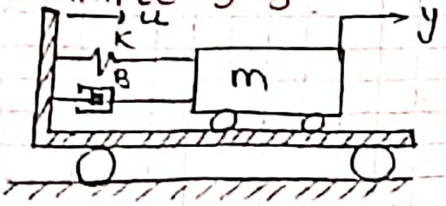
Camila Andrea Caceres Riaño - 20172005133.

EXAMPLE 3-3



$$m\ddot{y} = -b(\dot{y}-\dot{u}) - k(y-u)$$

$$m\ddot{y} + b\dot{y} + ky = b\dot{u} + ku$$

Se aplica transformada de laplace

$$(ms^2+bs+k)\,Y(s) = (bs+k)\,U(s)$$

→ Función de transferencia

$$G(s) = \frac{Y(s)}{U(s)} = \frac{(bs+k)}{ms^2+bs+k}$$

Tenemos:

$$\dddot{y} + a_1\ddot{y} + a_2 y = b_0\ddot{u} + b_1\dot{u} + b_2 u$$

donde:

$$a_1 = \frac{b}{m},\quad a_2 = \frac{k}{m},\quad b_0 = 0,\quad b_1 = \frac{b}{m},\quad b_2 = \frac{k}{m}$$

Referente a la ecuacion (3-35)

$$\beta_0 = b_0 = 0 \qquad \beta_2 = b_2 - a_1\beta_1 - a_2\beta_0 = \frac{k}{m} - \left(\frac{b}{m}\right)^2$$

$$\beta_1 = b_1 - a_1\beta_0 = \frac{b}{m}$$

Con respecto a la ecuación (2-34) se tiene

$$x_1 = y - \beta_0 u = y$$

$$x_2 = \dot{x}_1 - \beta_1 u = \dot{x} - \frac{b}{m}u$$

De la ecuación (2-36) se tiene

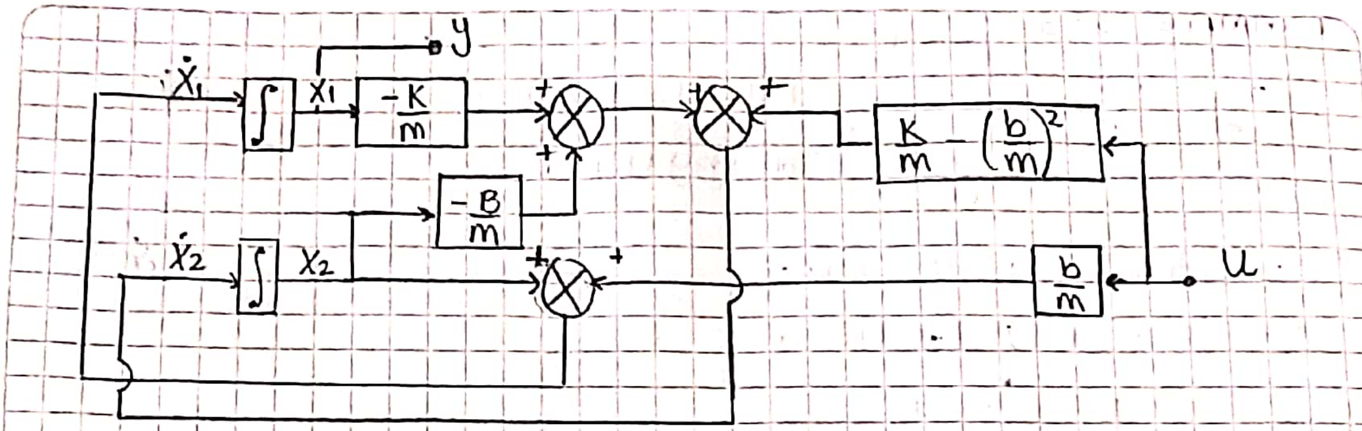
$$\dot{x}_1 = x_2 + \beta_1 u = x_2 + \frac{b}{m}u, \quad \dot{x}_2 = -a_2x_1 - a_1x_2 + \beta_2 u = -\frac{k}{m}x_1 - \frac{b}{m}x_2 + \left[\frac{k}{m} - \left(\frac{b}{m}\right)^2\right]u$$

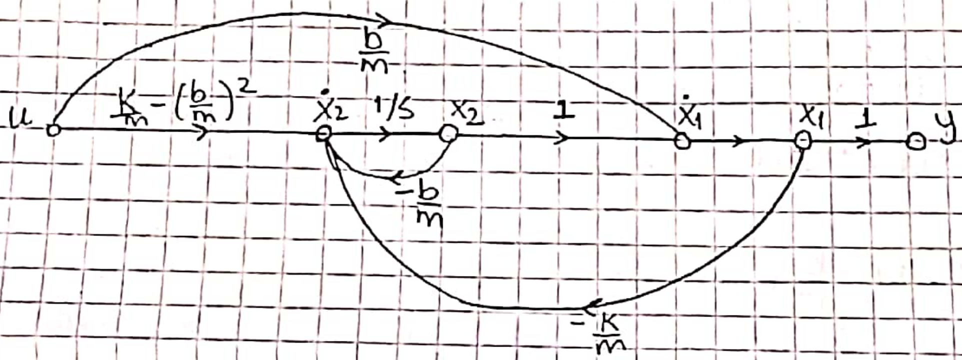
$$y = x_1$$

Se tiene entonces:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -\frac{k}{m} & -\frac{b}{m}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}\frac{b}{m}\\ \frac{k}{m} - \left(\frac{b}{m}\right)^2\end{bmatrix}u$$

$$y = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$





EJERCICIO A-3.9

$T = K_2 i_a$

$e_b = K_3 \frac{d\theta}{dt}$

Motor controlado por armadura.

