

## 6. Supervised Techniques III

DS-GA 3001, Text as Data  
Arthur Spirling

March 6, 2018

# Housekeeping

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- 2 Homework 2 out soon—due  $\sim$  March 25.

From Last Week: “99% accuracy”

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Culture

Machine Learning

f share this



## A.I. Algorithm Recognizes Terrorist Propaganda with 99 Percent Accuracy

The war on terror goes digital.

By [Kevin Litman-Navarro](#) on February 13, 2018

Filed Under [A.I.](#), [Algorithms](#) & [Data](#)

The UK-based company [ASI Data Science](#) unveiled a [machine learning](#) algorithm Wednesday that can identify terrorist propaganda videos with 99 percent accuracy.

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# Where Are We?

The map is a detailed black and white illustration of Middle-earth. It shows the continent of Middle-earth with various regions labeled, including Eriador, Mordor, Gondor, Rohan, and the Shire. It also depicts the Red Mountains of Erildor, the Misty Mountains, and the Great River Anduin. A compass rose and a scale bar are included in the bottom left corner.

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Now want to cover powerful, commonly used techniques from **machine learning** that appear in social science research: **SVM**, KNN, CART etc.

These techniques involve some important decisions about the **bias-variance** tradeoff, and the use of **(cross) validation** in checking model performance and selecting the best model.

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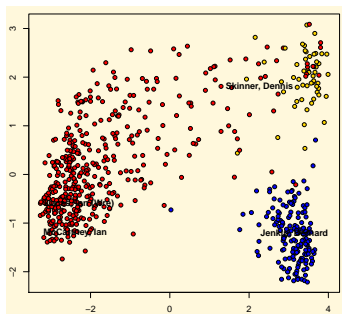
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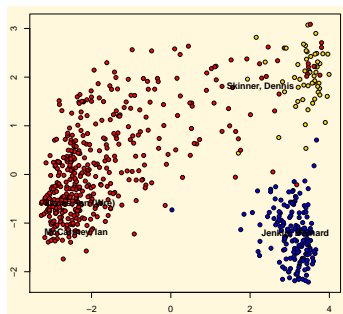




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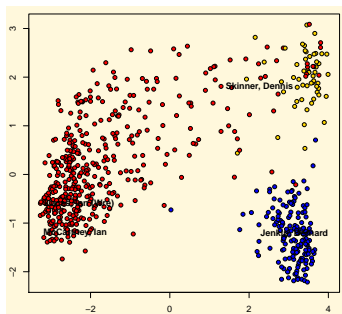


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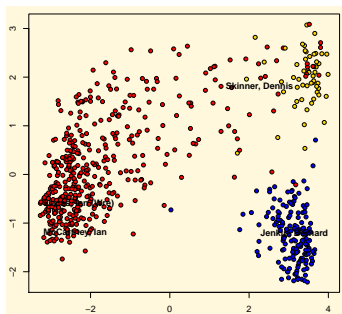


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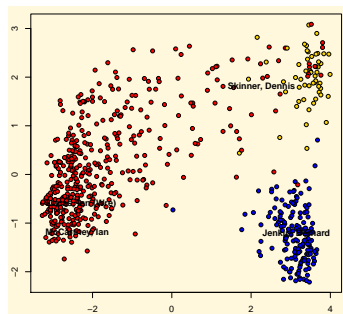
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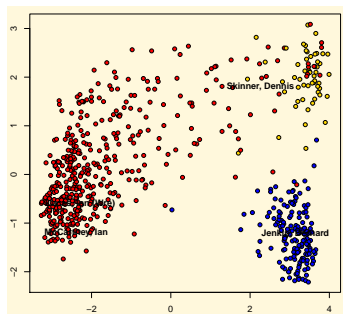
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**CRITIC REVIEWS FOR STAR WARS: EPISODE VII - THE FORCE AWAKENS**

All Critics (313) | Top Critics (48) | My Critics | Fresh (293) | Rotten (20)


 The new movie, as an act of pure storytelling, streams by with fluency and zip.


