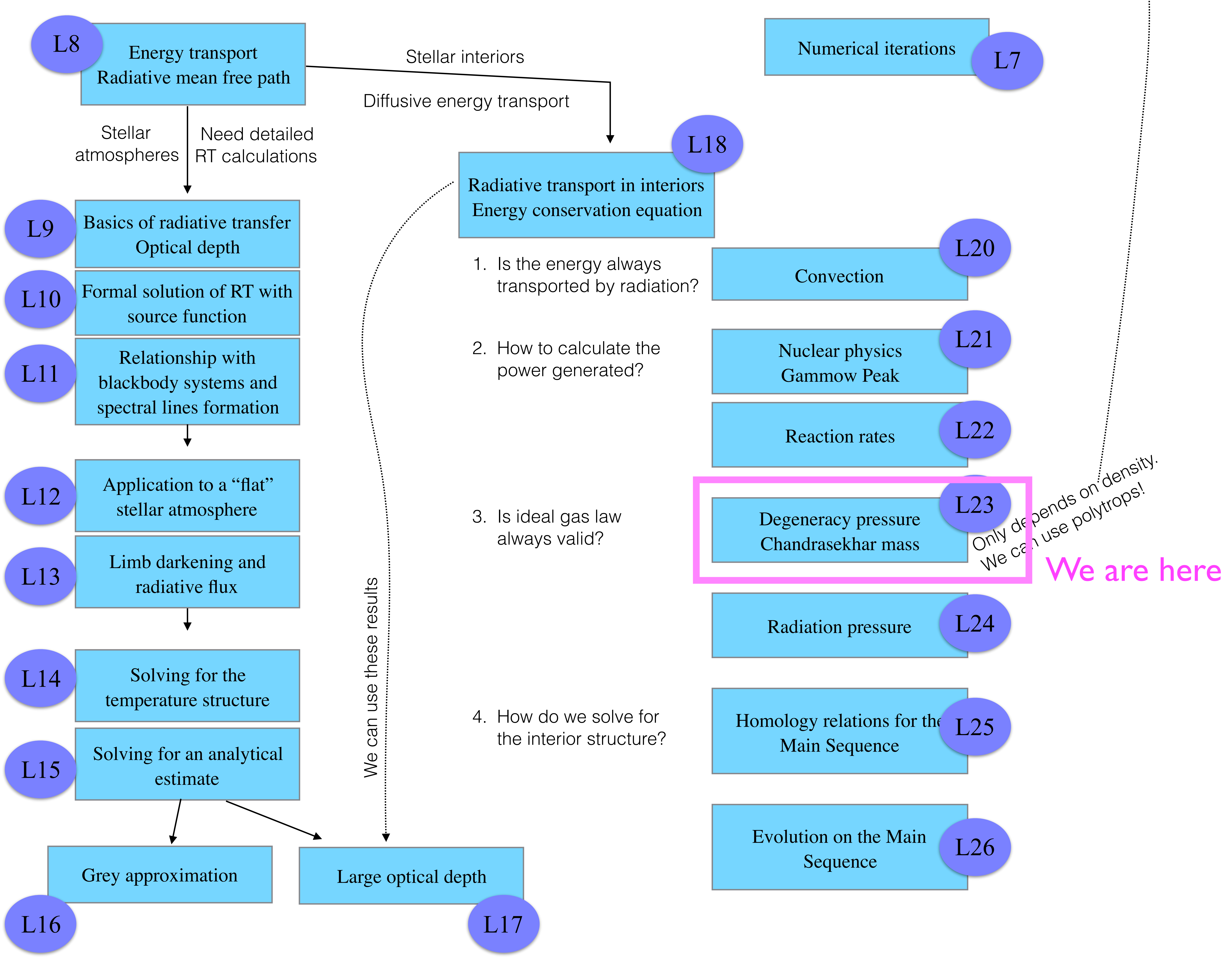
Week 12 Tuesday
L-21


L8
Energy transport
Radiative mean free path
Stellar interiors
Diffusive energy transport
Numerical iterations
L7
Stellar atmospheres
Need detailed RT calculations
L18
Radiative transport in interiors
Energy conservation equation
L9
Basics of radiative transfer
Optical depth
L10
Formal solution of RT with source function
L11
Relationship with blackbody systems and spectral lines formation
1. Is the energy always transported by radiation?
L20
Convection
2. How to calculate the power generated?
L21
Nuclear physics
Gammow Peak
L22
Reaction rates
L12
Application to a “flat” stellar atmosphere
3. Is ideal gas law always valid?
L23
Degeneracy pressure
Chandrasekhar mass
Only depends on density.
We can use polytrops!
We are here
L13
Limb darkening and radiative flux
L24
Radiation pressure
We can use these results
L14
Solving for the temperature structure
4. How do we solve for the interior structure?
L25
Homology relations for the Main Sequence
L15
Solving for an analytical estimate
L26
Evolution on the Main Sequence
Grey approximation
L16
Large optical depth
L17

