PENDULO

$$(M+m)\ddot{x} + ml\ddot{\theta} = u$$
$$(I+ml^2)\ddot{\theta} + ml\ddot{x} = mgl\theta$$

$$\rightarrow M\ddot{x} + m\ddot{x} + ml\ddot{\theta} = u$$
$$I\ddot{\theta} + ml^2\theta + ml\ddot{x} = mgl\theta$$

$q_1 = x$ $\qquad$ $q_3 = \theta$

$q_2 = \dot{q}_1 = \dot{x}$ $\qquad$ $q_4 = \dot{q}_3 = \dot{\theta}$

$\dot{q}_2 = \ddot{x}$ $\qquad$ $\dot{q}_4 = \ddot{\theta}$

$$(M+m)\ddot{x} + ml\ddot{\theta} = u$$
$$\ddot{x} = \frac{u - ml\ddot{\theta}}{M+m} = \frac{u}{M+m} - \frac{ml}{(M+m)}\ddot{\theta}$$
$$\frac{(I+ml^2)}{mgl}\ddot{\theta} + \frac{ml\ddot{x}}{mgl} = \theta = \left(\frac{I+ml^2}{mgl}\right)\ddot{\theta} + \left(\frac{1}{g}\right)\left(\frac{u}{M+m} - \frac{ml}{M+m}\ddot{\theta}\right)$$
$$\left(\frac{I+ml^2}{mg^2l} - \frac{ml}{g(M+m)}\right)\ddot{\theta} + \frac{1}{g(M+m)}u = \theta$$
$$\left(\frac{I+ml^2}{mg^2l} - \frac{ml}{g(M+m)}\right)\ddot{\theta} = \theta - \frac{1}{g(M+m)}u$$
$$\left(\frac{(I+ml^2)(g(M+m)) - ml(mg^2l)}{mg^2l(g(M+m))}\right)\ddot{\theta} = \theta - \frac{1}{g(M+m)}u$$
$$\frac{mg^2l(g(M+m))}{(I+ml^2)(g(M+m)) - ml(mg^2l)}\theta - \frac{mg^2l(g(M+m))}{mg^2l(g(M+m))^2}u$$
$$\frac{mg^2l(g(M+m))}{(I+ml^2)(g(M+m)) - ml(mg^2l)}\theta - \frac{1}{g(M+m)}u = \ddot{\theta}$$

$$\ddot{x} = \left(\frac{I+ml^2}{mgl}\right)\left(\left(\frac{mg^2 l(g(M+m)}{(I+ml^2)(g(M+m)-ml(mg^2 l)}\right)\right)\theta - \frac{I+ml^2}{mg^2 l(M+m)}u$$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{I+ml^2}{mgl}\left(\frac{mg^2 l(g(M+m)}{(I+ml^2)(g(M+m)-ml(mg^2 l))}\right) & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{mg^2 l(g(M+m)}{(I+ml^2)(g(M+m)-mg^2 l)} & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} + \dots$$

$$\dots + \begin{bmatrix} 0 \\ \frac{1}{g(M+m)} \\ 0 \\ -\frac{1}{g(M+m)} \end{bmatrix} u$$

$$\begin{bmatrix} x \\ \theta \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} + [0]u$$