Week 6 Tuesday
L11

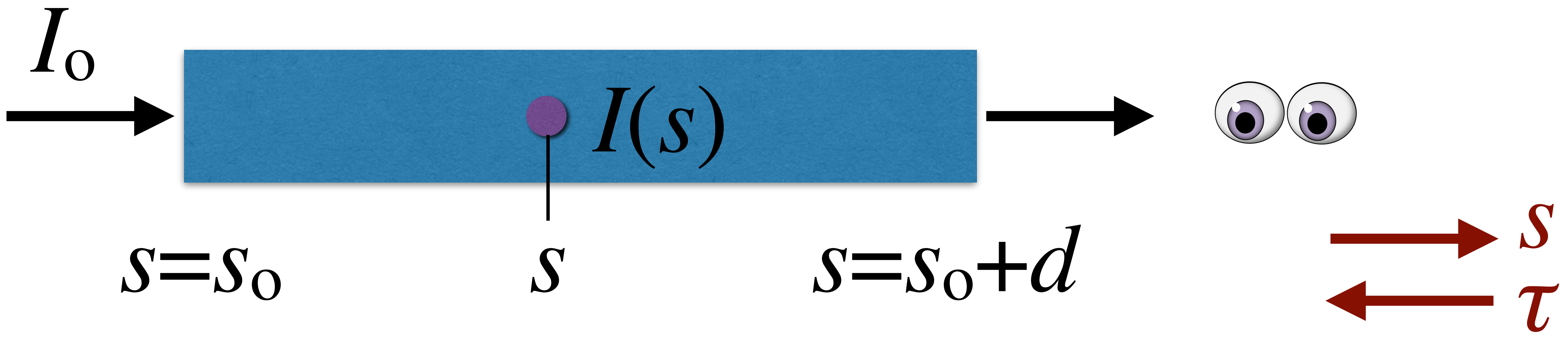

L8
Energy transport
Radiative mean free path
Stellar interiors
Diffusive energy transport
Numerical iterations
L7
Stellar atmospheres
Need detailed RT calculations
L18
Radiative transport in interiors
Energy conservation equation
L9
Basics of radiative transfer
Optical depth
L10
Formal solution of RT with source function
We are here
L11
Relationship with blackbody systems and spectral lines formation
L12
Application to a "flat" stellar atmosphere
L13
Limb darkening and radiative flux
L14
Solving for the temperature structure
L15
Solving for an analytical
We can use these results
1. Is the energy always transported by radiation?
L20
Convection
2. How to calculate the power generated?
L21
Nuclear physics
Gammow Peak
L22
Reaction rates
3. Is ideal gas law always valid?
L23
Degeneracy pressure
Chandrasekhar mass
Only depends on density.
We can use polytrops!
L24
Radiation pressure
4. How do we solve for the interior structure?
Homology relations for the Main Sequence
L25

Exam: week after the break -> sign up sheet (google doc?).

Exam questions will be posted on Thursday

Material for exam: up until this lecture inclusively (thermal radiation)

Change in intensity



Absorption + emission

$$-\frac{dI_\lambda(s)}{\kappa_\lambda(s)\rho(s)ds} = I_\lambda(s) - S_\lambda(s)$$

$$\frac{dI_\lambda(\tau_\lambda)}{d\tau_\lambda} = I_\lambda(\tau_\lambda) - S_\lambda(\tau_\lambda)$$

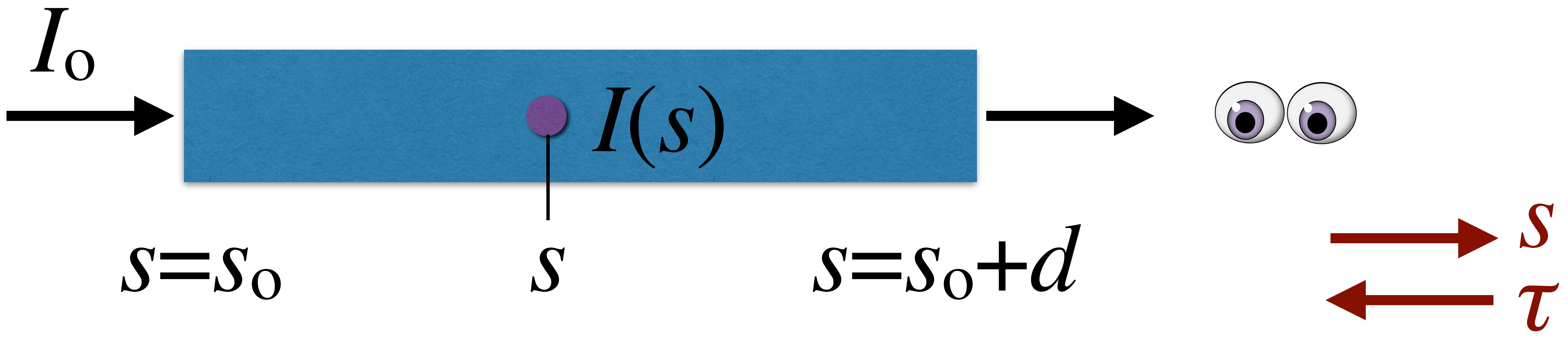
General Solution

$$I(\tau(s)) = I_o\, \mathrm{e}^{\tau(s)-\tau_o} + \int_{\tau'=\tau(s)}^{\tau'=\tau_o} S(\tau')\, \mathrm{e}^{\tau(s)-\tau'}\, d\tau'$$

Constant source function

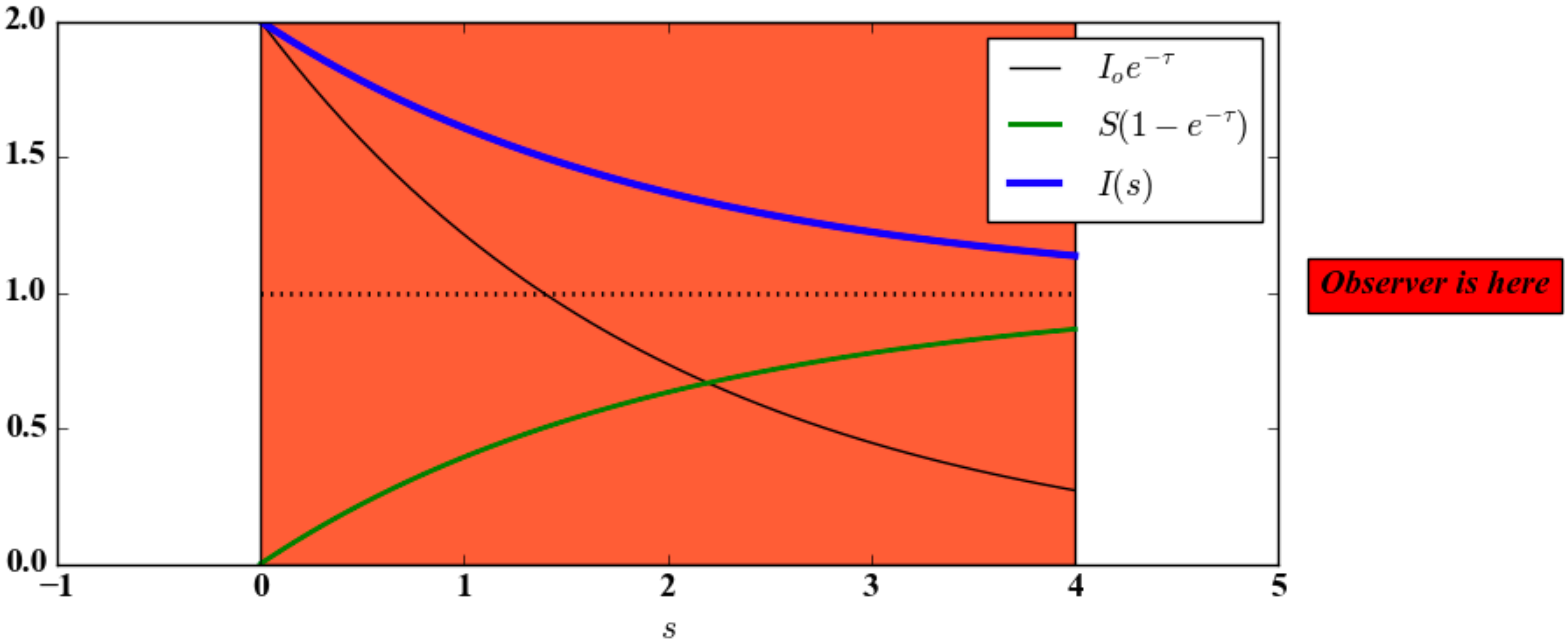
$$I(\tau(s)) = I_o\, \mathrm{e}^{\tau(s)-\tau_o} + S\left[1 - \mathrm{e}^{\tau(s)-\tau_o}\right]$$

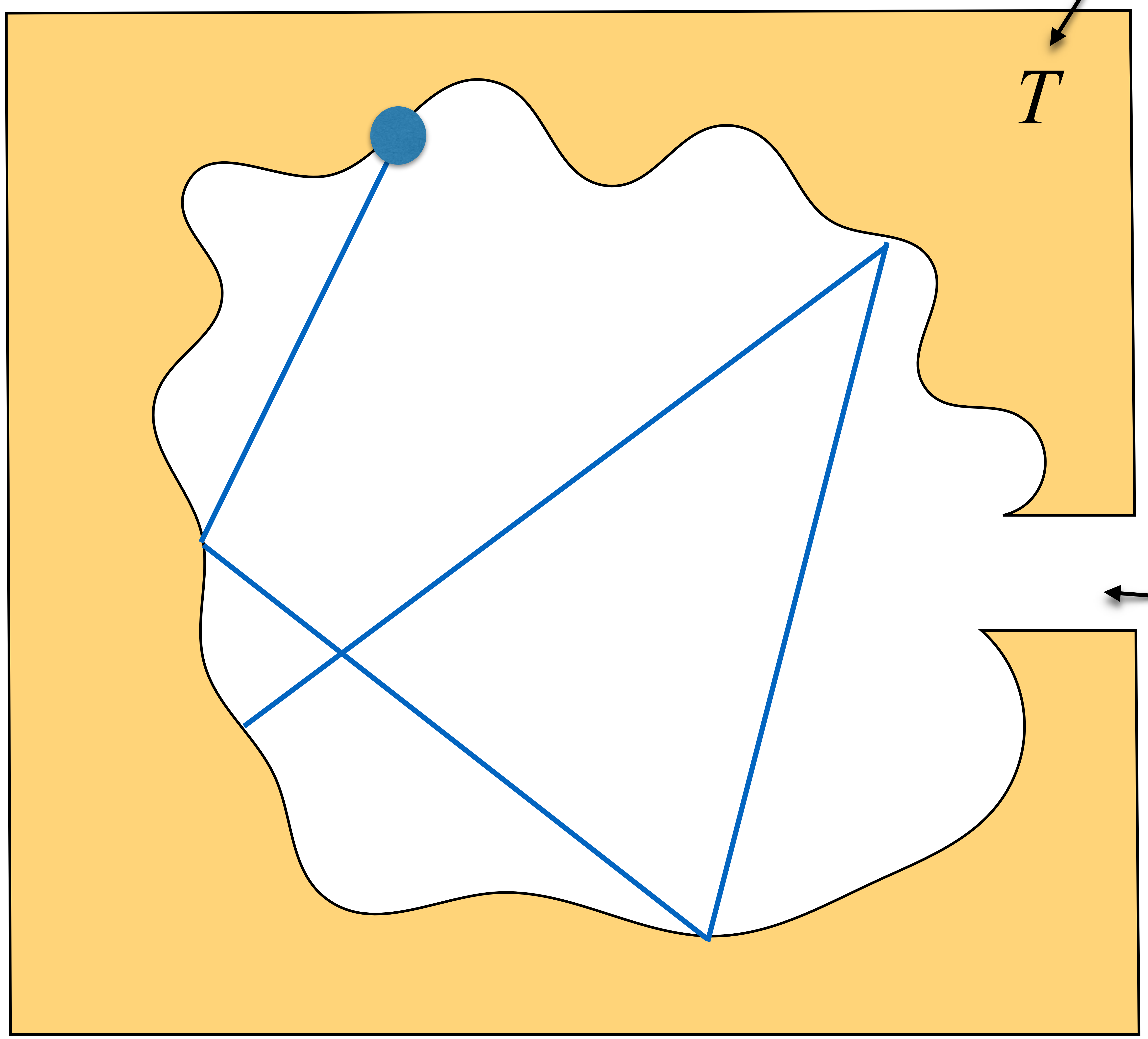
Change in intensity



Constant values

$$I(\tau(s)) = I_o\, \mathrm{e}^{\tau(s)-\tau_o} + S\left[1 - \mathrm{e}^{\tau(s)-\tau_o}\right]$$

