

# INSTRUMENTAL VARIABLE ESTIMATION (I)

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## Learning objectives

At the end of this lecture you will be able to

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- Define an instrumental variable
- Define the standard instrumental variable estimator
- Provide examples of commonly used instruments

## □ Key concepts

- Instrument
- Surrogate instrument
- Two-stage least squares estimator

Our goal: To estimate the average causal effect of treatment  $A$  on outcome  $Y$

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□ For example

- the effect of aspirin ( $A$ ) on blood pressure ( $Y$ )
- the effect of smoking cessation ( $A$ ) on weight gain ( $Y$ )

□ The average causal effect is the difference of counterfactual means  $E[Y^{a=1}] - E[Y^{a=0}]$

To validly estimate causal effects we will generally need to adjust for confounding

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For example

- Individuals with history of coronary heart disease are more likely to receive aspirin ( $A$ ) and also to have higher blood pressure ( $Y$ )
- Heavy smokers are less likely to quit smoking ( $A$ ) and also less likely to gain weight ( $Y$ )

## Methods to adjust for confounding

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- Stratification/Regression
- Matching
- Stratification/matching based on propensity scores
- Standardization/G-formula
- IP weighting
- G-estimation

□ All these methods require one unverifiable condition...

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Instrumental variables (I)

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## The measured confounders $L$ are sufficient to control all confounding

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“No unmeasured confounding” condition

Requires **conditional exchangeability**

- the outcome distribution in the treated if untreated is the same as the outcome distribution in the untreated, and vice versa, within levels of  $L$

$$Y^a \perp\!\!\!\perp A | L = l \quad \text{for all } a$$

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Instrumental variables (I)

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Conditional exchangeability is necessary for all the above methods, but...

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... it is also empirically **unverifiable**

- Data to test this condition are, by definition, unavailable

□ For example:

- We cannot prove that individuals who take and do not take aspirin are comparable after conditioning on the measured confounders
- because these individuals may still not be comparable with respect to unmeasured confounders

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Instrumental variables (I)

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## Meet instrumental variable (IV) estimation

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□ We can identify causal effects using IV estimation even if we do not measure the confounders!

□ Sounds like magic?

- Economists have used IV methods for a long time
- Epidemiologists are increasingly using IV methods

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Instrumental variables (I)

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## The “magic” of IV estimation applies to treatment-outcome confounders only

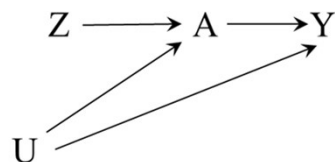
- For example, if there is time-varying selection bias (e.g., because of differential loss to follow-up), then additional adjustment is necessary
  - Such adjustment requires additional data
  - e.g., data on joint predictors of censoring and outcome to estimate IP weights and then implement IV estimation in the pseudo-population

Instrumental variables (I)

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## $Z$ is an instrument if it meets the 3 instrumental conditions

- i.  $Z$  is associated with treatment  $A$ 
  - relevance condition
- ii.  $Z$  affects the outcome  $Y$  only through treatment  $A$ 
  - exclusion restriction
- iii.  $Z$  does not share causes with the outcome  $Y$ 
  - no confounding for  $Z$



Instrumental variables (I)

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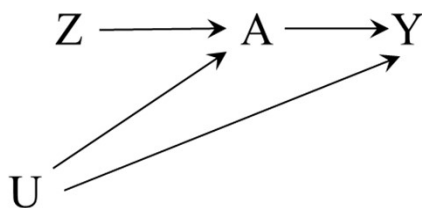
## Outline

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1. IV estimation in randomized experiments
2. IV estimation in observational studies
3. Application to smoking cessation study
  - Standard IV estimator
  - Two-stage estimator
4. Limitations of IV estimation
5. Conclusions

## Example: Double-blind randomized experiment

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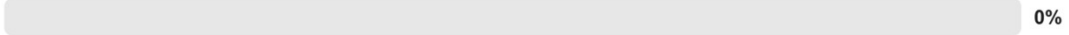