$$L_a \frac{di_a}{dt} + R_a i_a + e_b = e_a$$

$$J_0 \ddot{\theta} + b_0 \dot{\theta} = T = K_2 i_a$$

$$\frac{\theta(s)}{E_V(s)} = \frac{K_1 K_2}{s(L_a s + R_a)(J_0 s + b_0) + K_2 K_3 s}$$

$$C(s) = n\theta(s) \Rightarrow E_V(s) = K_0\left[R(s) - C(s)\right] = K_0 E(s)$$

$$G(s) = \frac{C(s)}{\theta(s)} \frac{\theta(s)}{E_V(s)} \frac{E_V(s)}{E(s)} = \frac{K_0 K_1 K_2 n}{s\left[(L_a s + R_a)(J_0 s + b_0) + K_2 K_3\right]}$$

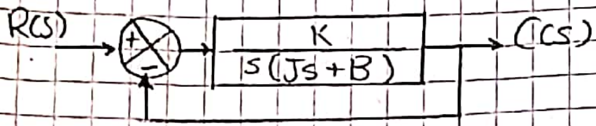
Cuando L_a es pequeña se tiene

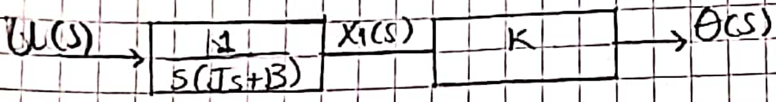
$$G(s) = \frac{K_0 K_1 K_2 n}{s\left[R_a (J_0 s + b_0) + K_2 K_3\right]} = \frac{K_0 K_1 K_2 n / R_a}{J_0 s^2 + \left(b_0 + \frac{K_2 K_3}{R_a}\right) s}$$

$J = J_0 / n^2, \quad B = [b_0 + K_2 K_3 / Ra)] / n^2$

$K = K_0 K_1 K_2 / n Ra$

$G(s) = \dfrac{K}{Js^2 + Bs} = \dfrac{K}{s(Js + B)} \rightarrow \dfrac{\Theta(s)}{Ev(s)} = \dfrac{K}{s(Js + B)}$





$\dfrac{X_1(s)}{U(s)} = \dfrac{1}{Js^2 + Bs}$

$(Js^2 + Bs) X_1(s) = U(s)$

$Js^2 X_1(s) + Bs X_1(s) = U(s)$

$J\ddot{x}_1 + B\dot{x}_1 = u(t)$

$\Theta(s) = K X_1(s)$

$\theta(t) = K x_1$

$x_1 = x_1$

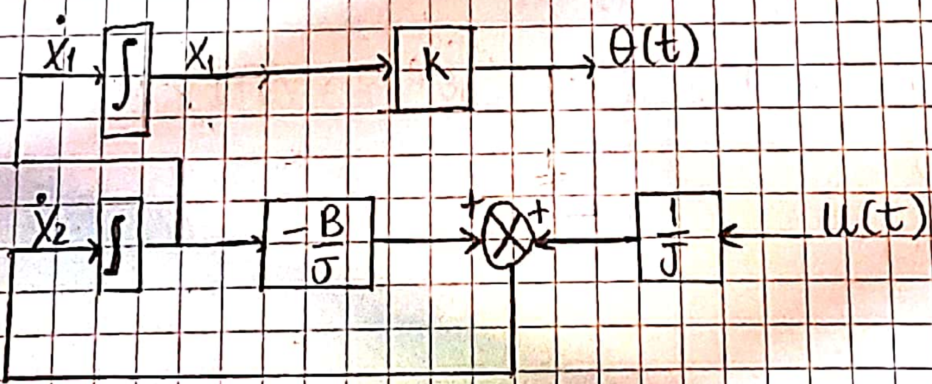
$x_2 = \dot{x}_1$

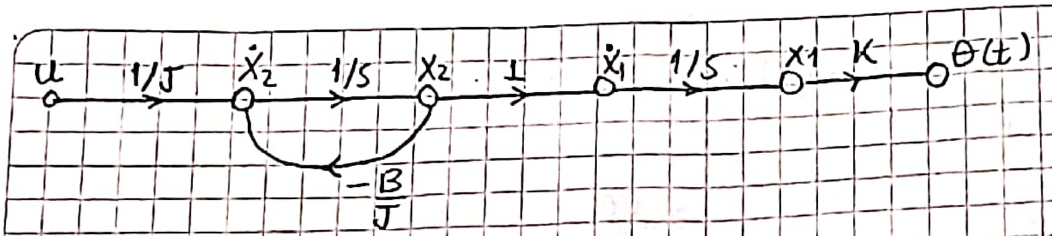
$\dot{x}_2 = \ddot{x}_1$

$\dot{x}_2 = \dfrac{u(t)}{J} - \dfrac{B}{J} x_2$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{B}{J} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{J} \end{bmatrix} u(t)$$

$$\theta(t) = \begin{bmatrix} K & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$