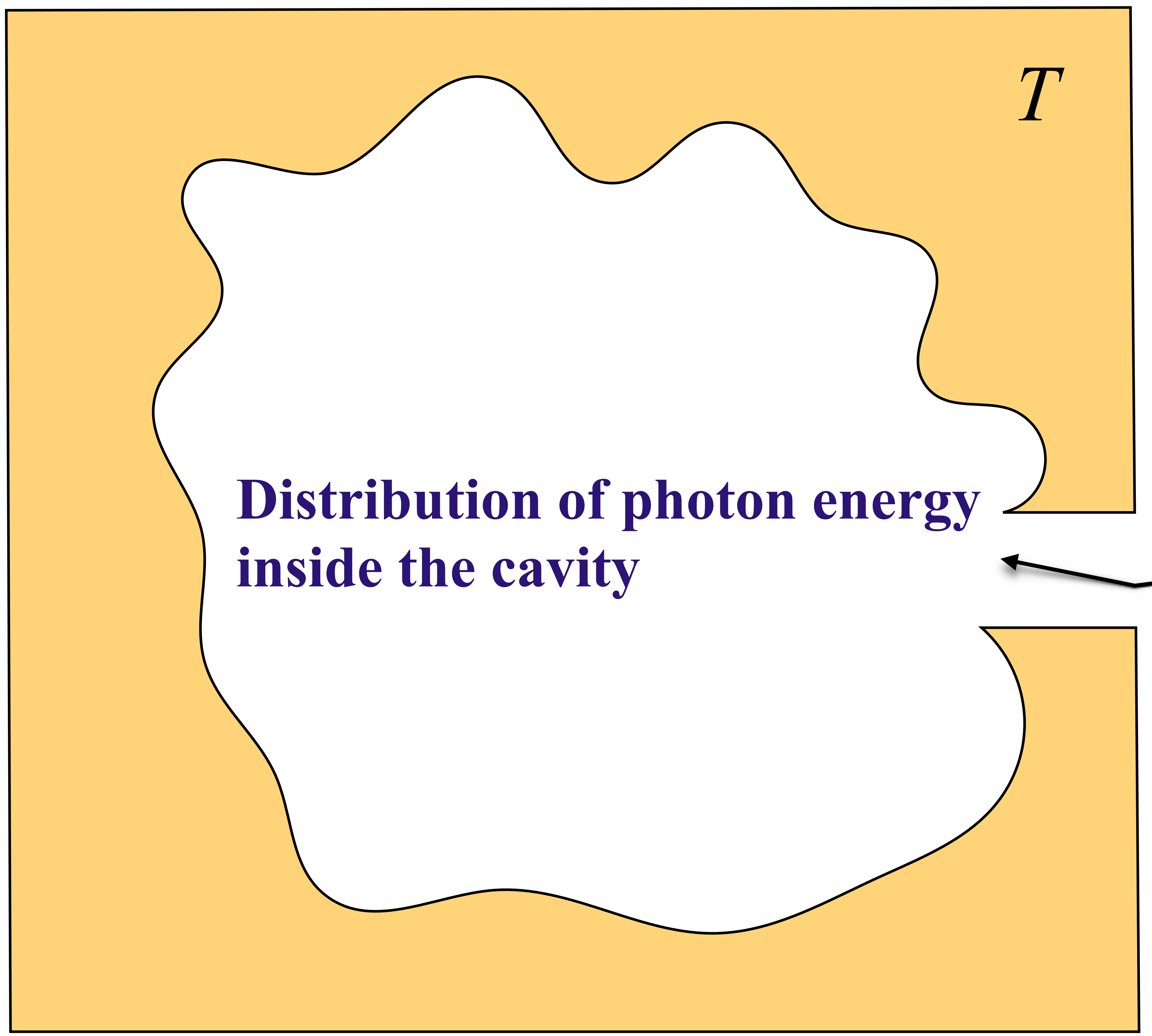




Enclosure maintained at a fixed temperature

Very small opening
(so that no significant energy is lost if a few photons managed to escape)

No <u>net</u> exchange of energy between the matter and the radiation field

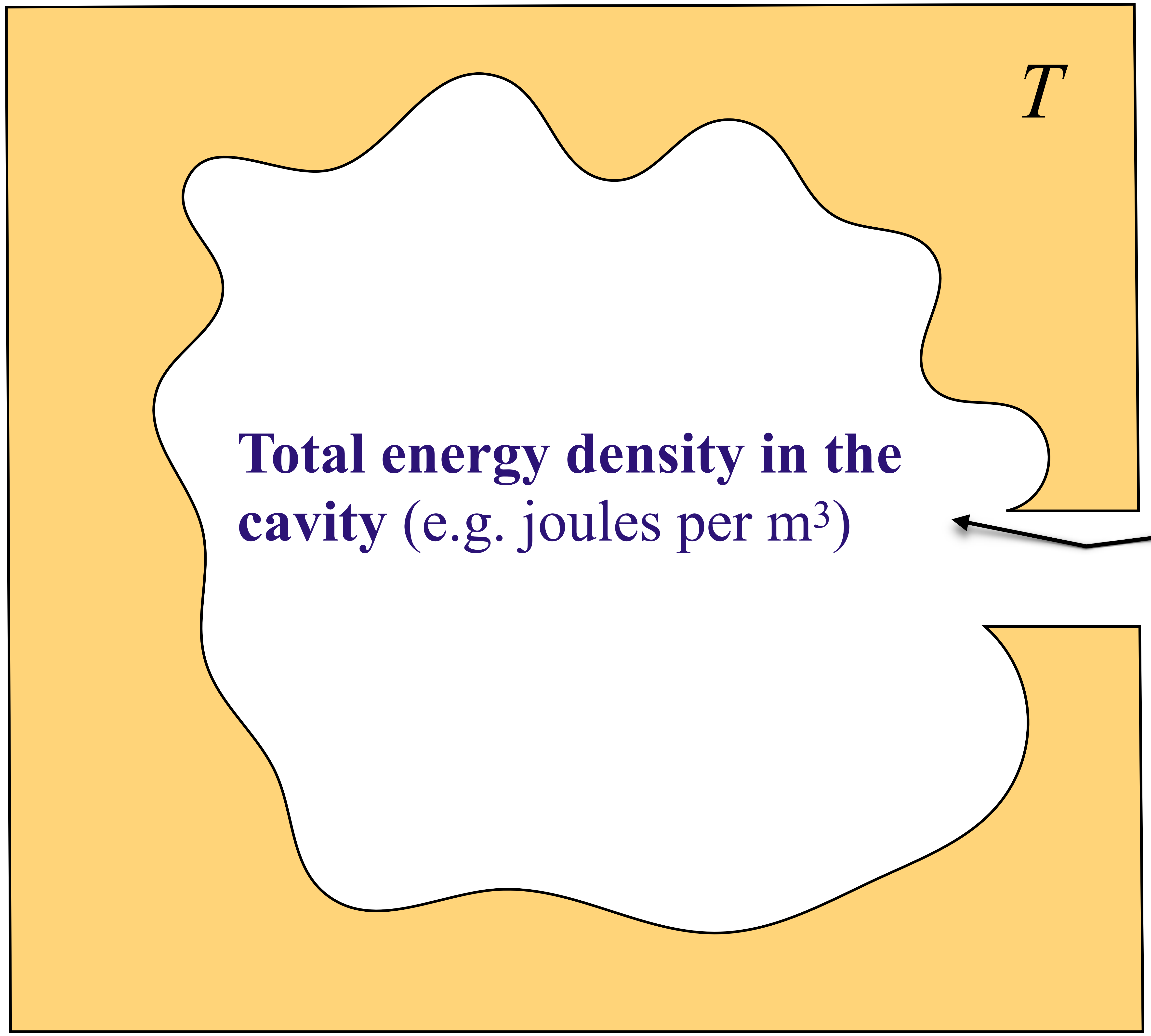


Energy density (*u*) per unit of photon energy (*E*)

e.g. joules per m^3 per keV

$$u(E) = \frac{8\pi}{(hc)^3}\frac{E^3}{e^{E/kT}-1}$$

(Undergrad Thermal Physics textbook by Schroeder, Sec 7.4)



$$U = \frac{8\pi}{(hc)^3}\int_0^\infty \frac{E^3}{e^{E/kT}-1}$$

$$= \frac{4}{c}\boxed{\frac{2\pi^4 k^4}{15h^3c^2}}T^4$$

Stefan-Boltzmann constant σ

$$U = \frac{4\sigma}{c}T^4$$

Energy density (*u*)
per unit of photon energy (*E*)

e.g. joules per m³ per keV

$$\int_0^\infty u(E)\ dE$$

$$= \frac{4\sigma}{c}T^4$$

Energy density (*u*)
per unit of photon frequency (*ν*)

e.g. joules per m³ per Hz

$$\int_0^\infty u(\nu)\ d\nu$$

$$= \frac{4\sigma}{c}T^4$$

Energy density (*u*)
per unit of photon wavelength (*λ*)

e.g. joules per m³ per nm

$$\int_\infty^0 u(\lambda)\ d\lambda$$

$$= \frac{4\sigma}{c}T^4$$

$$u(E)dE = u(\nu)d\nu = -u(\lambda)d\lambda$$

Energy density (u)
per unit of photon energy (E)

e.g. joules per m³ per keV

Energy density (u)
per unit of photon frequency (ν)

e.g. joules per m³ per Hz

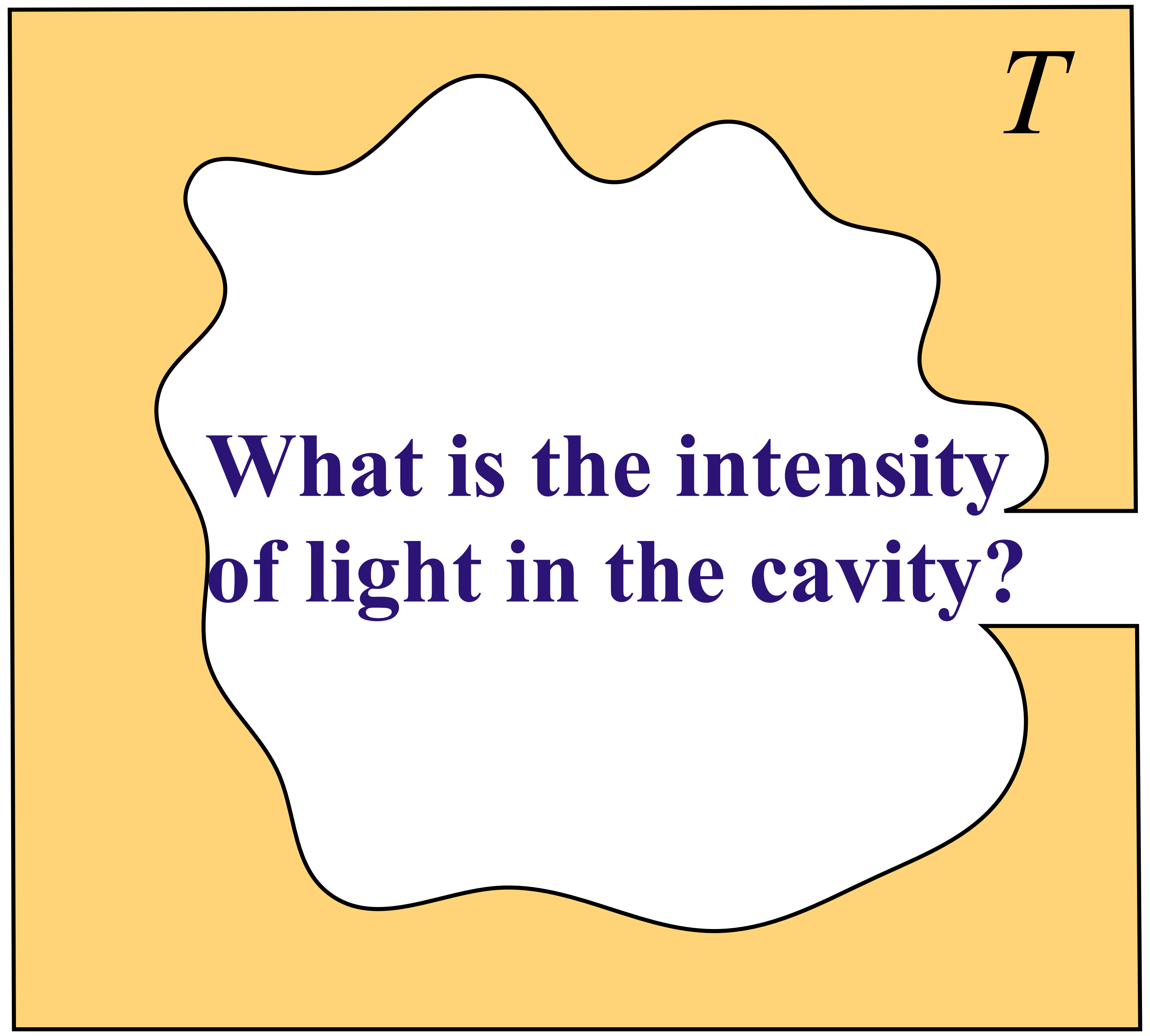
$$u(E)dE = u(\nu)d\nu$$

$$E = h\nu \qquad dE = hd\nu$$

$$\frac{dE}{d\nu} = h$$

$$u(\nu) = u(E)\frac{dE}{d\nu}$$

$$= \underbrace{\frac{8\pi}{(hc)^3}\frac{(\overset{(h\nu)}{E})^3}{e^{(\underset{(h\nu)}{E})/kT}-1}}_{\text{Still has units of energy}} \qquad \underbrace{\overset{h}{\frac{dE}{d\nu}}}_{\text{Units of energy per frequency}}$$



