Express Air Shipping Schedule

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1 Executive Summary

Given a weekly schedule of demands between the airports of Express Air, our consulting firm was tasked with finding an optimal, weekly schedule and initial placement of aircraft to minimize the total cost incurred by the firm. In addition, we analyzed the effect of increasing the number of fleet size with the total weekly cost for the company. Based on these results, we advised them to acquire additional number of aircraft such that the total weakly cost decreases. Finally, we concluded our paper by giving insightful remarks.

After modeling the firm's flight schedule as an integer linear program, we found that the optimal flight schedule has a total weekly cost of $17,925 (see Table 3 for this schedule). For the very first week, the firm should place 130 planes in Airport A, 630 in Airport B, and 440 in Airport C. We then observed that we can increase the fleet's size to 1390 before we stop seeing cost decreases; it stagnates at $15,125. We cannot provide any advice on what the purchase-price of these planes should be. However, these additional 190 planes should not be rented for any more than $10 per week, $20 for the first 90 aircraft.

2 Problem Overview

Express Air operates an aircraft fleet to run a cargo business among three airports, airports A, B and C. Each day, a certain amount of cargo that needs to be delivered between each origin-destination airport arrives into the system. The cargo operator for Express Air considers the amount of cargo that was not delivered the day before for each origin-destination pair and the amount of cargo that arrives on the current day and decides what portion of the cargo should be delivered. Naturally, the amount of cargo that can be delivered between an origin-destination pair depends on the number of aircraft at the origin. In addition to carrying cargo, the cargo operator has the option of repositioning aircraft empty between the airports. Repositioning aircraft becomes useful when there is imbalance between the amounts of inbound and outbound cargo from a particular location. The important point is the dynamic nature of the cargo operations. In particular, assuming that the travel time between each origin-destination pair is a full day, if the carrier carries a certain amount of cargo from airport A to B during day t, then the aircraft that is used for this purpose during day t becomes available at airport B at the beginning of day t+1.

Our goal is to develop a weekly schedule for Express Air's 1200 cargo planes that meets its demand requirements and that maximizes net income. Since demand comes at a fixed, weekly schedule,the firm can only improve its income by minimizing its weekly operational expenditures. The variable costs that Express Air incurs are from storing loads, as opposed to flying them, ($10 per load per day) and from flying empty cargo planes; the later has a variable cost per plane depending on the plane's route.

2.1 Data Description

Express Air has exactly 1200 aircraft at its disposal. Each aircraft can only have one of these three states per day: stay at the same airport for a whole day, transfer one full cargo load to the destination airport such that it arrives the next day, or reposition to a destination airport while not carrying cargo and arriving the next day.

There is a consistent weekly pattern for the cargo that needs to be delivered. Table 1 shows the amount of cargo arriving into the system on each day of the week that needs to be delivered between each origin-destination airport. All of the quantities in the table are in full aircraft loads. Each aircraft can carry either one aircraft load or no loads. For example, to carry 25 aircraft loads of cargo, you need 25 aircraft.

Express Air also provided us with the the unit costs the firm incurs for repositioning empty airplanes along all possible routes. Note that these costs are independent of direction; repositioning an airplane from A to B costs the same as repositioning it from B to A. They can be seen in Table 2.

Day / Origin-destination	Monday	Tuesday	Wednesday	Thursday	Friday
A-B	100	200	100	400	300
A-C	50	50	50	50	50
B-A	25	25	25	25	25
B-C	25	25	25	25	25
C-A	40	40	40	40	40
C-B	400	200	300	200	400

Table 1: Weekly demand schedule of Express Air.

3 Model

Our team has decided to model the problem as an integer linear program (ILP) keeping track of the firm's aircraft over the span of a week. We assume that the firm can only move its planes (both empty and loaded) during weekdays, so our model only contains 5 days, numbered 0 for Monday, 1 for Tuesday, and so on.

3.1 Constraints

To fit our model to the problem, and other real-world constraints, we decided to make sure our model obeyed the following rules:

1. The produced schedule must repeat every week.
2. No planes can appear/disappear at airports without having flown to/from said airport.
3. The total number of airplanes is 1200 (changed in analysis).
4. Cargo cannot appear/disappear at a site; it must arrive via our demand schedule and be flown away to be removed.
5. The number of loaded airplanes leaving an airport does not exceed the total load/cargo present at said airport at the time.

Airplane Pairs	Cost per Airplane (USD$)
A,B	7
A,C	3
A,B	6

Table 2: Unit costs of repositioning aircraft between different airports.

6. All loads scheduled to go from some airport to another on a certain day must be delivered before a week passes (5 days), a note on this later.

