A short implementation of the exponential function in the C-language

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1 About the exponential function

The exponential function takes an argument x and calculates the Euler's number to the power of the argument. Euler's number is $e = 2.71828$. The mathematical expression for the exponential function can be seen in equation 1.

$$y = e^x \tag{1}$$

2 Implementing the exponential function in the C-language

To make it easier for the computer to calculate the value of the exponential function, it is sometimes more useful to make use the Taylor expansion of the exponential function (equation 2):

$$y = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + ... = \Sigma_0^\infty x^n \tag{2}$$

To make it even easier for the computer to calculate, one can expand equation 2 until you get equation 3:

$$y = 1+x\cdot(1+x/2\cdot(1+x/3\cdot(1+x/4\cdot(1+x/5\cdot(1+x/6\cdot(1+x/7\cdot(1+x/8\cdot(1+x/9\cdot(1+x/10))))))))) \tag{3}$$

It should be noted that equation 3 is only an approximation to equation 1. To reduce errors and to have a precision of $\epsilon = 10^{-15}$, to things are done.

If $x < 0$, the expression to calculate becomes $e^x = \frac{1}{e^{-x}} = \frac{1}{y(-x)}$, where y is the expression from equation 3. This is done to avoid a negative argument in equation 3.

If $x > 1./8$, the expression to calculate becomes $e^x = (e^{x/2})^2 = (y(x/2))^2$, where y is the expression from equation 3.. This is done to make it possible to have a precision of $\epsilon = 10^{-15}$. If x becomes larger than 1./8, the precision becomes less than 10^{-15}. To take this into account, we divide by 2 until x becomes equal or smaller than 1./8, take the exponential of the that, and then take the result to the power of 2 * times we have divided by 2.

3 Test of the implemented exponential function in the C-language

To test if equation 3 is a good approximation to equation 1, both equations are plotted for $0 < x < 8$. The result can be seen in figure 1.

As it can be seen from figure 1, equation 3 is a good approximation to equation 1.

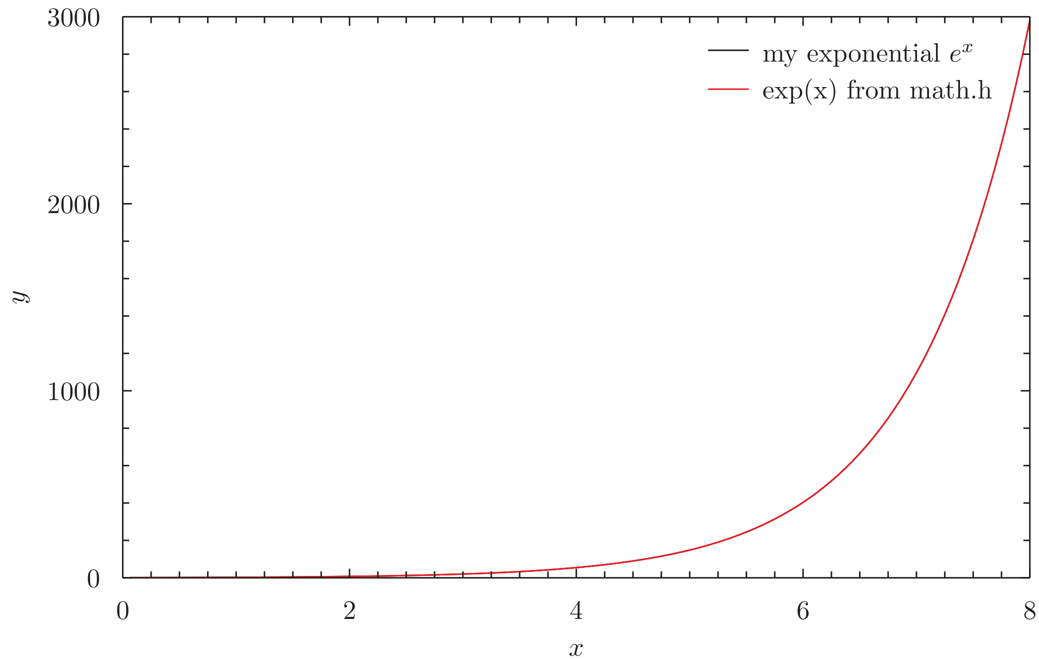

Figure 1: Plots of equation 3 (my exponential e^x) and equation 1 (exp(x) from math.h).