Simulation of plasticity

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Simulation of plasticity

- Definition

Plasticity = Ability of a material to undergo **permanent deformation**

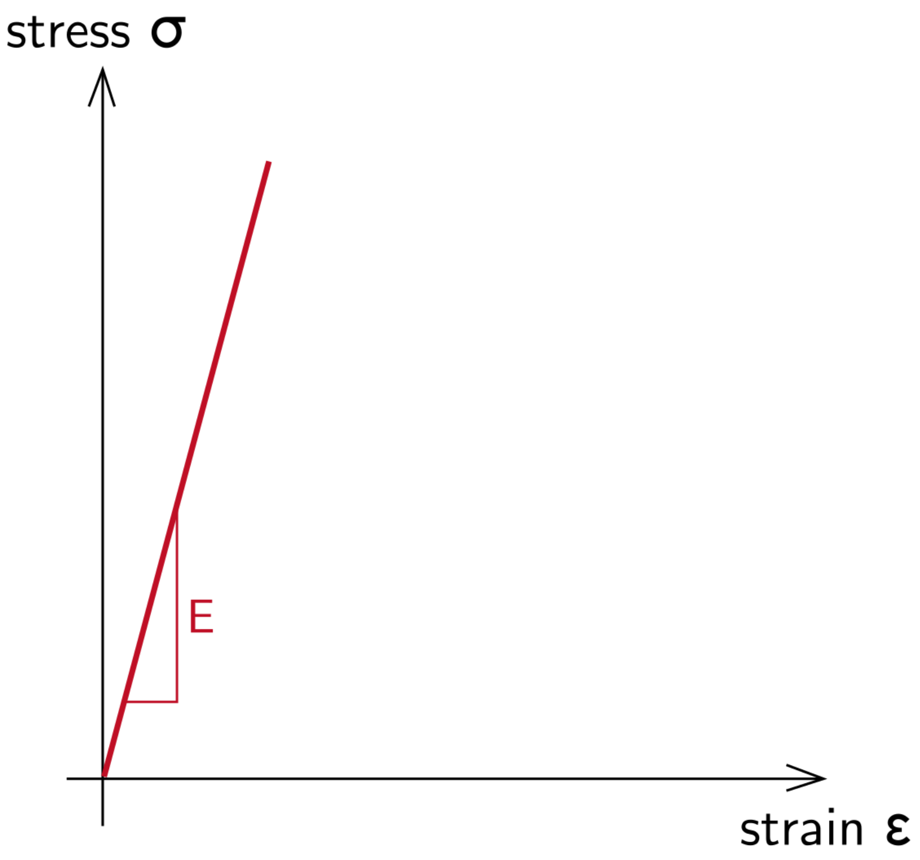
- *"Slippage" in the material structure at a microscale level*

Elasticity = Ability of a material to **recover its rest shape** after undergoing deformation

Simulation of plasticity

- Representation

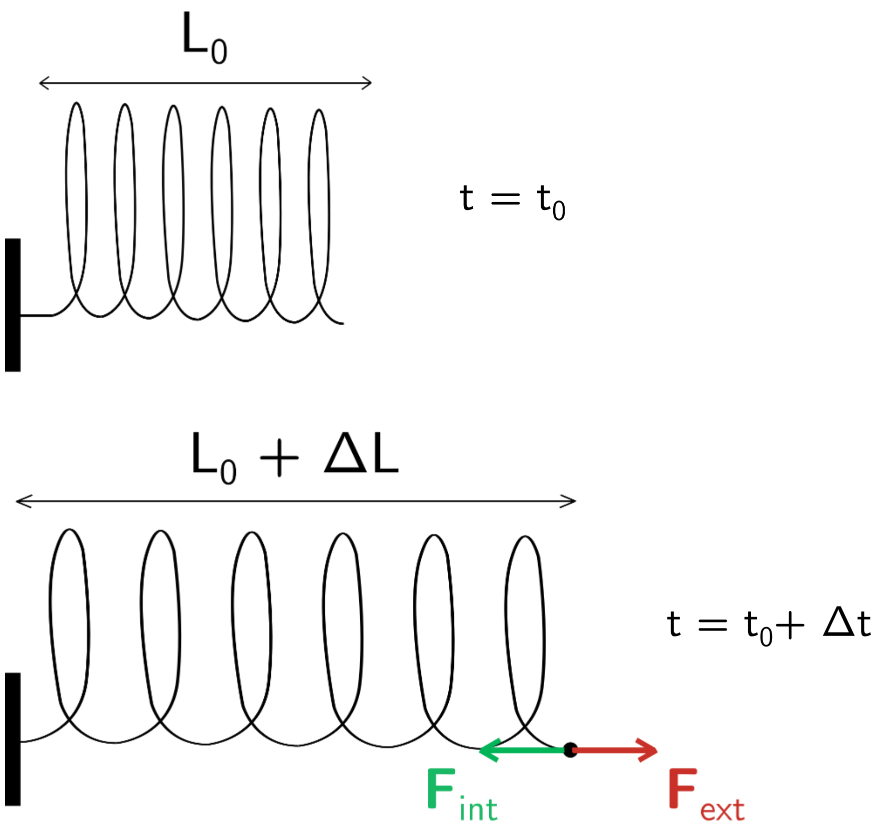
Stress-strain curve - *elasticity*

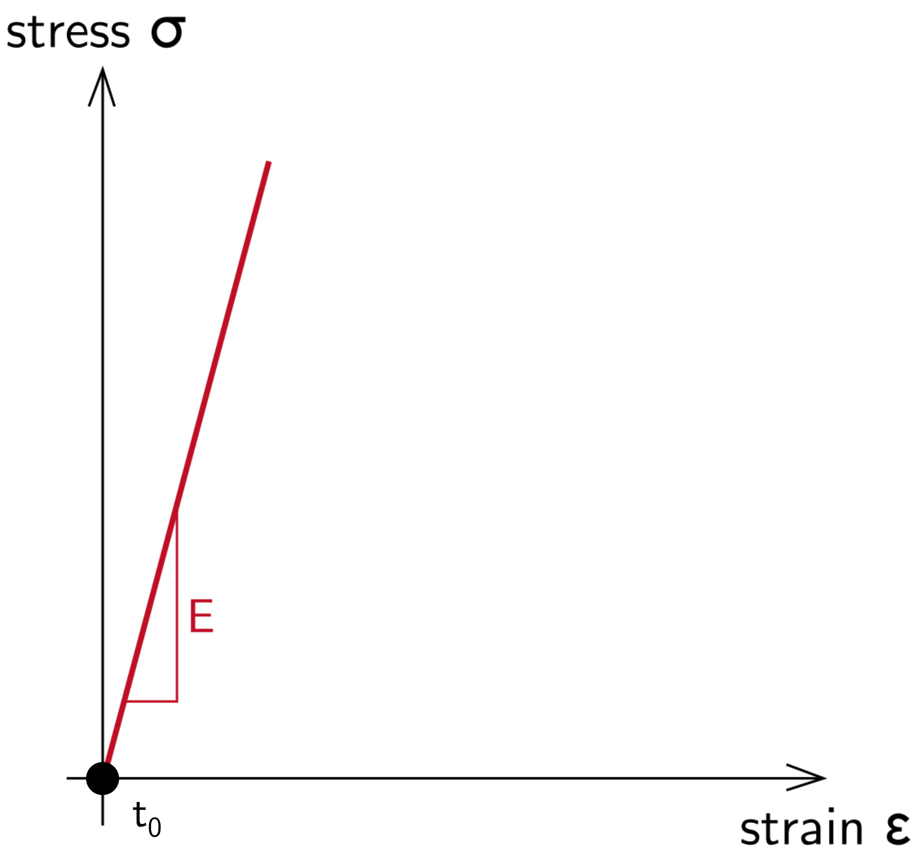


Simulation of plasticity

- Representation

Stress-strain curve - *elasticity*





Simulation of plasticity

- Representation

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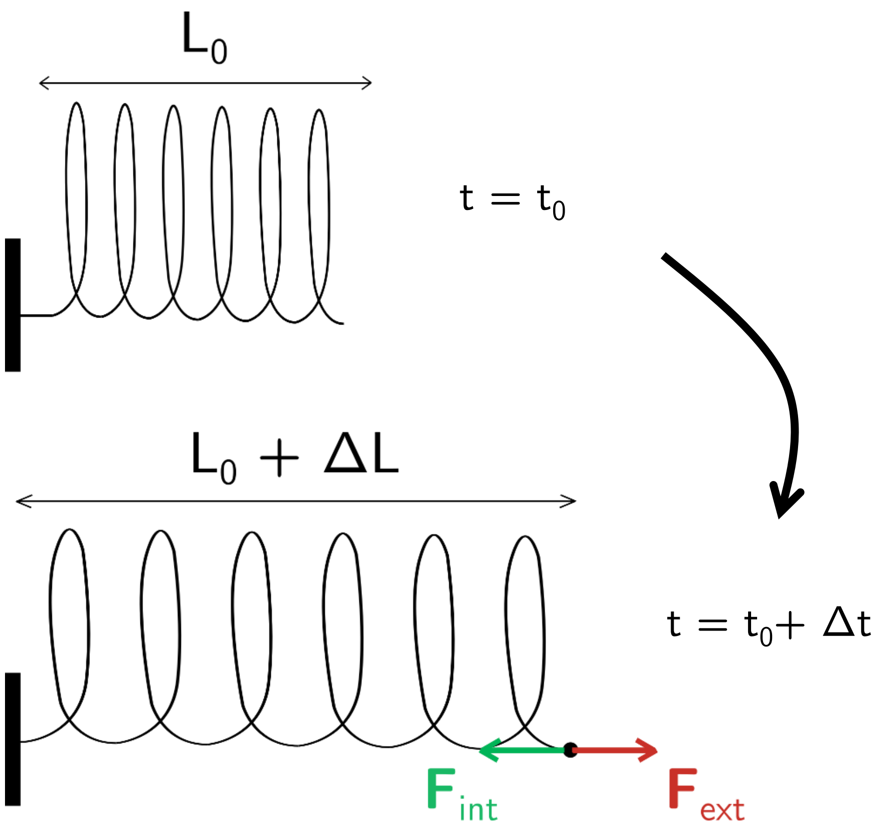
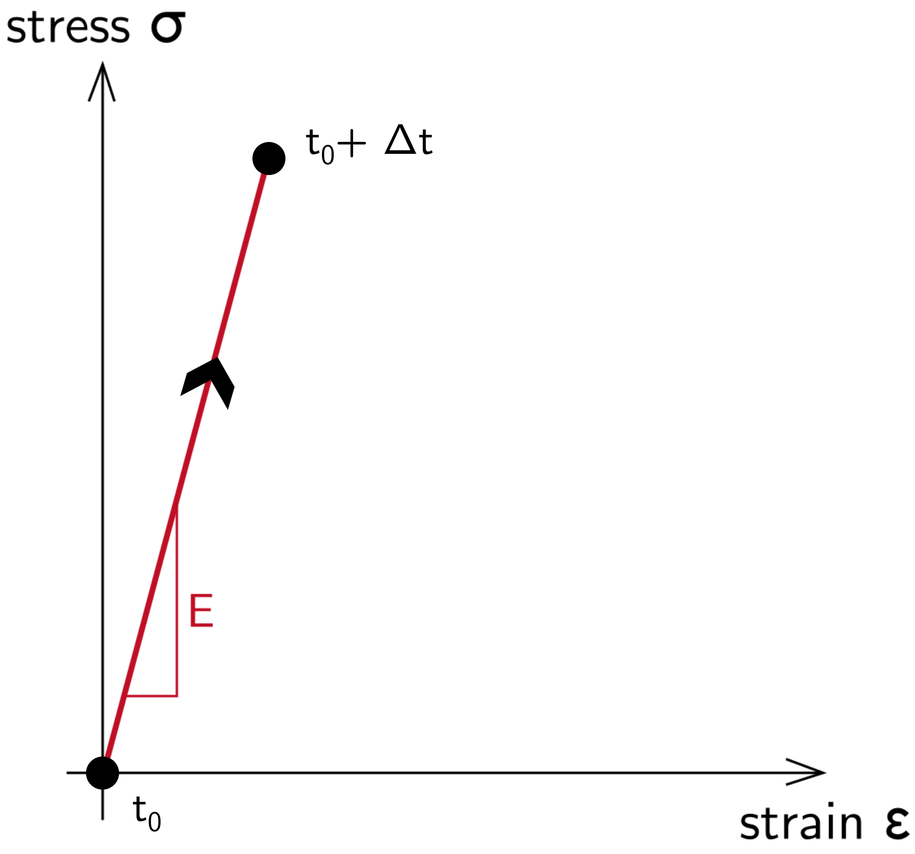
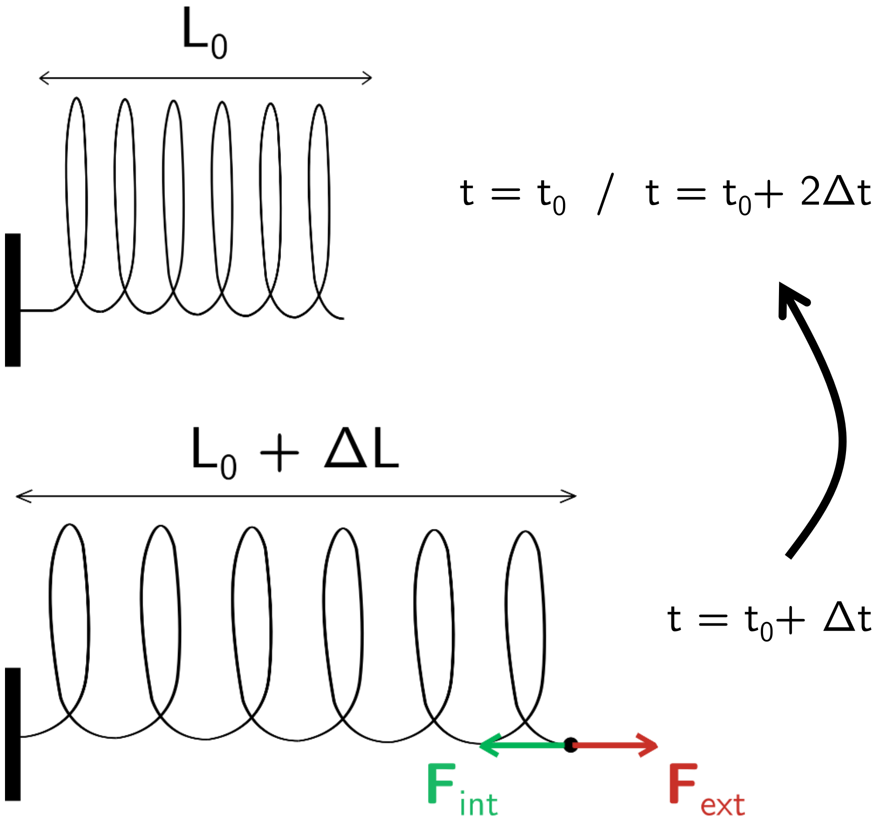


