Homework

1. Show the Lorentz invariance of the Klein-Gordon equation for scalars.

2. Given the Dirac Lagrangian density

$$\mathcal{L} = \bar{\Psi} \left(i c \hbar \gamma^{\mu} \partial_{\mu} - m_0 c^2 \right) \Psi$$

where $\bar{\Psi} = \Psi^{\dagger} \gamma^0$ is the adjoint spinor, calculate the energy momentum tensor

$$T^{\mu}{}_{\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_{\mu}\Psi)} \partial_{\nu} \Psi + \frac{\partial \mathcal{L}}{\partial(\partial_{\mu}\bar{\Psi})} \partial_{\nu} \bar{\Psi} - \delta^{\mu}_{\nu} \mathcal{L}$$

and interpret the results for $T^0{}_0$, $T^0{}_i$ and $T^{\mu}{}_{\mu}$

3. Construct the four-current density and equation of continuity in case of the Dirac equation. Prove the covariance of the continuity equation. Hint: You can start by showing that the current density transforms as a 4-vector.

4. ⓐ Using Maxwell's equations in 3-dimensions show that the electric field $\vec{E}$ is a vector and magnetic field $\vec{B}$ is an axial vector.

4. (b) As one can see, Maxwell's equations are not completely symmetric because although they include an electric charge density ρ_e and an electric current density $\vec{J}_e$, the equivalent magnetic quantities ρ_m and $\vec{J}_m$ are absent indicating that there are no magnetic monopoles. Introduce magnetic monopoles and write down the completely symmetric Maxwell's equations. Show that ρ_m must be a pseudoscalar and $\vec{J}_m$ an axial vector.

5. We showed in class that $\bar{\psi}\gamma^5\psi$ is a pseudoscalar. Now show that $\bar{\psi}\gamma^5\gamma^\mu\psi$ is a pseudovector (or axial vector).

Comment on the Lorentz and parity properties of the quantities

a) $\bar{\psi}\gamma^5\gamma^\mu\psi\,\bar{\psi}\gamma_\mu\psi$

b) $\bar{\psi}\gamma^5\psi\,\bar{\psi}\gamma^5\psi$

c) $\bar{\psi}\psi\,\bar{\psi}\gamma^5\psi$

d) $\bar{\psi}\gamma^5\gamma^\mu\psi\,\bar{\psi}\gamma^5\gamma_\mu\psi$

e) $\bar{\psi}\gamma^\mu\psi\,\bar{\psi}\gamma_\mu\psi$

6. The ρ^0 is a vector boson, i.e. a 1^- (spin parity) state. Explain why the decay $\rho^0 \to \pi^+\pi^-$ is allowed and why the decay $\rho^0 \to \pi^0\pi^0$ is forbidden.

Hints: 1) Wavefunctions of identical bosons must be symmetric under interchange of particles.

2) In case of identical particles, to exchange their positions is the same as to make parity transformation.

7. Griffiths 4.11: In the decay $\Delta^{++} \to p + \pi^+$, what is the minimum possible value of the orbital angular quantum number 'l'?

8. Griffiths 4.27: For two isospin $\frac{1}{2}$ particles show that $\vec{I}^{(1)} \cdot \vec{I}^{(2)} = \frac{1}{4}$ in the triplet state and $-3/4$ in the singlet.

9. Griffiths 4.29: Find the ratio of the cross-sections for the following reactions when the total centre of mass energy is 1232 MeV.

a) $\pi^- p \to K^0 \Sigma^0$, (b) $\pi^- p \to K^+ \Sigma^-$ (c) $\pi^+ p \to K^+ \Sigma^+$