June. 3rd 2019, Lille, France.

Screw Theory as a Unified Approach for Rigid, Soft Articulated and Soft Continuum Robots

Federico Renda[1,2*]

*federico.renda@ku.ac.ae

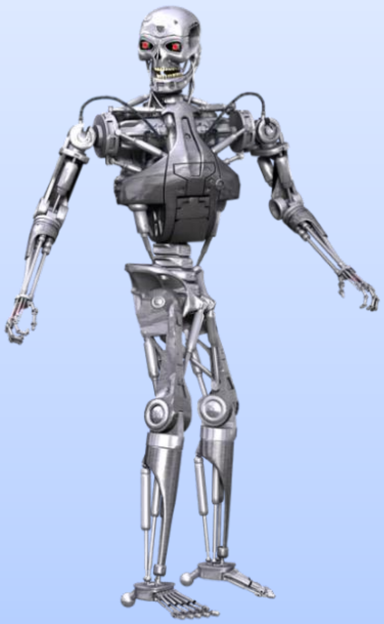
جامعة خليفة
Khalifa University

[1] Department of Mechanical Engineering, Khalifa University, Abu Dhabi, UAE.
[2] Khalifa University Center for Autonomous Robotic System (KU-CARS), Khalifa University, Abu Dhabi, UAE.

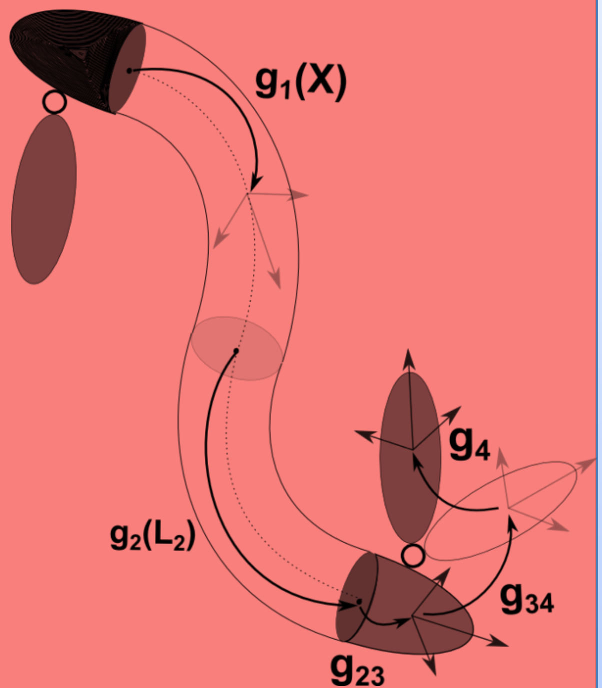
MODELLING OF SOFT-RIGID ROBOTS

DISCRETE COSSERAT APPROACH

RIGID-LINK ROBOTS

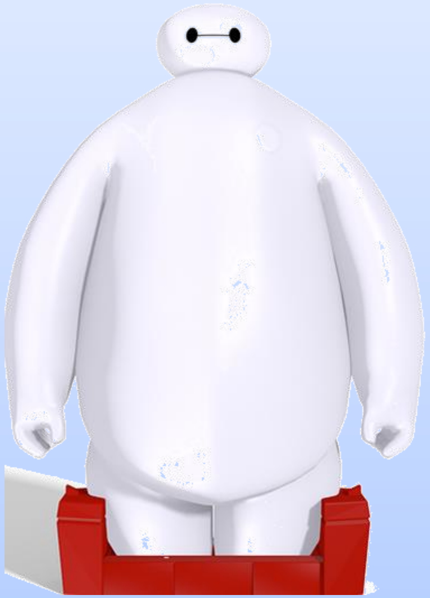


- Lumped Degrees of Freedom (usually revolute angles)
- Lagrangian Model



- Geometric approach based on the exponential map
- **Treats rigid, soft or hybrid robots indistinctly**

SOFT ROBOTS



- Distributed Degrees of Freedom
- Many different modeling approaches including constant curvature, continuous Cosserat and FEM approach

- F. Renda and L. Seneviratne, "A Geometric and Unified Approach for Modeling Soft-Rigid Multi-Body Systems with Lumped and Distributed Degrees of Freedom," *2018 IEEE International Conference on Robotics and Automation (ICRA)*, Brisbane, Australia, 2018, pp. 1567-1574.

LIE GROUP NOTATIONS

position – orientation	standard representation $\boldsymbol{g} = \begin{pmatrix} \boldsymbol{R} & \boldsymbol{u} \\ 0 & 1 \end{pmatrix} \in SE(3)$	Adjoint and coAdjoint representation $Ad_{\boldsymbol{g}} = \begin{pmatrix} \boldsymbol{R} & 0 \\ \widetilde{\boldsymbol{u}}\boldsymbol{R} & \boldsymbol{R} \end{pmatrix}, Ad_{\boldsymbol{g}}^{*} = \begin{pmatrix} \boldsymbol{R} & \widetilde{\boldsymbol{u}}\boldsymbol{R} \\ 0 & \boldsymbol{R} \end{pmatrix} \in \mathbb{R}^{6\times 6}$
velocity (body frame)	Lie Algebra element $\boldsymbol{g}^{-1}\dot{\boldsymbol{g}} = \widehat{\boldsymbol{\eta}} = \begin{pmatrix} \widetilde{\boldsymbol{w}} & \boldsymbol{v} \\ 0 & 0 \end{pmatrix} \in \mathfrak{se}(3)$	adjoint and coadjoint map $ad_{\boldsymbol{\eta}} = \begin{pmatrix} \widetilde{\boldsymbol{w}} & 0 \\ \widetilde{\boldsymbol{v}} & \widetilde{\boldsymbol{w}} \end{pmatrix}, ad_{\boldsymbol{\eta}}^{*} = \begin{pmatrix} \widetilde{\boldsymbol{w}} & \widetilde{\boldsymbol{v}} \\ 0 & \widetilde{\boldsymbol{w}} \end{pmatrix} \in \mathbb{R}^{6\times 6}$
	twist vector $\boldsymbol{\eta} = \begin{bmatrix} \boldsymbol{w} \\ \boldsymbol{v} \end{bmatrix} \in \mathbb{R}^{\boldsymbol{6}}$	Where $\widetilde{\boldsymbol{a}} = \begin{pmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{pmatrix}$
strain (body frame)	Lie Algebra element $\boldsymbol{g}^{-1}\boldsymbol{g}' = \widehat{\boldsymbol{\xi}} = \begin{pmatrix} \widetilde{\boldsymbol{k}} & \boldsymbol{p} \\ 0 & 0 \end{pmatrix} \in \mathfrak{se}(3)$	adjoint and coadjoint map $ad_{\boldsymbol{\xi}} = \begin{pmatrix} \widetilde{\boldsymbol{k}} & 0 \\ \widetilde{\boldsymbol{p}} & \widetilde{\boldsymbol{k}} \end{pmatrix}, ad_{\boldsymbol{\xi}}^{*} = \begin{pmatrix} \widetilde{\boldsymbol{k}} & \widetilde{\boldsymbol{p}} \\ 0 & \widetilde{\boldsymbol{k}} \end{pmatrix} \in \mathbb{R}^{6\times 6}$
	twist vector $\boldsymbol{\xi} = \begin{bmatrix} \boldsymbol{k} \\ \boldsymbol{p} \end{bmatrix} \in \mathbb{R}^{\boldsymbol{6}}$	

- J. M. Selig. *Geometric Fundamentals of Robotics*. Monographs in Computer Science. Springer New York, 2007.

EXPONENTIAL FUNCTIONS

	standard representation	exponential representation
Rigid motion	$\boldsymbol{g}(X) = \begin{pmatrix} \boldsymbol{R} & \boldsymbol{u} \\ 0 & 1 \end{pmatrix} \in SE(3)$	$\boldsymbol{g}(X) = e^{X\hat{\boldsymbol{\xi}}}$ $= \boldsymbol{I}_4 + X\hat{\boldsymbol{\xi}} + \frac{1}{\theta^2}(1-\cos(X\theta))\hat{\boldsymbol{\xi}}^2 + \frac{1}{\theta^3}(X\theta - \sin(X\theta))\hat{\boldsymbol{\xi}}^3$
Adjoint matrix	$Ad_{\boldsymbol{g}(X)} = \begin{pmatrix} \boldsymbol{R} & 0 \\ \tilde{\boldsymbol{u}}\boldsymbol{R} & \boldsymbol{R} \end{pmatrix} \in \mathbb{R}^{6\times 6}$	$Ad_{\boldsymbol{g}(X)} = e^{X ad_\xi}$ $= \boldsymbol{I}_6 + \frac{1}{2\theta}(3\sin(X\theta) - X\theta\cos(X\theta))ad_\xi$ $+ \frac{1}{2\theta^2}(4 - 4\cos(X\theta) - X\theta\sin(X\theta))ad_\xi{}^2$ $+ \frac{1}{2\theta^3}(\sin(X\theta) - X\theta\cos(X\theta))ad_\xi{}^3$ $+ \frac{1}{2\theta^4}(2 - 2\cos(X\theta) - X\theta\sin(X\theta))ad_\xi{}^4$
Tangent operator of the exponential map	$T_{\boldsymbol{g}}(X) = \int_0^X Ad_{\boldsymbol{g}(s)} ds$	$T_{\boldsymbol{g}}(X) = \int_0^X e^{s\,ad_\xi} ds$ $= X\boldsymbol{I}_6 + \frac{1}{2\theta^2}(4 - 4\cos(X\theta) - X\theta\sin(X\theta))ad_\xi$ $+ \frac{1}{2\theta^3}(4X\theta - X\theta\cos(X\theta) - 5\sin(X\theta))ad_\xi{}^2$ $+ \frac{1}{2\theta^4}(2 - 2\cos(X\theta) - X\theta\sin(X\theta))ad_\xi{}^3$ $+ \frac{1}{2\theta^5}(2X\theta - X\theta\cos(X\theta) - 3\sin(X\theta))ad_\xi{}^4$

- J. M. Selig. *Geometric Fundamentals of Robotics*. Monographs in Computer Science. Springer New York, 2007.

CONTINUOUS COSSERAT ROD

- Taking the equilibrium of a continuous section of an elastic rod and limiting the length of the section to zero, we obtain the PDE describing the rod dynamics

KINEMATICS

The space derivative of the roto-translation gives the strain

$$\boldsymbol{g}'(X) = \boldsymbol{g}\,(X)\hat{\boldsymbol{\xi}}(X)$$

DIFFERENTIAL KINEMATICS

Equality of mixed partial derivative of g

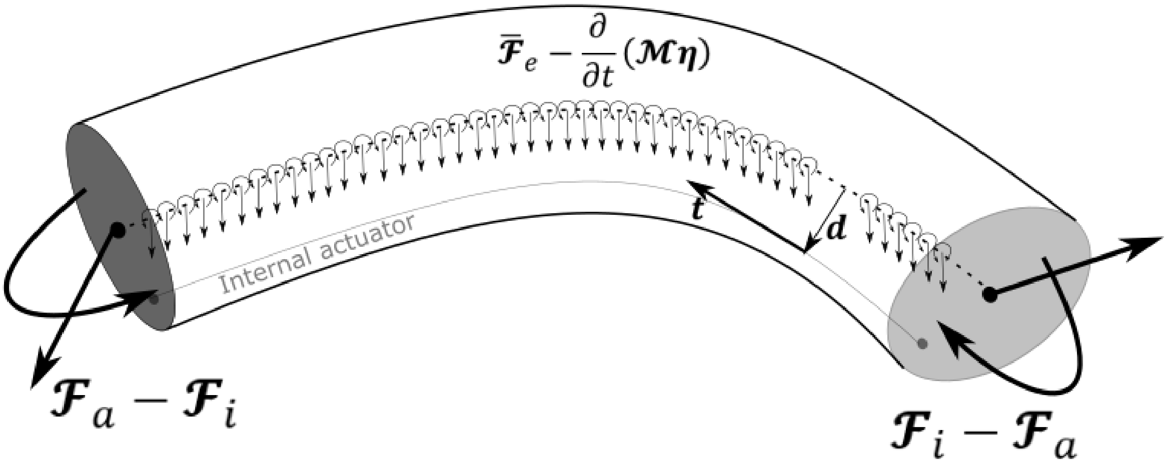
$$\boldsymbol{\eta}'(X) = \dot{\boldsymbol{\xi}}(X) - ad_{\xi(X)}\boldsymbol{\eta}(X)$$

COSSERAT ROD DYNAMICS

PDE describing a Cosserat Rod in local frame

$$\boldsymbol{\mathcal{M}}\dot{\boldsymbol{\eta}} + ad^*_{\boldsymbol{\eta}}\boldsymbol{\mathcal{M}}\boldsymbol{\eta} = \boldsymbol{\mathcal{F}}'_{i-a} + ad^*_{\boldsymbol{\xi}}\boldsymbol{\mathcal{F}}_{i-a} + \bar{\boldsymbol{\mathcal{F}}}_e$$

$$\boldsymbol{\mathcal{F}}_{i-a}(0) = -\boldsymbol{\mathcal{F}}_J(0) \qquad \boldsymbol{\mathcal{F}}_{i-a}(L) = -\boldsymbol{\mathcal{F}}_J(L)$$



INTERNAL ACTUATION

The internal actuation is given by the equilibrium of the actuator internal force

$$\boldsymbol{\mathcal{F}}_a = \begin{bmatrix} \boldsymbol{d} \times T\boldsymbol{t} \\ T\boldsymbol{t} \end{bmatrix}$$

- F. Renda, F. Boyer, J. Dias and L. Seneviratne. Discrete Cosserat Approach for Multi-Section Soft Manipulators Dynamics. *Robotics, IEEE Transactions on*, vol. 34, no. 6, pp. 1518-1533, Dec. 2018..