Energy density (u)
per unit of photon frequency (ν)
e.g. joules per m³ per Hz

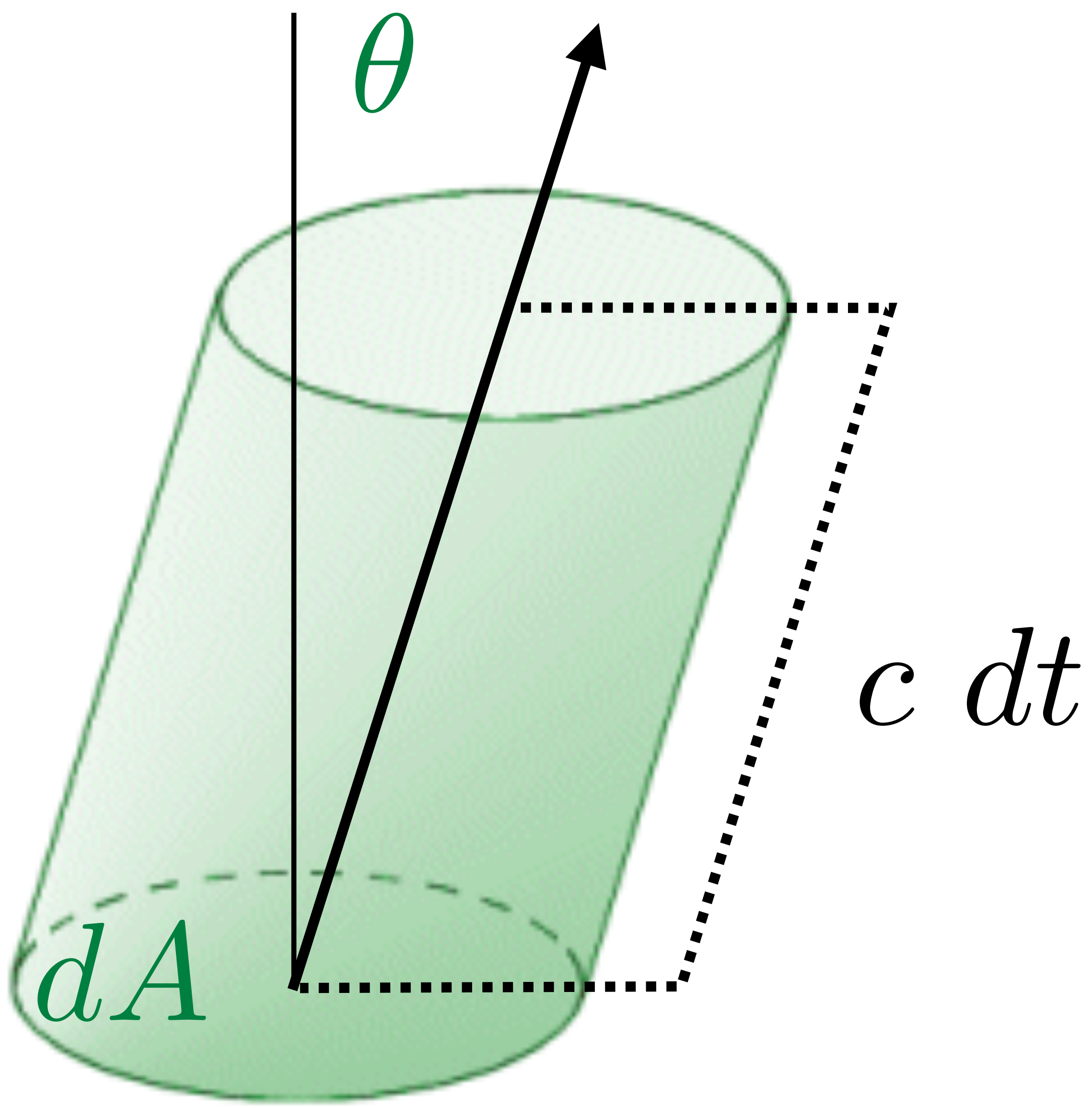
$$u(\nu) = \frac{8\pi h\nu^3}{c^3} \frac{1}{e^{h\nu/kT} - 1}$$

$$E = \int I_\nu \; d\nu \; dt \; dA \cos(\theta) \; d\Omega$$

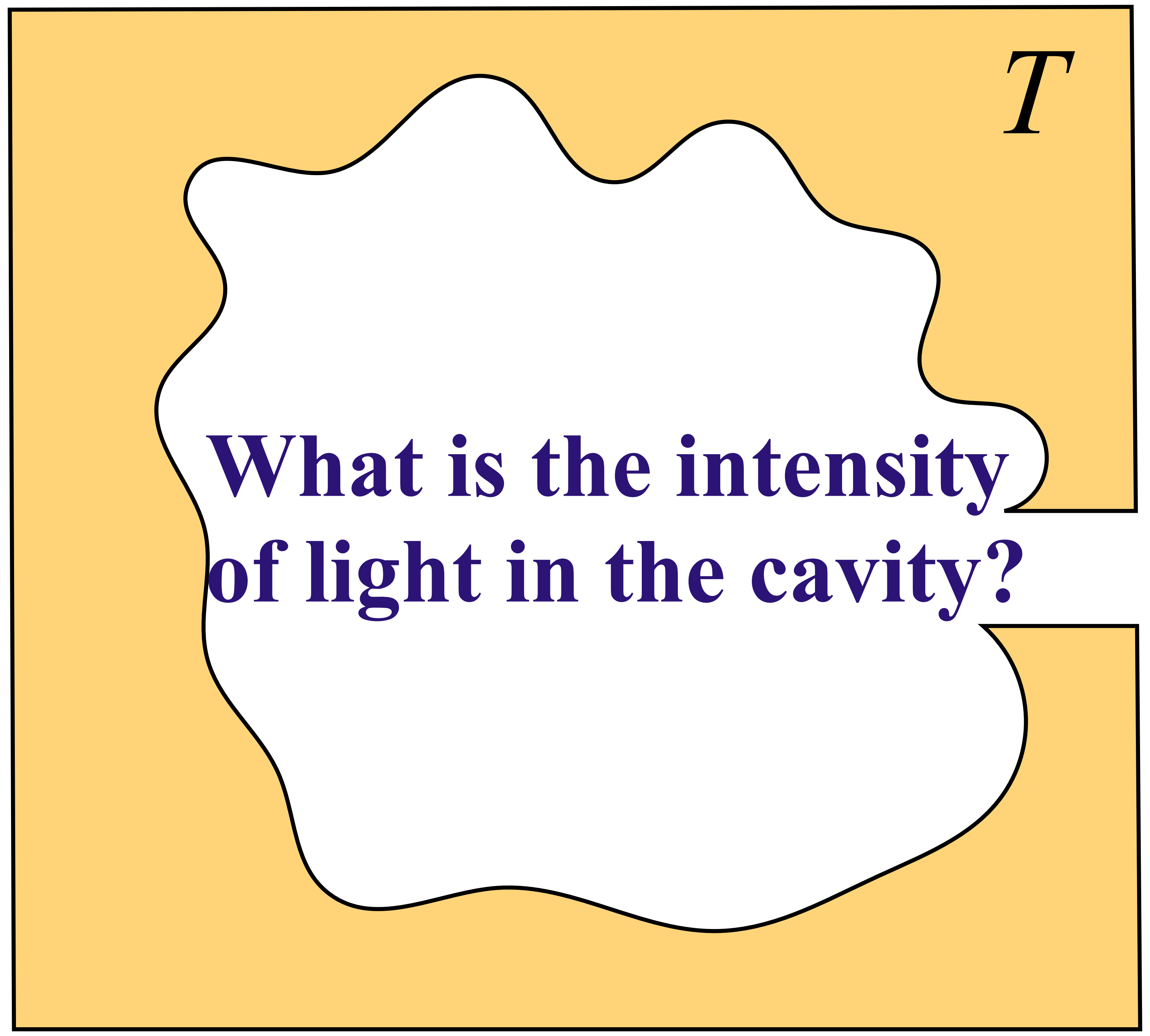
$$u(\nu) = \frac{E_{\text{Transported by the light rays}}}{d\nu dV}$$

$$= \int I_\nu \; \frac{d\nu \; dt \; dA \cos(\theta)}{d\nu \; c \; dt \; dA \cos(\theta)} \; d\Omega$$

$$= \frac{1}{c} \underbrace{\int I_\nu \; d\Omega}_{4\pi \; J_\nu}$$



$$dV = c \; dt \; dA \cos(\theta)$$

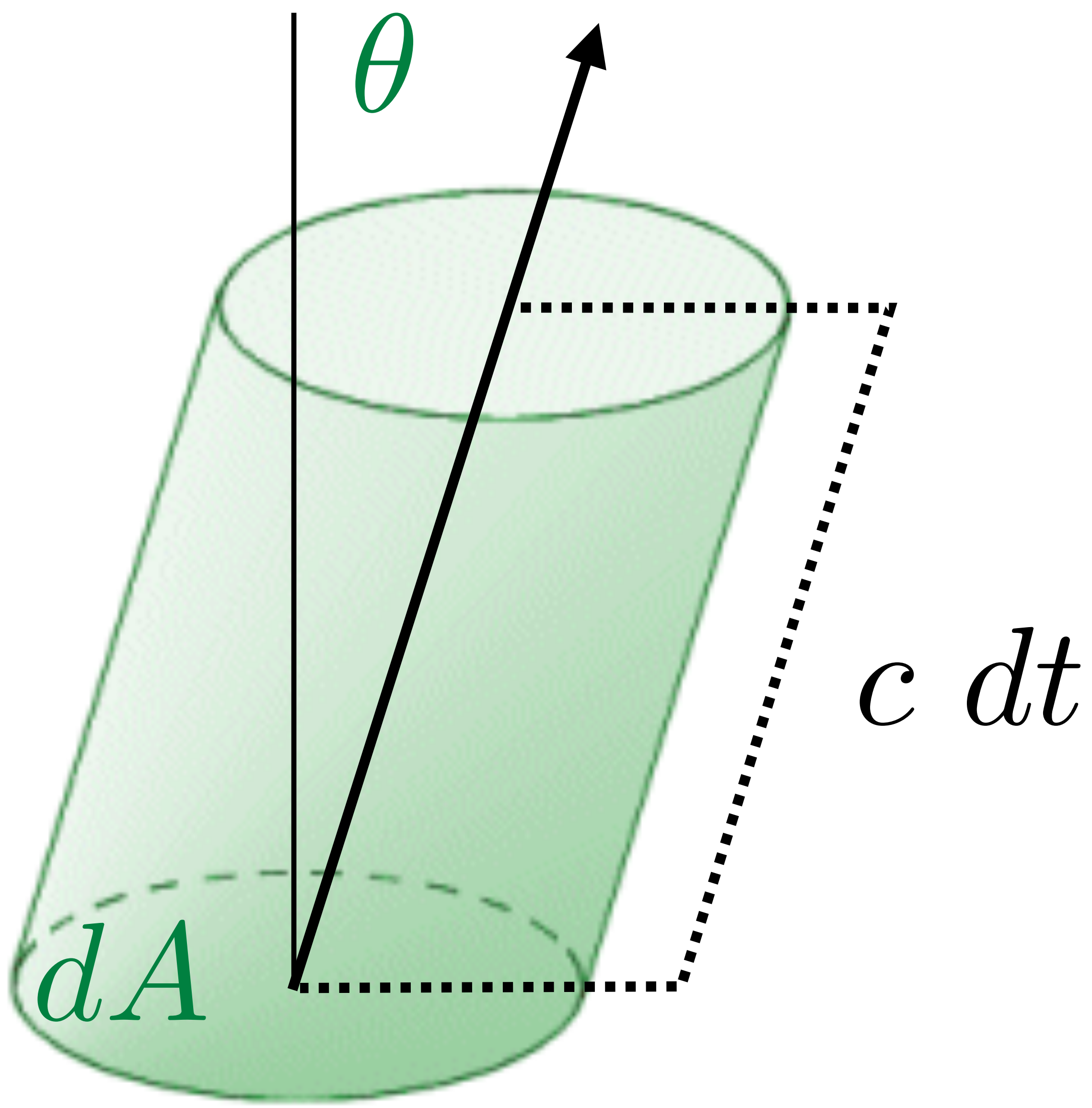


Energy density (*u*)
per unit of photon frequency (*ν*)
e.g. joules per m^3 per Hz

if radiation is isotropic

$$u(\nu) = \frac{8\pi h\nu^3}{c^3}\frac{1}{e^{h\nu/kT}-1} = \frac{4\pi}{c}J_\nu = \frac{4\pi}{c}I_\nu$$

$$I_{\nu,BB} = B_\nu(T) = \frac{c}{4\pi}u(\nu) = \frac{2h}{c}\frac{\nu^3}{e^{h\nu/kT}-1}$$



$$dV = c\ dt\ dA\cos(\theta)$$

One of the exam question: find B_λ
(beware: same principle as converting between $u(E)$ and $u(\nu)$)

