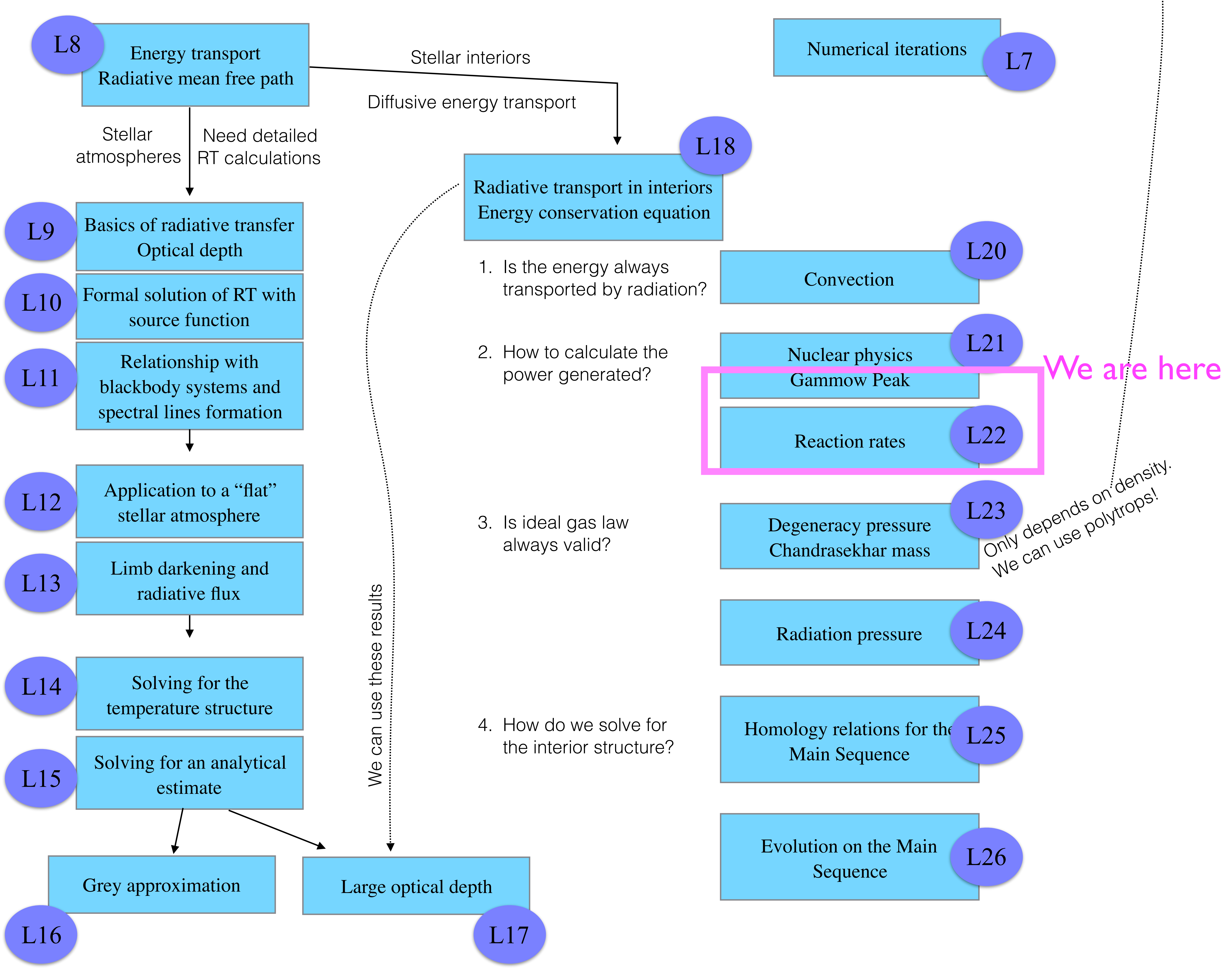
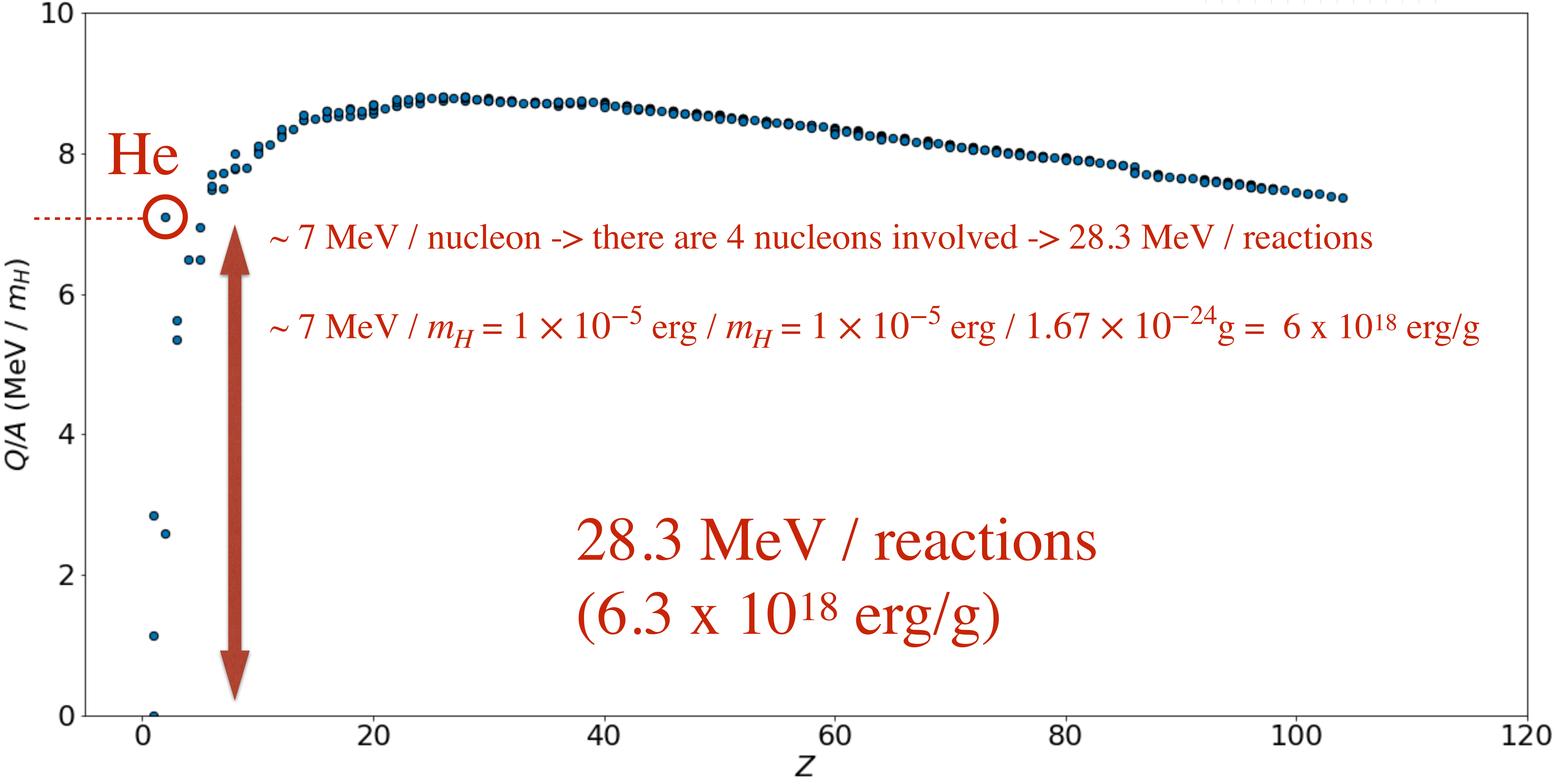
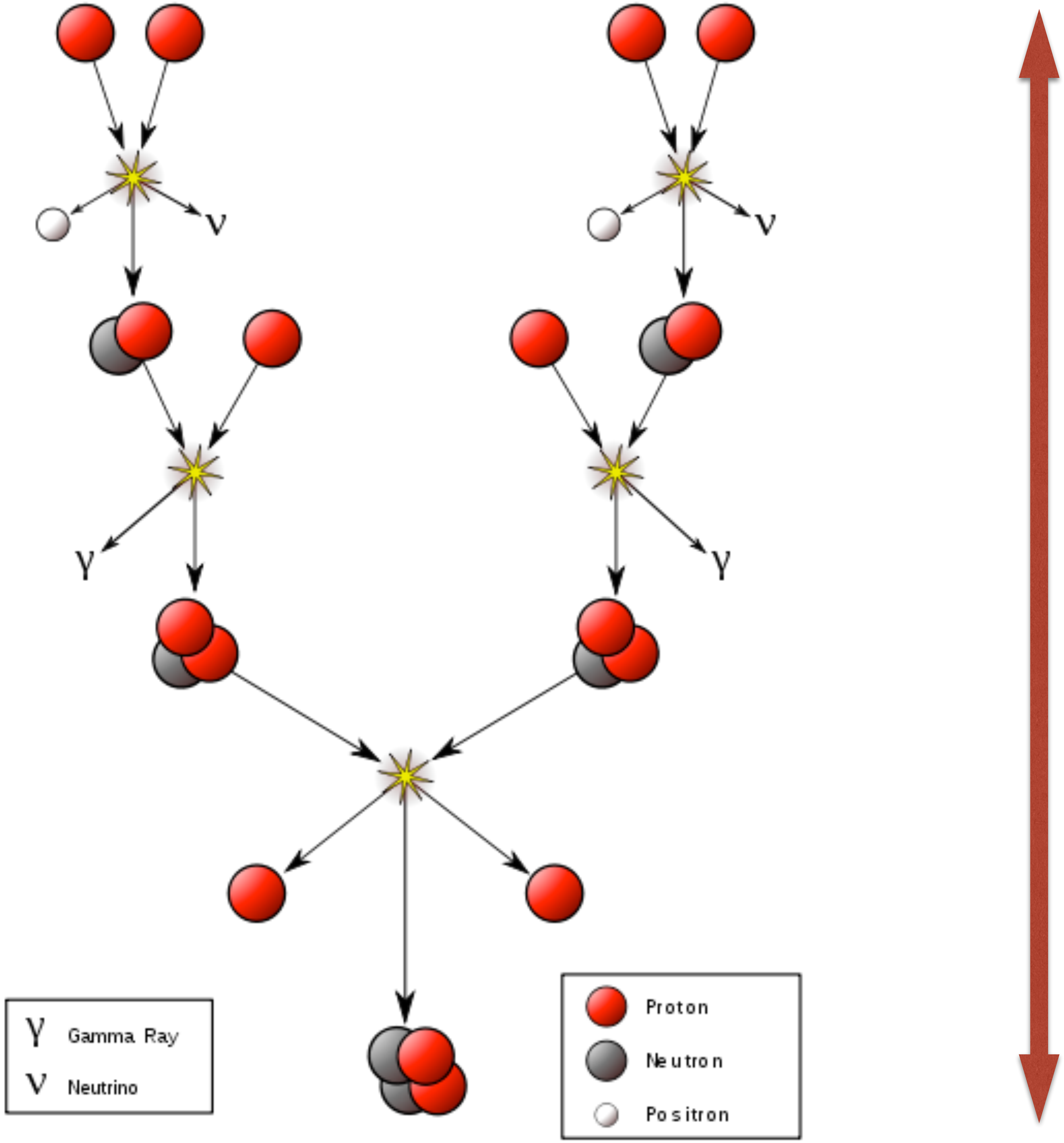
Week 11 Thursday
L-20
Nuclear reactions