FROM CONTINUUM TO DISCRETE COSSERAT

the Constant Strain Approach

Kinematics

- The robot arm is divided in N pieces of constant strain
- We move from a continuous strain field $\xi(X)$ to piecewise constant field $\xi_1,..\xi_N$

KINEMATICS

Continuous strain field

$$\boldsymbol{g}'_j(X) = \boldsymbol{g}_j(X)\hat{\boldsymbol{\xi}}_j(X)$$

$\int$

EXPONENTIAL MAP

constant strain field

$$\boldsymbol{g}_j(X) = e^{X\hat{\boldsymbol{\xi}}_j}$$

STRAIN TWIST

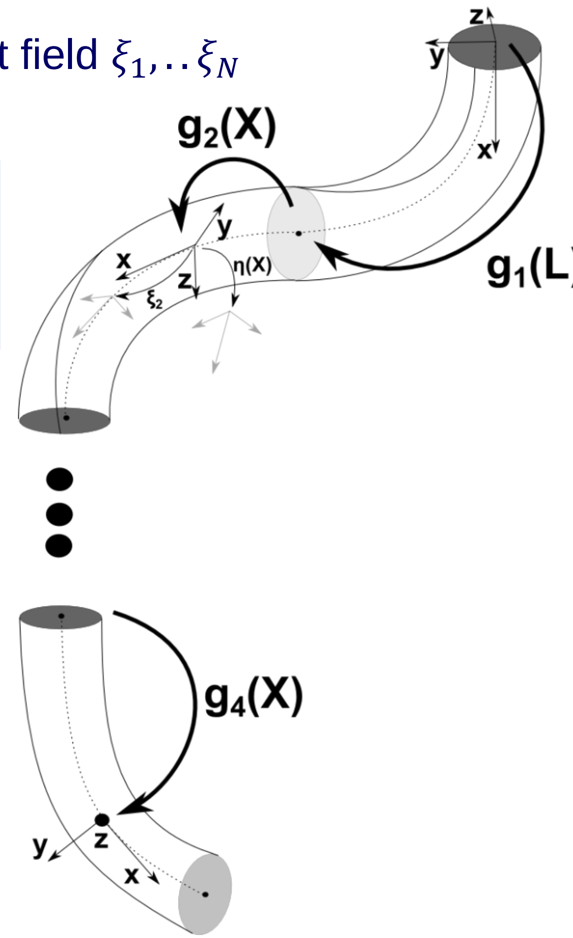
A strain twist belongs to a subspace of $\mathfrak{se}(3)$

$$\boldsymbol{\xi}_j = \boldsymbol{B}_j\boldsymbol{q}_j + \bar{\boldsymbol{\xi}}_j$$

POE FORMULA

Twist are expressed in the body frame

$$\boldsymbol{g}_{sj}(X) = e^{L\hat{\xi}_1}e^{L\hat{\xi}_2}\dots e^{X\hat{\xi}_j}$$



- F. Renda, F. Boyer, J. Dias and L. Seneviratne. Discrete Cosserat Approach for Multi-Section Soft Manipulators Dynamics. *Robotics, IEEE Transactions on*, vol. 34, no. 6, pp. 1518-1533, Dec. 2018..

GEOMETRIC INTERPRETATION

Screw-based Geometry

- Each section is an arc of screw, constant curvature circular arc is a special case

SCREW MOTION

$$\boldsymbol{g}(X) = e^{X\hat{\xi}}$$

CONSTANT STRAIN FIELD

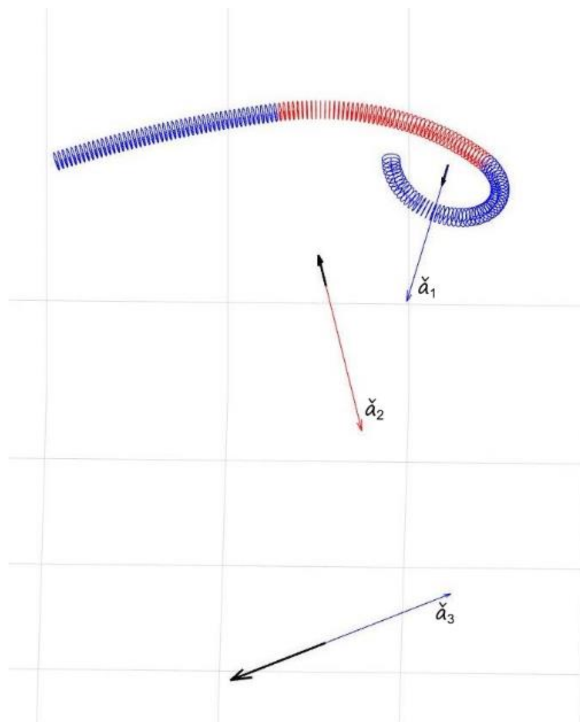
Axis $\hat{a}$, pitch h and magnitude m is uniquely determined by ξ

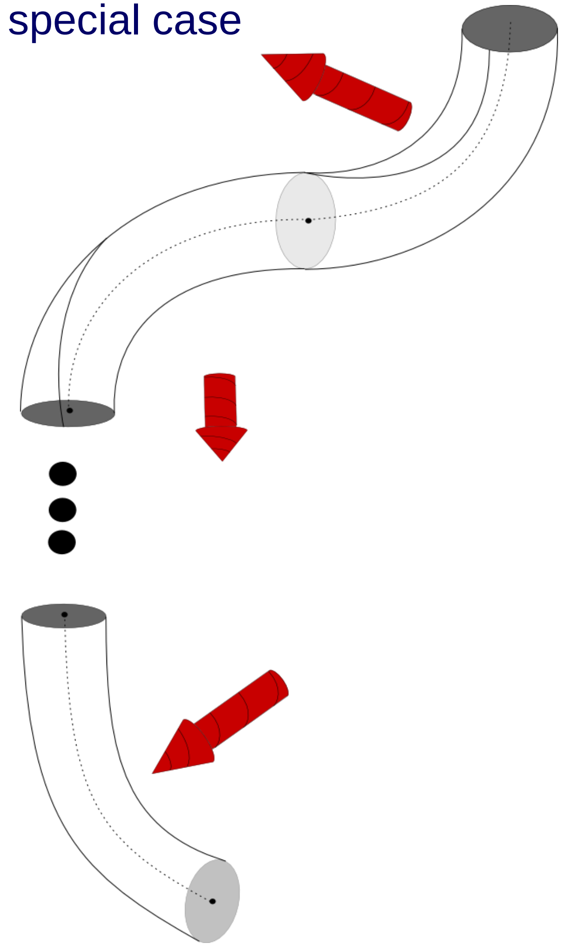
$$h = \frac{\boldsymbol{k}^T\boldsymbol{p}}{\theta^2}$$

$$\hat{a} = \frac{\widetilde{\boldsymbol{k}}\boldsymbol{p}}{\theta^2} + \boldsymbol{k}s$$

$$m = X\theta$$

Where $\theta^2 = \boldsymbol{k}^T\boldsymbol{k}$ and $s \in \mathbb{R}^+$





- F. Renda, M. Cianchetti, H. Abidi, J. Dias and L. Seneviratne. Screw-Based Modeling of Soft Manipulators with Tendon and Fluidic Actuation. *ASME Journal of Mechanism and Robotics*, 9(4):041012-041012-8, 2017.

FROM CONTINUUM TO DISCRETE COSSERAT

the Constant Strain Approach

Differential Kinematics

- Under the constant strain assumption, we can analytically integrate the velocity-strain relation, which yields to a soft robotics geometric Jacobian

DIFFERENTIAL KINEMATICS

continuous strain field

$$\boldsymbol{\eta}_j'(X) = \dot{\boldsymbol{\xi}}_j(X) - ad_{\boldsymbol{\xi}_j(X)}\boldsymbol{\eta}_j(X)$$

$\int$

DIFFERENTIAL MAP

constant strain field

$$\boldsymbol{\eta}_j(X) = Ad^{-1}_{\boldsymbol{g}_j(X)} T_{\boldsymbol{g}_j}(X)\boldsymbol{B}_j\dot{\boldsymbol{q}}_j$$

TANGENT OPERATOR OF THE EXPONENTIAL MAP

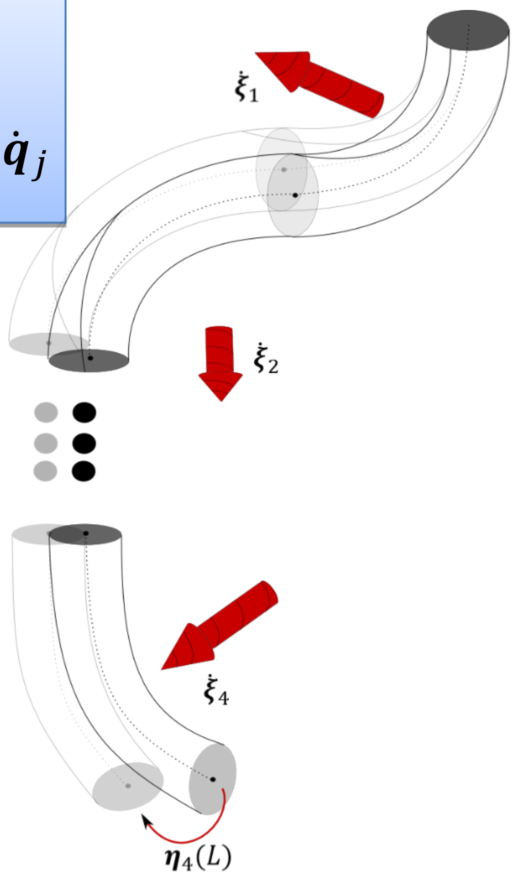
From time derivative of strain twist to velocity twist

$$T_{\boldsymbol{g}_j}(X) = \int_0^X e^{s\, ad_{\boldsymbol{\xi}_j}} ds$$

GEOMETRIC JACOBIAN

Soft robotics geometric Jacobian

$$\boldsymbol{\eta}_j(X) = \sum_{i=0}^{j} Ad^{-1}_{\boldsymbol{g}_i \cdots \boldsymbol{g}_j} T_{\boldsymbol{g}_i}\boldsymbol{B}_i\dot{\boldsymbol{q}}_i = \sum_{i=0}^{j} {}^{j}\boldsymbol{S}_i(X)\dot{\boldsymbol{q}}_i = \boldsymbol{J}_j(X)\dot{\boldsymbol{q}}$$



- F. Renda, F. Boyer, J. Dias and L. Seneviratne. Discrete Cosserat Approach for Multi-Section Soft Manipulators Dynamics. *Robotics, IEEE Transactions on*, vol. 34, no. 6, pp. 1518-1533, Dec. 2018..

FROM CONTINUUM TO DISCRETE COSSERAT

the Constant Strain Approach

Differential Kinematics

- Under the constant strain assumption, we can analytically integrate the velocity-strain relation, which yields to a soft robotics geometric Jacobian

GEOMETRIC JACOBIAN

Soft robotics geometric Jacobian

$$\boldsymbol{J}_1(X) = \left[Ad^{-1}_{\boldsymbol{g}_1(X)} T_{\boldsymbol{g}_1}(X)\boldsymbol{B}_1 \quad \mathbf{0} \quad \cdots\right]$$

$$\boldsymbol{J}_2(X) = \left[Ad^{-1}_{\boldsymbol{g}_1(L)\boldsymbol{g}_2(X)} T_{\boldsymbol{g}_1}(L)\boldsymbol{B}_1 \quad Ad^{-1}_{\boldsymbol{g}_2(X)} T_{\boldsymbol{g}_2}(X)\boldsymbol{B}_2 \quad \mathbf{0} \quad \cdots\right]$$

$$\boldsymbol{J}_3(X) = \left[Ad^{-1}_{\boldsymbol{g}_1(L)\boldsymbol{g}_2(L)\boldsymbol{g}_3(X)} T_{\boldsymbol{g}_1}(L)\boldsymbol{B}_1 \quad Ad^{-1}_{\boldsymbol{g}_2(L)\boldsymbol{g}_3(X)} T_{\boldsymbol{g}_2}(L)\boldsymbol{B}_2 \quad Ad^{-1}_{\boldsymbol{g}_3(X)} T_{\boldsymbol{g}_3}(X)\boldsymbol{B}_3 \quad \mathbf{0} \quad \cdots\right]$$

$$\vdots$$

- F. Renda, F. Boyer, J. Dias and L. Seneviratne. Discrete Cosserat Approach for Multi-Section Soft Manipulators Dynamics. *Robotics, IEEE Transactions on*, vol. 34, no. 6, pp. 1518-1533, Dec. 2018..