$$\frac{dM_r(r)}{dr} = 4\pi r^2 \rho(r)$$

$$\frac{dP(r)}{dr} = -\rho(r)\frac{GM_r(r)}{r^2}$$

$$\frac{dT(r)}{dr} = -\frac{3\kappa_R(r)\rho(r)}{64\pi\sigma}\frac{L_r(r)}{r^2 T(r)^3}$$

$$\frac{dL_r(r)}{dr} = 4\pi r^2 \rho(r)\epsilon(r)$$

$$M_r(r) \quad P(r) \quad L_r(r) \quad T(r)$$

$$\rho(r) \quad \mu(r) \quad \epsilon_{\rm nuc}(r) \quad \kappa_R(r)$$

$$P(r) = \frac{\rho(r)}{\mu(r)} kT(r)$$

$$\mu(r) = f(\text{comp}, T(r), P(r))$$

$$\kappa_R(r) = f(\text{comp}, T(r), P(r))$$

$$\epsilon_{\rm nuc}(r) = f(\text{comp}, T(r), P(r))$$

Other energy transport?

Ideal gas always valid?

Nuclear mechanism?

How to solve?

What changes with time?

Summary of derivation for degeneracy pressure:

Momentum is quantized:

$$\vec{p}_{x,y,z} = \frac{h\, n_{x,y,z}}{2L}$$

Relation between the last filled level (n_{Fermi}) and the total number of electrons:

$$n_F = \left(\frac{3N_{tot}}{\pi}\right)^{1/3}$$

Fermi momentum:

$$p(n_F) = \frac{h n_F}{2L} = \frac{h}{2}\left(\frac{3n_e}{\pi}\right)^{1/3}$$

Using the relation between concentration, density, and mean molecular weight:

$$= \frac{h}{2}\left(\frac{3}{\pi m_H}\right)^{1/3}\left(\frac{\rho}{\mu_e}\right)^{1/3}$$

Pressure is a momentum flux through a surface:

$$P_{(x)} = \int_0^{\infty} p_x\, v_x\, \underbrace{n(p)}_{\frac{\#\text{ of electron}}{dV\ dp}}\, 4\pi p^2\, dp$$

In 3D, and spherical coordinates:

$$P_e = \frac{8\pi}{3h^3}\int_0^{p_F} v\, p^3\, dp$$

Non-relativistic case:

$$v(p) = \frac{p}{m_e}$$

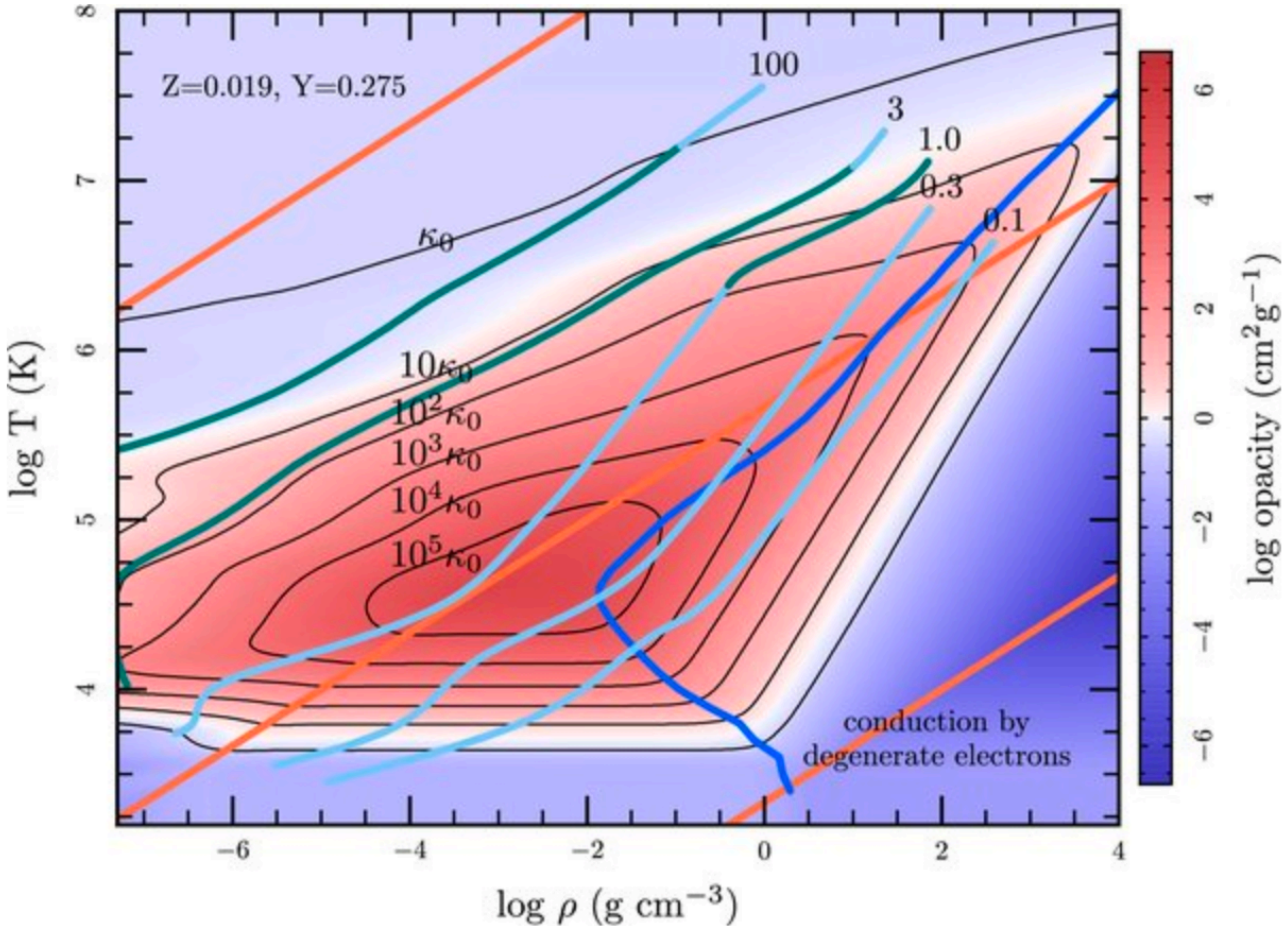
$$P_{deg}^{non\text{-}relativistic} = 1.00 \times 10^{13} \left(\frac{\rho}{\mu_e}\right)^{5/3} \; [dyne/cm^2]$$