L8
Energy transport
Radiative mean free path
Stellar interiors
Diffusive energy transport
Numerical iterations
L7
Stellar atmospheres
Need detailed RT calculations
L18
Radiative transport in interiors
Energy conservation equation
L9
Basics of radiative transfer
Optical depth
L10
Formal solution of RT with source function
L11
Relationship with blackbody systems and spectral lines formation
L12
Application to a "flat" stellar atmosphere
L13
Limb darkening and radiative flux
L14
Solving for the temperature structure
L15
Solving for an analytical estimate
Grey approximation
L16
Large optical depth
L17
We can use these results
1. Is the energy always transported by radiation?
Convection
L20
2. How to calculate the power generated?
Nuclear physics
Gammow Peak
L21
We are here
Reaction rates
L22
3. Is ideal gas law always valid?
Degeneracy pressure
Chandrasekhar mass
L23
Only depends on density.
We can use polytrops!
Radiation pressure
L24
4. How do we solve for the interior structure?
Homology relations for the Main Sequence
L25
Evolution on the Main Sequence
L26

$$Q(Z,N) = \left[\; Zm_p + Zm_e + Nm_n \;-\; m(A,Z) \;\right] c^2$$



The “pp” chain



28.3 MeV - neutrino

(~ 26.25 MeV)

Q: how many reactions per second should there be?