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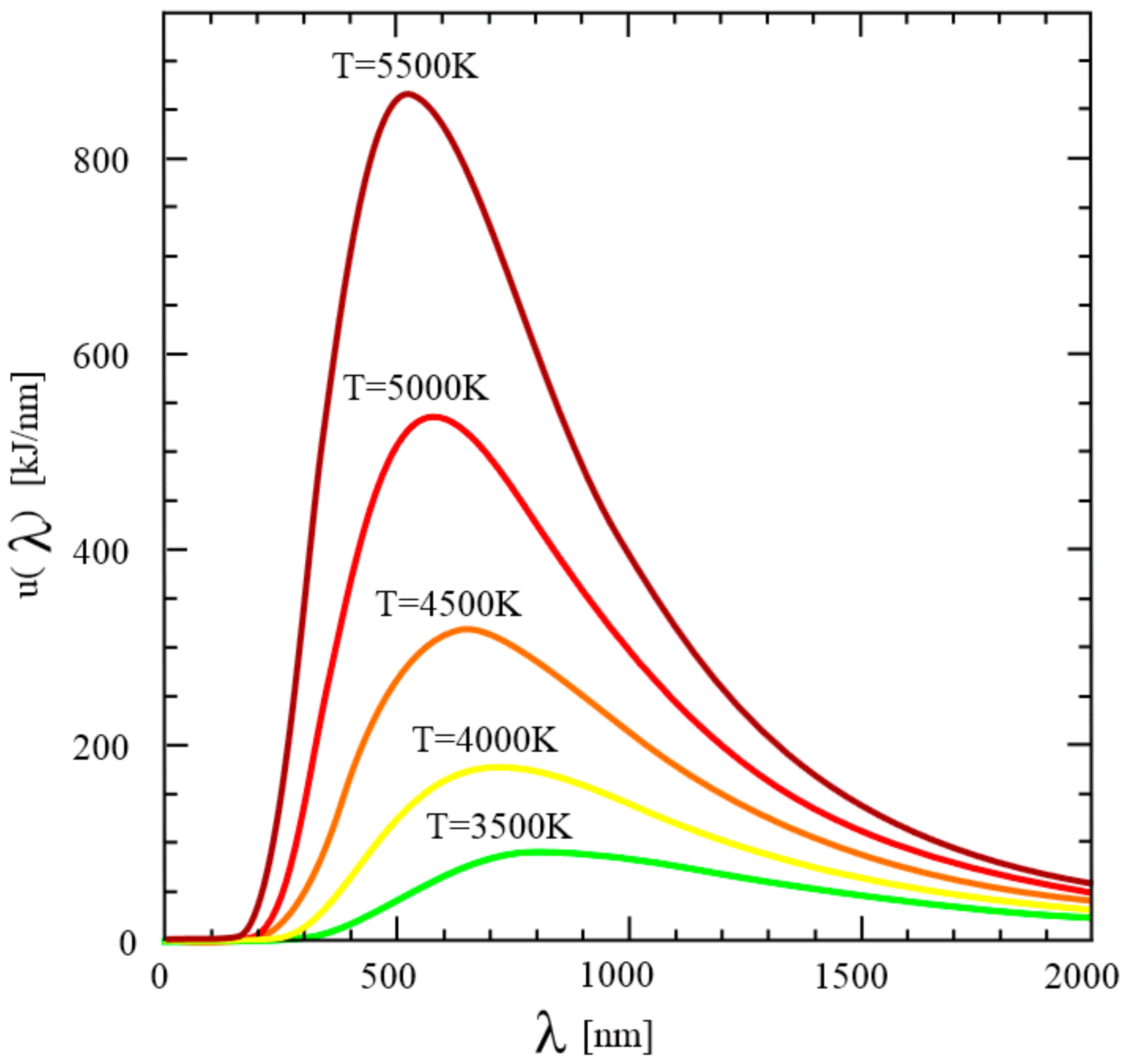
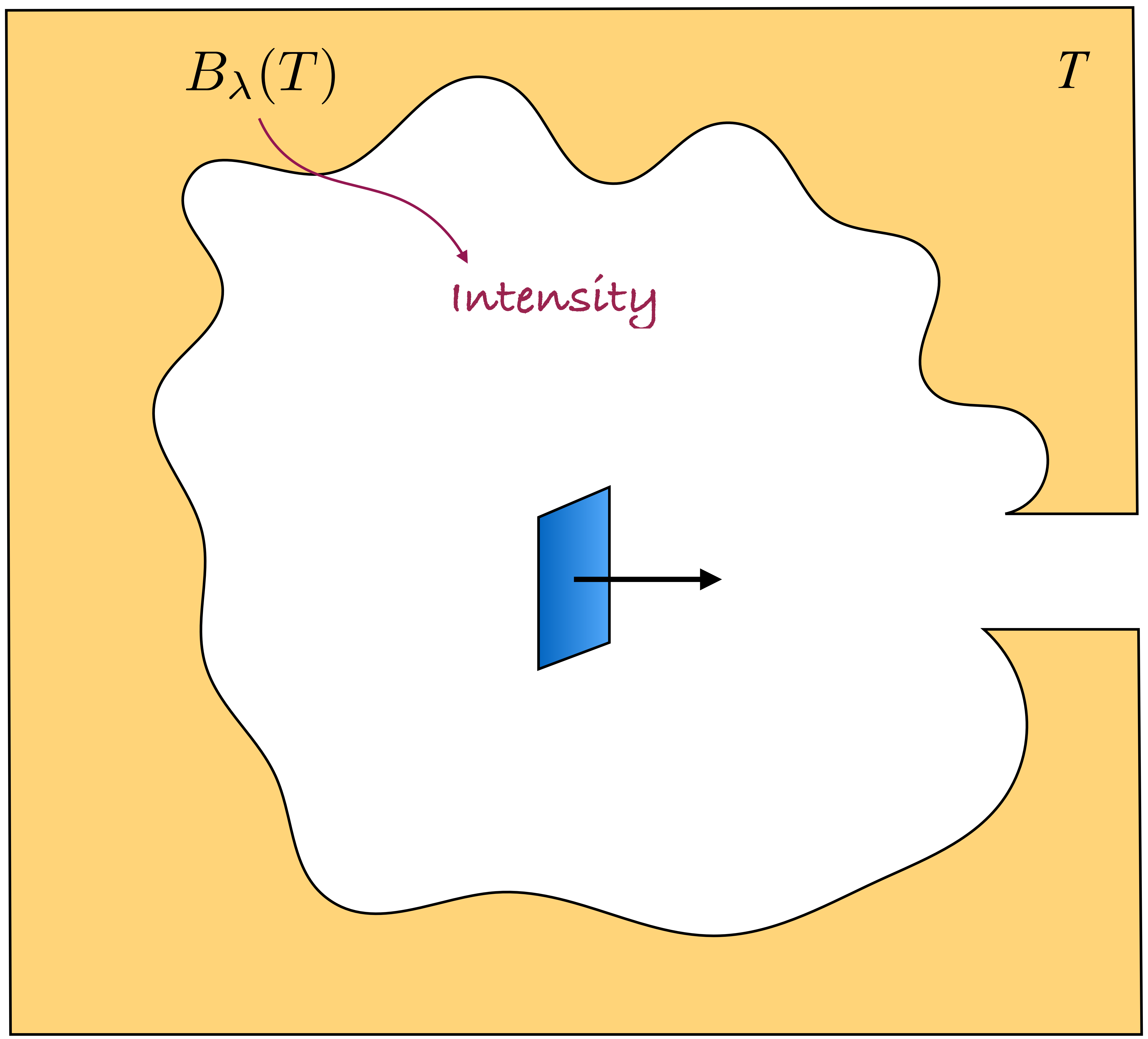
(beware: same principle as converting between $u(E)$ and $u(\nu)$)

(And it should look like that :))



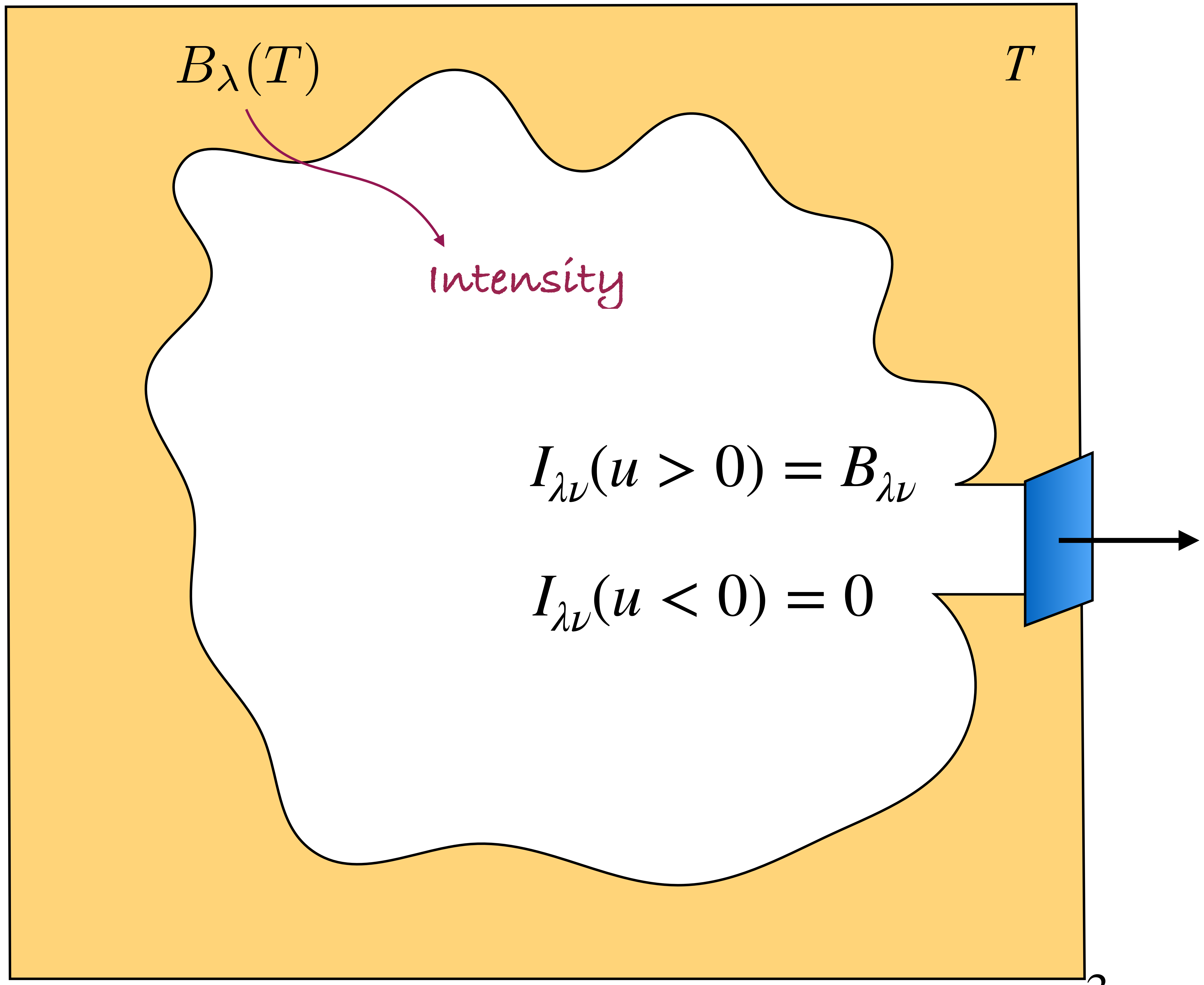
Flux through a surface in a blackbody radiation field



Isotropic radiation:

$$F = 0$$

Flux through a surface in a blackbody radiation field



$$F_{\lambda\nu} = 2\pi \int_{\cancel{-1}\ 0}^{+1} B_{\lambda\nu}\, u du$$

$$= 2\pi B_{\lambda\nu} \int_0^{+1} u du$$

$$= \pi B_{\lambda\nu}$$

$$F_{\rm tot} = \pi \int_0^\infty B_{\lambda\nu} d\nu$$

$$= \pi \int_0^\infty \frac{2h}{c^2} \frac{\nu^3}{e^{h\nu/kT} - 1} d\nu$$

$$= \frac{2\pi^5 k^4}{15 h^3 c^2} T^4$$

$$= \sigma T^4$$

$$I_{\nu,BB} = B_\nu(T) = \frac{c}{4\pi} u(\nu) = \frac{2h}{c} \frac{\nu^3}{e^{h\nu/kT} - 1}$$

Source function of a blackbody radiation field

Thermodynamical equilibrium: each photon absorbed is replaced by an emitted photon of the same wavelength. $dI_\lambda = 0$