FROM CONTINUUM TO DISCRETE COSSERAT

the Constant Strain Approach

Dynamics

- We obtain the generalized dynamics equations for a single soft body *i* by projecting the Cosserat rod dynamics onto the constrained motion space

COSSERAT ROD DYNAMICS

PDE describing a Cosserat Rod in local frame

$$\boldsymbol{\mathcal{M}}_i\dot{\boldsymbol{\eta}}_i + ad^*_{\boldsymbol{\eta}_i}\boldsymbol{\mathcal{M}}_i\boldsymbol{\eta}_i = \boldsymbol{\mathcal{F}}'_{i-a_i} + ad^*_{\boldsymbol{\xi}_i}\boldsymbol{\mathcal{F}}_{i-a_i} + \overline{\boldsymbol{\mathcal{F}}}_{e_i}$$

$$\boldsymbol{\mathcal{F}}_{i-a_i}(0) = -\boldsymbol{\mathcal{F}}_{J_i} \qquad \boldsymbol{\mathcal{F}}_{i-a_i}(L) = -Ad^*_{\boldsymbol{g}_{ij}}\boldsymbol{\mathcal{F}}_{J_j}$$

PROJECTION

Projection onto the constrained motion subspace

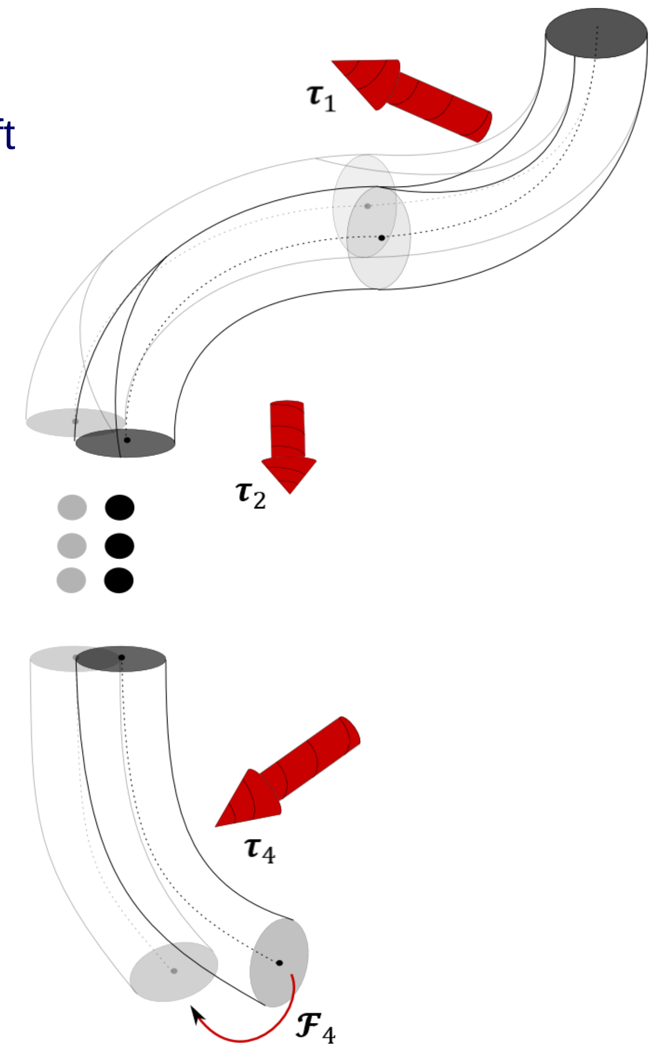
$$\int_{\mathbf{0}}^{\boldsymbol{L}} \boldsymbol{J}_i^T \overline{\boldsymbol{\mathcal{F}}}\, dX$$

KINEMATICS

Constrained kinematics

$$\boldsymbol{\eta}_i(X) = \boldsymbol{J}_i(X)\dot{\boldsymbol{q}}$$

$$\dot{\boldsymbol{\eta}}_i(X) = \boldsymbol{J}_i(X)\ddot{\boldsymbol{q}} + \dot{\boldsymbol{J}}_i(X)\dot{\boldsymbol{q}}$$

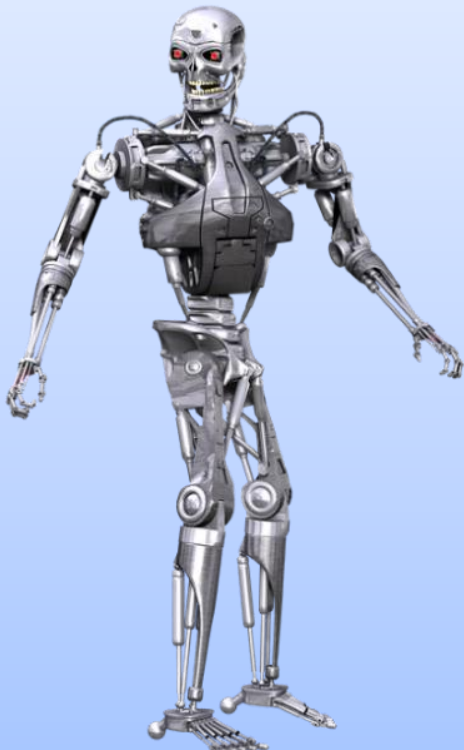


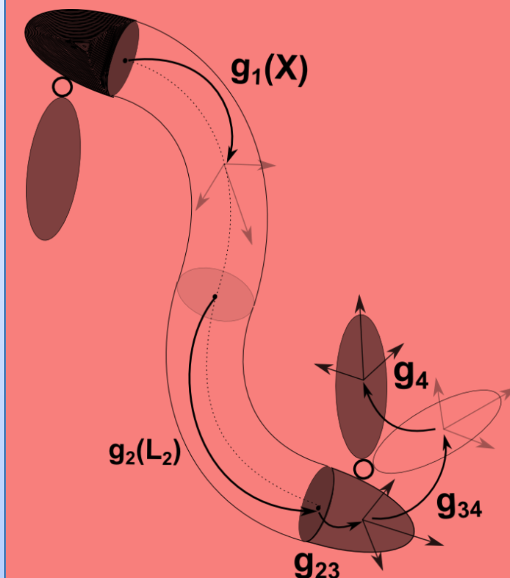
- F. Renda, F. Boyer, J. Dias and L. Seneviratne. Discrete Cosserat Approach for Multi-Section Soft Manipulators Dynamics. *Robotics, IEEE Transactions on*, vol. 34, no. 6, pp. 1518-1533, Dec. 2018.

MODELLING OF SOFT-RIGID ROBOTS

DISCRETE COSSERAT APPROACH (Piece-wise constant strains)

RIGID-LINK ROBOTS





SOFT ROBOTS

EXP MAP

$$\boldsymbol{g}_j(X) = e^{X\hat{\boldsymbol{\xi}}_j}$$

STRAIN TWIST

$$\boldsymbol{\xi}_j = \boldsymbol{B}_j\boldsymbol{q}_j + \bar{\boldsymbol{\xi}}_j$$

TANGENT OPERATOR OF THE EXPONENTIAL MAP

$$T_{\boldsymbol{g}_j}(X) = \int_0^X e^{s\,ad_{\xi_j}} ds$$

PROJECTION

$$\int_0^L \boldsymbol{J}_i^T \overline{\boldsymbol{\mathcal{F}}}\, dX$$

KINEMATICS

$$\boldsymbol{\eta}_i(X) = \boldsymbol{J}_i\dot{\boldsymbol{q}}$$

COSSERAT ROD DYNAMICS

$$\boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{C}\dot{\boldsymbol{q}} + \boldsymbol{K}(\boldsymbol{q} - \boldsymbol{q}^*) = \boldsymbol{\tau} + \boldsymbol{F}$$

$$\boldsymbol{\tau}_i = \boldsymbol{B}_i^T \int_0^L \boldsymbol{\mathcal{F}}_{a_i} dX \qquad \boldsymbol{K}_{ii} = L\boldsymbol{B}_i^T \boldsymbol{\Sigma}_i \boldsymbol{B}_i$$

- F. Renda and L. Seneviratne, "A Geometric and Unified Approach for Modeling Soft-Rigid Multi-Body Systems with Lumped and Distributed Degrees of Freedom," *2018 IEEE International Conference on Robotics and Automation (ICRA)*, Brisbane, Australia, 2018, pp. 1567-1574.

BROCKETT'S PRODUCT OF EXPONENTIALS FORMULA

KINEMATICS

Screw motion parametrized as a space trajectory with constant strain

$$\boldsymbol{g}'_j(X) = \boldsymbol{g}_j(X)\hat{\boldsymbol{\xi}}_j$$

$\int$

EXPONENTIAL MAP

We follow the trajectory up to X=1

$$\boldsymbol{g}_j = e^{\hat{\boldsymbol{\xi}}_j}$$

JOINT TWIST

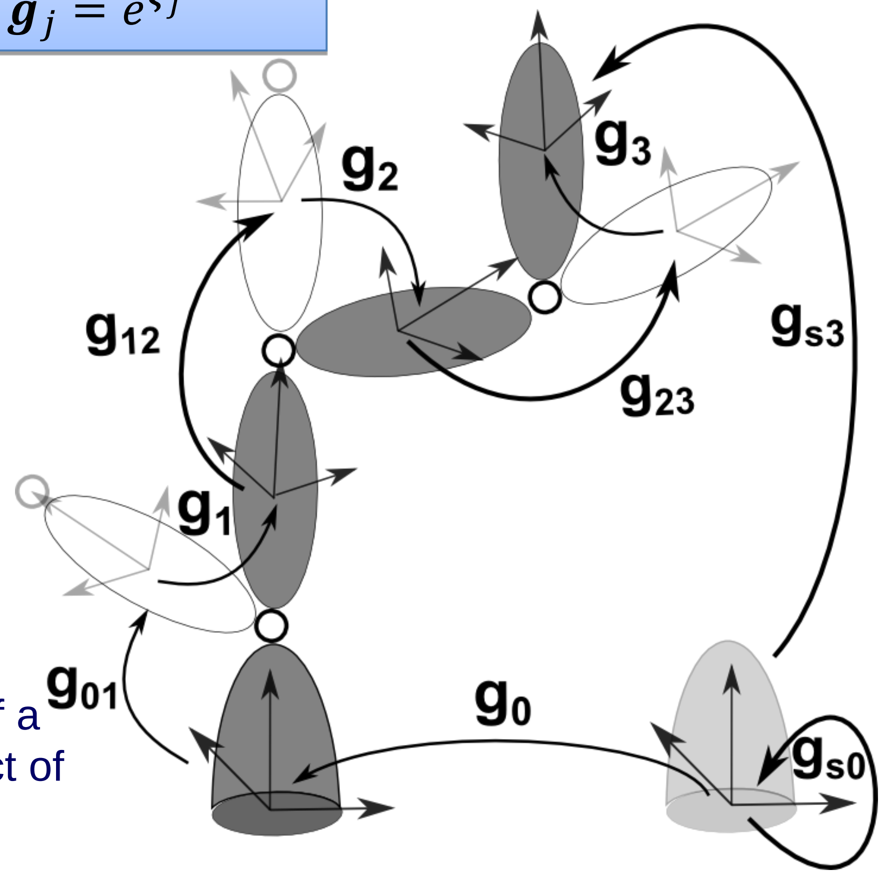
A joint twist belongs to a subspace of $\mathfrak{se}(3)$

$$\boldsymbol{\xi}_j = \boldsymbol{B}_j \boldsymbol{q}_j$$

POE FORMULA

Twist are expressed in the body frame

$$\boldsymbol{g}_{sj} = \boldsymbol{g}_{s0} e^{\hat{\boldsymbol{\xi}}_0} \boldsymbol{g}_{01} e^{\hat{\boldsymbol{\xi}}_1} \cdots \boldsymbol{g}_{ij} e^{\hat{\boldsymbol{\xi}}_j}$$



- Thanks to the exponential map the configuration of a multi-body system can be represented by a product of exponentials

- R. W. Brockett. Robotic manipulators and the product of exponentials formula. In *Mathematical Theory of Networks and Systems*, pages 120–129. Springer Berlin Heidelberg, 1984.
- R.M. Murray, Z. Li, and S.S. Sastry. *A Mathematical Introduction to Robotic Manipulation*. Taylor & Francis, 1994.

