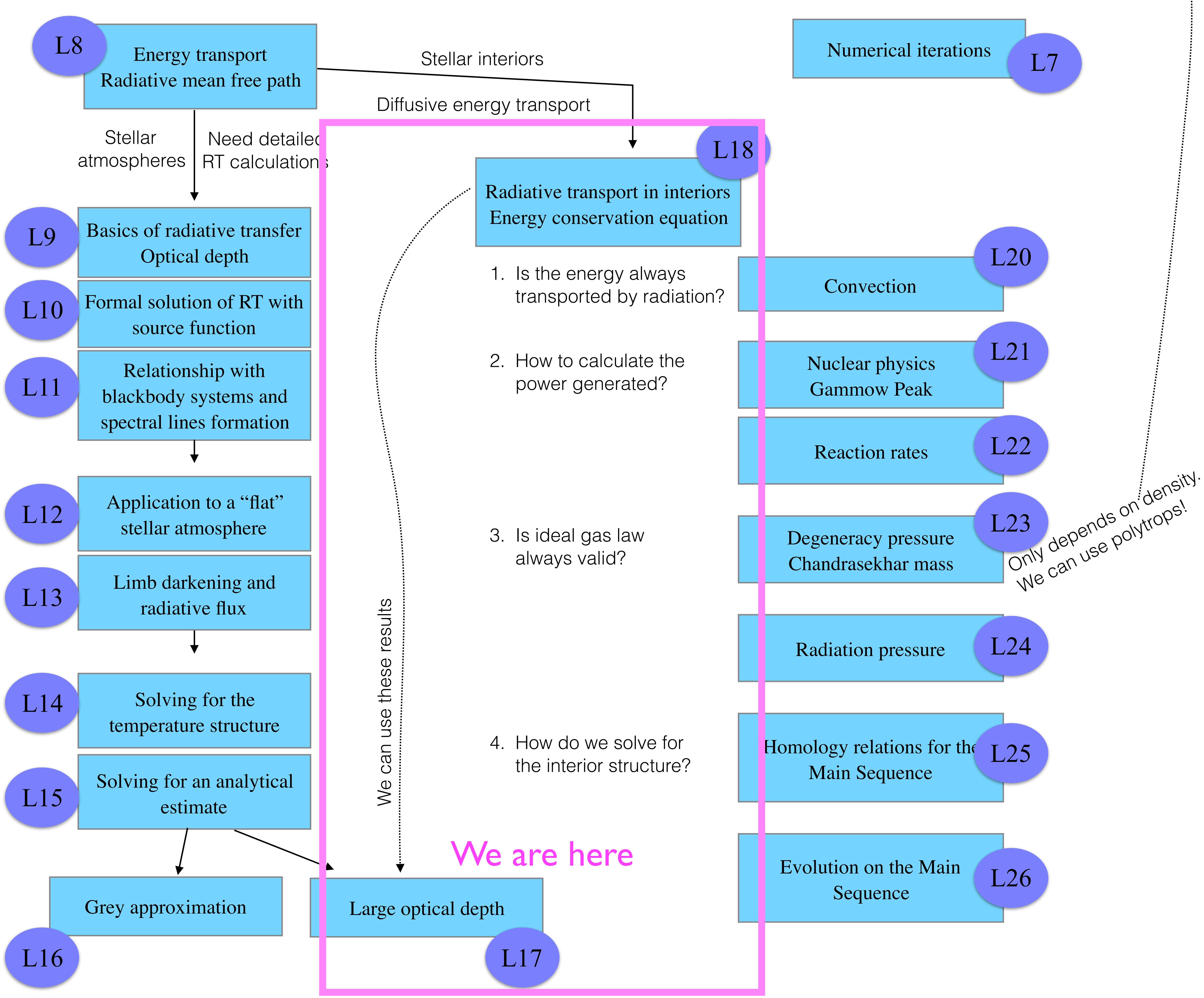
Week 10 Tuesday
L-17
Large optical depth
And interiors


L8
Energy transport
Radiative mean free path
Stellar interiors
Diffusive energy transport
Numerical iterations
L7
Stellar atmospheres
Need detailed RT calculations
L18
Radiative transport in interiors
Energy conservation equation
L9
Basics of radiative transfer
Optical depth
L10
Formal solution of RT with source function
L11
Relationship with blackbody systems and spectral lines formation
L12
Application to a "flat" stellar atmosphere
L13
Limb darkening and radiative flux
L14
Solving for the temperature structure
L15
Solving for an analytical estimate
L16
Grey approximation
Large optical depth
L17
We can use these results
We are here
1. Is the energy always transported by radiation?
2. How to calculate the power generated?
3. Is ideal gas law always valid?
4. How do we solve for the interior structure?
Convection
L20
Nuclear physics
Gammow Peak
L21
Reaction rates
L22
Degeneracy pressure
Chandrasekhar mass
L23
Only depends on density.
We can use polytrops!
Radiation pressure
L24
Homology relations for the Main Sequence
L25
Evolution on the Main Sequence
L26

Interiors

Photons don't go far =
energy transport is easier
(diffusion approximation)

We will have to include the energy created by nuclear reactions

Atmospheres

Transport of energy more tricky
(radiative transfer)

No energy production

Scale height is small:
Flat geometry approximation

We can do the same procedure (approximation for large τ) for all of the 'moment' equations, and also for the intensity solution.

$$\boxed{I(\tau, u) = S(\tau) + u \frac{dS(\tau')}{d\tau'}\bigg|_\tau} + 2!\ u^2 \frac{d^2 S(\tau')}{d\tau'^2}\bigg|_\tau + \ldots$$

Reminder

$$\boxed{J(\tau) = S(\tau)} + \frac{1}{3} \frac{d^2 S(\tau')}{d\tau'^2}\bigg|_\tau + \ldots$$

$$\boxed{\frac{F}{4\pi} = \frac{1}{3} \frac{dS(\tau')}{d\tau'}\bigg|_\tau} + \frac{1}{5} \frac{d^3 S(\tau')}{d\tau'^3}\bigg|_\tau + \ldots$$

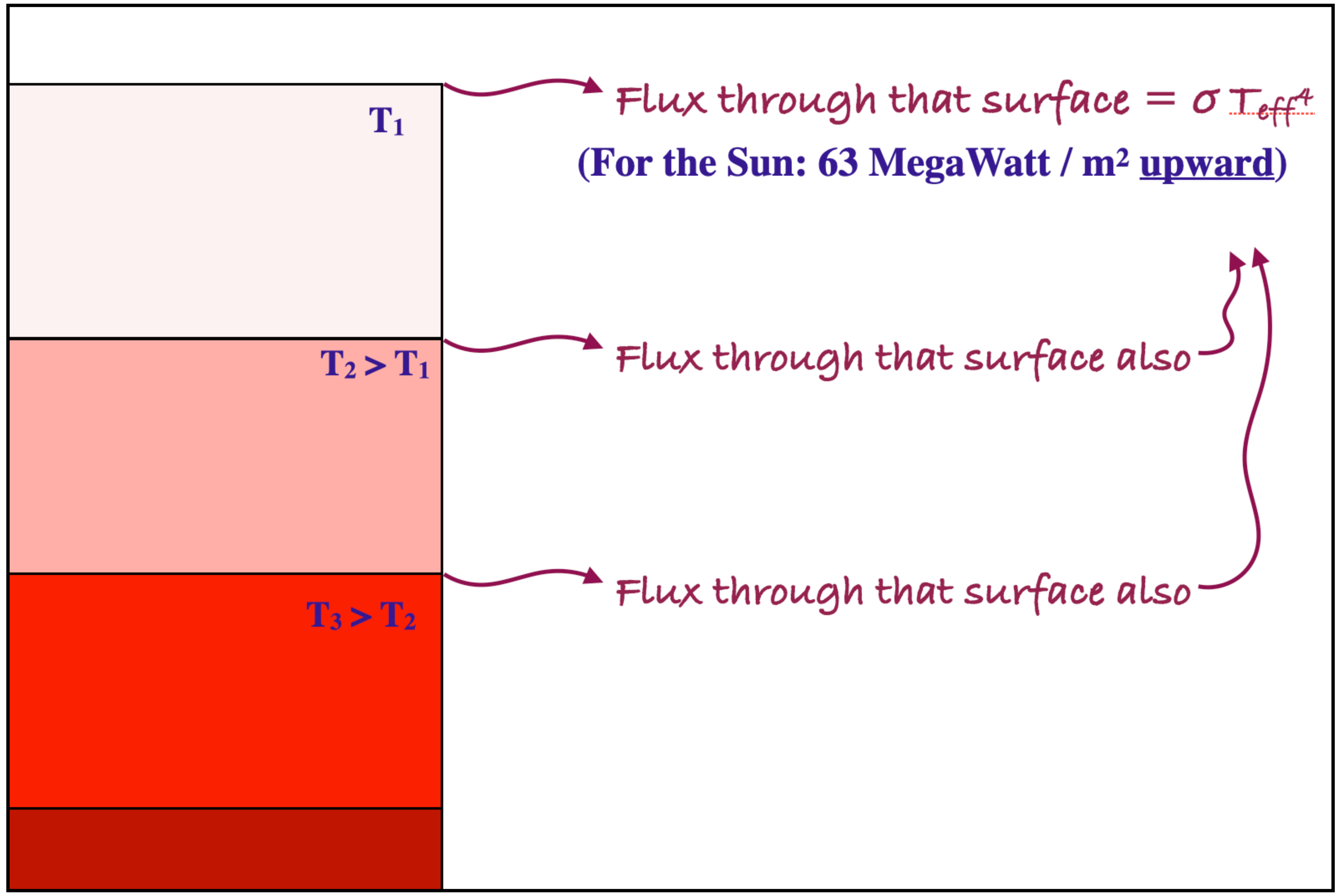
$$\boxed{K(\tau) = \frac{1}{3} S(\tau)} + \frac{1}{5} \frac{d^2 S(\tau')}{d\tau'^2}\bigg|_\tau + \ldots$$