[Full Review...](#) | December 21, 2015

 **Anthony Lane**  
New Yorker  
★ Top Critic


 At the end The Force Awakens looks more like a nostalgic film that will work as a transition to the new Star Wars' age. [Full Review in Spanish]


[Full Review...](#) | December 29, 2015

 **Salvador Franco Reyes**

 While Star Wars: The Force Awakens gets temporarily bogged down taking us back to the world that we left in 1983, it introduces us to the new and exciting torch-bearers of the franchise.

[Full Review...](#) | December 30, 2015

 **Blake Howard**  
Graffiti With Punctuation

 This film is a well-planned product that balances nostalgia with the capacity to attract new generations into the Star Wars universe. [Full Review in Spanish]

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So techniques may require careful tuning of **regularization parameters** to obtain good performance.

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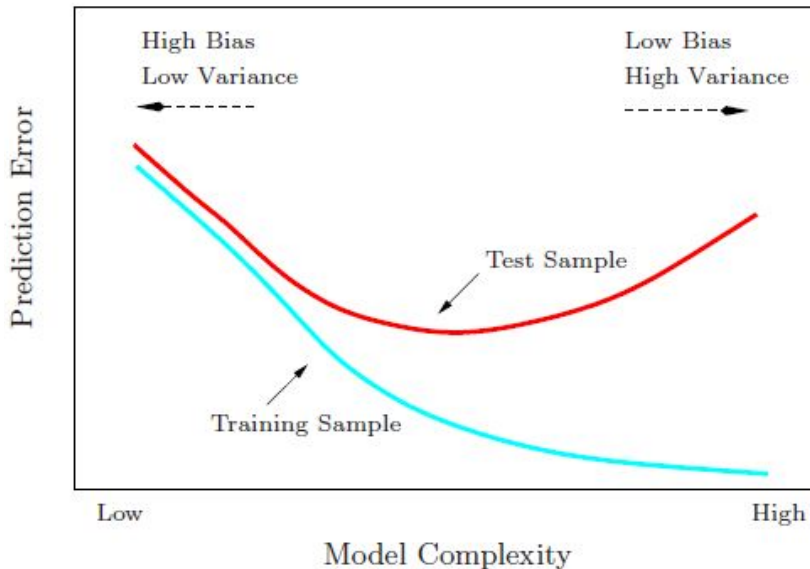
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# Bias-Variance Tradeoff (Hastie et al, p38)

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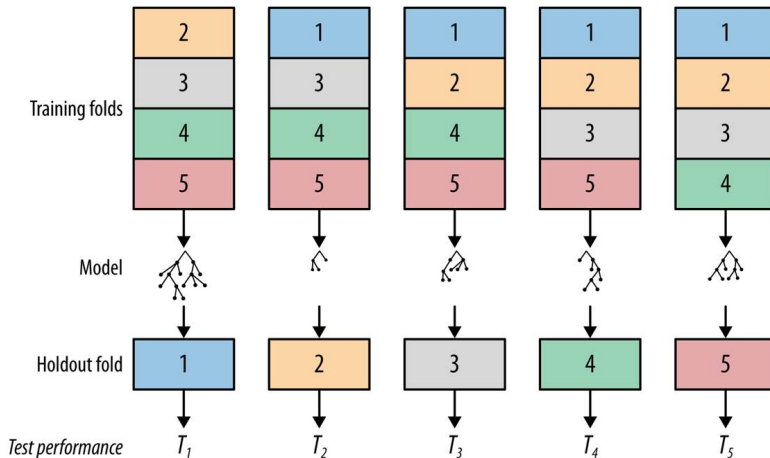
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Mean and standard deviation of test sample performance

# Motivating Example Diermeier et al, 2011

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Study the speeches of the 25 **most liberal** and the 25 **most conservative** senators (1989–2004),



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Have the (stemmed, stopped, weighted etc) **speech term matrix** for each Senator as  $X$ .

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What method to use?

