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by

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A COMPARISON BETWEEN UNIDIMENSIONAL AND MULTIDIMENSIONAL APPROACHES TO THE MEASUREMENT OF POVERTY¹

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Abstract

The paper presents a comparison between an unidimensional approach to the measurement of poverty, obtained as a function of observable income, and a multidimensional approach, defined on the basis of economic, social, demographic and cultural indicators. The comparison is carried out by means of rank correlation analysis, which stresses how the two approaches indicate the presence of two different sets of poor households. An analysis performed on 12 European countries shows that income based evaluation provides only partial insights on poverty condition.

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1. Introduction

Modern concept of poverty greatly differs from traditional view: actually sociologists, psychologists, economists and politicians refer to poverty as social exclusion, that is a wide and general notion, which includes many aspects other than income or wealth (Hagenaars, 1986; Dagum, 1989; Sen, 1992; World Bank, 2001). While unidimensional approach to the measurement of poverty refers only to one information variable, usually the income, in the multidimensional approach several indicators are used, in order to obtain a more exhaustive and useful measure. By means of the multidimensional analysis it is possible to identify the main causes of poverty and, therefore, to adopt socioeconomic policies to reduce poverty diffusion. A relevant step in the poverty analyses is to compare uni- and multidimensional approaches, in order to find if the set identified as poor on the basis of income information corresponds to the set identified as poor according to the multidimensional approach. If the differences are negligible, unidimensional analysis maintains an important role in poverty studies, on the other hand, when the two sets of poor display strong dissimilarities, it is necessary to choose the most adequate framework.

The paper presents a rank correlation analysis between uni- and multidimensional approaches, allowing a comprehensive comparison for household population in 12 European countries on the basis of European Community Household Panel data.

2. Methodology

This section follows the research program outlined in Dagum and Costa (2002) in order to compare uni- and multidimensional approaches to the measurement of poverty. The methodological tool is represented by correlation analysis, carried out by means of different measures (Balakrishnan and Rao, 1998*a,b*; Stuart et al., 1999): Bravais-Pearson correlation coefficient, Kendall correlation index, Spearman rank correlation index and Gini rank correlation, or cograduation, index.

The first step of the research refers to the unidimensional approach.

By using only the net income as information about poverty status of the household, the OECD equivalence scale is adopted in order to ensure comparability among incomes of households of different sizes. By indicating with n_e the number of adult equivalents, with n the household size and with n_a the number of adults (14 years or more), the OECD equivalence scale states that:

$$n_e = 1 + 0.7 * (n_a - 1) + 0.5 * (n - n_a) .$$

Table 1

Household size n , number of adults n_a and number of adults equivalents n_e according to OECD equivalence scale

n	1	2	2	3	3	3	4	4	4	5	5	5
n_a	1	2	1	3	2	1	4	3	2	4	3	2
n_e	1	1.7	1.5	2.4	2.2	2	3.1	2.9	2.7	3.6	3.4	3.2

Table 1 shows the relation between household size and number of adults equivalents for some values of n and n_a ; with respect to the simple per capita household income y_i / n_i , the transformation $y_{ei} = y_i / n_{ei}$ gives the per capita household income corrected by the effect of different household sizes, thus allowing to correctly compare household incomes.

Given an information variable and an equivalence scale it is possible to obtain the unidimensional list (list A) of the households by ascending order of their equivalent income y_{ei} , i.e. from the poorest to the richest household.

The last relevant element needed in the unidimensional approach is represented by a poverty line (Ravallion, 1998), that is a rule partitioning total population into poor and non-poor. In the following it is adopted the International Standard of Poverty Line (ISPL), which states that the poverty line for a two-person household corresponds to the per capita average income of the population, or, equivalently, that the individual poverty line is equal to 50% of the per capita average income of the population.

By using OECD equivalence scale, thus moving from y_i to $y_{ei} = y_i / n_{ei}$, the reference is represented by a one-member household and therefore the poverty line is defined as

$$\sum_{i=1}^N y_i / (2 * NN)$$

where N and NN are, respectively, the number of households and the number of individuals belonging to the population. The i -th household is considered poor if $y_{ei} \leq \sum_{i=1}^N y_i / (2 * NN)$.

Given an equivalence scale and a poverty line it is finally possible to obtain some synthetic unidimensional poverty measure, such as the head count ratio

$$H = q / N$$

that is the ratio of poor households, q , to total population of households, N .

The second step of the research takes into account the multidimensional approach to the measurement of poverty (Dagum and Costa, 2002).

