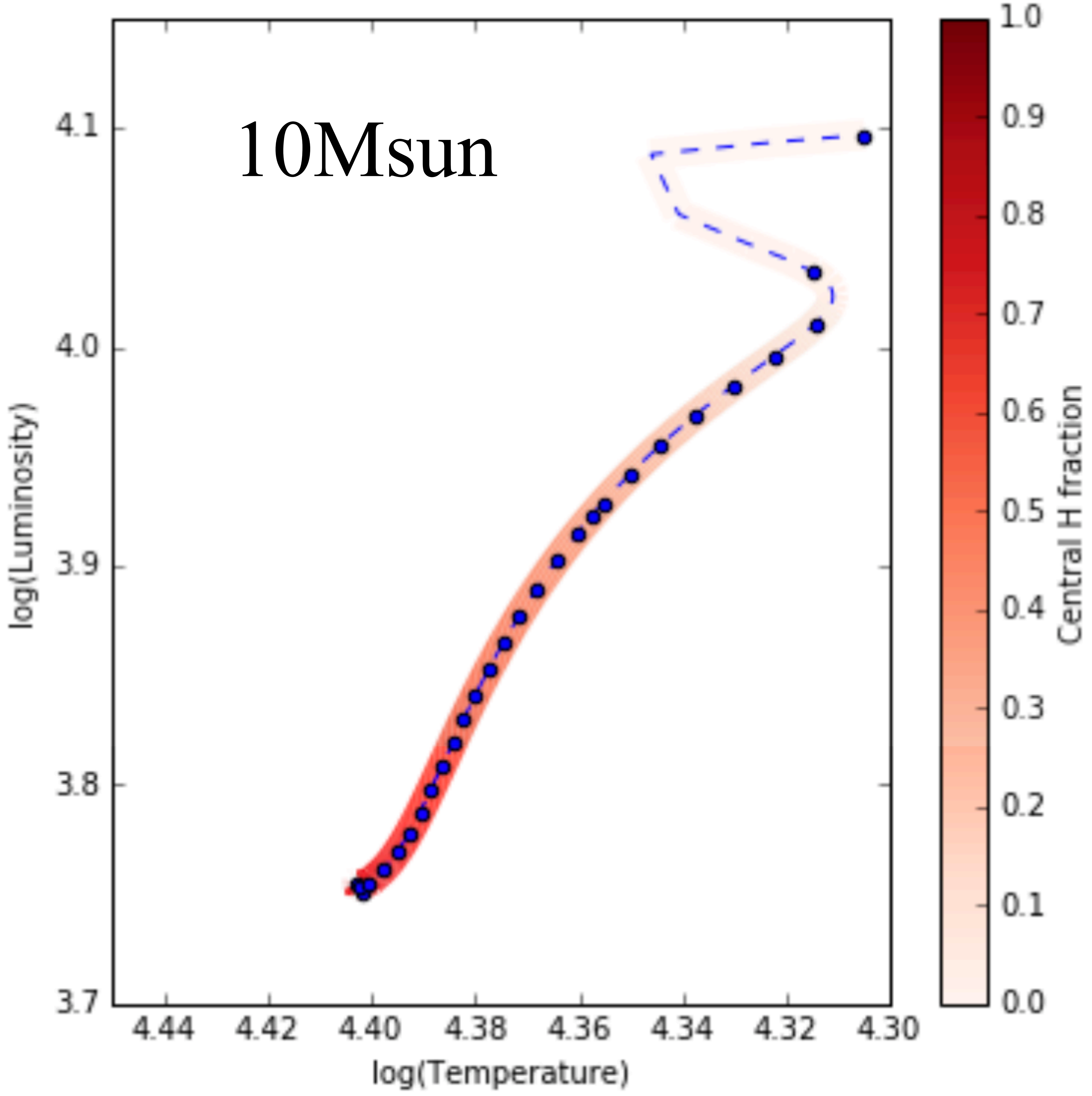
ρ_c goes up and P_c goes down

If P_c goes down, P_{env} also need to go down
(because of HS equilibrium)

$$P_{\rm env} = \frac{\rho_{\rm env} k T_{\rm env}}{\mu m_H}$$

Make ρ_{env} and T_{env} goes down
by expending the star (R goes up)

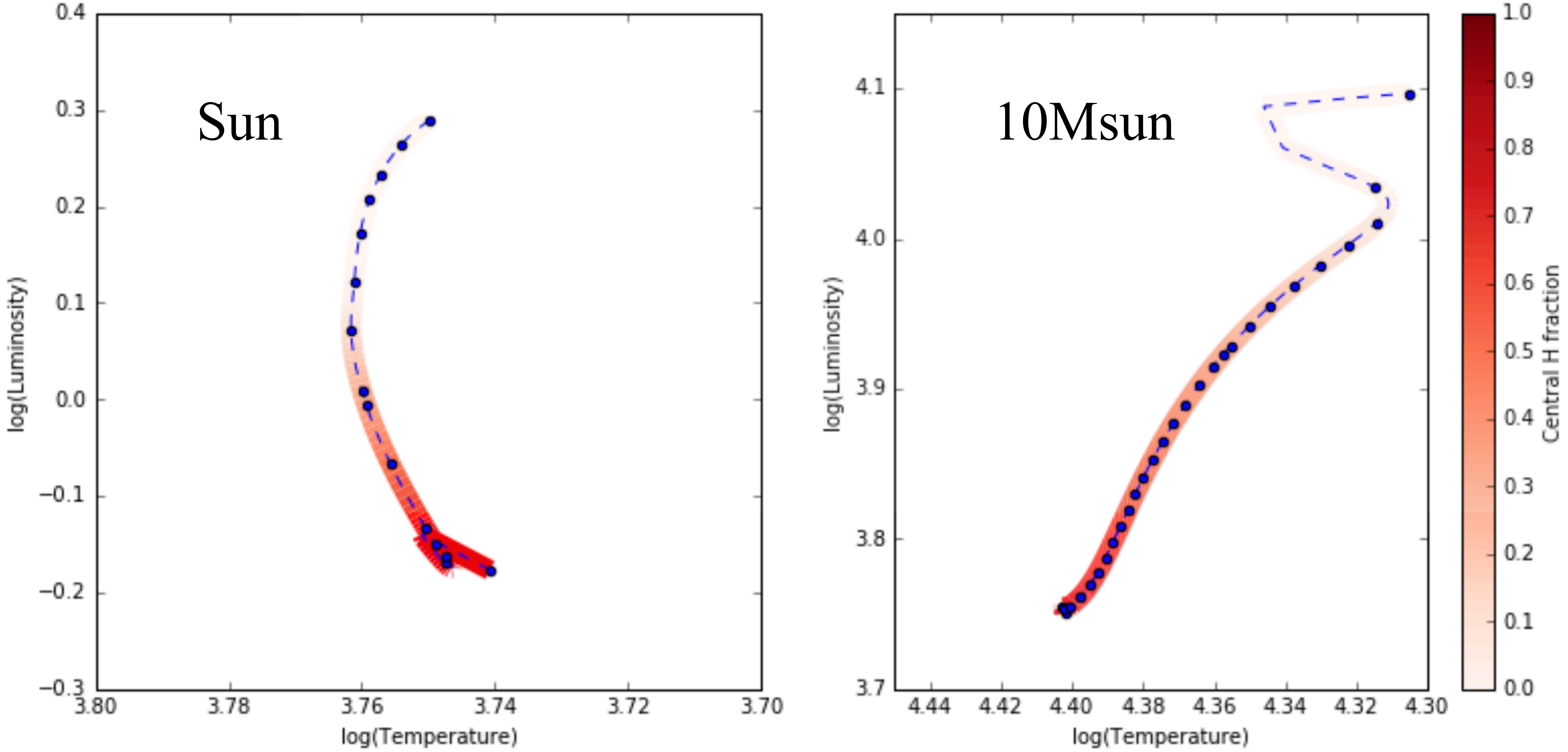
$$L_\star = 4\pi R_\star^2 \sigma T_{\rm eff}^4$$



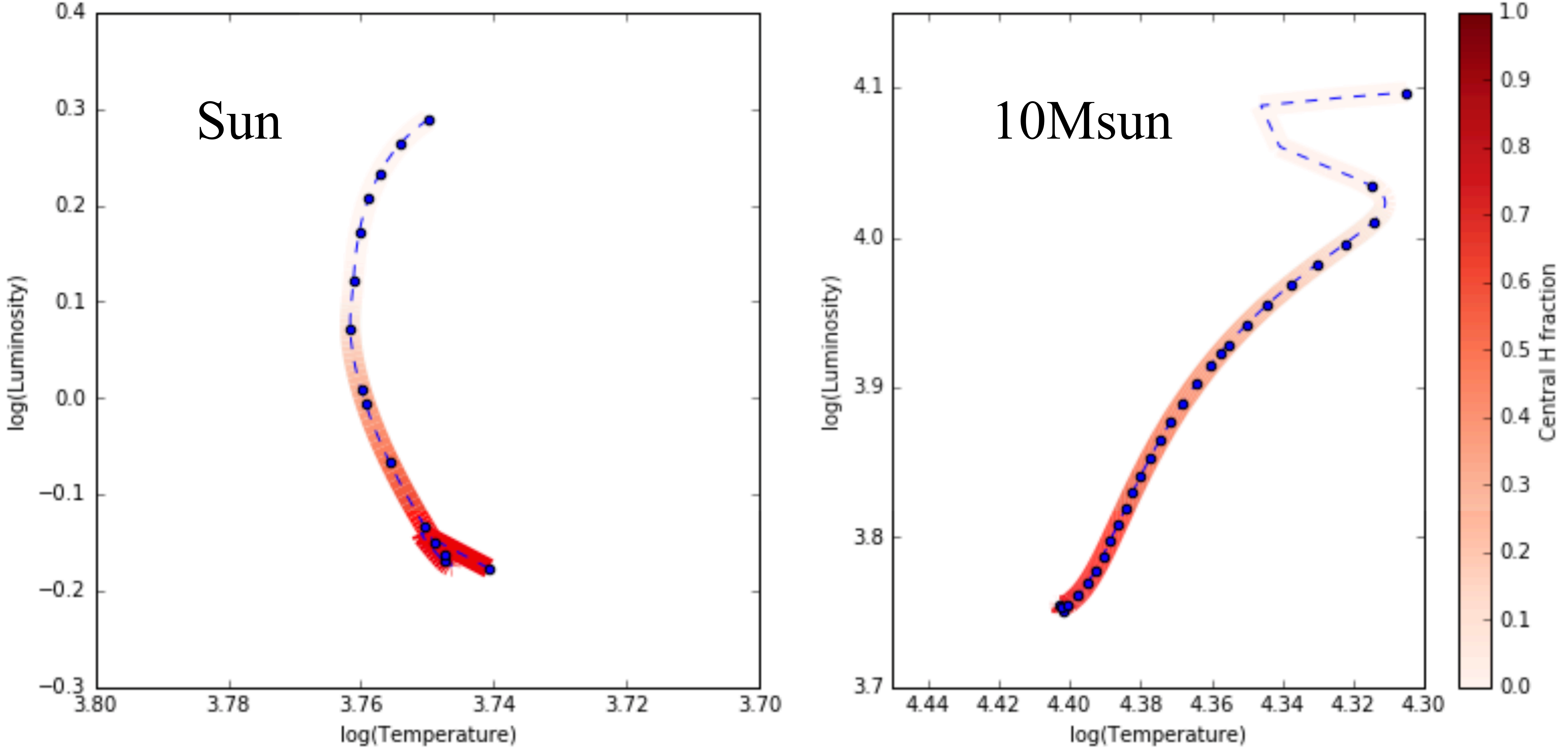
$$P_{\mathrm{env}} = \frac{\rho_{\mathrm{env}} k T_{\mathrm{env}}}{\mu m_H}$$

