Instrumental Variable Estimation: Technical Points

EPI289 Epidemiologic Methods III: Models for Causal Inference

1 Calculating the proportion of compliers

1.1 Compliance types

In the setting of a binary time-fixed instrument and a binary time-fixed treatment, we can describe participants as belonging to one of four mutually-exclusive "compliance types":

1. Always-takers
2. Never-takers
3. Compliers
4. Defiers

We cannot identify an individual's compliance type using observational data. This would require us to know both (1) their treatment level had their instrument been set to a value of 0 (i.e., $A^{Z=0}$) and (2) their treatment level had their instrument been set to a value of 1 (i.e., $A^{Z=1}$).

With observational data, each individual's instrument and treatment level would be compatible with two compliance types:

Table 1: Compliance types corresponding to each combination of observed instrument and treatment levels

	$A = 1$	$A = 0$
$Z = 1$	Always taker or complier	Never taker or defier
$Z = 0$	Always taker or defier	Never taker or complier

1.2 Compliance types under the monotonicity assumption

Under the monotonicity condition, we assume that there are no defiers in our study. Therefore, our table of compliance types based on the observed instrument and treatment levels can be simplified to the following:

Table 2: Compliance types corresponding to each combination of observed instrument and treatment levels under the monotonicity condition

	$A = 1$	$A = 0$
$Z = 1$	Always taker or complier	Never taker
$Z = 0$	Always taker	Never taker or complier

1.3 Calculating the proportion of compliers

If Z is randomized, we can assume the distribution of compliance types is the same across levels of Z. Therefore, under monotonicity, we can estimate the proportion of never takers or always takers by looking within levels of Z:

- **Proportion of never takers**: among individuals with $Z = 1$, never takers can be identified as those with $A = 0$

$$\text{Proportion of never takers} = \Pr[A = 0|Z = 1]$$

- **Proportion of always takers**: among individuals with $Z = 0$, always takers can be identified as those with $A = 1$

$$\text{Proportion of always takers} = \Pr[A = 1|Z = 0]$$

Therefore, the proportion of compliers can be calculated as:

$$\begin{aligned}
\text{Proportion of compliers} &= 1 - \text{Proportion of never takers} - \text{Proportion of always takers} \\
&= 1 - \Pr[A = 0|Z = 1] - \Pr[A = 1|Z = 0] \\
&= \Pr[A = 1|Z = 1] - \Pr[A = 1|Z = 0]
\end{aligned}$$

This quantity, $\Pr[A = 1|Z = 1] - \Pr[A = 1|Z = 0]$, is also the denominator of our standard IV estimator.

2 Instrumental variable estimation and selection bias

2.1 The third instrumental condition

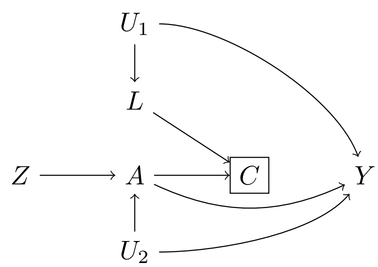
Informally, IV condition (iii) states that Z does not share causes with the outcome Y. More formally, this condition states that individuals with different levels of the proposed instrument must be exchangeable with respect to the outcome. Violations of exchangeability may be due to:

- Confounding (or a common cause) between the instrument and the outcome, or
- Selection bias (or conditioning on a collider) between the instrument and the outcome

Therefore, many forms of selection bias that threaten the validity of our confounding-adjusted estimate can also affect the validity of our IV estimate.

2.2 Examples of selection bias in instrumental variable estimation

Example 1. Consider a DAG with loss to follow-up in our IV analysis:



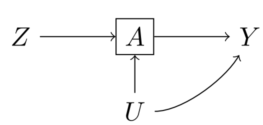
There are two biasing paths between Z and Y due to conditioning on C:

1. $Z \to A \to C \leftarrow L \leftarrow U_1 \to Y$ (Open path due to conditioning on collider C)
2. $Z \to A \leftarrow U_2 \to Y$ (Open path due to conditioning on C, which is downstream of collider A)

In this example, we can use inverse probability of censoring weights (assuming the variables in L have been measured) to create a pseudopopulation in which there is no censoring, and calculate the IV estimate in the pseudopopulation. However, we can only use nonstabilized weights to get unbiased IV estimates:

- **Nonstabilized weights** remove the $A \to C$ and $L \to C$ arrows. There is no selection (and therefore no selection bias) in the pseudopopulation.
- **Stabilized weights** remove only the $L \to C$ arrow. The existence of the $A \to C$ arrow in the pseudopopulation means that we are *still* conditioning on a variable (C) that is downstream of collider A, and therefore we still have selection bias (see biasing path #2, described above).

Example 2. Consider an IV analysis which excludes individuals with certain values of treatment A. This would correspond to the following DAG:

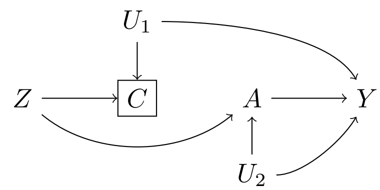


This analysis introduces selection bias by conditioning on treatment A, which is a collider. Because treatment A functions as a collider on the path from Z to Y, conditioning on A or a variable downstream of A (as we saw in Example 1) will introduce selection bias.

Example 3. Suppose our proposed instrument Z affects the probability of being selected into our study. This could arise because:

- Our proposed instrument affects the probability that we survive long enough to be enrolled into the study
- Our proposed instrument affects the probability of meeting the eligibility criteria for our study

In this scenario, our DAG may look like the following:



Again, we have introduced selection bias due to conditioning on a collider, C, which opens the biasing path $Z \to C \leftarrow U_1 \to Y$.

2.3 Additional readings

1. Swanson SA, Robins JM, Miller M, Hernán MA. Selecting on treatment: a pervasive form of bias in instrumental variable analyses. American journal of epidemiology. 2015 Feb 1;181(3):191-7.
2. Swanson SA. A practical guide to selection bias in instrumental variable analyses. Epidemiology. 2019 May 1;30(3):345-9.