RIGID-LINK ROBOTS GEOMETRIC JACOBIAN

- Thanks to the tangent operator of the exponential map a geometric Jacobian between joint space and Euclidean space is obtained

DIFFERENTIAL KINEMATICS

Equality of mixed partial derivative of g

$$\boldsymbol{\eta}'_j(X) = \dot{\boldsymbol{\xi}}_j - ad_{\boldsymbol{\xi}_j}\boldsymbol{\eta}_j(X)$$

$\int$

DIFFERENTIAL MAP

We follow the trajectory up to X=1

$$\boldsymbol{\eta}_j = Ad^{-1}_{\boldsymbol{g}_j} T_{\boldsymbol{g}_j} \boldsymbol{B}_j \dot{\boldsymbol{q}}_j$$

TANGENT OPERATOR OF THE EXPONENTIAL MAP

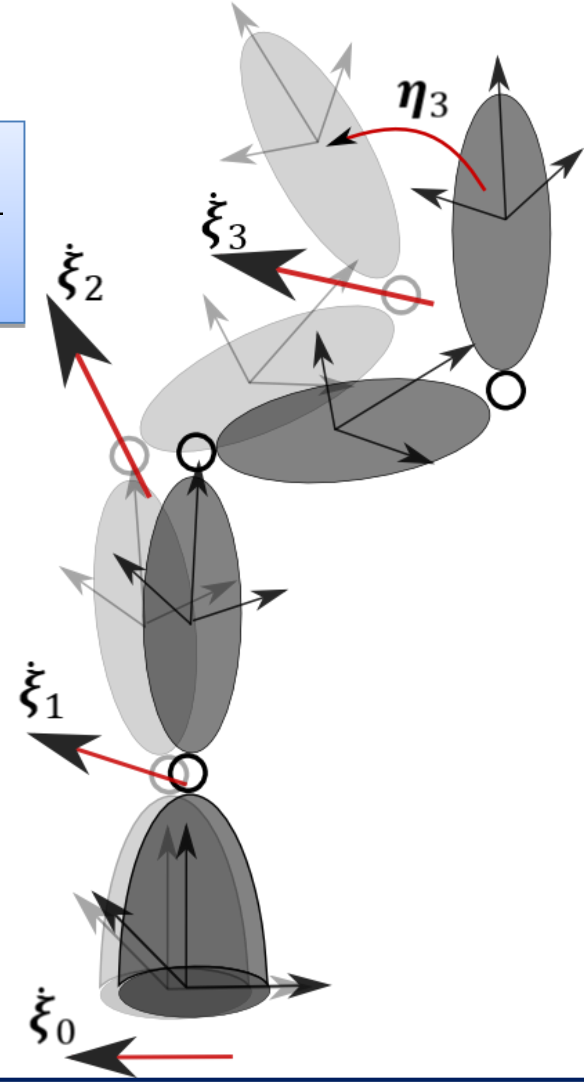
From time derivative of joint twist to velocity twist

$$T_{\boldsymbol{g}_j} = \int_0^1 e^{s\,ad_{\xi_j}} ds$$

GEOMETRIC JACOBIAN

Including multi-dimensional joints

$$\boldsymbol{\eta}_j = \sum_{i=0}^{j} Ad^{-1}_{\boldsymbol{g}_i \cdots \boldsymbol{g}_j} T_{\boldsymbol{g}_i} \boldsymbol{B}_i \dot{\boldsymbol{q}}_i = \sum_{i=0}^{j} {}^{j}\boldsymbol{S}_i \dot{\boldsymbol{q}}_i = \boldsymbol{J}_j \dot{\boldsymbol{q}}$$



RIGID-LINK ROBOT DYNAMICS

- We obtain the generalized dynamics equations for a single body *i* by projecting the N-E dynamics onto the constrained motion space

NEWTON-EULER EQUATION

N-E equation for body *I*

$$\boldsymbol{\mathcal{M}}_i \dot{\boldsymbol{\eta}}_i + ad^*_{\boldsymbol{\eta}_i} \boldsymbol{\mathcal{M}}_i \boldsymbol{\eta}_i = \boldsymbol{\mathcal{F}}_{J_i} - Ad^*_{\boldsymbol{g}_{ij}\boldsymbol{g}_j} \boldsymbol{\mathcal{F}}_{J_j} + \boldsymbol{\mathcal{F}}_{e_i}$$

PROJECTION

Projection onto the constrained motion subspace

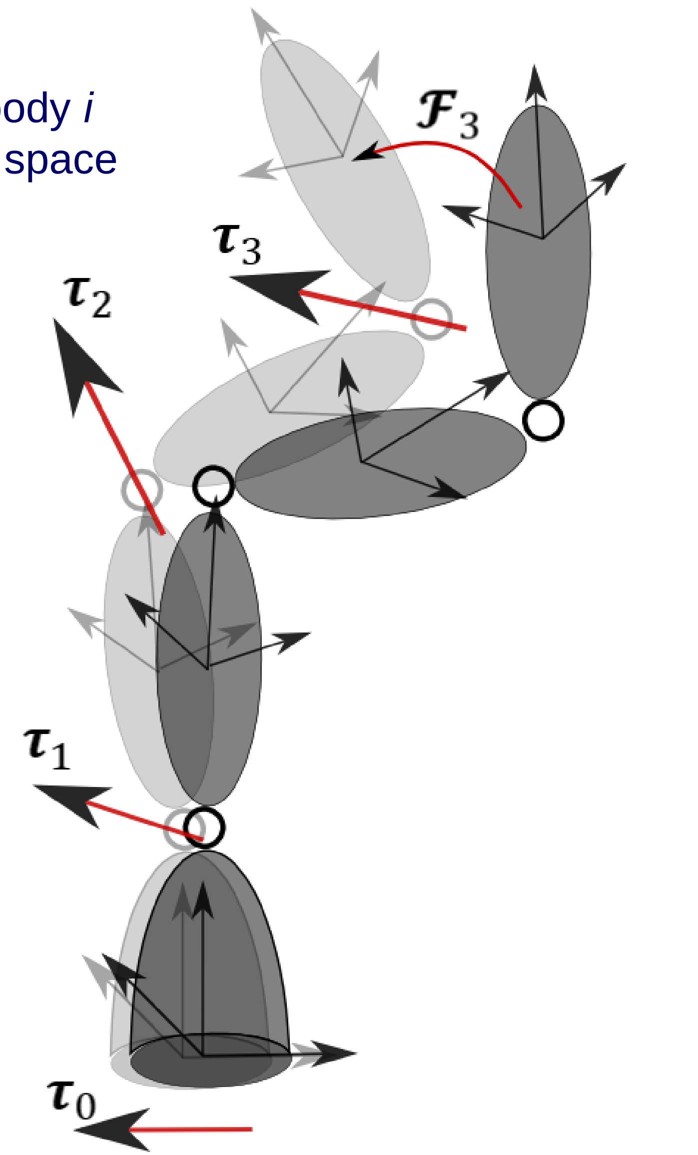
$$\boldsymbol{J}_i^T \boldsymbol{\mathcal{F}}$$

KINEMATICS

Constrained kinematics

$$\boldsymbol{\eta}_i = \boldsymbol{J}_i \dot{\boldsymbol{q}}$$

$$\dot{\boldsymbol{\eta}}_i = \boldsymbol{J}_i \ddot{\boldsymbol{q}} + \dot{\boldsymbol{J}}_i \dot{\boldsymbol{q}}$$



- R. Featherstone. *Rigid Body Dynamics Algorithms*. Springer US, 2008.

MODELLING OF SOFT-RIGID ROBOTS

DISCRETE COSSERAT APPROACH (Piece-wise constant strains)

RIGID-LINK ROBOTS

EXP MAP

$$\boldsymbol{g}_i = e^{\hat{\boldsymbol{\xi}}_i}$$

JOINT TWIST

$$\boldsymbol{\xi}_i = \boldsymbol{B}_i \boldsymbol{q}_i$$

TANGENT OPERATOR OF THE EXPONENTIAL MAP

$$T_{\boldsymbol{g}_i} = \int_0^1 e^{s\,ad_{\xi_i}} ds$$

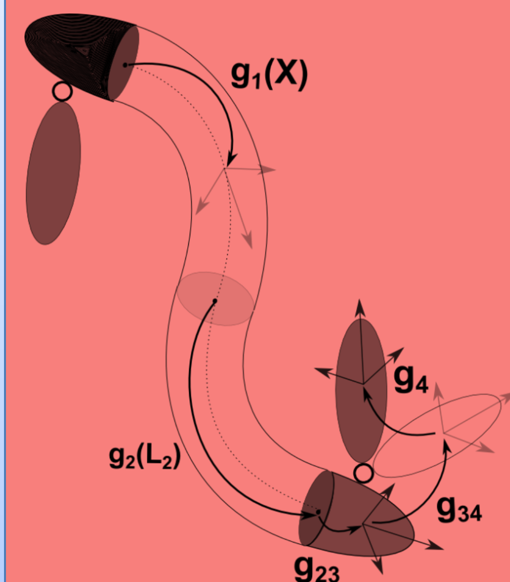
PROJECTION

$$\boldsymbol{J}_i^T \boldsymbol{\mathcal{F}}$$

KINEMATICS

$$\boldsymbol{\eta}_i = \boldsymbol{J}_i \dot{\boldsymbol{q}}$$

NEWTON-EULER EQUATION



SOFT ROBOTS

EXP MAP

$$\boldsymbol{g}_j(X) = e^{X\hat{\boldsymbol{\xi}}_j}$$

STRAIN TWIST

$$\boldsymbol{\xi}_j = \boldsymbol{B}_j \boldsymbol{q}_j + \bar{\boldsymbol{\xi}}_j$$

TANGENT OPERATOR OF THE EXPONENTIAL MAP

$$T_{\boldsymbol{g}_j}(X) = \int_0^X e^{s\,ad_{\xi_j}} ds$$

PROJECTION

$$\int_0^L \boldsymbol{J}_i^T \bar{\boldsymbol{\mathcal{F}}}\, dX$$

KINEMATICS

$$\boldsymbol{\eta}_i(X) = \boldsymbol{J}_i \dot{\boldsymbol{q}}$$

COSSERAT ROD DYNAMICS

$$\boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{C}\dot{\boldsymbol{q}} + \boldsymbol{K}(\boldsymbol{q} - \boldsymbol{q}^*) = \boldsymbol{\tau} + \boldsymbol{F}$$

$$\boldsymbol{\tau}_i = {}^i\boldsymbol{S}_i^T \boldsymbol{\mathcal{F}}_{J_i} \qquad \boldsymbol{K}_{ii} = {}^i\boldsymbol{S}_i^T \boldsymbol{\Sigma}_i \boldsymbol{B}_i \qquad \longleftrightarrow \qquad \boldsymbol{\tau}_i = \boldsymbol{B}_i^T \int_0^L \boldsymbol{\mathcal{F}}_{a_i} dX \qquad \boldsymbol{K}_{ii} = L\boldsymbol{B}_i^T \boldsymbol{\Sigma}_i \boldsymbol{B}_i$$

- F. Renda and L. Seneviratne, "A Geometric and Unified Approach for Modeling Soft-Rigid Multi-Body Systems with Lumped and Distributed Degrees of Freedom," *2018 IEEE International Conference on Robotics and Automation (ICRA)*, Brisbane, Australia, 2018, pp. 1567-1574.

EXTENDED JOINT KINEMATICS TABLE FOR SOFT-RIGID ROBOTS

	LUMPED				DISTRIBUTED				
	Joint	DoF	Base B	Screw Sys. $\mathfrak{m}$	Beam	DoF	Base B	Fixed Twist $\bar{\xi}$	Screw Sys. $\mathfrak{m}$
1 SYS.	Revolute,	1	$\begin{bmatrix}0\\0\\1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\0\\1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\\0\\0\\p\end{bmatrix}$	$\mathfrak{so}(2)$	Linear Spring	1	$\begin{bmatrix}0\\0\\0\\1\\0\\0\end{bmatrix}$	-	$\mathfrak{R}$
	Prismatic,			$\mathfrak{R}$					
	Helical			$\mathfrak{h}_p$					
2 SYS.	Cilindrical	2	$\begin{bmatrix}1&0\\0&0\\0&0\\0&1\\0&0\\0&0\end{bmatrix}$	$\mathfrak{so}(2)\times\mathfrak{R}$	Planar Constant Curvature	2	$\begin{bmatrix}0&0\\0&0\\1&0\\0&1\\0&0\\0&0\end{bmatrix}$	-	span$\{B\}$
3 SYS.	Planar	3	$\begin{bmatrix}0&0&0\\0&0&0\\1&0&0\\0&1&0\\0&0&1\\0&0&0\end{bmatrix}$	$\mathfrak{se}(2)$	Inextensible Constant Curvature	2	$\begin{bmatrix}0&0\\1&0\\0&1\\0&0\\0&0\\0&0\end{bmatrix}$	$\begin{bmatrix}0\\0\\0\\1\\0\\0\end{bmatrix}$	span$\{B, \bar{\xi}\}$
	Spherical		$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\\0&0&0\\0&0&0\\0&0&0\end{bmatrix}$	$\mathfrak{so}(3)$	Constant Curvature	3	$\begin{bmatrix}0&0&0\\1&0&0\\0&1&0\\0&0&1\\0&0&0\\0&0&0\end{bmatrix}$	-	span$\{B\}$
4 SYS.	-	-	-	-	Inextensible Kirchhoff-Love	3	$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\\0&0&0\\0&0&0\\0&0&0\end{bmatrix}$	$\begin{bmatrix}0\\0\\0\\1\\0\\0\end{bmatrix}$	span$\{B, \bar{\xi}\}$
					Kirchhoff-Love	4	$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\\0&0&0&0\\0&0&0&0\end{bmatrix}$	-	span$\{B\}$
6 SYS.	Free Motion	6	$\boldsymbol{I}_6$	$\mathfrak{se}(3)$	Simo-Reissner	6	$\boldsymbol{I}_6$	-	$\mathfrak{se}(3)$

- F. Renda and L. Seneviratne, "A Geometric and Unified Approach for Modeling Soft-Rigid Multi-Body Systems with Lumped and Distributed Degrees of Freedom," *2018 IEEE International Conference on Robotics and Automation (ICRA)*, Brisbane, Australia, 2018, pp. 1567-1574.