EJERCICIO 2.23

Motor controlado por armadura; procedemos a encontrar las variables Jm y Dm

$$J_M = J_a + J_L\left(\frac{N_1}{N_2}\right)^2 = 5 + 700\left(\frac{1}{10}\right)^2 = 12$$

$$D_m = D_a + D_L\left(\frac{N_1}{N_2}\right)^2 = 2 + 800\left(\frac{1}{10}\right)^2 = 10$$

Ahora las variables eléctricas

$$T = 500$$
$$\omega = 50$$
$$e_a = 100$$

$$\frac{K_t}{R_a} = \frac{T}{e_a} = \frac{500}{100} = 5 \quad ; \quad K_b = \frac{e_a}{\omega} = \frac{100}{50} = 2$$

$$\frac{\Theta m(s)}{Ea(s)} = \frac{5/12}{s\left\{s + \frac{1}{12}\left[10 + (5)(2)\right]\right\}} = \frac{0{,}417}{s(s+1{,}667)}$$

$$\xrightarrow{Ea(s)} \boxed{\frac{1}{s^2+1{,}667s}} \xrightarrow{X_1(s)} \boxed{0{,}417} \longrightarrow \Theta m(s)$$

$$\frac{X_1(s)}{Ea(s)} = \frac{1}{s^2+1{,}667s}$$

$$(s^2+1{,}667s)X_1(s) = Ea(s)$$

$$s^2X_1(s) + 1{,}667sX_1(s) = Ea(s)$$

$$\ddot{X}_1 + 1{,}667\dot{X}_1 = e_a(t)$$

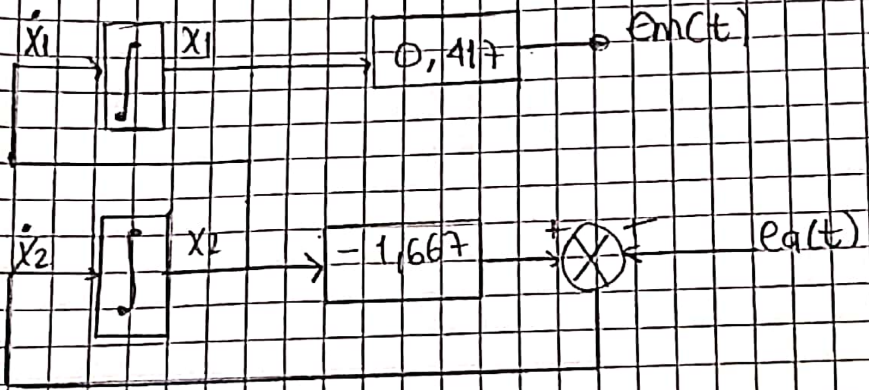
$$\Theta m(s) = 0{,}417\,X_1(s)$$

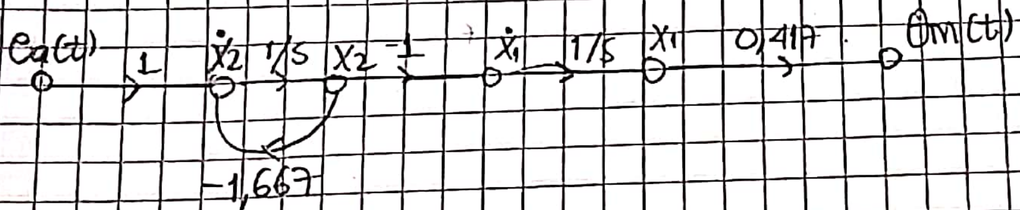
$$\Theta m(t) = 0{,}417\,X_1$$

$$X_1 = X_1$$
$$X_2 = \dot{X}_1$$
$$\dot{X}_2 = \ddot{X}_1$$
$$\dot{X}_2 = -1{,}667X_2 + e_a(t)$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & -1{,}667\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix} e_a(t)$$

$$\theta_m(t) = \begin{bmatrix}0{,}417 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$





La unica diferencia entre los ultimos dos ejercicios es que en uno no se usan valores reales mientras que en el otro se usan las variables.

Día	Mes	Año	Hora	Institución		
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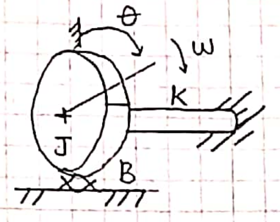
Profesor						

1.



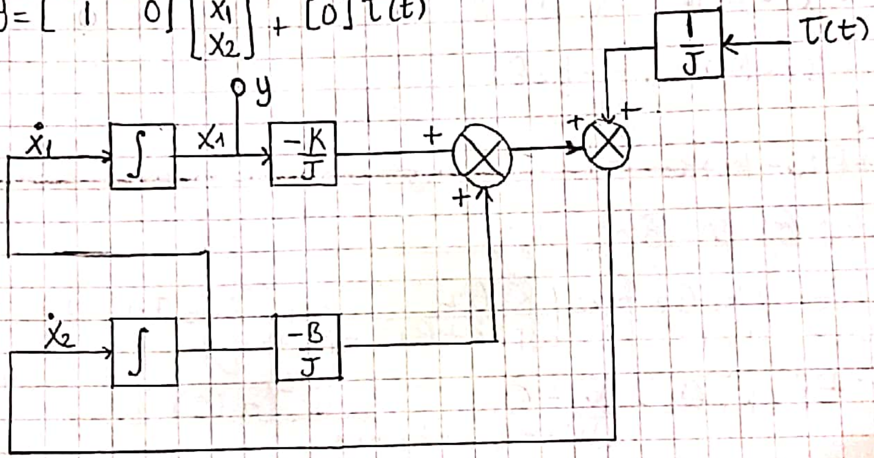
$$J\ddot{\theta} = \tau(t) - B\dot{\theta} - K\theta$$