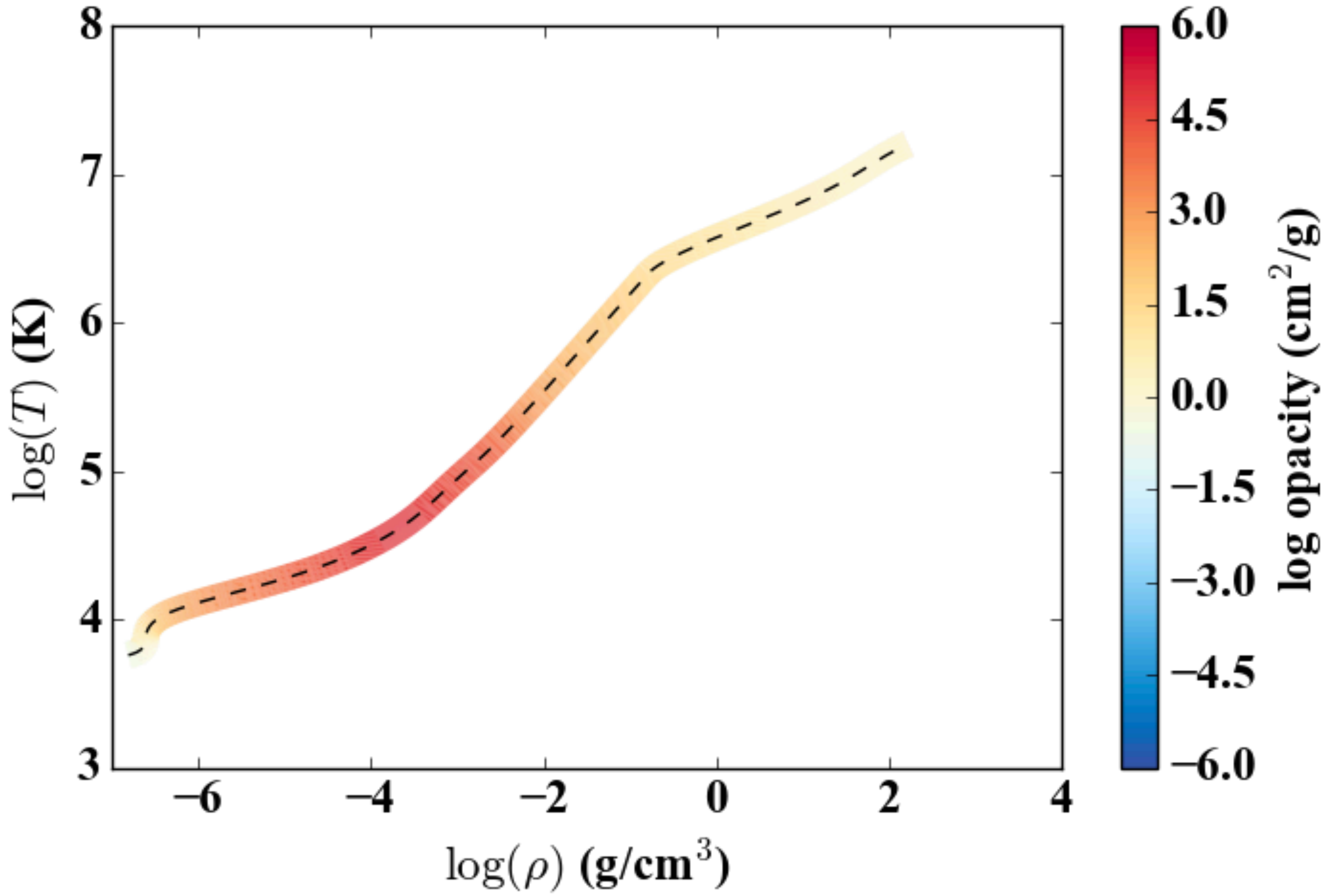
Relativistic case:

$$v \simeq c$$

$$P_{deg}^{relativistic} = 1.24 \times 10^{15} \left(\frac{\rho}{\mu_e}\right)^{4/3} \; dyn/cm^2$$

Remember this graph from the convection notebook:





At which ρ and T does the degeneracy pressure becomes important?