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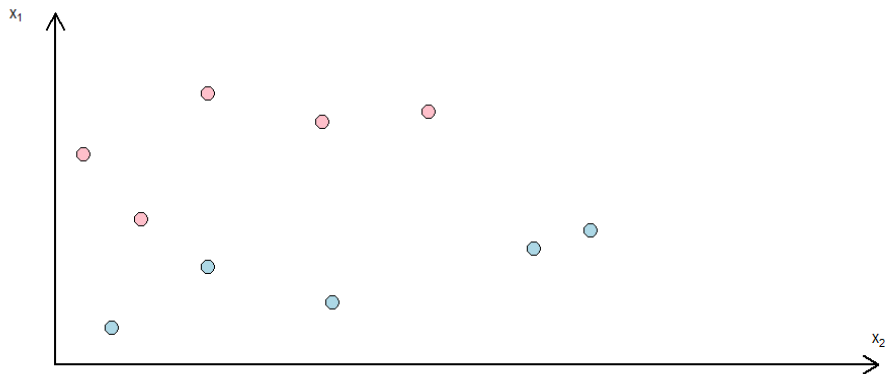
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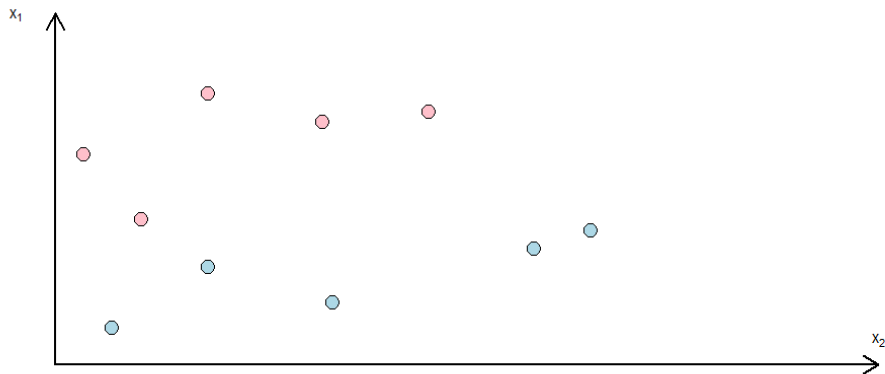
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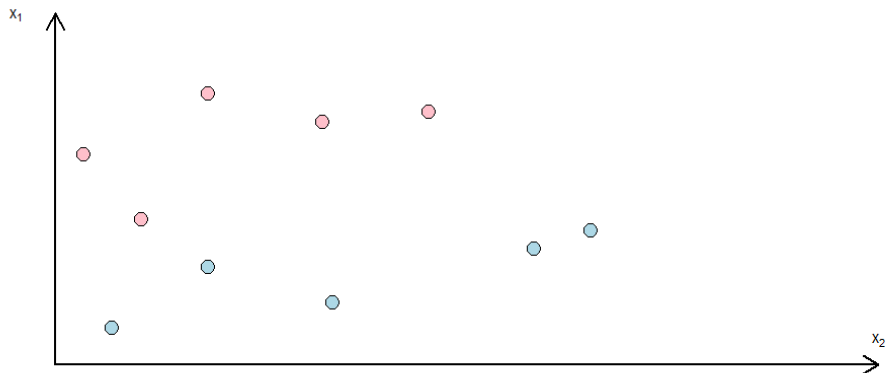