ILLUSTRATIVE EXAMPLE: LOPHOTRICUS

- Let's take a motile bacteria as illustrative example

REVOLUTE JOINT

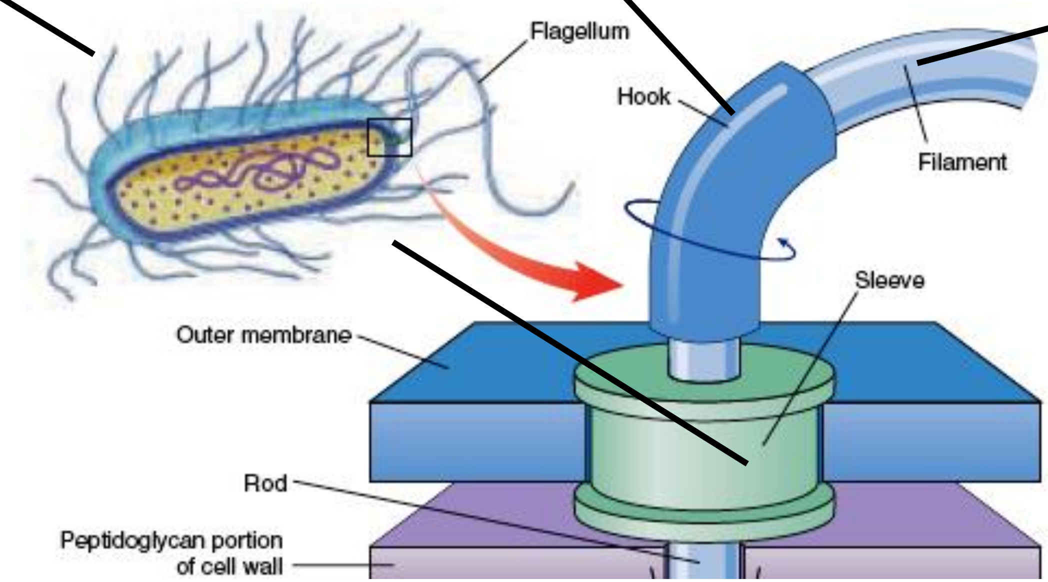
$$\boldsymbol{\xi}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \boldsymbol{q}_1$$

INEXTENSIBLE CONSTANT CURVATURE

$$\boldsymbol{\xi}_2 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \boldsymbol{q}_2 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

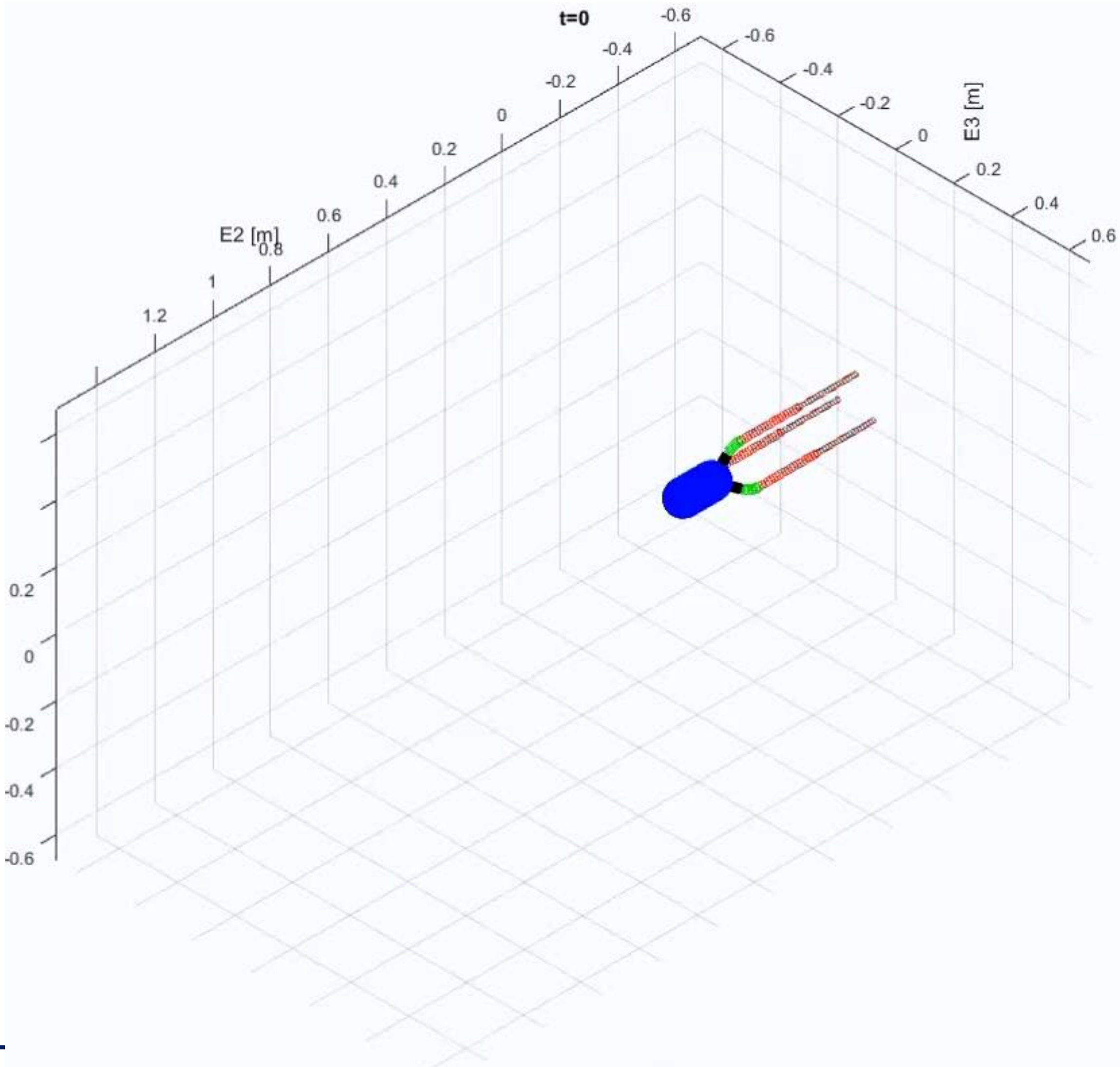
KIRCHHOFF-LOVE

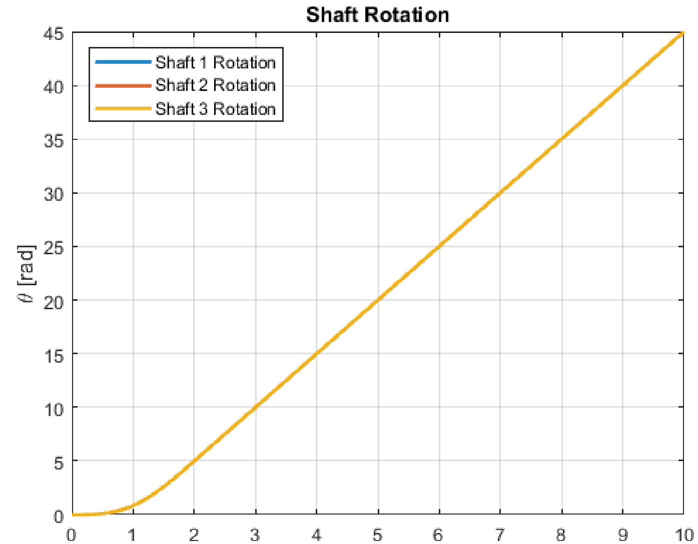
$$\boldsymbol{\xi}_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \boldsymbol{q}_3$$

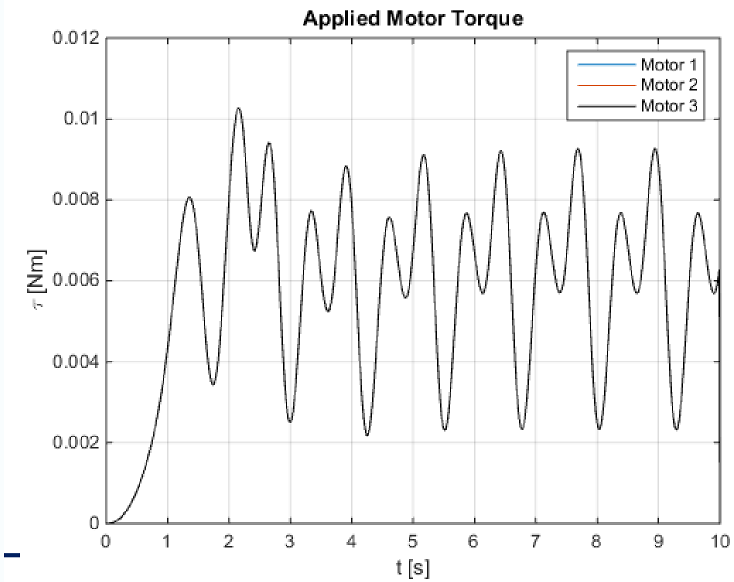


- F. A. Samatey, H. Matsunami, K. Imada, S. Nagashima, T. R. Shaikh, D. R. Thomas, J. Z. Chen, D. J. Derosier, A. Kitao and K. Namba. Structure of the bacterial flagellar hook and implication for the molecular universal joint mechanism. *Nature*, 431(7012):1062–1068, 2004.

ILLUSTRATIVE EXAMPLE: LOPHOTRICUS STRAIGHT MOTION

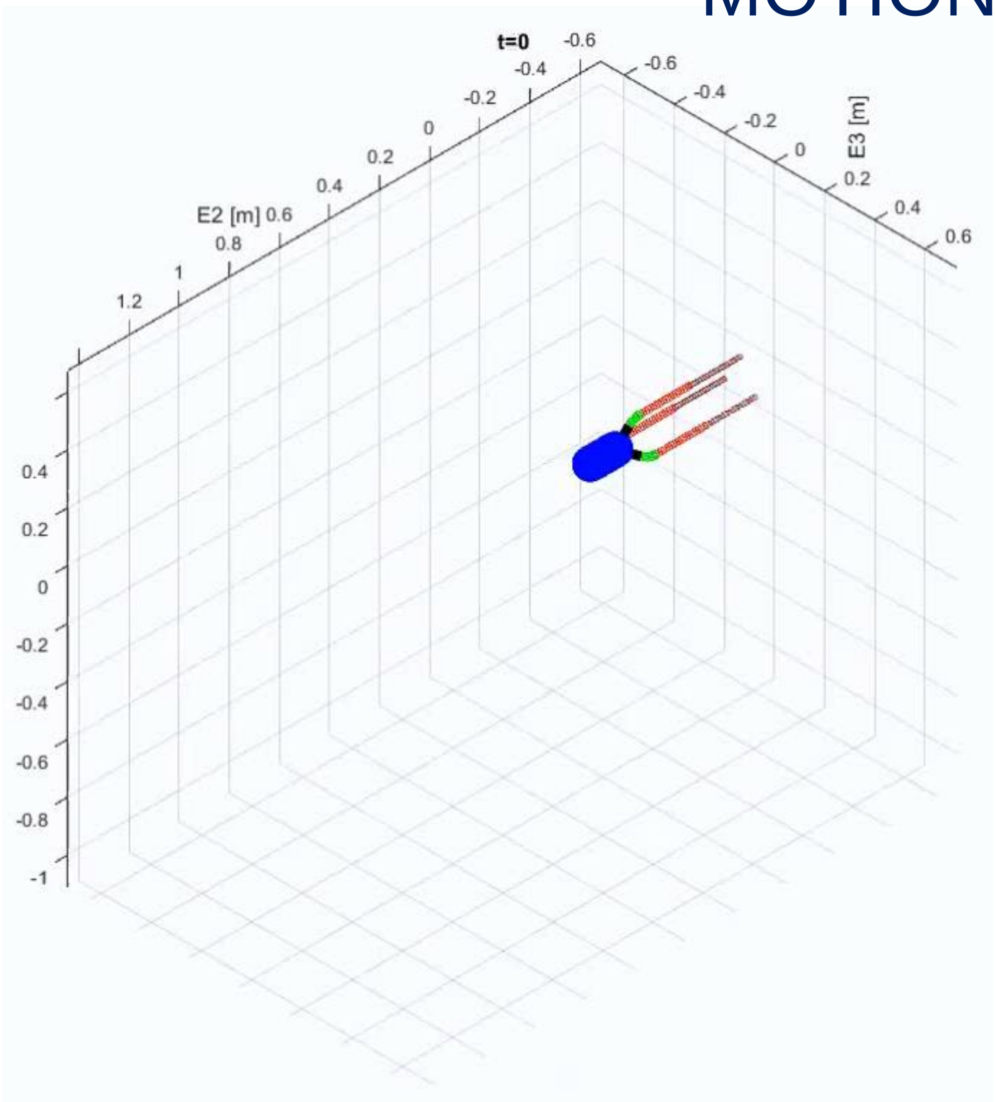


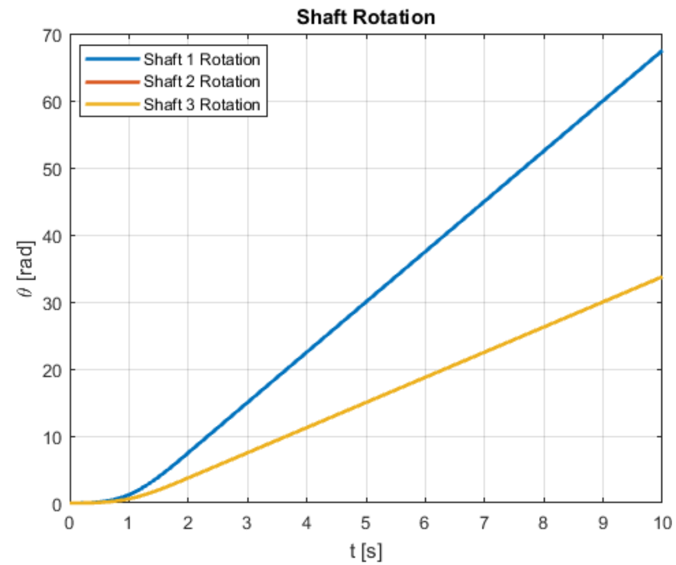


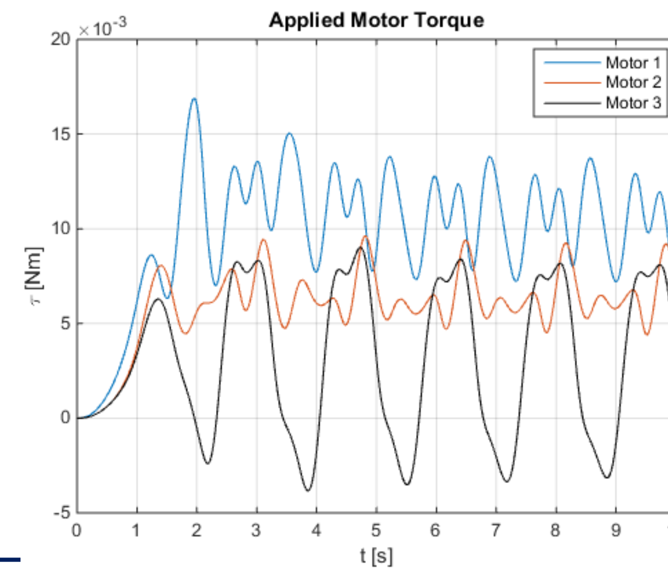


- F. Renda and L. Seneviratne, "A Geometric and Unified Approach for Modeling Soft-Rigid Multi-Body Systems with Lumped and Distributed Degrees of Freedom," *2018 IEEE International Conference on Robotics and Automation (ICRA)*, Brisbane, Australia, 2018, pp. 1567-1574.