Make ρ_{env} and T_{env} goes down by expending the star (R goes up)

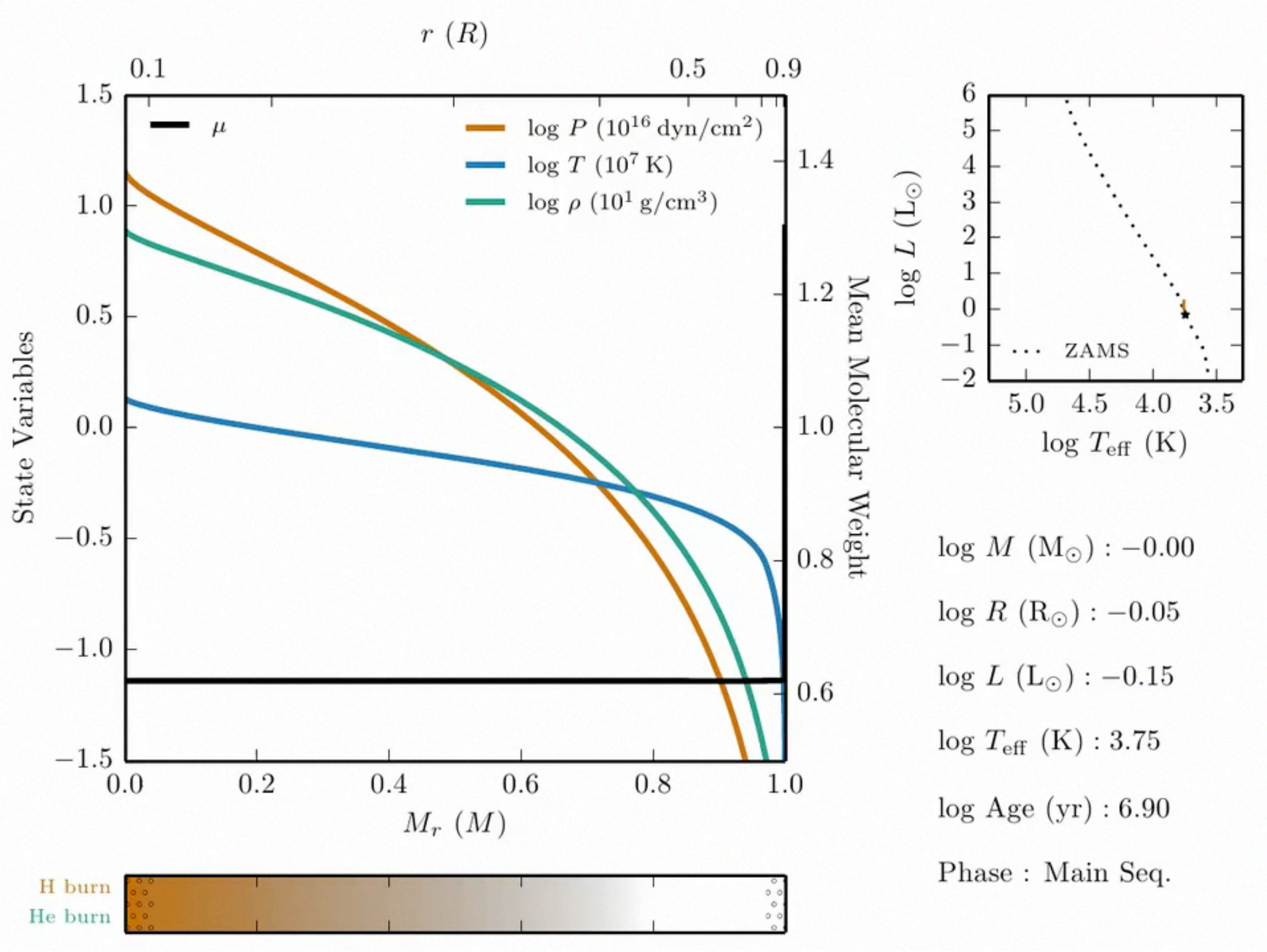
$$L_\star = 4\pi R_\star^2 \sigma T_{\mathrm{eff}}^4$$



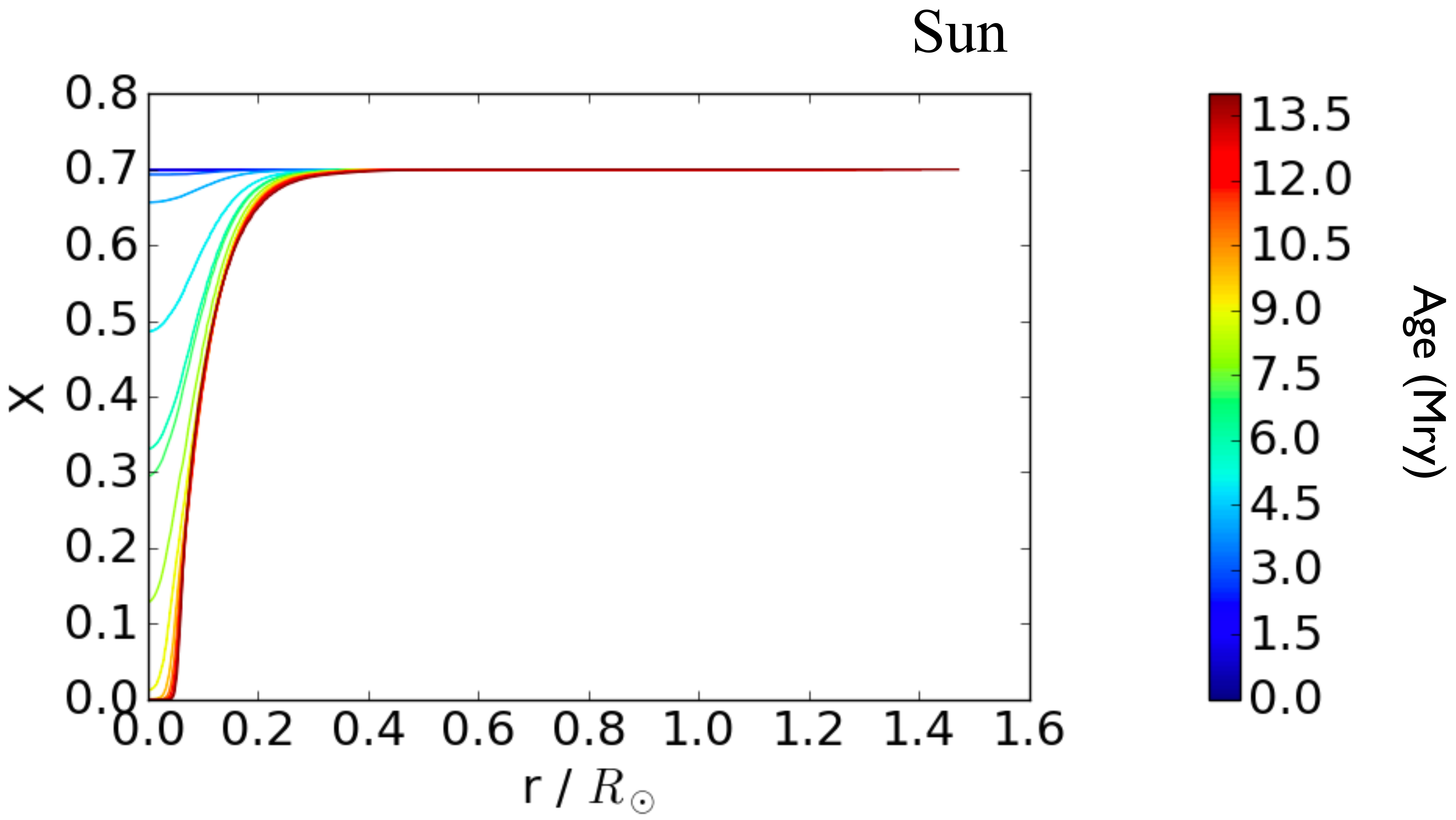
Sun
10Msun
log(Luminosity)
log(Temperature)
Central H fraction



Sun
10Msun
log(Luminosity)
log(Temperature)
0.4
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
3.80
3.78
3.76
3.74
3.72
3.70
4.1
4.0
3.9
3.8
3.7
4.44
4.42
4.40
4.38
4.36
4.34
4.32
4.30
Central H fraction
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0



r (R)
0.1
0.5
0.9
μ
log P (10^16 dyn/cm^2)
log T (10^7 K)
log ρ (10^1 g/cm^3)
State Variables
Mean Molecular Weight
M_r (M)
H burn
He burn
log L (L_⊙)
log T_eff (K)
ZAMS
log M (M_⊙) : −0.00
log R (R_⊙) : −0.05
log L (L_⊙) : −0.15
log T_eff (K) : 3.75
log Age (yr) : 6.90
Phase : Main Seq.



Sun
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
X
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
r / $R_\odot$
13.5
12.0
10.5
9.0
7.5
6.0
4.5
3.0
1.5
0.0
Age (Mry)

1. $$\frac{dM_r(r)}{dr} = 4\pi r^2 \rho(r)$$

2. $$\frac{dP(r)}{dr} = -\rho(r)\frac{GM_r(r)}{r^2}$$

3. $$\frac{dT(r)}{dr} = -\frac{3\kappa_R(r)\rho(r)}{64\pi\sigma}\frac{L_r(r)}{r^2 T(r)^3}$$

4. $$\frac{dL_r(r)}{dr} = 4\pi r^2 \rho(r)\epsilon(r)$$

$$P = \frac{\rho k T}{\mu}$$

μ: higher when more He,
higher when neutral

$\varkappa_R$: higher when neutral

ε_{PP}: $\propto T^4$

ε_{CNO}: $\propto T^{20}$

1. Why will our Sun run out of Hydrogen in the middle of the core first?

Because T is higher in the center, burns H faster.

2. Once that happens, sketch the variation of *dL/dr*

Gradient of *dL/dr* = 0, the *L* becomes zero progressively over the core. H burns in a "shell" where there is still some H.