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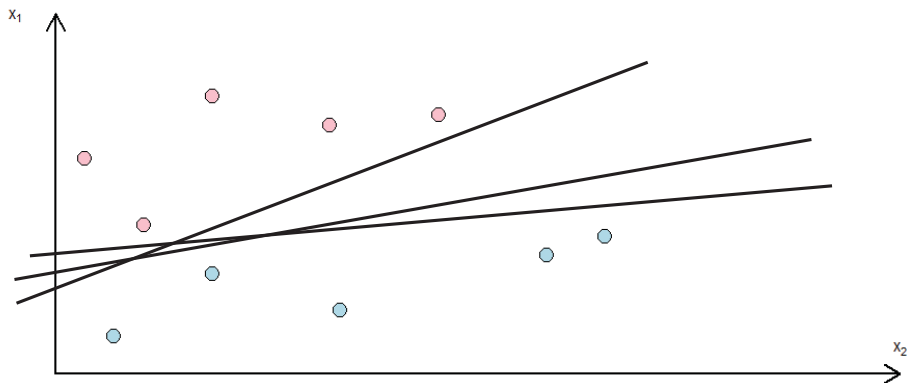
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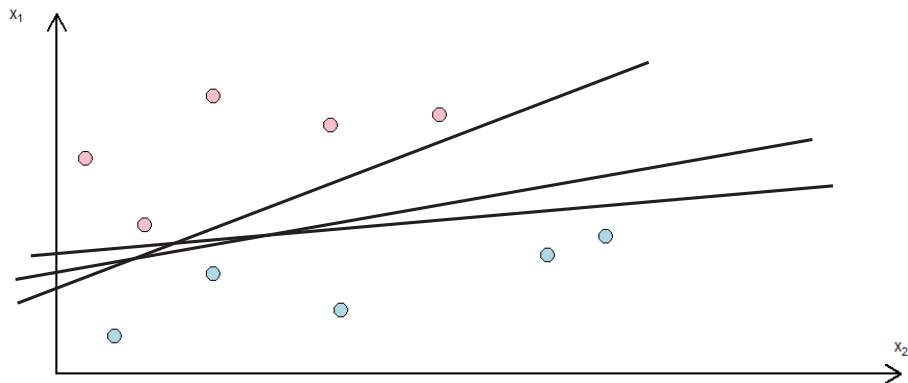


Are the parties linearly separable? Where could you draw the line?

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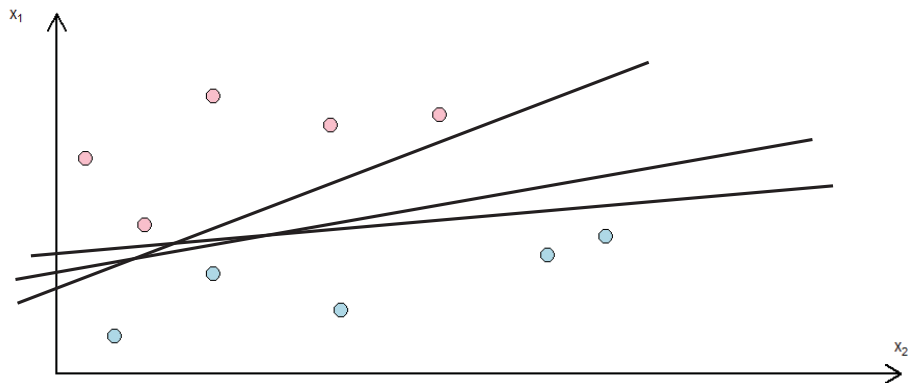


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→ That optimal line—the separating hyperplane—is the **maximum margin** hyperplane. It will maximize the **margin** of the training data.

# Partner Exercise

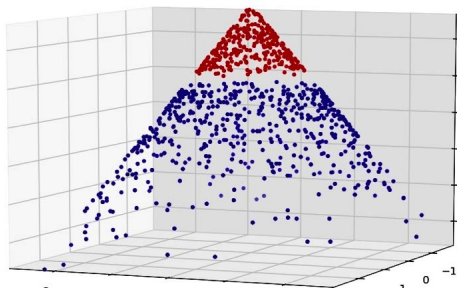
# Partner Exercise

Consider the figure.

It's a situation where each Senator's features are of three dimensions (rather than two).

How could we (optimally) separate the data in a linear way?