3.2 Key Definitions

In our model we introduce four types of decision variables. Three of them correspond to what we do with certain aircraft at certain airports at a specified day. The last one simply accounts for the load at each airport.

- $Y_{f|d}$: the number of airplanes that stay at airport f on day d
- $X_{f\to t|d}$: the number of airplanes that carry load from airport f to airport t on day d where $f \neq t$
- $Z_{f\to t|d}$: the number of empty airplanes repositioned from airport f to airport t on day d where $f \neq t$
- $L_{f\to t|d}$: the held cargo load that is present at airport f on the start of day d. That is, the load present after cargo has been flown off for the day.

We also use the following notation for defining certain constants given to us:

- Days are represented as numbers, where Monday is 0, Tuesday is 1, Wednesday is 2, Thursday is 3, and Friday is 4.
- Ω is the set of all airports in the system ($\{A, B, C\}$)
- $D_{f\to t|d}$ is the demand that arrives to airport f destined for airport t at the start of day d
- $K_{f,t}$ is the cost of repositioning a single, empty airplane from airport f to airport t.

3.3 Resulting Model

The model described above can be modeled with the following integer linear program:

$$\text{Minimize} \qquad \sum_{d=0}^{4}\sum_{f\in\Omega}\sum_{t\in\Omega|t\neq f}\left(10 * L_{f\to t|d} + K_{f,t}Z_{f\to t|d}\right)$$

$$\text{Such that} \qquad \sum_{t\in\Omega}\sum_{f\in\Omega\setminus\{t\}}\Big[X_{f\to t|4} + Z_{f\to t|4}\Big] + \sum_{t\in\Omega} Y_{t|4} = 1200 \tag{1}$$

$$Y_{t|d} + \sum_{f\in\Omega\setminus\{t\}}\big[X_{f\to t|d} + Z_{f\to t|d}\big] = Y_{t|d+1} + \sum_{f\in\Omega\setminus\{t\}}\big[X_{t\to f|d+1} + Z_{t\to f|d+1}\big] \qquad \forall t \in \Omega,\ d = 0, .., 4 \tag{2}$$

$$L_{f\to t|d} = L_{f\to t|d-1} + D_{f\to t|d} - X_{f\to t|d} \qquad \forall d = 0, .., 4,\ f, t \in \Omega|f \neq t \tag{3}$$

$$X_{f\to t|d} \leq L_{f\to t|d-1} + D_{f\to t|d} \qquad \forall f \in \Omega,\ d = 0, .., 4 \tag{4}$$

$$L_{f|d}, Y_{f|d}, X_{f\to t|d}, Z_{f\to t|d} \ \in \mathbb{N} \qquad \forall d = 0, 1, .., 4,\ f, t \in \Omega|t \neq f \tag{5}$$

4 Results

When optimizing the linear program above, we obtain the schedule indicated by the tables in Table 3, which yields a minimum weekly cost of **$17925**. Although the schedule displays some anomalies at a first-glance, it makes decent sense overall. Furthermore, these anomalies are only present because our generated weekly schedule is not for all weeks, not just the first one.

	Day 0	Day 1	Day 2	Day 3	Day 4
A->B	100	200	100	400	300
A->C	30	70	50	50	50
B->A	25	25	25	25	25
B->C	25	25	25	25	25
C->A	40	40	40	40	40
C->B	520	270	300	200	210

	Day 0	Day 1	Day 2	Day 3	Day 4
A->B	0	0	0	0	0
A->C	0	0	0	0	0
B->A	205	325	145	285	65
B->C	255	245	165	175	485
C->A	0	0	0	0	0
C->B	0	0	0	0	0

	Day 0	Day 1	Day 2	Day 3	Day 4
Airport A	0	0	240	0	0
Airport B	0	0	110	0	0
Airport C	0	0	0	0	0

Table 3: Weekly schedule for airplane movement. Top table schedules movement of airplanes with cargo. The second schedules repositioning of empty airplanes. The bottom table schedules airplanes to stay at airports for specified days. Note that here Day 0 is Monday, Day 1 is Tuesday, and so on.

	Day 0	Day 1	Day 2	Day 3	Day 4
A->B	0	0	0	0	0
A->C	20	0	0	0	0
B->A	0	0	0	0	0
B->C	0	0	0	0	0
C->A	0	0	0	0	0
C->B	70	0	0	0	190

Table 4: Table showing the weekly amount of cargo present at specific airports by the end-of-day. The held-loads are also separated by destination.