The multidimensional poverty ratio of the i -th household, μ_{Bi} , $i = 1, \dots, N$, i.e. the degree of membership of the i -th household to the set of poor households, is obtained from the results in Costa (2002), where are also explained the choice of the m indicators of poverty, X_1, \dots, X_m , the choice of the weights w_1, \dots, w_m , attached to each indicator X_j , and the choice of the degrees of membership of the i -th household to the set of poor households with respect to the j -th indicator. The poverty ratios μ_{Bi} allow to obtain the multidimensional list (list B) of the households by descending order of μ_{Bi} , i.e. from the poorest to the richest household.

Given the poverty ratios μ_{Bi} , it is possible to calculate the multidimensional poverty ratio for the population, μ_B , as the average³ of μ_{Bi} :

$$\mu_B = \sum_{i=1}^N \mu_{Bi} / N.$$

The third step of the research deals with the comparison between uni- and multidimensional approaches by means of four correlation measures which are applied to lists A and B.

Correlation analysis is carried out on the ranks p_i , $i = 1, \dots, N$, of list A, derived from the unidimensional approach, and on the ranks π_i , $i = 1, \dots, N$, of list B, obtained in the multidimensional framework. Rank correlation analysis aims to assess the similarity between the ranks related to the two approaches and also allows to evaluate the similarity between the ranks of specific subsets of households, that is an extension of particular interest for the case of the subset of poor households.

The first standard correlation measure used is the Bravais-Person correlation coefficient r applied to the ranks (p_i, π_i) :

$$r = \frac{\sum_{i=1}^N (p_i - \bar{p})(\pi_i - \bar{\pi})}{\sqrt{\sum_{i=1}^N (p_i - \bar{p})^2} \sqrt{\sum_{i=1}^N (\pi_i - \bar{\pi})^2}}$$

where \bar{p} and $\bar{\pi}$ are, respectively, the mean of p_i and the mean of π_i .

The second correlation measure is represented by Kendall's τ , which can be expressed as

$$\tau = \frac{C - D}{N(N-1)/2}$$

³ Working with survey data usually requires to resort to a weighted average, where the weights are attached to the sample observations.

where **C** is the number of concordant pairs of ranks (\mathbf{p}_i, π_i), and **D** is the number of discordant pairs of ranks (\mathbf{p}_i, π_i). Kendall's index does not consider the amount of the differences between \mathbf{p}_i and π_i and therefore can be less informative than other measures, such as Bravais-Person **r**.

A further important rank correlation measure is represented by Spearman index:

$$S = \frac{1 - 6 * \sum_{i=1}^N (\mathbf{p}_i - \pi_i)^2}{N(N^2 - 1)}$$

Finally, the fourth correlation measure considered is the Gini rank correlation, or cograduation, index:

$$G = \frac{\sum_{i=1}^N |\mathbf{p}_i - \pi'_i| |\mathbf{p}_i - \pi_i|}{N^2 / 2}$$

where π'_i is the reverse rank of list B, that is the rank of multidimensional list of the households by ascending order, i.e. from the richest to the poorest.

All indexes range between -1 and 1 , reaching their maximum (minimum) value for perfect positive (negative) correlation and assuming value zero for the absence of correlation between \mathbf{p}_i and π_i .

3. Data

The data used in this study are derived from the 2001 release⁴ of the European Community Household Panel (Eurostat, 1996a,b, 2001a,b) and refer to household total net income, household size, number of adults (14 years or more) and sample weight. These variables contain all information for unidimensional approach, while the multidimensional poverty ratio μ_{Bi} for each household is obtained from the results of a previous research (Costa, 2002).

On the basis of ECHP data, 12 European countries (Germany, Denmark, The Netherlands, Belgium, France, United Kingdom, Ireland, Italy, Greece, Spain, Portugal, Austria) are analyzed for 1998.

Tables 2 and 3 report for the case of Germany a subset of observations of the four variables of interest:

- (i) the household equivalent income y_{ei} ,
- (ii) its rank \mathbf{p}_i ,
- (iii) the multidimensional poverty ratio μ_{Bi} ,
- (iv) its rank π_i .

⁴ ECHP UDB, version of December 2001.