Can we still use a line?



from <http://www.edvancer.in/>

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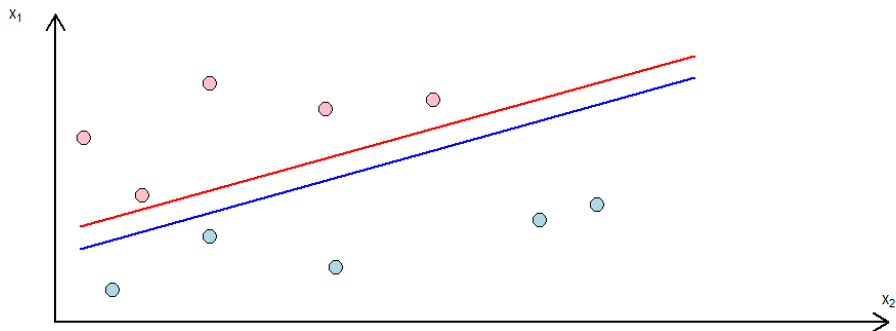
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**NB** The hyperplane cannot be anywhere other than **equidistant** because then it will break the rule about ensuring the **largest minimum distance**.

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This will give us the two hyperplanes  $H_R$  and  $H_D$ , and the line equidistant between them will be our classifier.

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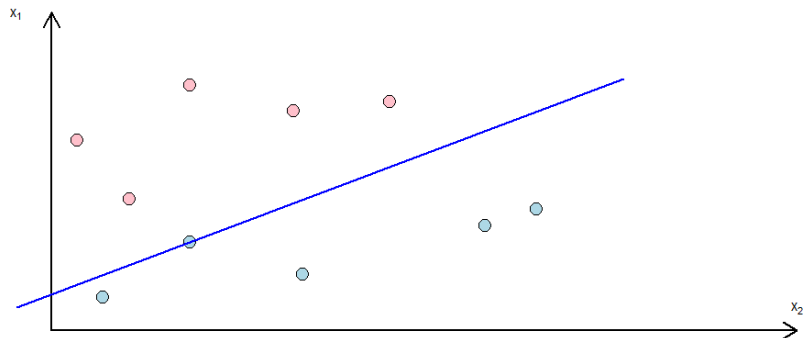
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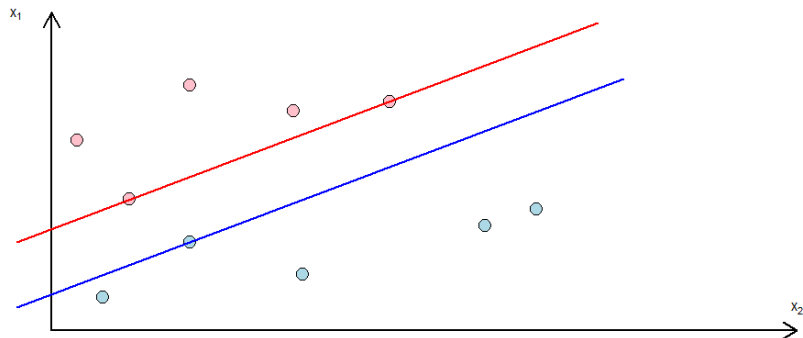


