CSE 105: Homework Set 6

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1. Turing Machine Construction

   M = "On input string w:
   1. Sweep along the tape left to right until a 1 is found. If the first symbol encountered was a blank or if no 1's were found, reject.

   2. Cross off the 1 which was found and check if there are at least two 0's by zig zagging along the tape. If at least two 0's exist, then cross off two 0's. If there were less than two zeros found, reject.

   3. Sweep along the tape from left to right until a 1 is found. If all of the symbols were found to be crossed off, accept. If no 1's were found, reject. If a 1 was found repeat steps 2 and 3."

2. Proof of Closure Under Intersection
   For any two decidable languages L_1 and L_2, let M_1 and M_2 be the TM's that recognizes them. We can construct a TM M' that decides the intersection of L_1 and L_2.

   M' = "On input string w:
   1. Run M_1 and M_2 alternatively on w step by step. If either halt and reject, reject. If both accept on the same input, accept."

   This construction is correct because if both M_1 and M_2 accept w on the same input then that must mean that M' also accepts w since they will both reach their accepting states after a finite number of steps. An intersection means that M' will only accept the strings that are common among both M_1 and M_2. That is why M' must reject if either of the machines running reject the input.

3. Enumeration and Decidability

In order to prove that this statement is true we must show that
(1) The enumerator enumerates the language in lexicographic order $\implies$ The language is decidable
(2) The language is decidable $\implies$ The enumerator enumerates it in lexicographic order.

In order to prove (1) we must construct a TM that is decidable in terms of an enumerator and then show why the construction is correct. For the rest of this proof, note that that M denotes the Turing Machine, E denotes the enumerator and $s_1, s_2, ...s_i$ denotes the strings in Σ^*. The $<$ operator is used to show when a string is lexicographically less than another string.

M = "On input string w:
1. Run E in lexicographic order. Every time E outputs a string, compare it with w.

2. If w appears in the output of E, accept.

3. If the lexicographic value of the current string in E's output is greater than the lexicographic value of w, reject

4. If there are no more strings to be read and the input has not yet been accepted, reject. "

This Turing Machine runs E in lexicographic order and must be decidable since this construction will always halt on any input. If the input is recognized by the TM, it will automatically be accepted. Otherwise, it will be rejected. If the language is finite, it will always eventually reject because at some point it will run out of strings to try in the language. The lexicographic criterion becomes useful if the language is infinite. If it is, then that means as the machine runs there will always be some value recognized by the language that is lexicographically greater than the current input being checked. If that is found then the machine should not need to continue further as it will know that it will never find a value equal to the input string as it will always enumerate through strings that are lexicographically greater than the input. So in this case it must reject as well.

In order to prove (2) we must construct an Enumerator in lexicographic order in terms of a decidable Turing Machine. This can be defined as follows:

E = "Ignore the input
1. Repeat the following for i = 1, 2, 3...

2. Run M for i steps of each input $s_1, s_2, ..., s_i$ where $s_1 < s_2 < ... < s_i$

3. If any of the computations accept, print out the corresponding s_j"

This language is decidable since the TM will always run for a finite amount of i steps. That ensures that the machine will never infinitely loop but will also still attempt to recognize the ith string in the language. It is also being printed in lexicographic order because of the criterion $s_1 < s_2 < ... < s_i$. Therefore (2) must be true.

Since we have proven (1) and (2) we have shown that the statement is true.