Table 2

Multidimensional poverty ratio μ_{Bi} , rank of μ_{Bi} , equivalent income y_{ei} , rank of y_{ei} for increasing ranks of μ_{Bi}

Germany				
$F(\pi_i)\%$	π_i , rank of μ_{Bi}	μ_{Bi}	p_i , rank of y_{ei}	y_{ei}
0	1	0.937	18	2541.0
0	2	0.906	107	7054.4
0	3	0.847	174	8777.3
0	4	0.837	269	10544.1
0	5	0.828	295	10800.0
0	6	0.819	73	5946.0
0	7	0.817	63	5400.0
0	8	0.817	154	8400.0
0	9	0.792	311	11040.0
0	10	0.788	102	6955.0
1	56	0.657	710	14675.3
5	289	0.482	3029	25667.0
10	612	0.345	1065	16784.0
25	1487	0.207	705	14661.7
50	2986	0.097	1976	20975.9
75	4319	0.056	1964	20934.1
90	5028	0.029	3016	25588.0
95	5305	0	2075	21412.6
99	5542	0	5472	70847.0
100	5561	0	5546	104652.4
100	5562	0	5548	108607.1
100	5563	0	5552	118020.0
100	5564	0	5559	141474.1
100	5565	0	5561	144102.4
100	5566	0	5565	155877.0
100	5567	0	5566	193699.0
100	5568	0	5567	205328.0
100	5569	0	5568	231142.3
100	5570	0	5569	243729.0

Table 3

Multidimensional poverty ratio μ_{Bi} , rank of μ_{Bi} , equivalent income y_{ei} , rank of y_{ei} for increasing ranks of y_{ei}

Germany				
$F(p_i)\%$	π_i , rank of μ_{Bi}	μ_{Bi}	p_i , rank of y_{ei}	y_{ei}
0	1235	0.233	1	82.0
0	1170	0.238	2	97.0
0	35	0.694	3	140.0
0	32	0.703	4	185.0
0	182	0.541	5	208.0
0	1255	0.231	6	447.1
0	1485	0.207	7	972.0
0	1236	0.233	8	1033.5
0	348	0.442	9	1294.1
0	126	0.575	10	1500.0
1	1019	0.262	36	3946.0
5	1175	0.238	226	9764.0
10	273	0.489	464	12769.0
25	3456	0.085	1452	18686.5
50	1591	0.191	2991	25457.1
75	3962	0.066	4363	34287.7
90	4148	0.061	5158	47482.3
95	4159	0.061	5357	57211.4
99	1425	0.212	5541	97381.8
100	5565	0	5561	144102.4
100	1792	0.177	5562	146134.1
100	4983	0.031	5563	151402.0
100	4984	0.031	5564	152423.5
100	5566	0	5565	155877.0
100	5567	0	5566	193699.0
100	5568	0	5567	205328.0
100	5569	0	5568	231142.3
100	5570	0	5569	243729.0
100	4616	0.056	5570	391460.0

The first column of Table 2 shows selected values of the cumulative distribution of μ_{Bi} , the second and the fourth columns contain, respectively, multidimensional list B and unidimensional list A, which will be used in the rank correlation analysis. Observations presenting the same value of μ_{Bi} are sorted by increasing size of y_{ei} : that is only a simplifying procedure which ensures a more clear and regular synthesis of the relation between p_i and π_i , without influencing neither its direction or its importance.

By comparing column 2 to column 4 in Table 2 it is immediate to observe how the highest ranks of μ_{Bi} generally correspond to the highest ranks of equivalent income, while, on the contrary, the lowest ranks of μ_{Bi} do not correspond to the lowest ranks of equivalent income.

Analogously to Table 2, Table 3 reports multidimensional poverty ratio μ_{Bi} , rank of μ_{Bi} , equivalent income y_{ei} and rank of y_{ei} for increasing ranks of y_{ei} : from Table 3 it is possible to observe how, for the lowest ranks of y_{ei} , the differences between p_i and π_i are even more accentuated than on the case of Table 3. Similarly to the previous case, observations presenting the same value of y_{ei} are sorted by decreasing size of μ_{Bi} .

Data for the other countries are reported in Table A.1 in the Appendix and regularly confirm the pattern outlined for Germany: while highest ranks of μ_{Bi} correspond to highest ranks of y_{ei} , lowest ranks of μ_{Bi} generally differ from lowest ranks of y_{ei} .

Multidimensional and unidimensional approaches seem to define two different sets of poor households: a more complete analysis, carried out on the basis of rank correlation, will be illustrated in the next paragraph.

4. Results

The results of the first step of the research refer to unidimensional poverty ratio. On the basis of OECD equivalence scale, the international standard of poverty line is obtained dividing the per capita average income of the population by 2. For Germany the per capita average income and the ISPL in 1998 are, respectively, equal to 19698.28 and to 9849.14. Consequently, all incomes equal or less than 9849.14 indicate a poor household belonging to the unidimensional set q . From Table 2 it is possible to observe how, in the set of the 10 poorest households according to multidimensional approach, 7 households have an equivalent income lower than 9849.14, and therefore are to be considered poor also in the unidimensional approach.