ILLUSTRATIVE EXAMPLE: LOPHOTRICUS SINK MOTION



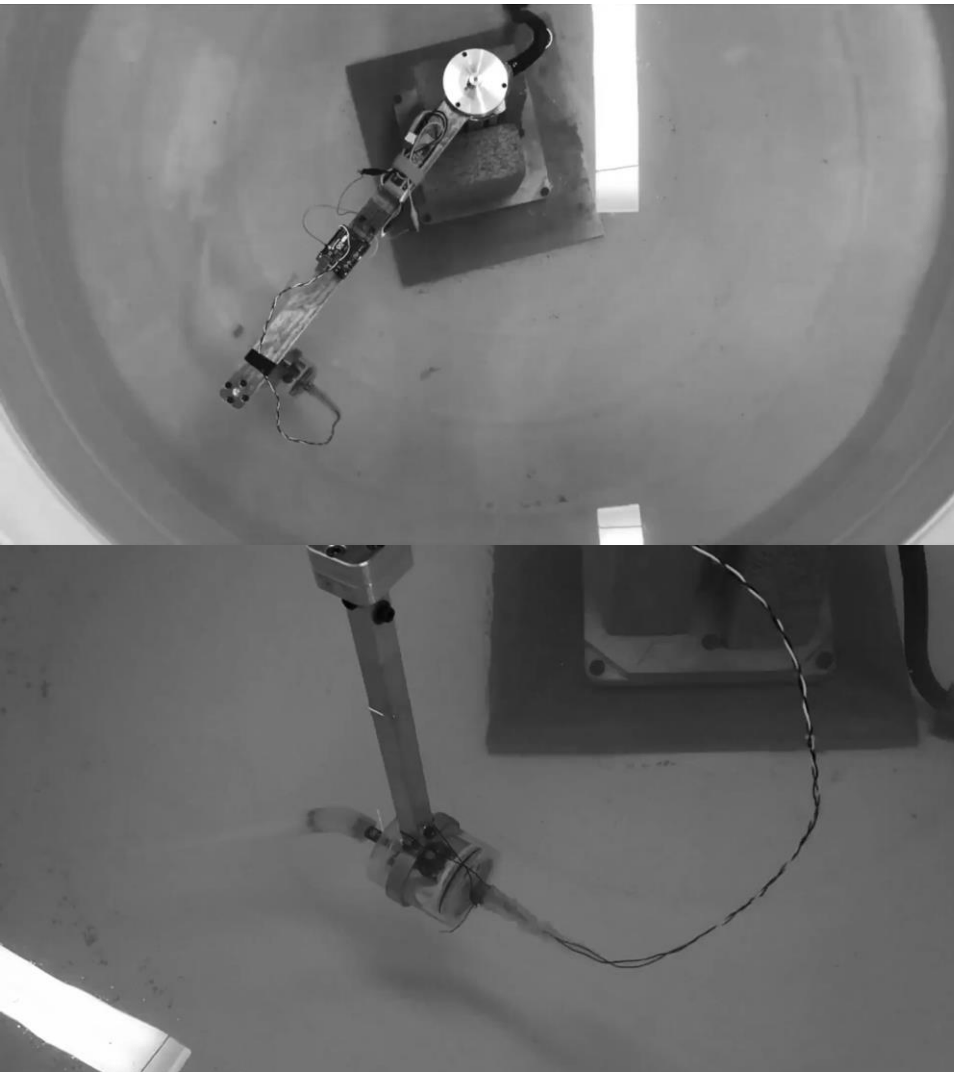




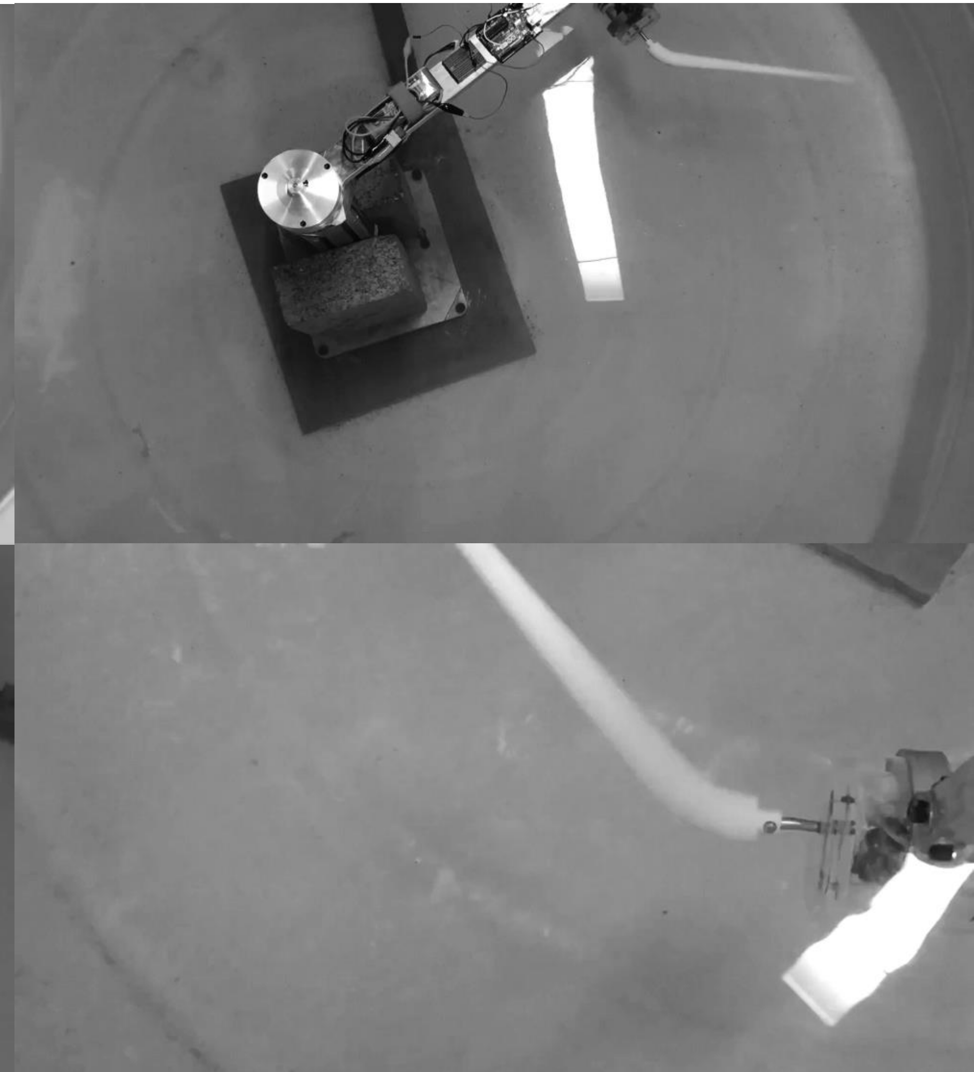
- F. Renda and L. Seneviratne, "A Geometric and Unified Approach for Modeling Soft-Rigid Multi-Body Systems with Lumped and Distributed Degrees of Freedom," *2018 IEEE International Conference on Robotics and Automation (ICRA)*, Brisbane, Australia, 2018, pp. 1567-1574.

SELF-PROPULSION RESULTS (I)

Soft Flagellum (Dragon Skin 30)

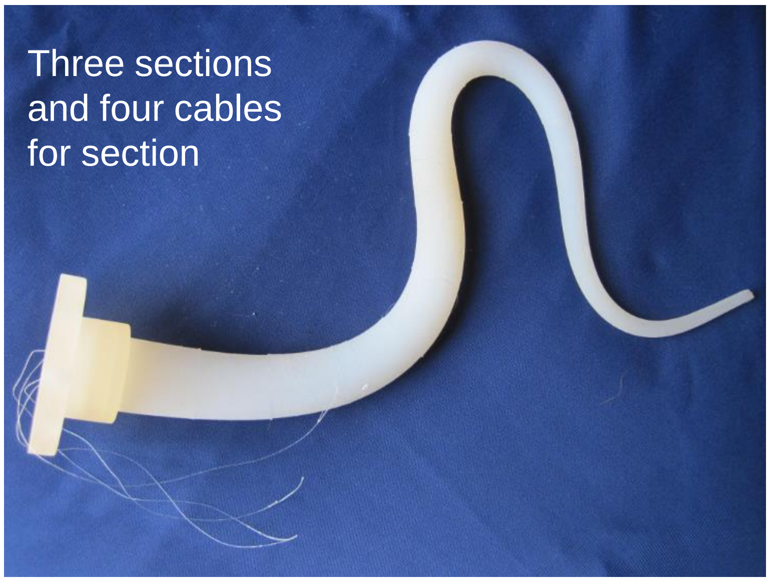


Rigid Flagellum (ABS)



OCTOPUS ARM MANIPULATOR

- The model has been applied to a cable driven octopus-like manipulator
- The friction between the cables and the silicone body is not considered