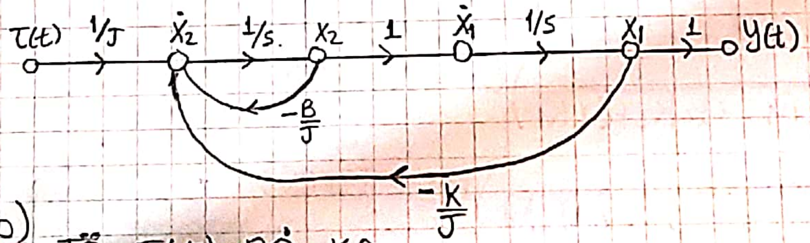
a)

$$X_1 = \theta$$
$$X_2 = \dot{\theta} = \dot{X}_1$$
$$\dot{X}_2 = \ddot{\theta} = \frac{1}{J}\tau(t) - \frac{B}{J}X_2 - \frac{K}{J}X_1$$

$$\begin{bmatrix}\dot{X}_1\\ \dot{X}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -\frac{K}{J} & -\frac{B}{J}\end{bmatrix}\begin{bmatrix}X_1\\ X_2\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{J}\end{bmatrix}\tau(t)$$

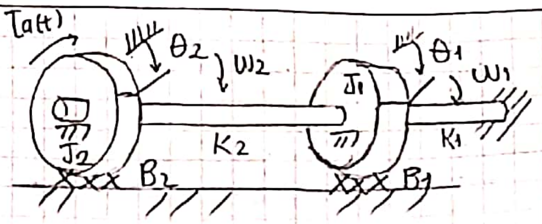
$$y = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}X_1\\ X_2\end{bmatrix} + [0]\,\tau(t)$$





b)

$$J\ddot{\theta} = \tau(t) - B\dot{\theta} - K\theta$$
$$J\ddot{\theta} + B\dot{\theta} + K\theta = \tau(t)$$
$$Js^2\Theta(s) + Bs\Theta(s) - K\Theta(s) = T(s)$$
$$(Js^2 + Bs - K)\Theta(s) = T(s)$$
$$\left(s^2 + \frac{B}{J}s - \frac{K}{J}\right)\Theta(s) = \frac{1}{J}T(s) \Rightarrow \frac{\Theta(s)}{T(s)} = \frac{1/J}{s^2 + \frac{B}{J}s - \frac{K}{J}}$$



$\theta_2 > \theta_1$

$J_2\ddot{\theta}_2 + B_2\dot{\theta}_2 + K_2(\theta_2 - \theta_1) = Ta(t)$

$J_2\ddot{\theta}_2 + B_2\dot{\theta}_2 + K_2\theta_2 - K_2\theta_1 = Ta(t)$ (1)

$J_1\ddot{\theta}_1 + B_1\dot{\theta}_1 + K_2(\theta_2 - \theta_1) + K_1\theta_1 = \emptyset$

$J_1\ddot{\theta}_1 + B_1\dot{\theta}_1 + K_2\theta_2 - K_2\theta_1 + K_1\theta_1 = \emptyset$ (2)

a) Hacemos transformada de laplace

$J_2S^2\theta_2 + B_2S\theta_2 + K_2\theta_2 - K_2\theta_1 = Ta(s)$ (3)

$J_1S^2\theta_1 + B_1S\dot{\theta}_1 + K_2\theta_2 - (K_2 - K_1)\theta_1 = \emptyset$ (4)

De (4) tenemos

$(J_1S^2 + B_1S - (K_2 - K_1))\theta_1 + K_2\theta_2 = \emptyset$

$J_1S^2 + B_1S - (K_2 - K_1))\theta_1 = -K_2\theta_2$

$\theta_1 = \dfrac{K_2\theta_2}{J_1S^2 + B_1S - K_2 + K_1}$ (5)

Reemplazamos (5) en (3)

$J_2S^2\theta_2 + B_2S\theta_2 + K_2\theta_2 - K_2\left(\dfrac{K_2\theta_2}{J_1S^2 + B_1S - K_2 + K_1}\right) = Ta(s)$

$J_2S^2\theta_2 + B_2S\theta_2 + K_2\theta_2 - \dfrac{K_2^2\theta_2}{J_1S^2 + B_1S - K_2 + K_1} = Ta(s)$

$S^2\theta_2 + \dfrac{B_2}{J_2}S\theta_2 - \dfrac{K_2\theta_2}{J_2(J_1S^2 + B_1S - K_2 + K_1)} = Ta(s)$

$\left(S^2 + \dfrac{B_2S}{J_2} - \dfrac{K_2}{J_2J_1S^2 + J_2B_1S - J_2K_2 + J_2K_1}\right)\theta_2 = Ta(s)$