The ratio of the poor households q to total population of households N gives an important unidimensional poverty ratio, the head count ratio, which for Germany is equal to 0.059.

The head count ratio by country is reported on Table 5 and indicates that, in the set of 12 European countries analyzed in this study, Portugal, Greece and Spain are the poorest, while Denmark, Austria and the Netherlands are the richest countries.

In the second step of the research the multidimensional poverty ratio for the population μ_B is obtained as a weighted average of the multidimensional poverty ratio for the household μ_{Bi} , where the weights are the sample weights attached to the survey observations.

The multidimensional approach does not possess a poverty line partitioning total population into poor and non-poor: in order to individuate poor households it is necessary to calculate the rank π^* corresponding to $F(\pi^*)=\mu_B$.

For Germany $\mu_B=0.152$, and $F(\pi^*)=\mu_B$ for $\pi^*=952$: therefore in the multidimensional approach the German poor households are the first 952 by descending order of μ_{Bi} . From Table 3 it is possible to observe how, in the set of the 10 poorest households according to unidimensional approach, only 5 are also poor in the multidimensional framework, having a rank π_i lower than 952.

Table 4

Values observed in the multidimensional approach variables for 5 German households

Household	I, $p_I=1$,	II, $p_{II}=2$,	III, $p_{III}=6$,	IV, $p_{IV}=7$,	V, $p_V=8$,
Variables	$\pi_I=1235$	$\pi_{II}=1170$	$\pi_{III}=1255$	$\pi_{IV}=1485$	$\pi_V=1236$
Observed income	82.0	97.0	760.1	972.0	1757.0
Equivalent income	82.0	97.0	447.1	972.0	1033.5
Household size	1	1	2	1	2
Number of adults	1	1	2	1	2
Household residence:					
- Number of rooms					
without kitchen	3	2	3	2	4
- Heating	yes	yes	yes	yes	yes
- Bath or shower	yes	yes	yes	yes	yes
- Indoor flushing toilet	yes	yes	yes	yes	yes
Reference person:					
-level of education	ISCED 3	ISCED 3	ISCED 0-2	ISCED 0-2	ISCED 3
-principal activity	Retired	Employer in paid apprenticeship	Retired	Self- employment, manager	Retired
Household type	Female under 65	Female under 65	2 adults both under 65	Male under 65	2 adults both under 65

*For the level of education, ISCED 3 corresponds to second stage of secondary level education, ISCED 0-2 corresponds to less than second stage of secondary education.

In order to assess the poverty condition of these 5 households it is possible to observe analytically (Table 4) the values which they present in the variables used for the multidimensional analysis.

The household with the lowest equivalent income, $y_e=82$, is a one-adult, retired female under 65 household, with the second stage of secondary level education achieved, living in a dwelling of 4 rooms, with heating, bath and indoor flushing toilet: the only poverty signal originates from equivalent income, while other variables do not detect any condition of deprivation. Also for the other four cases illustrated in Table 4 the same pattern is repeated: the only clear poverty signal comes from equivalent income, while other variables indicate a non-poor condition. The more exhaustive and complete information provided by multidimensional approach allows to correctly classify these households as non-poor units.

The multidimensional poverty ratio by country is reported on Table 5 and indicates Portugal, Spain and Greece as the poorest countries (the same results given by the head count ratio), while Denmark, France and United Kingdom are the richest.

Table 5

Unidimensional H and multidimensional μ_B poverty ratios by country

Country	D	DK	NL	B	F	UK	IRL	I	GR	SP	P	A
H	0.059	0.033	0.046	0.059	0.065	0.113	0.046	0.075	0.144	0.122	0.159	0.041
μ_B	0.152	0.102	0.130	0.142	0.127	0.127	0.160	0.154	0.172	0.177	0.191	0.142

In the two approaches Denmark and Portugal maintain the same position, respectively at the top and at the bottom of the list. It also interesting to observe how in all countries, the unidimensional poverty ratio indicates a lower diffusion of poverty with respect to the multidimensional measure.

The global and aggregate measures reported on Table 5 provide only partially insights on the comparison between uni- and multidimensional approaches and, moreover, on the socio-economic policies able to reduce poverty. The key point in poverty analyses is not to establish how many are the poor households, but who are they.

These objectives require a more exhaustive analysis, which can be performed on the basis of the rank correlation indexes presented in paragraph 2.