Reaction rate $r = \dfrac{\#\ \text{of reaction}}{\text{second}\ \ \text{volume}}$

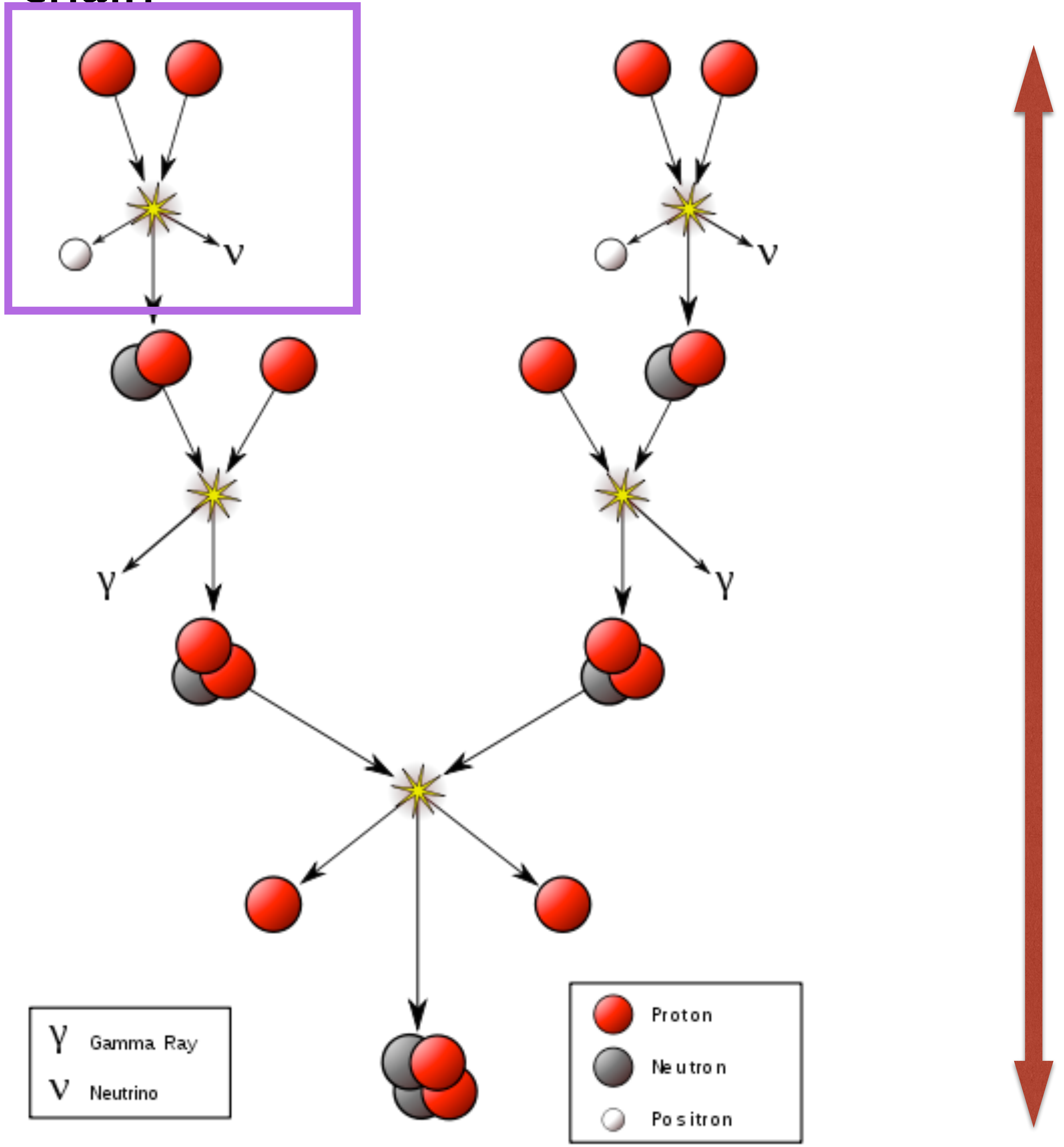
Energy per reaction

$$\epsilon = \frac{\dfrac{\text{Energy}}{\text{second}}}{\text{unit of mass}} = \frac{r \quad Q}{\rho}$$

$$\frac{\text{mass}}{\text{volume}}$$

The "pp" chain

Let's concentrate on this first reaction here



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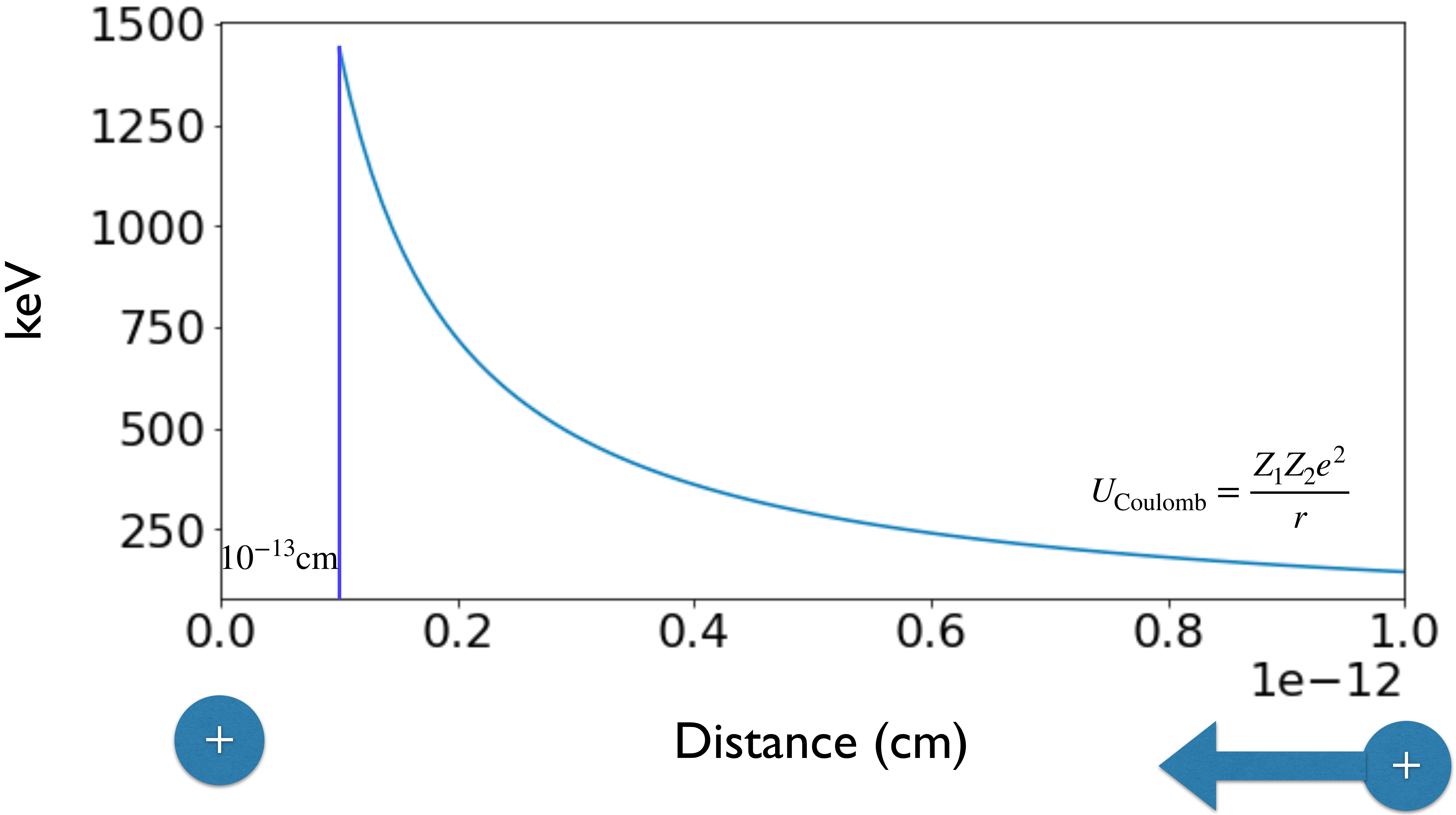