3. What happens to the temperature in the core?

***dT/dt* is proportional to *L*, which become zero. The core becomes isothermal.**

1. No hydrogen burning in the core

2. No energy production = isothermal core

3. Isothermal core, but need a pressure gradient

$$\frac{dP(r)}{dr} = -\rho(r)g(r)$$

4. Density (and then degenaracy pressure) take a hit for the team = core contracts

$$P(r) = \frac{\rho(r)kT(r)}{\mu(r)m_H}$$

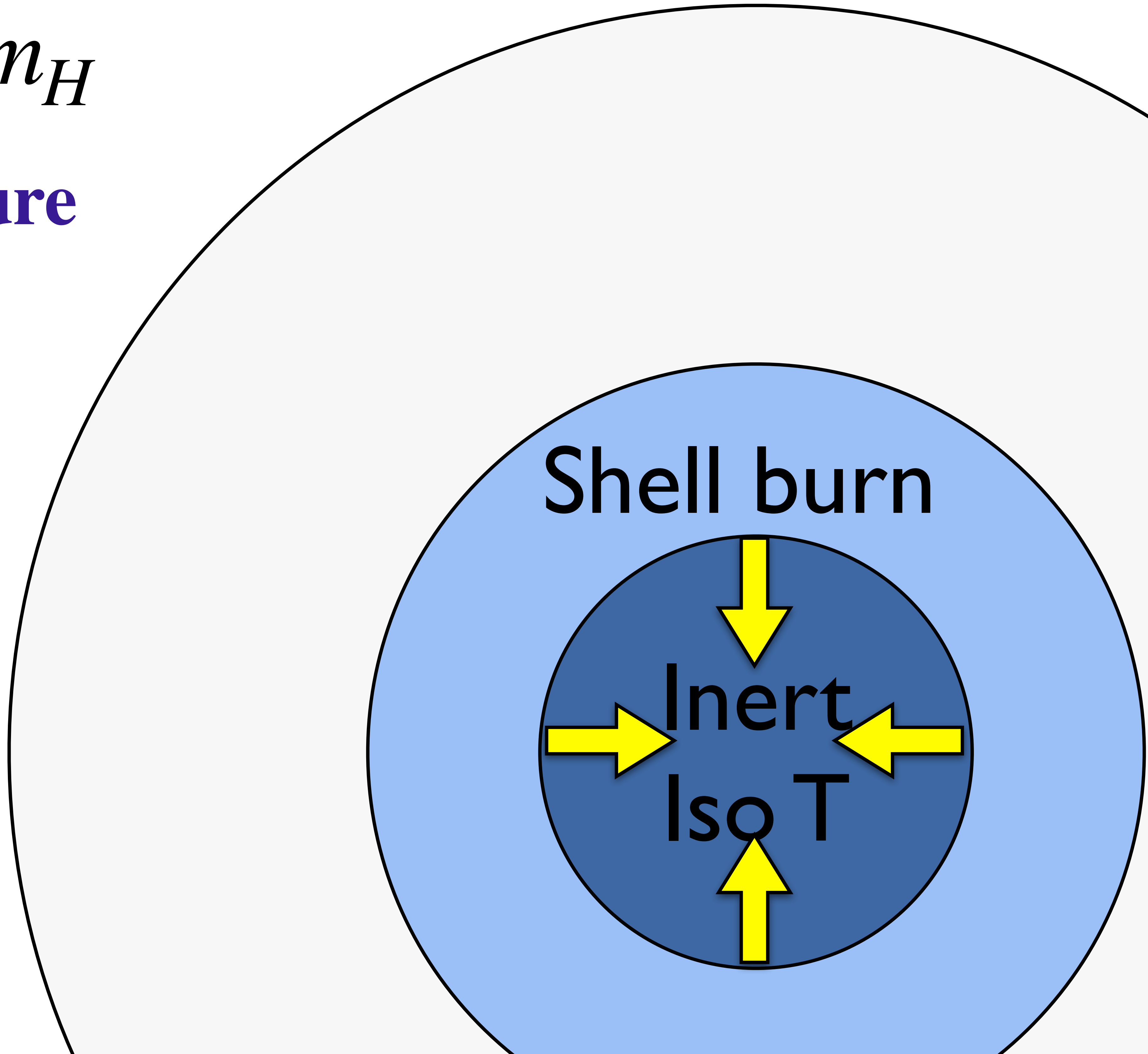
Not helping with the gradient

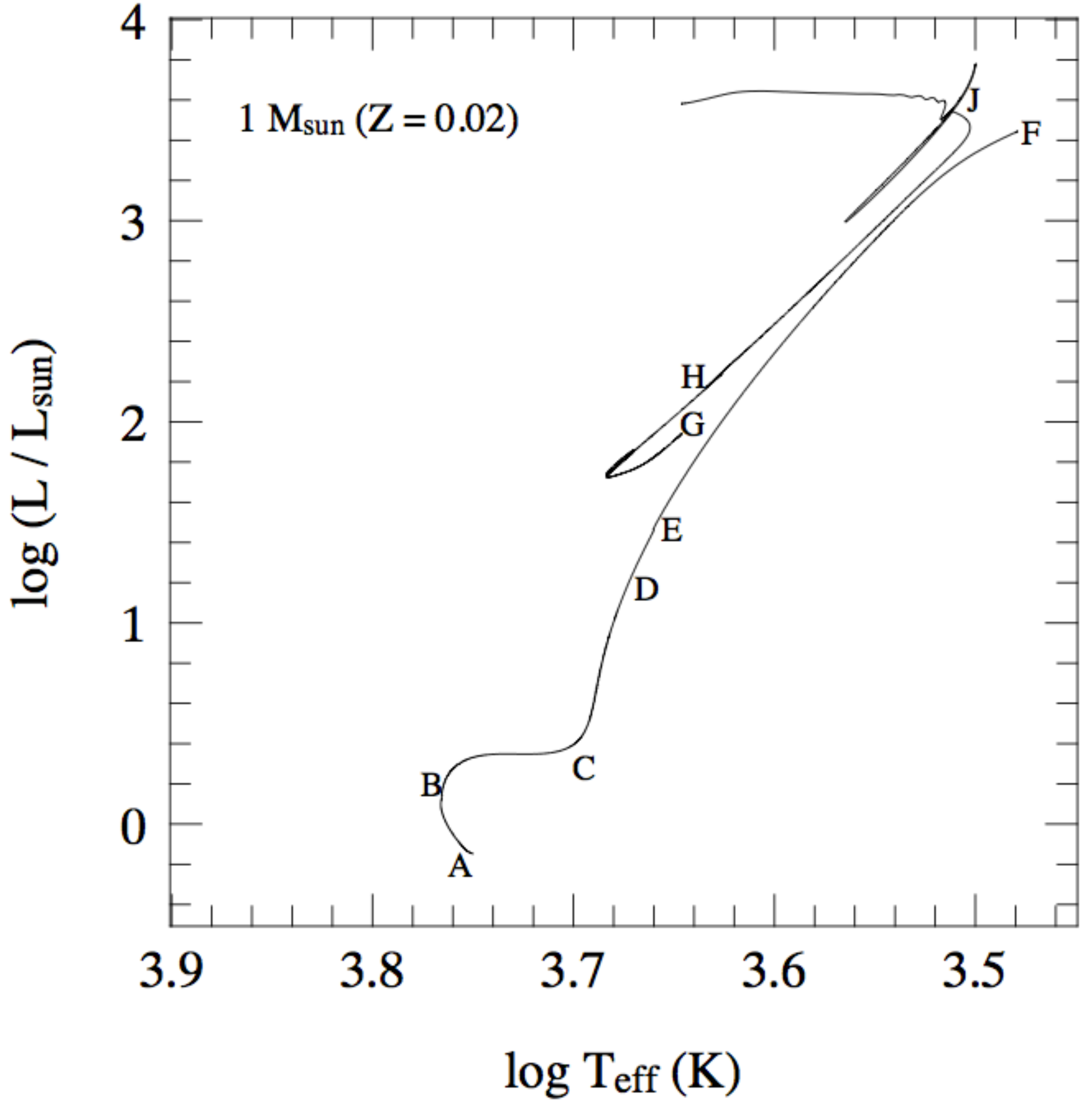
5. Burning shell has to follow the contraction = higher temperature

6. Luminosity of the shell doesn't want to increase too much = pressure drops

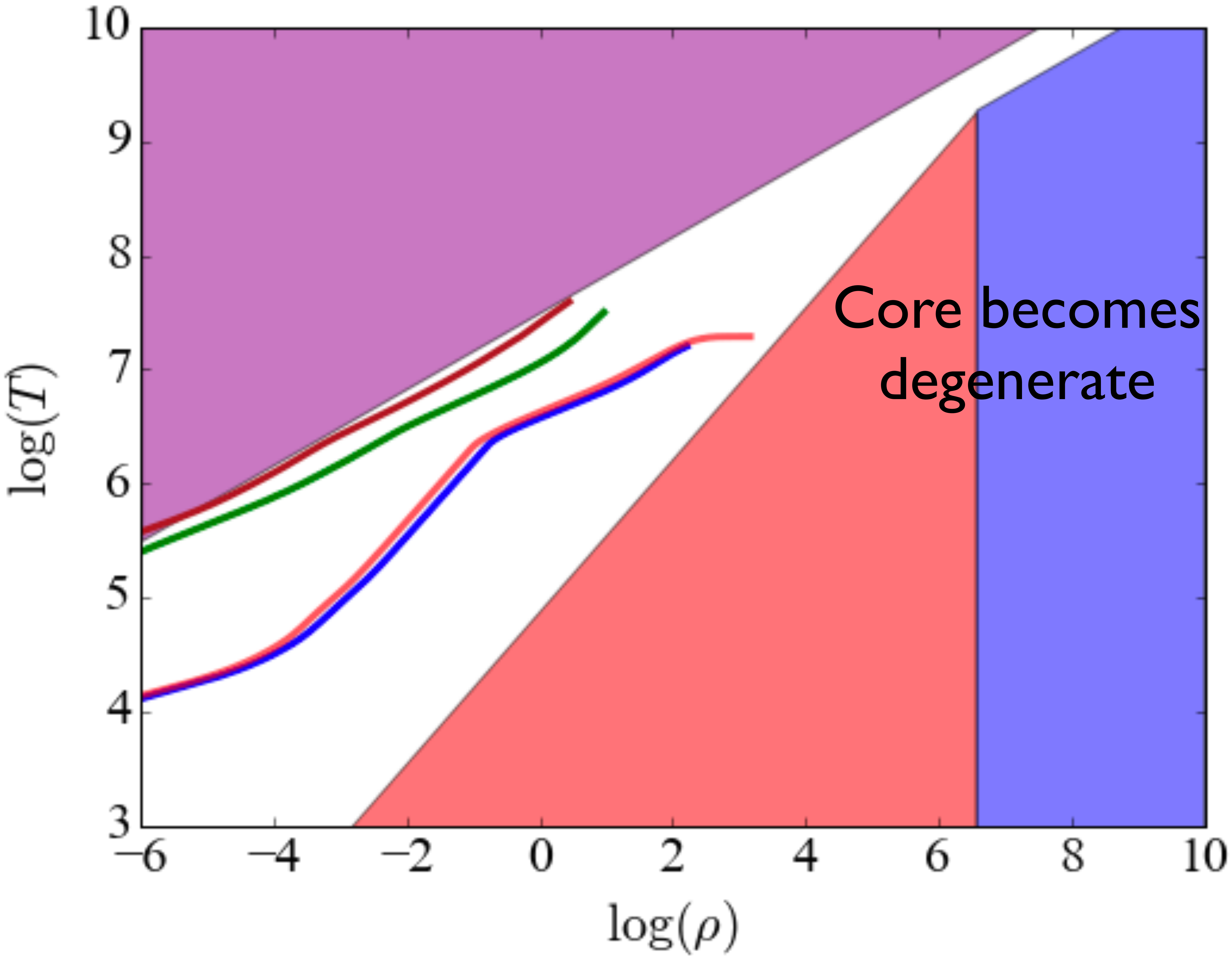
7. Pressure drops in the shell = pressure drops in the enveloppe