Table 6 reports the results of rank correlation analysis related to Germany. The first column shows selected values of the cumulative distribution of μ_{Bi} , $F(\mu_{Bi})$, with μ_{Bi} in decreasing order, while the following four columns contain the values of the four rank correlation indexes.

Table 6**Rank correlation between multidimensional poverty ratio μ_{Bi} and equivalent income y_{ei}**

$F(\mu_{Bi})$	Germany			
	Rank correlation index			
	Kendall	Bravais	Spearman	Gini
5	0.2680	0.3574	0.3010	0.2969
10	0.3174	0.4610	0.3333	0.4489
25	0.2390	0.2930	0.0902	0.2583
50	0.3422	0.4597	0.3625	0.4173
75	0.4081	0.5604	0.5008	0.4813
90	0.4207	0.5925	0.5672	0.4935
95	0.4370	0.6127	0.5918	0.5059
100	0.4597	0.6396	0.6229	0.5244

The rank correlation indexes for the whole population, that is for $F(\mu_{Bi}) = 100$, are, respectively:

0.4597, 0.6396, 0.6229 and 0.5244.

If only the poorest 5% of total population is analyzed, that is for $F(\mu_{Bi}) = 5$, the rank correlation indexes are, respectively:

0.2680, 0.3574, 0.3010 and 0.2969.

From Table 6 it is possible to observe how, moving from $F(\mu_{Bi}) = 100$ to $F(\mu_{Bi}) = 5$, there is a strong decrease in rank correlation, clearly indicating that only few of the households identified as poor in the multidimensional analysis correspond to the households which are poor with respect to their income.

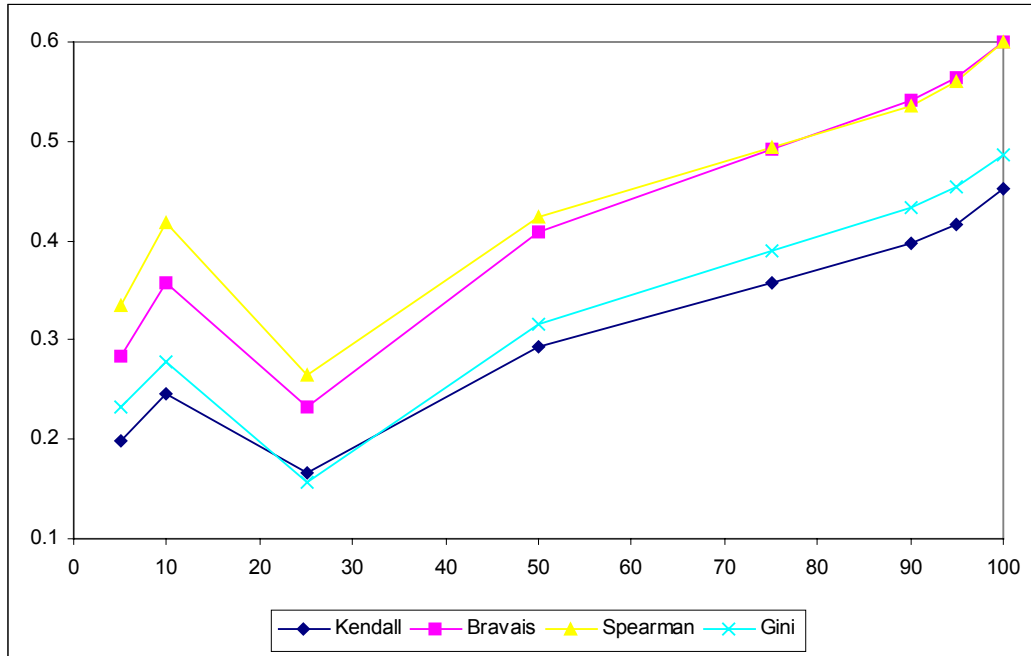
Rank correlation results for the other countries here analysed are reported in Table A.2 in the Appendix. In all countries the same pattern is easily detectable, with rank correlation indexes which decrease from $F(\mu_{Bi}) = 100$ to $F(\mu_{Bi}) = 5$, generally reaching the minimum value for $F(\mu_{Bi}) = 25$. For Greece are also present negative rank correlation indexes for $F(\mu_{Bi}) = 25$ and $F(\mu_{Bi}) = 10$.

As a synthesis of the values of the rank correlation indexes, Figure 1 illustrates their mean in the 12 European countries for some values of $F(\mu_{Bi})$.