1500
1250
1000
750
500
250
keV
10^{-13}cm
$U_{\text{Coulomb}} = \frac{Z_1 Z_2 e^2}{r}$
0.0
0.2
0.4
0.6
0.8
1.0
1e−12
Distance (cm)
+
+

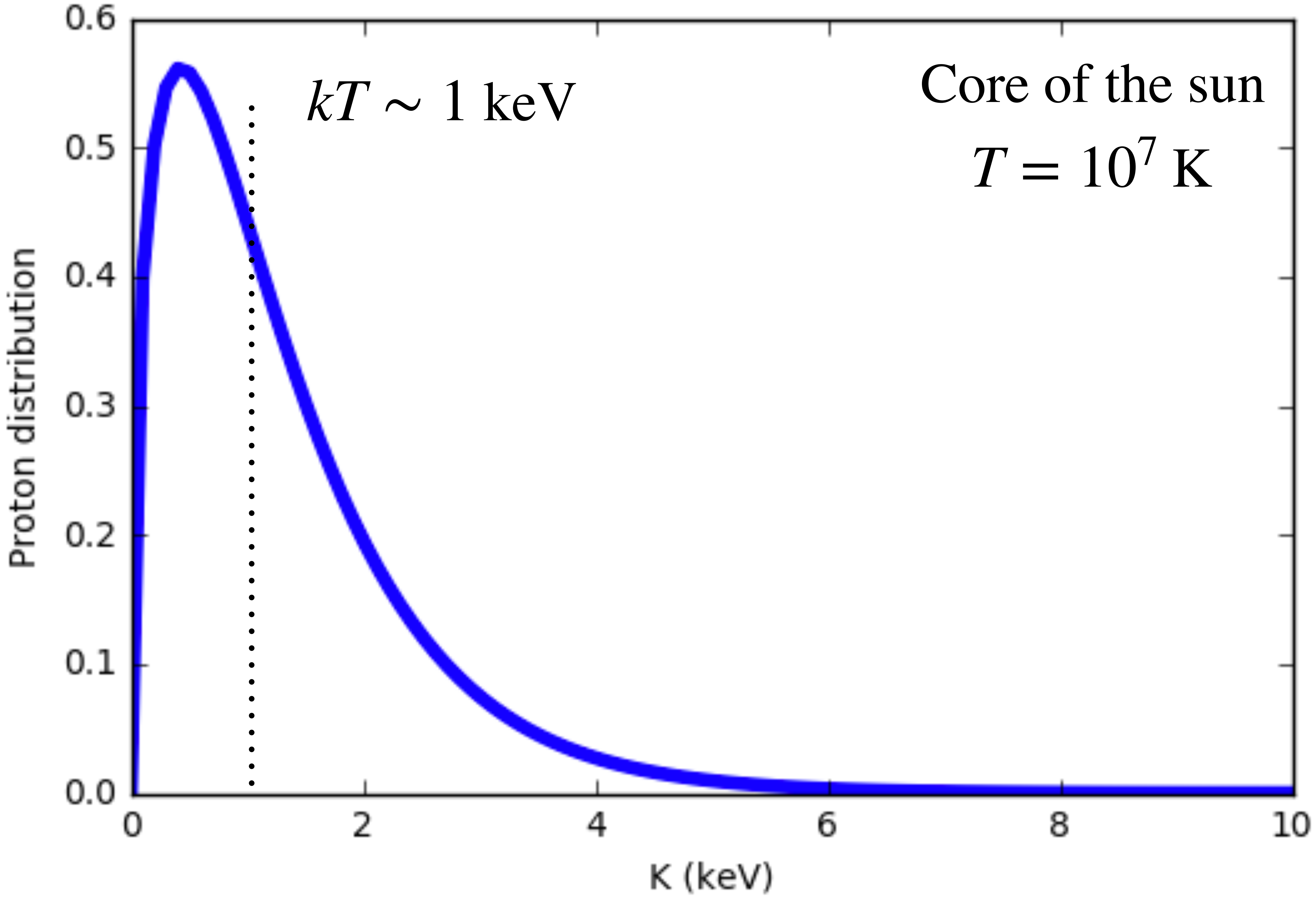
Maxwell-Boltzmann distribution of energy