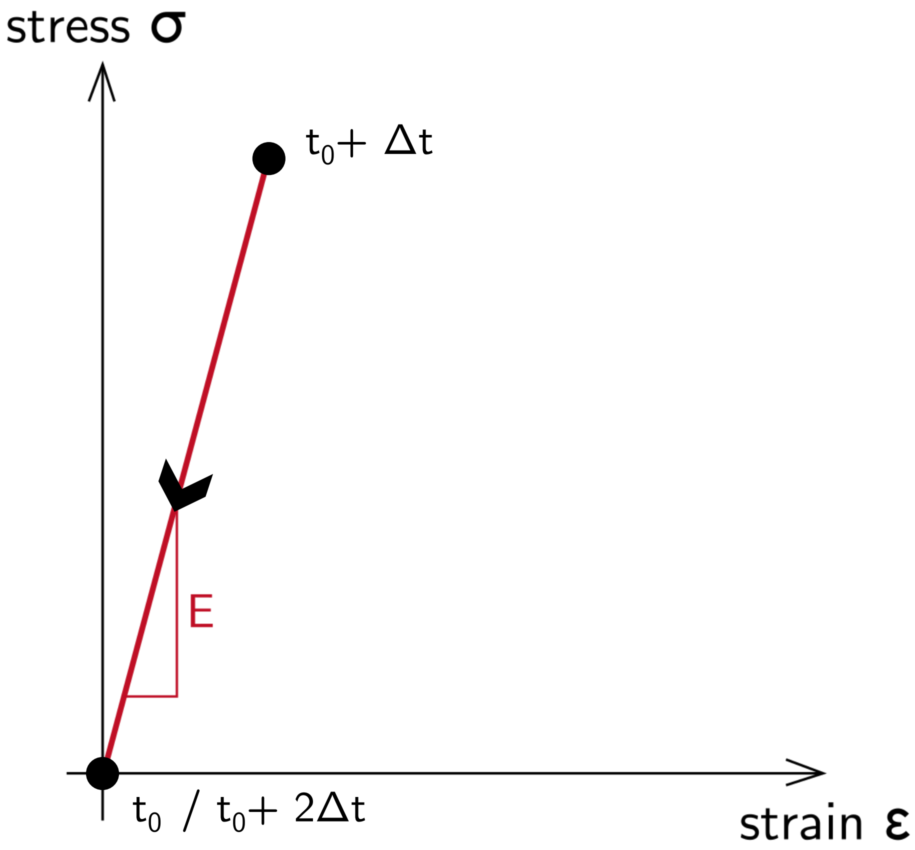


Simulation of plasticity

- Representation

Stress-strain curve - *elasticity*

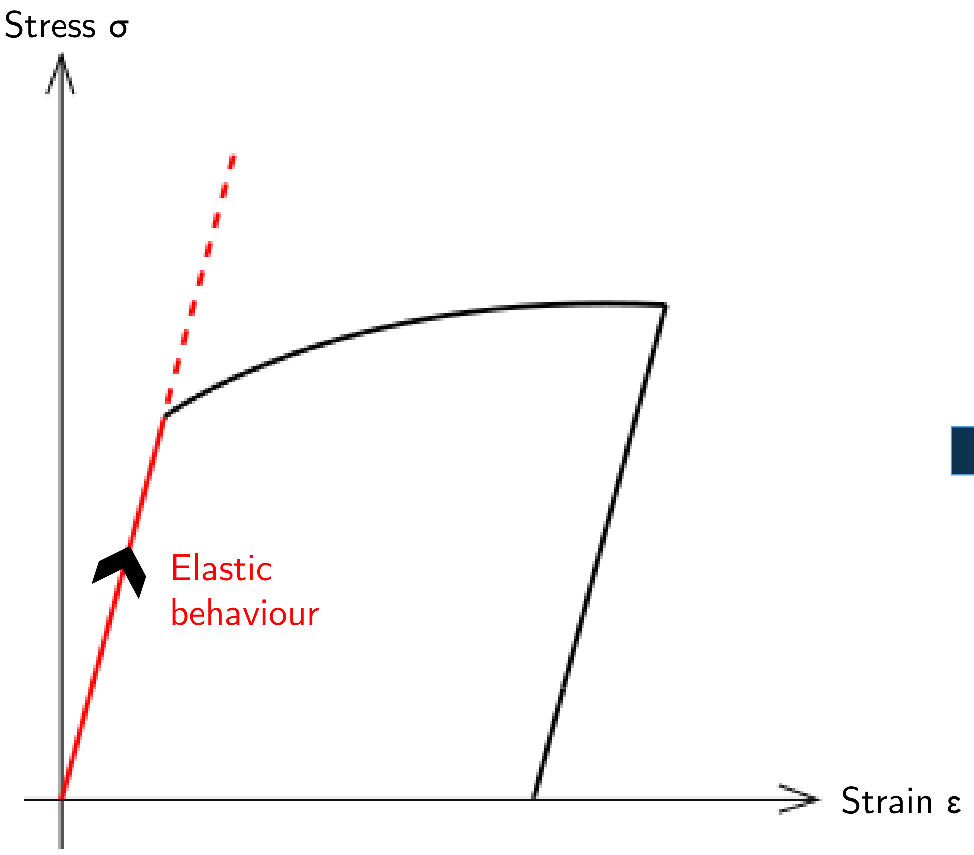


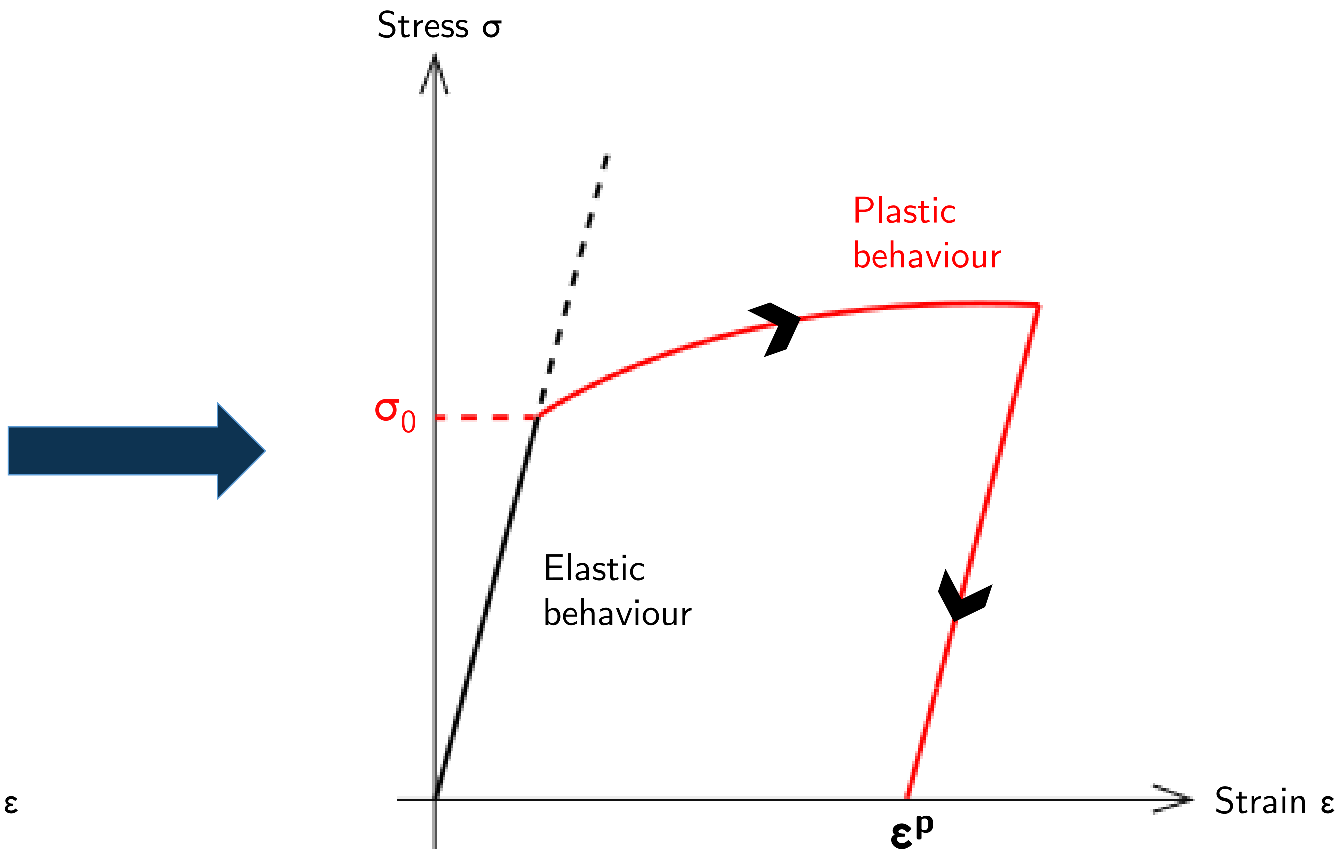


Simulation of plasticity

- Representation

Stress-strain curve - *plasticity*





Simulation of plasticity

- Computational modelling

Displacement field

$$u$$

Deformation gradient

$$F = \nabla u$$

Strain tensor ($\underline{\underline{\varepsilon}}$)

$$\underline{\underline{\varepsilon}} = \frac{1}{2}(F + F^T) - I$$

(Small strain theory)

Stress tensor ($\underline{\underline{\sigma}}$)

$$\underline{\underline{\sigma}} = \varphi(\underline{\underline{\varepsilon}})$$

Internal forces

Virtual work principle
(spatial integration)

Simulation of plasticity

- Computational modelling

Elasticity

Displacement field u

↓

Deformation gradient $F = \nabla u$

↓

Strain tensor ($\underline{\underline{\varepsilon}}$) $\underline{\underline{\varepsilon}} = \frac{1}{2}(F + F^T) - I$

(Small strain theory)

↓

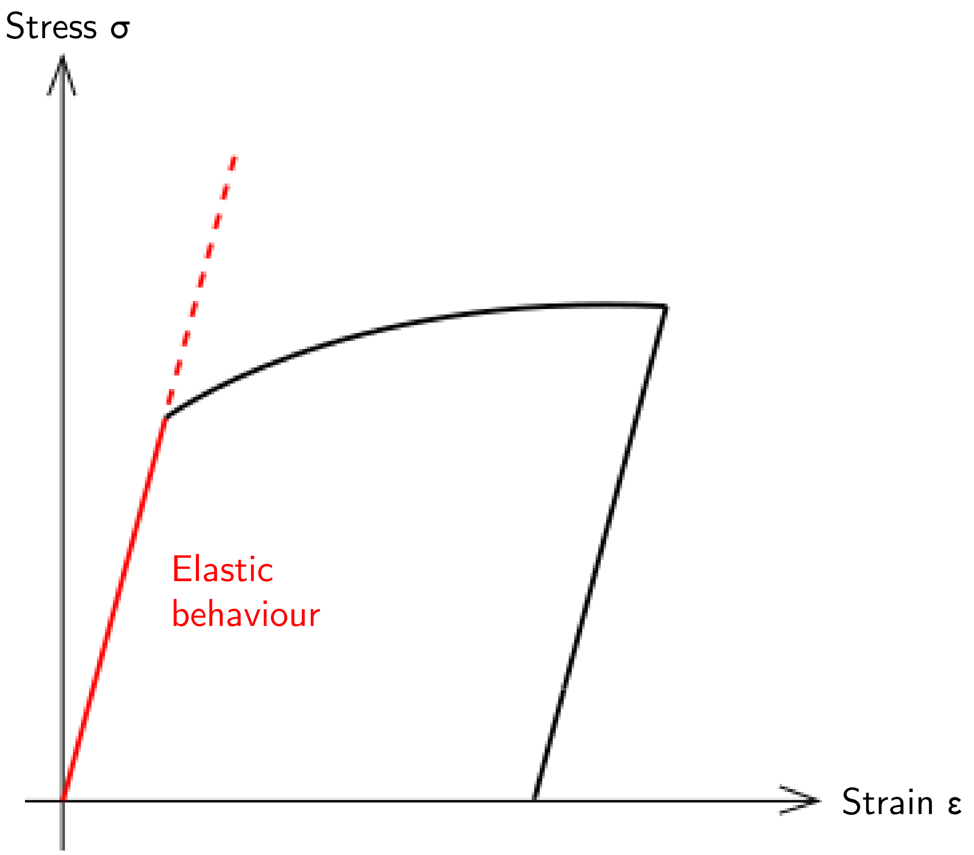
Stress tensor ($\underline{\underline{\sigma}}$) $\underline{\underline{\sigma}} = C * \underline{\underline{\varepsilon}}$

(Generalised Hooke's law)

↓

Internal forces — Virtual work principle

(spatial integration)