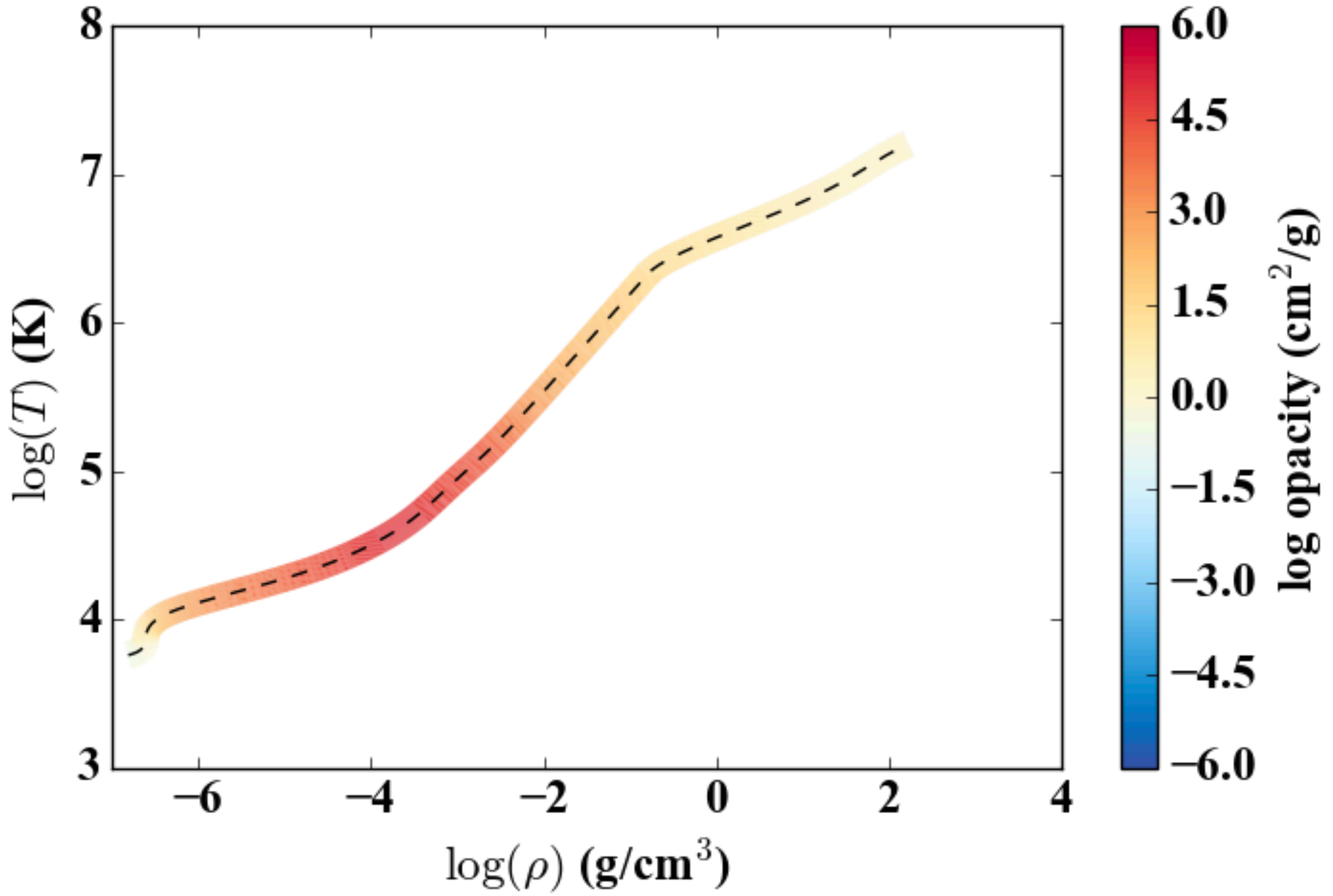
1. Transition from non-relativistic to relativistic degeneracy pressure

$$\left(\frac{\rho}{\mu_e}\right)_{\text{transition}} \approx 1.9 \times 1\ldots$$

2. Transition from ideal gas pressure to degeneracy pressure

$$\left(\frac{\rho}{\mu_e}\right)_{\substack{\text{trans}\\\text{non-rel}}} = 2.5 \times 10^{-8}\ T^{3/2} \left[\frac{g}{cm^3}\right]$$

$$\left(\frac{\rho}{\mu_e}\right)_{\substack{\text{trans}\\\text{rel}}} = 2.95 \times 10^{-22}\ T^{3} \left[g/cm^3\right]$$



$$P_{deg}^{\text{non-relativistic}} = 1.00 \times 10^{13} \left(\frac{\rho}{\mu_e}\right)^{5/3} \left[dyne/cm^2\right]$$

$$P_{deg}^{\text{relativistic}} = 1.24 \times 10^{15} \left(\frac{\rho}{\mu_e}\right)^{4/3} \; dyn/cm^2$$

$$P_{deg}^{\text{non-relativistic}} = 1.00 \times 10^{13} \left(\frac{\rho}{\mu_e}\right)^{5/3} \ [dyne/cm^2]$$

$$P_{deg}^{\text{relativistic}} = 1.24 \times 10^{15} \left(\frac{\rho}{\mu_e}\right)^{4/3} \ dyn/cm^2$$