We can do the same procedure (approximation for large τ) for all of the ‘moment’ equations, and also for the intensity solution.

We can use this (yah!)

$$\boxed{\frac{F}{4\pi} = \frac{1}{3}\frac{dS(\tau')}{d\tau'}\bigg|_{\tau}}$$

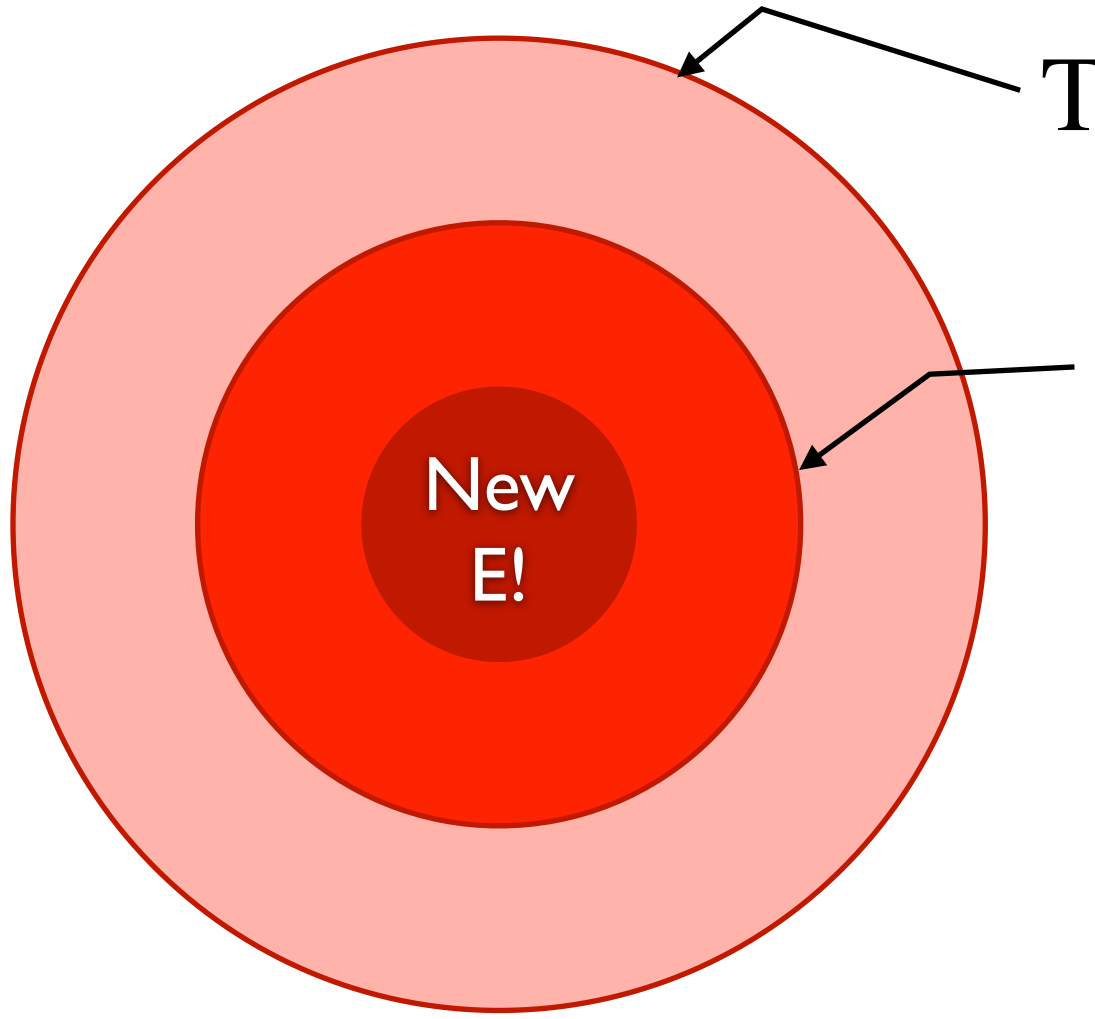
We can do the same procedure (approximation for large τ) for all of the 'moment' equations, and also for the intensity solution.



We can use this (yah!)