Simulation of plasticity

- Computational modelling

Displacement field — u

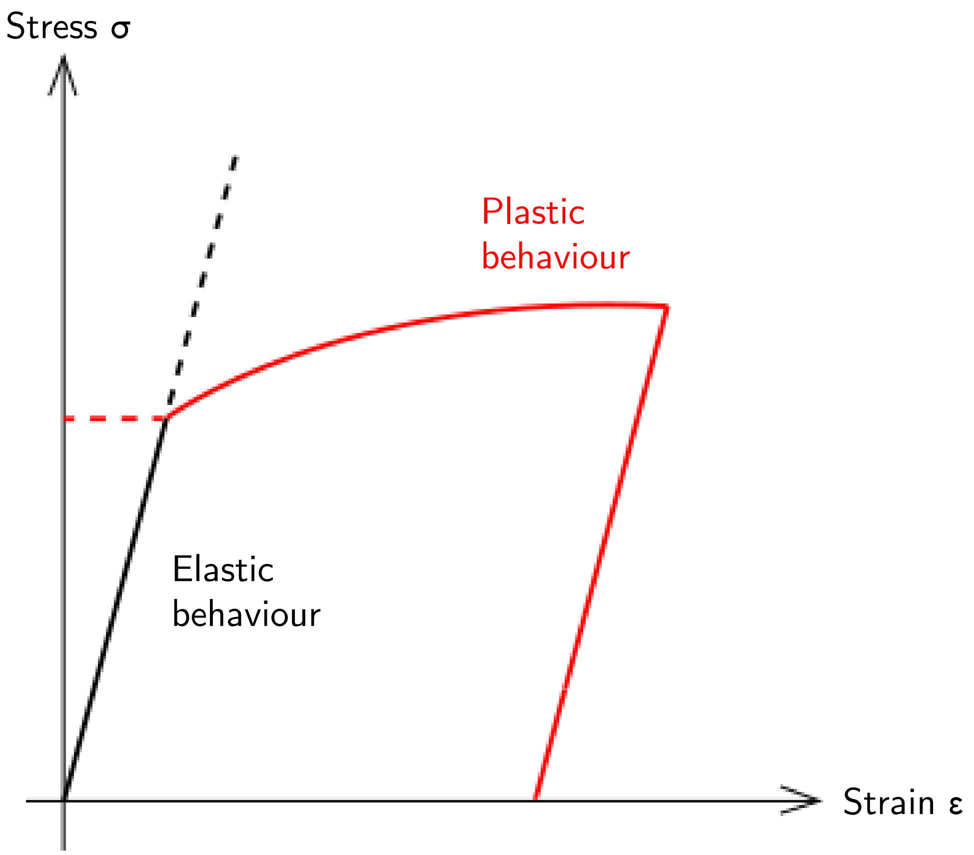
Deformation gradient — $F = \nabla u$

Strain tensor ($\underline{\underline{\varepsilon}}$) — $\underline{\underline{\varepsilon}} = \frac{1}{2}(F + F^T) - I$
(Small strain theory)

Stress tensor ($\underline{\underline{\sigma}}$) — $\underline{\underline{\sigma}} = \varphi(\underline{\underline{\varepsilon}})$
(Nonlinear behaviour law)

Internal forces — Virtual work principle
(spatial integration)

Plasticity



Simulation of plasticity

- Computational modelling

Yield criterion

Threshold above which plastic deformation occurs

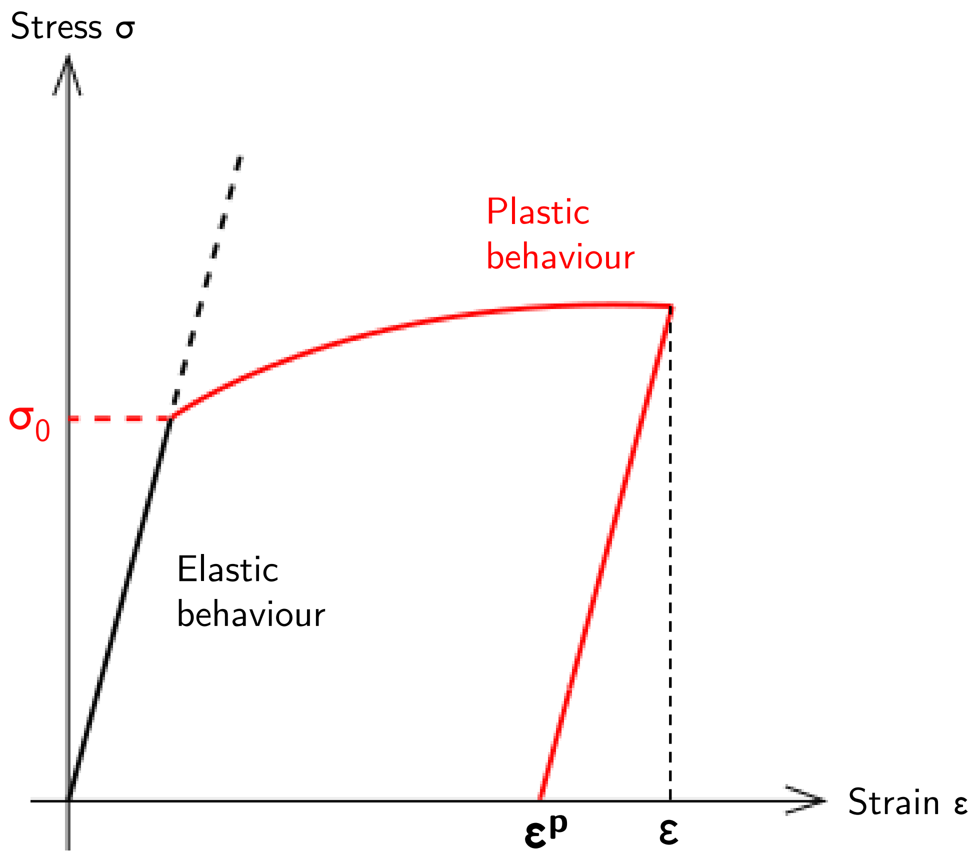
- *Von Mises yield function (metals)*

Flow rule

Evolution of plastic strain (energy dissipation)

- *Associative flow rule*

Plasticity



Simulation of plasticity

- Computational modelling

Yield criterion : **Von Mises yield function**

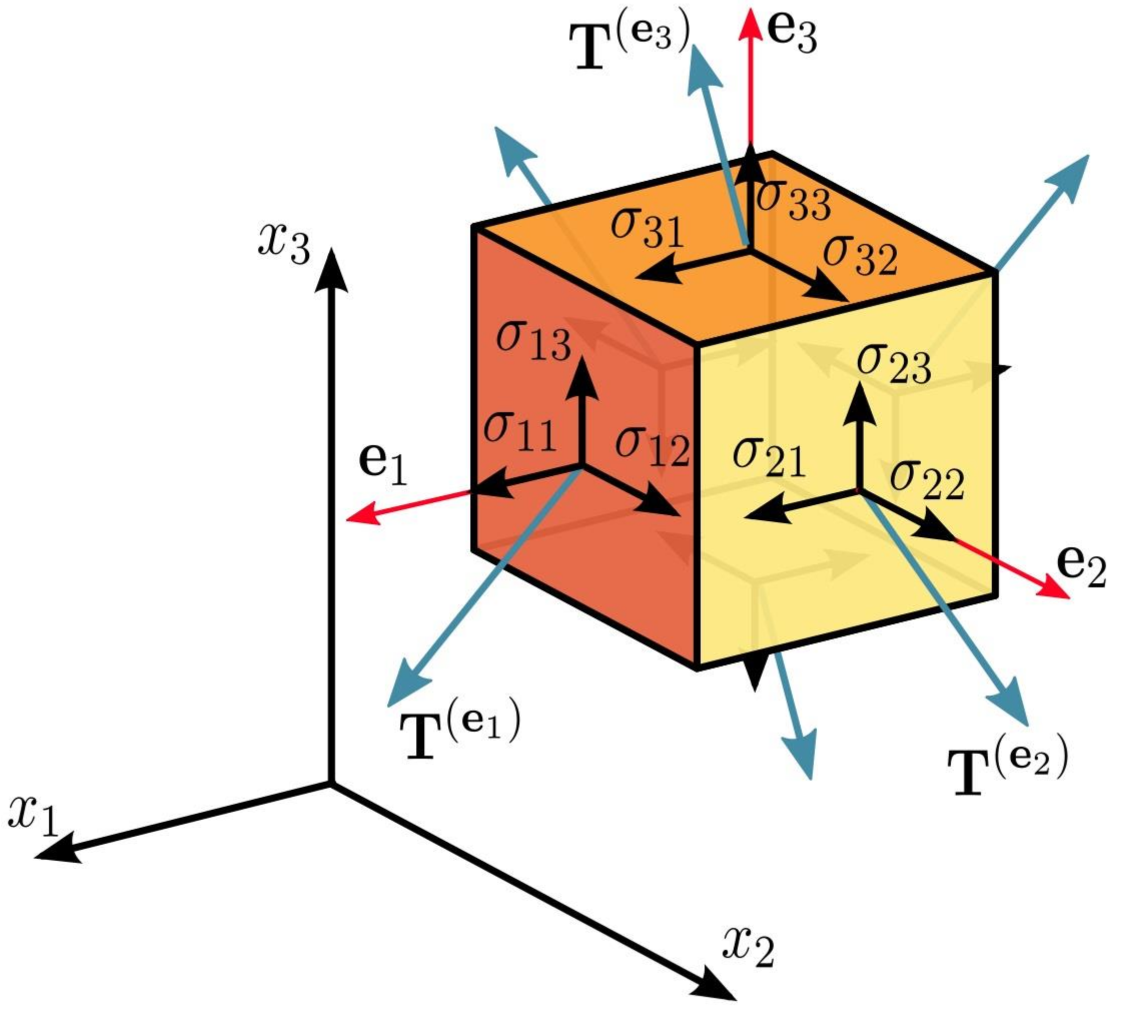
$$f : \begin{cases} \mathbb{R}^6 \longrightarrow \mathbb{R} \\ \underline{\underline{\sigma}} \longmapsto f(\underline{\underline{\sigma}}) \end{cases}$$

Simulation of plasticity

- Computational modelling

Stress tensor $\underline{\underline{\sigma}}$

$$\underline{\underline{\sigma}} = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix}$$



Simulation of plasticity

- Computational modelling

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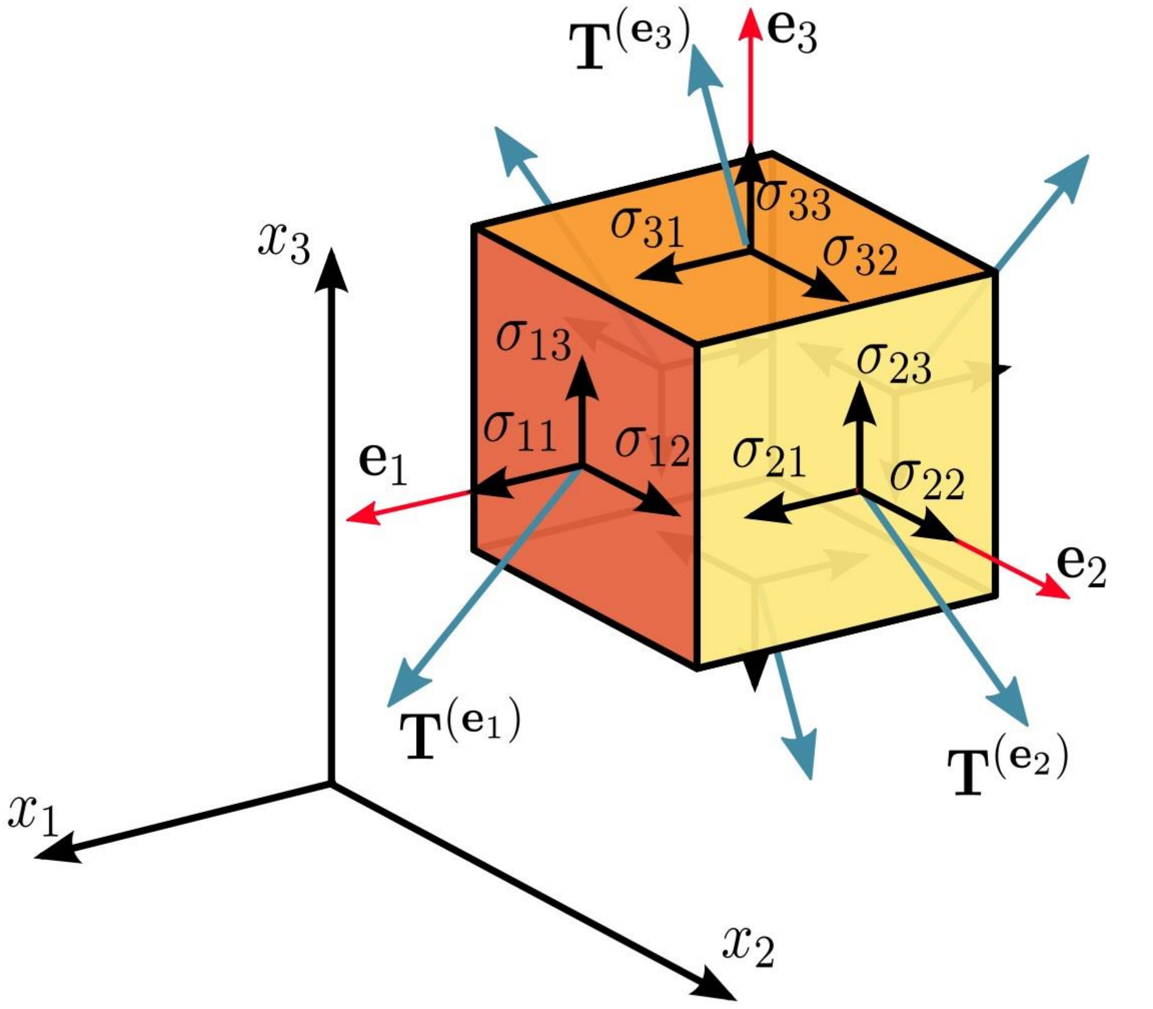
$$\underline{\underline{\sigma}} = \begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{xz} \\ \sigma_{xy} \end{pmatrix}$$ *Voigt notation*



Simulation of plasticity

- Computational modelling

Yield criterion : **Von Mises yield function**

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- Computational modelling

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Simulation of plasticity

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Equivalent stress $\sigma^{eq}()$

Simulation of plasticity

- Computational modelling

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Yield stress

Simulation of plasticity

- Computational modelling

  Yield criterion : **Von Mises yield function** – example in 1D

$$\begin{cases} \underline{\underline{\sigma}} = \sigma \in \mathbb{R} \\ f(\underline{\underline{\sigma}}) = \sigma - \sigma_0 \end{cases}$$