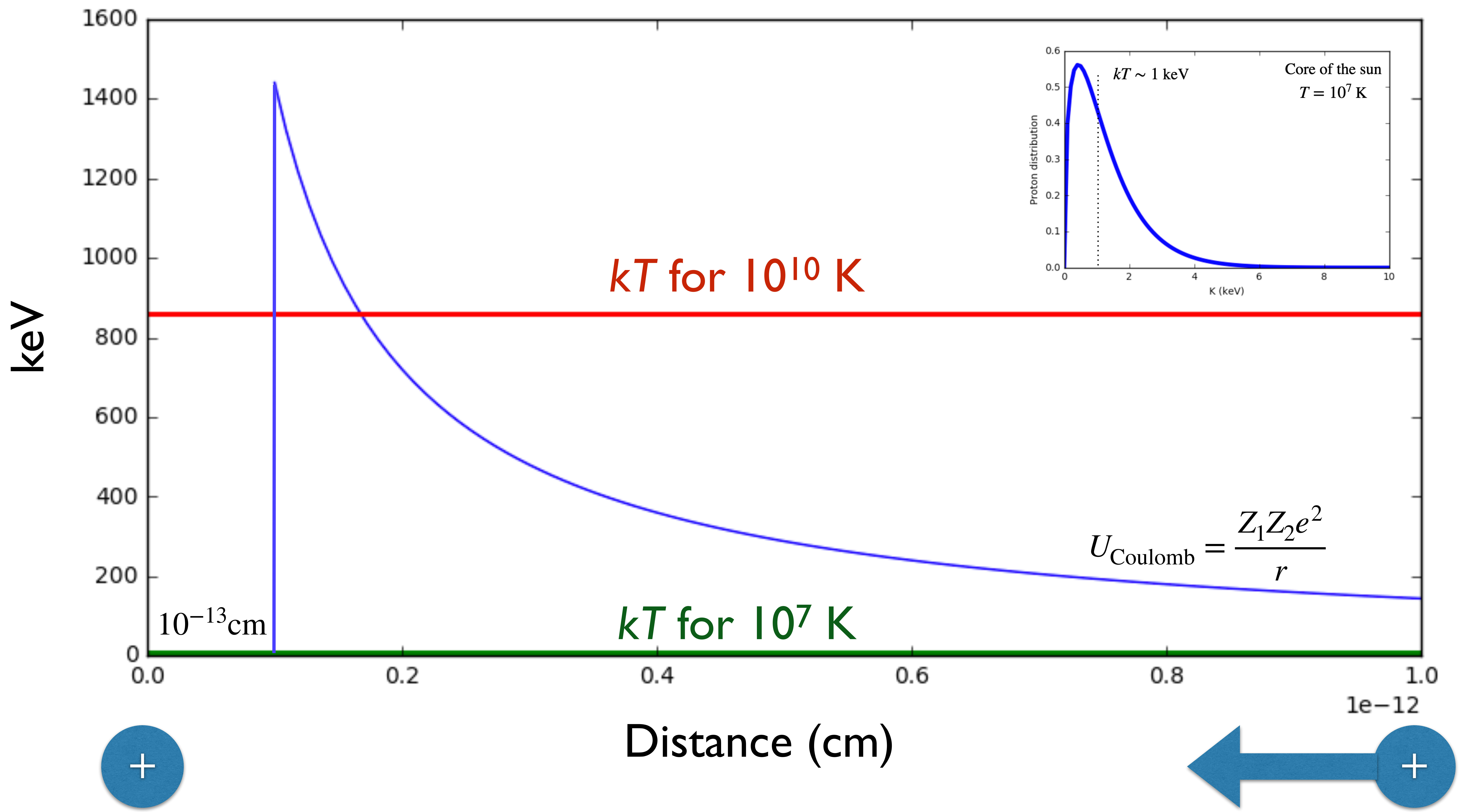
$$f(v) = \left(\frac{m}{2\pi kT}\right)^{3/2} 4\pi v^2 \mathrm{e}^{\frac{-mv^2}{2kT}}$$

$$k_B \sim 10^{-7}\ \mathrm{keV / K}$$

$$f(E)dE = f(v)dv$$

$$f(E) = \left(\frac{E}{\pi}\right)^{1/2} \frac{2}{(kT)^{3/2}} \mathrm{e}^{-\frac{E}{kT}}$$