The "busiest" routes (in terms of demand) are going from A and C to B, which also have very high demands. From this, it then makes sense that Airport B, which has much less weekly demand from the other two airports, consistently sends a large number of planes back to Airports A and C; in fact, it is the only airport from which repositioning of empty aircraft occurs.

4.1 Strange Anomaly Due to Steady-State Schedule

The only aspect of these schedules that may appear surprising is the schedule for loaded-aircraft flights from Airport C, which, on a few occasions, appears to send off more cargo than what's available there at the time. However, it is important to remember, that this schedule is meant to be repeatable for multiple weeks, not just the first one. Therefore, this effect is simply due to the fact that there is cargo from the previous week that needs to get shipped.

For example, Our plane schedule indicates that on Monday, 520 loads should be moved from Airport C to Airport B, which only has a demand of 400 on that day. However, Table 4 indicates that by end-of-day Friday, 190 loads of cargo for this flight are left at Airport C. This combined with the 400 loads introduced on Monday add up to 590, which is more than the 540 scheduled for this day.

4.2 Initial Aircraft Placement

Because of this anomaly, the initial placement of aircraft to satisfy our schedule is not completely trivial. We must take the following into account: the demand schedule in Table 1, our flight schedule from Table 3, and the resulting cargo-holding schedule in Table 4. The main insight from looking at all three tables is that 120 additional planes scheduled to fly from Airport C to Airport B should initially just be stationed at the destination airport on the very first Monday and remain their for that day.

Thus, to satisfy this schedule, Express Air should, for the very first day, place 130 planes in Airport A, 630 planes in Airport B, and 440 in Airport C.

5 Analysis

5.1 Increasing Fleet Size

In addition to developing this schedule, Express Air also would like to know how increasing its fleet size could benefit the company. To investigate this, we ran our model with different fleet sizes (different Constraint 1) in the range $[1200, 1430]$. The results can be seen in Figure 1.

The results were that, for the first 90 planes Express Air procured in addition to their current fleet of 1200, each additional plane decreased weekly operational costs by \$20. Further increases only result in decreases of \$10 per aircraft until stagnating at \$15,125 with a fleet of 1390 aircraft. Any further increase in the fleet size has no change on operational expenditures.

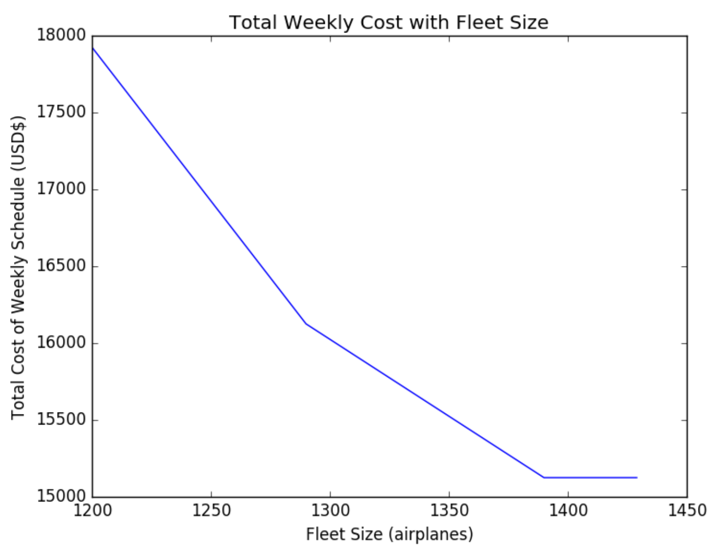

Figure 1: Plot showing the minimum total weekly cost of Express Air's operations as a function of the number of aircraft in the firm's fleet

5.2 Recommendations

The firm should not acquire much more than 190 additional aircraft unless it wants to prepare for increasing demands or plane failures in the future. If the firm decides to rent airplanes, it should rent the first 90 at no more than \$20 per week and the rest for no more than \$10 per week.

6 Concluding Remarks

In this investigation, we were tasked with developing an optimal routing schedule for Express Air to minimize its weekly operational expenditures. We created a schedule for the firm's current demand schedule. We also discussed how adding additional aircraft lowers this minimal expense until reaching a minimum of 190 additional planes. We strongly recommend that the firm try to obtain an additional 190 number of aircraft, or at least 90.

Future work should be done into investigating the life-spans of cargo aircraft available in the market. We currently cannot recommend a set purchase price for aircraft since we do not know how long they would be in service. The only thing we can say is that the firm should avoid spending more than \$10w, where w is the number of weeks a plane would be in use, (\$20w if the fleet size is at most 1290 planes). If Express Air decides to pursue this option, it may also want to consider buying multiple models of aircraft, with different capacities and life-spans.

7 Acknowledgements

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