Simulation of plasticity

- Computational modelling

Yield criterion : **Von Mises yield function** – example in 1D

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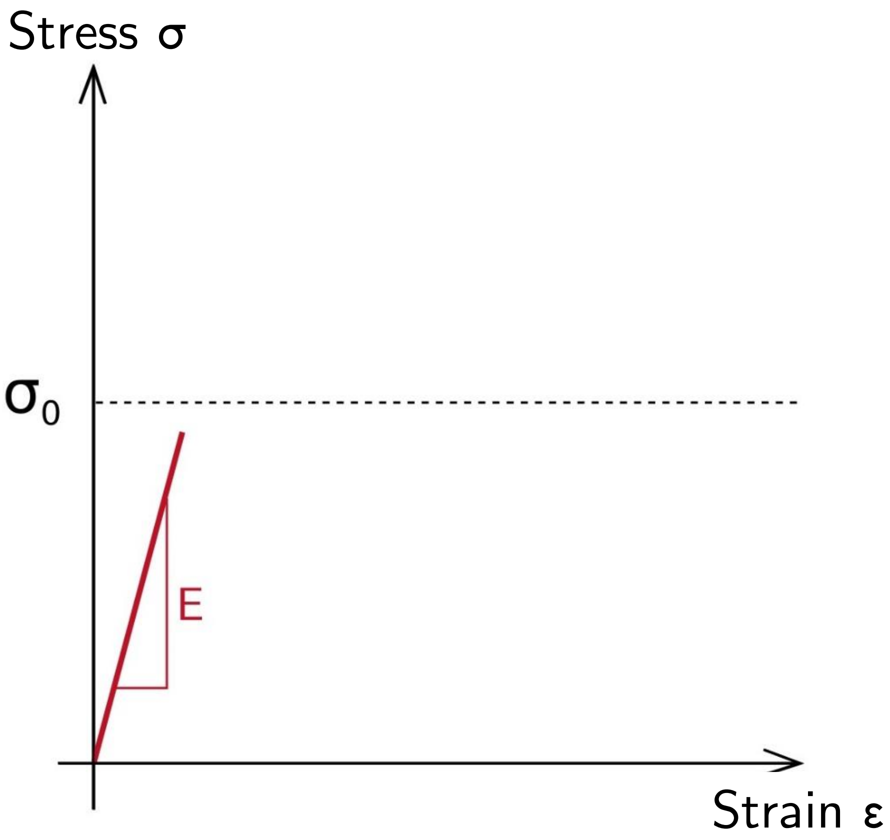

$f(\sigma) < 0$: elastic behaviour

Simulation of plasticity

- Computational modelling

Yield criterion : **Von Mises yield function** – example in 1D

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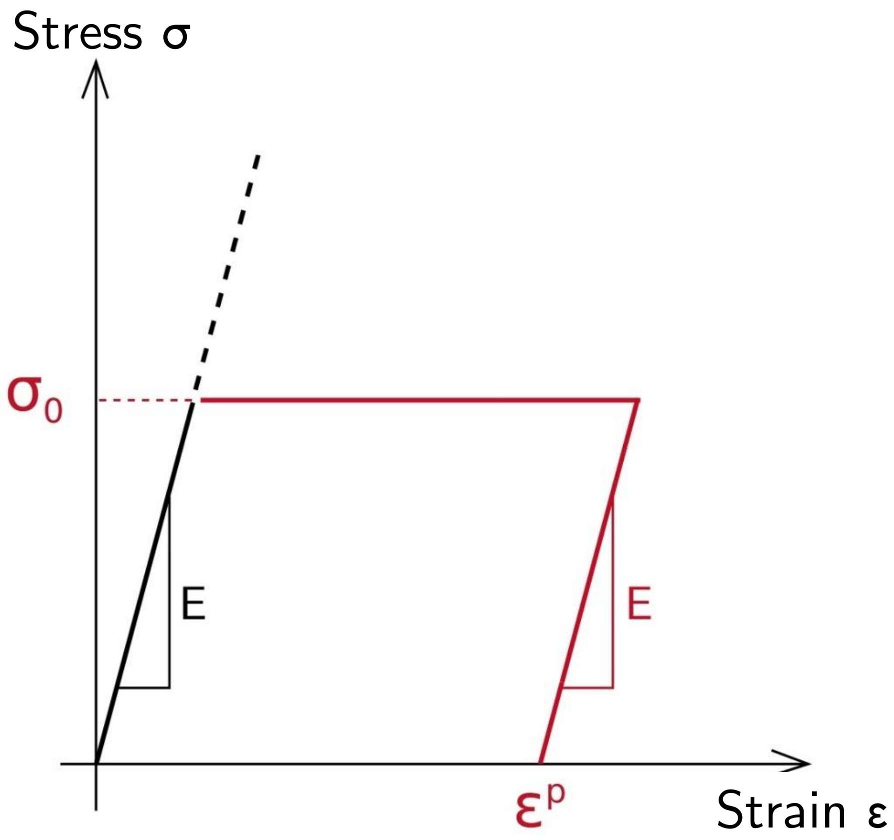

$f(\sigma) < 0$: elastic behaviour

$f(\sigma) = 0$: plastic behaviour

Simulation of plasticity

- Computational modelling

Yield criterion : **Von Mises yield function** – example in 1D

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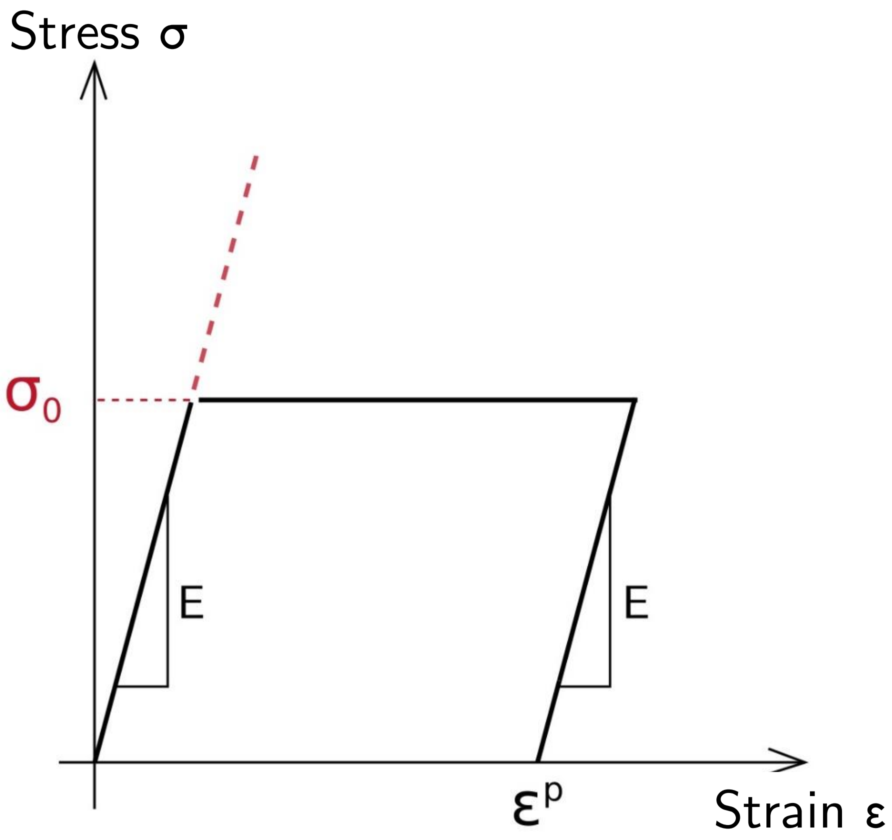


$f(\sigma) < 0$: elastic behaviour

$f(\sigma) = 0$: plastic behaviour

$f(\sigma) > 0$: unrealistic behaviour

Simulation of plasticity

- Computational modelling

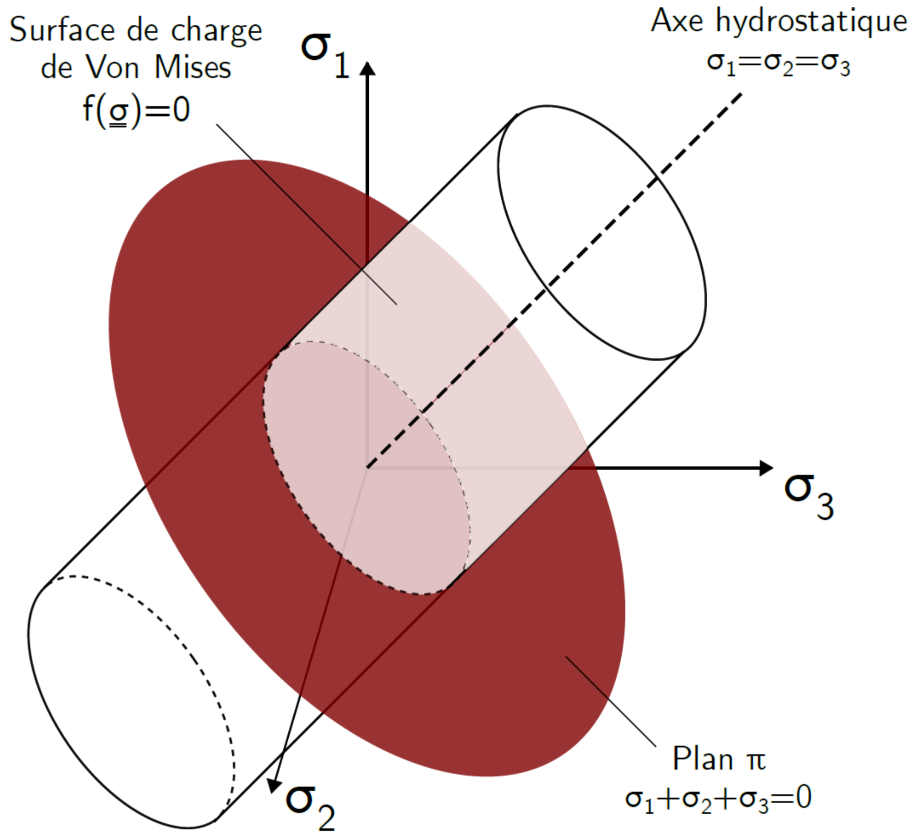
Yield surface : $f(\underline{\underline{\sigma}})=0$