Kendall's and Gini's measures give quite similar results and report a lower correlation between p_i and π_i than Bravais-Pearson and Spearman indexes. All indexes, however, clearly signal a decreasing rank correlation for decreasing values of $F(\mu_{Bi})$, with a minimum value corresponding to $F(\mu_{Bi})=25$.

Figure 1

Mean of rank correlation indexes in the 12 countries for some values of $F(\mu_{Bi})$



5. Conclusion

Perception of poverty changes greatly in the last decades, leading to a wide theoretical debate, which states that income and wealth provide insufficient information on poverty condition. The adoption of a more general and multidimensional definition of poverty requires to adequate methodological tools for the measurement of poverty, actually generally still obtained on the basis of income only.

The paper illustrates a comparison between traditional unidimensional approach to the measurement of poverty and a new multidimensional approach, obtained by taking into account economic, social, demographic and cultural indicators. The results of a rank correlation analysis allow to demonstrate that the two approaches define two different sets of poor households.

Therefore any socio-economic policy to reduce poverty developed on the basis of income information is likely to not achieve its proposed goals, being addressed to socioeconomic units which are, in effect, non-poor. Only in the framework of the multidimensional approach it is possible to correctly individuate the set of the poor and to formulate actions able to reduce poverty.

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Appendix

Table A.1

Multidimensional poverty ratio μ_{Bi} , rank of μ_{Bi} , equivalent income y_{ei} , rank of y_{ei} for increasing sizes of μ_{Bi}

Denmark				
$F(\mu_{Bi})$	π_i , rank of μ_{Bi}	μ_{Bi}	p_i , rank of y_{ei}	y_{ei}
1	21	0.500	37	41600.0
5	125	0.329	434	81037.1
10	245	0.237	652	93176.5
25	574	0.137	1067	113011.1
50	1140	0.065	1684	144851.2
75	1732	0.041	1761	150624.0
90	2136	0.021	2330	260942.9
95	2257	0	1701	146148.1
99	2369	0	2304	239070

The Netherlands				
$F(\mu_{Bi})$	π_i , rank of μ_{Bi}	μ_{Bi}	p_i , rank of y_{ei}	y_{ei}
1	39	0.517	163	10326.0
5	202	0.363	2647	28112.5
10	412	0.298	1623	21348
25	1050	.0200	1031	18276.0
50	2204	0.106	2883	30145.3
75	3389	0.016	1364	20004.7
90	4102	0	3504	36768.2
95	4309	0	4061	46782.5
99	4474	0	4419	73252.0

Table A.1 (continued)

Belgium				
$F(\mu_{Bi})$	π_i , rank of μ_{Bi}	μ_{Bi}	p_i , rank of y_{ei}	y_{ei}
1	26	0.579	32	119546.7
5	111	0.426	14	45229.0
10	218	0.327	1640	611771.2
25	562	0.191	675	384364.7
50	1146	0.104	2365	1449373.0
75	1762	0.053	2211	960000.0
90	2168	0.020	1262	514193.6
95	2290	0	1392	541381.0
99	2426	0	2263	1050000.0
France				
$F(\mu_{Bi})$	π_i , rank of μ_{Bi}	μ_{Bi}	p_i , rank of y_{ei}	y_{ei}
1	53	0.585	39	8326.0
5	275	0.422	1910	63632.0
10	531	0.320	15	1651.8
25	1304	0.178	3448	89655.6
50	2697	0.071	1524	56478.0
75	4106	0.029	4915	140747.8
90	4954	0.021	5173	163270.0
95	5258	0	4473	117911.5
99	5470	0	5364	201831.2
United Kingdom				
$F(\mu_{Bi})$	π_i , rank of μ_{Bi}	μ_{Bi}	p_i , rank of y_{ei}	y_{ei}
1	54	0.492	102	2117.2
5	234	0.371	2424	9072.0
10	439	0.303	1385	6175.9
25	1101	0.186	1091	5424
50	2171	0.091	1102	5466.3
75	3299	0.033	1698	7071.1
90	4011	0	2523	9410.4
95	4246	0	3875	15642.3
99	4437	0	4376	26632.9

Table A.1 (continued)