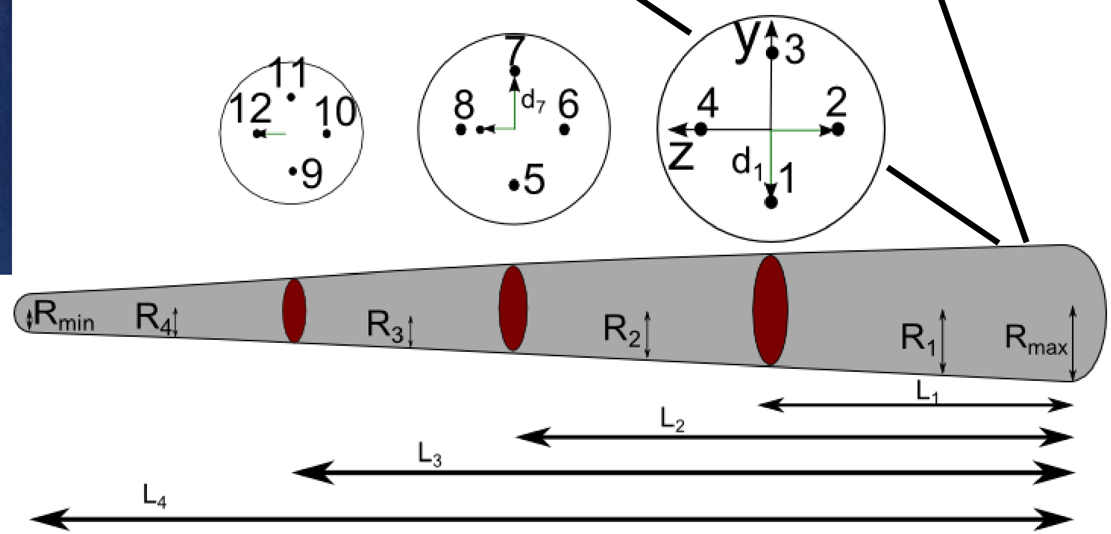


PARALLEL CABLES ACTUATION

$$\boldsymbol{\tau}_1 = L\boldsymbol{\mathcal{F}}_{a_1}$$

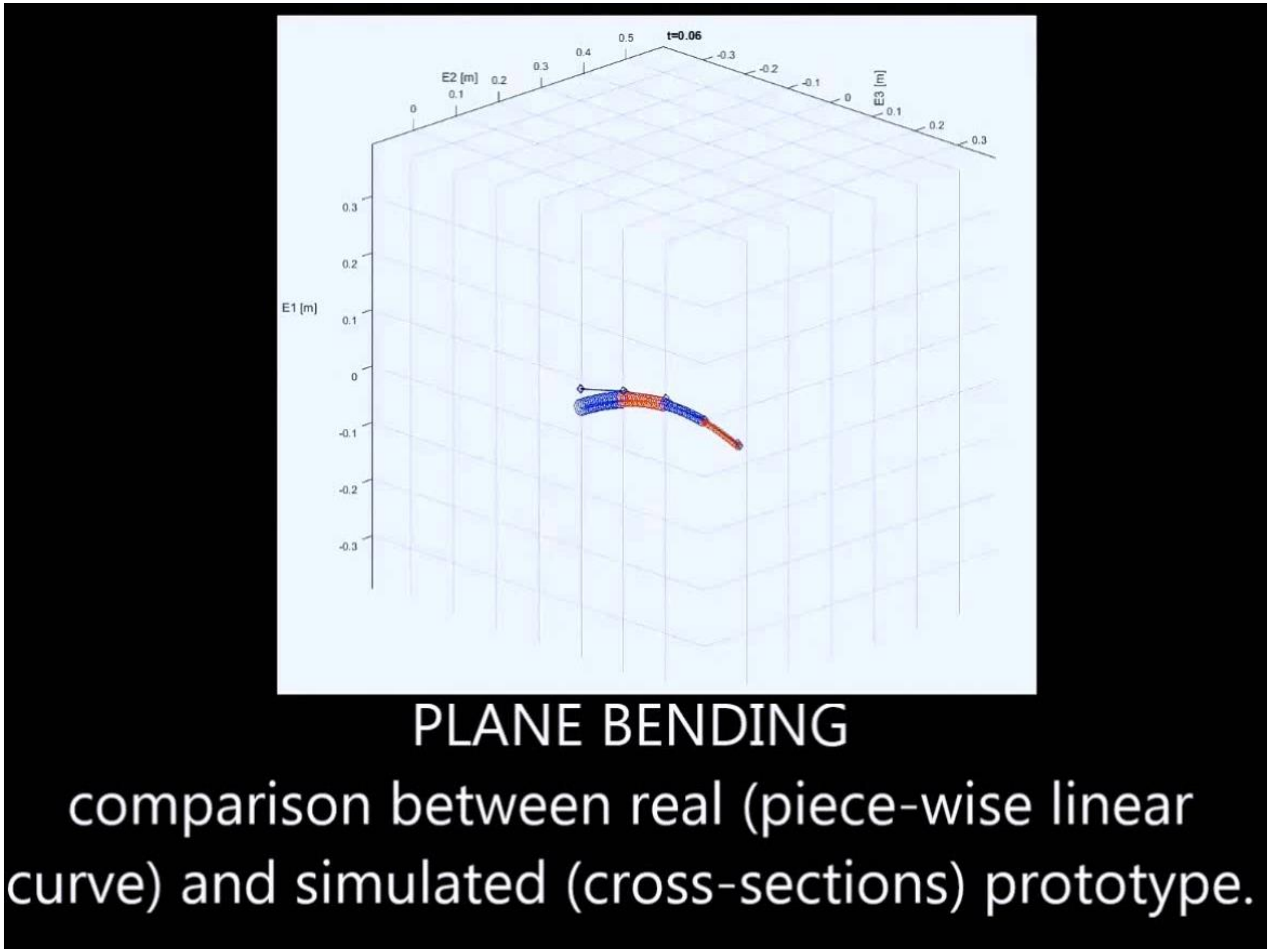
SIMO-REISSNER

$$\boldsymbol{\xi}_1 = \boldsymbol{q}_1$$



- F. Renda, M. Giorelli, M. Calisti, M. Cianchetti and C. Laschi "Dynamic Model of a Multibending Soft Robot Arm Driven by Cables" *Robotics, IEEE Transactions on* , vol.30, no.5, pp.1109,1122, Oct. 2014.

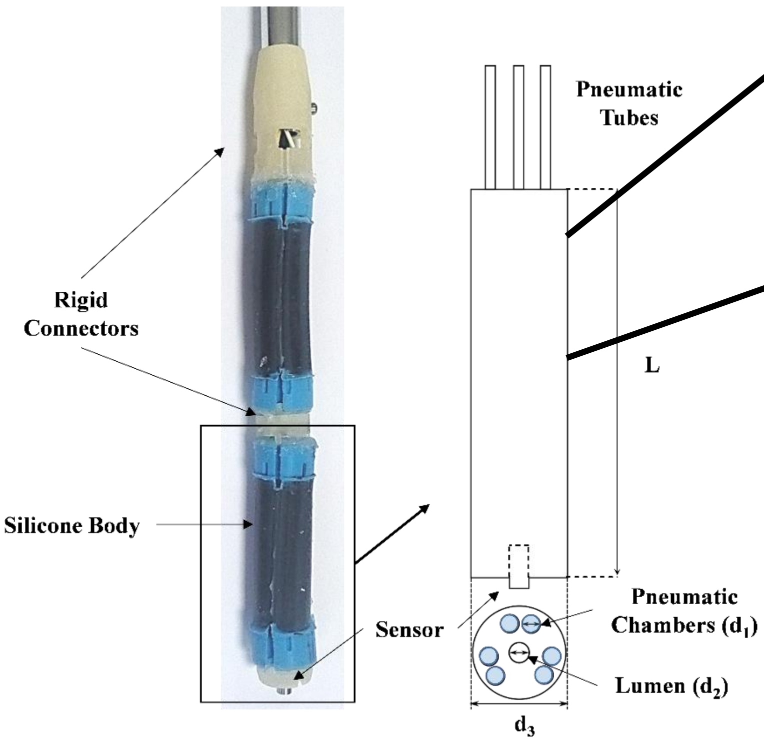
OCTOPUS ARM EXPERIMENTAL RESULTS



- F. Renda, F. Boyer, J. Dias and L. Seneviratne. Discrete Cosserat Approach for Multi-Section Soft Manipulators Dynamics. *Robotics, IEEE Transactions on*, vol. 34, no. 6, pp. 1518-1533, Dec. 2018.

STIFF-FLOP MANIPULATOR

- The model has been used to solve the inverse statics of the Stiff-Flop medical manipulator



PARALLEL FLUIDIC ACTUATION

$$\boldsymbol{\tau}_1 = L\boldsymbol{\mathcal{F}}_{a_1}$$

SIMO-REISSNER

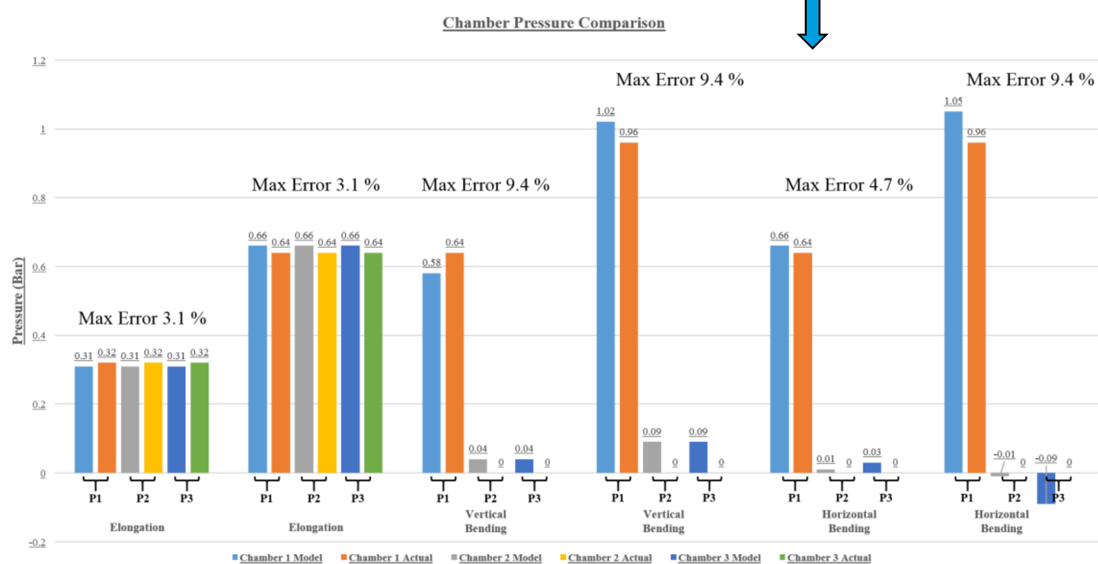
$$\boldsymbol{\xi}_1 = \boldsymbol{q}_1$$

INVERSE KINEMATICS

$$\boldsymbol{\xi}_1 = \log(\boldsymbol{g}_1(L))/L$$

STATICS

$$\mathbf{0} = \boldsymbol{\tau} - \boldsymbol{K}(\boldsymbol{q} - \boldsymbol{q}^*) + \boldsymbol{F}$$



- F. Renda, M. Cianchetti, H. Abidi, J. Dias and L. Seneviratne. Screw-Based Modeling of Soft Manipulators with Tendon and Fluidic Actuation. *ASME Journal of Mechanism and Robotics*, 9(4):041012-041012-8, 2017.

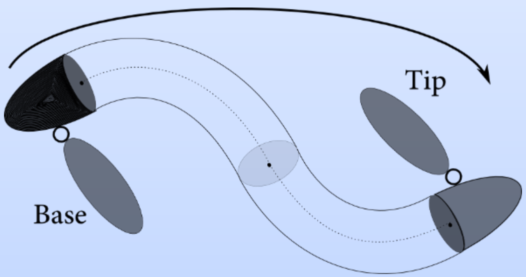
RECURSIVE NEWTON-EULER ALGHORITHM

the floating-base inverse dynamics problem

- The recursive N-E algorithms for floating-base multi-body dynamics can be extended to hybrid (soft-rigid) systems

FIRST PASS: KINEMATICS

Recursive calculation of the relative (to the base) kinematics quantities

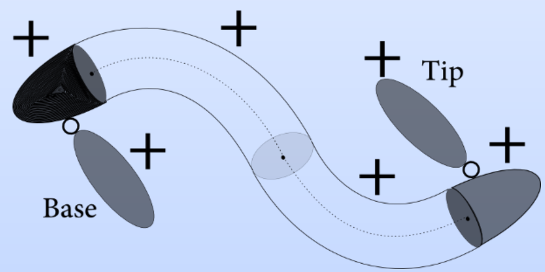


$$\boldsymbol{\eta}_j(X) = Ad^{-1}_{\boldsymbol{g}_{ij}\boldsymbol{g}_j}\boldsymbol{\eta}_i + {}^j\boldsymbol{S}_j\dot{\boldsymbol{q}}_j$$

$$\begin{aligned}\dot{\boldsymbol{\eta}}^r_j(X) = {} & Ad^{-1}_{\boldsymbol{g}_{ij}\boldsymbol{g}_j}\dot{\boldsymbol{\eta}}^r_i + {}^j\boldsymbol{S}_j\ddot{\boldsymbol{q}}_j \\ & + ad_{Ad^{-1}_{\boldsymbol{g}_{ij}\boldsymbol{g}_j}\boldsymbol{\eta}_i}{}^j\boldsymbol{S}_j\dot{\boldsymbol{q}}_j\end{aligned}$$

SECOND STEP: DYN. SUM

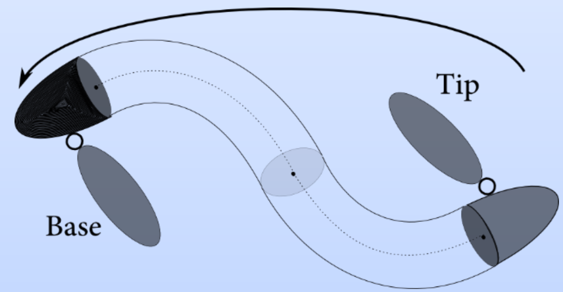
Summation of all the dynamics equations expressed in the base frame



$$\boldsymbol{0} = \sum_i \left({}^0\boldsymbol{\mathcal{M}}^t_i\right)\dot{\boldsymbol{\eta}}_0 + \sum_i \left(\int_0^L {}^0\boldsymbol{\mathcal{M}}_i\, {}^0\dot{\boldsymbol{\eta}}^r_i\, dX + {}^0\boldsymbol{\mathcal{P}}^t_i\right)$$

THIRD PASS: TORQUES

Recursive calculation of the actuation forces/torques



PROJECTION

$$\int_{\boldsymbol{0}}^{\boldsymbol{L}} {}^i\boldsymbol{S}_i^T\, \overline{\boldsymbol{\mathcal{F}}}\, dX$$

COSSERAT ROD DYNAMICS

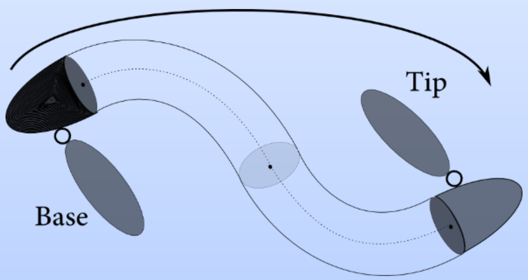
- F. Renda and L. Seneviratne, "A Geometric and Unified Approach for Modeling Soft-Rigid Multi-Body Systems with Lumped and Distributed Degrees of Freedom," *2018 IEEE International Conference on Robotics and Automation (ICRA)*, Brisbane, Australia, 2018, pp. 1567-1574.
- R. Featherstone. *Rigid Body Dynamics Algorithms*. Springer US, 2008.

RECURSIVE NEWTON-EULER ALGHORITHM

the floating-base forward dynamics problem

FIRST PASS: KINEMATICS

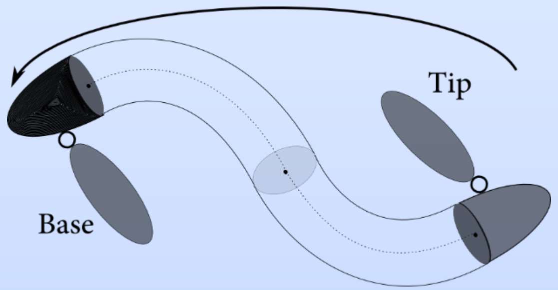
Recursive calculation of the kinematics quantities



$$\boldsymbol{\eta}_j(X) = Ad^{-1}_{g_{ij}g_j}\boldsymbol{\eta}_i + {}^j\boldsymbol{S}_j\dot{\boldsymbol{q}}_j$$

SECOND PASS: ART. BODY

Recursive calculation of the articulated body inertia and force



ART. BODY EQ.

$$\boldsymbol{\mathcal{F}}_{J_j} = \boldsymbol{\mathcal{M}}^A_j\dot{\boldsymbol{\eta}}_j + \boldsymbol{\mathcal{P}}^A_j$$

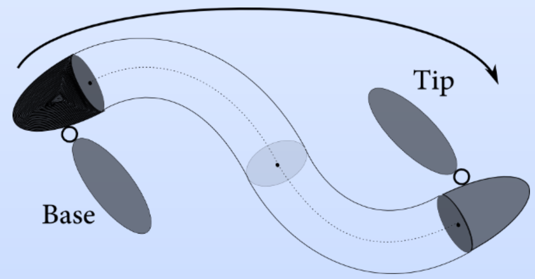
PROJECTION

$$\int_0^L {}^i\boldsymbol{S}_i^T\bar{\boldsymbol{\mathcal{F}}}\,dX$$

COSSERAT ROD DYNAMICS

THIRD PASS: ACCEL.