Simulation of plasticity

- Computational modelling

Yield surface : $f(\underline{\underline{\sigma}})=0$

3D – principal stress space

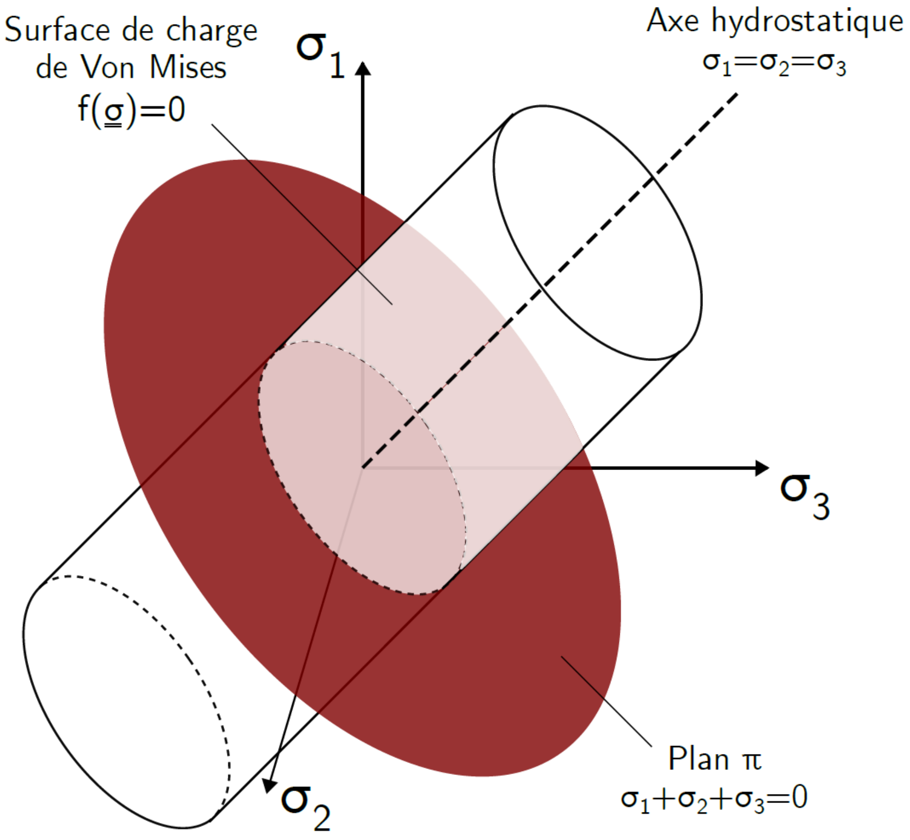


Simulation of plasticity

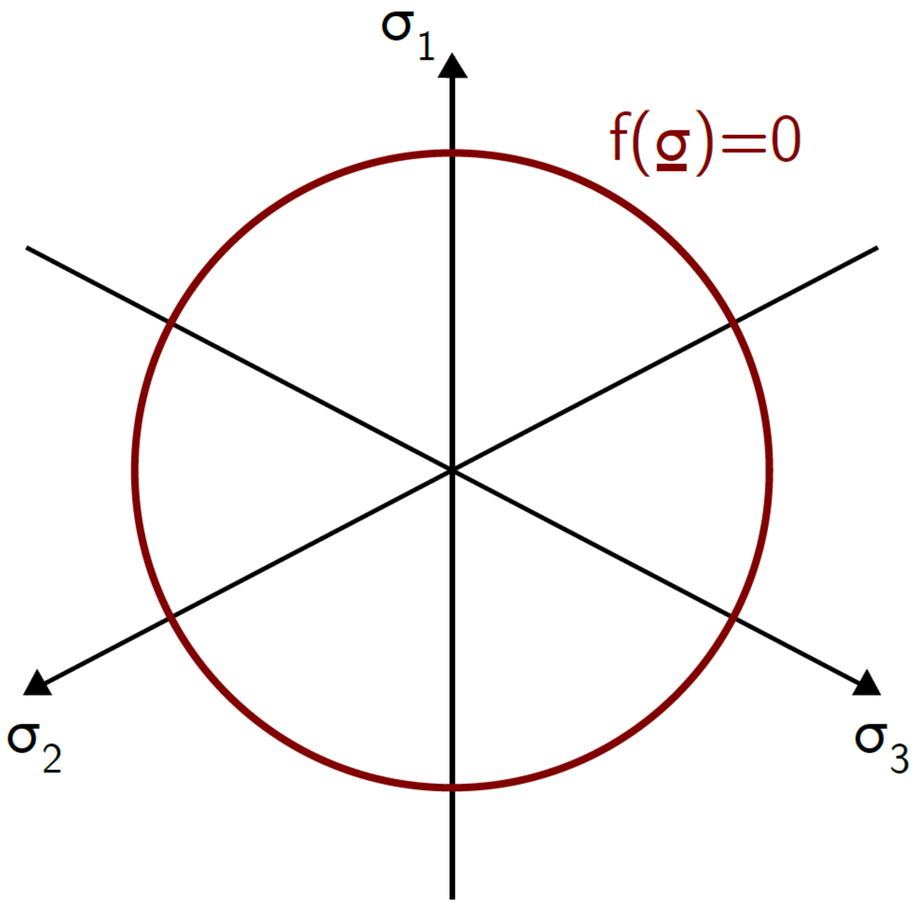
- Computational modelling

Yield surface : $f(\underline{\sigma})=0$

3D – principal stress space



2D – π plane



Simulation of plasticity

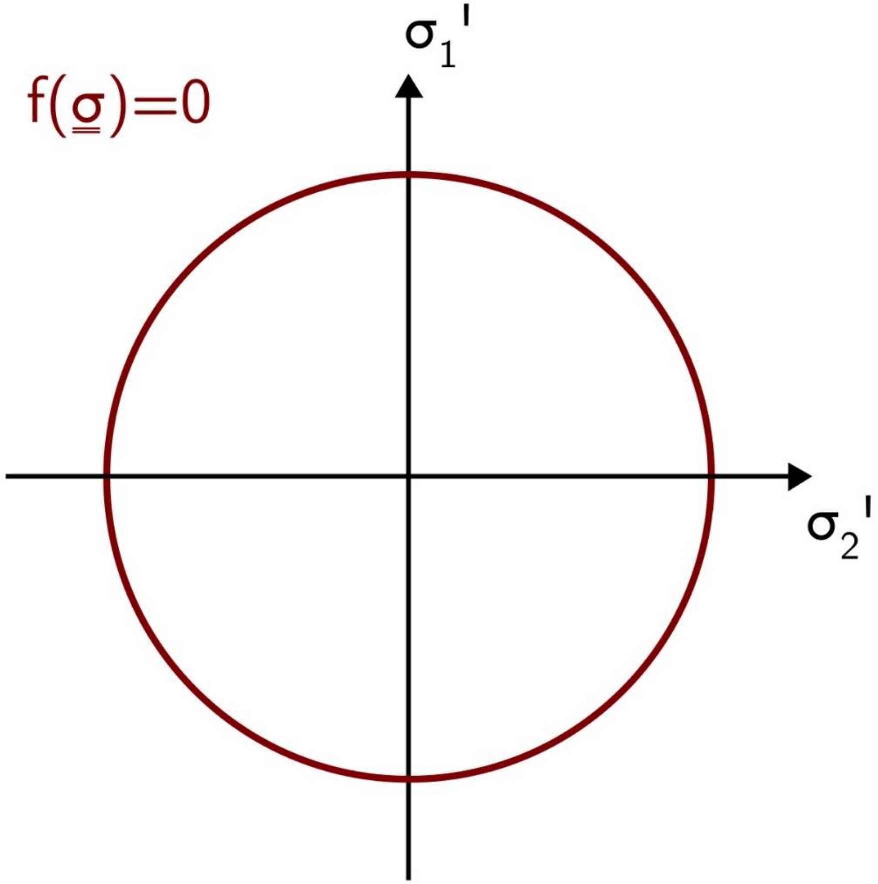
- Computational modelling

  Radial return algorithm

Simulation of plasticity

- Computational modelling

  Radial return algorithm



Simulation of plasticity

- Computational modelling

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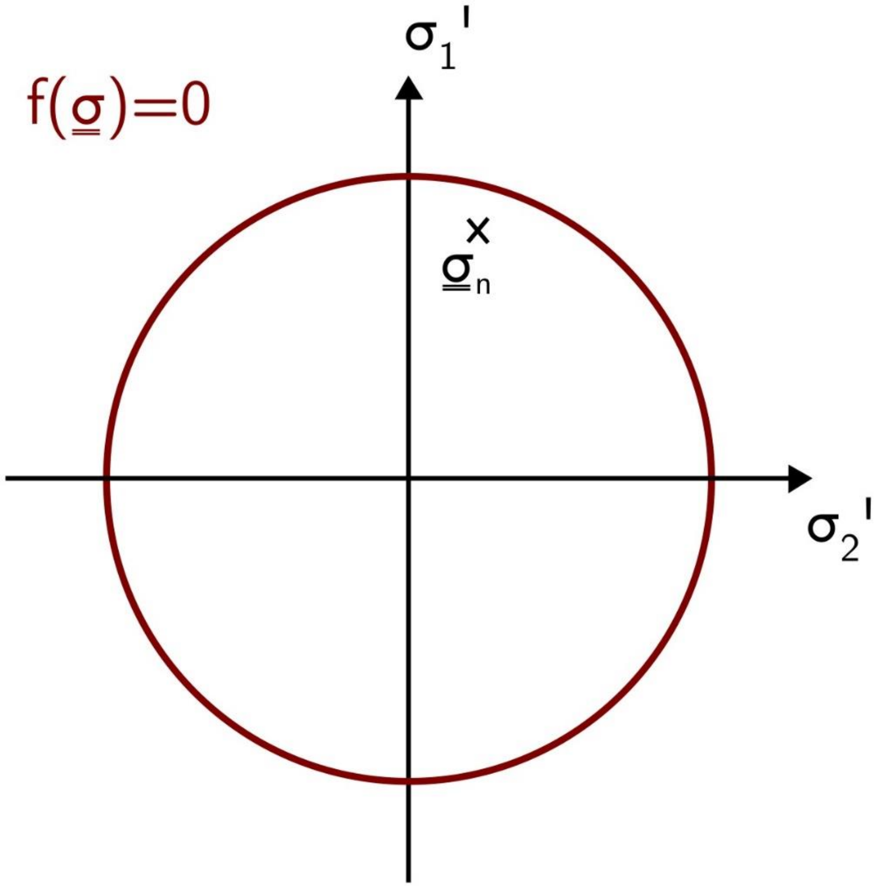
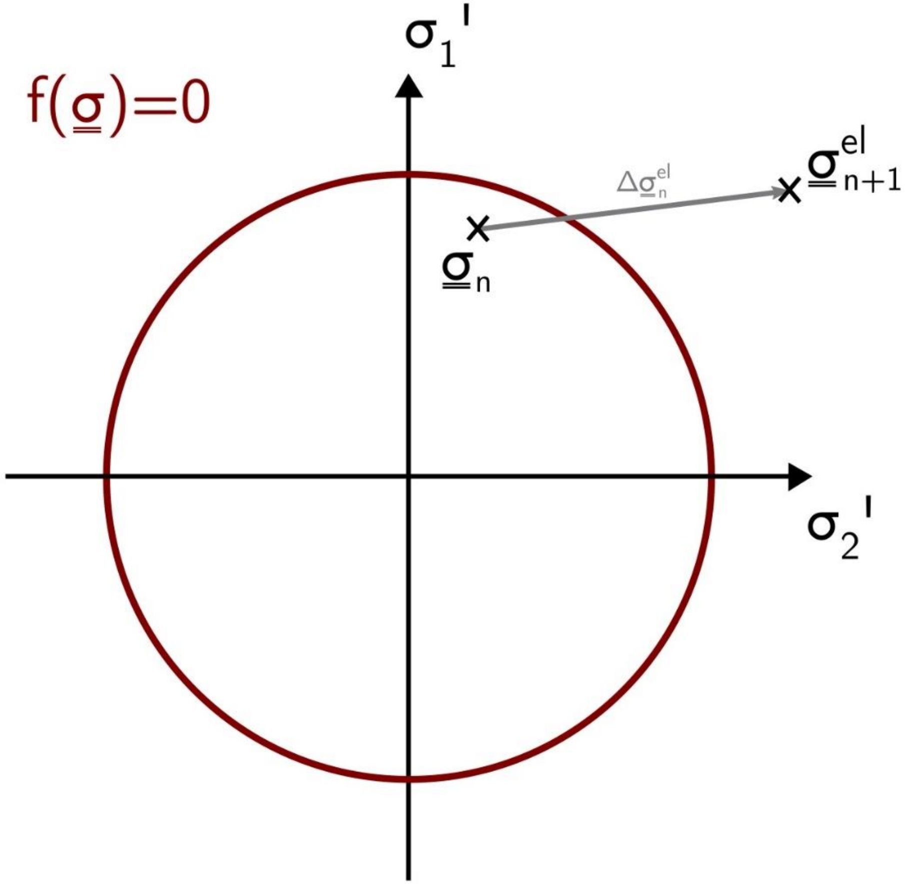


$\underline{\underline{\sigma}}_n$: stress state at step n $(\mathrm{f}(\underline{\underline{\sigma}}_n) < 0)$

Simulation of plasticity

- Computational modelling

Radial return algorithm



$\underline{\underline{\sigma}}_n$: stress state at step n ($\mathrm{f}(\underline{\underline{\sigma}}_n) < 0$)

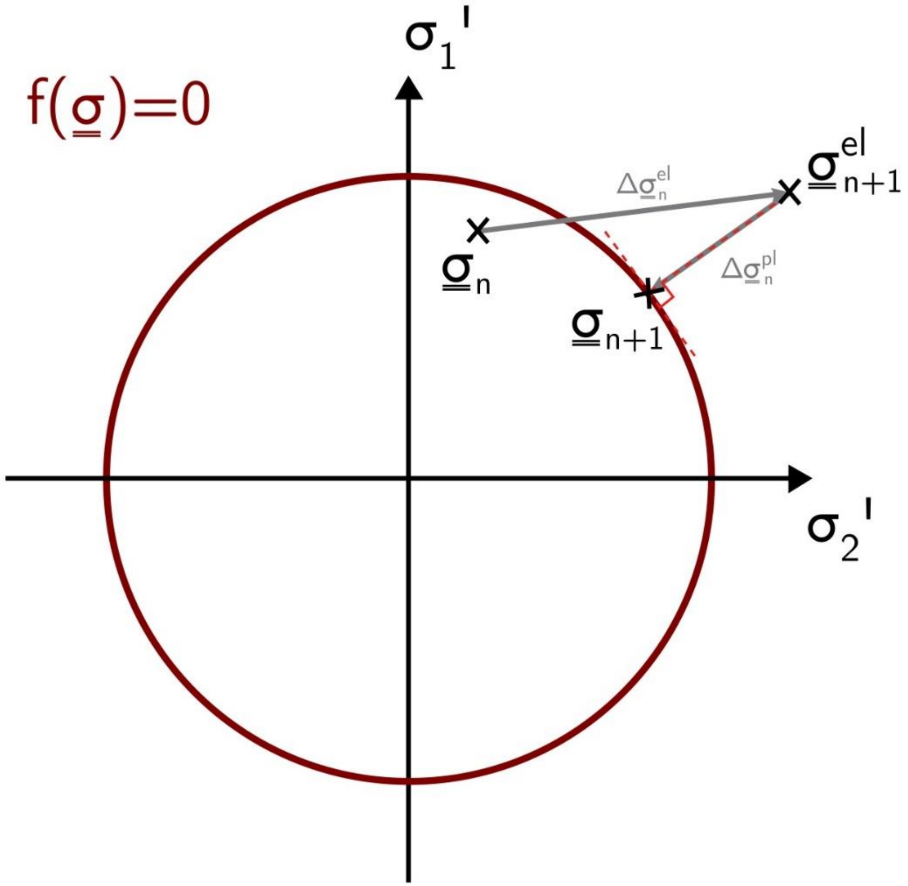
$\Delta\underline{\underline{\sigma}}_n^{el}$: elastic increment

$\underline{\underline{\sigma}}_{n+1}^{el}$: elastic predictor ($\mathrm{f}\left(\underline{\underline{\sigma}}_{n+1}^{el}\right) > 0$)

Simulation of plasticity

- Computational modelling

Radial return algorithm



$\underline{\underline{\sigma}}_n$: stress state at step *n* $(\mathrm{f}(\underline{\underline{\sigma}}_n) < 0)$

$\Delta\underline{\underline{\sigma}}_n^{el}$: elastic increment

$\underline{\underline{\sigma}}_{n+1}^{el}$: elastic predictor $\left(\mathrm{f}\left(\underline{\underline{\sigma}}_{n+1}^{el}\right) > 0\right)$

$\Delta\underline{\underline{\sigma}}_n^{pl}$: plastic correction

$\underline{\underline{\sigma}}_{n+1}$: stress state at step *n+1* $(\mathrm{f}(\underline{\underline{\sigma}}_{n+1}) = 0)$