On the board: this implies that $S_\lambda = I_\lambda$

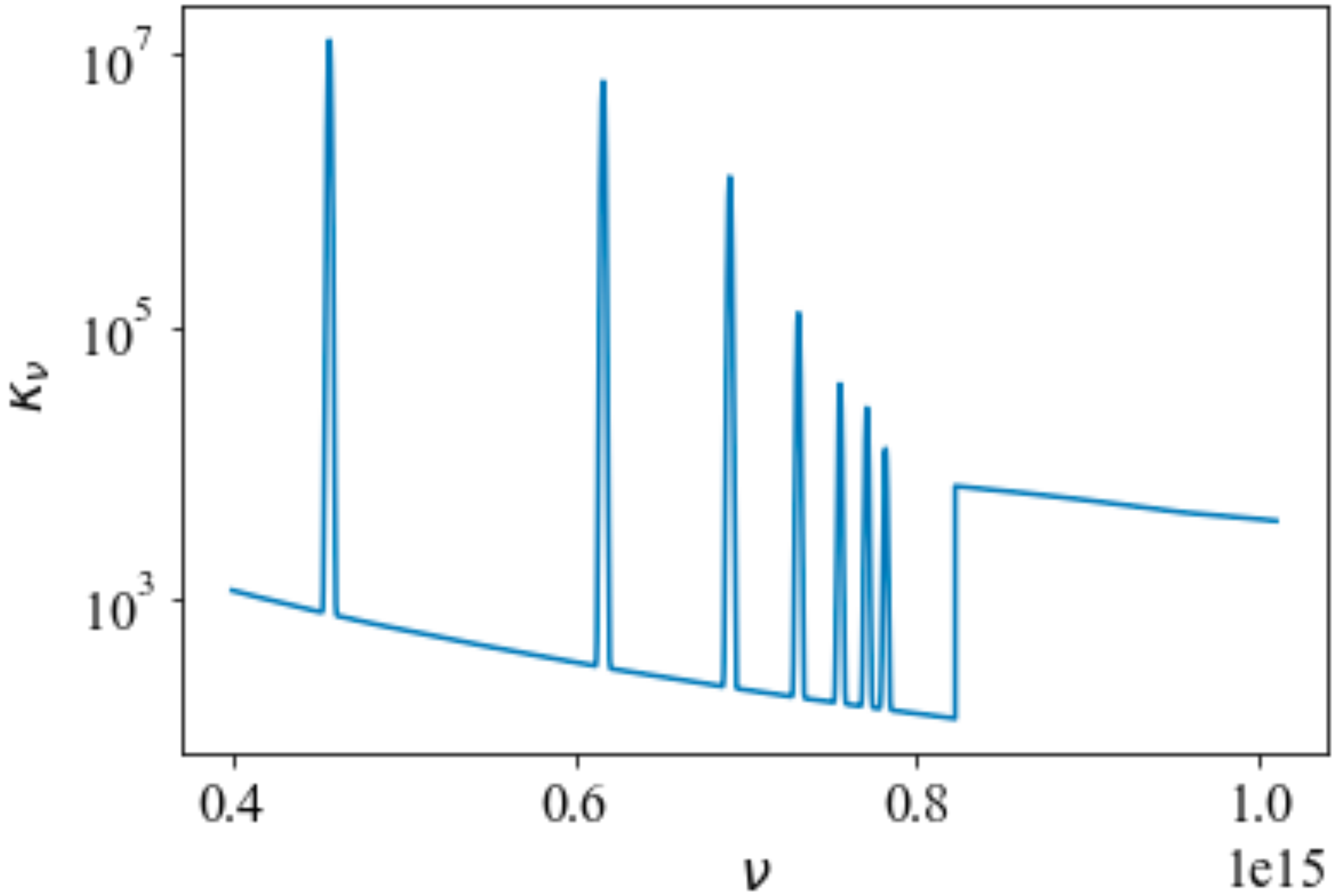
In the more general case where there is both absorption k_λ and scattering σ_λ processes

On the board: this implies that $S_\lambda = \dfrac{k_\lambda B_\lambda + \sigma_\lambda J_\lambda}{k_\lambda + \sigma_\lambda}$

The opacity wavelength dependance:

Q: what are the spikes?
Q: what is the step?

Example of pure hydrogen

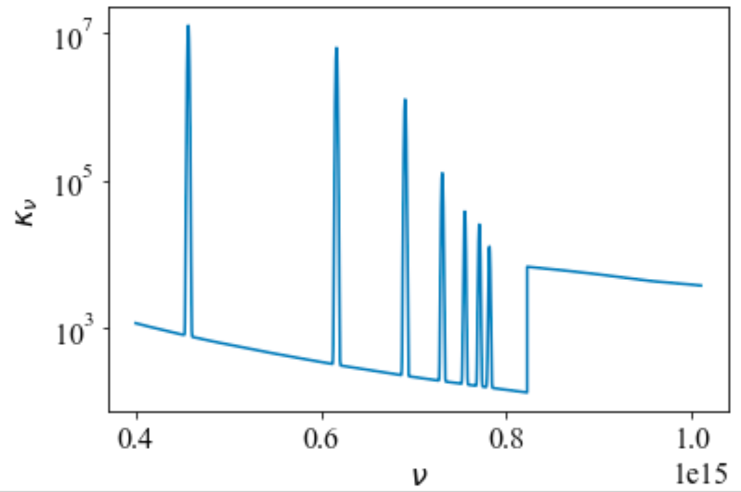


Example 1

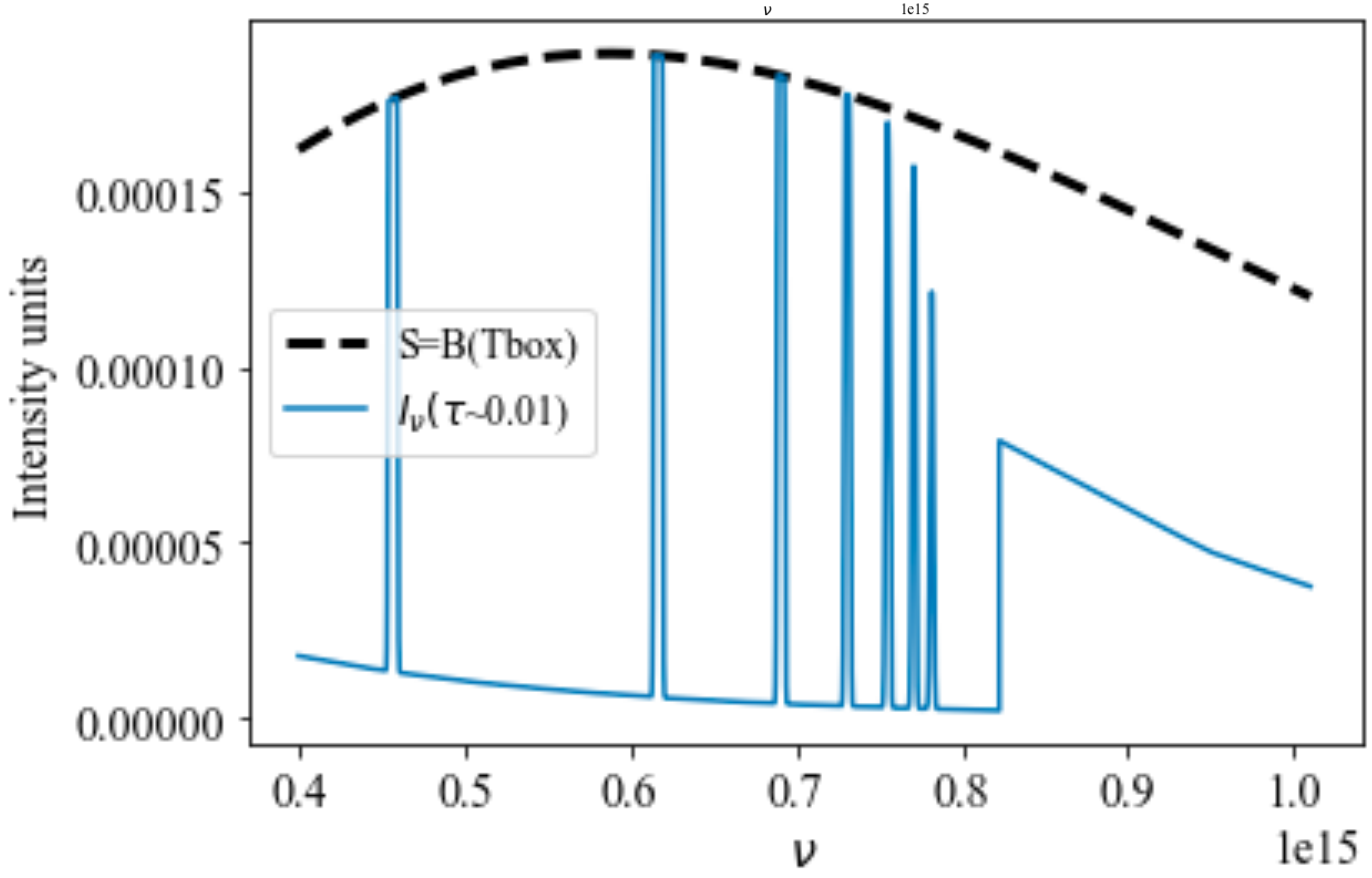
$$I(s) = I_o e^{\tau(s)-\tau_o} + S_o\left[1 - e^{\tau(s)-\tau_o}\right]$$

$$I(\mathrm{obs}) = + B_\nu(T_{\mathrm{box}})\left[1 - e^{-\tau_o}\right]$$

Box with $S_\nu = B_\nu(T_{\mathrm{box}})$



a. The optical depth of the box is small



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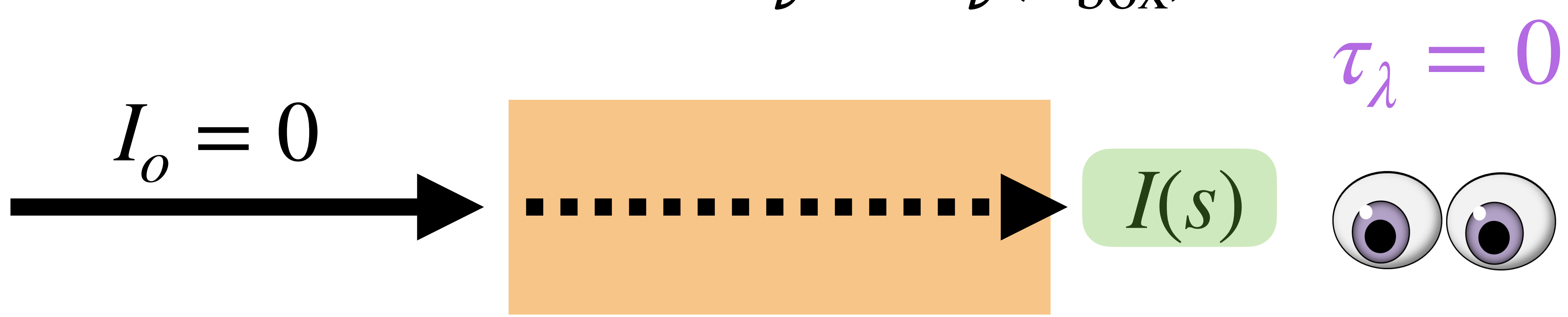
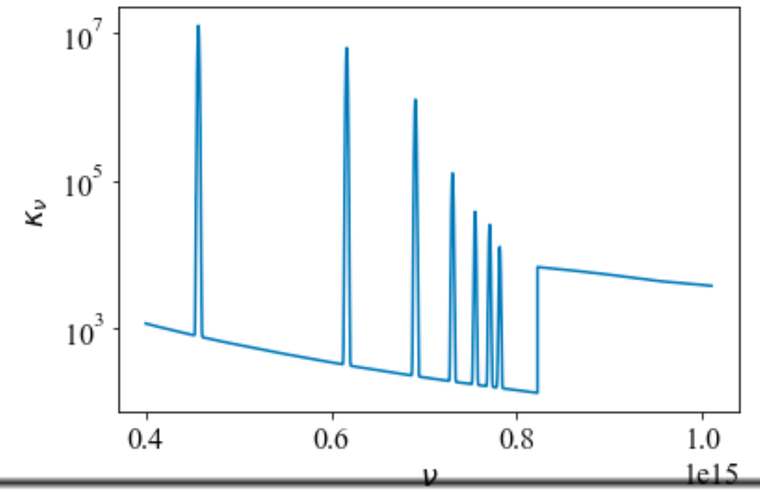
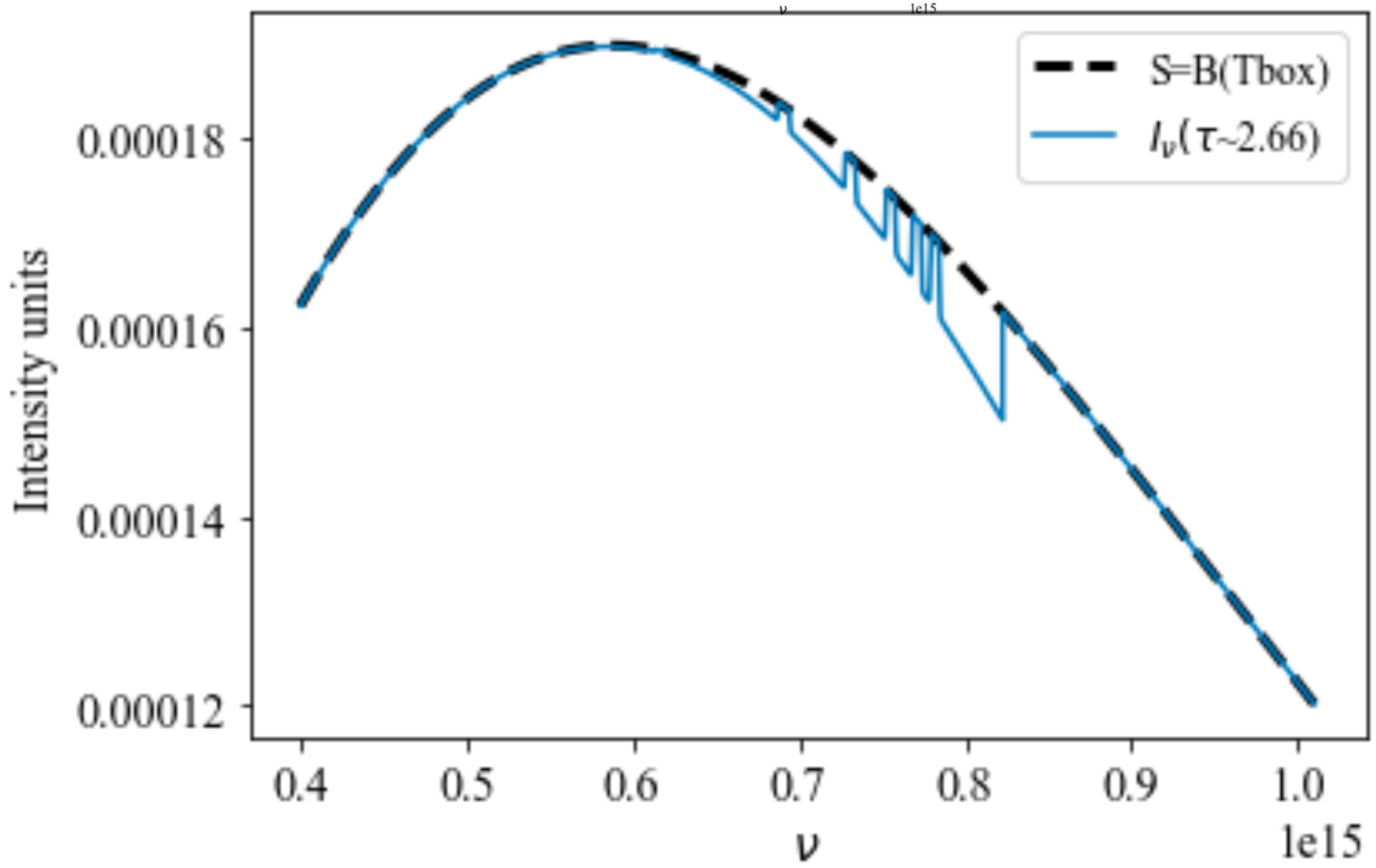