Ireland				
$F(\mu_{Bi})$	π_i , rank of μ_{Bi}	μ_{Bi}	p_i , rank of y_{ei}	y_{ei}
1	12	0.627	236	3461.3
5	101	0.438	380	3829.0
10	190	0.364	133	2935.0
25	509	0.238	589	4297.0
50	1169	0.111	1009	5597.5
75	1915	0.055	1848	9384.8
90	2326	0.020	2374	14867.0
95	2466	0	1975	10295.9
99	2571	0	2530	22017.1
Italy				
$F(\mu_{Bi})$	π_i , rank of μ_{Bi}	μ_{Bi}	p_i , rank of y_{ei}	y_{ei}
1	59	0.585	102	1764.7
5	295	0.427	4742	23000.0
10	590	0.334	3644	17550.0
25	1473	0.198	1914	11334.7
50	2946	0.099	4588	22028.3
75	4419	0.057	1967	11503.1
90	5302	0.038	5799	47847.1
95	5597	0.018	3116	15466.7
99	5833	0	5360	29010.0
Greece				
$F(\mu_{Bi})$	π_i , rank of μ_{Bi}	μ_{Bi}	p_i , rank of y_{ei}	y_{ei}
1	40	0.532	440	630808.0
5	209	0.410	279	495000.0
10	407	0.352	1568	1320000.0
25	1059	0.232	3262	2752941.0
50	2125	0.142	2374	1800000.0
75	3129	0.082	1651	1375294.0
90	3691	0.044	3657	3540000.0
95	3855	0.032	3791	4128067.0
99	4008	0	3323	2819362.0

Table A.1 (continued)

Spain				
$F(\mu_{Bi})$	π_i , rank of μ_{Bi}	μ_{Bi}	p_i , rank of y_{ei}	y_{ei}
1	33	0.560	68	50746.5
5	203	0.432	2378	868125.0
10	469	0.359	1916	779800.0
25	1261	0.251	1146	609734.7
50	2627	0.139	2130	815961.2
75	4041	0.090	4266	1565792.0
90	4887	0.047	5011	2378824.0
95	5141	0.022	4396	1666667.0
99	5302	0	5128	2690177.0
Portugal				
$F(\mu_{Bi})$	π_i , rank of μ_{Bi}	μ_{Bi}	p_i , rank of y_{ei}	y_{ei}
1	55	0.611	23	32127.0
5	272	0.433	3849	1415000.0
10	516	0.353	3166	1009511.0
25	1146	0.247	892	426166.7
50	2286	0.152	2404	758925.8
75	3428	0.110	4408	2716000.0
90	4229	0.072	3387	1103230.0
95	4421	0.054	4544	4414118.0
99	4560	0.019	4555	4970000.0
Austria				
$F(\mu_{Bi})$	π_i , rank of μ_{Bi}	μ_{Bi}	p_i , rank of y_{ei}	y_{ei}
1	32	0.510	462	102500.0
5	148	0.373	167	73200.0
10	273	0.317	352	94175.9
25	739	0.202	237	85264.7
50	1435	0.099	1053	139825.5
75	2161	0.048	1193	148510.0
90	2605	0.038	2435	263000.0
95	2758	0.022	2269	236584.0
99	2871	0	2333	246628.4

Table A.2**Rank correlation between multidimensional poverty ratio μ_{Bi} and equivalent income y_{ei}**

Denmark				
F(μ_{Bi})	Rank correlation index			
	Kendall	Bravais	Spearman	Gini
5	0.2570	0.3463	0.3300	0.3369
10	0.2777	0.4423	0.4476	0.3757
25	0.0655	0.1170	0.1830	0.0337
50	0.2770	0.4080	0.4979	0.2761
75	0.3361	0.5175	0.5629	0.3931
90	0.4187	0.6159	0.6296	0.4873
95	0.4088	0.5830	0.5950	0.4747
100	0.4575	0.6296	0.6352	0.5142
The Netherlands				
F(μ_{Bi})	Rank correlation index			
	Kendall	Bravais	Spearman	Gini
5	0.2584	0.3644	0.5272	0.2428
10	0.2578	0.3038	0.4573	0.1930
25	0.2261	0.3251	0.4515	0.2366
50	0.2925	0.4173	0.4721	0.3104
75	0.2993	0.4225	0.4407	0.3196
90	0.3352	0.4575	0.4684	0.3494
95	0.3796	0.5139	0.5246	0.3910
100	0.4289	0.5748	0.5859	0.4402

Table A.2 (continued)