$$\boxed{\frac{F}{4\pi} = \frac{1}{3}\frac{dS(\tau')}{d\tau'}\bigg|_{\tau}}$$

But…



There is 10^{26} W in $4\pi R_\star^2$

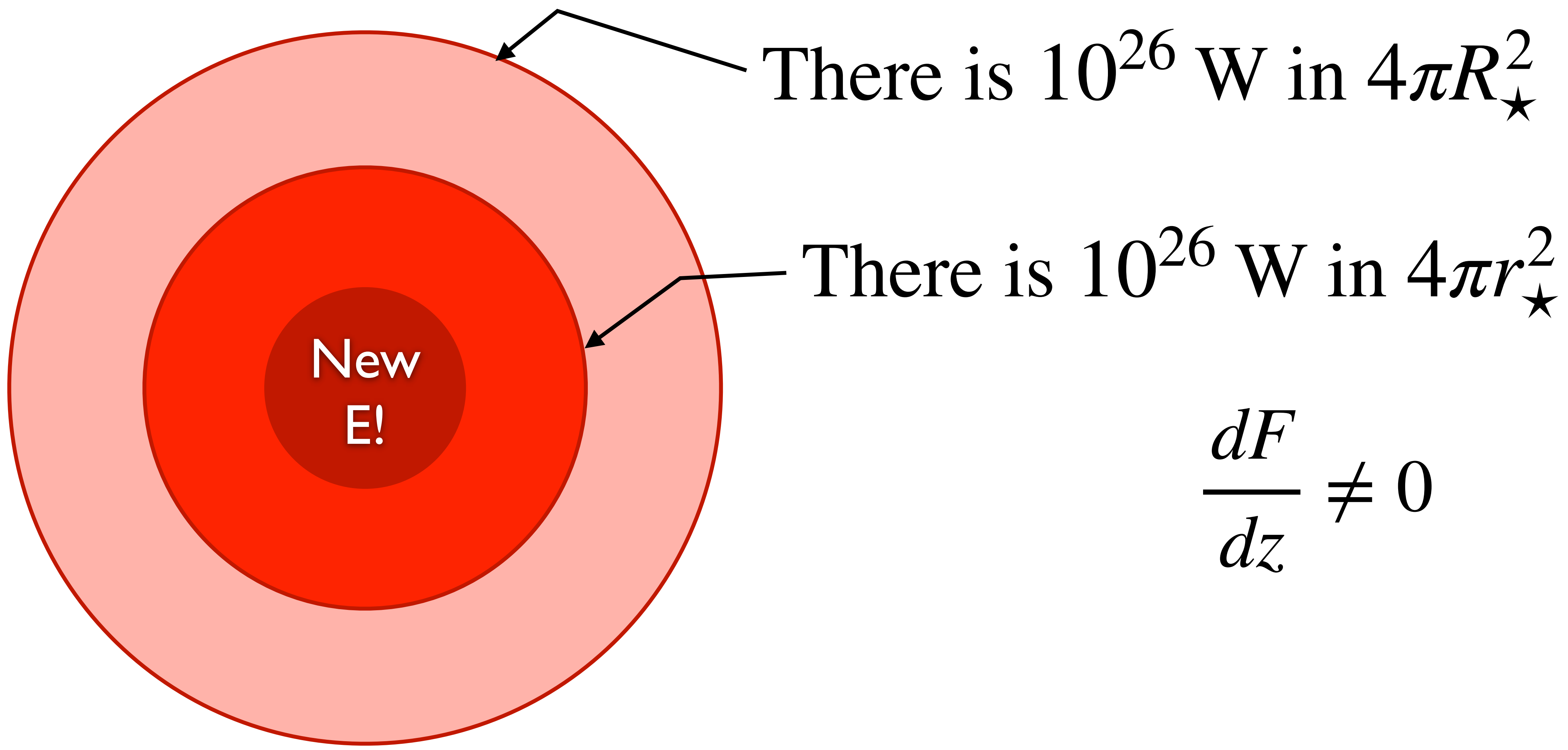
There is 10^{26} W in $4\pi r_\star^2$

$$\frac{dF}{dz} \neq 0$$

The "enclosed luminosity" $L_r(r)$:

Energy per second (erg/s or Watt)
going through a spherical surface at radius r

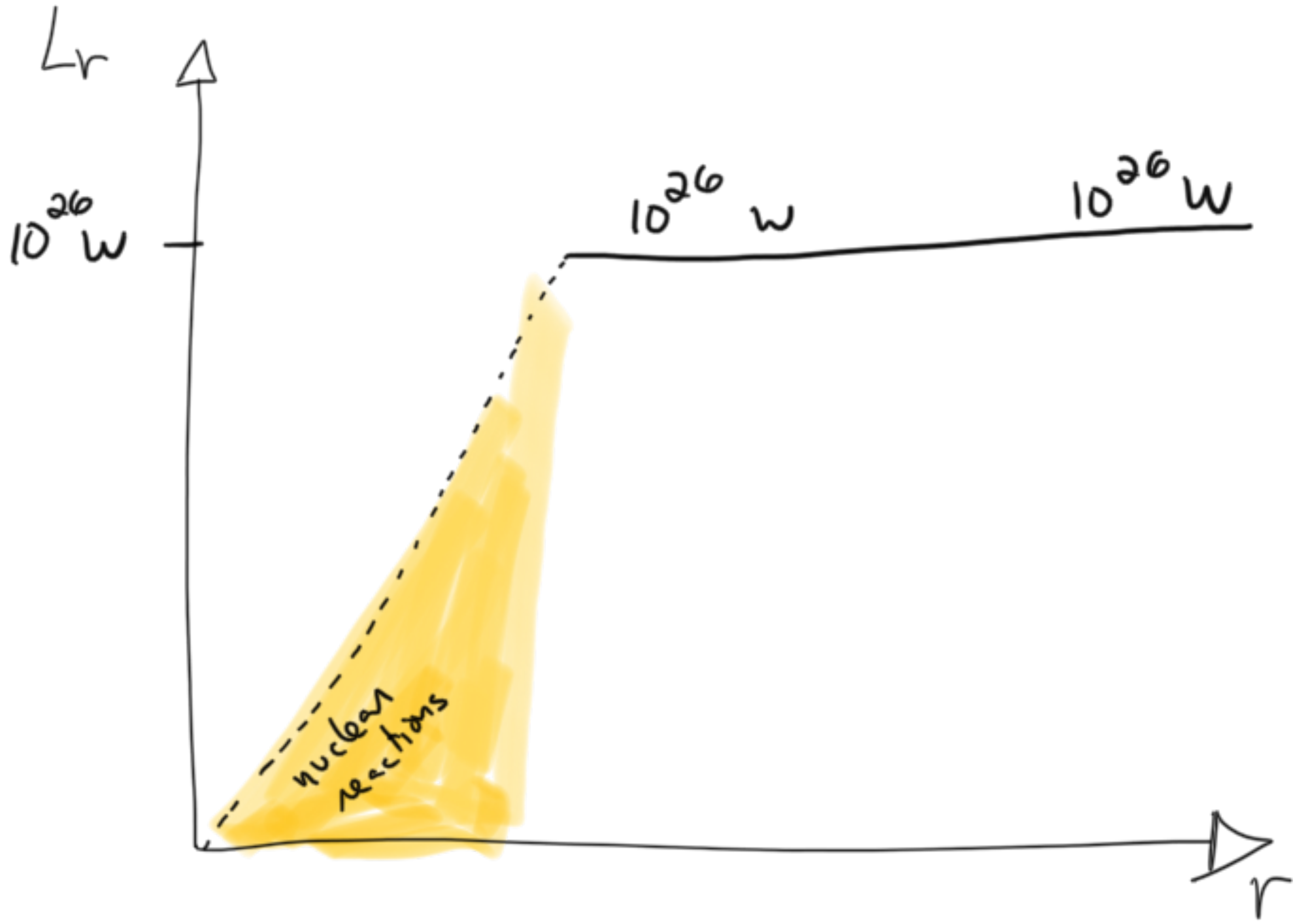
$$L_r(r) = 4\pi r^2 \tilde{F}(r)$$



$$\frac{dF}{dz} \neq 0$$

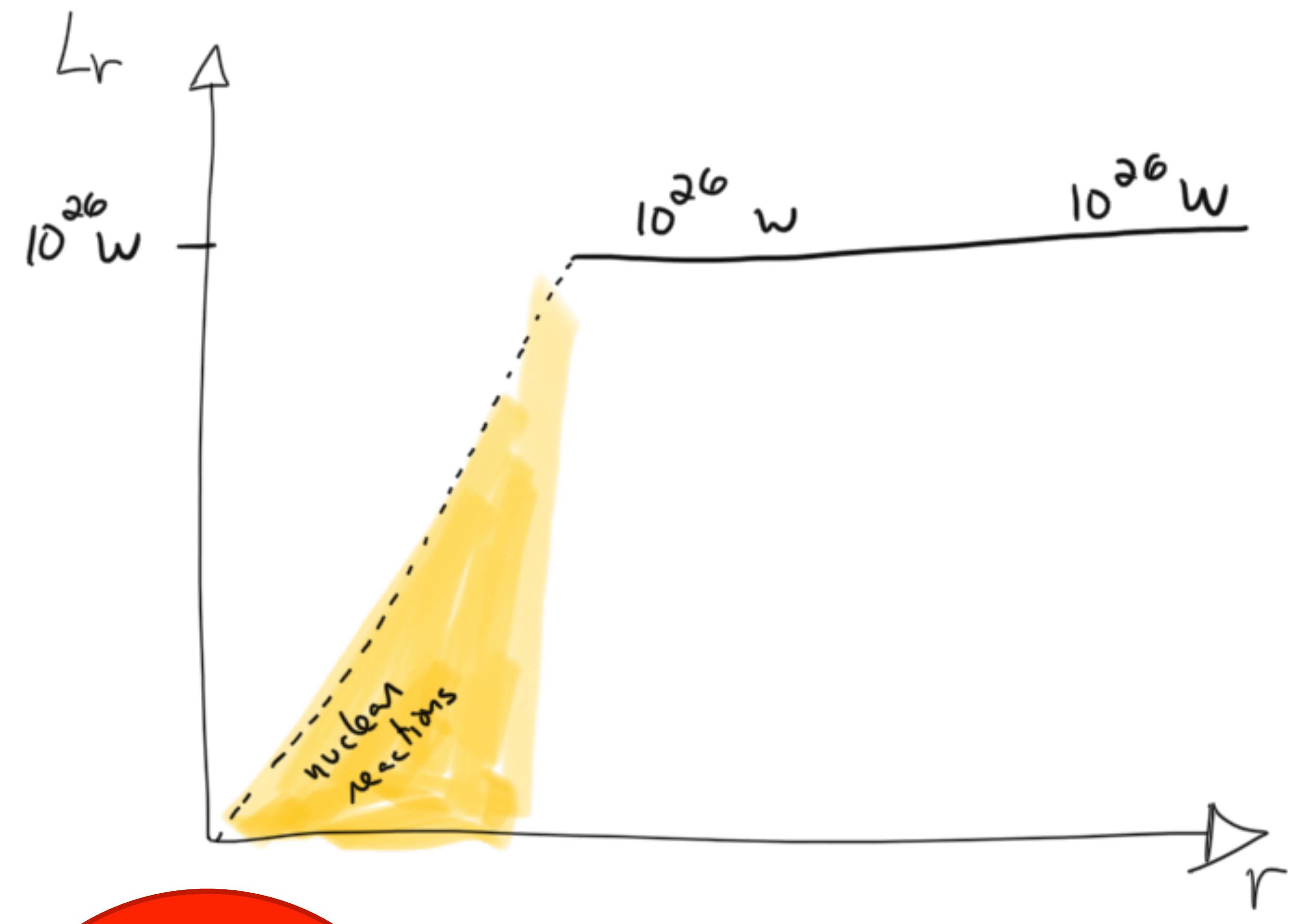
Plan:

1. Let's sketch $L_r(r)$
2. How can we quantify the change in $L_r(r)$ in the core
3. How can we relate $L_r(r)$ to the $T(r)$ through the flux at high τ



Plan:

1. Let's sketch $L_r(r)$
2. **How can we quantify the change in $L_r(r)$ in the core**
3. How can we relate $L_r(r)$ to the $T(r)$ through the flux at high τ



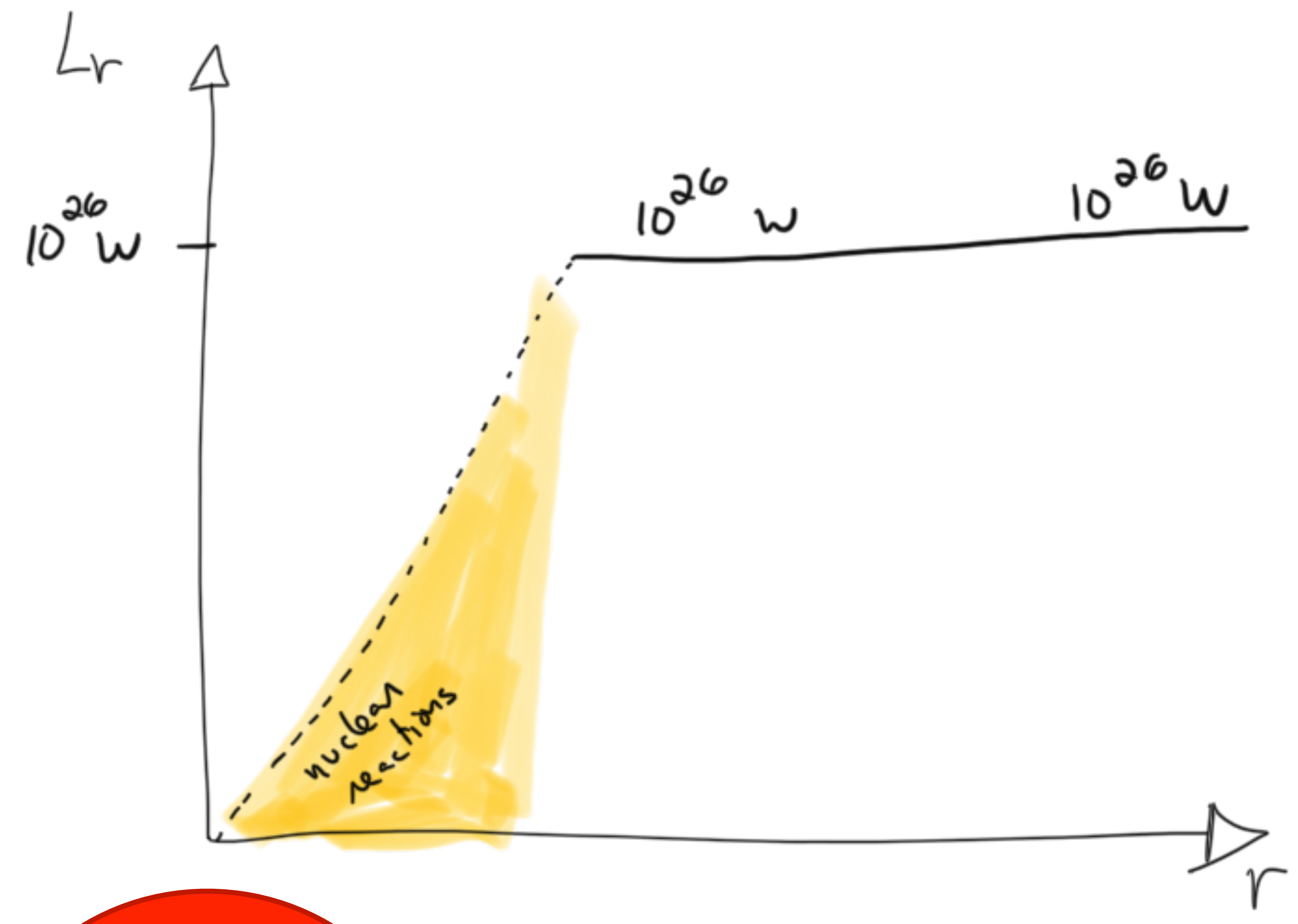
How much (nuclear) energy is added to L_r by one thin shell?
(On the board)

$$\frac{dL_r(r)}{dr} = 4\pi r^2 \rho(r) \epsilon(r)$$

Q: what is $\epsilon(r)$ in the atmosphere of the Sun?
Q: why is $L_r(r = 0)$ zero?

Plan:

1. Let's sketch $L_r(r)$
2. How can we quantify the change in $L_r(r)$ in the core
3. **How can we relate $L_r(r)$ to the $T(r)$ through the flux at high τ**



$$L_r(r) = 4\pi r^2 \tilde{F}(r)$$

$$\frac{F}{4\pi} = \frac{1}{3}\frac{dS(\tau')}{d\tau'}\bigg|_{\tau}$$

On the board: transform the flux equation to get:

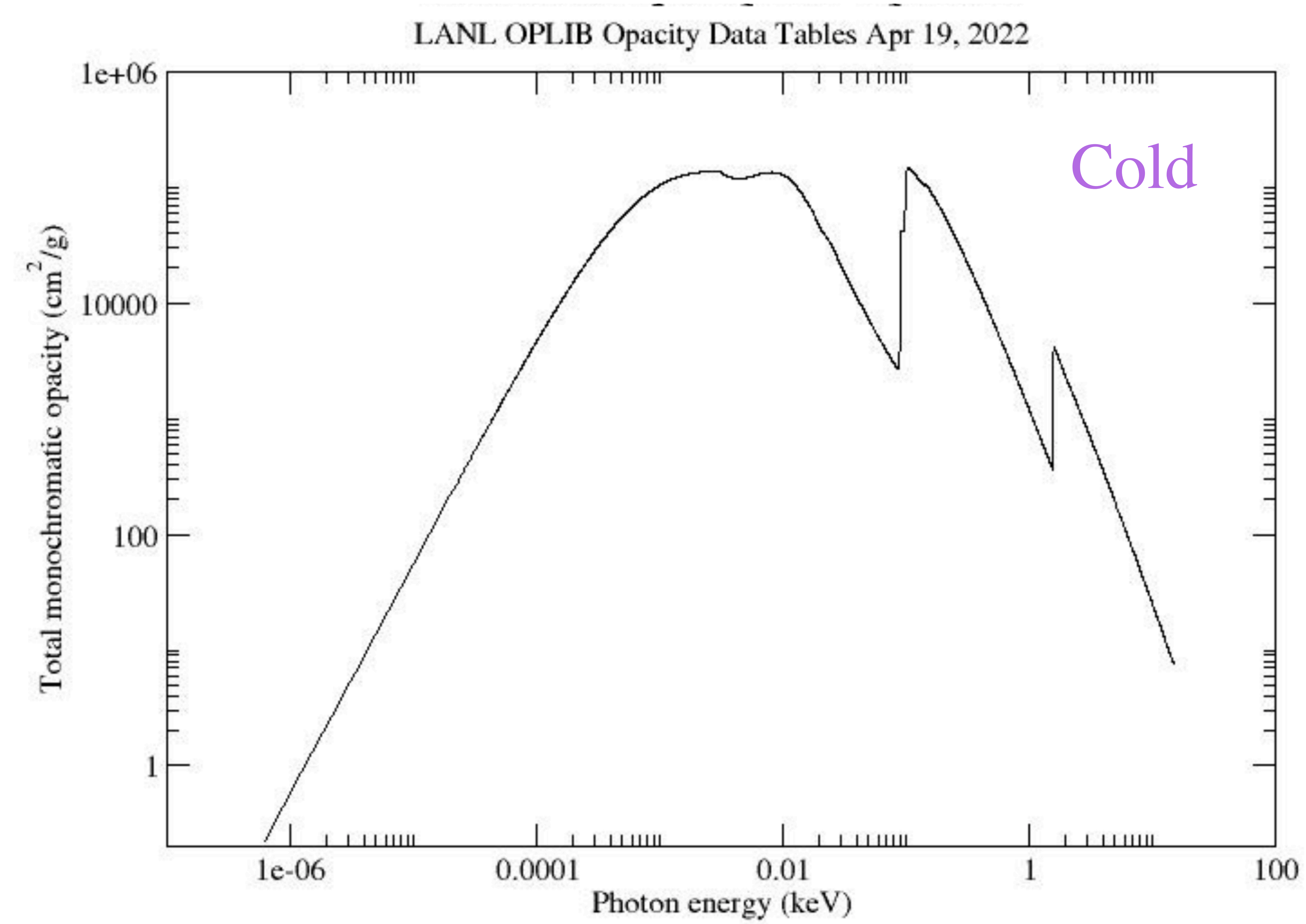
$$\tilde{F}(r) = -\frac{4\pi}{3\rho}\frac{dT}{dr}\int_0^\infty \frac{1}{\kappa_\lambda}\frac{dB_\lambda}{dT}d\lambda$$