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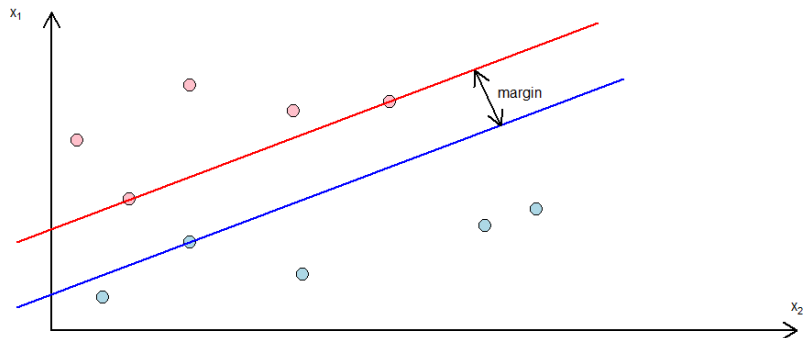
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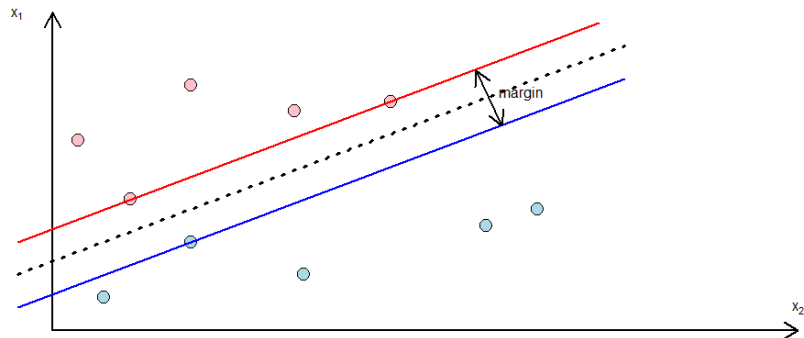
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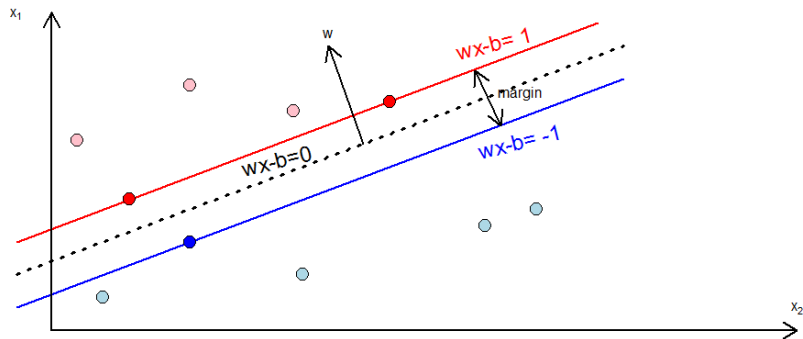
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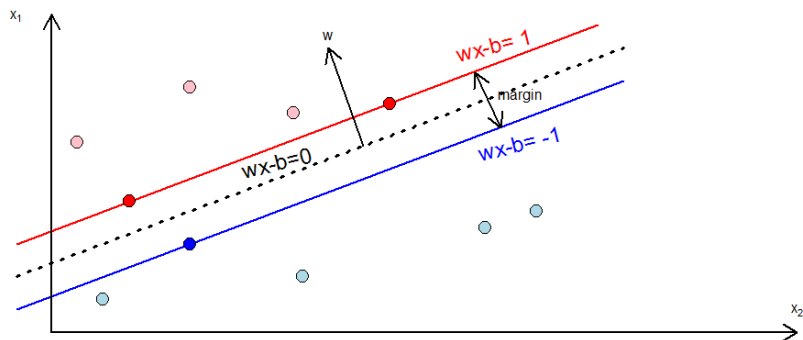


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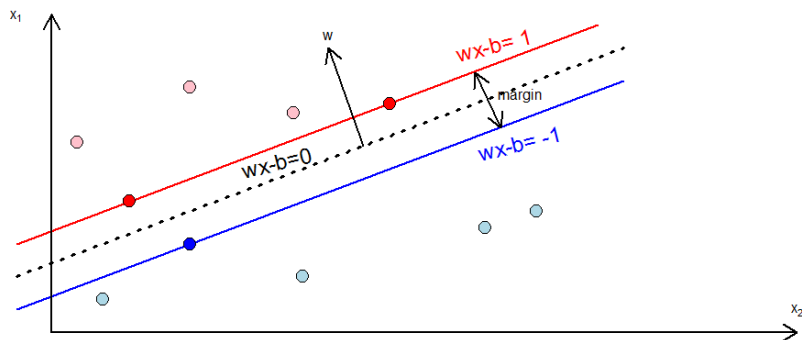


