Aluna Camila Braz Soares

Prova 1 - MatD48 Planejamento de Experimento

① Mostre que

$$E\left[\frac{SQ_{ERRO}}{N-a}\right] = \sigma^2$$

$$E(QME) = E\left(\frac{SQ_E}{N-a}\right) = \frac{1}{N-a} E(SQE) = \frac{1}{N-a} E(SQTotal - SQTrat) \qquad 2\mu(\tau_i + \varepsilon_{ij})$$

$$= \frac{1}{N-a}\left[\underbrace{E(SQTotal)}_{①} - \underbrace{E(SQTrat)}_{②}\right] \qquad (\mu + (\tau_i + \varepsilon_{ij}))^2$$

$$E(C) = E\left[\frac{(\sum_i\sum_j Y_{ij})^2}{N}\right] = \frac{1}{N} E\left[(\sum_i\sum_j Y_{ij})^2\right] = \frac{1}{N} E\left[(\sum_i\sum_j (\mu + \tau_i + \varepsilon_{ij}))^2\right]$$

$$= \frac{1}{N} E\left[(N\mu + n\sum_i \tau_i + \sum_{ij}\varepsilon_{ij})^2\right]$$

$$= \frac{1}{N} E\left[N^2\mu^2 + 2N\mu n\sum_i \tau_i + 2N\mu\sum_{ij}\varepsilon_{ij} + n^2\tau_i^2 + 2n\tau_i\sum_{ij}\varepsilon_{ij} + \sum_{ij}\varepsilon_{ij}^2\right]$$

$$= \frac{1}{N}\left[N^2\mu^2 + Var(\sum_{ij}\varepsilon_{ij}) + E(\sum_{ij}\varepsilon_{ij})^2\right]$$

$$= N\mu^2 + \sigma^2$$

$$E(SQTotal) = E\left[\sum_i\sum_j Y_{ij}^2 - C\right] = E\left[\sum_i\sum_j Y_{ij}^2\right] - E(C)$$

$$= E\left[\sum_{ij}(\mu + \tau_i + \varepsilon_{ij})^2\right] - E(C) =$$

$$E\left[\sum_{ij}(\mu^2 + 2\mu\tau_i + 2\mu\varepsilon_{ij} + \tau_i^2 + 2\tau_i\varepsilon_{ij} + \varepsilon_{ij}^2\right] - E(C)$$

$$= E\left[N\mu^2 + 2\mu n\cancel{\sum_i \tau_i}^{0} + 2\mu\cancel{\sum_{ij}\varepsilon_{ij}}^{0} + n\sum_i \tau_i^2 + 2\sum_i \cancel{\tau_i}^{0}\sum_{ij}\varepsilon_{ij} + \sum_{ij}\varepsilon_{ij}^2\right] - E(C)$$

$$= \cancel{N\mu^2} + n\sum_i \tau_i^2 + N\sigma^2 - \cancel{N\mu^2} - \sigma^2$$

$$= n\sum_i \tau_i^2 + \sigma^2(N-1)$$

$$E(SQTrat) = E\left[\frac{1}{n}\sum_i \left(\sum_j y_{ij}\right)^2 - C\right] = \frac{1}{n}\left\{E\left[\sum_i\left(\sum_j(\mu+\tau_i+\varepsilon_{ij})\right)^2\right] - E(C)\right\}$$

$$= \frac{1}{n}\left\{E\left[\sum_i\left(n^2\mu^2 + 2n\mu\left(n\tau_i + \sum_j \varepsilon_{ij}\right) + \left(n\tau_i + \sum_j \varepsilon_{ij}\right)^2\right)\right] - E(C)\right\}$$

$$= \frac{1}{n}\left\{E\left[\sum_i\left(n^2\mu^2 + 2n^2\mu\tau_i + 2n\mu\sum_j \varepsilon_{ij} + n^2\tau_i^2 + 2n\tau_i\sum_j\varepsilon_{ij} + \left(\sum_j \varepsilon_{ij}\right)^2 - E(C)\right\}\right.$$

$$= \frac{1}{n}\left\{a n^2\mu^2 + 2n^2\mu\overset{\to 0}{\cancel{\sum_i \tau_i}} + 2n\mu\overset{\to 0}{\cancel{\sum_{ij}\varepsilon_{ij}}} + n^2\sum_i \tau_i^2 + 2n\overset{\to 0}{\cancel{\sum_i \tau_i \sum_{ij}\varepsilon_{ij}}} + \sum_i\left(\sum_j \varepsilon_{ij}\right)^2 - E(C)\right\}$$

$$= \cancel{N\mu^2} + n\sum_i \tau_i^2 + a\sigma^2 - (\cancel{N\mu^2} + \sigma^2)$$

$$= n\sum_i \tau_i^2 + \sigma^2(a-1).$$

$$E(QME) = \frac{1}{N-a}E(SQRes) = \frac{1}{N-a}\left[E(SQTotal) - E(SQTrat)\right]$$

$$= \frac{1}{N-a}\left[\left(n\cancel{\sum_i \tau_i^2} + \sigma^2(N-1)\right) - \left(n\sum_i \tau_i^2 + \sigma^2(a-1)\right)\right]$$

$$= \frac{1}{N-a}\left[\sigma^2(N-\cancel{1}) - \sigma^2(a-\cancel{1})\right] = \frac{1}{(\cancel{N-a})}\sigma^2(\cancel{N-a}) = \sigma^2 \;//$$

③ $N = \sum_{i=1}^{a} n_i$

$$SQTrat = N(\hat{\tau}_1^2 + \dots + \hat{\tau}_a^2) = N\left[\left(\frac{Y_1}{N} - \hat{\mu}\right)^2 + \dots + \left(\frac{Y_a}{N} - \hat{\mu}\right)^2\right]$$

$$= N\left[\left(\frac{Y_1^2}{N^2} - \frac{2}{N}Y_1\hat{\mu} + \hat{\mu}^2\right) + \dots + \left(\frac{Y_a^2}{N^2} - 2\frac{Y_a}{N}\hat{\mu} + \hat{\mu}^2\right)\right]$$

$$= (\Sigma_i Y_i)^2/N - (\Sigma_i \Sigma_j Y_{ij})^2/N.$$

$$= \Sigma_i Y_i^2/n_i - (\Sigma_i \Sigma_j Y_{ij})^2/N = \Sigma_i y_{i\cdot}/n_i - y_{\cdot\cdot}^2/N$$

④

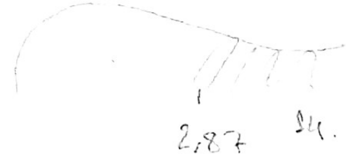
a) H_0: A resistência da fibra de algodão é igual ($r_1 = r_2 = r_3 = r_4 = r_5$)

H_1: A resistência de pelo menos uma fibra de algodão é diferente. ($r_i \neq r_j$ para algum $i \neq j$).

b) Erro tipo I: Afirmar que as resistências das fibras de algodão são diferentes quando na verdade são iguais.

Erro tipo II: Afirmar que as resistências das fibras são iguais, quando na verdade são diferentes.

c)



	g.l.	SQ	QM	F
Trat	4	475,76	118.94	14,757
Res	20	161,20	8,06	
Total	24	636,96		

d) $F_{(0,05;4,20)} = 2,87$

Como $F_{cal} = 14,757 > F_{tab} = 2,87$, o engenheiro deverá rejeitar H_0 com 95% de significância.

(5) p-valor = 0,7084618

Como p-valor = 0,71 é maior que 0,05, não devemos rejeitar a hipótese nula, portanto, podemos afirmar ao nível de 5% que as variâncias são iguais.