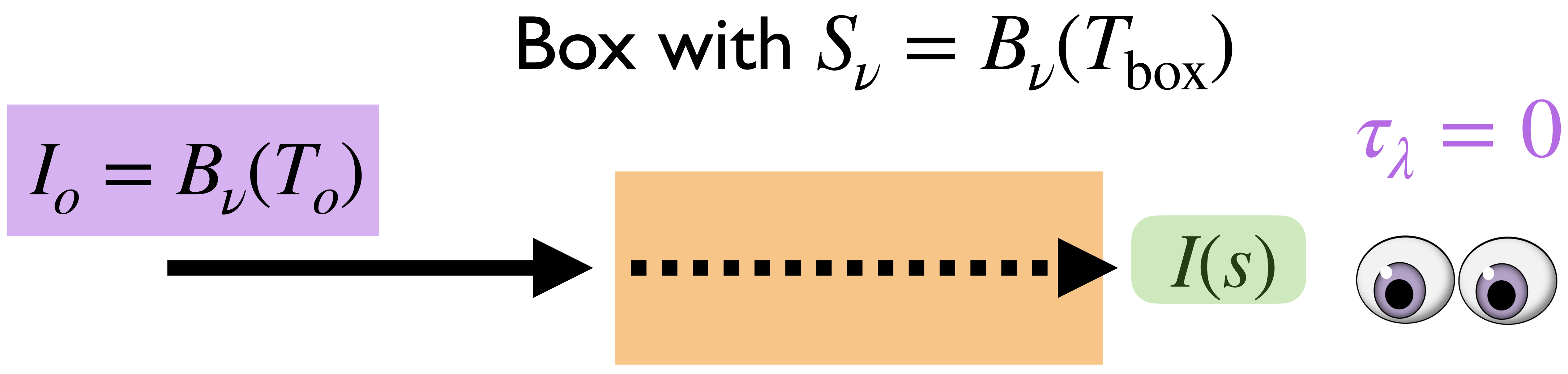
b. The optical depth of the box is large

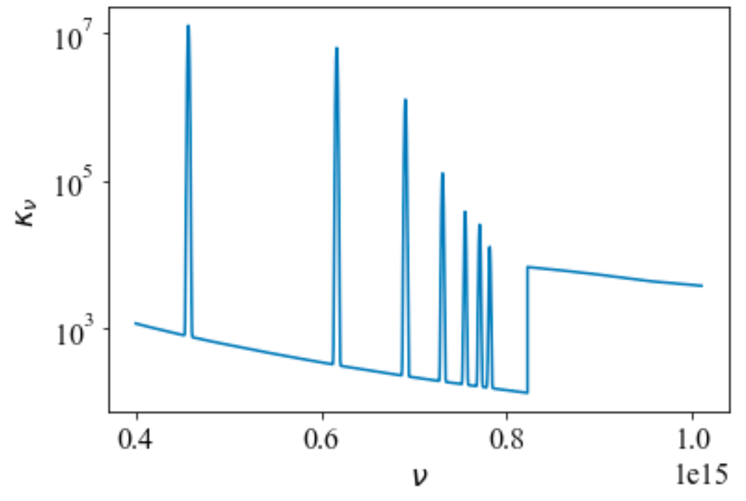


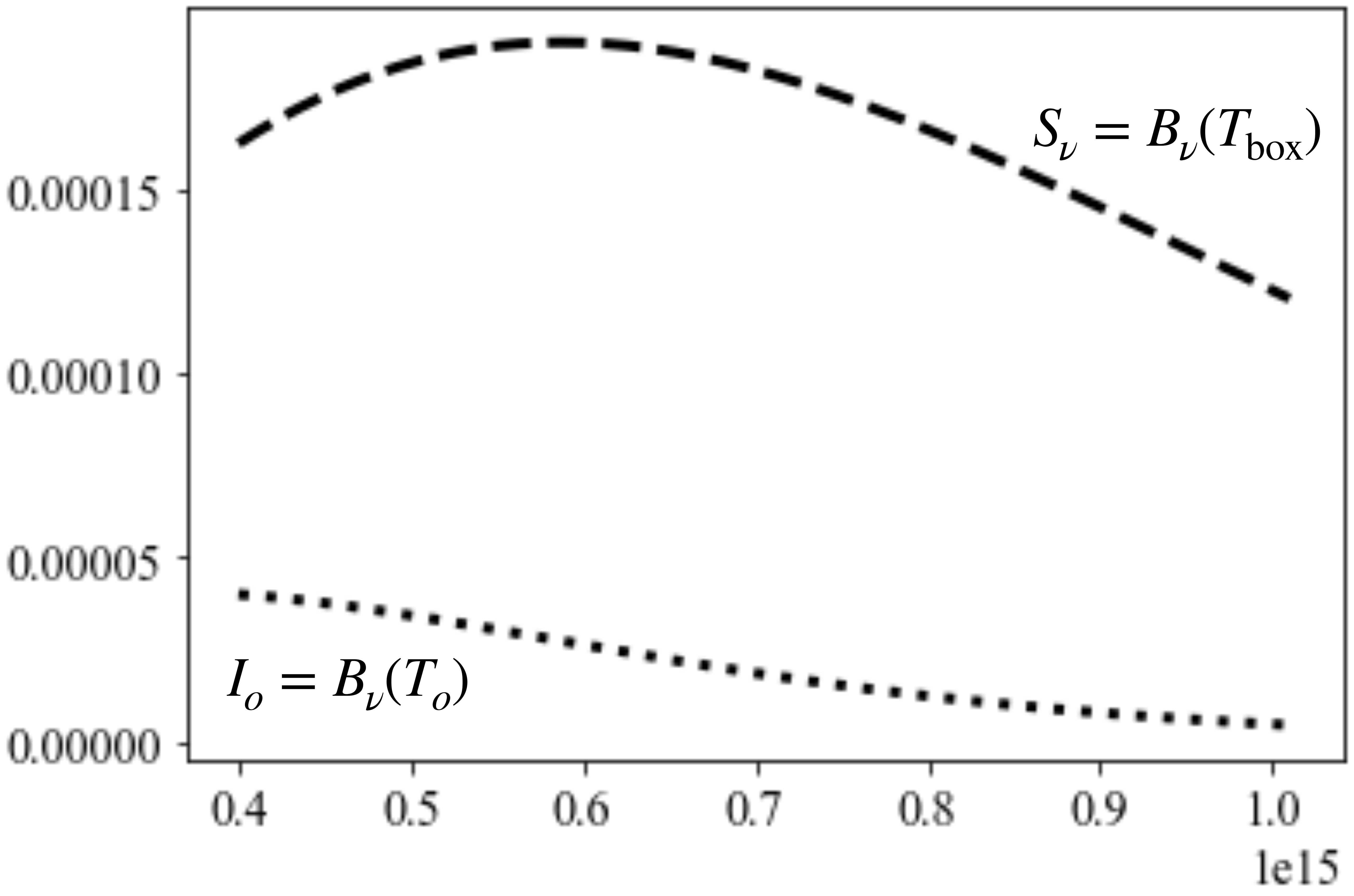
Example 2: $T_{\rm box} > T_o$

$$I(s) = I_o e^{\tau(s)-\tau_o} + S_o\left[1 - e^{\tau(s)-\tau_o}\right]$$

$$I(\text{obs}) = B_\nu(T_o)e^{-\tau_o} + B_\nu(T_{\rm box})\left[1 - e^{-\tau_o}\right]$$







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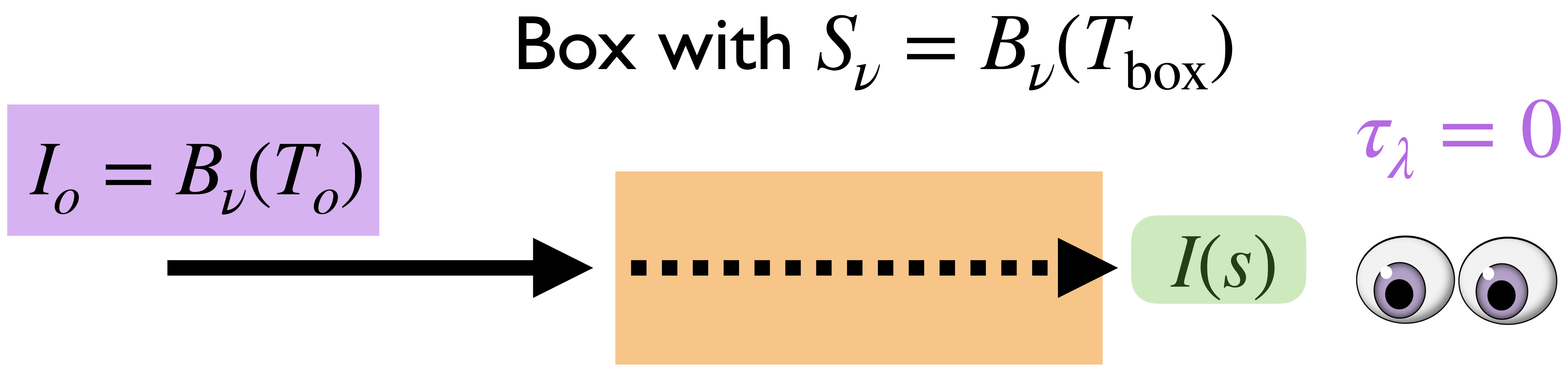