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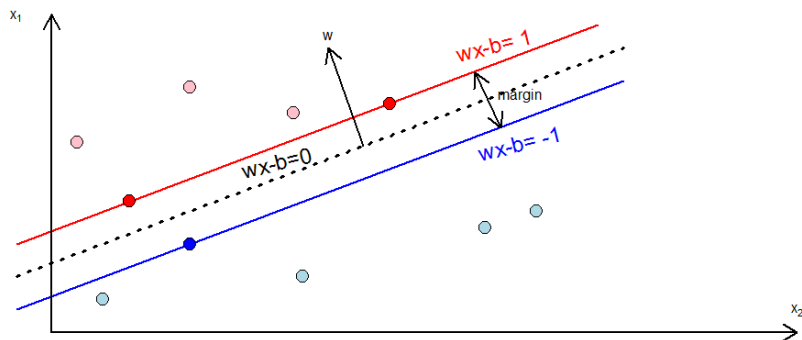
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| Words               |                       |                  |                     |
|---------------------|-----------------------|------------------|---------------------|
| Liberal             |                       | Conservative     |                     |
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| Ethanol: -198.92    | Nursing: -109.38      | CFTC: 187.16     | everglades: 102.87  |
| Wealthiest: -159.74 | Providence: -108.73   | surtax: 151.81   | tower: 101.67       |
| Collider: -142.28   | Arctic: -108.30       | marriage: 145.79 | tripartisan: 101.23 |
| WIC: -140.14        | Orange: -107.98       | cloning: 141.71  | PRC: 102.90         |
| ILO: -139.89        | Glaxo: -107.81        | tritium: 133.49  | scouts: 97.55       |
| Handgun: -129.01    | Libraries: -107.70    | ranchers: 132.95 | nashua: 99.32       |
| Lobbyists: -128.95  | Disabilities: -106.44 | BTU: 121.92      | ballistic: 97.22    |
| Enron: -127.71      | Prescription: -106.31 | grazing: 121.59  | salting: 94.28      |
| Fishery: -127.30    | NIH: -105.52          | unfunded: 120.82 | abortion: 91.94     |
| Hydrogen: -122.59   | Lobbying: -105.35     | catfish: 120.82  | NTSB: 93.81         |
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| PTSD: -119.87       | Trident: -104.15      | unborn: 111.88   | PAC: 92.85          |
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- 1 Does that imply that making conservative Senators use the word 'handgun' more often will make them more liberal? What does your answer suggest about **prediction** vs **explanation** with supervised techniques?
- 2 what is the (most likely) problem in the causal claim that  $X \rightarrow Y$  in the Diermeier et al study?

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BTW RLR can cope well with noise, and (hard margin) SVM will struggle if there is no linear separability...

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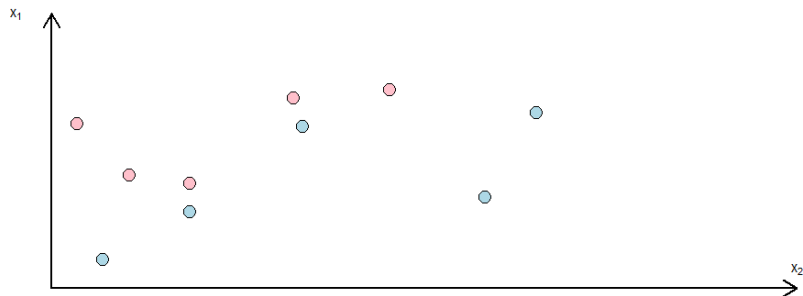
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Hyperplane(s) will be drawn in way that is more sensitive to 'bigger' mistakes in classification.

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$1 - f(x)y$  is  $1 - (-2)(+1)$  in first case and  $1 - (-100)(+1)$  in second case. Hinge loss larger in second case!