$\left(\dfrac{J_2S^2 + B_2S}{J_2} - \dfrac{K_2}{J_2J_1S^2 + J_2B_1S - J_2K_2 + J_2K_1}\right)\theta_2 = Ta(s)$

$\left(\dfrac{J_2K_2 - J_2S^2 + B_2S(J_2J_1S^2 + J_2B_1S - J_2K_2 + J_2K_1)}{J_2^2J_1S^2 + J_2^2B_1S - J_2^2K_2 + J_2^2K_1}\right)\theta_2 = Ta(s)$

$\dfrac{\theta_2(s)}{Ta(s)} = \dfrac{J_2(J_2J_1S^2 + J_2B_1S - J_2K_2 + J_2K_1)}{J_2K_2 - J_2S^2 + B_2S(J_2J_1S^2 + J_2B_1S - J_2K_2 + J_2K_1)}$

$\dfrac{\theta_2(s)}{Ta(s)} = \dfrac{J_2(J_2J_1S^2 + J_2B_1S - J_2K_2 + J_2K_1)}{-J_2S^2 + B_2S + J_2K_2(J_2J_1S^2 + J_2B_1S - J_2K_2 + J_2K_1}$

(b)

$$x_1 = \theta_2$$

$$x_2 = \dot{\theta}_2 = \dot{x}_1$$

$$\dot{x}_3 = \ddot{\theta}_1$$

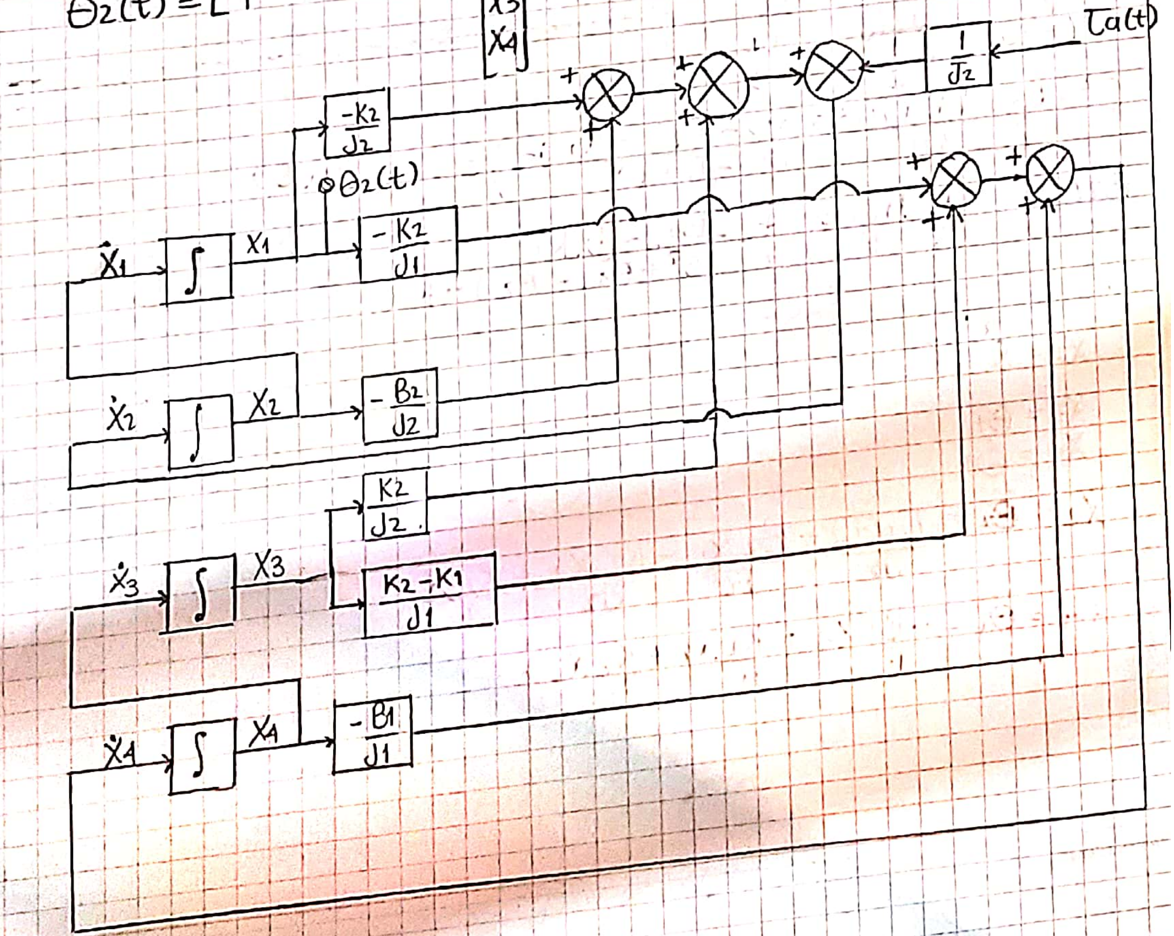
$$x_4 = \dot{\theta}_1$$

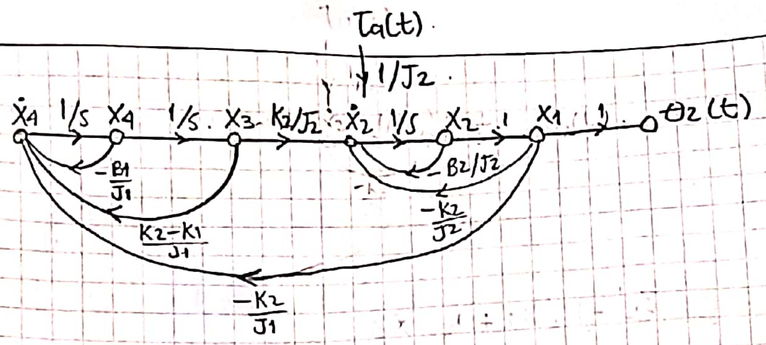
$$\dot{x}_2 = \ddot{\theta}_2 = \frac{T_a(t)}{J_2} - \frac{B_2}{J_2}x_2 - \frac{K_2}{J_2}x_1 + \frac{K_2}{J_2}x_3$$