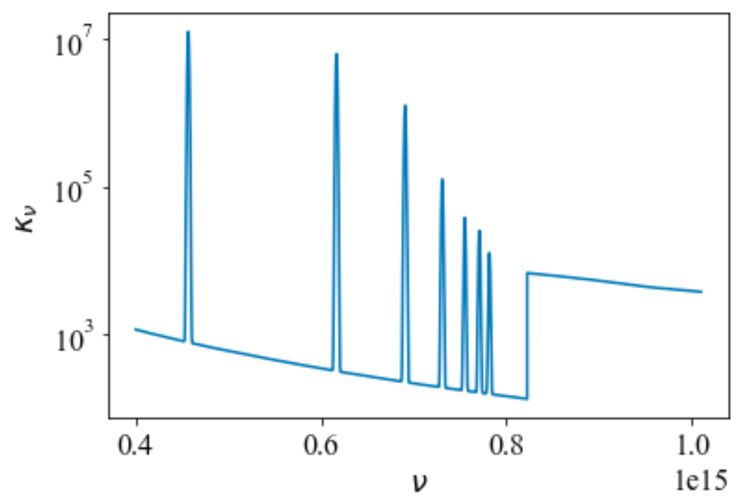

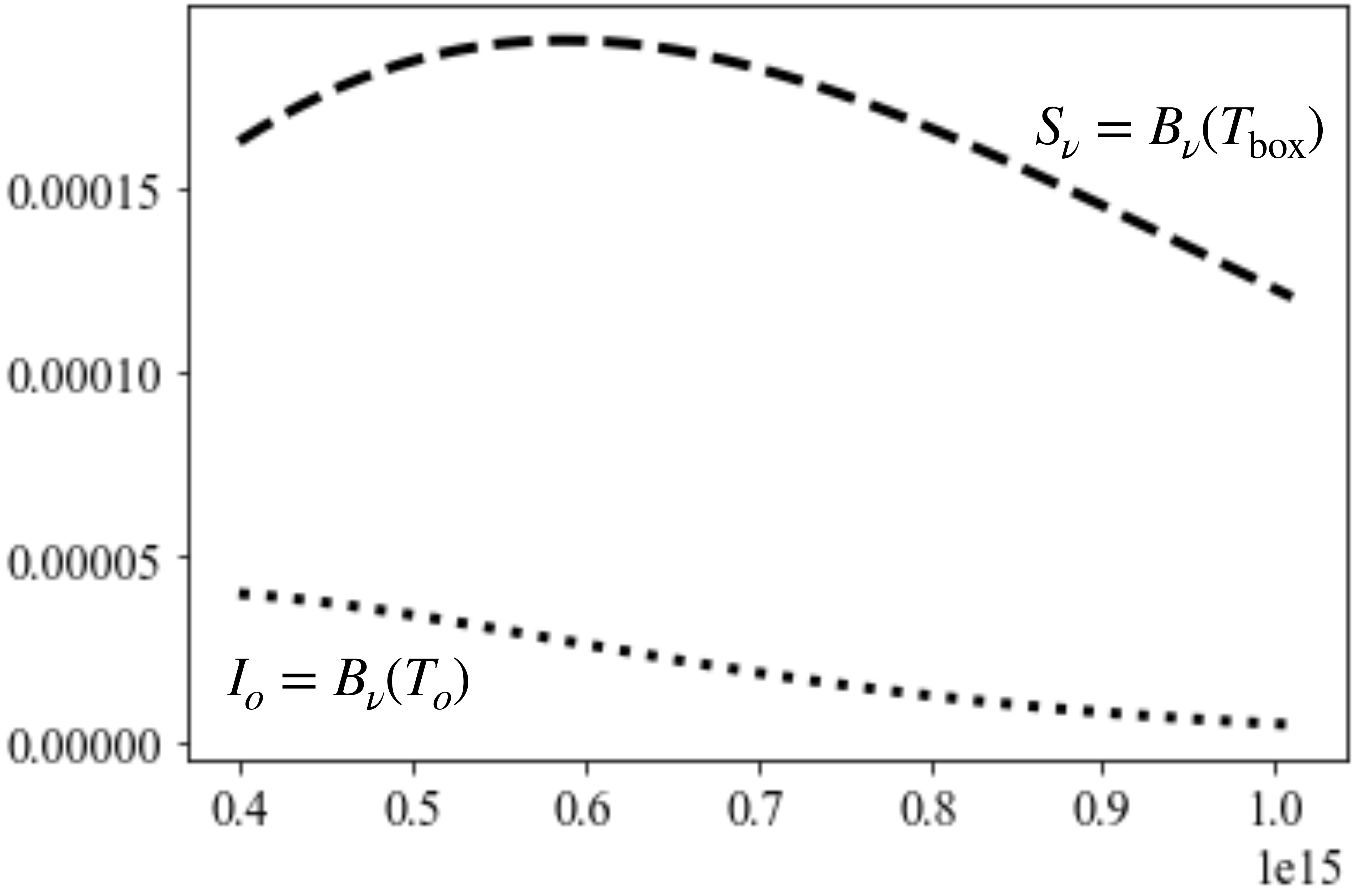

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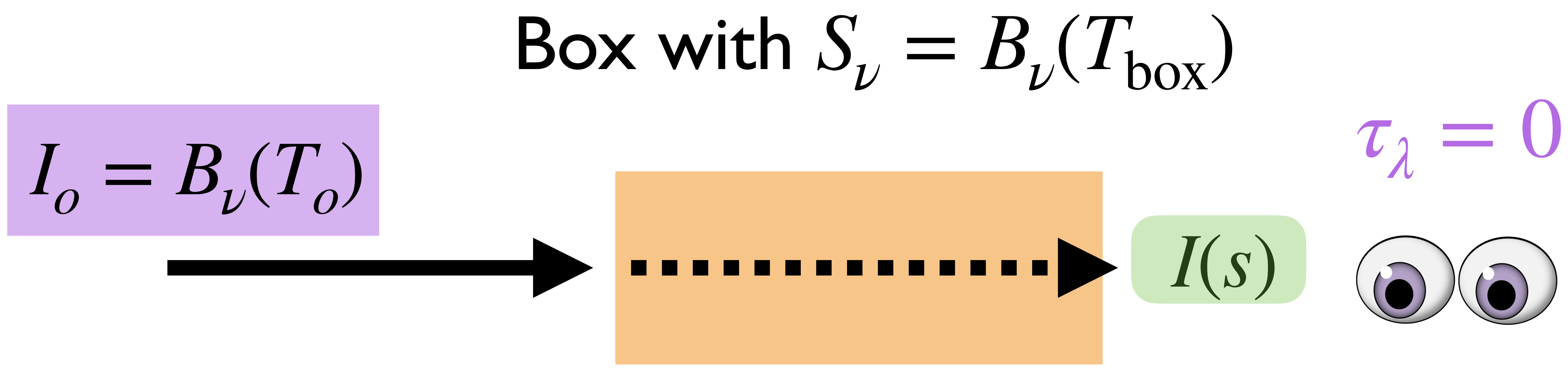

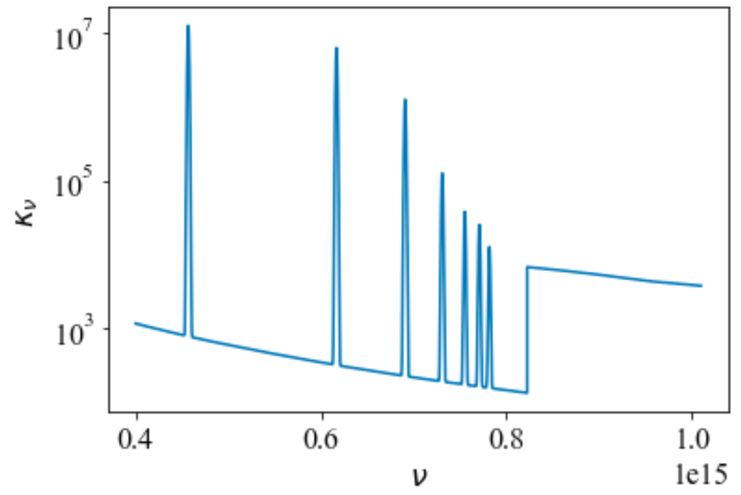

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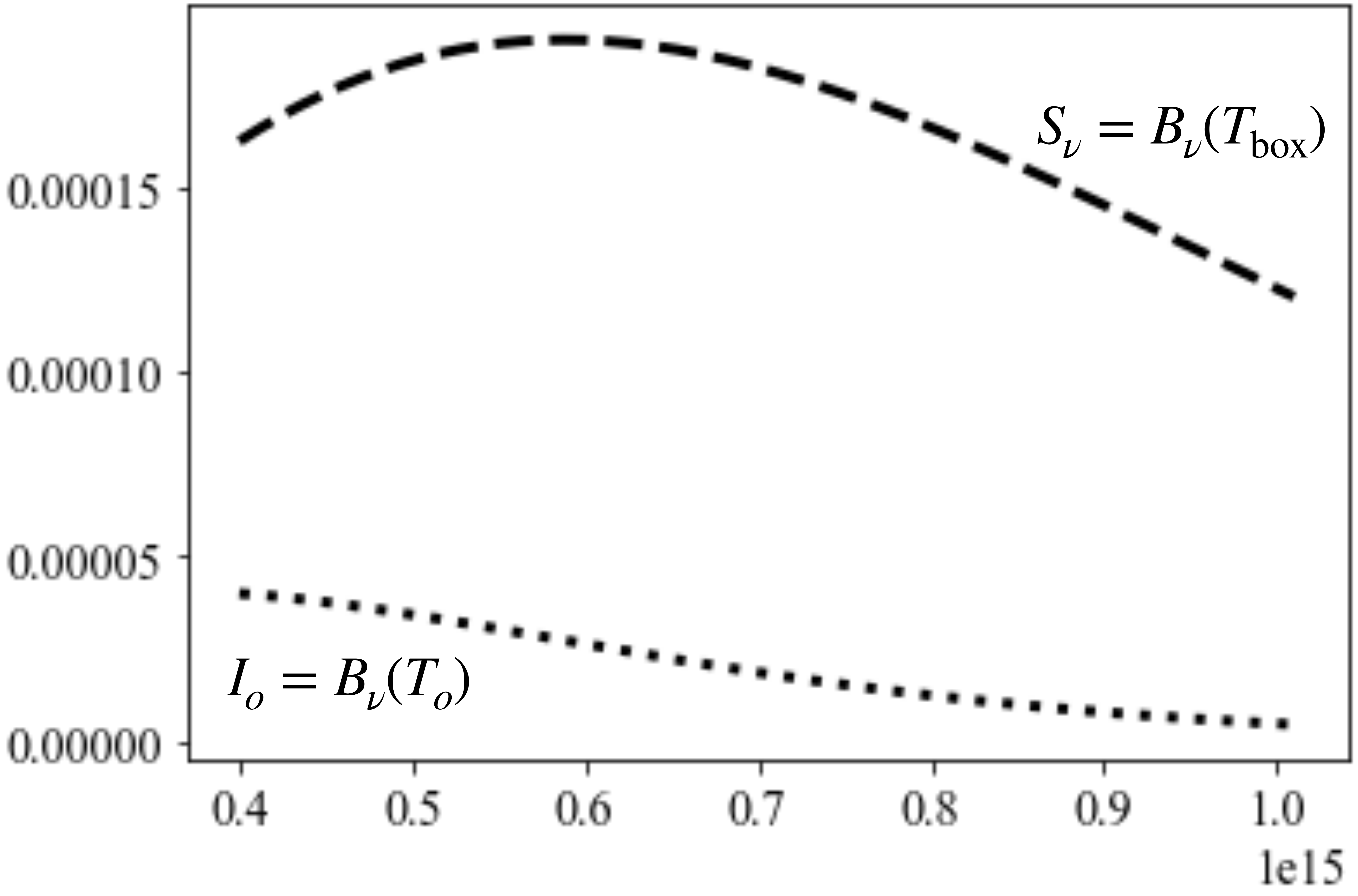

Example 3: $T_{\rm box} < T_o$

$$I(s) \quad = \quad I_o e^{\tau(s)-\tau_o} \; + \; S_o\left[1 - e^{\tau(s)-\tau_o}\right]$$

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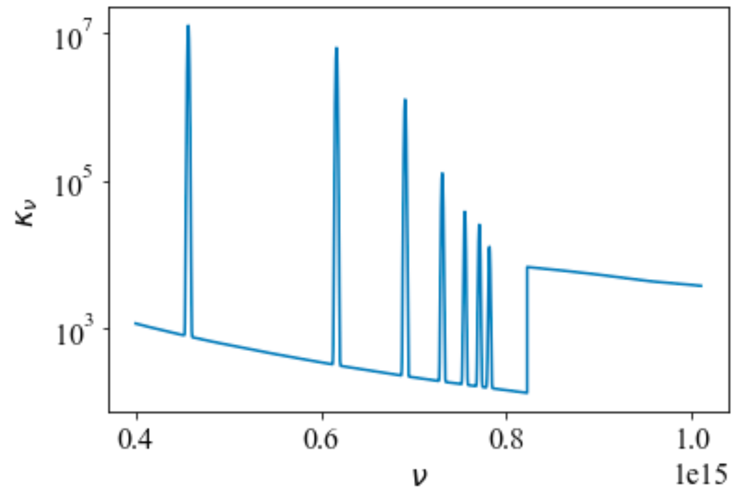