Belgium				
F(μ_{Bi})	Rank correlation index			
	Kendall	Bravais	Spearman	Gini
5	0.1135	0.1702	0.3320	0.1539
10	0.2929	0.3893	0.5582	0.2604
25	0.1983	0.2807	0.4288	0.2019
50	0.3611	0.5310	0.6078	0.3903
75	0.3744	0.5354	0.5849	0.4037
90	0.3888	0.5417	0.5713	0.4177
95	0.3909	0.5467	0.5729	0.4246
100	0.4259	0.5772	0.5895	0.4515
France				
F(μ_{Bi})	Rank correlation index			
	Kendall	Bravais	Spearman	Gini
5	0.1843	0.2330	0.2322	0.1879
10	0.3346	0.4800	0.5173	0.3696
25	0.2021	0.2786	0.3704	0.1990
50	0.3014	0.4333	0.4587	0.3299
75	0.3960	0.5361	0.5387	0.4233
90	0.4179	0.5529	0.5512	0.4333
95	0.4269	0.5657	0.5606	0.4521
100	0.4696	0.6149	0.6112	0.4900
United Kingdom				
F(μ_{Bi})	Rank correlation index			
	Kendall	Bravais	Spearman	Gini
5	0.1486	0.2184	0.1595	0.1987
10	0.1901	0.2643	0.3187	0.2163
25	0.1911	0.2488	0.3036	0.1847
50	0.2419	0.3413	0.3879	0.2504
75	0.3200	0.4485	0.4729	0.3396
90	0.3482	0.4888	0.4965	0.3817
95	0.3749	0.5220	0.5256	0.4067
100	0.4308	0.5832	0.5837	0.4563

Table A.2 (continued)

Ireland				
F(μ_{Bi})	Rank correlation index			
	Kendall	Bravais	Spearman	Gini
5	0.0590	0.0429	0.3797	0.0037
10	0.1423	0.2468	0.5691	0.1543
25	0.3391	0.4800	0.6864	0.2803
50	0.3726	0.5590	0.6528	0.4185
75	0.3816	0.5688	0.5716	0.4728
90	0.4297	0.6431	0.6193	0.5497
95	0.4451	0.6646	0.6367	0.5755
100	0.4859	0.7052	0.6787	0.6117
Italy				
F(μ_{Bi})	Rank correlation index			
	Kendall	Bravais	Spearman	Gini
5	0.1430	0.2411	0.5391	0.1333
10	0.2273	0.3437	0.5271	0.2425
25	0.1557	0.2454	0.3213	0.1569
50	0.2562	0.3524	0.3561	0.2642
75	0.3160	0.4259	0.4532	0.3274
90	0.3691	0.4939	0.5131	0.3813
95	0.4014	0.5322	0.5457	0.4100
100	0.4246	0.5609	0.5693	0.4332
Greece				
F(μ_{Bi})	Rank correlation index			
	Kendall	Bravais	Spearman	Gini
5	0.1861	0.2876	0.2144	0.2404
10	-0.0352	-0.0216	-0.0592	-0.0549
25	0.0143	-0.0412	-0.2092	-0.0665
50	0.3088	0.3258	0.2231	0.2973
75	0.3885	0.4587	0.4099	0.3953
90	0.4386	0.5416	0.5188	0.4539
95	0.4553	0.5732	0.5589	0.4775
100	0.4833	0.6095	0.6010	0.5054

Table A.2 (continued)

Spain				
F(μ_{Bi})	Rank correlation index			
	Kendall	Bravais	Spearman	Gini
5	0.2069	0.4497	0.6491	0.2723
10	0.3642	0.5339	0.6310	0.3906
25	0.1221	0.2091	0.2619	0.1364
50	0.2295	0.3441	0.3436	0.2554
75	0.3486	0.4927	0.4728	0.3936
90	0.4076	0.5614	0.5393	0.4560
95	0.4285	0.5927	0.5721	0.4832
100	0.4567	0.6245	0.6112	0.5071
Portugal				
F(μ_{Bi})	Rank correlation index			
	Kendall	Bravais	Spearman	Gini
5	0.4505	0.5685	0.3066	0.6081
10	0.3150	0.4303	0.2272	0.4191
25	0.0554	0.0960	0.0646	0.0591
50	0.2813	0.3342	0.3244	0.2618
75	0.3767	0.4499	0.4374	0.3536
90	0.4103	0.4903	0.4504	0.3873
95	0.4364	0.5301	0.4992	0.4185
100	0.4642	0.5809	0.5598	0.4597
Austria				
F(μ_{Bi})	Rank correlation index			
	Kendall	Bravais	Spearman	Gini
5	0.1125	0.1329	0.0557	0.1245
10	0.2769	0.4154	0.4847	0.3165
25	0.1798	0.2584	0.2309	0.2126
50	0.2560	0.4004	0.3949	0.3133
75	0.3419	0.4906	0.4903	0.3790
90	0.3733	0.5166	0.5113	0.4004
95	0.3996	0.5422	0.5365	0.4209
100	0.4327	0.5827	0.5788	0.4520

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