$$\dot{x}_4 = \ddot{\theta}_1 = -\frac{B_1}{J_1}x_4 - \frac{K_2}{J_1}x_1 + \frac{(K_2-K_1)}{J_1}x_3$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0\\ -\frac{K_2}{J_2} & -\frac{B_2}{J_2} & \frac{K_2}{J_2} & 0\\ 0 & 0 & 0 & 1\\ -\frac{K_2}{J_1} & 0 & \frac{K_2-K_1}{J_1} & -\frac{B_1}{J_1}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}0\\ 1/J_2\\ 0\\ 0\end{bmatrix} T_a(t)$$

$$\theta_2(t) = \begin{bmatrix}1 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + [0]\,T_a(t)$$





③ $J_2\ddot{\theta}_2 + B_2\dot{\theta}_2 + K_2\theta_2 - K_2\theta_1 = Ta(t)$

$J_1\ddot{\theta}_1 + B_1\dot{\theta}_1 + K_2\theta_2 - K_2\theta_1 = 0$

ⓐ Transformada de laplace

$J_2 S^2\theta_2 + B_2 S\theta_2 + K_2\theta_2 - K_2\theta_1 = ta(t)$

$J_1 S^2\theta_1 + B_1 S\theta_1 + K_2\theta_2 - K_2\theta_1 = 0$

$(J_1 S^2 + B_1 S - K_2)\theta_1 = -K_2\theta_2$

$$\theta_1 = \frac{-K_2\theta_2}{J_1S^2 + B_1S - K_2}$$

Reemplazando se tiene:

$$\left(\frac{S^2 + B_2S}{J_2} - \frac{K_2}{J_2(J_1S^2 + B_1S - K_2)}\right)\theta_2 = Ta(S)$$

$$\left(\frac{(S^2 + B_2S)J_2(J_1S^2 + B_1S - K_2) - K_2J_2}{J_2(J_1J_2S^2 + J_2B_1S - J_2K_2)}\right)\theta_2 = Ta(S)$$

$$\frac{\theta_2(S)}{ta(S)} = \frac{J_2J_1J_2S^2 + J_2B_1S - J_2K_2}{(S^2 + B_2S)J_2(J_1S^2 + B_1S - K_2) - K_2J_2}$$

b.

$X_1 = \theta_2$

$X_2 = \dot{X}_1 = \dot{\theta}_2$

$X_3 = \theta_1$

$X_4 = \dot{\theta}_1$

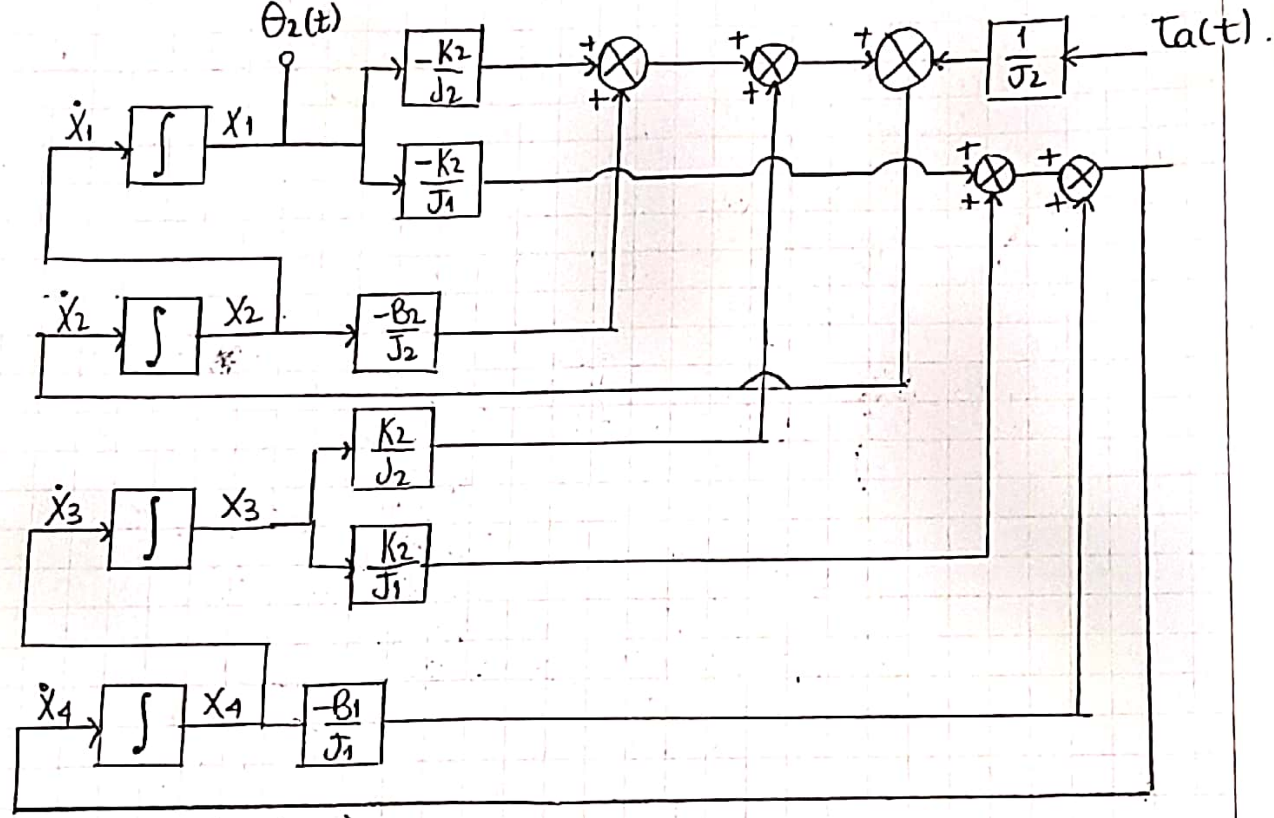
$$\dot{X}_2 = \ddot{\theta}_2 = \frac{Ta(t)}{J_2} - \frac{B_2X_2}{J_2} - \frac{K_2X_1}{J_2} + \frac{K_2X_3}{J_2}$$