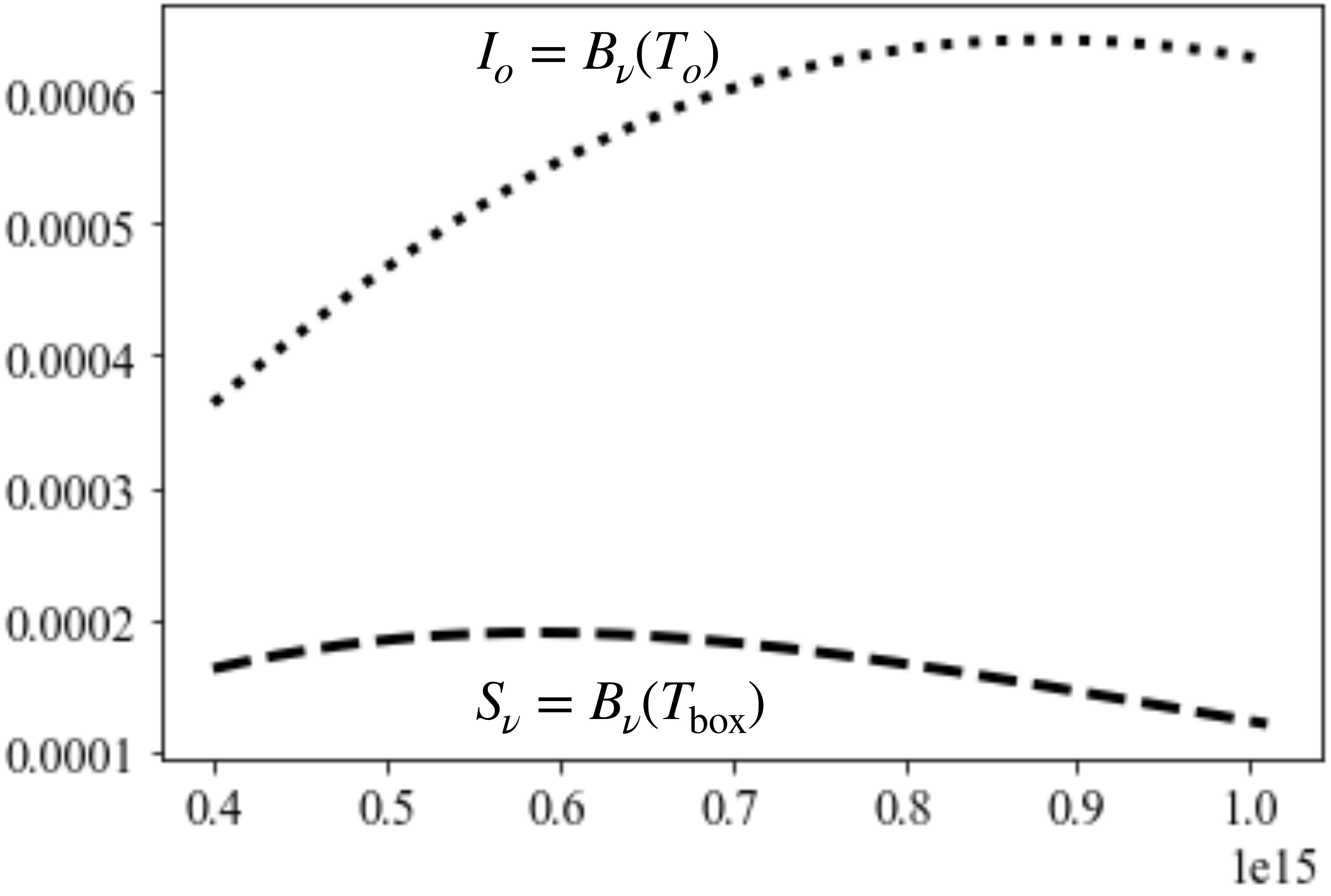

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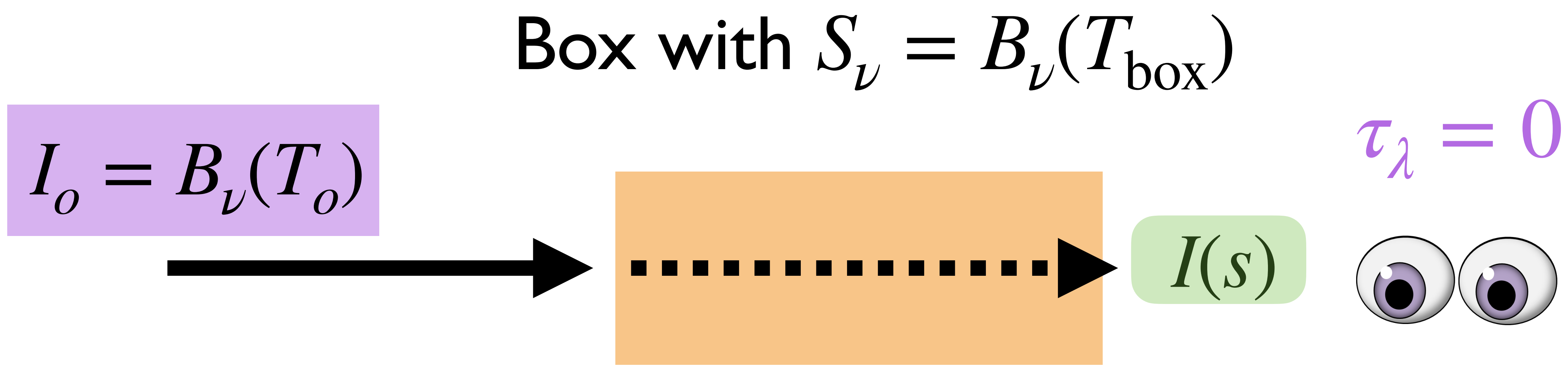

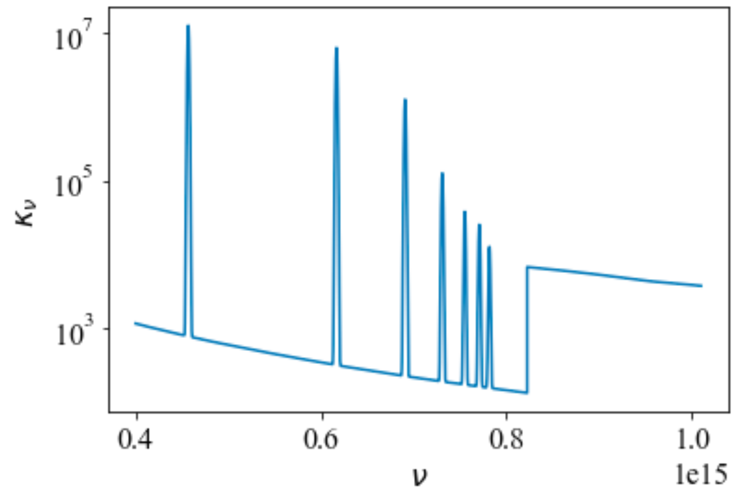

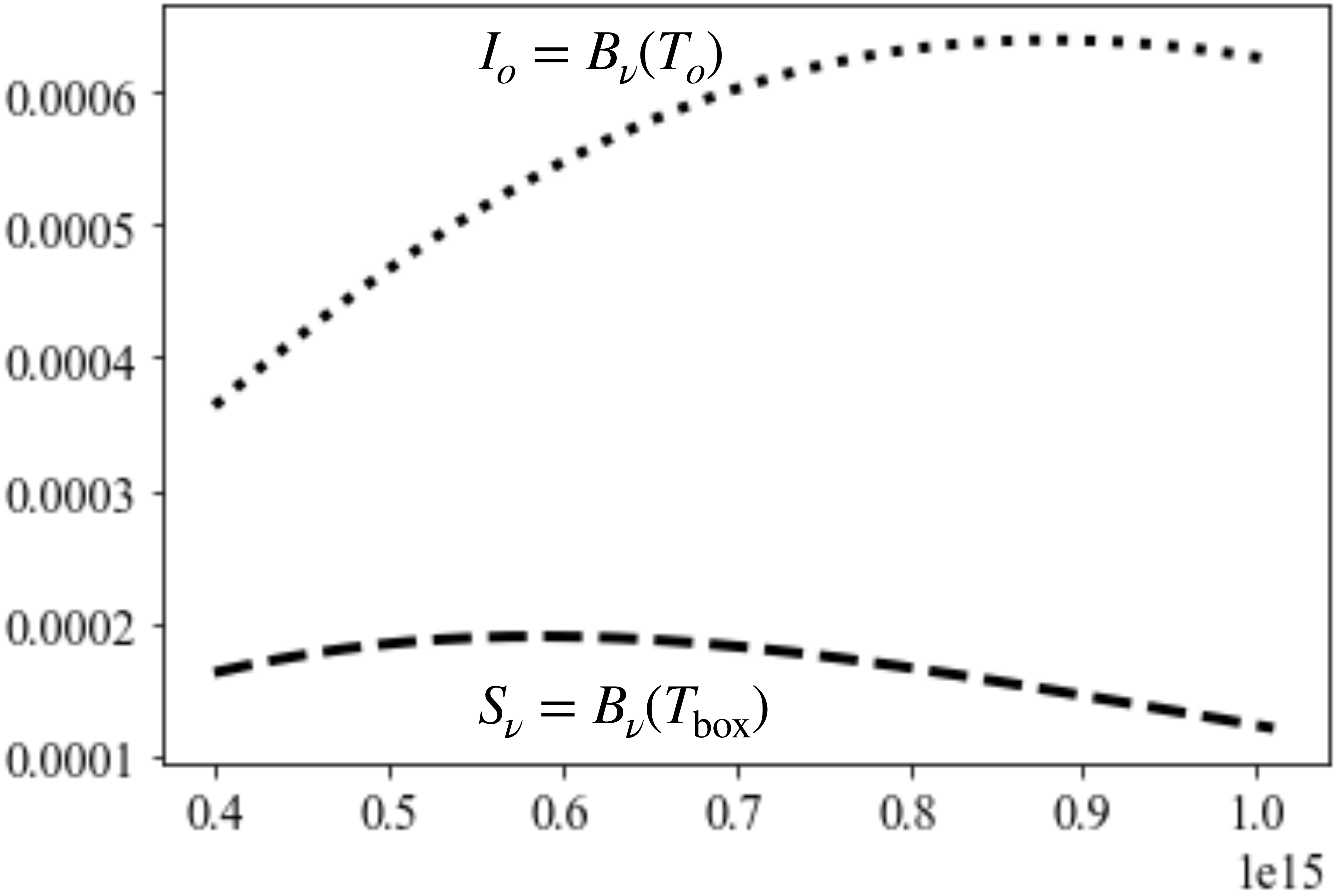

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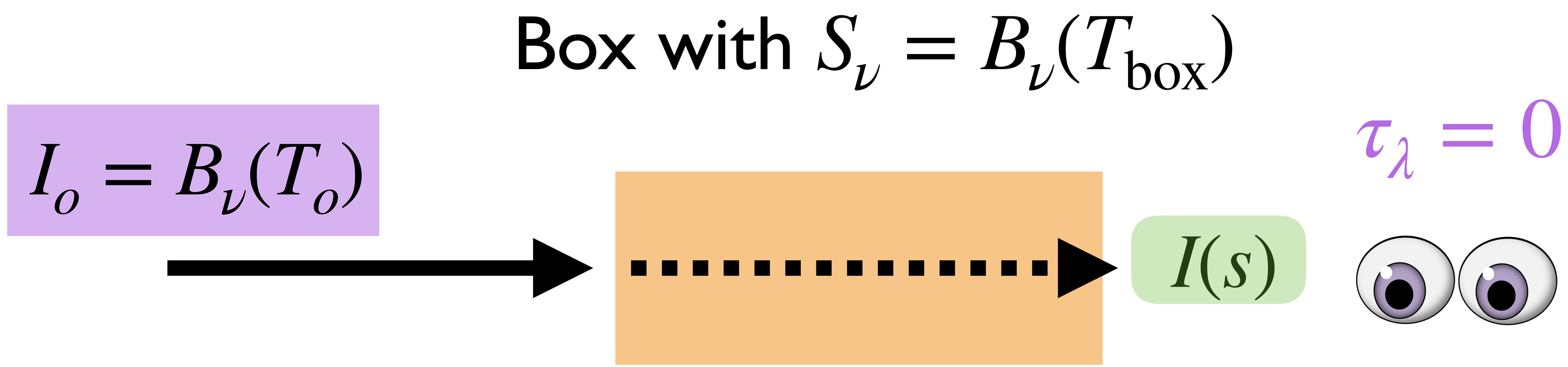

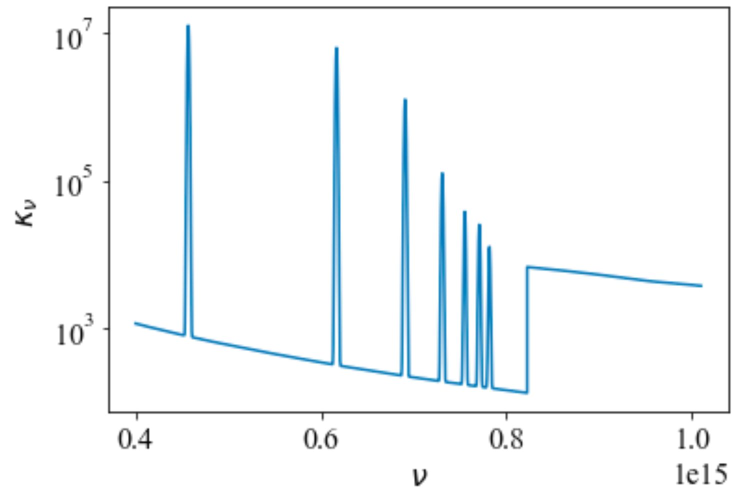

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