- ☐  $Z$ : assigned treatment
  - Aspirin (1: yes, 0: no)
- ☐  $A$ : actual treatment
  - Aspirin (1: yes, 0: no)
- ☐  $Y$ : outcome
  - Blood pressure
- ☐  $U$ : unmeasured factors
  - Healthy lifestyle

**The Z-A association is a valid estimator  
of the effect of Z on A**

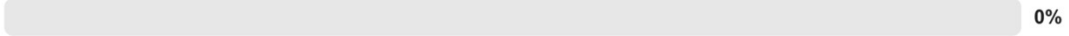
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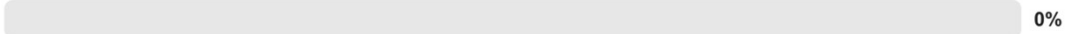
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**The Z-Y association is a valid estimator  
of the effect of Z on Y**

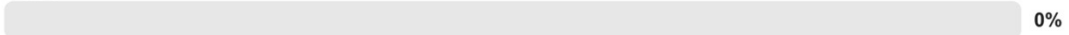
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## We can estimate the intention-to-treat effect

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- That is, the effect of being assigned to aspirin vs. no aspirin
  - regardless of whether trial participants adhered to their assignment
  - this is the effect of  $Z$  on  $Y$
- However, the intention-to-treat effect is hard to interpret because it critically depends on the degree of adherence
  - the effect of  $Z$  on  $A$

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Instrumental variables (I)

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## We would like to estimate the per-protocol effect

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- That is, the effect of actually receiving aspirin vs. no aspirin
  - the effect if trial participants adhered to their assignment
  - this is the effect of  $A$  on  $Y$

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Instrumental variables (I)

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The A-Y association is a valid estimator  
of the effect of A on Y

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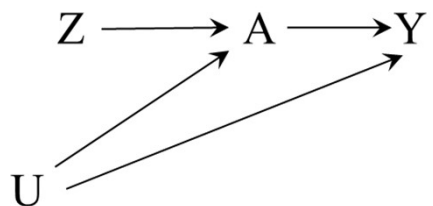
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## IV estimation to the rescue



- IV estimation can be used to estimate the effect of  $A$  on  $Y$  as long as we have an instrument  $Z$ 
  - even if no confounders  $U$  are measured

## How can IV estimation work?

- By taking advantage of two average causal effects that can be estimated without bias
  - The effect of  $Z$  on  $Y$
  - The effect of  $Z$  on  $A$

Instrumental variables (I)

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Two effects can be directly estimated because they are unconfounded

Effect of  $Z$  on  $Y$

$$E[Y|Z=1] - E[Y|Z=0]$$

□ on additive scale

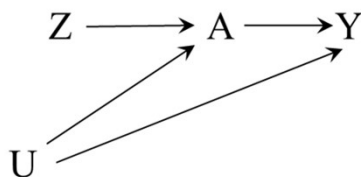
□ Intention-to-treat effect

Effect of  $Z$  on  $A$

$$E[A|Z=1] - E[A|Z=0]$$

□ on additive scale

□ Adherence (compliance)



Instrumental variables (I)

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The average causal effect of  $A$  on  $Y$   
 $E[Y^{a=1}] - E[Y^{a=0}]$  is the standard IV estimand

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$$\frac{\text{Effect of } Z \text{ on } Y}{\text{Effect of } Z \text{ on } A} = \frac{\text{ITT effect}}{\text{Adherence}} = \frac{E[Y|Z = 1] - E[Y|Z = 0]}{E[A|Z = 1] - E[A|Z = 0]}$$

- Intention-to-treat effect in the numerator is inflated by a measure of adherence in the denominator
  - If full compliance, the denominator is 1 and the intent-to-treat effect equals the effect of  $A$  on  $Y$
  - See Chapter 16 for a proof under additional conditions