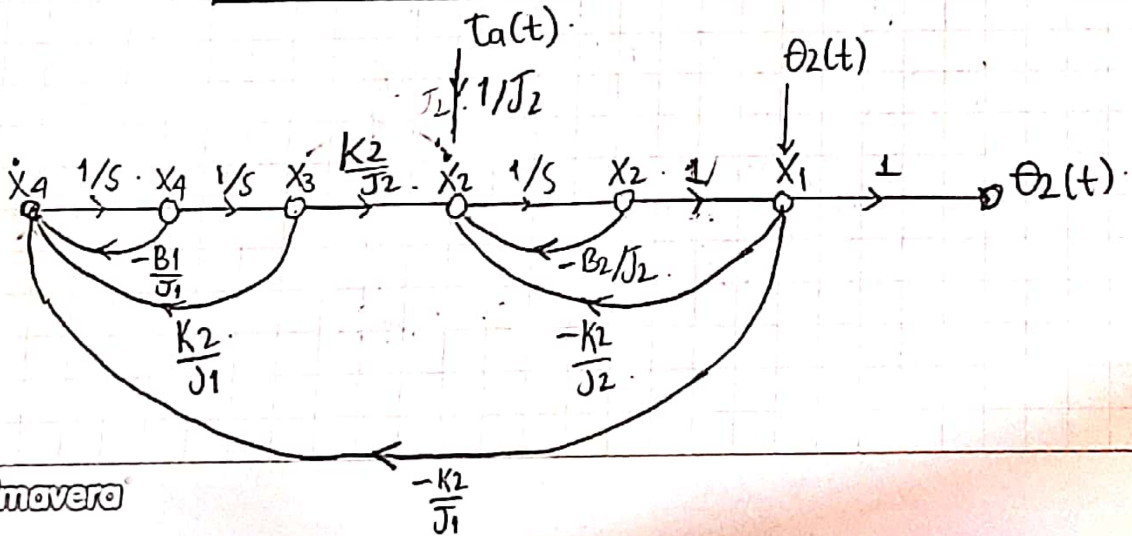
$$\dot{X}_4 = \ddot{\theta}_1 = -\frac{B_1}{J_1}X_4 - \frac{K_2}{J_1}X_1 + \frac{K_2}{J_1}X_3$$

Observaciones

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix}=\begin{bmatrix}0 & 1 & 0 & 0\\ -\frac{K_2}{J_2} & -\frac{B_2}{J_2} & \frac{K_2}{J_2} & 0\\ 0 & 0 & 0 & 1\\ -\frac{K_2}{J_1} & 0 & \frac{K_2}{J_1} & -\frac{B_1}{J_1}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}+\begin{bmatrix}0\\ 1/J_2\\ 0\\ 0\end{bmatrix}T_a(t).$$

$$\theta_2(t)=\begin{bmatrix}1 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}+[0]T_a(t).$$





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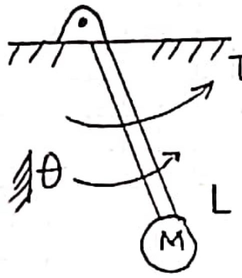


$$(M+m)\ddot{x} + mL\ddot{\theta} = Ta(t) \quad ①$$
$$J\ddot{\theta} + mL^2\theta + mL\ddot{x} + B\dot{x} = mgL\theta \quad ②$$

a

De ② tenemos

$$mL\ddot{x} = mgL\theta - J\ddot{\theta} - mL^2\theta - B\dot{x}$$
$$\ddot{x} = \frac{mg\cancel{L}}{m\cancel{L}}\theta - J\ddot{\theta} - \frac{m L^{\cancel{2}}}{m\cancel{L}}\theta - \frac{B}{mL}\dot{x}$$
$$\ddot{x} = g\theta - J\ddot{\theta} - L\theta - \frac{B}{mL}\dot{x} = (g-L)\theta - J\ddot{\theta} - \frac{B}{mL}\dot{x}$$