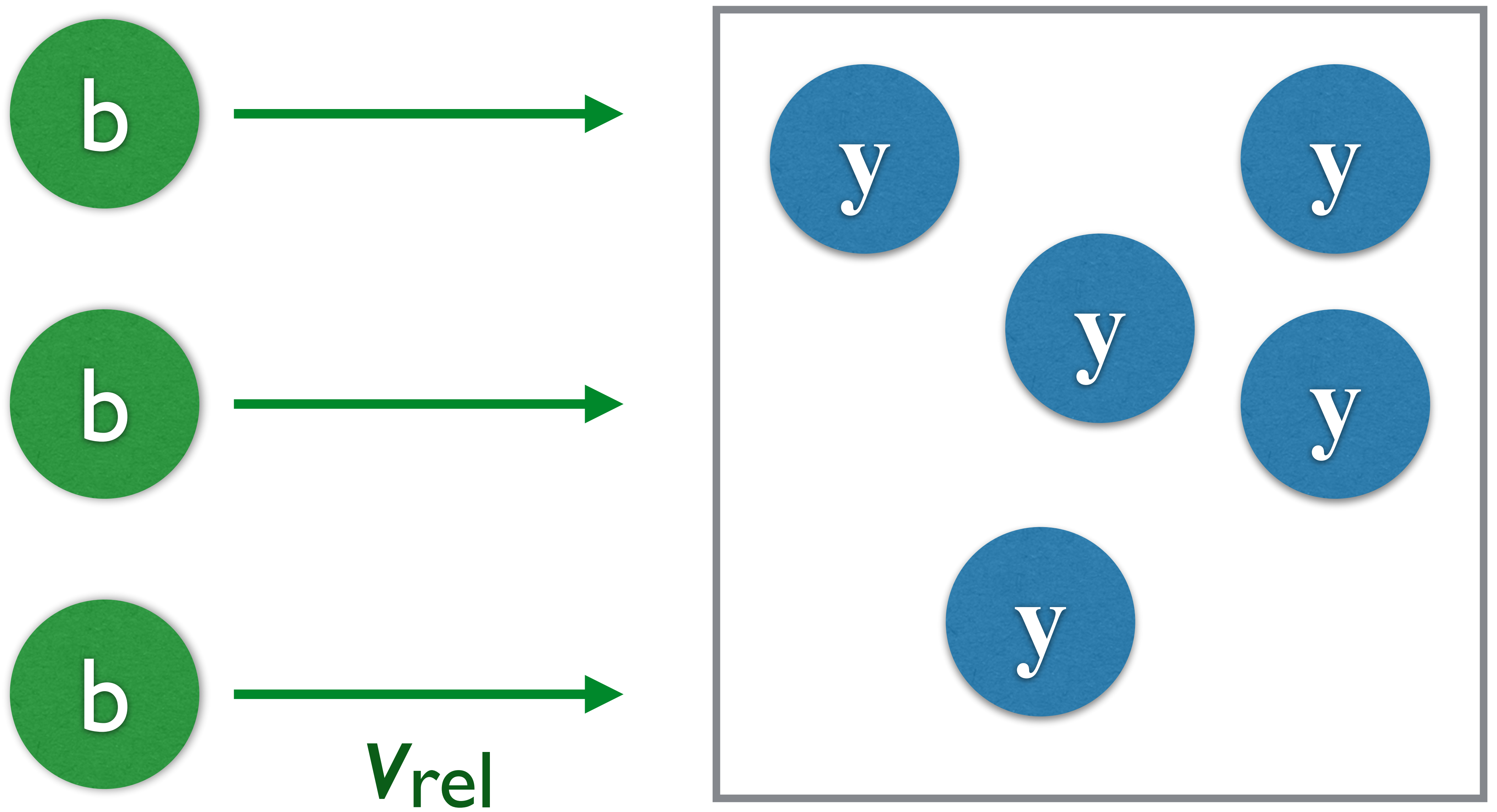




keV
1600
1400
1200
1000
800
600
400
200
0
0.0
0.2
0.4
0.6
0.8
1.0
1e−12
Distance (cm)
kT for 10^10 K
kT for 10^7 K
10^−13 cm
U_Coulomb = Z_1 Z_2 e^2 / r
kT ~ 1 keV
Core of the sun
T = 10^7 K
Proton distribution
K (keV)

Very quickly here (full math in the textbooks)

$$r_{by} = \boxed{\sigma_{by}(v)\ v_{\rm rel}}\ n_b\ n_y$$



Distribution of velocities:
Let's take a weighted mean

$$r_{by} = <\sigma_{by}(v)\ v_{\rm rel}>\ n_b\ n_y$$

$$\epsilon = \frac{<\sigma_{by}(v)\ v_{\rm rel}>\ n_b\ n_y\ Q}{\rho} = \frac{<\sigma_{by}(v)\ v_{\rm rel}>\ \frac{X_b \rho}{A_b m_H}\ \frac{X_y \rho}{A_y m_H}\ Q}{\rho}$$

Very quickly here (full math in the textbooks)

Intrinsic to collision
(small variation with E, compared to the exponential terms)

$$< \sigma_{by}(v) v_{rel} > = \left(\frac{8}{\pi \mu_{red} (kT)^3} \right)^{1/2} \int_0^\infty S(E) \;\; e^{-E/kT} \;\; e^{-b/E^{1/2}} \;\; dE$$

Tail of the MB distribution

Tunneling prob.

$$b = (8\mu_{red})^{1/2} \; \frac{\pi^2 Z_1 Z_2 e^2}{h}$$

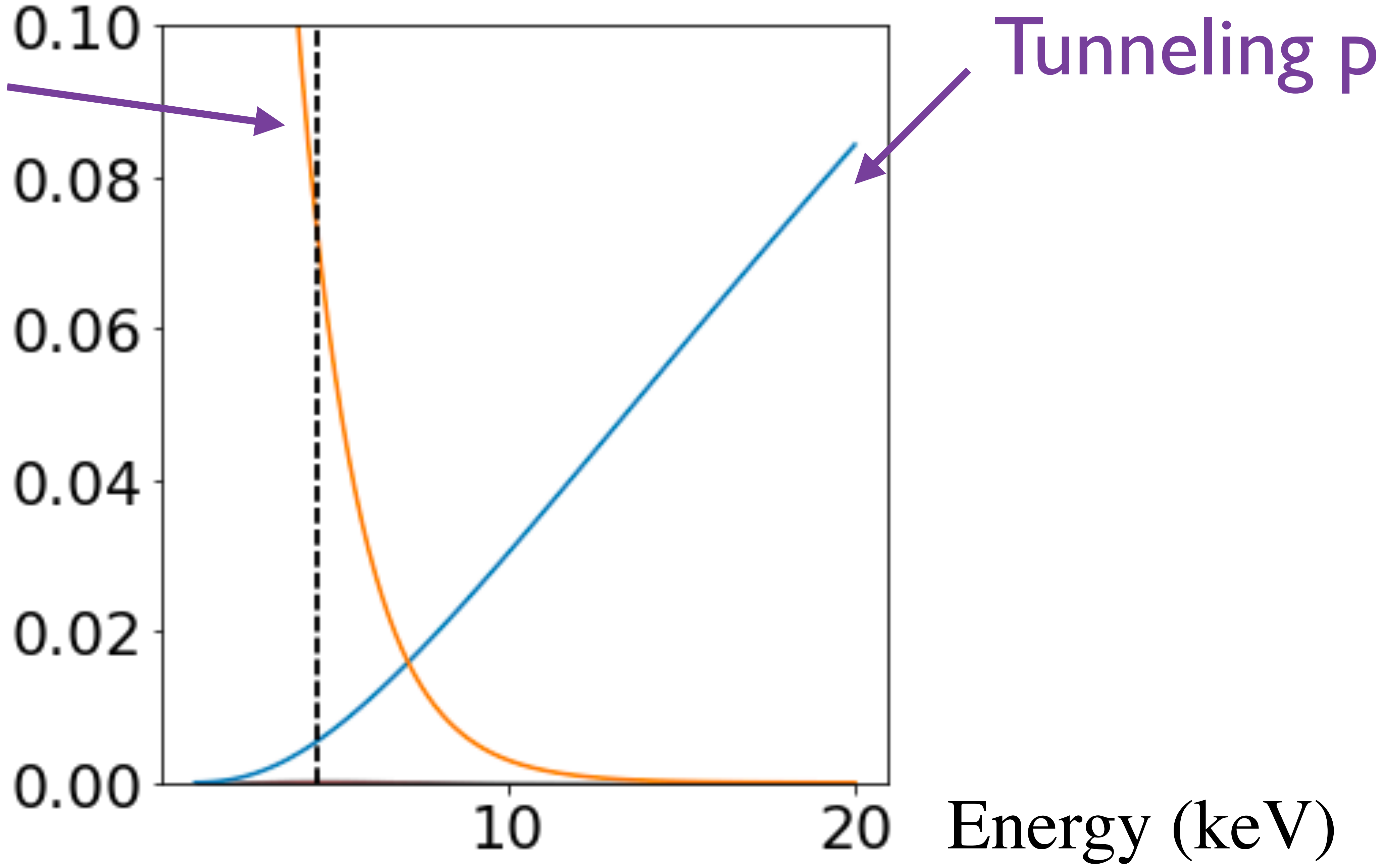
Very quickly here (full math in the textbooks)

Intrinsic to collision
(small variation with E, compared to the exponential terms)

$$< \sigma_{by}(v) v_{\rm rel} > = \left(\frac{8}{\pi \mu_{\rm red} (kT)^3} \right)^{1/2} \int_0^\infty S(E) \;\; e^{-E/kT} \;\; e^{-b/E^{1/2}} \;\; dE$$

Tail of the MB distribution

Tunneling prob.