8. For the pressure in the envelop to go down, temperature and/or density have to go down = envelop expends.

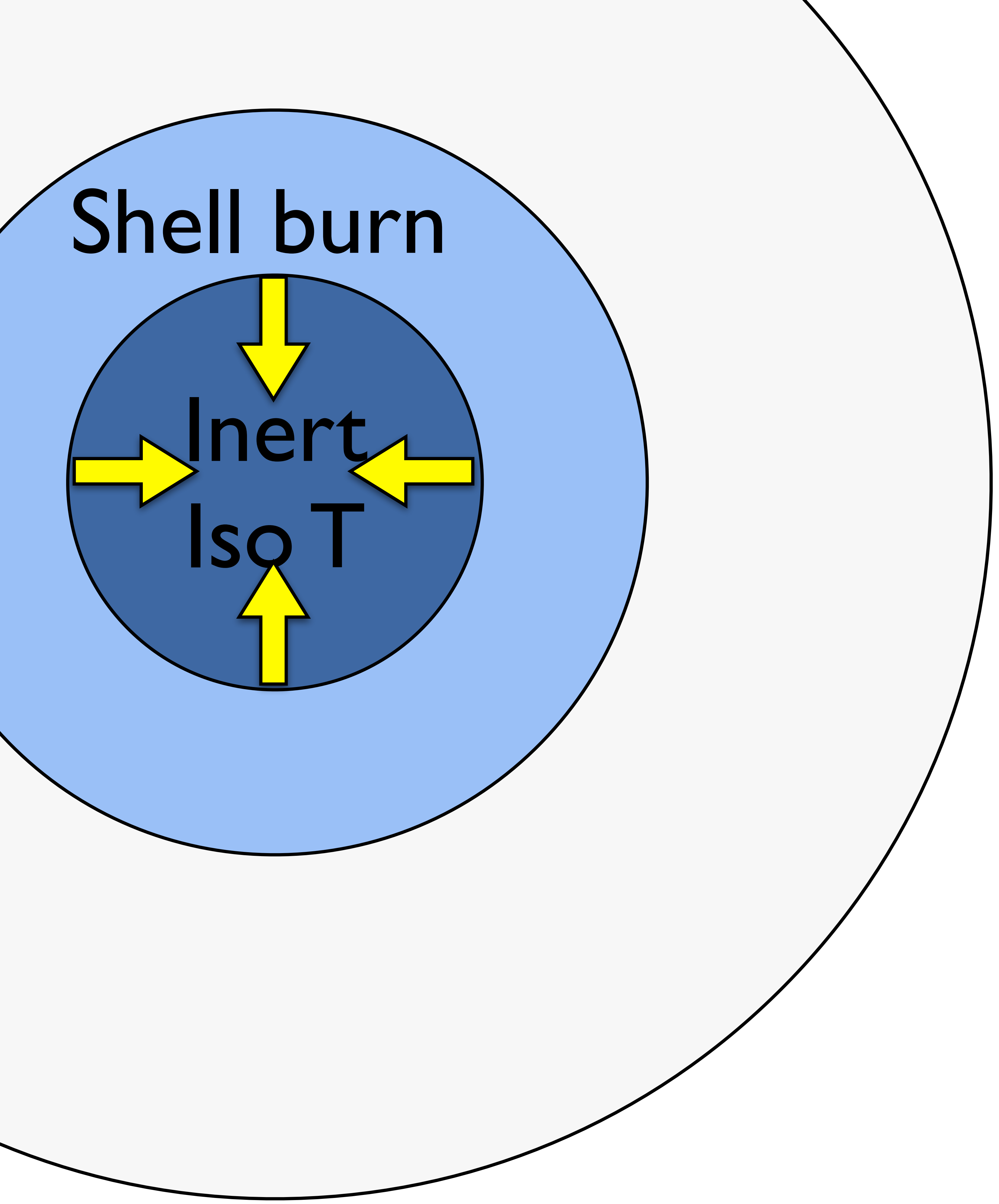



1 Msun (Z = 0.02)
log (L / Lsun)
log Teff (K)
4
3
2
1
0
3.9
3.8
3.7
3.6
3.5
A
B
C
D
E
F
G
H
J

The helium flash
(for low mass stars)

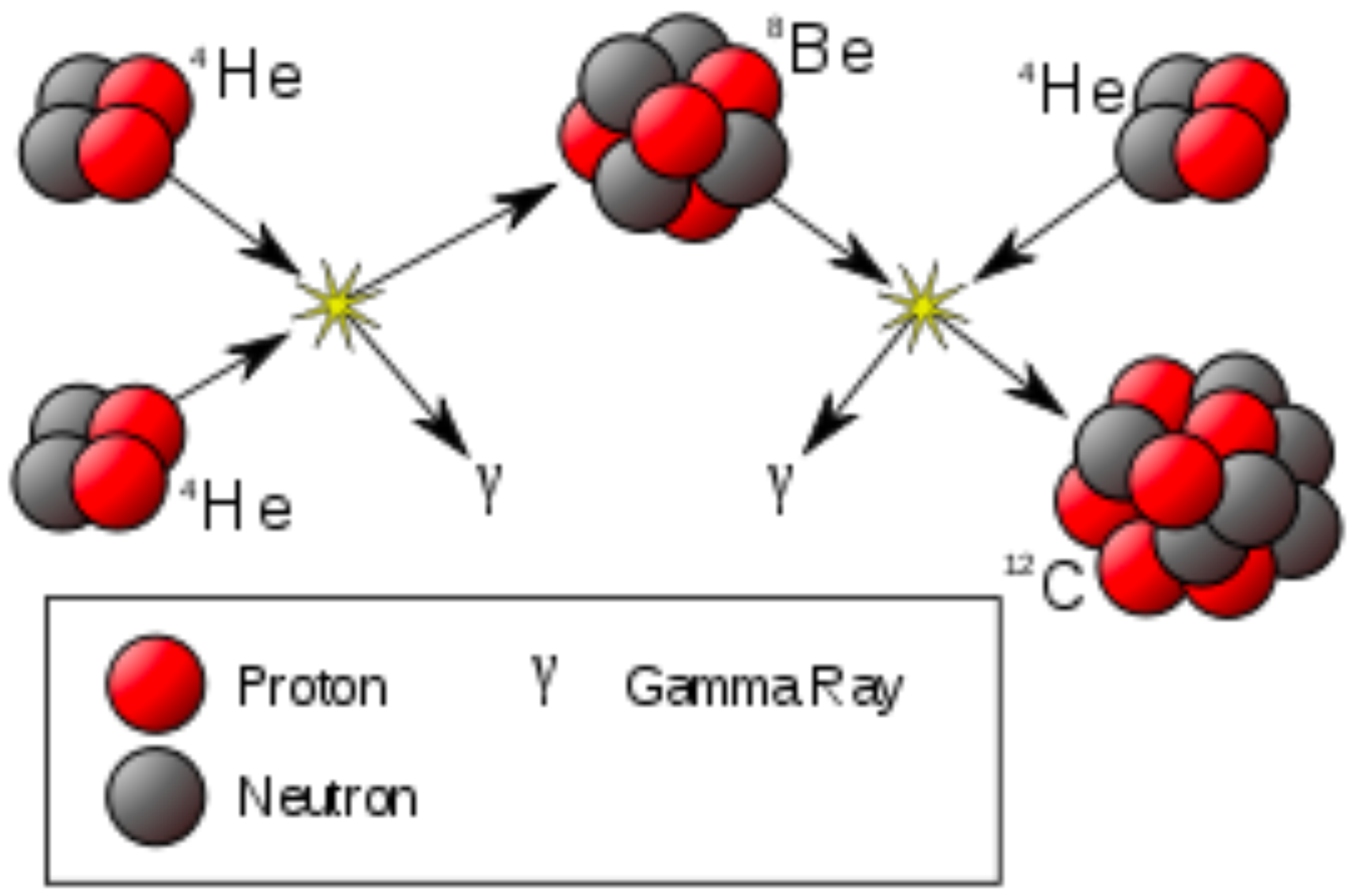


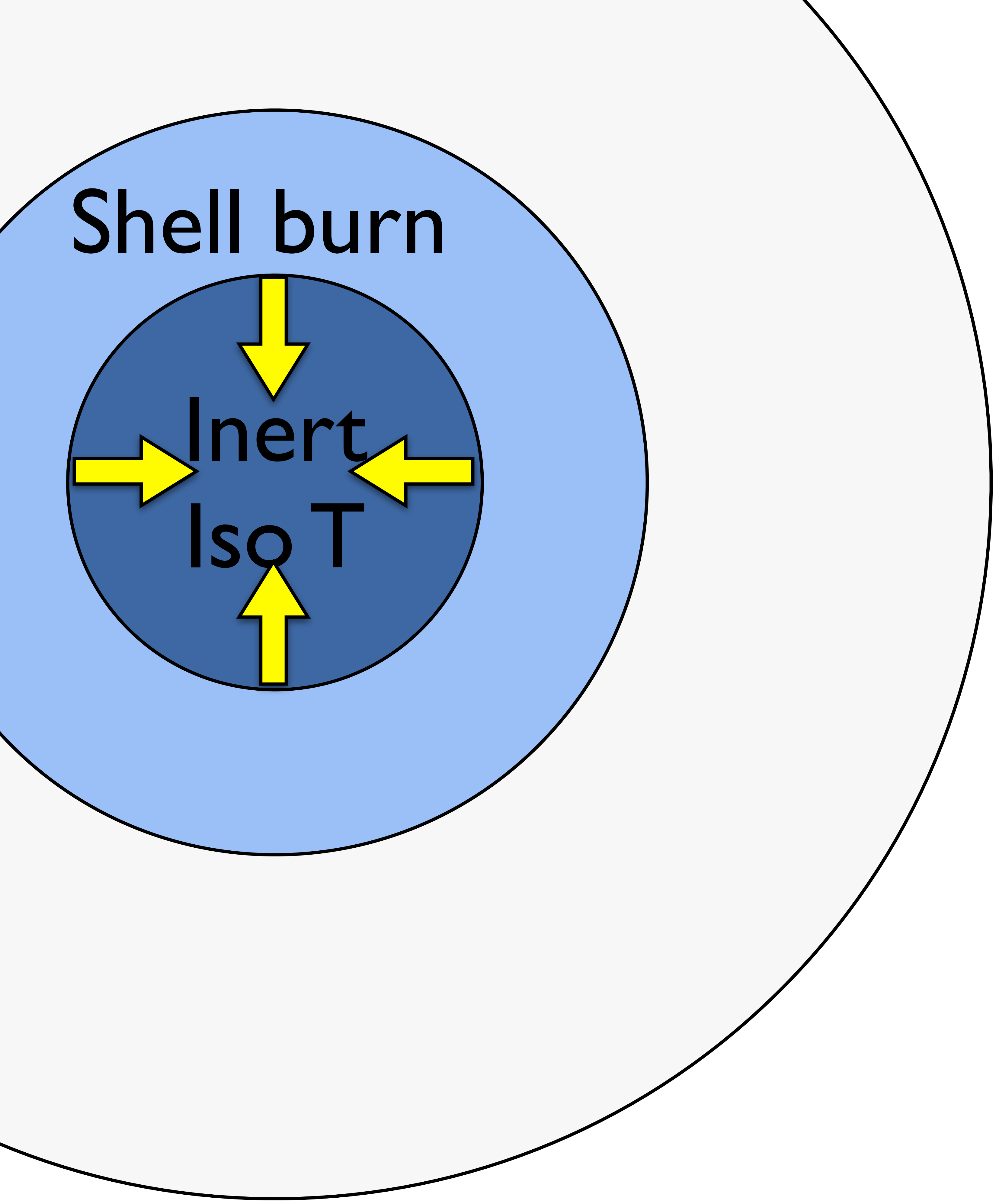
Pressure does not rise with temperature



When the core contract, temperature goes up (but not the pressure)