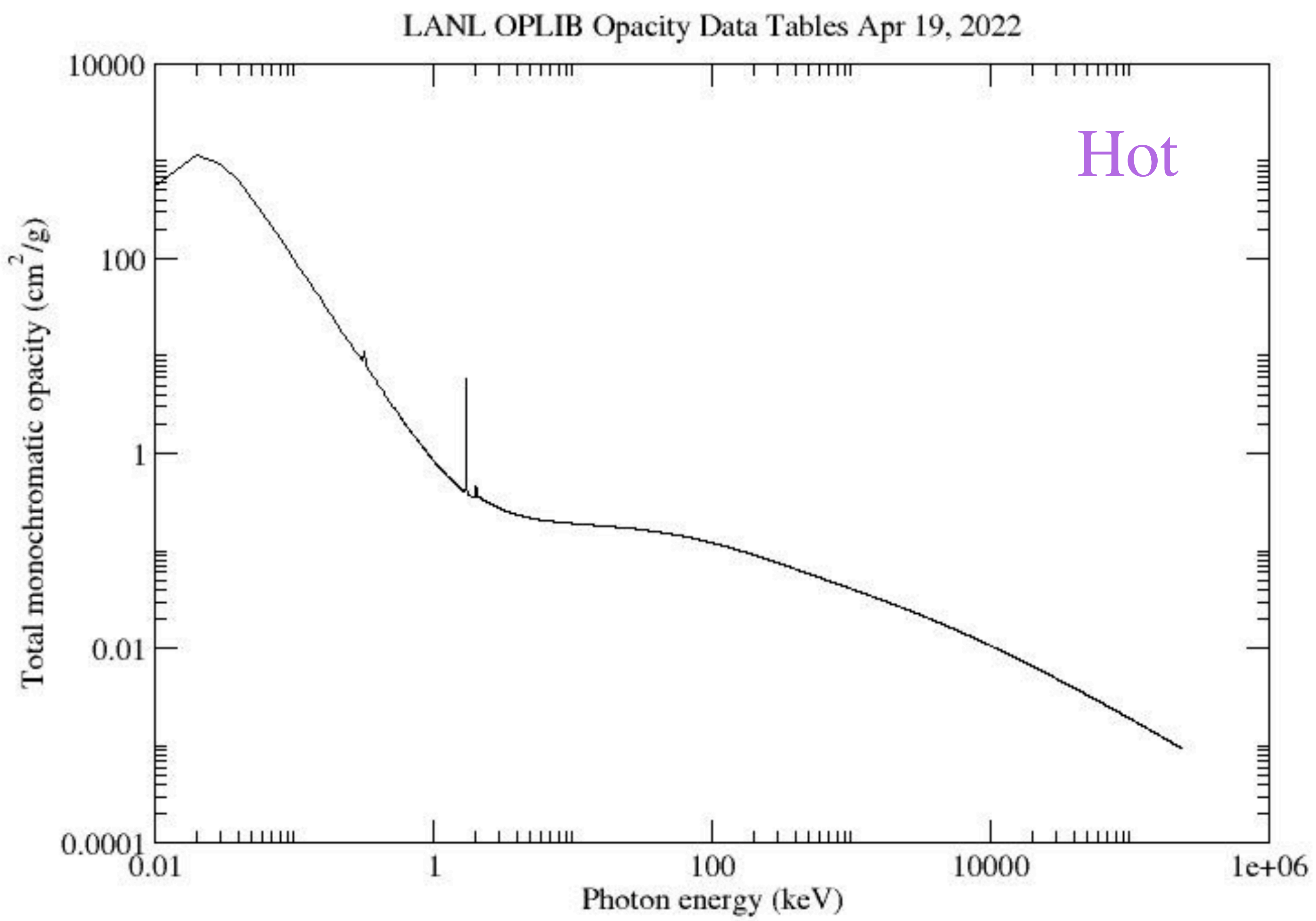
$$\tilde{F}(r) = -\frac{4\pi}{3\rho}\frac{dT}{dr}\int_0^\infty \frac{1}{\kappa_\lambda}\frac{dB_\lambda}{dT}d\lambda$$

This is an analytic equation (we can derivate B_λ with respect to T, and we can also integrate over λ.

$\kappa_\lambda(z) = f(\text{composition}, \mathrm{T(z)})$

Beuh — at each layer, need a full wavelength dependent calculation….