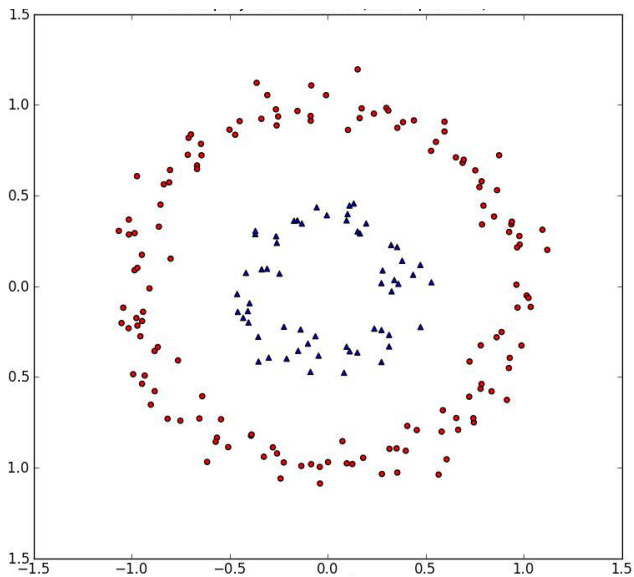
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from [www.eric-kim.net](http://www.eric-kim.net)

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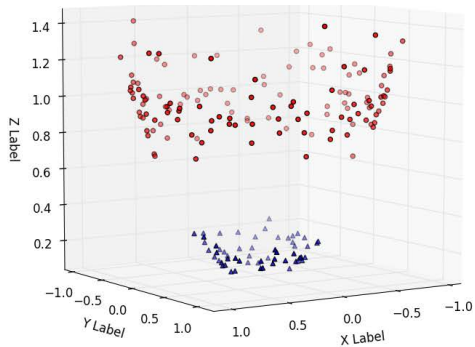
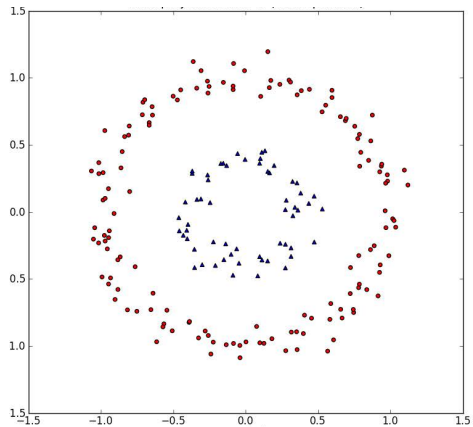
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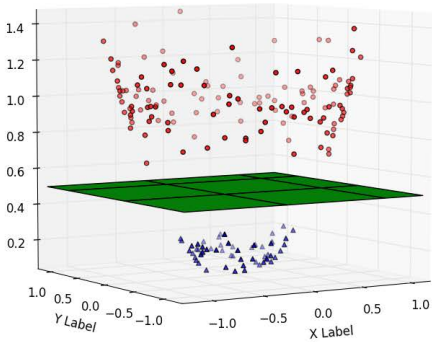
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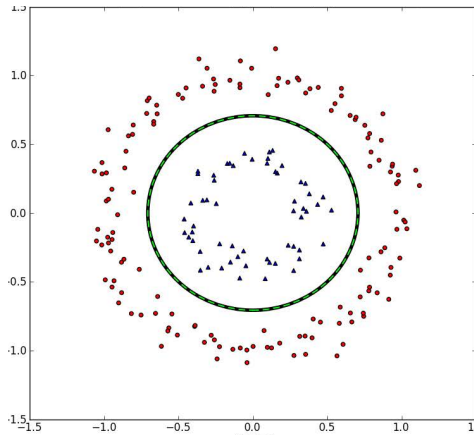
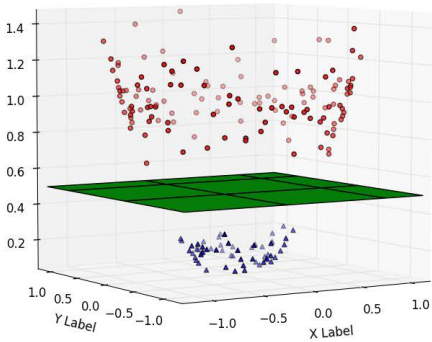
→ Results in a **non-linear** hyperplane once back in 2 dimensions.



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For text analysis, **string kernels** use a function  $K(a, b)$  to implicitly calculate the distance between strings of characters via the number of subsequences they have in common.

# Partner Exercise

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Using the ideas we discussed at the start of lecture, how should one go about picking a kernel (from the large variety on offer) for the problem at hand?