When you reach 10^8K, the "triple-alpha" process starts





When the core contract, temperature goes up (but not the pressure)

When you reach 10^8K, the "triple-alpha" process starts

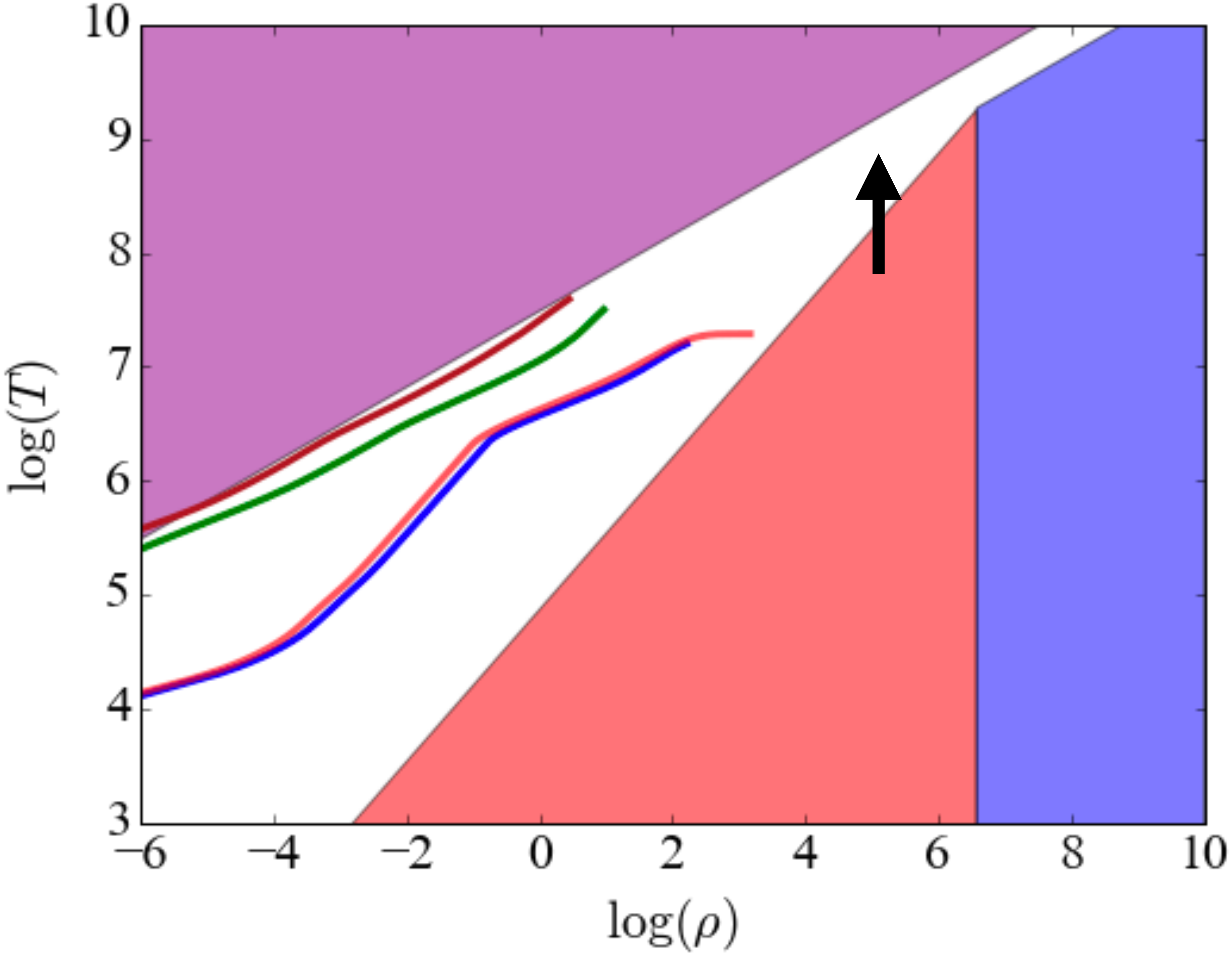
New source of energy -> core temperature rise

But pressure doesn't change -> gravity still wins

Contraction continues -> temperature rise

Temperature rise -> fusion becomes more effective

Thermonuclear "runaway" until....

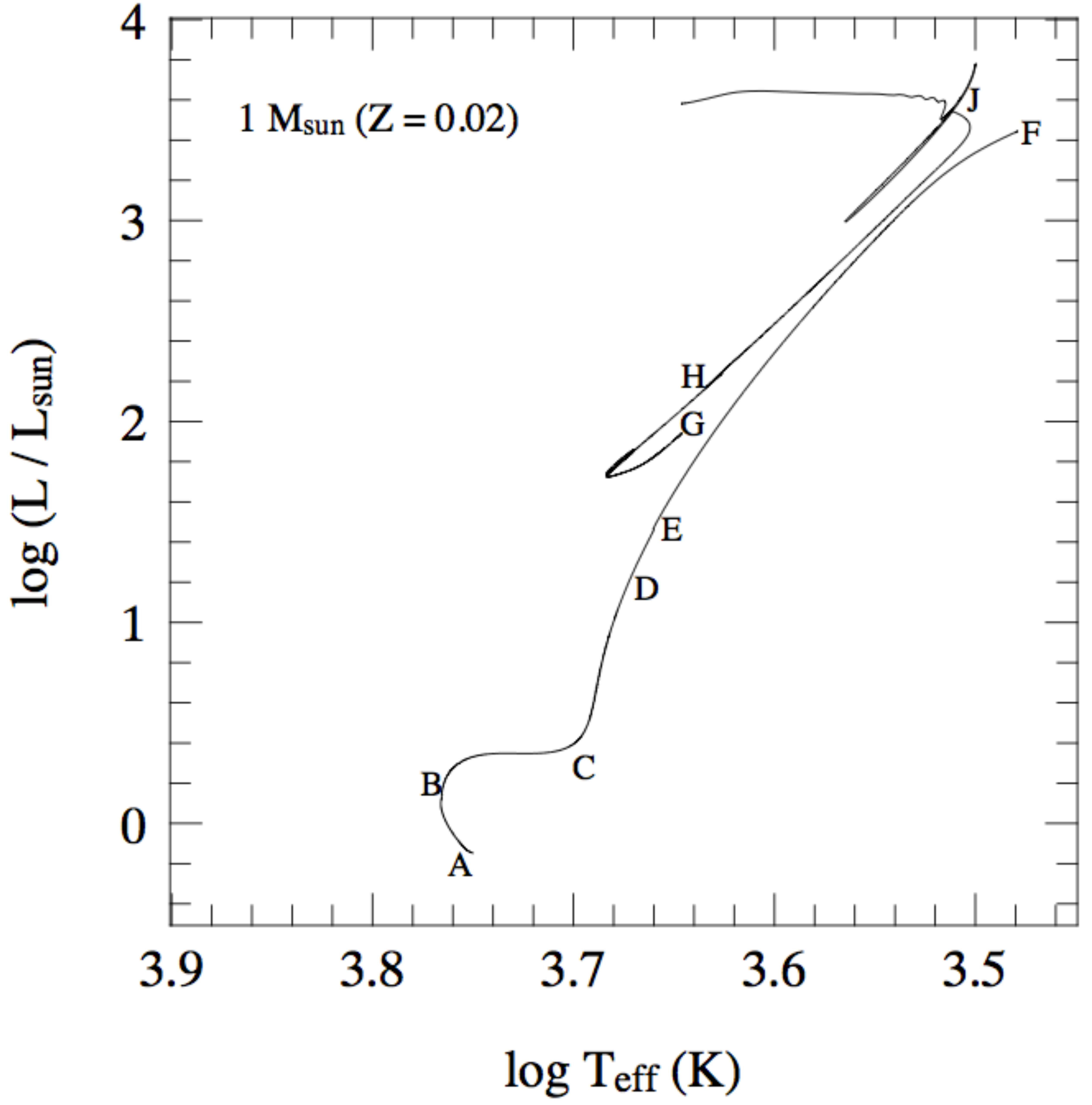


Ideal gas law takes over.

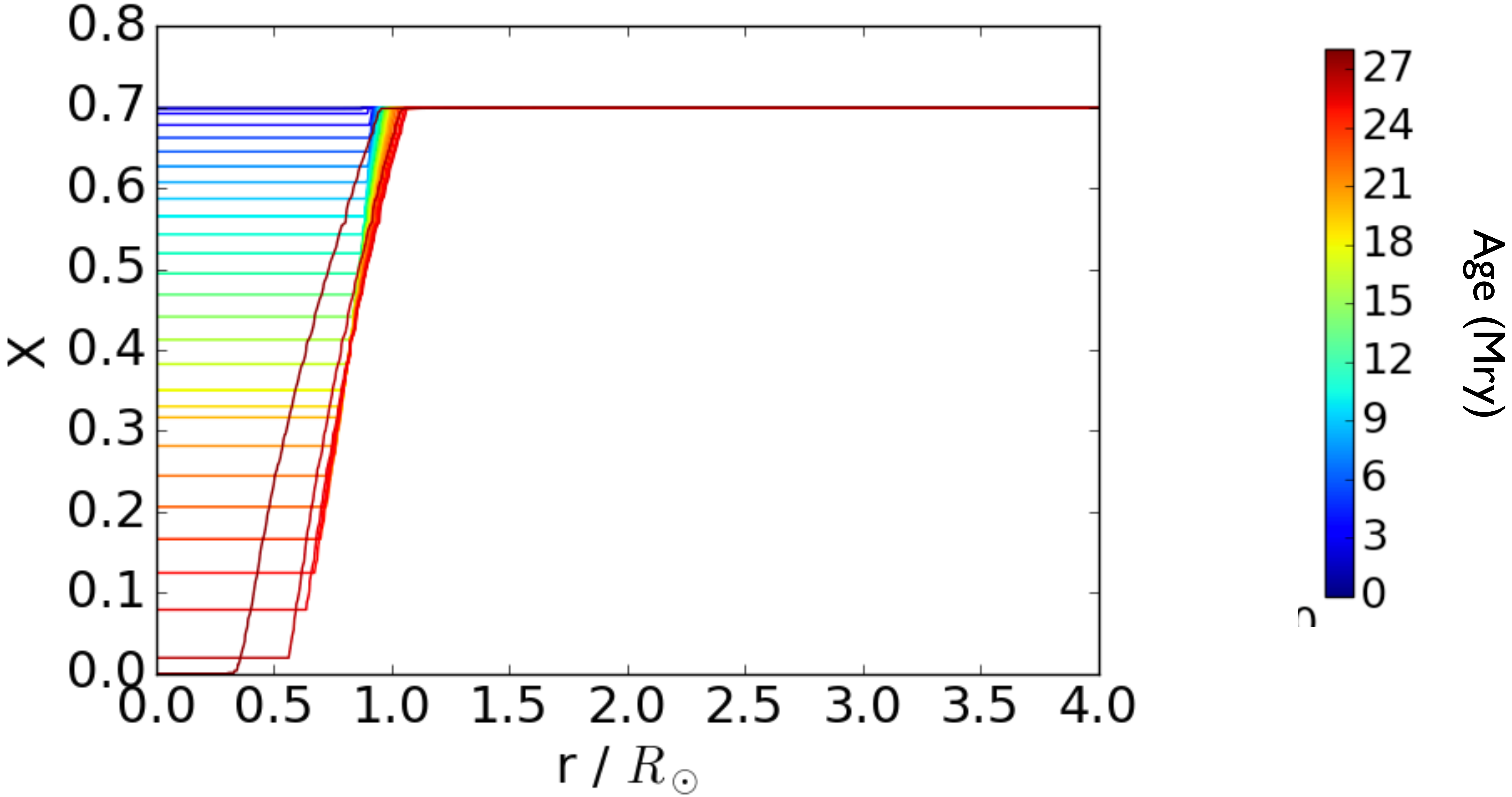
Pressure goes: "hooooottttt!!!!"