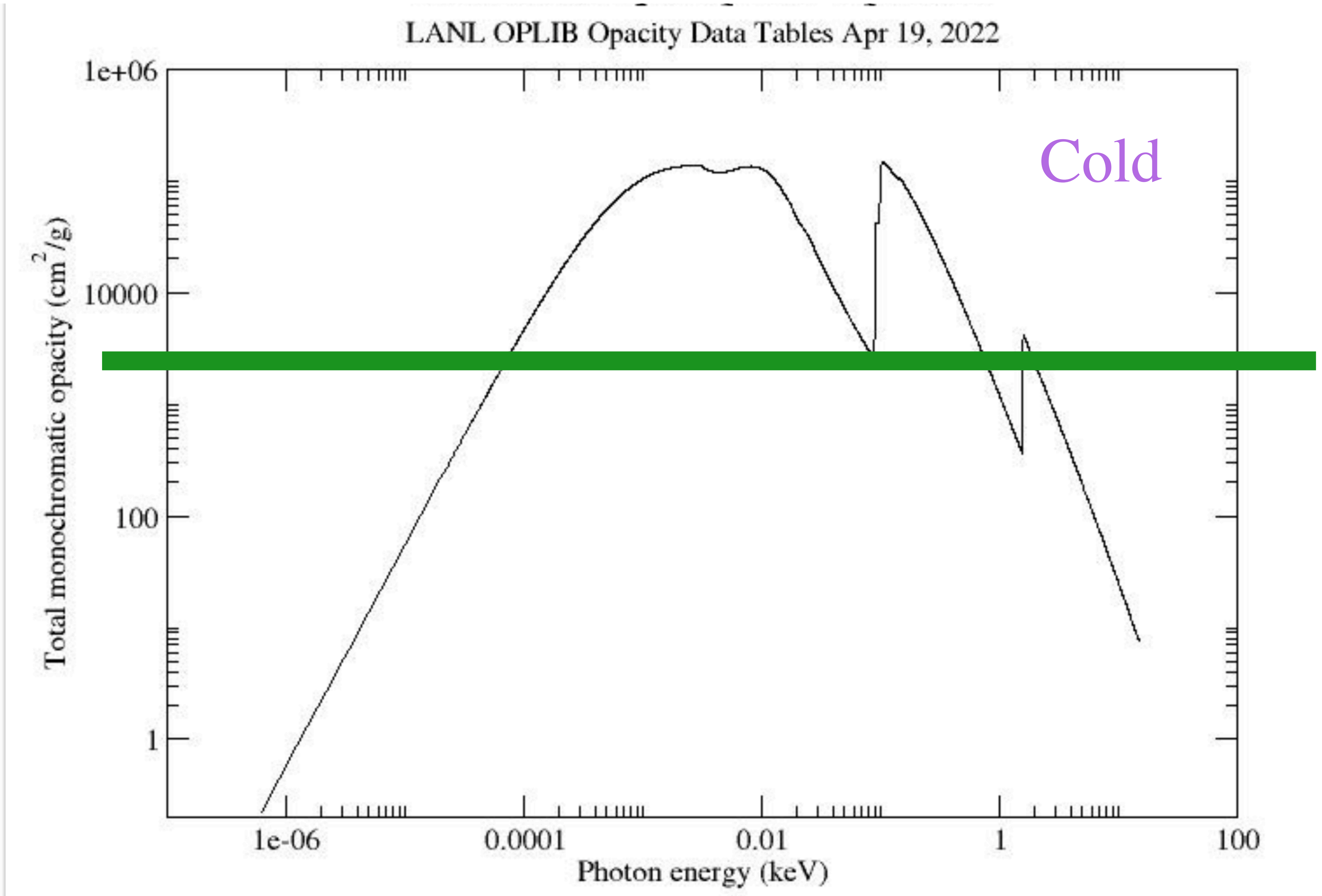


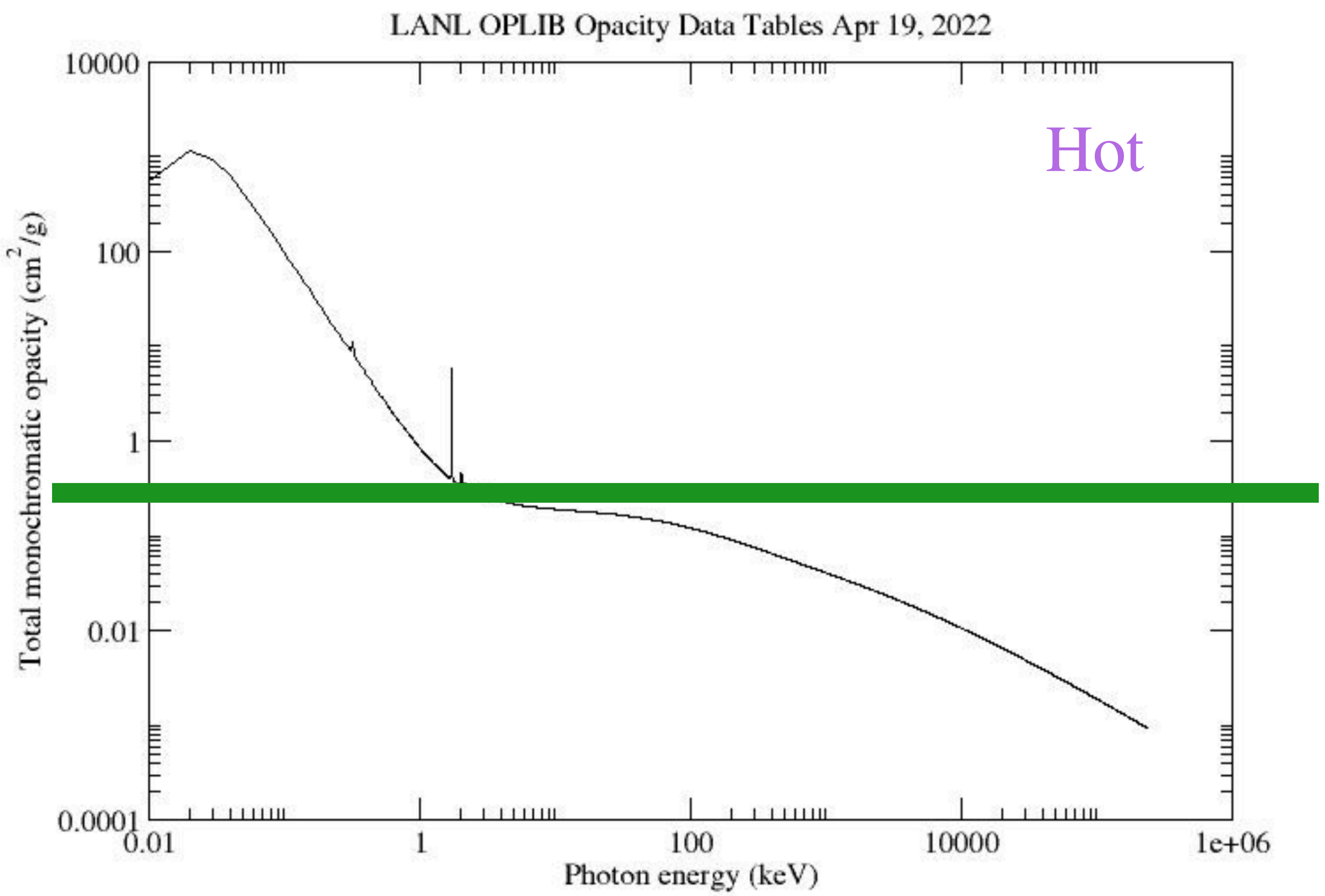


$$\tilde{F}(r) = -\frac{4\pi}{3\rho}\frac{dT}{dr}\int_0^\infty \frac{1}{\kappa_\lambda}\frac{dB_\lambda}{dT}d\lambda$$

This is an analytic equation (we can derivate B_λ with respect to T, and we can also integrate over λ.

How about we take an average value over all λ so that we can pull it out of the integral?





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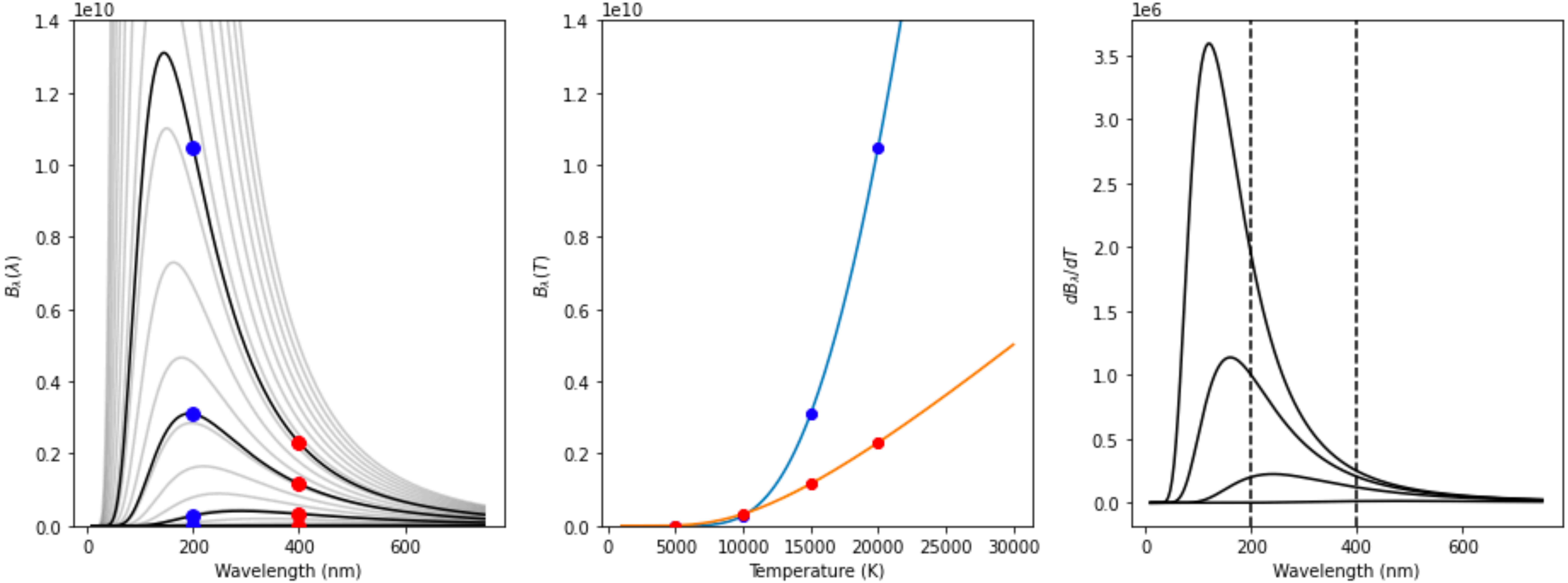

