CSE 105: Homework Set 3

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1. Regular Expression Construction

   Part 1A:
   One possible regex is:
   $$\%(Alpha_L)^*\%$$

   Part 1B:
   This can be defined by the regular expression
   $$(0 \cup 1)(0 \cup 1)(0)(\{0,1\}^*)$$

2. The Complement of A Regular Expression
   A proof that the complement exists for a regular expression can be shown with four propositions.

   (1) By Lemma 1.55, if a language is a regular expression then it is regular
   (2) It the language is regular then it is possible to construct a DFA that describes it
   (3) As shown in a previous homework, it was proved that swapping the accept and nonaccept states of a DFA yields its compliment. Therefore, the compliment of the DFA that describes L(R) exists. Let's call this DFA R'.
   (4) Since a DFA, R', exists such that R' is the compliment of machine R, then that means a regular expression L(R') must exist as a corollary of Lemma 1.56. This is because R' is regular, which means there must be a regular language that describes it.

3. The Pumping Lemma
   Lets assume for a contradiction that the language is regular. Choose a string S accepted by the language which may be described by the 3 sub-strings x, y, z.
   $$S = xyz = a^p b^{2p}$$

where $|y|>0$ and $|xy| \leq p$ and $xy^iz \in L$.

These constraints imply that $xy = a^p$
or more specifically that $x = a^r, y = a^s, z = a^tb^{2p}$ where $r + s + t = p$.

If we choose a string such as xyyz, then this violates the condition that $xy^iz \in L$
since xyyz yields $a^{r+s+s+t} = a^q$ for xyy and $2q \neq 2p$