The newly increase pressure overpower gravity, core expands

Core expands -> envelop contracts

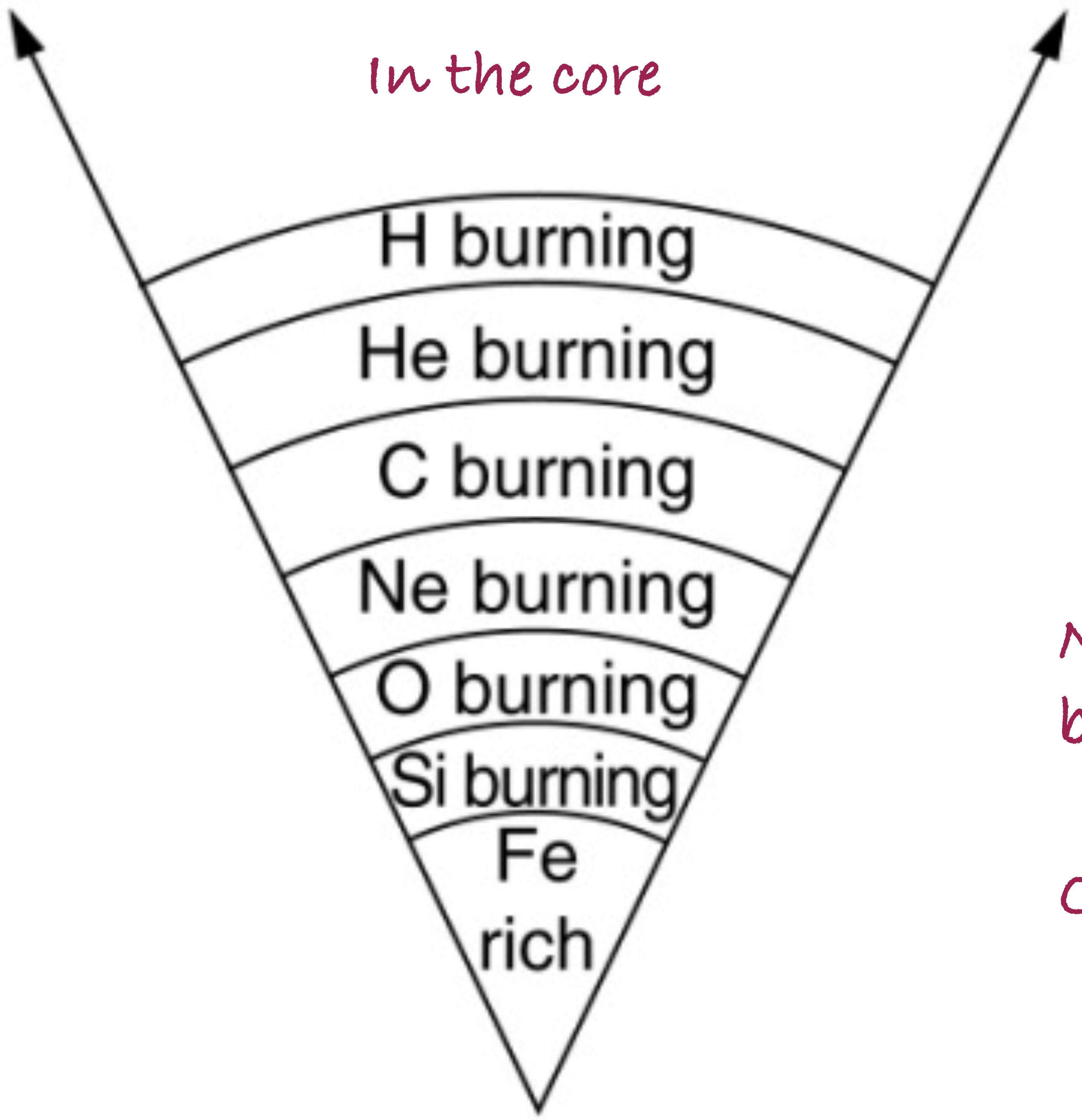

1 Msun (Z = 0.02)
log (L / Lsun)
log Teff (K)
4
3
2
1
0
3.9
3.8
3.7
3.6
3.5
A
B
C
D
E
F
G
H
J

10Msun



0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
X
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
r / $R_\odot$
27
24
21
18
15
12
9
6
3
0
Age (Mry)

For massive stars



Massive stars do get hot enough before the core stabilizes.