$$(M+m)\left(g\theta - J\ddot{\theta} - L\theta - \frac{B}{mL}\dot{x}\right) + mL\ddot{\theta} = Ta(t).$$
$$Mg\theta + mg\theta - MJ\ddot{\theta} - mJ\ddot{\theta} - ML\theta - mL\theta - \frac{B}{mL}M\dot{x} - \frac{Bm}{mL}\dot{x} + mL\ddot{\theta} = Ta$$
$$(Mg + mg - ML - mL)\theta + (-MJ - mJ + mL)\ddot{\theta} - \left(\frac{BM}{mL} + \frac{B}{L}\right)\dot{x} = Ta.$$
$$(mL - MJ - mJ)\ddot{\theta} = Ta - (Mg + mg - ML - mL)\theta + \left(\frac{BM}{mL} + \frac{B}{L}\right)\dot{x}$$
$$\ddot{\theta} = \frac{Ta}{mL-MJ-mJ} - \left(\frac{Mg + mg - M - mL}{mL - MJ - mJ}\right)\theta + \left(\frac{BM/mL + B/L}{mL - MJ - mJ}\right)\dot{x}$$

b

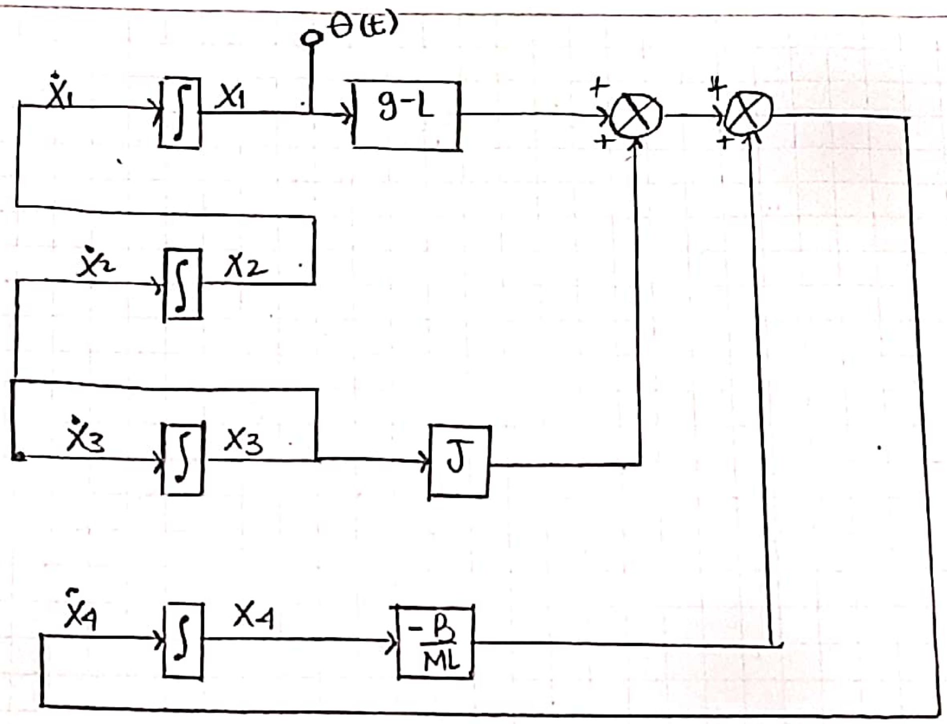
$x_1 = \theta$ $\quad x_4 = \dot{x}$

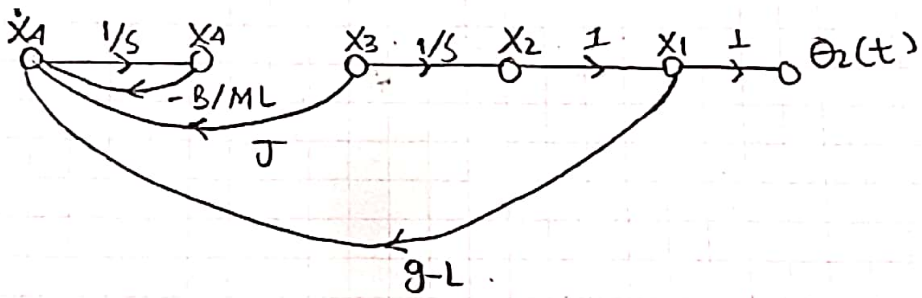
$x_2 = \dot{\theta} = \dot{x}_1$

$\dot{x}_3 = \ddot{\theta} = x_2$ $\quad \dot{x}_4 = \ddot{x}$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0\\ g-L & 0 & J & -\frac{B}{ML}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}0\\0\\0\\0\end{bmatrix} Ta(t)$$

$$\theta = \begin{bmatrix}1 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + [0]\, Ta(t)$$


θ(t)
$\dot{X}_1$
X_1
g-L
$\dot{X}_2$
X_2
$\dot{X}_3$
X_3
J
$\dot{X}_4$
X_4
$-\frac{B}{ML}$


$\dot{X}_4$
1/S
X_4
X_3
1/S
X_2
1
X_1
1
$\theta_2(t)$
-B/ML
J
g-L