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Intrinsic to collision
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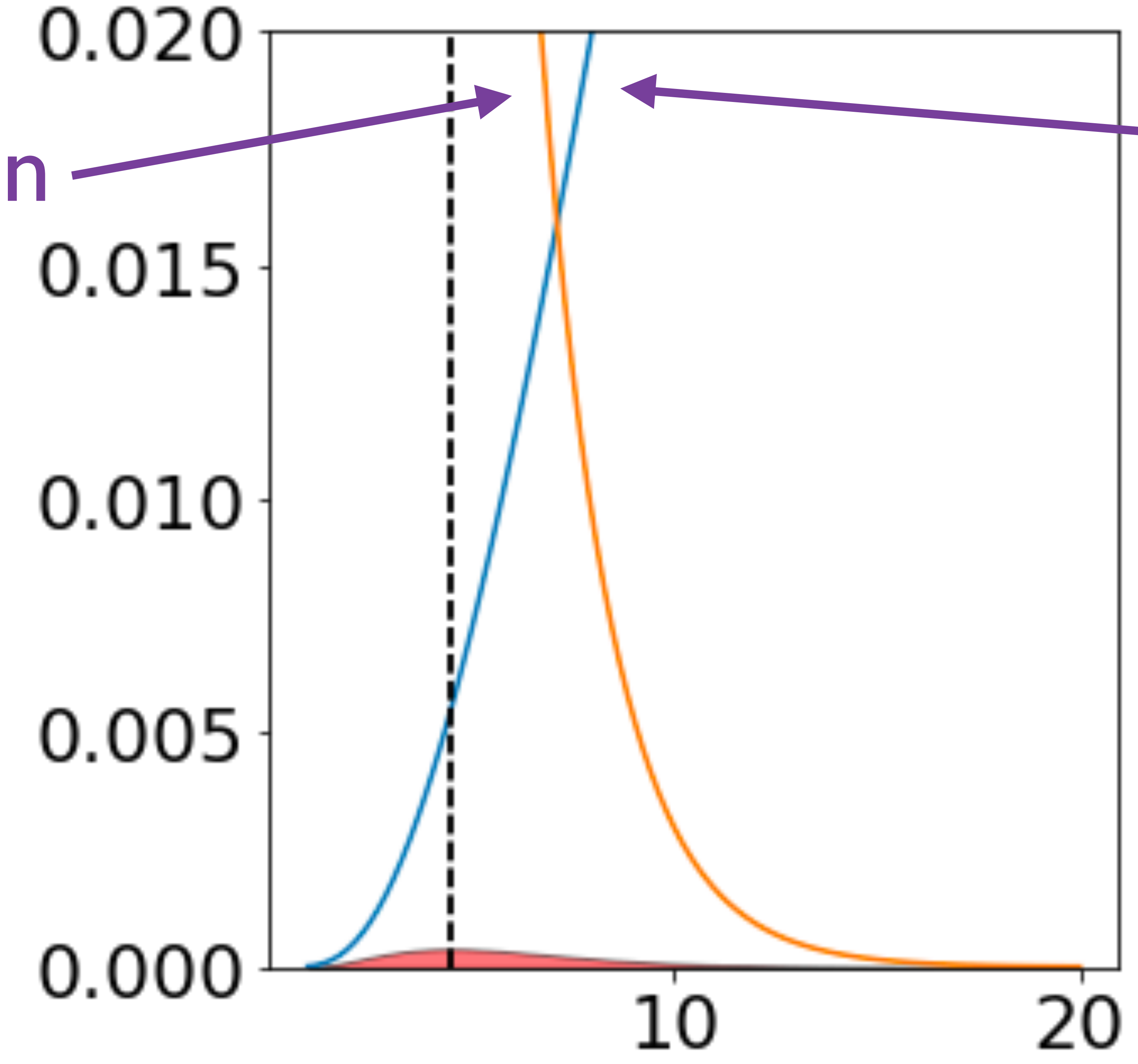
$$< \sigma_{by}(v) v_{\rm rel} > = \left(\frac{8}{\pi \mu_{\rm red} (kT)^3} \right)^{1/2} \int_0^\infty S(E) \; e^{-E/kT} \; e^{-b/E^{1/2}} \; dE$$

Tail of the MB distribution

Tunneling prob.



"Gammov peak"

$$\epsilon_{\mathrm{PP\,I}} = 2.38 \times 10^{6}\, g_{\mathrm{PP}}\, \rho\, X_H^2\, T_6^{-2/3}\ \mathrm{e}^{-33.8/T_6^{1/3}}$$

$$g_{\mathrm{PP}} = 1 + 0.0123\, T_6^{1/3} + 0.0109\, T_6^{2/3} + 0.0009\, T_6$$

$${}^{1}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{2}_{1}\mathrm{H} + e^{+} + \nu_e$$

$${}^{2}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{3}_{2}\mathrm{He} + \gamma$$

69%

$${}^{3}_{2}\mathrm{He} + {}^{3}_{2}\mathrm{He} \rightarrow {}^{4}_{2}\mathrm{He} + 2\,{}^{1}_{1}\mathrm{H}$$

(PP I)

31%

$${}^{3}_{2}\mathrm{He} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{7}_{4}\mathrm{Be} + \gamma$$

99.7%

$${}^{7}_{4}\mathrm{Be} + e^{-} \rightarrow {}^{7}_{3}\mathrm{Li} + \nu_e$$

$${}^{7}_{3}\mathrm{Li} + {}^{1}_{1}\mathrm{H} \rightarrow 2\,{}^{4}_{2}\mathrm{He}$$

(PP II)