Recursive calculation of the bodies' acceleration



PROJECTION

$$\int_0^L {}^i\boldsymbol{S}_i^T\bar{\boldsymbol{\mathcal{F}}}\,dX$$

COSSERAT ROD DYNAMICS

$$\begin{aligned}\dot{\boldsymbol{\eta}}_j(X) = {} & Ad^{-1}_{g_{ij}g_j}\dot{\boldsymbol{\eta}}_i + {}^j\boldsymbol{S}_j\ddot{\boldsymbol{q}}_j \\ & + ad_{Ad^{-1}_{g_{ij}g_j}\boldsymbol{\eta}_i}{}^j\boldsymbol{S}_j\dot{\boldsymbol{q}}_j\end{aligned}$$

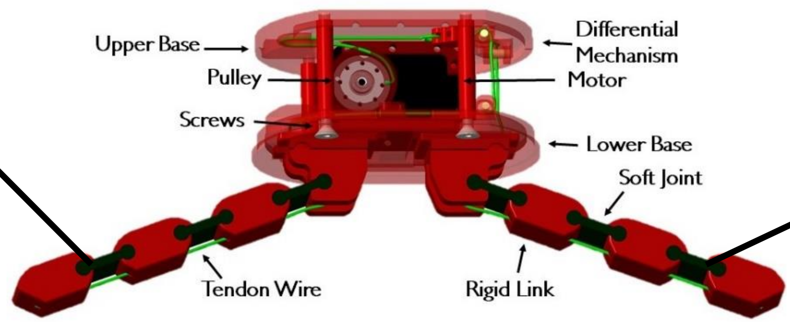
- F. Renda and L. Seneviratne, "A Geometric and Unified Approach for Modeling Soft-Rigid Multi-Body Systems with Lumped and Distributed Degrees of Freedom," *2018 IEEE International Conference on Robotics and Automation (ICRA)*, Brisbane, Australia, 2018, pp. 1567-1574.
- R. Featherstone. *Rigid Body Dynamics Algorithms*. Springer US, 2008.

COMPLIANT GRIPPER

- Manipulator’s ellipsoids can be calculated to optimize the gripping performance

EXTERNAL CABLE ACTUATION

$$\boldsymbol{\tau}_3 = {}^3\boldsymbol{S}_3^T \boldsymbol{\mathcal{F}}_{T_3}$$

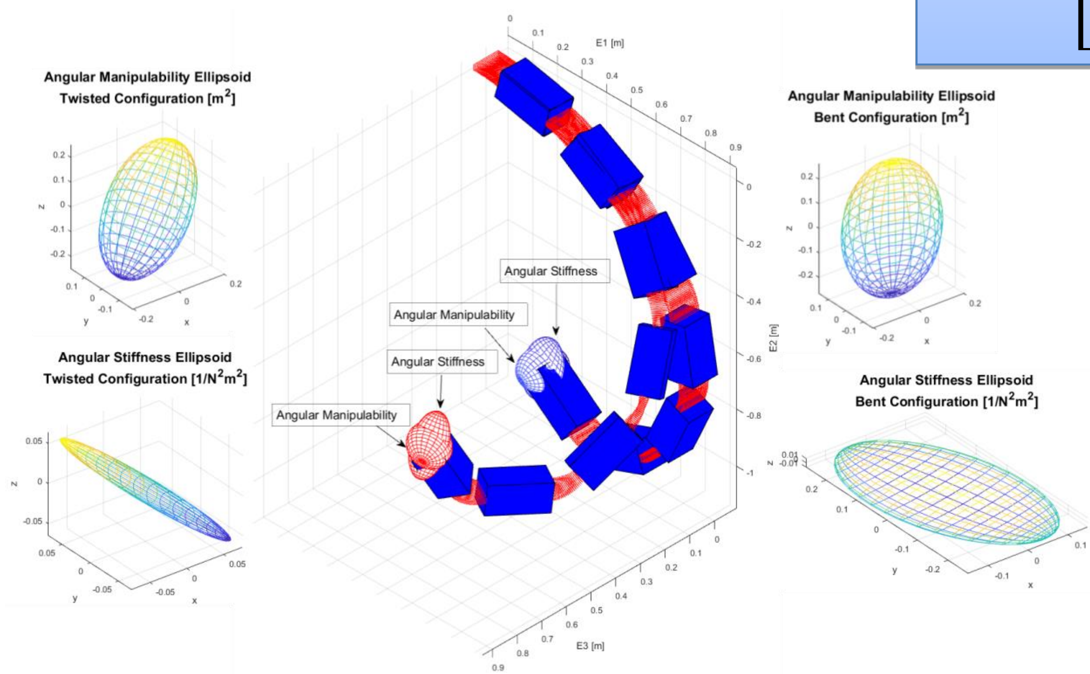


INEX. CONST. TORSION AND BENDING

$$\boldsymbol{\xi}_3 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \boldsymbol{q}_3 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

MANIPULABILITY ELLIPSOID

$$(\boldsymbol{J}_w \boldsymbol{J}_w^T)^{-1}$$



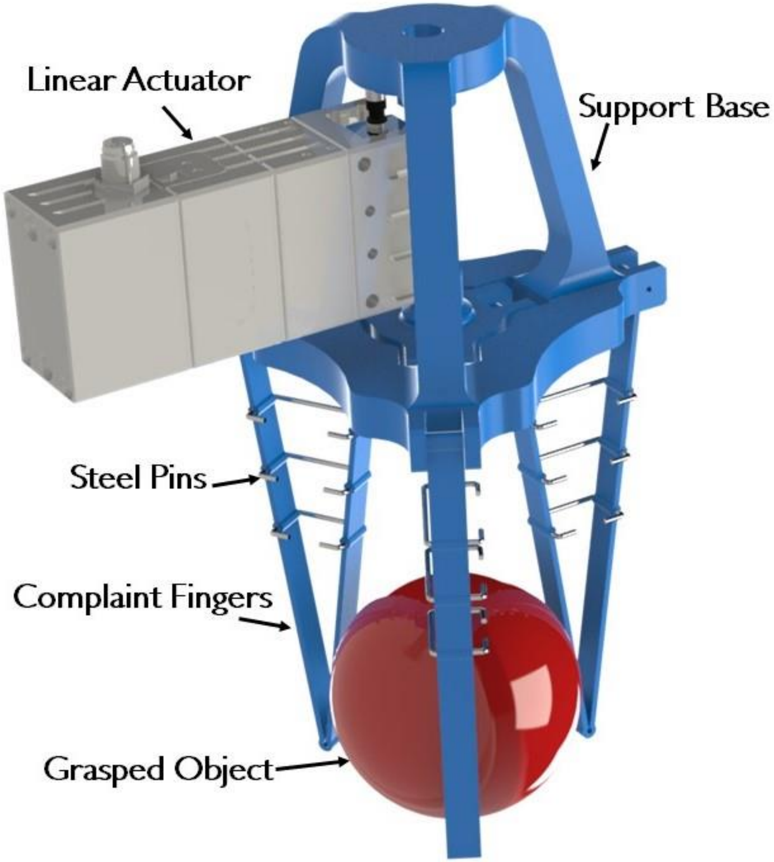
STIFFNESS ELLIPSOID

$$(\boldsymbol{J}_w \boldsymbol{K}^{-1} \boldsymbol{J}_w^T)^2$$

- I. Hussain et al., "Modeling and Prototyping of an Underactuated Gripper Exploiting Joint Compliance and Modularity," in *IEEE Robotics and Automation Letters*, vol. 3, no. 4, pp. 2854-2861, Oct. 2018.

CLOSED-CHAIN COMPLIANT GRIPPER (I)

- Adopting the technique developed for closed-chain multi-body systems, the model can be used for closed-chain soft-rigid manipulators like the FinRay finger



PFAFFIAN CONSTRAIN

The closed-chain joints restrict the motion space of the corresponding open-chain system

$$\begin{bmatrix} \boldsymbol{B}_h^{\perp T}\left(\boldsymbol{J}_{p_h} - \boldsymbol{J}_{s_h}\right) \\ \boldsymbol{B}_k^{\perp T}\left(\boldsymbol{J}_{p_k} - \boldsymbol{J}_{s_k}\right) \\ \vdots \\ \boldsymbol{B}_l^{\perp T}\left(\boldsymbol{J}_{p_l} - \boldsymbol{J}_{s_l}\right) \end{bmatrix} \dot{\boldsymbol{q}} = \boldsymbol{A}(\boldsymbol{q})\dot{\boldsymbol{q}} = \boldsymbol{0}$$

- Muddasar Anwar, Toufik Al Khawli, Irfan Hussain, Dongming Gan, Federico Renda, (2019) "Modeling and prototyping of a soft closed-chain modular gripper", *Industrial Robot: the international journal of robotics research and application*, Vol. 46 Issue: 1, pp.135-145

CLOSED-CHAIN COMPLIANT GRIPPER (II)

CLOSED-CHAIN DYNAMIC EQUATION

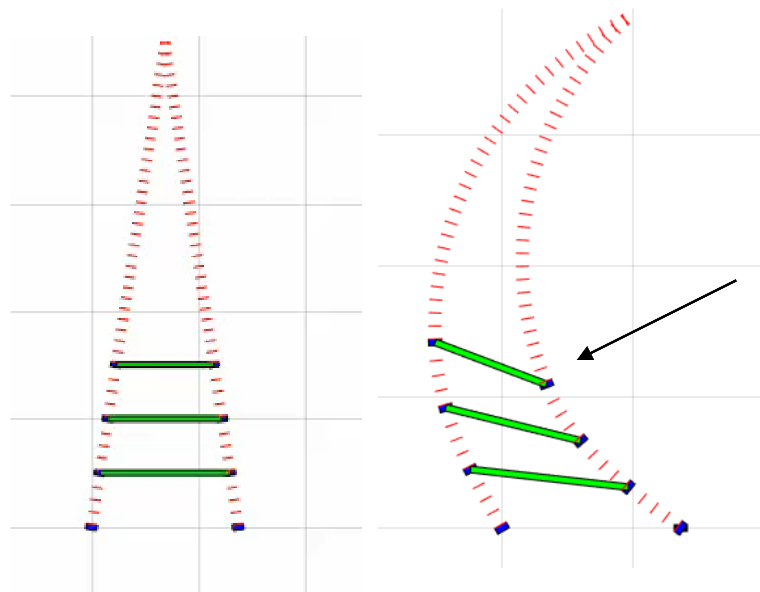
The open-chain dynamics is augmented with the constrain forces of the (passive) closed-chain joints

$$\begin{cases} \boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{C}\dot{\boldsymbol{q}} + \boldsymbol{K}(\boldsymbol{q} - \boldsymbol{q}^*) = \boldsymbol{\tau} + \boldsymbol{F} + \boldsymbol{A}^T\boldsymbol{\lambda} \\ \boldsymbol{A}\ddot{\boldsymbol{q}} + \dot{\boldsymbol{A}}\dot{\boldsymbol{q}} = 0 \end{cases} \Rightarrow$$

$$\boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{C}\dot{\boldsymbol{q}} + \boldsymbol{K}(\boldsymbol{q} - \boldsymbol{q}^*) = \boldsymbol{P}\boldsymbol{\tau} + \boldsymbol{F} + \boldsymbol{A}^T\left(\boldsymbol{A}\boldsymbol{M}^{-1}\boldsymbol{A}^T\right)^{-1}\left[\boldsymbol{A}\boldsymbol{M}^{-1}(\boldsymbol{C}\dot{\boldsymbol{q}} - \boldsymbol{F}) - \dot{\boldsymbol{A}}\dot{\boldsymbol{q}}\right]$$

Where $\boldsymbol{P} = \boldsymbol{I} - \boldsymbol{A}^T\left(\boldsymbol{A}\boldsymbol{M}^{-1}\boldsymbol{A}^T\right)^{-1}\boldsymbol{A}\boldsymbol{M}^{-1}$ projects away joint wrenches that act on the constraints without doing any work on the system

Dynamics



Statics

- Irfan Hussain, Muddasar Anwar, Dongming Gan, Federico Renda, "Modeling, Analysis and Prototyping of Closed Chain Fingers Inspired by Fin Ray Effect.", *Robotics, IEEE Transactions on*, 2019 (in preparation).

CONCLUSION

- A discrete Cosserat approach (piece-wise constant strain) is used to build a **geometric and unified modeling framework for rigid, soft or hybrid (soft-rigid) robots**

- The model is in fact a **generalization to soft and hybrid systems of the geometric theory of rigid robotics,** characterized by the exponential map

Benefits

- No restrictions on the form of the internal strain energy
- Good generality inherited by the Cosserat rod theory
- Suitable for control purpose
- Unique framework to transduce traditional robotics results to the soft robotics practice

Limitations

- Cross-section deformations are kinematically not allowed
- Only one dimensional media are considered
- No off-the-shelf software available

THANK YOU!

جامعـــة خليفـــة
Khalifa University

federico.renda@ku.ac.ae