Core gets very massive

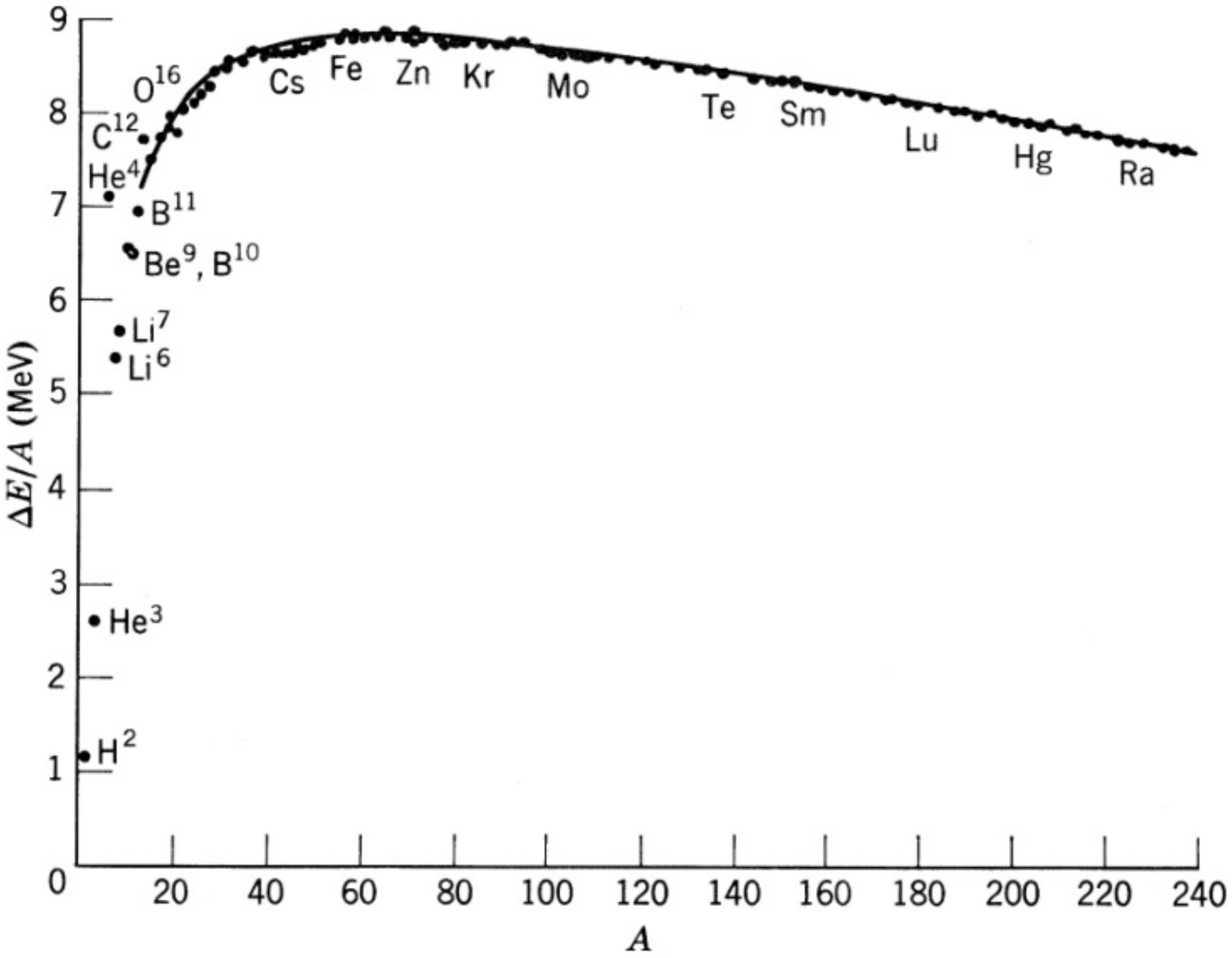
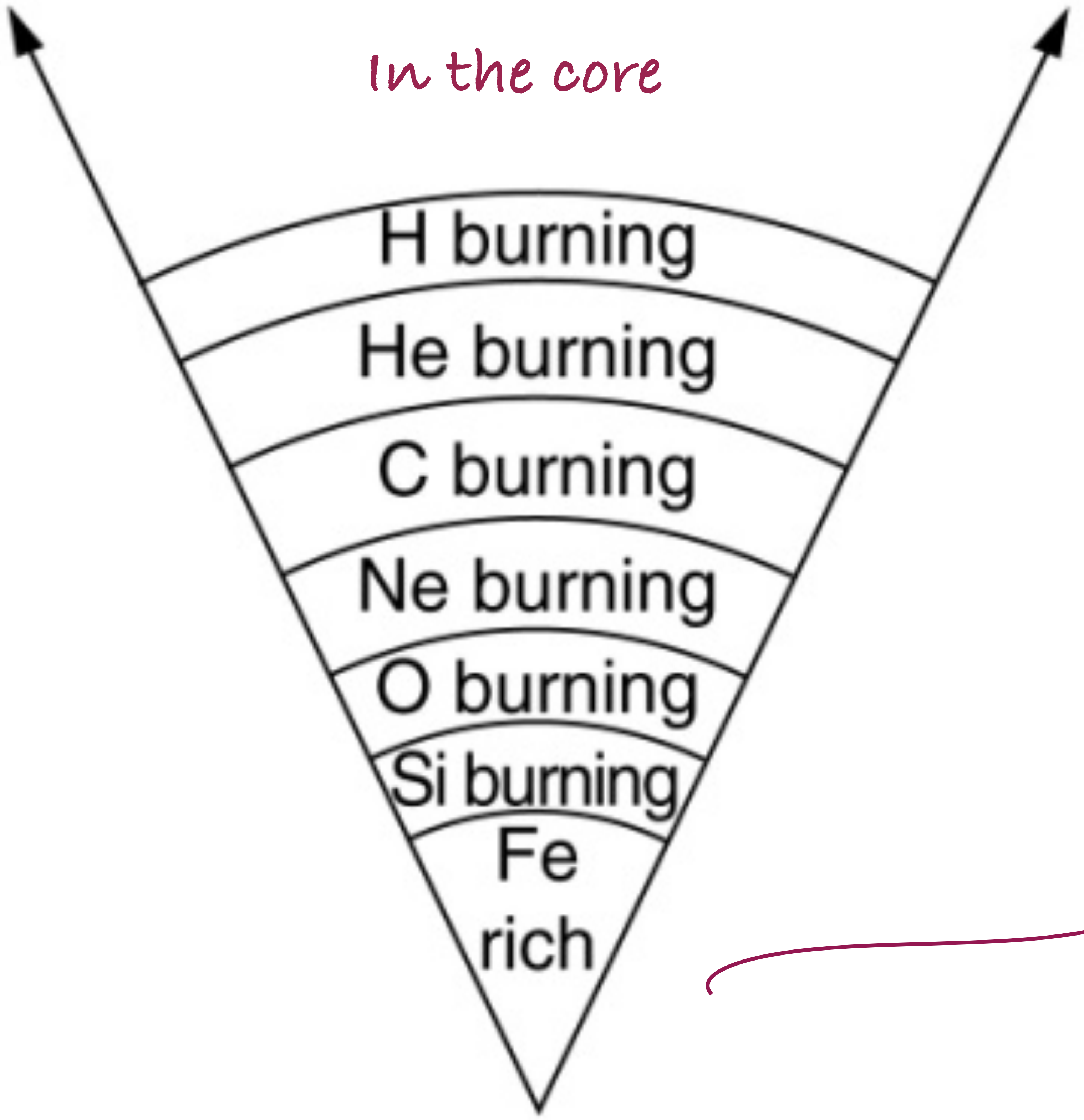
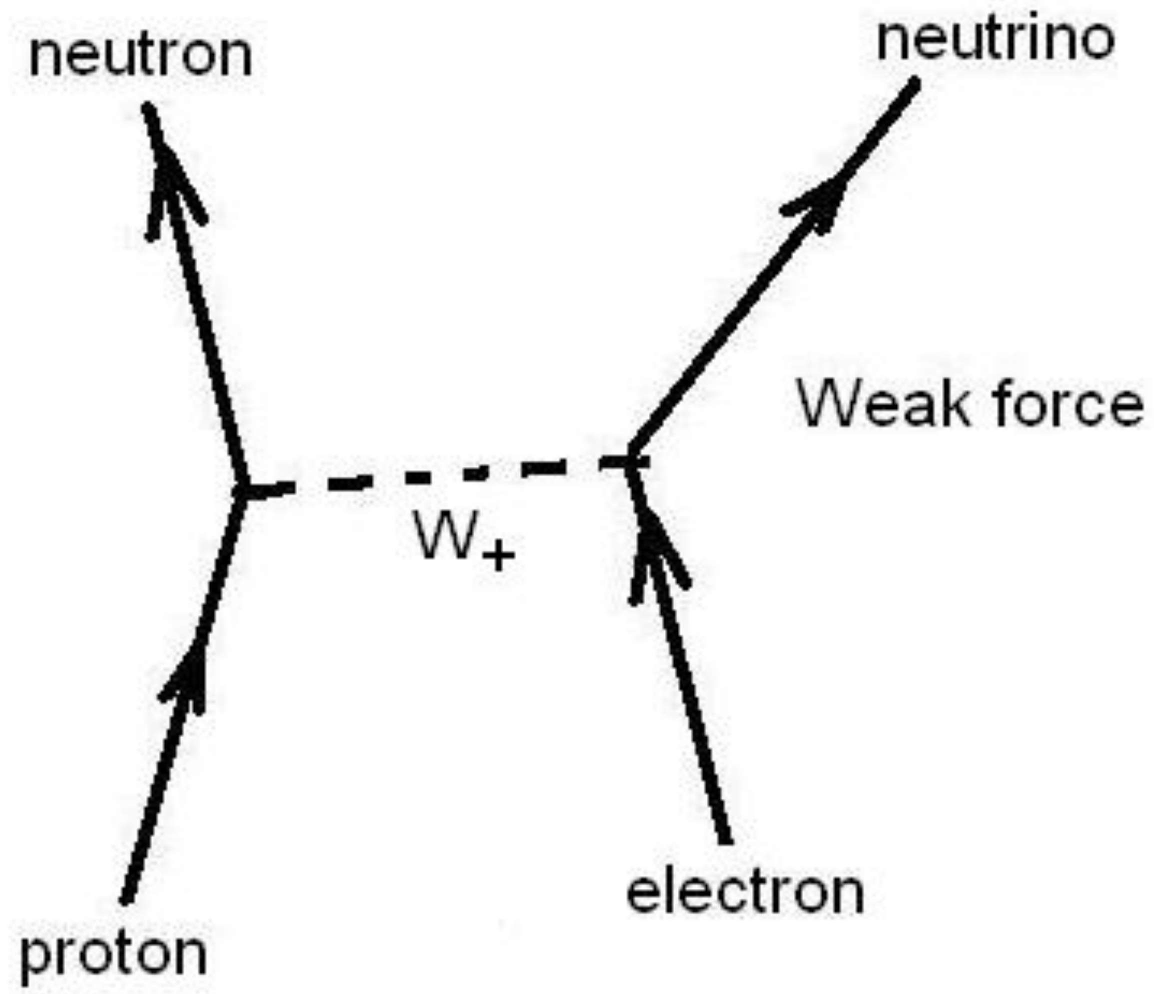
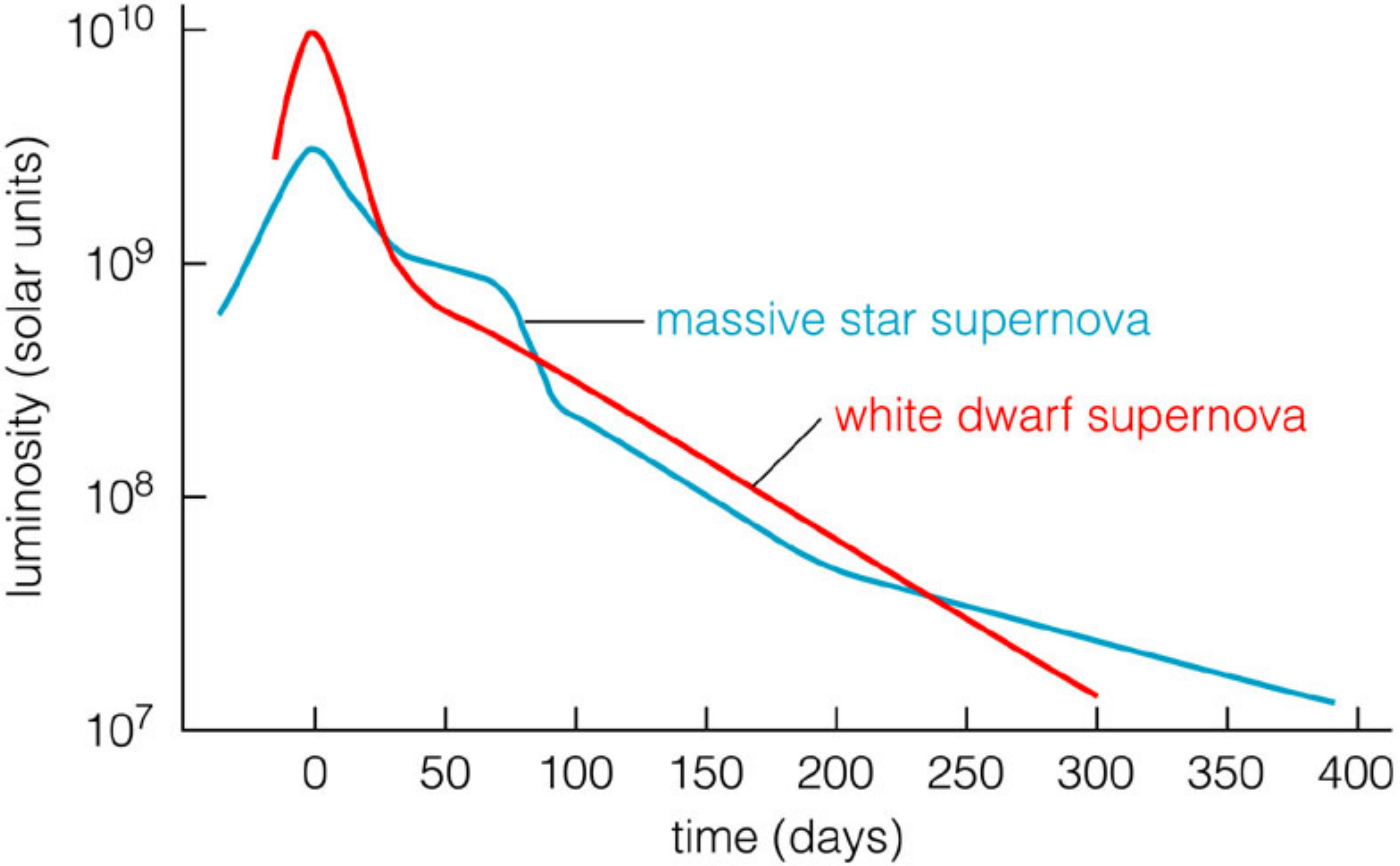


Figure 6.1 The average binding nuclear energy per nucleon $\Delta E/A$ as a function of the number of nucleons A in the various nuclei shown. The solid curve represents the results using the semiempirical mass formula (see (optional) Section 6.3.1). Reproduced with permission from Eisberg, R. and Resnick, R., *Quantum Physics of Atoms, Molecules, Solids, Nuclei and Particles*, John Wiley & Sons, Ltd, New York (1985).