0.3%

$${}^{7}_{4}\mathrm{Be} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{8}_{5}\mathrm{B} + \gamma$$

$${}^{8}_{5}\mathrm{B} \rightarrow {}^{8}_{4}\mathrm{Be} + e^{+} + \nu_e$$

$${}^{8}_{4}\mathrm{Be} \rightarrow 2\,{}^{4}_{2}\mathrm{He}$$

(PP III)

Notes:
$T_6 = T\,[\mathrm{K}] / 10^6\,\mathrm{K}$
The constant in front has implicit units of erg cm^3 / g^2 (so that the result is in erg/g if the density is in g/cm^3).

$$\epsilon_{\rm CNO} = 8.67 \times 10^{27}\, g_{\rm CNO}\, \rho\, X_H\, X_{\rm CNO}\, T_6^{-2/3}\ {\rm e}^{-152.83/T_6^{1/3}}$$

$$g_{\rm CNO} = 1 + 0.0027\, T_6^{1/3} - 0.00778\, T_6^{2/3} - 0.000149\, T_6$$

$C^{12} + H^{1} \rightarrow N^{13}$

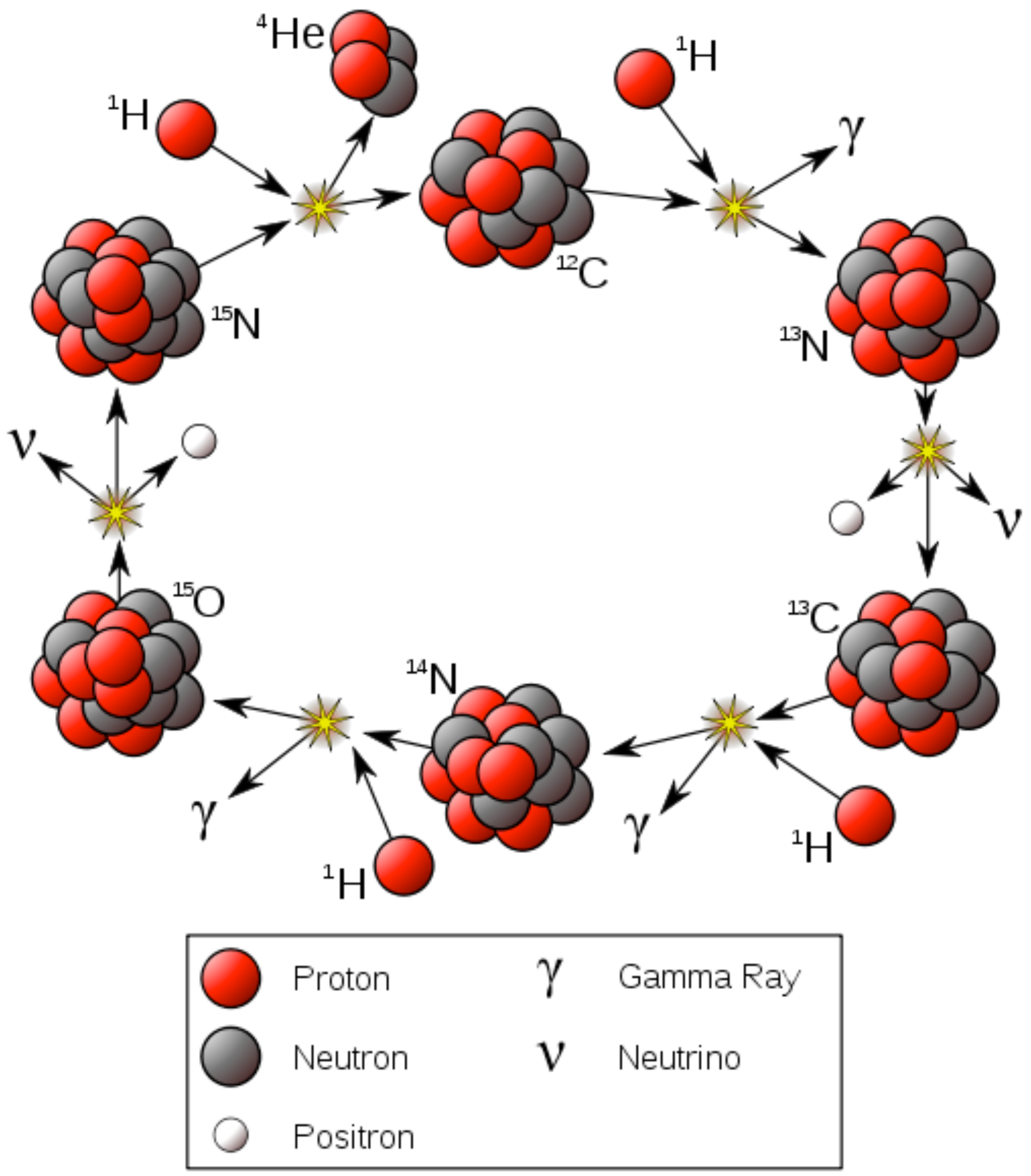
$N^{13} \rightarrow C^{13} + e^{+} + \nu$

$C^{13} + H^{1} \rightarrow N^{14}$

$N^{14} + H^{1} \rightarrow O^{15}$

$O^{15} \rightarrow N^{15} + e^{+} + \nu$

$N^{15} + H^{1} \rightarrow C^{12} + He^{4}$



$$\frac{dM_r(r)}{dr} = 4\pi r^2 \rho(r)$$

$$\frac{dP(r)}{dr} = -\rho(r)\frac{GM_r(r)}{r^2}$$

$$\frac{dT(r)}{dr} = -\frac{3\kappa_R(r)\rho(r)}{64\pi\sigma}\frac{L_r(r)}{r^2 T(r)^3}$$

$$\frac{dL_r(r)}{dr} = 4\pi r^2 \rho(r)\epsilon(r)$$

$$M_r(r) \quad P(r) \quad L_r(r) \quad T(r)$$

$$\rho(r) \quad \mu(r) \quad \epsilon_{\mathrm{nuc}}(r) \quad \kappa_R(r)$$

$$P(r) = \frac{\rho(r)}{\mu(r)} kT(r)$$

$$\mu(r) = f(\mathrm{comp}, T(r), P(r))$$

$$\kappa_R(r) = f(\mathrm{comp}, T(r), P(r))$$

$$\epsilon_{\mathrm{nuc}}(r) = f(\mathrm{comp}, T(r), P(r))$$

Other energy transport?

Ideal gas always valid?

Nuclear mechanism?

How to solve?

What changes with time?