Polytrops

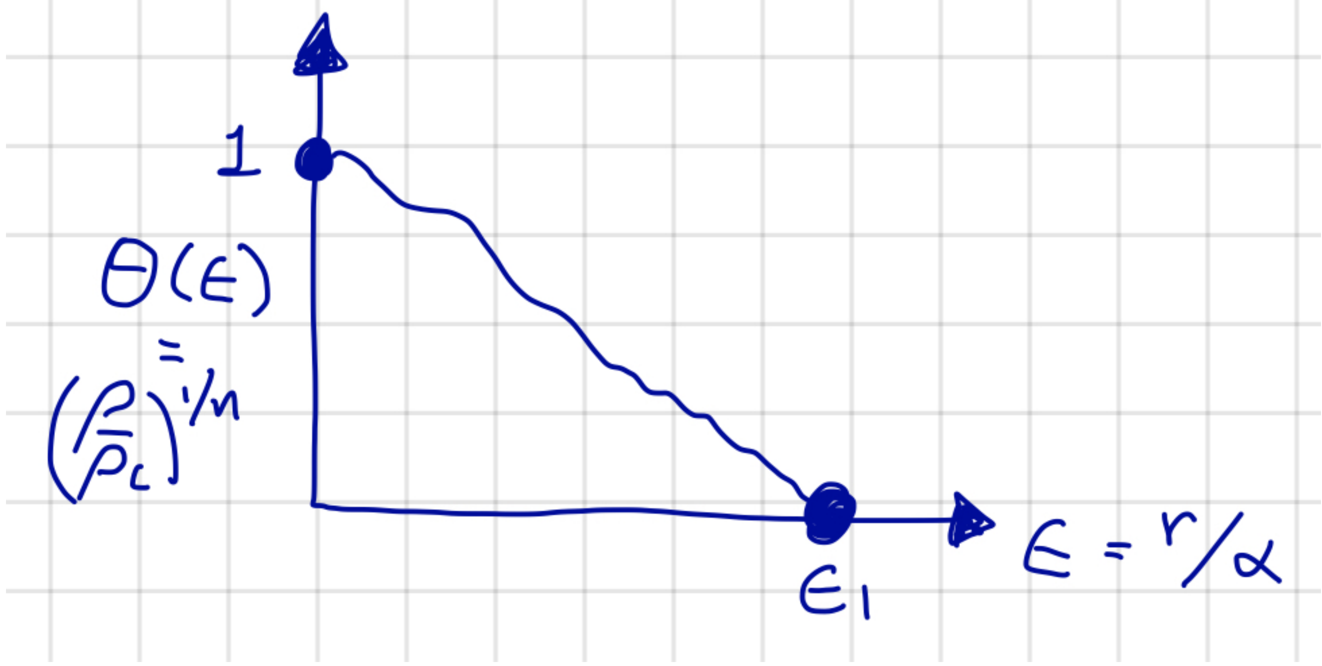
$$\boxed{P = k\rho^{1+1/n}}$$

$$P_{deg}^{non\text{-}relativistic} = 1.00 \times 10^{13} \left(\frac{\rho}{\mu_e}\right)^{5/3} \; [dyne/cm^2]$$

$$P_{deg}^{relativistic} = 1.24 \times 10^{15} \left(\frac{\rho}{\mu_e}\right)^{4/3} \; dyn/cm^2$$

Polytrops

$$\boxed{P = K\rho^{1+1/n}}$$



n

if we pick a

$P_c \quad \rho_c \quad \boxed{R_\star}$

$\boxed{K}$ known

we can find the $M_\star$

1 $$R_\star = \alpha\epsilon_1 = \left(\frac{(n+1)P_c}{4\pi G\rho_c^2}\right)^{1/2} \epsilon_1$$

2 $$P_c = K\rho_c^{(n+1)/n}$$

3 $$M_\star = -\frac{1}{(4\pi)^{1/2}} \left(\frac{n+1}{G}\right)^{3/2} \frac{P_c^{3/2}}{\rho_c^2} \, \epsilon_1^2 \, \theta'(\epsilon_1)$$

2. At home: Use the properties of polytrope to understand non-relativistic white dwarfs

White dwarfs are the remnant of solar mass object, once the fusion stops. WD are generally made of elements heavier than He, e.g. carbon or oxygen depending on their evolution. In the case were no H or He is present in a gas, the electronic mean molecular weigth $\mu_e \sim 2$.

These objects remain in hydrostatic equilibrium because of non-relativistic degeneracy pressure, of the form

$$P_e = 1.00 \times 10^{13} \left(\frac{\rho}{\mu_e}\right)^{5/3}.$$

The numerical constant is such that the pressure is in dyn/cm2 for a density given in g/cm3.

You probably notice that the pressure does not depend on temperature, just like a polytrop!

This would correspond to a polytrop of index n=3/2, with $K = 1.00 \times 10^{13}/\mu_e^{5/3}$. For such a polytrop, $\epsilon_1 = 3.654$, and $(-\theta(\epsilon_1)') = 0.20330$, according to Henson Table 7.1.

3. At home: Do the same as #2, for a relativistic white dwarf.