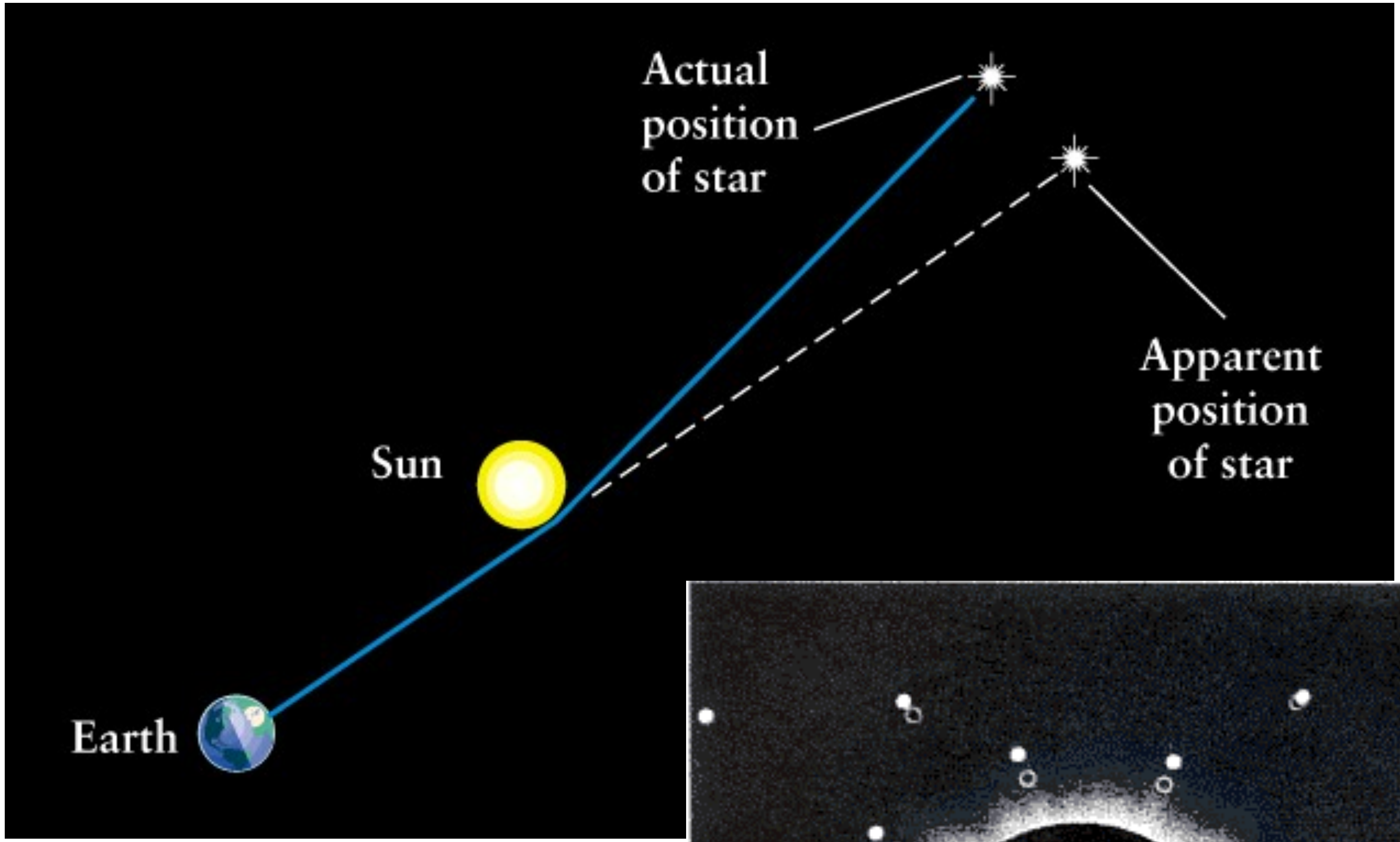

In the core
H burning
He burning
C burning
Ne burning
O burning
Si burning
Fe
rich


neutron
neutrino
Weak force
W_+
proton
electron

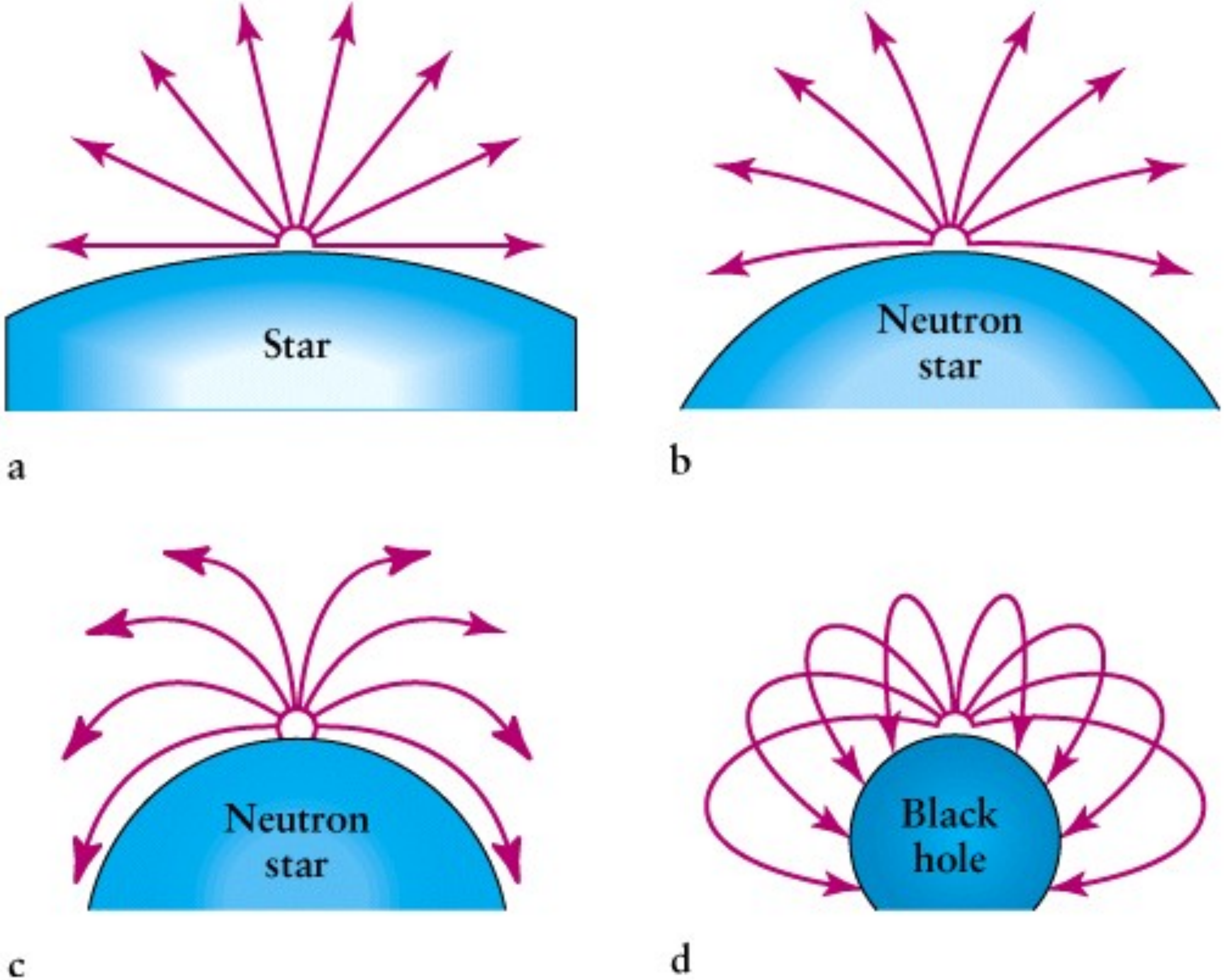


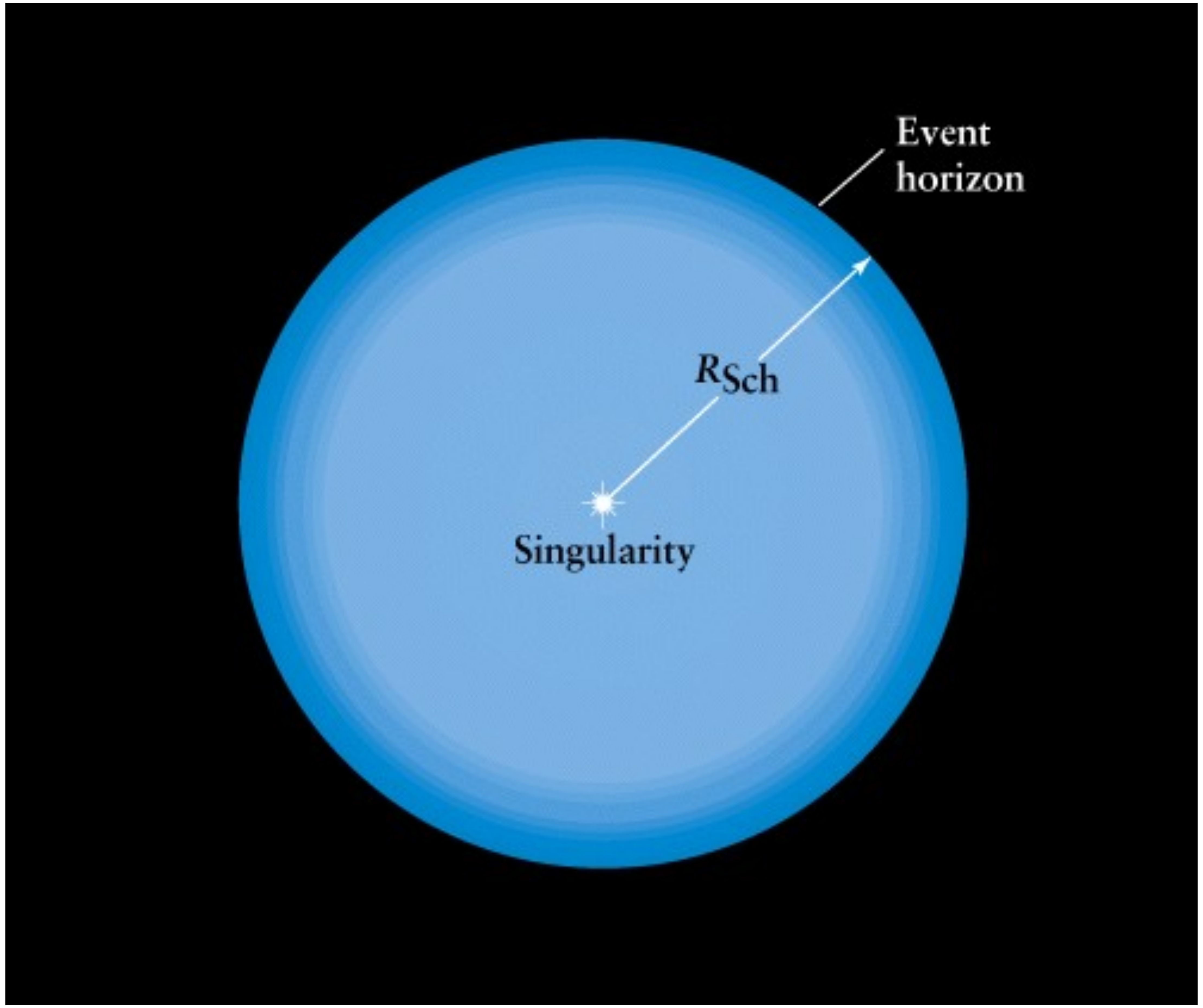
10^{10}
10^9
10^8
10^7
luminosity (solar units)
0
50
100
150
200
250
300
350
400
time (days)
massive star supernova
white dwarf supernova

log10 [Period derivative]
Period (s)
1 kyr
100 kyr
10 Myr
10^14 G
10^13 G
10^12 G
10^11 G
AXP
SGR
CCO
INS
RRAT
magnetic axis
radio beam
rotation axis
outer acceleration gap
inner acceleration gap
Neutron star
open field lines
closed field lines
light cylinder


Actual
position
of star
Apparent
position
of star
Sun
Earth


Eclipsed Sun


Star
a
Neutron
star
b
Neutron
star
c
Black
hole
d



$$R_S = 3.0 \frac{M}{M_{sun}} \text{ km}$$