## IV estimands for the average causal effect of $A$ on $Y$

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- The standard IV estimand measures the effect on the additive scale for a dichotomous instrument
 

$$\frac{E[Y|Z = 1] - E[Y|Z = 0]}{E[A|Z = 1] - E[A|Z = 0]}$$

  - Difference of means or risks
- Other IV estimands yield the effect on the multiplicative scale
  - Ratio of means or risks

## Outline

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2. IV estimation in observational studies
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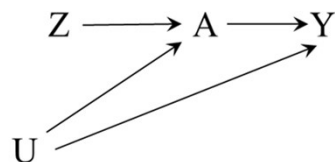
Instrumental variables (I)

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  - no confounding for  $Z$



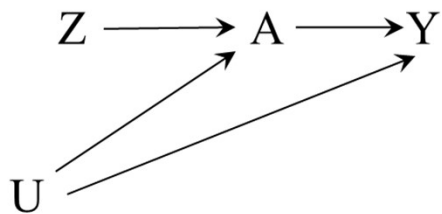
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Instrumental variables (I)

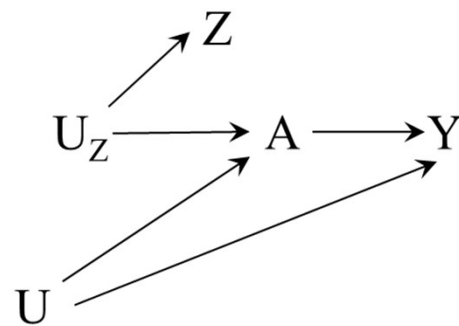
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## Instrument Z

Causal instrument



Surrogate instrument



Instrumental variables (1)

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For surrogate instruments, the Z-A association is a valid estimator of the effect of Z on A

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For surrogate instruments, the Z-Y association is a valid estimator of the effect of Z on Y

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For surrogate instruments, the A-Y association is a valid estimator of the effect of A on Y

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## A surrogate instrument $Z$ can also be used for IV estimation

- i.  $Z$  is associated with treatment  $A$ 
  - It is OK if the  $Z - A$  association is not a valid estimator of the effect of  $Z$  on  $A$
  - As long as the causal instrument  $U_Z$  is
- ii.  $Z$  affects the outcome  $Y$  only through treatment  $A$ 
  - It is OK if  $Z$  does not affect the outcome  $Y$  through  $A$
  - As long as the causal instrument  $U_Z$  only affects the outcome through treatment  $A$
- iii.  $Z$  does not share causes with the outcome  $Y$ 
  - It is OK if  $Z$  and  $Y$  share the causal instrument  $U_Z$  as a cause
  - $U_Z$  cannot share common causes with outcome  $Y$

Instrumental variables (I)

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Z in the graph below is a valid surrogate instrument

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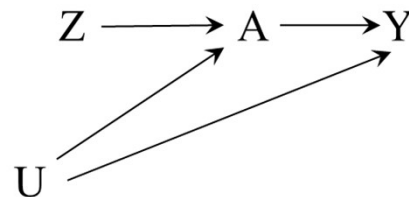
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## Proposed instruments in observational studies Genetic variants

- $A$ : Alcohol intake (1: heavy drinking, 0: mild/no drinking)
- $Y$ : Coronary heart disease
- $Z$ : Genetic variants associated with alcohol metabolism, e.g., ALDH2 polymorphisms in Asian populations



### □ Mendelian randomization

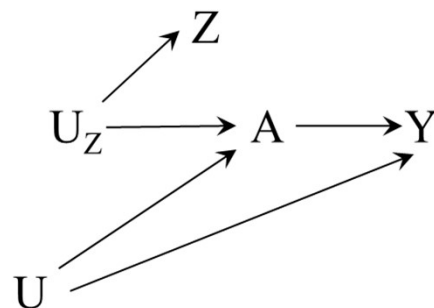
- Katan 1986, Davey Smith and Ebrahim 2004, Chen et al 2008...