Simulation of plasticity

- Computational modelling

  Hardening

Simulation of plasticity

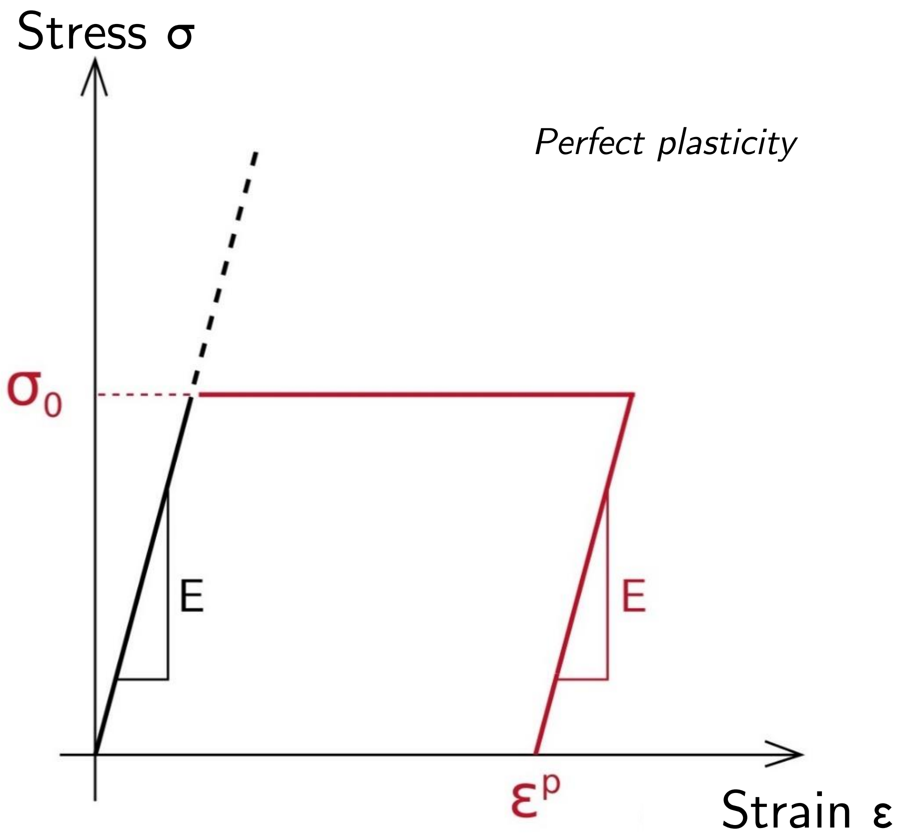
- Computational modelling

  Hardening – illustration with 1D example

Simulation of plasticity

- Computational modelling

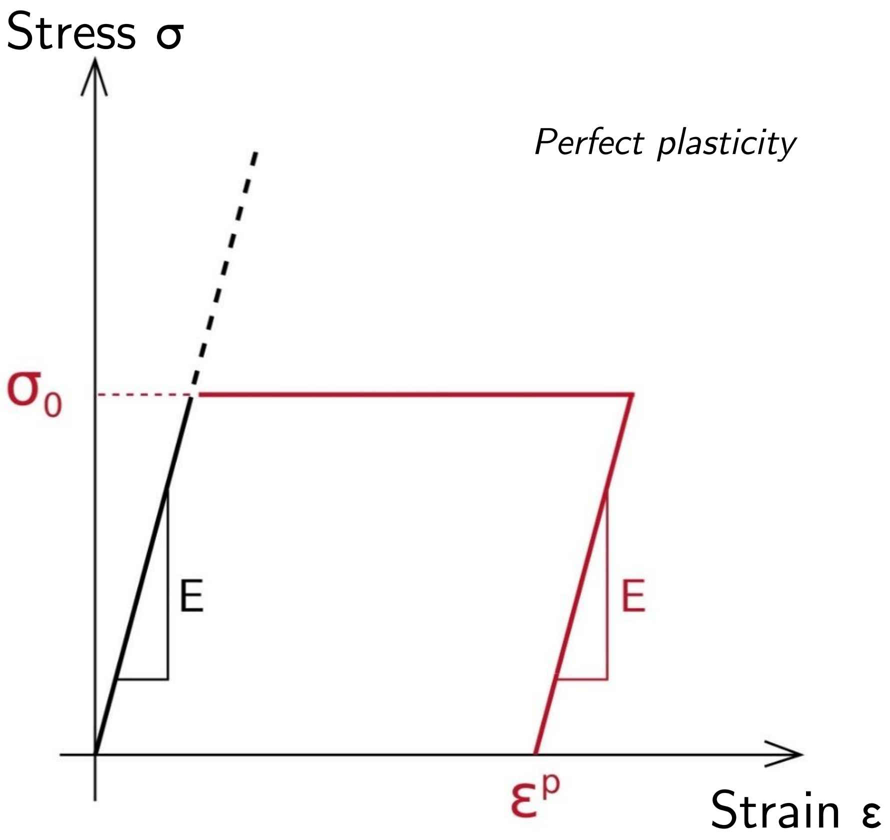
  Hardening – illustration with 1D example

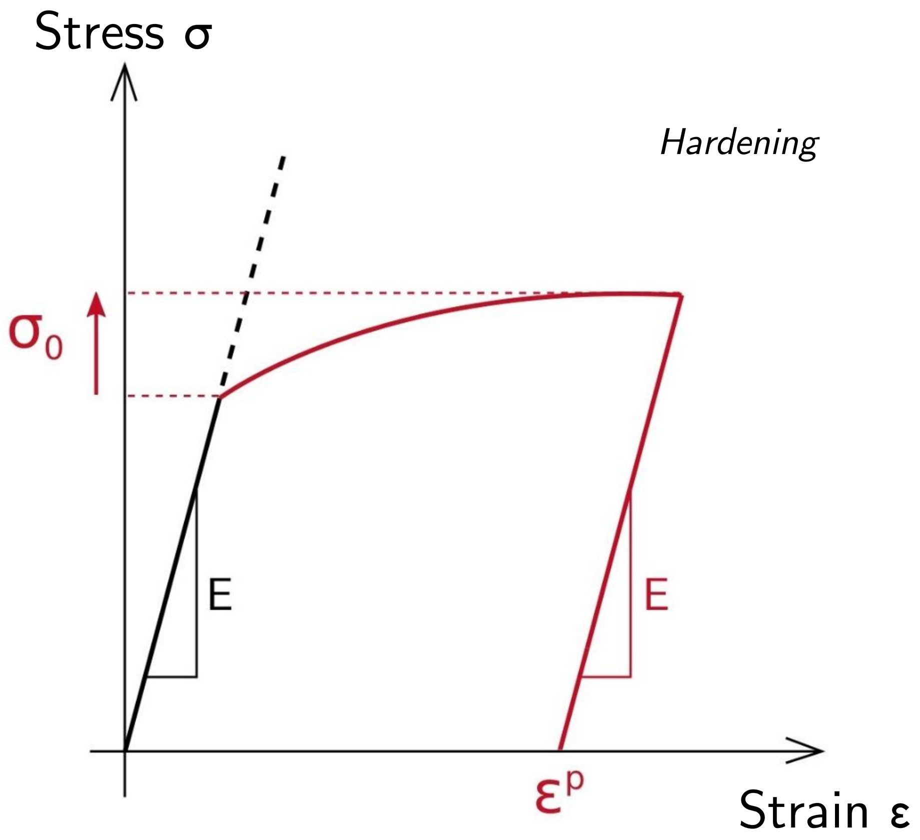


Simulation of plasticity

- Computational modelling

  Hardening – illustration with 1D example





Simulation of plasticity

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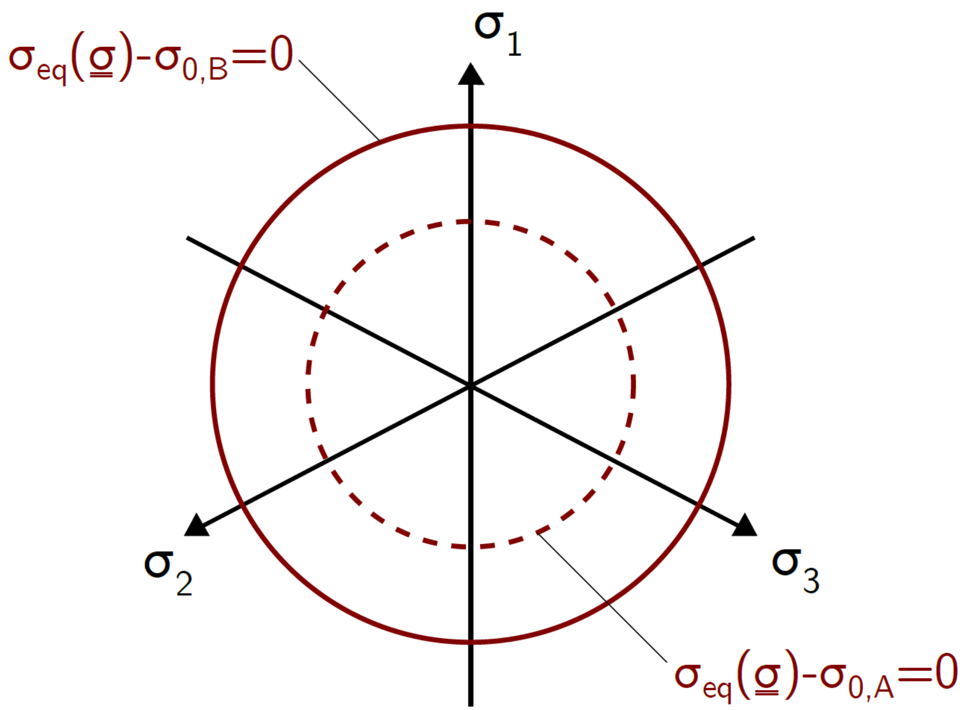
Simulation of plasticity