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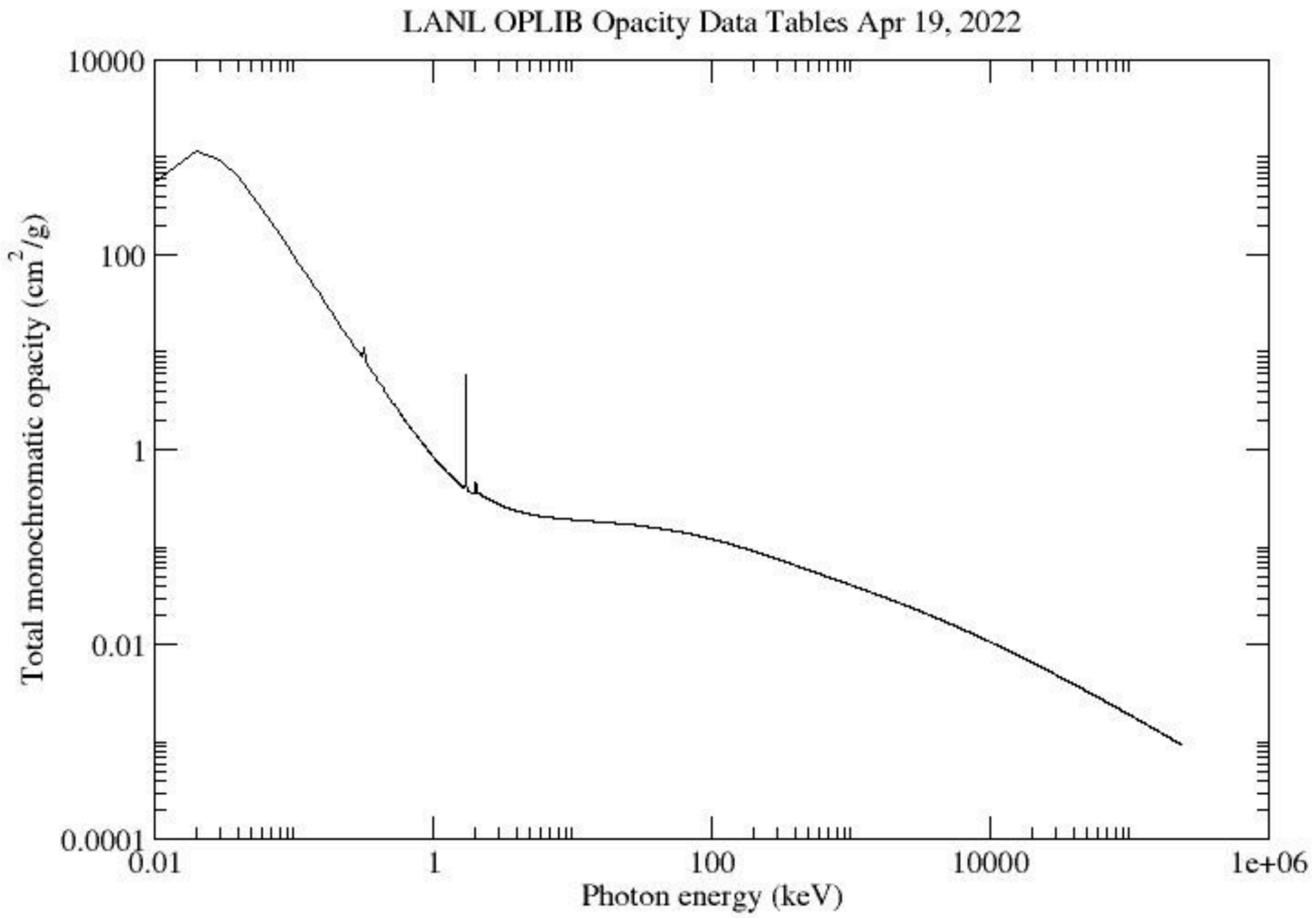
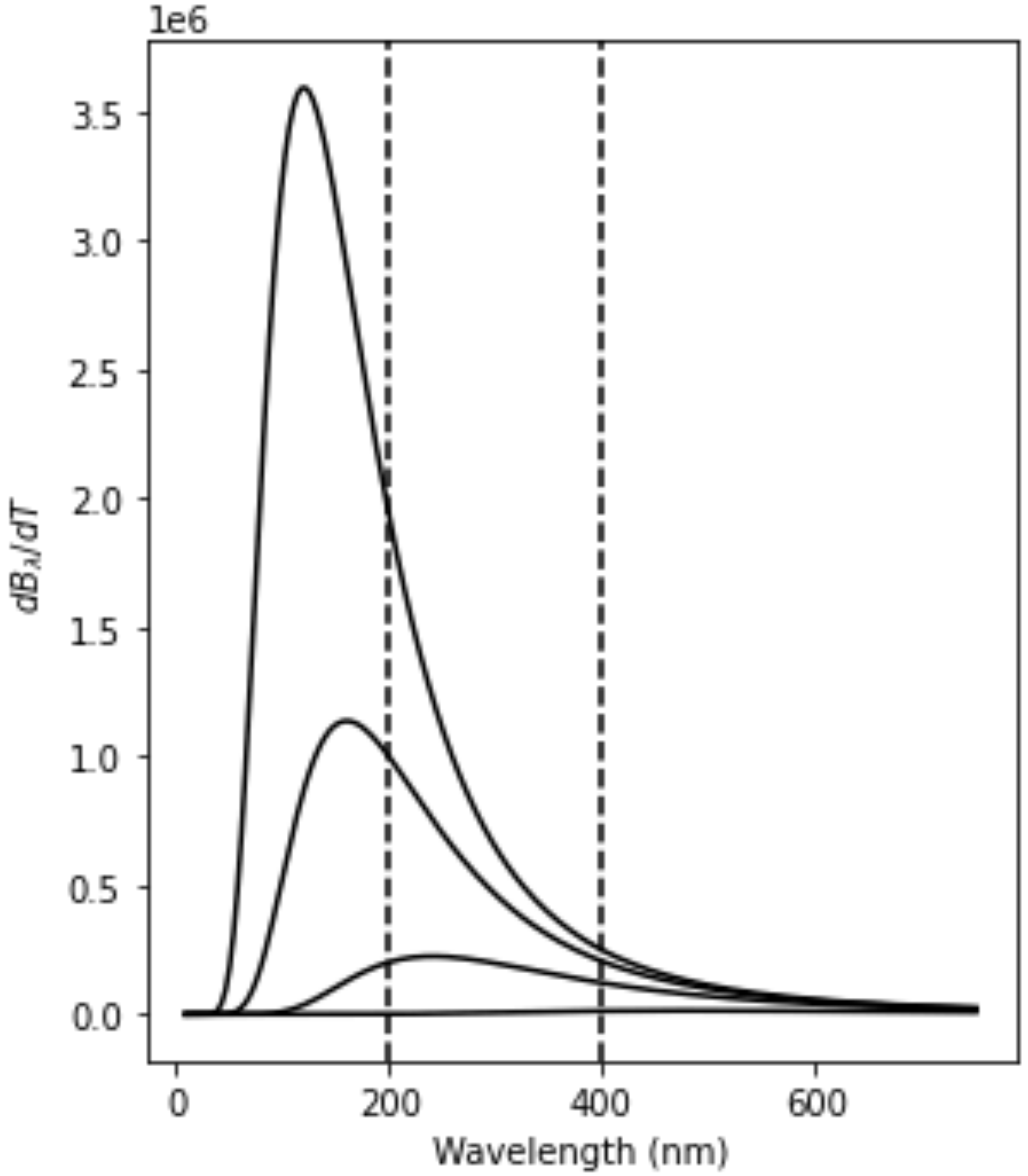