Instrumental variables (I)

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## Proposed instruments in observational studies Genetic variants (surrogate)

- $A$ : Dietary calcium
- $Y$ : Osteoporosis
- $U_Z$ : Lactose intolerance gene
- $Z$ : Self-reported lactose intolerance



- Causal instrument  $U_Z$  is unmeasured, instrument  $Z$  is a proxy for  $U_Z$

Instrumental variables (I)

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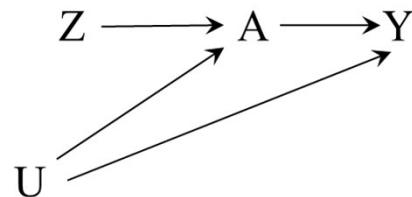


## Proposed instruments in observational studies

### Preference

#### □ Lung cancer patients

- $A$ : Type of chemotherapy
- $Y$ : 5-year mortality
- $Z$ : Physician's preference for type of chemotherapy



#### □ Pharmacoepidemiology / Outcomes research

- Korn and Baumrind 1998, Earle et al 2001, Brookhart et al 2006...

Instrumental variables (I)

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Just out of curiosity, do you think this proposed instrument is a valid instrument?

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(A) Yes

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(B) No

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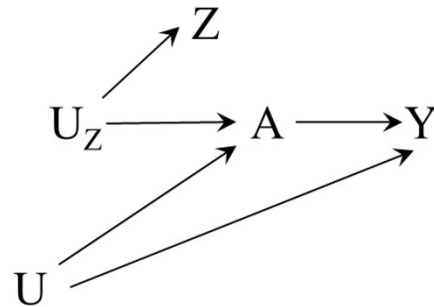
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## Proposed instruments in observational studies

### Preference (surrogate)

#### □ Lung cancer patients

- $A$ : Type of chemotherapy
- $Y$ : 3-year mortality
- $U_Z$ : Physician's preference for type of chemotherapy
- $Z$ : Proportion of patients recently treated by that physician who received  $A=1$



#### □ Causal instrument $U_Z$ is unmeasured, instrument $Z$ is a proxy for $U_Z$

Instrumental variables (I)

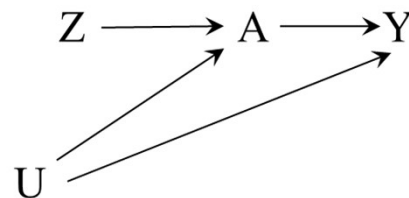
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## Proposed instruments in observational studies

### Access: physical distance

#### □ Patients with myocardial infarction

- $A$ : Invasive procedures
- $Y$ : Mortality
- $Z$ : Distance from patient's residence to hospital with capability for invasive procedures



□ McClellan et al 1994

Instrumental variables (I)

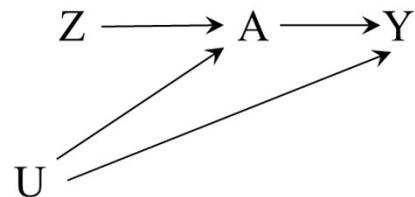
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## Proposed instruments in observational studies

### Access: price

#### □ Patients with myocardial infarction

- $A$ : Cigarette smoking cessation
- $Y$ : Health outcome (physical functional status)
- $Z$ : Cigarette price
  - Leigh and Schembri 2004



Instrumental variables (I)

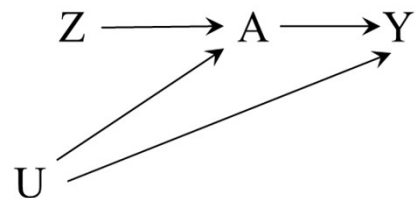
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## Proposed instruments in observational studies

### Access: calendar time

#### □ HIV-infected patients

- $A$ : Type of antiretroviral therapy (monotherapy, combination therapy, cART)
- $Y$ : AIDS or death
- $Z$ : Calendar period (1990-93, 1994-95, 1996-97)
  - Hoover et al 1994, Detels et al 1998



Instrumental variables (I)