- Computational modelling

Hardening

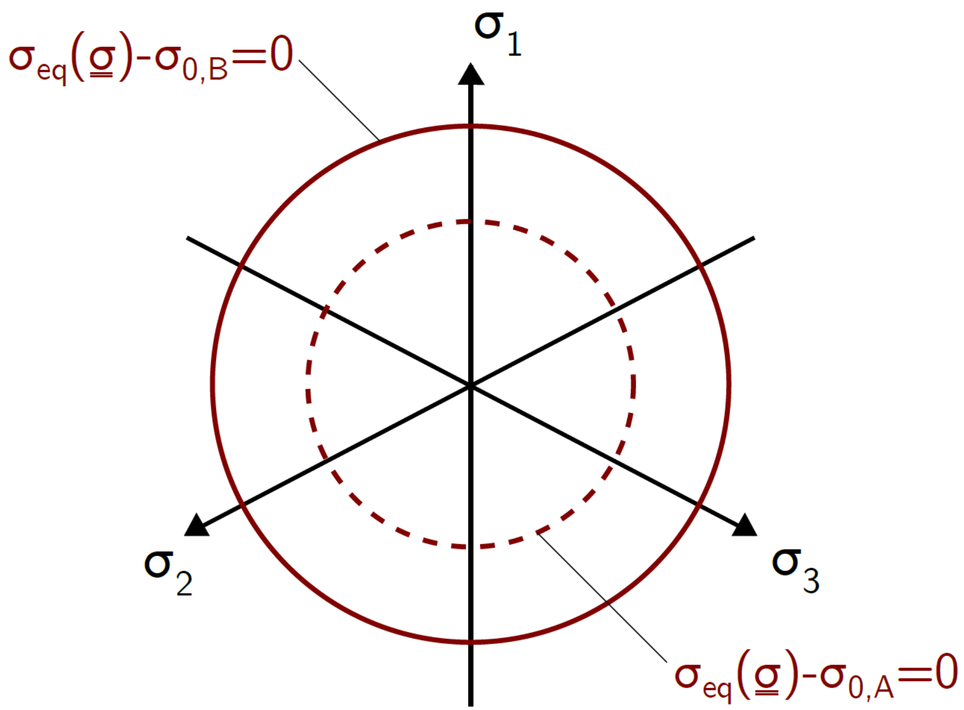


Isotropic hardening

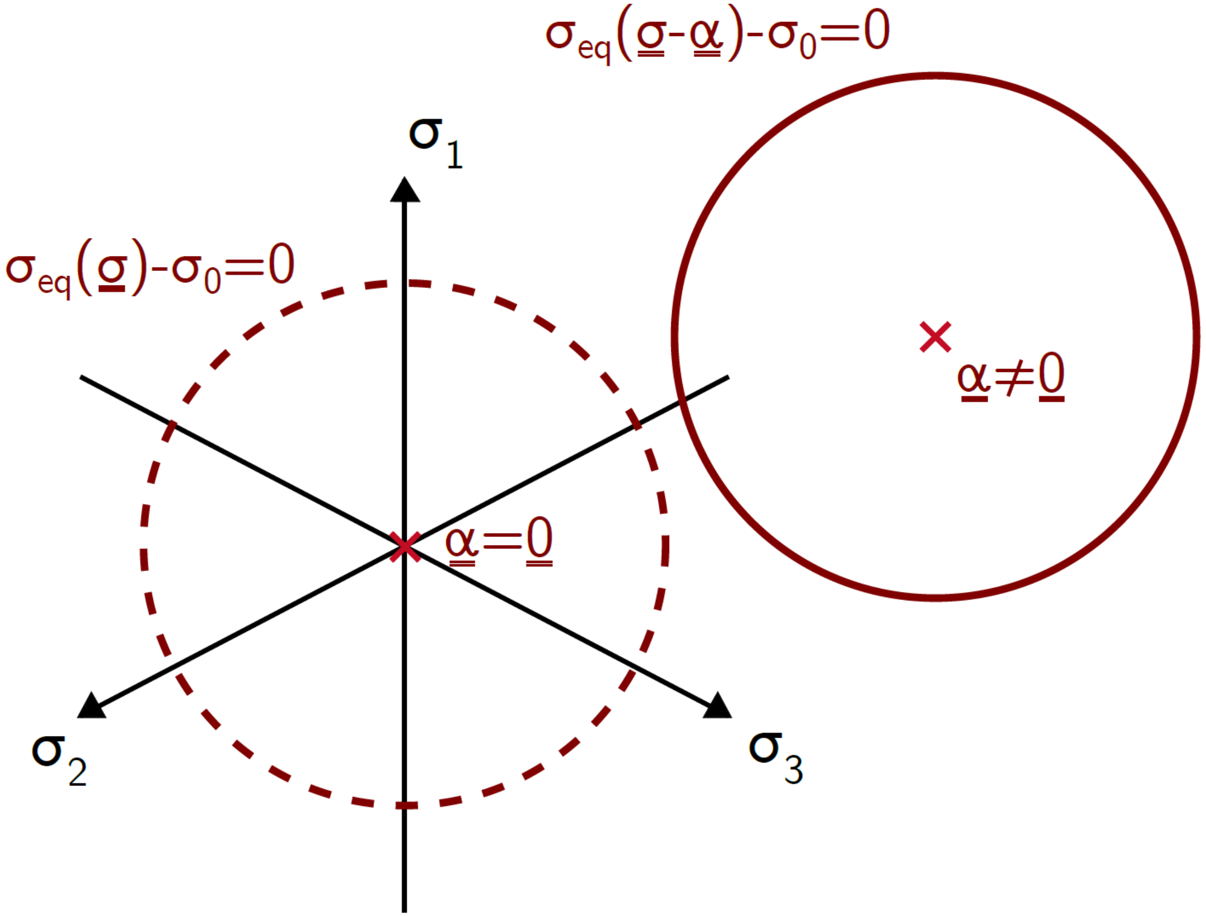
Simulation of plasticity

- Computational modelling

Hardening



Isotropic hardening



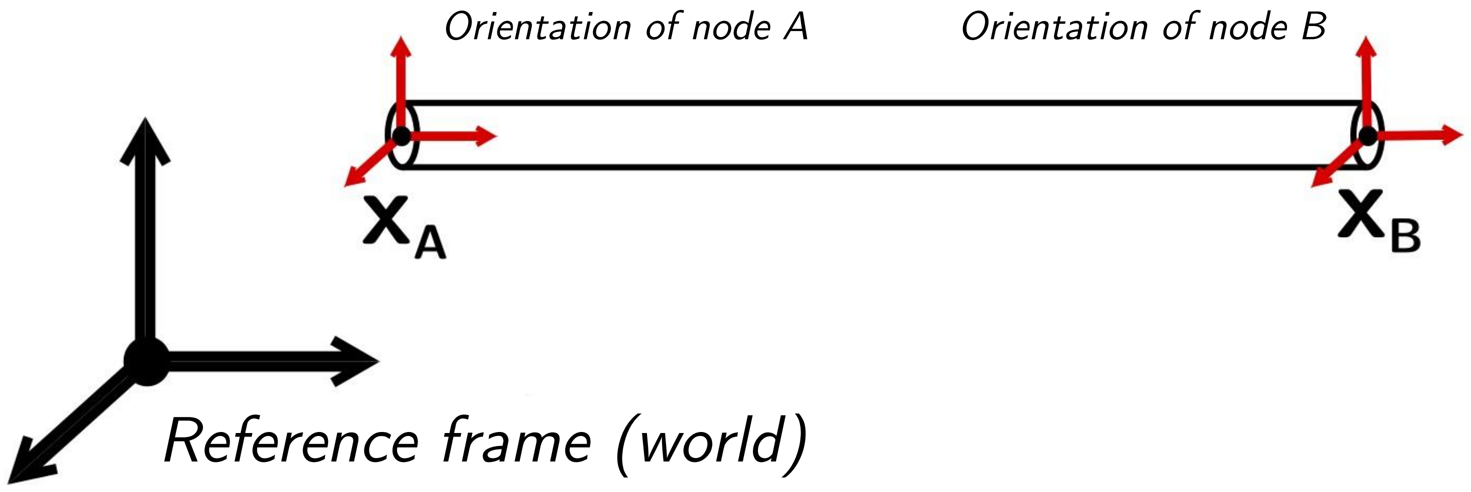
Kinematic hardening

Simulation of plasticity

- Example: application to coronary stent expansion simulation

Simulation of plasticity

- Example: application to coronary stent expansion simulation



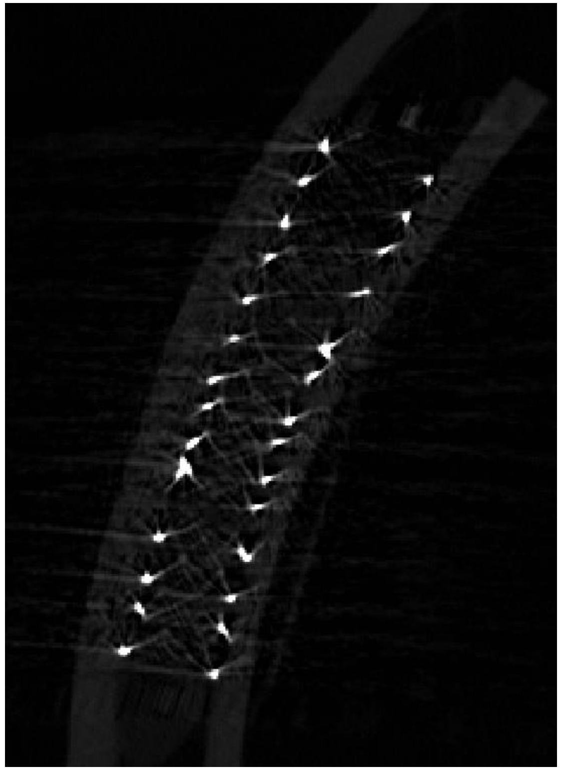
12 degrees of freedom **beam elements**

- 6 DoFs for position
- 6 DoFs for orientation

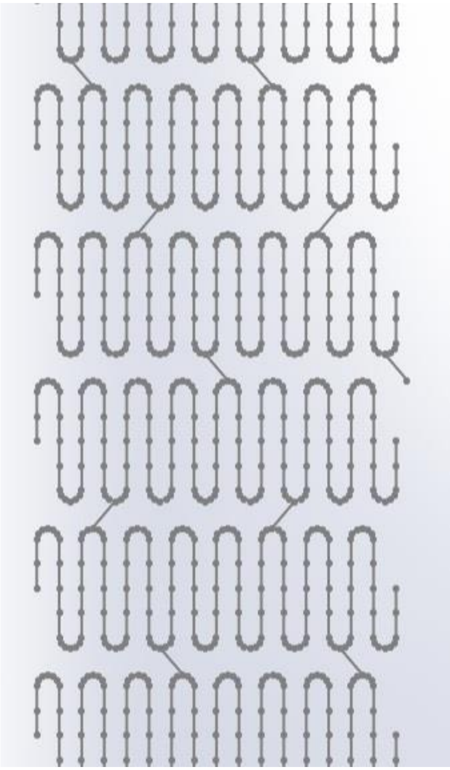
Simulation of plasticity

- Example: application to coronary stent expansion simulation

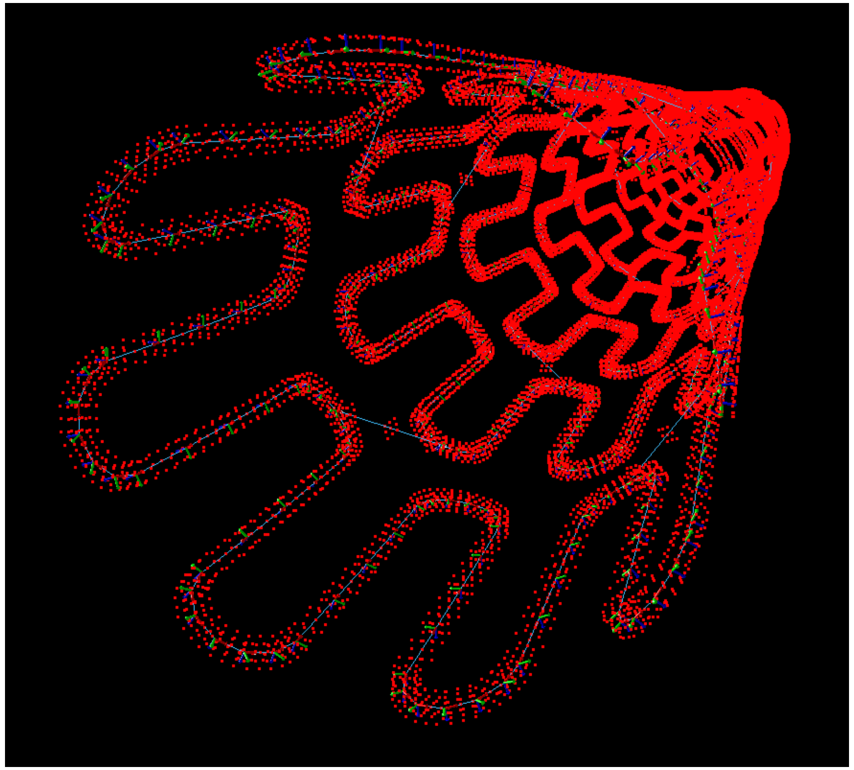
micro-CT acquisition of 3D geometry



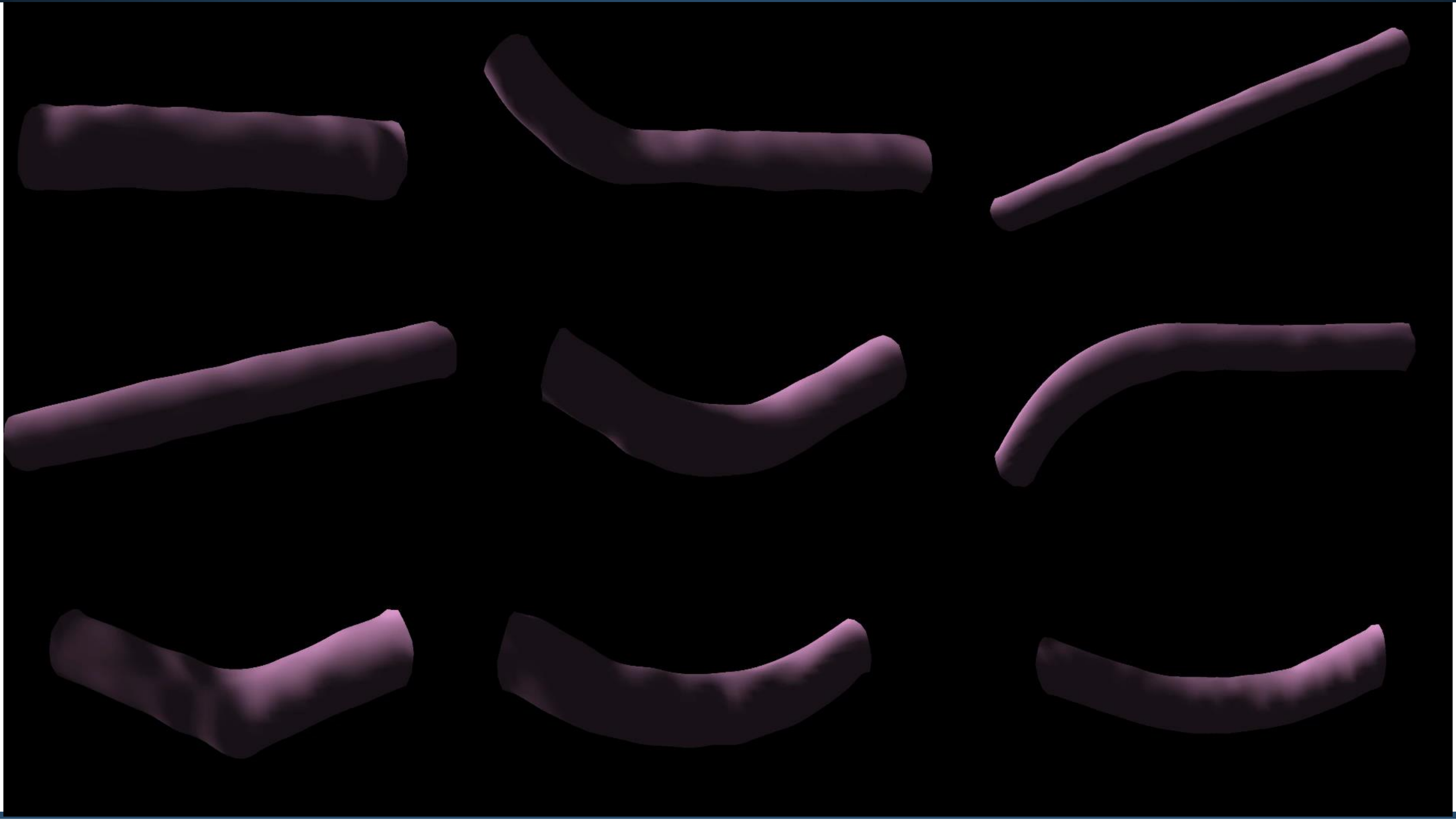
Flat mesh (CAD)



Reconstruction with plastic beam elements (SOFA)