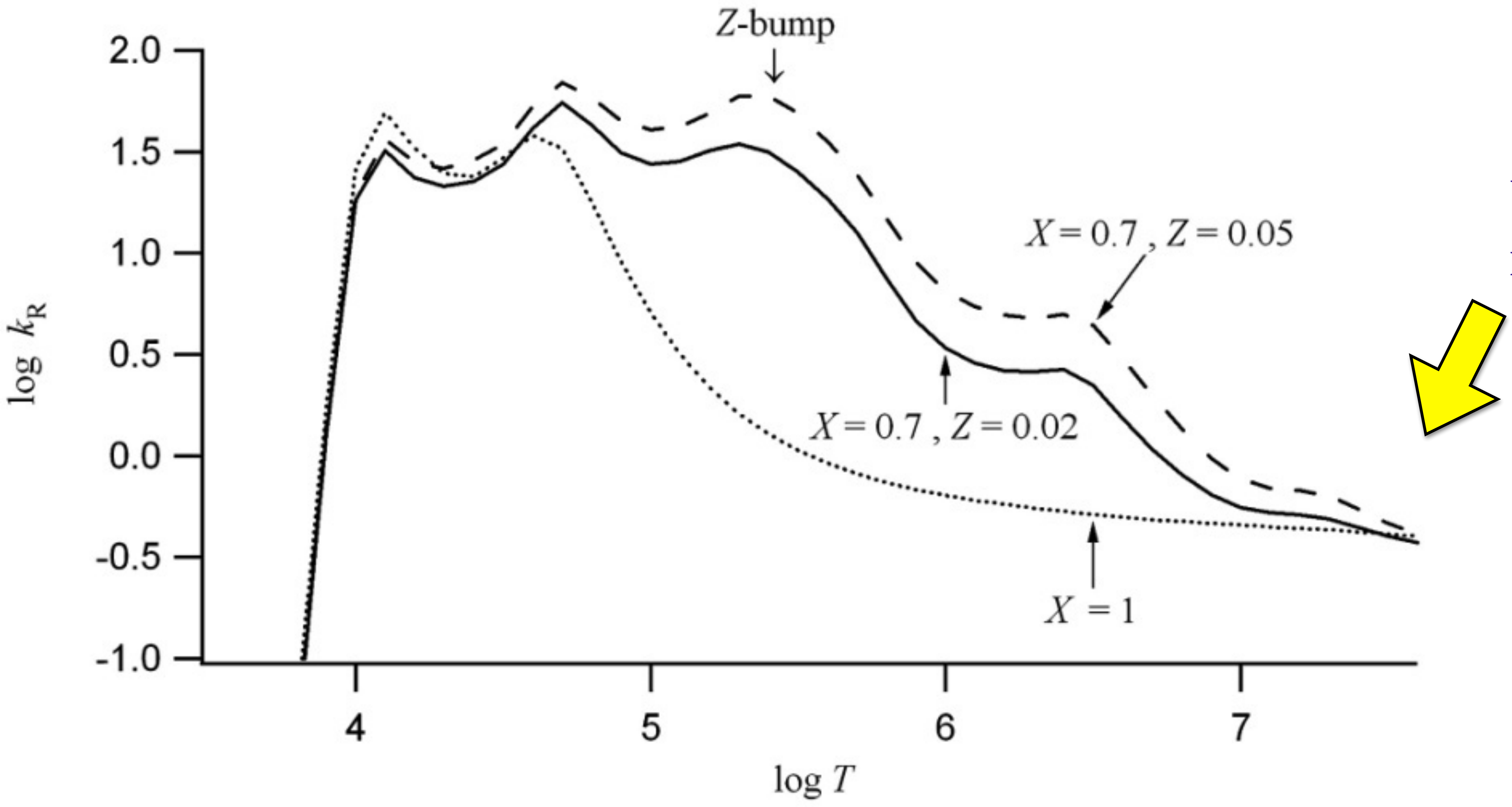
Weighted average:

$$\frac{1}{\kappa_R} = \frac{\int_0^\infty \frac{1}{\kappa_\lambda}\frac{dB_\lambda}{dT}d\lambda}{\int_0^\infty \frac{dB_\lambda}{dT}d\lambda}$$



Rosseland Mean Opacity

$$\frac{1}{\kappa_R} = \frac{\int_0^\infty \frac{1}{\kappa_\lambda} \frac{dB_\lambda}{dT} d\lambda}{\int_0^\infty \frac{dB_\lambda}{dT} d\lambda}$$



Why do they all meet here?

Figure 3.17 Rosseland mean opacity as a function of temperature from the Opacity Project data. The densities used are those for log R = –3. Three curves are shown for different abundances that are defined in the figure. Also identified in the figure is the position of the Z-bump.

$$\tilde{F}(r) = -\frac{4\pi}{3\rho}\frac{dT}{dr}\int_0^\infty \frac{1}{\kappa_\lambda}\frac{dB_\lambda}{dT}d\lambda$$

On the board: replace κ_λ with κ_R, so that we can pull it out of the integral

$$\frac{dT(r)}{dr} = -\frac{3\kappa_R(r)\rho(r)}{64\pi\sigma}\frac{L_r(r)}{r^2T(r)^3}$$

$$M_r(r) \quad P(r) \quad L_r(r) \quad T(r)$$

$$\rho(r) \quad \mu(r) \quad \epsilon_{\text{nuc}}(r) \quad \kappa_R(r)$$

$$\frac{dM_r(r)}{dr} = 4\pi r^2 \rho(r)$$

$$\frac{dP(r)}{dr} = -\rho(r)\frac{GM_r(r)}{r^2}$$

$$\frac{dT(r)}{dr} = -\frac{3\kappa_R(r)\rho(r)}{64\pi\sigma}\frac{L_r(r)}{r^2 T(r)^3}$$

$$\frac{dL_r(r)}{dr} = 4\pi r^2 \rho(r)\epsilon(r)$$

$$P(r) = \frac{\rho(r)}{\mu(r)} kT(r)$$

$$\mu(r) = f(\text{comp}, T(r), P(r))$$

$$\kappa_R(r) = f(\text{comp}, T(r), P(r))$$

$$\epsilon_{\text{nuc}}(r) = f(\text{comp}, T(r), P(r))$$

$$\frac{dM_r(r)}{dr} = 4\pi r^2 \rho(r)$$

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Other energy transport?

$$\frac{dL_r(r)}{dr} = 4\pi r^2 \rho(r)\epsilon(r)$$

$M_r(r) \quad P(r) \quad L_r(r) \quad T(r)$

$\rho(r) \quad \mu(r) \quad \epsilon_{\rm nuc}(r) \quad \kappa_R(r)$

$$P(r) = \frac{\rho(r)}{\mu(r)}kT(r)$$

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$\rho(r) \quad \mu(r) \quad \epsilon_{\rm nuc}(r) \quad \kappa_R(r)$

Other energy transport?

$$\boxed{P(r) = \frac{\rho(r)}{\mu(r)} kT(r)}$$

Always valid?

$$\mu(r) = f(\text{comp}, T(r), P(r))$$

$$\kappa_R(r) = f(\text{comp}, T(r), P(r))$$

$$\epsilon_{\rm nuc}(r) = f(\text{comp}, T(r), P(r))$$

$$\frac{dM_r(r)}{dr} = 4\pi r^2 \rho(r)$$

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$$M_r(r) \quad P(r) \quad L_r(r) \quad T(r)$$

$$\rho(r) \quad \mu(r) \quad \epsilon_{\text{nuc}}(r) \quad \kappa_R(r)$$

Other energy transport?

Ideal gas always valid?

$$P(r) = \frac{\rho(r)}{\mu(r)} kT(r)$$

$$\mu(r) = f(\text{comp}, T(r), P(r))$$

$$\kappa_R(r) = f(\text{comp}, T(r), P(r))$$

$$\boxed{\epsilon_{\text{nuc}}(r) = f(\text{comp}, T(r), P(r))}$$ Nuclear mechanism?

$$\frac{dM_r(r)}{dr} = 4\pi r^2 \rho(r)$$

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$$\frac{dL_r(r)}{dr} = 4\pi r^2 \rho(r)\epsilon(r)$$

$$\boxed{\begin{array}{cccc} M_r(r) & P(r) & L_r(r) & T(r) \\ \rho(r) & \mu(r) & \epsilon_{\rm nuc}(r) & \kappa_R(r) \end{array}}$$

Other energy transport?

Ideal gas always valid?

Nuclear mechanism?

How to solve?

$$P(r) = \frac{\rho(r)}{\mu(r)} kT(r)$$

$$\mu(r) = f(\mathrm{comp}, T(r), P(r))$$

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Other energy transport?

Ideal gas always valid?

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How to solve?

What changes with time?