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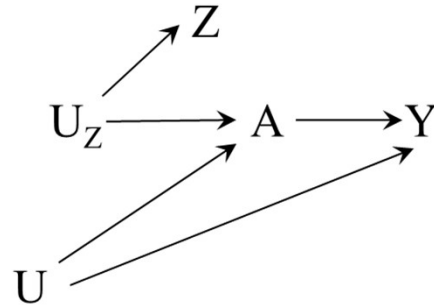
## Proposed instruments in observational studies

### Access (surrogate)

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- Physical distance, price, calendar period, can also be seen as surrogates of the causal instrument accessibility  $U_Z$

- Distance  $Z$  is an approximation to actual driving time (e.g., traffic patterns may be important too)
- Cigarette price in a state  $Z$  is an approximation to price at the closest store
- Calendar time  $Z$  is an approximation to accessibility to a treatment because there might be a fuzzy boundary of availability



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Instrumental variables (I)

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Instrumental variables (I)

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## Study population

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- ☐ 1629 cigarette smokers
- ☐ Aged 25-74 years when interviewed in 1971-75 (baseline)
- ☐ Interviewed again in 1982
- ☐ Known sex, age, race, weight, height, education, alcohol use, and smoking intensity at both baseline and follow-up visits, and who answered the general medical history questionnaire at baseline

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Instrumental variables (I)

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## Key variables

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<b>Treatment A</b>	Quit smoking between baseline and 1982 1: yes, 0: no
<b>Continuous outcome Y</b>	Weight gain, kg Weight in 1982 minus baseline weight Available for 1566 individuals
<b>Proposed instrument Z</b>	Price of a pack of cigarettes in 1982 in the state of birth in 2008 US dollars 1: greater than \$1.50, 0: \$1.50 or lower
<b>Y and Z available for 1476 individuals</b>	For simplicity, IV analysis restricted to them

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Instrumental variables (I)

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## Reminder

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- Average causal effect of  $A$  on  $Y$  is  $E[Y^{a=1}] - E[Y^{a=0}]$ 
  - on the additive scale
  
- where
  - $E[Y^{a=1}]$  is the mean weight gain if everybody had quit smoking
  - $E[Y^{a=0}]$  is the mean weight gain if nobody had quit smoking

## Data analysis

### Standard IV estimator

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- $\hat{E}[Y|Z=1] - \hat{E}[Y|Z=0]$ 
  - $= 2.686 - 2.536 = 0.1503$
- $\hat{E}[A|Z=1] - \hat{E}[A|Z=0] =$ 
  - $0.2578 - 0.1951 = 0.0627$
  
- The IV estimate of  $E[Y^{a=1}] - E[Y^{a=0}]$  is
  - $0.1503 / 0.0627 = 2.4 \text{ kg}$     *See iv.R, lines 6-11*

## Data analysis

### Two-stage least-squares estimator

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- Stage 1: Fit a linear model for treatment
  - $E[A|Z] = \alpha_0 + \alpha_1 Z$
  - Generate the predicted values  $\hat{E}[A|Z]$  for each individual
- Stage 2: Fit a linear model for the outcome
  - $E[Y|Z] = \beta_0 + \beta_1 \hat{E}[A|Z]$
- The parameter estimate of  $\beta_1$  is the IV estimate
  - 2.4 kg; 95% CI (-36.5, 41.3) *See iv.R, lines 16-24*

## Standard estimator and two-stage estimator are equivalent IV estimators

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- But the two-stage estimator is more frequently used because it makes it easier to
  - introduce covariates
  - handle continuous treatments
  - consider multiple instruments simultaneously
- However, the two-stage estimator makes strong parametric assumptions
  - Robins's g-estimators of structural mean models avoid some of those assumptions but have been rarely used

## Isn't it amazing?

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- NO DATA on confounders are needed!
- If we find an instrument
  - conditional exchangeability of the treated and untreated is not necessary for causal inference from observational data
- Why have we wasted our time with confounding adjustment methods that require exchangeability?

## Progress report

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1. Introduction to modeling
2. Stratified analysis:
  - outcome regression
  - propensity scores
3. Standardization
4. Inverse probability weighting
  - Marginal structural models
5. Instrumental variable estimation  
to be continued...