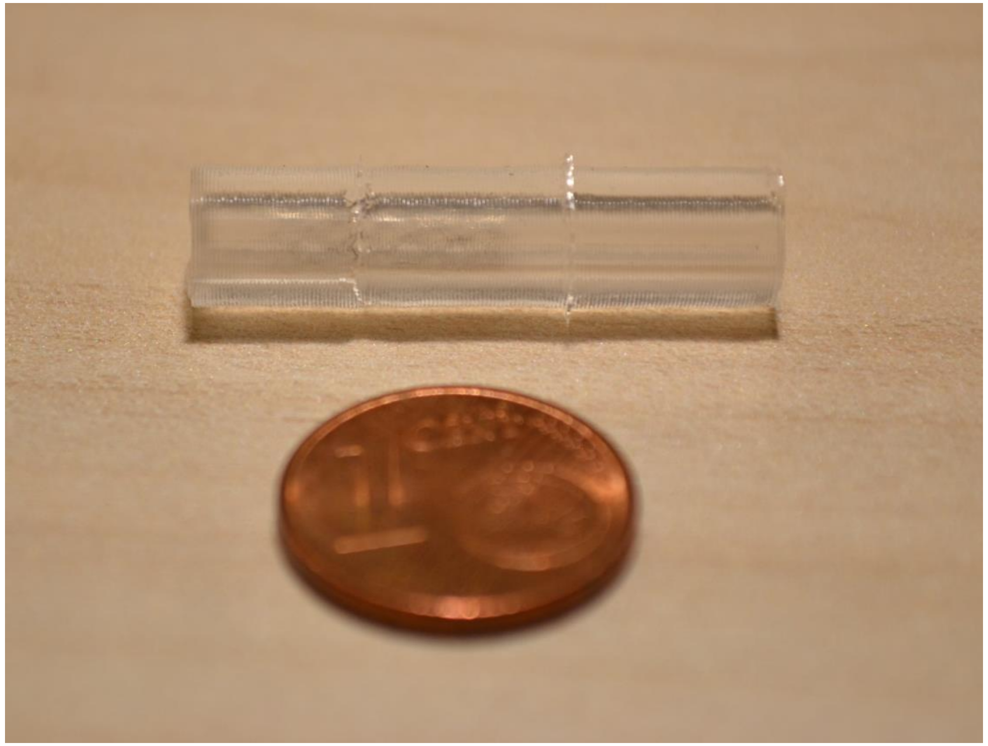
Simulation of plasticity

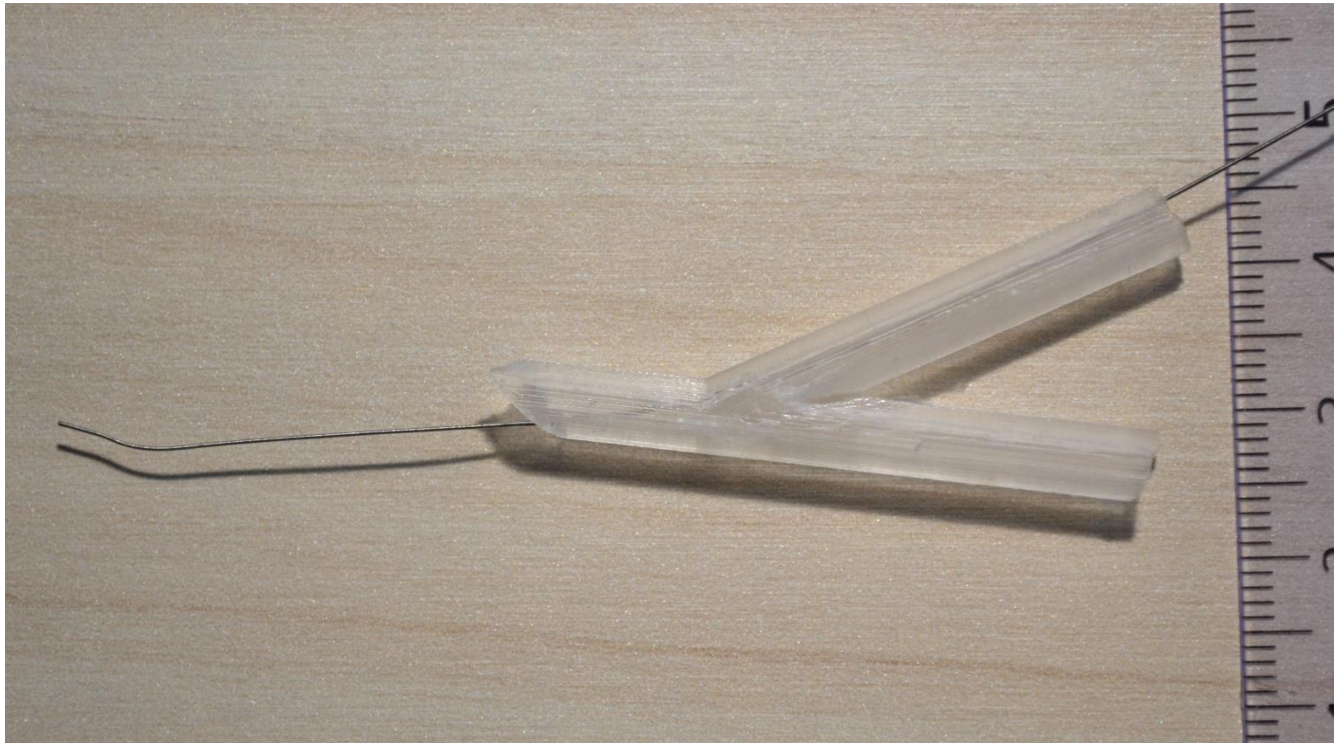


Simulation of plasticity

- Example: application to coronary stent expansion simulation

  Silicone coronary artery phantoms from 3D-printed moulds

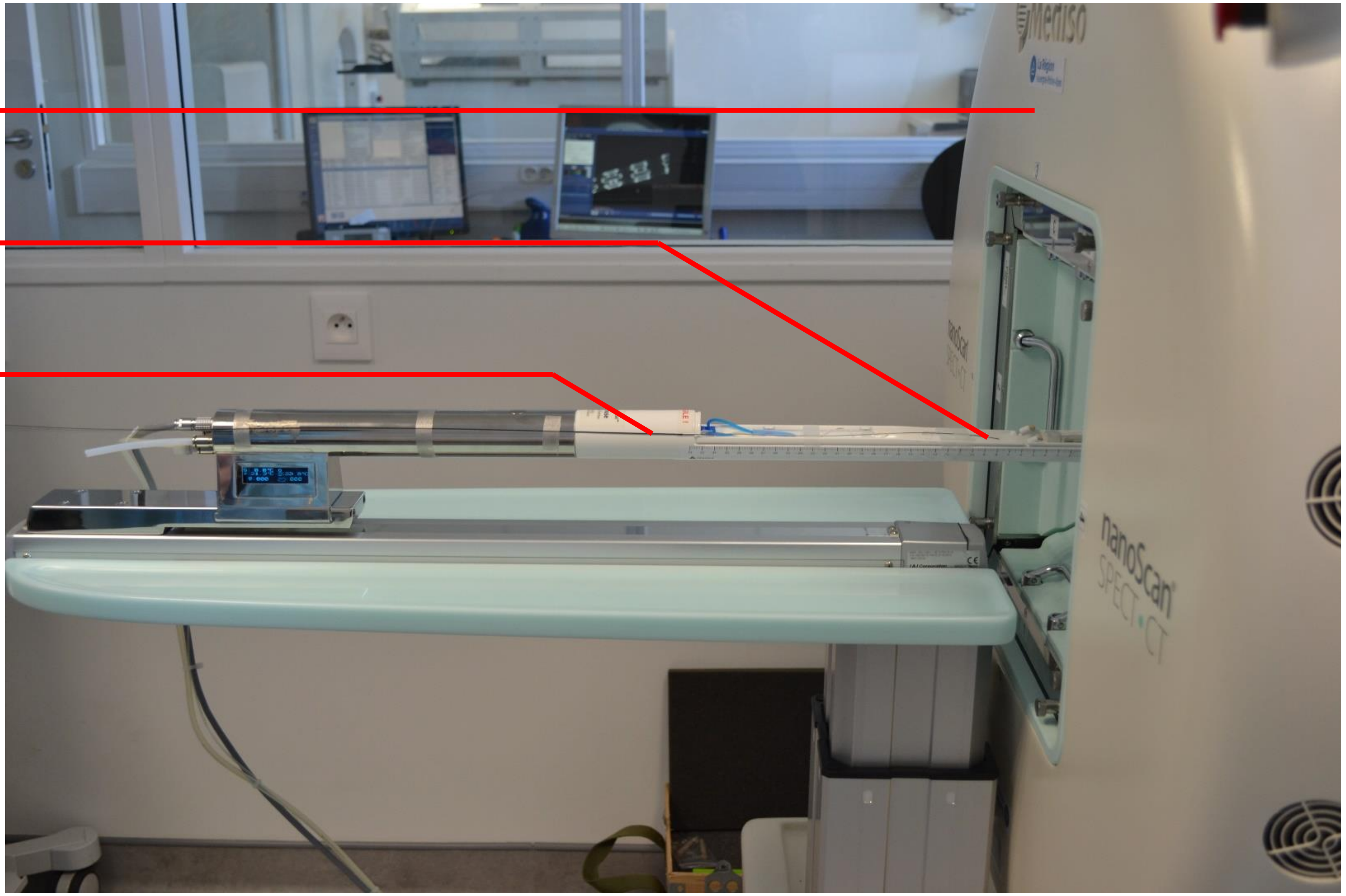




Simulation of plasticity

- Example: application to coronary stent expansion simulation

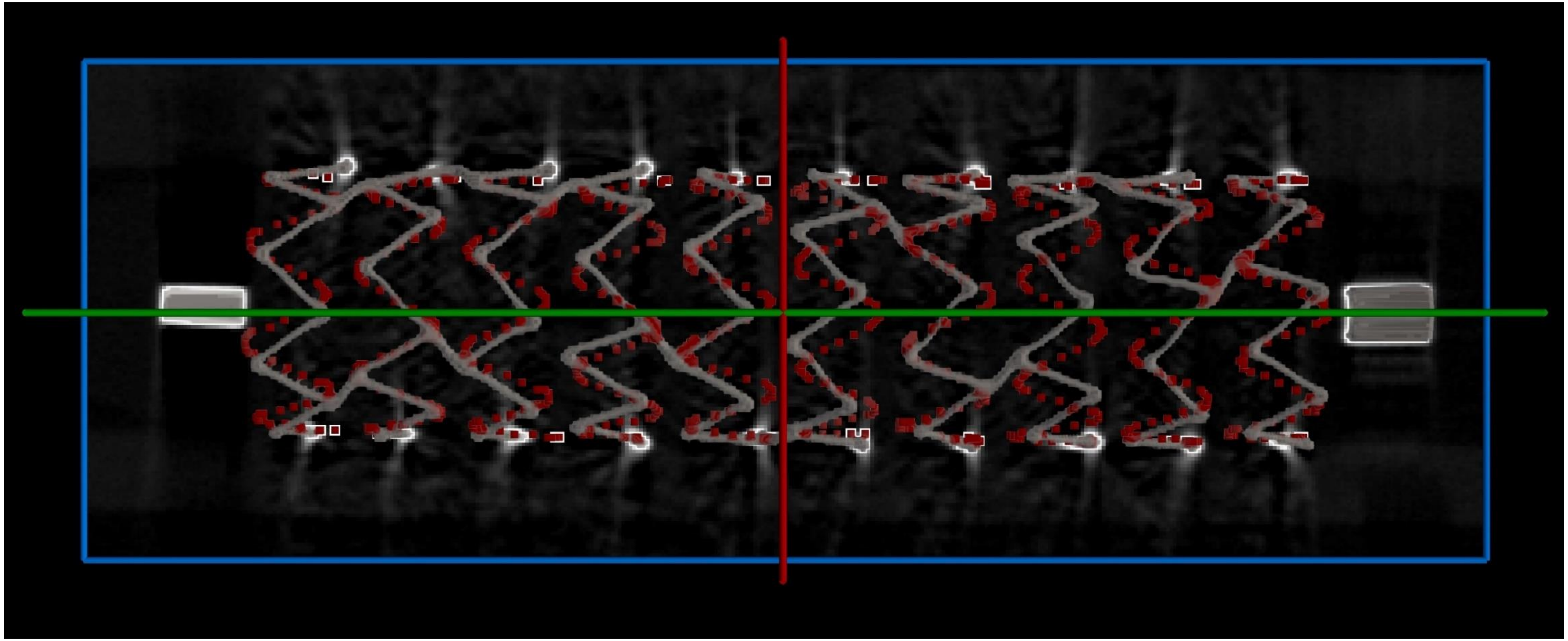
Stent deployment under micro-CT acquisition



Simulation of plasticity

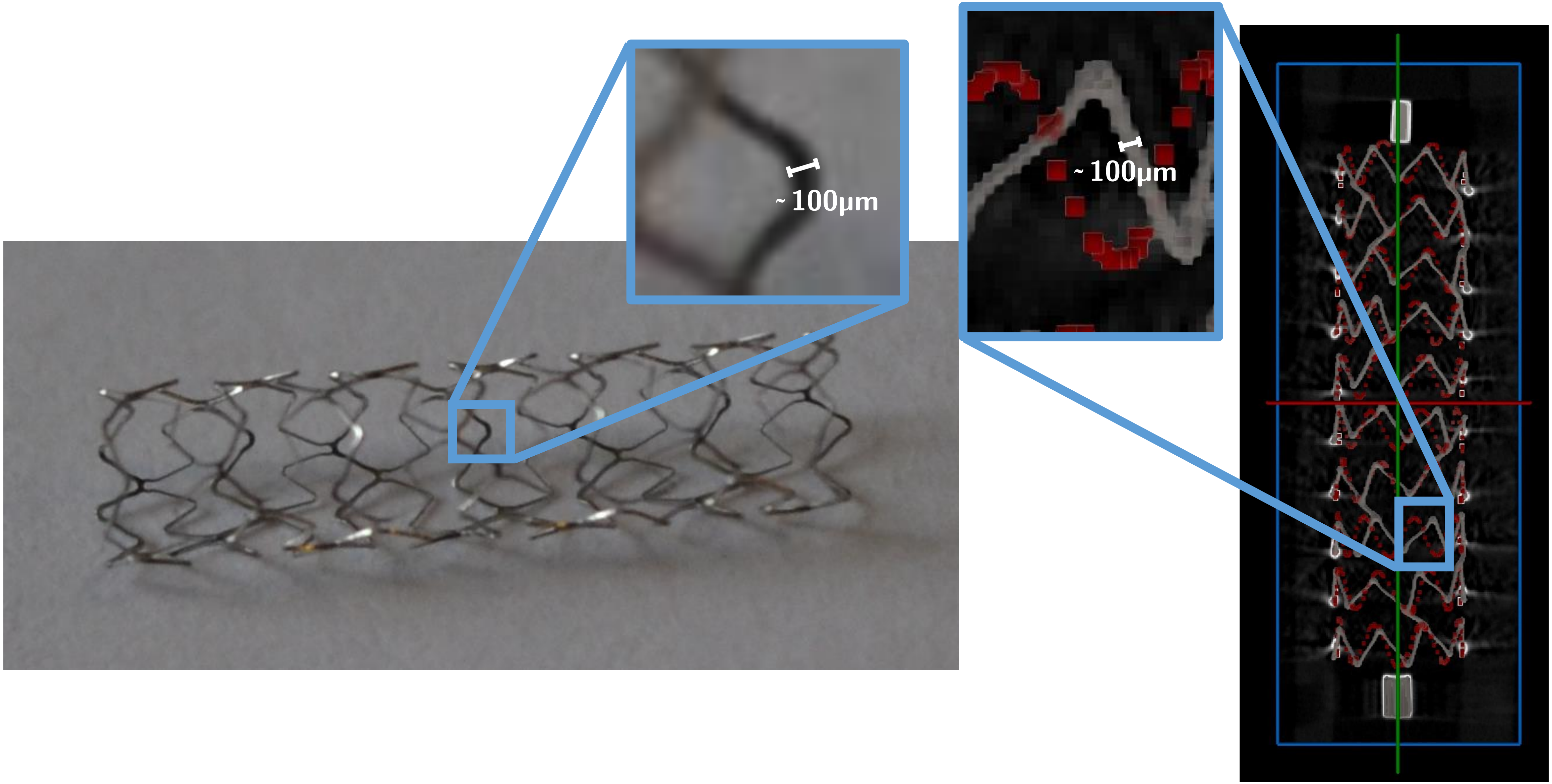
- Example: application to coronary stent expansion simulation

Rigid registration (CT/simulation output)



CT data Simulated data

Simulation of plasticity



Simulation of plasticity

- Example: application to coronary stent expansion simulation